Econometrics project

Relation between inflation and unemployment

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Abstract

The purpose of this paper is to investigate the relationship between inflation and unemployment rate within the Phillips curve context. We explored our study on the United States and Germany between 1971 and 2020. The relationship between inflation and unemployment has traditionally been an inverse correlation. However, this relationship is more complicated than it appears at first glance, and it has broken down on a number of occasions over the past 50 years. Since inflation and employment (and unemployment) are some of the most closely monitored economic indicators, we'll delve into their relationship and how they affect the overall economy. We will analyze our study through Phillips curve on either a short run or a long run approach. Over the long-term, higher inflation would not benefit the economy through a lower rate of unemployment. By the same token, a lower rate of inflation should not inflict a cost on the economy through a higher rate of unemployment. Since inflation has no impact on the unemployment rate in the long term, the long-run Phillips curve morphs into a vertical line at the natural rate of unemployment. For the long run we will introduce the concept of the NAIRU (Non-Accelerating Inflation Rate of Unemployment) in the Section 2, which is defined as the rate of unemployment when the rate of inflation is stable.

Keywords: Inflation, Unemployment, Phillips Curve

Contents

1 Introduction

The observed inverse relationship between unemployment and inflation, first discovered by William Phillips (1958) on his article "The Relationship between Unemployment and the Rate of Change of Money Wage Rates in the UK, 1861 to 1957", has come to be known as the Phillips curve. Since then, unemployment and inflation as economic concepts have been a central topic in macroeconomics. These two concepts are considered as key factors in the process of economic development of every country. All government programs are conducted focusing on policies that keep on stable price levels and low unemployment rates.
However, the failure to explain economic phenomena of the crisis of 1970s had created serious doubts about the validity of Phillips curve. The Phillips curve idea was openly criticized by the Monetarist school, among them Milton Friedman. Friedman (1968) argued that there is only short run trade-off between the inflation rate and the unemployment rate, but for the long run he introduced the concept of the NAIRU (Non-Accelerating Inflation Rate of Unemployment). NAIRU is defined as the rate of unemployment when the rate of inflation is stable. Therefore, the long run Phillips curve is vertical and there is no trade-off between unemployment and inflation (see Phelps 2006).
The purpose of this paper is to examine the existence of Phillips curve in Germany and USA using annual time series data for the period 1971-2020. The structure of the paper is as follows: Section 2 presents the literature review. Section 3 presents the correlation between inflation and unemployment. Section 4 analyzes methods and empirical results. Section 5 gives concluding remarks.

2 Literature Review

An extensive and expanding volume of both theoretical and empirical studies exists on the relationship between inflation and unemployment across developed and developing countries, over varying sample periods and different econometric approaches. However, the issue still remains controversial for the policy makers. The results seem to vary from country to country due to the different structure of their domestic economies and the continuous changes in economic conditions.
Changes in monetary conditions are often believed to have a significant impact on real economic variables, such as output and employment, through the classical Phillips curve relationship (Vermeulen 2017). Among prominent economists who support the existence of the Phillips curve are Samuelson and Solow (1960).
Samuelson and Solow (1960) examined the relationship between the rate of inflation and unemployment rate for a twenty-five year period (1934 to 1958) for the case of the United States (US). The results of their study revealed an inverse relationship between inflation and unemployment. It is worth mentioning that these researchers were the first who championed the Phillips curve as a policy tool. In addition, Gordon (1971) confirmed Phillips relation in the US using macroeconomic data for the pre-1970s and the post-1970s periods. The fact that there exists an inverse relationship between unemployment and inflation was criticized by Phelps (1967) and Friedman (1968). Phelps (1967) and Friedman (1968) supported that as the Phillips curve shifts over time, the equilibrium rate of unemployment is independent from the rate of inflation. Therefore there is only short run trade-off between inflation and unemployment rate. For the long run they introduced the concept of NAIRU (Non-Accelerating Inflation Rate of Unemployment). NAIRU refers to a level of unemployment, below which inflation rises. The stability of original Phillips curve was disputed in the early 1970s, where the US economy faced high inflation and unemployment rate si-

multaneously (stagflation). Later, other researchers argued against the Phillips hypothesis (Lucas, 1976; Okun, 1975).

Contrary to Friedman (1968) and Phelps (1967), Modigliani and Tarantelli (1976) argued that no "natural" unemployment rate exists and that the Phillips curve does not disappear in the long run. Modigliani and Tarantelli (1976) argued that the Phillips curve shifts upward and become steeper every contract renewal. Lucas (1976) argued that could be a trade-off relationship between unemployment and inflation under the condition that the workers do not expect the policymakers to create an artificial situation of high inflation combined with low unemployment. In a different case, employees would predict high inflation and an increase in wages would be possible. In such a case, high unemployment and high inflation could coexist, which is known as "Lucas Critique" (see also Zaman et al., 2011; Dritsaki and Dritsaki, 2012).

In 1975, Okun commented that "Phillips curve has become an unidentified flying object" (p. 353)". However, in the 1990's, Phillips curve came to the front giving mixed results. Alogoskoufis and Smith (1991) supported empirically the "Lucas Critique". On the other hand, Fisher and Seater (1993), King and Watson (1994) and Fair (2000) find a long run inflation unemployment trade-off.

Marcelino and Mizon (1999) examined the relationship between wages, prices, productivity, inflation and unemployment in Italy, Poland and the UK between the 1960's and the early 1990's. They investigated the labor markets of these countries and found that there are significant changes in the structures of the relationships between wages-prices and unemployment-inflation for the period 1979-80. They concluded that although there are important changes in the labor markets of the examined countries, taking into account a greater degree of flexibility, there are no common characteristics among them.

Recent advances in data analysis methods allow a more in depth examination of the Phillips curve hypothesis. Schreiber and Wolters (2007) investigated the relationship between unemployment and inflation, using a vector autoregressive model (VAR) and a vector error correction model (VECM), for the case of Germany over the period 1977-2002. Their study revealed a negative relationship between unemployment and inflation, both in the short and in the long run, for the examined period.

Del Boca et al. (2008) examined the existence of Phillips curve in Italy using data covering the period 1978-2000. The results of the study showed that a trade-off relationship exists only during low inflation and stable aggregate supply. Del Boca et al. (2008) captured the effects of structural changes and asymmetries on the estimated parameters of alternative Phillips equations using the Kalman filter.

A similar study, conducted by Russell and Banerjee (2008), examined the Phillips curve hypothesis assuming non-stationarity in the series. They found that there is a positive relation between inflation and unemployment rate in the short run for the case of US during the period 1952-2004.

Islam et al. (2011) used the ARDL bound approach to examine the existence and stability of Phillips curve for North Cyprus using data covering the period 1978-2007. The results of their analysis showed the existence of Phillips curve both in the short and in the long run. In addition, a stable relation is confirmed.

Karahan et al. (2012) investigated the relationship between unemployment and inflation for Turkey over the period 2006-2011. The ARDL bounds tests indicated that, in the short run, unemployment has a negative impact on inflation. However, the results did not reveal any causal relation between the two variables in the long run supporting the views of Friedman (1967) and Phelps (1967), not advocating the "hysteresis effect".

Dritsaki and Dritsaki (2012) investigated the Phillips curve hypothesis in Greece using data for the period 1980-2010. Their results revealed a long run relationship and a causal relationship between unemployment and inflation. In addition, their results showed that shocks in inflation cause a reduction on unemployment for the first years, following by a slight rise for the remaining years.

3 Data Analysis

3.1 Descriptive statistics

The descriptive statistics for all variables are shown in the table below.
We can see from **Table 3.1.1** that the average inflation rates in the USA are higher than those in Germany: 3.91 vs 2.59. While the average unemployment rates in the USA are lower than those in Germany: 6.27 vs 7.04. From these values we can say that in the period of consideration the economy of the USA is in general better than that of Germany.

Germany	Inflation	Unemployment
Mean	2.59	7.04
Median	2.00	7.40
Maximum	7.03	11.72
Minimum	-0.13	0.40
Std. Dev	1.89	2.82
Skewness	0.80	-0.44
Kurtosis	-0.45	-0.53
Observations	50	50
USA	Inflation	Unemployment
Mean	3.91	6.27
Median	3.09	5.90
Maximum	13.55	9.71
Minimum	-0.36	3.67
Std. Dev	2.91	1.59
Skewness	1.52	0.54
Kurtosis	1.96	-0.58
Observations	50	50

Table 3.1.1: descriptive statistics of the data on Germany and on USA

3.2 Correlation between inflation and unemployment

In this section, we present correlations between the two variables. The correlation matrix and the graphs of inflation rate and unemployment rate for the period 1971 - 2020 are following.
The inverse relationship between inflation and unemployment in the case of Germany is most obvious during the period 1982 - 2008 (see **Figure 2**). This correlation is the evidence which is traditionally associated with Phillips curve hypothesis and significantly different from zero (see **Figure 1**). In the case of the United States, before the crisis of 2008 the inverse relation is not that obvious, instead from 2008 until the end of the period of consideration the Phillips curve is followed (see **Figure 3**).

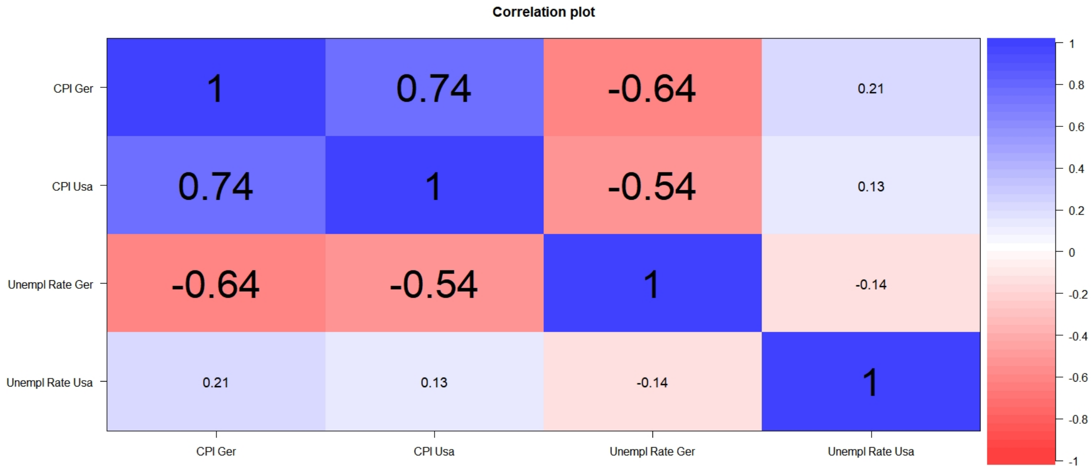


Figure 1: Correlation matrix (1971 - 2020)

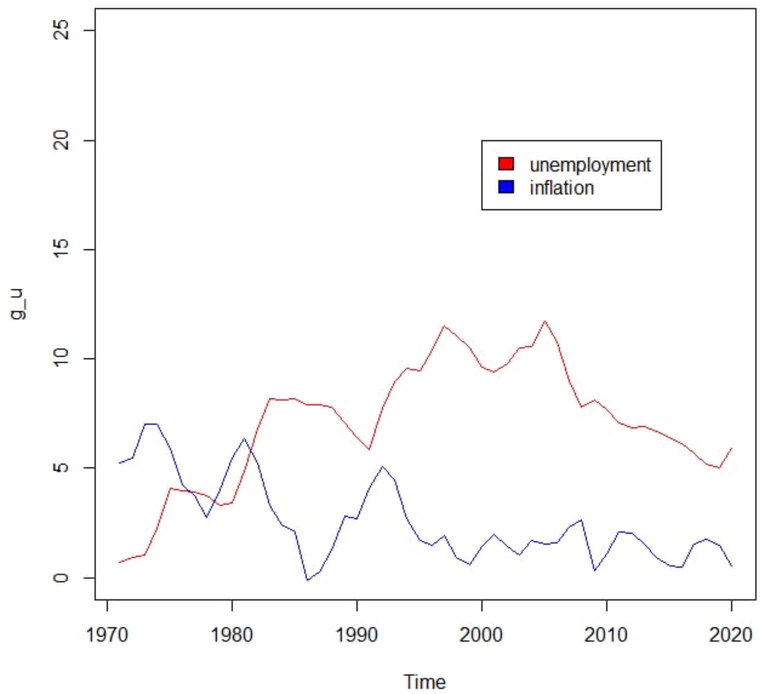


Figure 2: Inflation & Unemployment rates Germany (1971 - 2020)

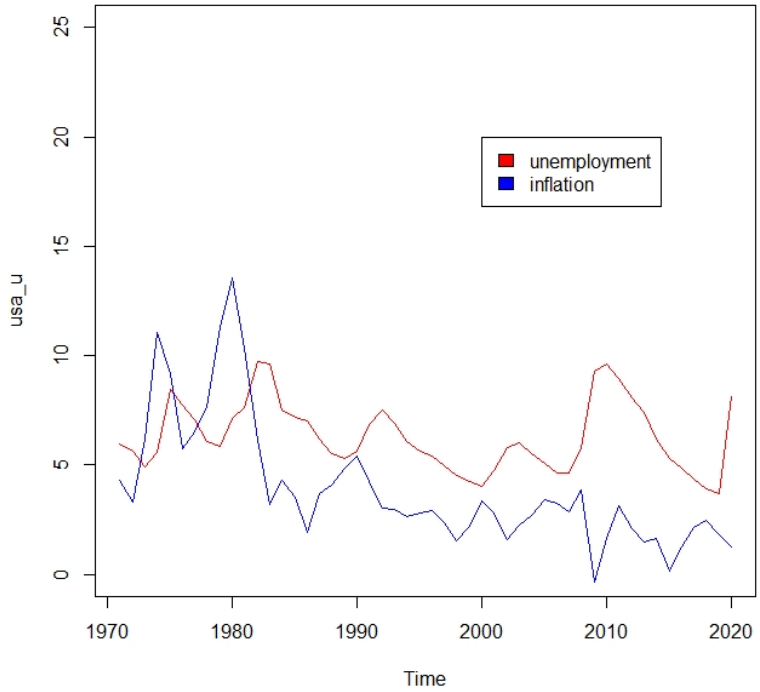


Figure 3: Inflation & Unemployment rates USA (1971 - 2020)

4 Methods and empirical results

4.1 Econometric Methodology

After presenting the descriptive statistics for the variables, the paper involves into the following objectives:

- The first is to examine the stationarity of the variables using the Augmented Dickey Fuller test (ADF) (1979) and the test proposed by Kwiatkowski et al. (KPSS) (1992).
- If the variables are integrated of order one then Johansen's (1998) cointegration test is the most appropriate to be used. In the case that the variables do not have the same integration order, Pesaran et al. (2001) cointegration test is the most appropriate. The Johansen's cointegration method, except the integration I(1) of the variables, requires a large number of observations in order to give robust results. Another test for the cointegration that we will use is the Engle-Granger test. In this study we apply the ARDL approach.
- The third step is to estimate the long run and short run relationship between the variables through models such as AR, MA and ARMA using time series.
- The last step is to check the causal relationship between the variables using a dynamic vector error correction model (VECM). In this paper, the Granger causality technique is applied.

4.1.1 Unit Root Tests

We begin our analysis by checking the stationary properties of the variables included in the study. We apply the test suggested by Augmented Dickey - Fuller (ADF) (1979). For simplicity, in the tables below we show the empirical results only of the case 0 lags.
First we applied the test on the original model with 3 lags, and the p-values show that we reject the hypothesis of stationarity at significance level 10% in all three cases (see **Table 4.1.1** and **Table 4.1.2**). Then we adjusted the variables by considering the first difference and the values of p-value obtained lead to reject the null hypothesis at significance level 10%, i.e. we can conclude with the stationarity hypothesis of our model (see **Table 4.1.3** and **Table 4.1.4**).
We check the stationarity considering another test called KPSS. For simplicity, we do not report the empirical results of this test, since the results are very similar to the previous test (i.e. we always accept the stationarity hypothesis, because the p-values are always higher than the significance level 10%).

CPI Germany	ADF	p-value
no drift no trend	-1.71	0.0849
with drift no trend	-1.99	0.3338
with drift and trend	-2.75	0.2680
Unem Germany	ADF	p-value
no drift no trend	0.013	0.6430
with drift no trend	-2.32	0.2059
with drift and trend	-1.31	0.8490

Table 4.1.1: ADF Test CPI and Unemployment Germany

CPI USA	ADF	p-value
no drift no trend	-1.42	0.1673
with drift no trend	-2.10	0.2900
with drift and trend	-3.41	0.0639
Unem USA	ADF	p-value
no drift no trend	-0.35	0.5390
with drift no trend	-2.62	0.0977
with drift and trend	-2.59	0.3312

Table 4.1.2: ADF Test CPI and Unemployment USA

ΔCPI Germany	ADF	p-value
no drift no trend	-5.13	0.01
with drift no trend	-5.13	0.01
with drift and trend	-5.07	0.01
ΔUnem Germany	ADF	p-value
no drift no trend	-4.03	0.01
with drift no trend	-4.04	0.01
with drift and trend	-4.28	0.01

Table 4.1.3: ADF Test ΔCPI and ΔUnemployment Germany

ΔCPI USA	ADF	p-value
no drift no trend	-5.91	0.01
with drift no trend	-5.85	0.01
with drift and trend	-5.81	0.01
ΔUnem USA	ADF	p-value
no drift no trend	-4.09	0.01
with drift no trend	-4.03	0.01
with drift and trend	-3.93	0.02

Table 4.1.4: ADF Test ΔCPI and ΔUnemployment USA

4.1.2 Autoregressive Distributed Lag (ARDL) Cointegration Analysis

In this section we consider an ARDL model with a cointegration method. The autoregressive distributed lag cointegration technique as a general vector of order p is shown below:

$$L = (INF_t, UNGAP_t) \tag{1}$$

The ARDL that we apply to our study is the following:

$$\begin{aligned} \Delta UNGAP_t = \beta_{01} + \delta_{11} UNGAP_{t-1} + \delta_{21} INF_{t-1} + \sum_{i=1}^{p} \alpha_{1i} \Delta UNGAP_{t-i} + \\ + \sum_{i=0}^{q} \alpha_{2i} \Delta INF_{t-i} + \epsilon_{1t} \end{aligned} \tag{2}$$

$$\begin{aligned} \Delta INF_t = \beta_{02} + \delta_{12} INF_{t-1} + \delta_{22} UNGAP_{t-1} + \sum_{i=1}^{p} \alpha_{1i} \Delta INF_{t-i} + \\ + \sum_{i=0}^{q} \alpha_{2i} \Delta UNGAP_{t-i} + \epsilon_{2t} \end{aligned} \tag{3}$$

where Δ is the first difference operator, β_{01} and β_{02} are constant, ϵ_{1t} and ϵ_{2t} are "well behaved" random disturbance terms. The error terms are independent and identically distributed.
The optimal lag values are p and q, they are chosen by applying the information criteria. In this paper we use only AIC and BIC. According to the previous model, we have:

$$UNGAP_t = \beta_{01} + \sum_{i=1}^{p} \alpha_{1i} UNGAP_{t-i} + \sum_{i=0}^{q} \alpha_{2i} INF_{t-i} + \mu_{1t} \tag{4}$$

$$INF_t = \beta_{02} + \sum_{i=1}^{p} \alpha_{1i} INF_{t-i} + \sum_{i=0}^{q} \alpha_{2i} UNGAP_{t-i} + \mu_{2t} \tag{5}$$

We verify the co-integration with Johansen Test and with Engle-Granger Test for both equation (1) and (2).
The null hypothesis is the case of no co-integration among the variables, the alternative hypothesis is the case of co-integration.

$$H_0 : \delta_{11} = \delta_{21} = 0 \tag{6}$$

$$H_1 : \delta_{11} \neq \delta_{21} \neq 0 \tag{7}$$

$$H_0 : \delta_{21} = \delta_{22} = 0 \tag{8}$$

$$H_1 : \delta_{21} \neq \delta_{22} \neq 0 \tag{9}$$

Once the coitegration is confirmed, we estimate the long run:

$$UNGAP_t = \beta_{01} + \sum_{i=1}^{p} \delta_{11} UNGAP_{t-i} + \sum_{i=0}^{q} \delta_{21} INF_{t-i} + e_{1t} \tag{10}$$

After considering the long run relation, we consider the short run:

$$\Delta INF_t = \beta_{02} + \sum_{i=1}^{p} \delta_{12} \Delta INF_{t-i} + \sum_{i=0}^{q} \delta_{22} \Delta UNGAP_{t-i} + e_{2t} \tag{11}$$

$$\Delta UNGAP_t = \beta_{01} + \sum_{i=1}^{p} \delta_{11} \Delta UNGAP_{t-i} + \sum_{i=0}^{q} \delta_{21} \Delta INF_{t-i} + e_{1t} \tag{12}$$

We also consider an error correction model (ECM) to investigate the short run dynamics of the respective variables towards the long run equilibrium. So the ECM integrates the short run coefficient with the long run coefficient without losing long run information. The dynamic of unrestricted ECM is shown in the next equations:

$$\Delta UNGAP_t = \beta_{01} + \sum_{i=1}^{p} \alpha_{1i} \Delta UNGAP_{t-i} + \sum_{i=0}^{q} \alpha_{2i} \Delta INF_{t-i} + \lambda_1 ECM_{t-1} + \epsilon_{1t} \tag{13}$$

$$\Delta INF_t = \beta_{02} + \sum_{i=1}^{p} \alpha_{1i} \Delta INF_{t-i} + \sum_{i=0}^{q} \alpha_{2i} \Delta UNGAP_{t-i} + \lambda_2 ECM_{t-1} + \epsilon_{1t} \tag{14}$$

where ECM_{t-1} is the error correction term, which should be negative and statistically significant. Moreover this coefficient indicates the speed of the adjustment to the long run equilibrium after short run shocks.
Only in the case of the USA we introduce a dummy variable, that is equal to 1 if there is a recession regarding the GDP, 0 otherwise. We introduce this factor because during the period of consideration many changes in unemployment and inflation occurred. We have the following model:

$$INF_t = \beta_{02} + \sum_{i=1}^{p} \alpha_{1i} INF_{t-i} + \sum_{i=0}^{q} \alpha_{2i} UNGAP_{t-i} + dum_t + \epsilon_{1t} \tag{15}$$

$$\Delta INF_t = \beta_{02} + \sum_{i=1}^{p} \alpha_{1i} \Delta INF_{t-i} + \sum_{i=0}^{q} \alpha_{2i} \Delta UNGAP_{t-i} + dum_t + \epsilon_{1t} \tag{16}$$

where dum_t is the dummy variable.

4.1.3 Valutation of the optimal lag length

In the following tables we find the optimal lag length for each model using AIC and BIC information criteria. We choose the model with the lowest value of AIC or BIC. For simplicity we show empirical results up to 2 lags (in some cases we report up to 3 lags).

long run CPI Germany	AIC	BIC
MA(1)	167.9484	175.5157
AR(1)	132.7222	140.2895
ARMA(1, 1)	134.7153	144.1744
ARMA(2, 2)	129.9203	143.0187
ARMA(2, 1)	127.9206	139.1478
AR(2)	125.9713	135.3273

Table 4.1.5 long run CPI Germany: AR(2) is selected

long run Un. Germany	AIC	BIC
MA(1)	219.0868	226.6541
AR(1)	116.7905	124.3577
ARMA(1, 1)	107.7365	117.1956
ARMA(2, 2)	104.8782	117.9766
ARMA(2, 1)	105-2802	116.5074
AR(2)	213.8861	223.2421

Table 4.1.6 long run Un. Germany: ARMA(2, 1) is selected

long run CPI USA	AIC	BIC
MA(1)	250.7520	258.3193
AR(1)	193.1527	200.7200
ARMA(1, 1)	193.7364	203.1955
ARMA(2, 2)	191.6712	204.7696
ARMA(2, 1)	190.0775	201.3048
AR(2)	190.7232	200.0792

Table 4.1.7 long run CPI USA: AR(2) is selected

long run Un. USA	AIC	BIC
MA(1)	179.0106	186.5778
AR(1)	155.1819	162.7492
ARMA(1, 1)	143.6912	153.1503
ARMA(2, 2)	137.7064	150.8048
ARMA(2, 1)	136.3774	147.6046
AR(2)	143.2583	152.6143

Table 4.1.8 long run Un. USA: ARMA(2, 1) is selected

short run CPI Germany	AIC	BIC
MA(1)	133.5309	141.0157
AR(1)	133.4610	140.9458
ARMA(1, 1)	132.9344	142.2904
ARMA(2, 2)	128.3670	141.3181
ARMA(2, 1)	127.8356	138.9365
AR(2)	128.1456	137.3964

Table 4.1.9 short run CPI Germany: AR(2) is selected

short run Un. Germany	AIC	BIC
MA(1)	117.7885	125.2733
AR(1)	112.9678	120.4526
ARMA(1, 1)	108.0277	117.3837
ARMA(2, 2)	108.9390	121.8900
ARMA(2, 1)	106.9505	118.0514
AR(2)	109.2882	118.5389

Table 4.1.10 short run Un. Germany: ARMA(1, 1) is selected

short run CPI USA	AIC	BIC
MA(1)	190.2409	197.7257
AR(1)	191.7369	199.2217
ARMA(1, 1)	191.4477	200.8037
ARMA(2, 2)	185.6561	198.6071
ARMA(2, 1)	183.7284	194.8292
AR(2)	182.5003	191.7511
AR(3)	172.6109	183.5828

Table 4.1.11 short run CPI USA: AR(3) is selected

short run Un. USA	AIC	BIC
MA(1)	150.9595	158.4443
AR(1)	153.5917	161.0765
ARMA(1, 1)	148.5228	157.8788
ARMA(2, 2)	149.2071	162.1582
ARMA(2, 1)	147.4497	158.5506
AR(2)	150.6734	159.9242
AR(3)	146.6489	157.6208

Table 4.1.12 short run Un. USA: AR(3) is selected

4.1.4 Valuation of the optimal lag length

Empirical results of Johansen Test and Engle-Granger Test are shown in the tables below. In the case of Johansen Test we can say that the co-integration rank is equal to 1, while for Engle-Granger Test we consider only the no trend case. For each country we have the value of the Johansen test that is lower than the critical statistical value in case of r <= 1 and higher than the critical statistical value in case of r = 0. So we can conclude that the co-integration rank is equal to 1 (see **Table 4.1.13, Table 4.1.14**). In the Engle-Granger test we obtain values of p-value around 5-6% so we have to reject the null hypothesis at significance level 10%, hence we have co-integration for the variables (see **Table 4.1.15, Table 4.1.16**).

Germany	Test	10pct	5pct	1pct
r<=1	5.40	6.50	8.18	11.65
r=0	25.21	15.66	17.95	23.52

Table 4.1.13 Johansen Test Germany

USA	Test	10pct	5pct	1pct
r<=1	6.14	6.50	8.18	11.65
r=0	21.92	15.66	17.95	23.52

Table 4.1.14 Johansen Test USA

Germany	lag	EG	p-value
Y = X+1	6	-2.8946	0.0543

Table 4.1.15 Engle-Granger Test Germany

USA	lag	EG	p-value
Y = X+1	5	-2.8634	0.0592

Table 4.1.16 Engle-Granger Test USA

4.2 Valuation of the Models

We look at some characteristics of the models, in the following we report only models with inflation as dependent variable.
The Long Run models are significant, since p-values are very low and the R^2 are quite high. Moreover we note that the values of g_u and USA_u are negative and significant for the two models. Then we can conclude that both are good models (see **Table 4.2.1** and **Table 4.2.2**).
The Short Run models are significant, since the p-values are sufficiently low, but the R^2 are not so high. The coefficient of Δg_u and ΔUSA_u are negative, but they are not so significant as they are in the long run models above. So we cannot say that they are good models (see **Table 4.2.5** and **Table 4.2.6**).
The model with dummy variable is significant, because the p-value is low and the R^2 is higher than the case without dummy. Moreover the dummy variable is significant, and also the negative estimate value of unemployment respects the relation of the Phillips curve (see **Table 4.2.7** and **Table 4.2.8**).
The ECM_{t-1} in the first case is negative and statistically significant, which implies a long run relationship between the examined variables. This means that in the short term the deviations from the long run equilibrium are adjusted by 74% every year (see **Table 4.2.3**).
The ECM_{t-1} in the second case is positive and statistically significant which does not imply a long run relationship between the examined variables (see **Table 4.2.4**).

AR(2) LR GER	Estimate	standard error	t-value	p-value
(Intercept)	1.87161	0.54441	3.438	0.00129
g_u	-0.15798	0.05543	-2.850	0.00663
g_{cpi1}	1.03352	0.13662	7.565	1.71e-09
g_{cpi2}	-0.34139	0.12798	-2.667	0.01066
$R^2{}_{adj}$	0.7875			
p-value	1.799e-15			

Table 4.2.1 AR(2) long run Germany

AR(2) LR USA	Estimate	standard error	t-value	p-value
(Intercept)	2.5303	1.0077	2.511	0.0158
USA_u	-0.3180	0.1772	-1.795	0.0795
USA_{cpi1}	1.0124	0.1409	7.184	6.14e-09
USA_{cpi2}	-0.1555	0.1543	-1.008	0.3190
$R^2{}_{adj}$	0.6877			
p-value	8.138e-12			

Table 4.2.2 AR(2) long run USA

ECM Germany	Estimate	standard error	t-value	p-value
ect1	-0.7421	0.1770	-4.192	0.000214
constant	-0.1482	0.1349	-1.098	0.280451
Δg_{u1}	-0.1762	0.1989	-0.886	0.382365
Δg_{cpi1}	-0.9400	0.1775	-5.295	9.22e-06
Δg_{u2}	-0.1462	0.1954	-0.748	0.460113
Δg_{cpi2}	-1.2993	0.2289	-5.677	3.09e-06
Δg_{u3}	-1.6474	0.3142	-5.243	1.07e-05
Δg_{cpi3}	-1.6474	0.3142	-5.243	1.07e-05
Δg_{u4}	-0.5750	0.2017	-2.851	0.00767
Δg_{cpi4}	-1.6421	0.3952	-4.155	0.000237
Δg_{u5}	-0.4685	0.2089	-2.243	0.032181
Δg_{cpi5}	-2.0778	0.4471	-4.647	5.89e-05
$R^2{}_{adj}$	0.4931			
p-value	0.0003679			

Table 4.2.3 ECM Germany

ECM USA	Estimate	standard error	t-value	p-value
ect1	0.75698	0.29998	2.523	0.016462
constant	-0.05846	0.23740	-0.246	0.806960
ΔUSA_{u1}	0.07695	0.28920	0.266	0.791783
ΔUSA_{cpi}	-0.76668	0.16068	-4.771	3.39e-05
ΔUSA_{u2}	0.47678	0.32336	1.474	0.149563
ΔUSA_{cpi2}	-0.94910	0.18418	-5.153	1.09e-05
ΔUSA_{u3}	0.08865	0.36001	0.246	0.806975
ΔUSA_{cpi3}	-1.02199	0.24347	-4.198	0.000183
ΔUSA_{u4}	0.30680	0.37197	0.825	0.415232
ΔUSA_{cpi4}	-1.08978	0.29354	-3.713	0.000732
$R^2{}_{adj}$	0.4484			
p-value	0.0003892			

Table 4.2.4 ECM USA

ARMA(1, 1) SR GER CPI	Estimate	standard error	t-value	p-value
(Intercept)	-0.03233	0.13300	-0.243	0.809
Δg_{u}	-0.25107	0.19171	-1.310	0.197
Δg_{cpi1}	0.23089	0.14764	1.564	0.125
Δg_{u1}	-0.30625	0.19860	-1.542	0.130
$R^2{}_{adj}$	0.1713			
p-value	0.01025			

Table 4.2.5 ARMA(1, 1) short run Germany

AR(2) SR USA	Estimate	standard error	t-value	p-value
(Intercept)	-0.18724	0.21777	-0.860	0.3949
ΔUSA_u	-0.39185	0.20208	-1.939	0.0594
ΔUSA_{cpi1}	0.14155	0.14060	1.007	0.3200
ΔUSA_{cpi2}	-0.25304	0.12919	-1.959	0.0570
ΔUSA_{cpi3}	-0.08298	0.13619	-0.609	0.5457
$R^2{}_{adj}$	0.1845			
p-value	0.01422			

Table 4.2.6 AR(3) short run USA

AR(2) DUMMY USA LR	Estimate	standard error	t-value	p-value
(Intercept)	1.40918	1.00528	1.402	0.16800
USA_u	-0.46971	0.21922	-2.143	0.03771
USA_{u1}	0.29643	0.20568	1.441	0.15662
USA_{cpi1}	0.82925	0.08866	9.353	5.05e-12
dum_t	1.86382	0.64487	2.890	0.00596
$R^2{}_{adj}$	0.7308			
p-value	7.516e-13			

Table 4.2.7 AR(2) with dummy variable USA Long run

AR(2) DUMMY USA SR	Estimate	standard error	t-value	p-value
(Intercept)	-0.3269	0.2398	-1.363	0.18004
USA_u	-0.3715	0.1984	-1.873	0.06805
USA_{cpi1}	0.2013	0.1273	1.582	0.12117
USA_{cpi2}	-0.3749	0.1312	-2.857	0.00663
dum_t	1.6017	0.6423	2.494	0.01667
$R^2{}_{adj}$	0.2861			
p-value	0.001037			

Table 4.2.8 AR(2) with dummy variable USA Short Run

4.2.1 Diagnostic Tests

One of the most important and crucial assumptions in the bounds testing (ARDL) approach is that the error terms of equations (2) and (3) have to be serially independent and normally distributed. So, in order to check the validity and reliability of the estimation results, several diagnostics are performed. The diagnostic tests for heteroskedasticity include ARCH test, Breusch Pagan test and Ramsey RESET specification test.
In all cases we note that results from ARCH and Breusch Pagan tests are very similar, as expected. Only for Germany we have homoskedasticity (in both long run and short run cases), since we accept the null hypothesis of Breusch-Pagan (and ARCH) test at significance level of 10% (see **Tables 4.2.9** and **4.2.11**). Instead in the USA cases we have heteroskedasticity (it is more evident in the cases with the dummy variable than in the cases without it) for the opposite reason (see **Tables 4.2.10** and **4.2.12**).
In the long run models we do not need extra terms (i.e. y^2, y^3,...) to describe them, because we accept the null hypothesis of RESET test (see **Tables 4.2.9**, **4.2.10** and **4.2.13**), while for the short run we accept the alternative hypothesis, always considering significance level

of 10% (see **Tables 4.2.11**, **4.2.12** and **4.2.14**).

long run CPI Germany	statistic value	p-value
Breusch Pagan	1.6650	0.6445
ARCH	1.1926	0.5508
RESET	1.0692	0.3520

Table 4.2.9 long run Germany

long run CPI USA	statistic value	p-value
Breusch Pagan	6.4126	0.0932
ARCH	4.0708	0.1306
RESET	2.3687	0.1054

Table 4.2.10 long run USA

short run CPI Germany	statistic value	p-value
Breusch Pagan	1.3207	0.7242
ARCH	1.7388	0.4192
RESET	5.3527	0.0084

Table 4.2.11 short run Germany

short run CPI USA	statistic value	p-value
Breusch Pagan	8.6177	0.0714
ARCH	6.9329	0.0741
RESET	20.819	5.221e-07

Table 4.2.12 short run USA

Dummy USA LR	statistic value	p-value
Breusch Pagan	15.232	0.0042
ARCH	7.4485	0.0241
RESET	1.3283	0.2756

Table 4.2.13 long run with dummy variable USA

Dummy USA SR	statistic value	p-value
Breusch Pagan	10.4830	0.0330
ARCH	6.05940	0.0483
RESET	19.7510	9e-07

Table 4.2.14 short run with dummy variable USA

4.2.2 Stability Test

The existence of the co-integration does not imply that the estimated coefficients are stable. So we must introduce the Wald Test to verify the stability of the coefficients. In all cases we reject the hypothesis of the test at significance level of 10%, so the estimated parameters are stable (see **Table 4.2.15**).

Wald test	statistic value	p-value
LR GER CPI	177.2	0.0000
LR GER UN	631.4	0.0000
LR USA CPI	106.5	0.0000
LR USA UN	97.9	0.0000
SR GER CPI	12.8	0.0050
SR GER UN	21.6	7.8e-05
SR USA CPI	14.2	0.0067
SR USA UN	13.1	0.0110
USA DUMMY LR	34.6	5.6e-07
USA DUMMY SR	22.4	0.0002

Table 4.2.15 Wald Test

4.2.3 Causality Test

The objective is to understand the causality relationship between unemployment and inflation using Granger causality Test. For Germany, unemployment implies inflation, and vice versa, because the p-values of the test are lower than the significance level of 10% in both cases, so we have a causality relationship between them (see **Table 4.2.16**). For the USA, we have that in only one case exists this causality relationship: unemployment implies inflation, but does not hold the vice versa, since the p-value for the opposite implication is greater than 10% (see **Table 4.2.17**).

Dependent Variable	CPI Germany	Unemployment Germany
CPI Germany		3.4223 (0.04176)
Unemployment Germany	3.3413 (0.04479)	

Table 4.2.16 Causality Test Germany

Dependent Variable	CPI USA	Unemployment USA
CPI USA		1.1508 (0.3259)
Unemployment USA	3.3174 (0.04573)	

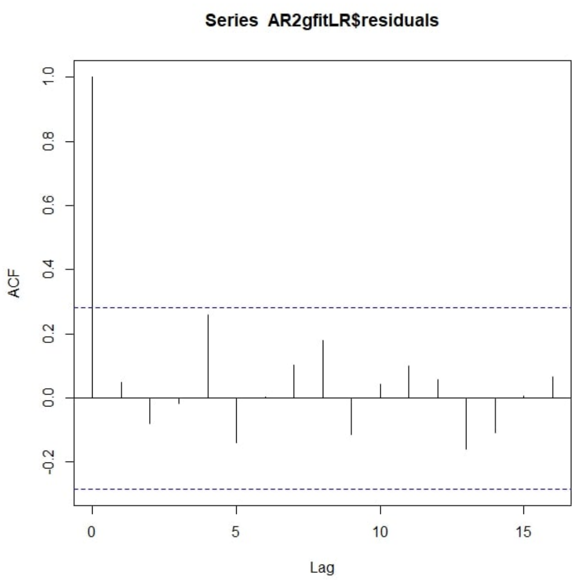
Table 4.2.17 Causality Test USA

4.2.4 Auto-correlation

To analyze the auto-correlation, we consider 3 tests: Durbin-Watson, Box-Ljung and Box-Pierce. For all these tests we consider as null hypothesis no auto-correlation.
In all cases we accept the null hypothesis, because we have p-values greater than the significance level of 10% (see **Tables 4.2.18-25**), so we do not have any auto-correlation in the model. This fact is highlighted from the ACF plots below tables, where we note that only rarely the values of ACF cross over the threshold.

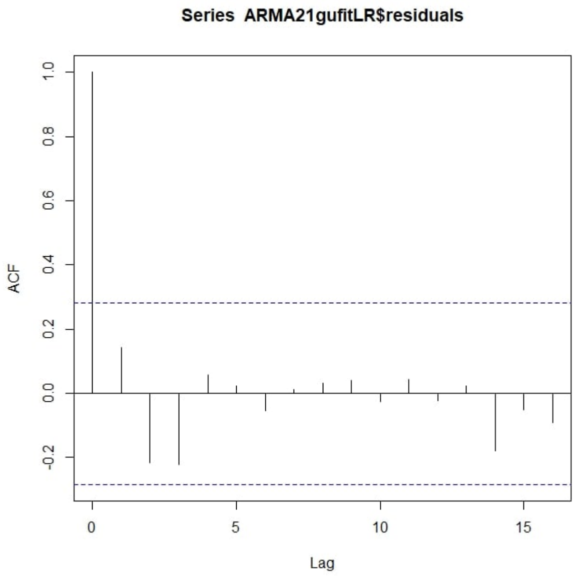
long run CPI Germany	statistic value	p-value
Durbin Watson	1.7812	0.1392
Box Ljung	5.1701	0.5222
Box Pierce	4.5483	0.6029

Table 4.2.18 ACF Tests for long run CPI Germany



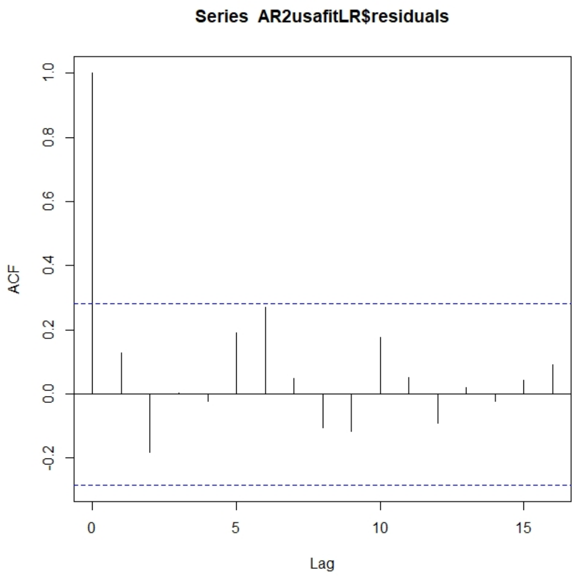
long run Un. Germany	statistic value	p-value
Durbin Watson	1.6517	0.0568
Box Ljung	6.4530	0.3744
Box Pierce	5.8825	0.4365

Table 4.2.19 ACF Tests for long run Un. Germany



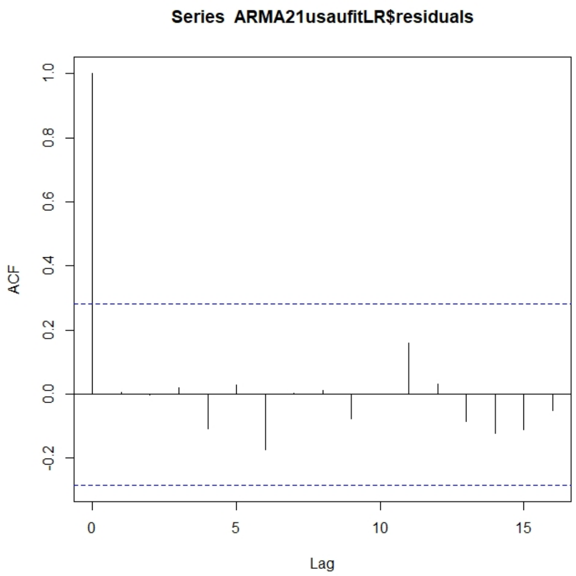
long run CPI USA	statistic value	p-value
Durbin Watson	1.6905	0.0908
Box Ljung	8.731	0.1893
Box Pierce	7.5966	0.2692

Table 4.2.20 ACF Tests for long run CPI USA



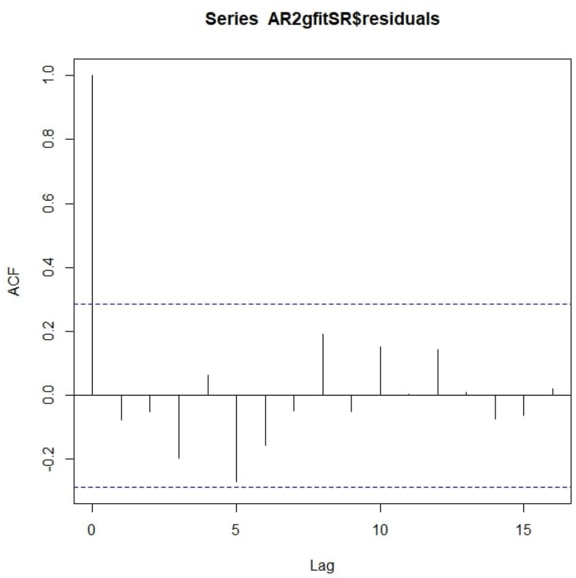
long run Un. USA	statistic value	p-value
Durbin Watson	1.6537	0.0609
Box Ljung	2.3992	0.8796
Box Pierce	2.0427	0.9157

Table 4.2.21 ACF Tests for long run Un. USA



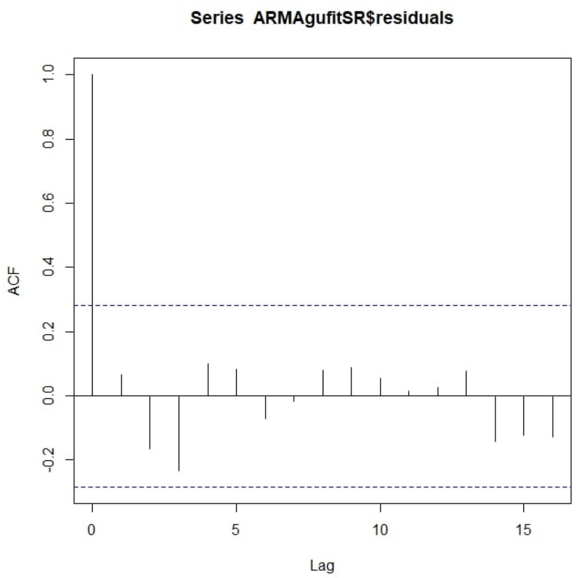
short run CPI Germany	statistic value	p-value
Durbin Watson	2.1475	0.6431
Box Ljung	8.0697	0.2330
Box Pierce	7.0074	0.3202

Table 4.2.22 ACF Tests for short run CPI Germany



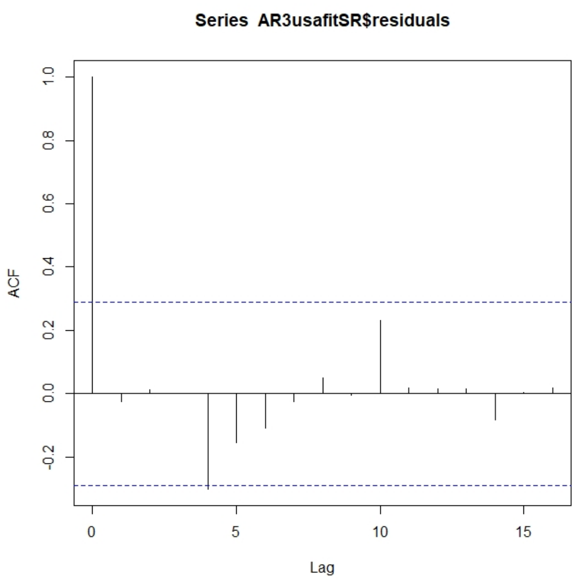
short run Un. Germany	statistic value	p-value
Durbin Watson	1.8368	0.2476
Box Ljung	5.7621	0.4504
Box Pierce	5.1796	0.5210

Table 4.2.23 ACF Tests for short run Un. Germany



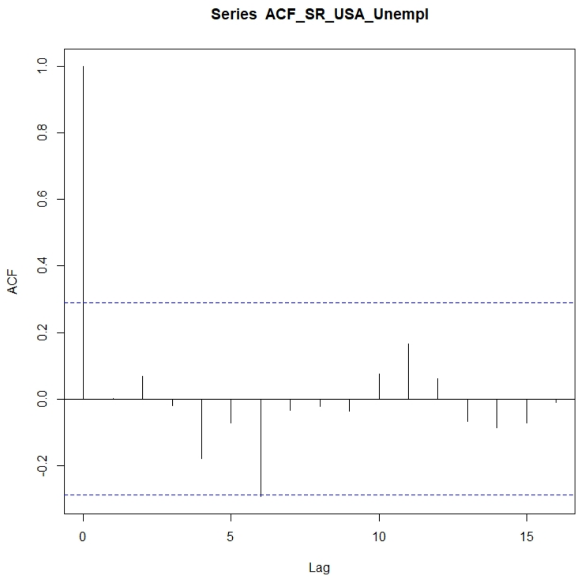
short run CPI USA	statistic value	p-value
Durbin Watson	2.0177	0.4890
Box Ljung	6.7026	0.3492
Box Pierce	5.8138	0.4444

Table 4.2.24 ACF Tests for short run CPI USA



short run Un. USA	statistic value	p-value
Durbin Watson	1.5680	0.0628
Box Ljung	6.9439	0.3261
Box Pierce	5.8834	0.4364

Table 4.2.25 ACF Tests for short run Un. USA



5 Conclusions

This study investigates the Phillips curve for long and short term, using the data during the period 1971-2020 in the USA and Germany. The implication of the low unemployment can have important implications for monetary policy. The results of this paper, based on co-integration relation, confirms that there exists a relation between inflation and unemployment either in Germany or in the USA. Looking at the Granger Causality Test, there is causality indirect relation between unemployment and inflation rate, with direction from unemployment to inflation in the case of the Germany and in the opposite way for the USA. A stable Phillips curve opens opportunities for the Central Bank to adopt monetary policies that would keep inflation and unemployment at political and social acceptable rates. German and American government, as a matter of necessity and concern, should continue to improve the macroeconomic policies for a sustainable economic framework that will enhance the domestic output, while continuing controlling inflation. The study recommends that the German government should pay attention to its findings in order to tackle unemployment issue, less present in the USA case, and encourages it to conduct active labor market programs to reduce unemployment level through the creation of productive and labor intensive projects (the replace of foreign labor with local labor could be the starting point). The results of the paper agree with these of Islam et al., 2011, who found that a stable Phillips curve exists for the case of North Cyprus for the period 1978-2007. Policy makers could make use of this paper for their future policy-making decisions which would stabilize the price level by controlling inflation and at the same time, living within an unemployment rate consistent with inflation.

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