



Contents lists available at ScienceDirect

## Finance Research Letters

journal homepage: [www.elsevier.com/locate/frl](http://www.elsevier.com/locate/frl)

## The measure of model risk in credit capital requirements

Roberto Baviera<sup>\*</sup>

Department of Mathematics, Politecnico di Milano, 32 p.zza Leonardo da Vinci, Milano I-20133, Italy

## ARTICLE INFO

## JEL classification:

C51  
G21  
G28  
G32

## Keywords:

Regulatory capital  
estimation risk  
VaR  
IRB approach  
LGD-PD dependency  
scaling factor

## ABSTRACT

Credit capital requirements in Internal Rating Based approaches require the calibration of two key parameters: the probability of default and the loss-given-default. This letter considers the uncertainty about these two parameters and models this uncertainty in an elementary way: it shows how this estimation risk can be computed and properly taken into account in regulatory capital.

We analyse two standard real datasets: one composed by all corporates rated by Moody's and one limited only to the speculative grade ones. We statistically test model hypotheses on the two key parameters and we observe that parameter dependency raises substantially the tail risk in capital requirements. The results are striking with a required increase in regulatory capital in the range 38% – 66%. This study draws also a clear policy implication: we suggest to reintroduce the *scaling factor* in credit capital requirements – factor that has been removed in Basel III Accord – to take into account model risk.

## 1. Introduction

Following the Great Financial crisis of 2007, model risk in the capital requirement of financial institutions has emerged as a key public concern. The set of international banking rules require banks to hold a minimum capital as a buffer against future loss exposures. The variability of these future losses depends, on the one hand, on a model via some stochastic risk factors and, on the other hand, on the uncertainty related to the selected model.<sup>1</sup> In this letter we focus on the model risk associated to credit regulatory capital.

Credit capital requirement (also known as capital adequacy) in banks is often determined via an Internal Rating Based (IRB) approach under the Basel II capital accord and the modifications that have followed.<sup>2</sup> A peculiarity of the IRB approach is that the modeling framework is established by the regulator while model calibration is left to the banks. In the IRB approach, regulators base the capital requirement on the value-at-risk (VaR) of bank's credit portfolio calculated using the Asymptotic Single Risk Factor (ASRF) model, introduced by Gordy (2003). Thus, model risk in credit capital requirements consists in the estimation noise of the parameters, i.e. the risk arising from errors in model parameters when we cannot rely on the assumption that the parameters of the model are known with certainty. The characteristics of credit exposures are captured by two parameters for each obligor in bank's portfolio: the obligor's probability of default (PD) and his loss-given-default (LGD). The other model parameter, the correlation between obligors' assets, is established in Basel requirements for IRB as a deterministic function of the default probability (cf. Basel Committee, 2005, p.13). Both parameters (PD and LGD) are the corresponding forecast over a one-year time horizon; they are calibrated with

<sup>\*</sup> Tel.: +39-02-2399 4575.

E-mail address: [roberto.baviera@polimi.it](mailto:roberto.baviera@polimi.it).

<sup>1</sup> Banks adopt internal models (IMs) both in the asset and in the liability side of the balance sheet. E.g. for liabilities, recently Dal Borgo (2020) investigates the impacts of IMs for deposits. All IMs carry a model risk: in this letter we consider assets, focusing on credit capital requirements.

<sup>2</sup> The other approach, the Standardized one, presents no model risk.

<https://doi.org/10.1016/j.frl.2021.102064>

Received 15 October 2020; Received in revised form 4 April 2021; Accepted 10 April 2021

Available online 16 April 2021

1544-6123/© 2021 Published by Elsevier Inc.

Through-the-Cycle values, i.e. long term default and recovery rates, often provided by rating agencies.<sup>3</sup>

Should capital requirement for credit risk account for parameter uncertainty? Albeit the relevance of this question is well known, there is a relative paucity of empirical studies that measure model risk in credit capital requirements. The problem has been introduced by Löffler (2003), even before the details of the IRB approach were introduced by the Basel Committee: he has analysed the impact on the  $\alpha$ -quantile of two homogeneous reference portfolios, one rated BBB and another one B. After the seminal paper of Löffler (2003), the main contribution is due to Tarashev (2010), who has formalized the model risk approach in the ASRF, clarifying that the *correct* capital requirement reflects all potential losses, whose uncertainty includes the imperfect information about risk parameters.

In both studies, the impact of parameter uncertainty on measures of tail risk is analysed on the basis of a stylized credit portfolio that is homogeneous, i.e. characterized by the same exposure and the same parameters (PD and LGD) for each obligor. We follow the same approach in this letter.

We assume some probability density functions (hereinafter p.d.f.) of model parameters and statistically test these distributional hypotheses on a real dataset. Having the parameters' p.d.f., it is possible to evaluate how much their uncertainty impacts capital requirements.

In Löffler (2003) PD and LGD were considered independent, while LGD was considered a constant parameter in Tarashev (2010). However, it is quite reasonable to observe a relationship between PD and LGD (or equivalently recovery). The economic reason is rather simple: if an economy experiences a recession, on the one hand, the observed frequency of corporate defaults increases and, on the other hand, recoveries decrease because the assets of failed companies are sold when many other firms have defaulted and when few buyers are available at extremely discounted prices (fire sale).<sup>4</sup> In this letter, we consider this dependency, estimate statistically it and identify the impact on capital requirement: we show that this dependency is the most relevant source of model risk in credit capital requirements.

The contributions of this letter to the existing literature are threefold: i) we indicate some distributional assumptions for model parameters in capital requirements and statistically test them on a real dataset, ii) we consider model risk for credit capital requirements within the IRB approach and analyse the impact of parameter dependency in capital requirements, and iii) we draw some policy implications for credit capital requirements.

The rest of the letter is divided as follows. In Section 2, we briefly recall the credit regulatory capital, describing a *naïve* approximation and the *correct* implementation of the requirements in the IRB approach. In Section 3, we describe the estimation risk methodology: we introduce a distributional assumption for model parameters and statistically test it. In Section 4, we discuss the consequences on capital requirements. Section 5 concludes focusing on policy implications. Finally, at the end of this letter, we recall the notation and the abbreviations we use.

## 2. The capital requirement in the IRB approach

In the IRB approach, the capital requirement is the 1-year unexpected loss VaR at the  $\alpha$  confidence level, i.e. it is the maximum portfolio unexpected loss that is exceeded within a year with probability  $\alpha$  ( $\alpha$ -quantile). The capital adequacy per unit exposure at default (hereinafter regulatory capital or RC) is

$$RC = VaR_\alpha[L] - \mathbb{E}[L],$$

where  $L$  is the *portfolio loss rate*, i.e. the ratio of total losses to total portfolio exposure at default.<sup>5</sup> The value of  $\alpha$  is equal to 99.9% as established by the Basel Committee for credit risk (cf. Basel Committee, 2005, p.11).

Bank capital rules are based on the ASRF model (Gordy, 2003).<sup>6</sup> In this section we briefly recall the main modeling results; the notation follows closely the one in Tarashev (2010).

We focus on model risk for a homogeneous portfolio as in Löffler (2003) and Tarashev (2010). The ASRF model, applied to a homogeneous portfolio of  $n$  obligors, describes log-assets of a generic obligor  $i$  as

$$X_i = \sqrt{\rho}M + \sqrt{1-\rho}\varepsilon_i \quad i = 1, \dots, n$$

and  $M, \varepsilon_i$  are i.i.d. st.n. rvs.

The variable  $M$  is a common credit risk factor representing the market,  $\varepsilon_i$  is an obligor-specific risk component, while  $\rho$  is the cor-

<sup>3</sup> More precisely two are the IRB approaches (Foundation and Advanced). Under the Foundation IRB approach, banks supply their own estimates of PD, while the other parameters are supervisory values set by the Basel Committee. Under the Advanced IRB approach, banks supply both PD and LGD (see e.g. Hull, 2012; Wernz, 2020).

<sup>4</sup> A dependency between PD and LGD has been first pointed out by Frye (2000) for non-financial issuers domiciled in the USA in the time interval 1982-1997, then a positive correlation between PD and LGD has been identified and measured by Altman et al. (2005) in the speculative grade USA bond market. Recently, Hurlin et al. (2018) have suggested the relevance –in capital requirement determination– of the correlation between LGD and the exposure at default.

<sup>5</sup> In this letter, the exposure at default (EAD) is not considered a source of model risk; i.e. we focus on a traditional banking activity where either the EAD stemming from credit lines is negligible w.r.t. direct lending via loans or credit line EAD is considered a known (non-stochastic) quantity, e.g. through a credit conversion factor. Furthermore, IRB maturity adjustment is assumed equal to one.

<sup>6</sup> Several are the possible extensions of the single factor model (see e.g. Tang et al., 2019, and references therein). In this letter we focus on the ASRF model and the estimation risk of credit requirements in Basel formulation.

relation between obligors' assets. A default of an obligor in one year occurs with a probability  $PD$  and produces a loss-given-default  $LGD$ . Obligor  $i$  defaults when  $X_i$  is below some threshold  $k$ , that is often referred to as the default point (see, e.g. [Tarashev, 2010](#), p.2066). The threshold  $k$  is then chosen s.t.  $\mathbb{P}(X_i < k) = PD$ ; this implies  $k = \Phi^{-1}(PD)$ , with  $\Phi$  the standard normal (st.n.) cumulative distribution function.

The IRB approach considers the case with a large number of obligors  $n$ : the financial literature refers to this limiting portfolio as an "asymptotic portfolio" (see, e.g. [Gordy, 2003](#); [Tarashev, 2010](#)). In this case, the expected loss conditional on the common risk factor  $M$  and given the parameters  $PD$  and  $LGD$ , is

$$\mathbb{E}[L|M, PD, LGD] = LGD \cdot \Phi\left(\frac{k - \sqrt{\rho} M}{\sqrt{1-\rho}}\right). \quad (1)$$

Moreover,  $\rho$  is a deterministic function of  $PD$  as established by the Basel Committee for IRB models (cf. [Basel Committee, 2005](#), p.13). In the case of corporate, sovereign, and bank exposures this function is

$$\rho(PD) = 0.12 \cdot \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} + 0.24 \cdot \left(1 - \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}}\right), \quad (2)$$

and a similar relation holds in the other cases.

A naïve approximation of IRB (see [Tarashev, 2010](#)) accounts for the credit risk factor  $M$  but treats the  $PD$  and the  $LGD$  as known and equal to  $\widehat{PD}$  and  $\widehat{LGD}$ , the point estimates of the respective parameters. In this special case, the above setup reduces to a single risk factor model and the capital requirement becomes

$$RC^{naive} = \widehat{LGD} \cdot \Phi\left(\Phi^{-1} \frac{(\widehat{PD}) - \sqrt{\rho(\widehat{PD})} \Phi^{-1}(1-\alpha)}{\sqrt{1-\rho(\widehat{PD})}}\right) - EL^{naive} \quad (3)$$

where

$$EL^{naive} := \widehat{LGD} \cdot \widehat{PD}$$

is the expected loss and  $\rho(PD)$  is given by (2). As already discussed by [Tarashev \(2010\)](#), the simplicity of this analytical closed formula justifies the popularity of this naïve IRB approach among practitioners.

In general, parameters could carry a significant estimation noise that cannot be neglected. Thus, the *correct* capital requirement in IRB is

$$RC = VaR_\alpha[\mathbb{E}[L|M, PD, LGD]] - \mathbb{E}[L], \quad (4)$$

where  $\mathbb{E}[L|M, PD, LGD]$  is reported in (1). This result can be proven as in [Tarashev \(2010, proposition 1, p.2067\)](#).

Moreover, as already pointed out by [Tarashev \(2010\)](#),  $M$  is independent from parameters. Since the uncertainty about  $M$  refers to the *ex-post* realization of the credit risk factor, while parameters are the best *ex-ante* estimation given past data, the assumed temporal independence of the risk factor  $M$  implies that it is independent from parameter uncertainty.

Hence, there is no closed formula for the *correct* capital requirement (4) but it can be easily obtained via a Monte Carlo simulation. It requires i) to simulate  $N_{sim}$  values for the unknown parameters and the common risk factor, ii) to compute the loss for each simulation and iii) to determine the  $\alpha$ -quantile and the mean of the loss distribution. Let us mention that the expected loss  $\mathbb{E}[L]$  is equal to  $EL^{naive}$  if the two parameters are independent, but this result does not hold true in general.

The aim of this research is to consider a p.d.f. for the set of parameters in the Basel IRB approach, to statistically test the distributional assumptions and to evaluate the impact of the uncertainty of each parameter on the capital requirement and in particular the impact of  $PD$ - $LGD$  dependency.

In this letter, we model the default point  $k$  and the loss-given-default as Gaussian rvs; a distributional assumption that can be easily tested on a real dataset. We consider

$$\begin{cases} k & \sim \mathcal{N}(\widehat{k}, \sigma_k^2) \\ LGD & \sim \mathcal{N}(\widehat{LGD}, \sigma_{LGD}^2) \end{cases}.$$

As already mentioned,  $\widehat{PD} := \mathbb{E}[PD]$  and  $\widehat{LGD} := \mathbb{E}[LGD]$ , while  $\sigma_k^2$  and  $\sigma_{LGD}^2$  are respectively the variance of  $k$  and of  $LGD$ . The value of  $\widehat{k}$  can be obtained inverting

$$\widehat{PD} = \mathbb{E}[\Phi(k)] = \int_{-\infty}^{\infty} dx \frac{e^{-x^2/2}}{\sqrt{2\pi}} \Phi\left(\widehat{k} + \sigma_k x\right) \simeq \Phi(\widehat{k}) - \frac{\sigma_k^2}{2} \cdot \frac{\widehat{k}}{\sqrt{2\pi}} e^{-\widehat{k}^2/2}, \quad (5)$$

where the right term is obtained via a Taylor expansion in  $\sigma_k$  up to the third order.

What really matters is the increase in capital requirement w.r.t. the *naïve* IRB approach.

As already mentioned, the regulatory capital refers only to the amount of capital an institution must hold against unexpected loss. Regulators recognize that expected losses are usually covered by the way a financial institution prices its products; thus, if the bank computes its RC according to the *naïve* IRB, only  $EL^{naïve}$  are the expected losses covered by reserves when pricing products.

In presence of estimation noise, the institution must hold an *excess loss reserve* to cover for an increase in both the unexpected and the expected loss. This *excess loss reserve* is the additional required capital in the case a bank has considered a *naïve* approach; it is the difference between the VaR in the case with estimation noise and the one in the *naïve* IRB. In particular, the relevant quantity is the (regulatory capital) *add-on*. It is defined as the ratio between the *excess loss reserve* (inclusive of the expected loss correction) and the RC in the *naïve* approximation

$$add-on := \frac{(RC - RC^{naïve}) + (\mathbb{E}[L] - EL^{naïve})}{RC^{naïve}}. \quad (6)$$

In this letter we focus on this percentage increase in  $RC^{naïve}$  induced by parameter uncertainty: we first consider the *add-on* generated by each parameter one at a time and then we analyse all parameters together.

### 3. The dataset

We analyse a dataset provided by Moody's Investor Service on annual LGD rates for defaulted senior unsecured corporate bonds and on annual corporate default rates (Ou et al., 2020, exhibit 29 & exhibit 41).<sup>7</sup> Two are the default rates considered: the first set includes all corporates rated by Moody's (hereinafter "All Ratings" or "AR") while the second is limited only to firms who have a speculative grade at the beginning of the default year (hereinafter "Speculative Grade" or "SG"). The dataset reports an annual value for the period 1983-2019 (37 years); it is used by several financial institutions either in the determination of regulatory capital or in the definition of benchmarks for measuring IRB parameters.

In this letter, we estimate the empirical properties of one-year LGD and PD via the observed default rates in this dataset. Table 1 contains descriptive statistics about annual LGD and PD data (both AR and SG) for the whole time window.

As already mentioned in the introduction, we analyse the distribution for  $k = \Phi^{-1}(PD)$  and LGD. The empirical distribution of LGD is shown in Figure 1. It looks well described by a normal distribution; moreover, modeling LGD with a Gaussian rv, the probability to observe "non-financial" LGD values –either negative or greater than 1– appears negligible.

The Gaussian property of a sample can be verified in several ways: the simplest is probably the Quantile-Quantile plot. The Quantile-Quantile plot of each parameter is shown in Figure 2. Because all the three plots tend to be close to a straight line, it seems that the p.d.f. of each parameter follows tightly a normal distribution. It is also possible to verify from a quantitative perspective the normality hypothesis via a statistical procedure. The Shapiro-Wilk test allows to determine if the null hypothesis of univariate normality is a reasonable assumption regarding the population distribution of a random sample (see, e.g. Shapiro and Wilk, 1965; Royston, 1982). Moreover, Royston (1983) has extended the Shapiro-Wilk hypothesis test to the bivariate case to verify composite normality.

The results for the Shapiro-Wilk test statistic  $W$  and the p-value are reported in Table 2. We do not reject the null hypothesis of normality with a 10% threshold. Notice that all p-values are above 50%. Thus, we can consider normal the marginal distribution of each parameter; furthermore, the couple  $LGD - k$  follows a bivariate normal distribution in both cases.

Parameter estimators  $\hat{k}$  and  $\sigma_k$  are reported in Table 3. The value of  $\hat{k}$  is obtained from the corresponding  $\widehat{PD}$  via equation (5); in both cases (AR and SG) it is identical to the sample mean up to the third decimal digit.

We can easily verify whether LGD and  $k$  are correlated. In Figure 3 we show the scatter-plot of the couples  $LGD - k$  and a linear regression that fits the data considering All Ratings. We can reject the uncorrelated hypothesis with a p-value  $6.12 \cdot 10^{-07}$ . The scatter plot in the Speculative Grade case looks similar; the uncorrelated hypothesis is rejected with a p-value  $8.85 \cdot 10^{-05}$  in this case. The estimated Pearson correlation  $\rho_{LGD-k}$  between LGD and  $k$  is reported (with the 95%-confidence interval) in Table 4. The value of Pearson correlation appears to be quite-high in both cases.

### 4. The measurement of model risk in capital requirements

In this section we analyse the impact on capital requirements stemming from parameter uncertainty.

First, we compute the regulatory capital per unit exposure at default in the *naïve* approximation for two homogeneous portfolios, respectively one AR and another one SG. The RC in the *naïve* IRB approach (3) is obtained for a homogeneous portfolio considering the mean values  $\widehat{LGD}$  and  $\widehat{PD}$ . The capital requirements in the two cases of interest are shown in Table 5.

Then, the uncertainty due to the estimation of LGD and PD is taken into account measuring the *add-on* (6) in different cases. We analyse one parameter at a time (and impose the other parameter equal to its expected value) and both parameters at the same time, considering both the independent and the correlated case. In this way we can "isolate" each contribution to the *add-on*. Table 6 shows

<sup>7</sup> LGD rates are obtained via Trading prices recoveries. Trading prices recoveries are the trading prices of defaulted bonds in the distressed debt market shortly after the default event: Moody's Investor Service reports them as the prices at which they trade about 30 days after default, as a percent of their face value. Only for distressed exchanges, Trading price recoveries correspond to the exchange value at the default date.

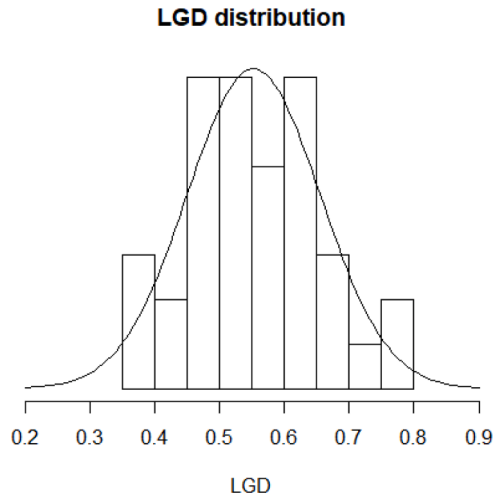


Fig. 1. Density for LGD in the dataset and for a normal distribution with same mean and std.

Table 1

Descriptive statistics for annual loss-given-default rates for senior unsecured corporate bond (LGD), annual corporate default rates for all rated firms ( $PD_{AR}$ ) and for speculative grade firms ( $PD_{SG}$ ). The data are collected at world level in the time window 1983-2019 (37 years). We report min, max, mean, median, standard deviation (std) and number of observations.

|           | min    | max    | mean   | median | std    | obs |
|-----------|--------|--------|--------|--------|--------|-----|
| LGD       | 36.25% | 78.81% | 55.26% | 54.76% | 10.25% | 37  |
| $PD_{AR}$ | 0.35%  | 5.00%  | 1.59%  | 1.25%  | 1.01%  | 37  |
| $PD_{SG}$ | 0.94%  | 12.09% | 4.30%  | 3.54%  | 2.62%  | 37  |

the results obtained considering either one parameter at time or the two parameters simultaneously. When analysing the results obtained with one parameter at time, it is interesting to observe that, in line with the literature, the largest contribution is due to the uncertainty in PD.

When considering the impact of both parameters, we observe that the independent case underestimates significantly the estimation risk. Parameter dependency, that cannot be neglected from a statistical point of view, has the most relevant impact: it determines the most relevant contribution to the *add-on* in capital adequacy.<sup>8</sup> The values of *add-on* appear very large: the *correct* RC, that takes into account estimation noise, is significantly greater than the one computed with the *naïve* approach. We obtain an increase in the required capital larger than 38%, if All Ratings are considered, and almost equal to 2/3, if we consider a credit portfolio composed only by Speculate Grade corporates. This is the main result of this study.

We also perform two robustness tests. First, we consider a confidence level  $\alpha$  equal to 99%, as it is considered in both Löffler (2003) and Tarashev (2010). Even if the regulatory capitals with this different  $\alpha$  are significantly lower, the *add-ons* look rather similar to the ones obtained with the  $\alpha$  imposed by regulators for credit risk.

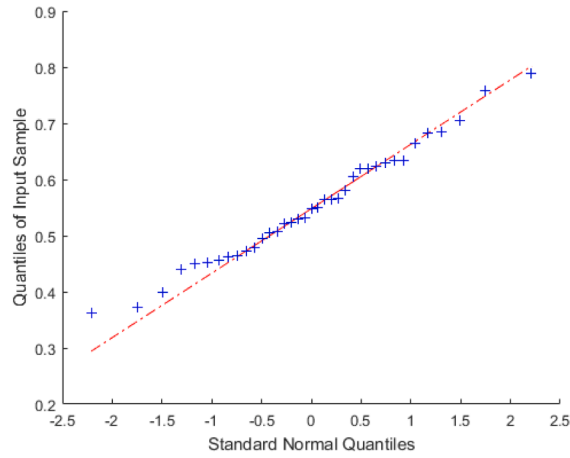
Second, we verify the impact of granularity (see, e.g. Gordy and Lütkebohmert, 2007), i.e. we check whether we observe, for a finite number of obligors, a significant deviation from the asymptotic portfolio case. We have considered a small credit portfolio composed by 50 obligors as in Löffler (2003). We obtain slightly higher capital requirements, but the measure of model risk is similar to the asymptotic case. Both robustness tests support our empirical findings: numerical results are available upon request.

## 5. Conclusions and policy implications

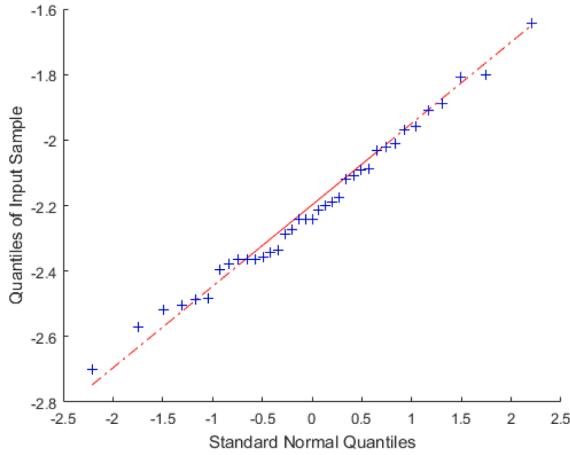
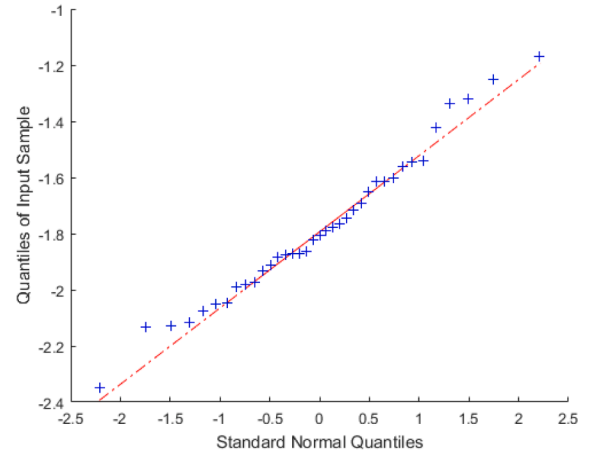
It is common practice by risk managers to rely on a *naïve* IRB approach for capital requirement, where parameters are estimated with the long term averages of historical rates. This *naïve* approximation is necessarily a downward biased estimate of the *correct* regulatory capital, because estimation noise is neglected.

In this letter, we show how to incorporate the inevitable uncertainty about the forecasted parameter values in measures of portfolio credit risk: such parameter forecasting depends on statistical hypotheses that should be tested on real datasets. A correct quantification of capital requirements reveals that ignoring estimation noise leads to a substantial understatement of the regulatory capital; in

<sup>8</sup> The expected loss correction gives a small, but not negligible, contribution to the *add-on* when the two parameters are correlated, with a correction to the *excess loss reserve* equal to  $6 \cdot 10^{-4}$  for AR and to  $14 \cdot 10^{-4}$  for SG (i.e. in RC terms of Table 5), that correspond – in *add-on* terms – to 0.79% for AR and to 1.18% for SG, respectively.



(a) LGD

(b)  $k_{AR}$ (c)  $k_{SG}$ **Fig. 2.** Quantile-Quantile (Q-Q) plot for (a)  $LGD$ , (b)  $k_{AR}$  and (c)  $k_{SG}$ .**Table 2**

Shapiro-Wilk test outcome on  $LGD$  and default point  $k$ ;  $W$  is the Shapiro-Wilk test statistics. We never reject the null hypothesis of normality.

|                            | $W$   | $p$ -value |
|----------------------------|-------|------------|
| $LGD$                      | 0.983 | 0.840      |
| $k_{AR}$                   | 0.987 | 0.941      |
| $k_{SG}$                   | 0.979 | 0.706      |
| composite $LGD$ - $k_{AR}$ | 0.973 | 0.509      |
| composite $LGD$ - $k_{SG}$ | 0.984 | 0.856      |

particular, we show that parameters' dependency plays the most relevant role in capital adequacy.

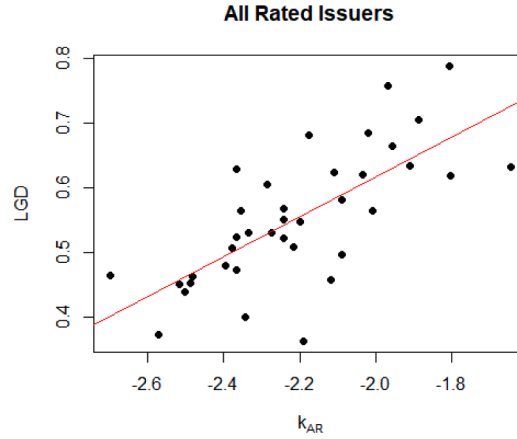
This study highlights the importance of the measure of model risk when a *naïve* approximation is implemented in credit capital requirements. We propose to capture model risk via a succinct measure, termed (regulatory capital) *add-on*, which is an incremental capital charge to  $RC^{naïve}$  for the estimation risk in IRB approaches.

This *add-on* ranges from 38% (All Ratings) up to 66% (Speculative Grade), where this second value could be an important benchmark for model risk. We have shown that IRB models could be subject to significant model risk; we expect that this risk could be particularly relevant during periods of financial distress, which are when several obligors are downgraded (even to speculative grade)

**Table 3**

Parameter estimators  $\hat{k}$  and  $\sigma_k$  in the All Ratings and in the Speculative Grade cases. The values for  $\widehat{LGD}$  and  $\sigma_{LGD}$  are reported in Table 1.

|            | AR     | SG     |
|------------|--------|--------|
| $\hat{k}$  | -2.208 | -1.778 |
| $\sigma_k$ | 0.237  | 0.268  |



**Fig. 3.** Scatter-plot of  $LGD - k_{AR}$  and a linear regression that fits the data (with an Adjusted R-squared of 0.500). We observe that the two parameters are positively correlated.

**Table 4**

Pearson correlation between  $LGD$  and  $k$  considering All Ratings and only Speculative Grade firms. We report the estimator  $\rho_{LGD-k}$  and the 95%-confidence interval (CI).

|                        | $\rho_{LGD-k}$ | CI             |
|------------------------|----------------|----------------|
| All Ratings (AR)       | 0.717          | (0.511, 0.844) |
| Speculative Grade (SG) | 0.599          | (0.342, 0.773) |

**Table 5**

Regulatory capital (RC) as a fraction of the total exposure at default in the *naïve* case for a homogeneous credit portfolio with All Ratings and a credit portfolio with only Speculative Grade firms.

|              | AR     | SG     |
|--------------|--------|--------|
| $RC^{naïve}$ | 0.0866 | 0.1224 |

**Table 6**

Regulatory capital *add-on* due to parameter uncertainties, computed via a Monte Carlo with  $N_{sim} = 10^7$  simulations. First, we consider the *add-on* due to  $LGD$  and  $k$  separately, keeping the other parameter constant. Then, on the one hand, we consider the two parameters independent and, on the other hand, correlated with the Pearson correlation  $\rho_{LGD-k}$  estimated in previous section. The most important contribution to the regulatory capital *add-on* is due to the dependency between the two parameters in both cases (AR and SG). The contribution of parameter uncertainty to capital requirements appears startling with an increase in the regulatory capital in the range 38% – 66%.

|          |               | AR            | SG            |
|----------|---------------|---------------|---------------|
| $LGD$    | (only)        | 5.63%         | 9.12%         |
| $k$      | (only)        | 12.22%        | 28.87%        |
| $LGD, k$ | (independent) | 18.67%        | 39.54%        |
| $LGD, k$ | (correlated)  | <b>38.48%</b> | <b>65.97%</b> |



at the same time. Unfortunately, these periods of financial distress are the ones when a capital adequacy is most needed.

At first glance, this result could be a cause for concern due to the documented degree of model risk in credit capital requirements.

However, this result should not take us by surprise, but it should allow drawing some policy implications for capital requirements. For market risk, an adjustment buffer is taken into account via a multiplication factor  $m_c$  imposed by the regulators. Regulatory capital is calculated as  $m_c$  times the measured VaR, where the minimum value for  $m_c$  is 3. It has been shown that such a multiple is in line with the model risk adjustment buffer for several market risks (see, e.g. Boucher et al., 2014, and references therein).

Also for credit risk the Basel II Accord required, if the regulator found that the regulatory capital was too low, to apply a multiplication factor (named *scaling factor*) –greater than 1– to the result of the credit VaR calculations, factor that corresponds to a –greater than 0– *add-on* (see, e.g. Hull, 2012, p.275). In Basel III, the Committee has agreed to remove this *scaling factor* (cf. Basel Committee, 2017, p.6). The main conclusion of this study from a financial policy perspective is that, to cope with the associated model risk, regulators should reintroduce the *scaling factor* at least equal to 1.4, when a bank prefers to stuck with a *naïve* regulatory computation.

Ultimately, this analysis has laid bare the weaknesses of the *naïve* approximation in capital requirement and it has provided a measure of model risk on regulatory capital. The non-negligible results observed in terms of *add-on* induce us to consider carefully model risk impacts on regulatory capital for banks' portfolios. A better understanding of estimation risk in IRB approaches should lead to more robust policymaking in credit risk capital requirements.

As for future research, two main promising directions are evident.

First, Brezigar-Masten et al. (2021) have suggested a new measurement of credit risk for non-financial small and medium-sized enterprises based on the number of days past due; they use supervisory data from the credit national register of the Bank of Slovenia. They experience an improvement in credit risk identification and a reduction in the fluctuations of IRB capital requirements; it would be interesting to measure model risk in terms of capital requirements, as presented in this letter, also in their case.

Second, we have suggested in the Introduction that the observed positive correlation between PD and LGD may arise when all banks engage in similar behaviour in an economic downturn; it would be worthy to study whether this dependency is due to inherently pro-cyclical parameters that would overexpose the financial system to the same risk.<sup>9</sup> Let us emphasize that the relevance of taking into account pro-cyclicality is not limited to the soundness of individual institutions but it has also consequences in terms of macro-prudential policy, i.e. when safeguarding the stability of the financial system as a whole.

## Abbreviations

|        |  |
|--------|--|
| AR     | All Ratings, all corporates rated by Moody's |
| ASRF   | Asymptotic Single Risk Factor model          |
| cf.    | compare (Latin: confer)                      |
| CI     | 95%-confidence interval                      |
| e.g.   | for example (Latin: exempli gratia)          |
| i.e.   | that is (Latin: id est)                      |
| i.i.d. | independent identically distributed          |
| IM     | Internal Model                               |
| IRB    | Internal Rating Based approach               |
| p.d.f. | probability density function                 |
| rv     | random variable                              |
| SG     | Speculative Grade corporates                 |
| s.t.   | such that                                    |
| std    | standard deviation                           |
| st.n.  | standard normal                              |
| VaR    | value-at-risk                                |
| w.r.t. | with respect to                              |

## Notation

| Symbol                | Description   |
|-----------------------|---|
| $\varepsilon_\bullet$ | obligor-specific risk component, modeled as a st.n. rv          |
| $\mathbb{E}[\bullet]$ | expected value  |
| $EL^{naïve}$          | expected loss $\mathbb{E}[L]$ in the <i>naïve</i> approximation |
| $\Phi(\bullet)$       | cumulative distribution function of the st.n. rv                |
| $k$                   |   |

(continued on next page)

<sup>9</sup> Our findings appear consistent with a growing stream of literature – mainly in Europe – showing that risk-sensitive capital regulation may reinforce banks pro-cyclical behaviour. For instance, Malovaná (2021) documents the pro-cyclicality of risk weights for credit exposures in the Czech banking sector under the IRB approach, while Cannata et al. (2011) have shown pro-cyclical risk weights for credit risk using supervisory data for Italian banks. Behn et al. (2016) find evidence that German bank regulation amplifies the pro-cyclical nature of bank lending behaviour after 2008; also Fraisse et al. (2020), considering French data from the credit national register, show that IRB capital requirements exacerbate temporarily the pro-cyclicality.



(continued)

| Symbol                       | Description  |
|------------------------------|--|
| $L$                          | default point, defined as $\Phi^{-1}(PD)$  |
| $LGD$                        | portfolio loss rate, i.e. total losses per unit exposure at default                      |
| $\widehat{LGD}$              | loss-given-default   |
| $M$                          | mean loss-given-default  |
| $\mathcal{N}(\mu, \sigma^2)$ | market risk variable, modeled as a st.n. rv  |
| $n$                          | Gaussian distribution with mean $\mu$ and variance $\sigma^2$                            |
| $N_{sim}$                    | number of obligors in the reference portfolio  |
| $PD$                         | number of simulations in the Monte Carlo method  |
| $\widehat{PD}$               | annual probability of default, estimated with the annual default rate in a real database |
| $\rho$                       | mean probability of default  |
| $\rho_{LGD-k}$               | obligors' asset correlation  |
| $RC$                         | Pearson correlation between LGD and $k$  |
| $RC^{naive}$                 | (correct) regulatory capital   |
| $\sigma_k^2$                 | regulatory capital in the <i>naive</i> approximation                                     |
| $\sigma_{LGD}^2$             | variance of the $k$ parameter  |
| $X_i$                        | variance of the $LGD$ parameter  |
|                              | log-asset for the $i^{th}$ obligor   |

## Acknowledgments

The authors thank all participants to the seminar at the European Investment Bank (EIB). We are grateful in particular to M. Azzone, G. Bonavolontá, S. Borgioli, M. Boukerroui, S. Gaal, J. Hlinicky, A. Lauria, A. Nassigh, A. Ozsoy, O. Reichmann, S. Scandizzo and P. Tychon for useful comments.

The authors acknowledge EIB financial support under the EIB Institute Knowledge Programme. The findings, interpretations and conclusions presented in this letter are entirely those of the authors and should not be attributed in any manner to the EIB. Any errors remain those of the authors.

## References

- Altman, E.I., Brady, B., Resti, A., Sironi, A., 2005. The link between default and recovery rates: Theory, empirical evidence, and implications. *The Journal of Business* 78 (6), 2203–2228.
- Basel Committee, 2005. An explanatory note on the Basel II IRB risk weight functions. Technical Report. Bank for International Settlements, Basel Committee on Banking Supervision.
- Basel Committee, 2017. High-level summary of Basel III reforms. Technical Report. Bank for International Settlements, Basel Committee on Banking Supervision.
- Behn, M., Haselmann, R., Wachtel, P., 2016. Procyclical capital regulation and lending. *The Journal of Finance* 71 (2), 919–956.
- Boucher, C.M., Danielsson, J., Kouontchou, P.S., Maillet, B.B., 2014. Risk models-at-risk. *Journal of Banking & Finance* 44, 72–92.
- Brezigar-Masten, A., Masten, I., Volk, M., 2021. Modeling credit risk with a tobit model of days past due. *Journal of Banking & Finance* 122, 105984.
- Cannata, F., Casellina, S., Quagliariello, M., 2011. The myths and truths about Basel II cyclical. *Risk* 24 (3), 65.
- Dal Borgo, M., 2020. Internal models for deposits: Effects on banks' capital and interest rate risk of assets. *Journal of Banking & Finance* 105940.
- Fraisse, H., Lé, M., Thesmar, D., 2020. The real effects of bank capital requirements. *Management Science* 66 (1), 5–23.
- Frye, J., 2000. Depressing recoveries. *Risk Magazine* 13 (November), 108–111.
- Gordy, M.B., 2003. A risk-factor model foundation for ratings-based bank capital rules. *Journal of Financial Intermediation* 12 (3), 199–232.
- Gordy, M.B., Lütkebohmert, E., 2007. Granularity adjustment for Basel II. Technical Report. Bundesbank Series 2 Discussion Paper No. 2007/01.
- Hull, J., 2012. Risk management and financial institutions, 733. John Wiley & Sons.
- Löffler, G., 2003. The effects of estimation error on measures of portfolio credit risk. *Journal of Banking & Finance* 27 (8), 1427–1453.
- Malovaná, S., 2021. The pro-cyclicality of risk weights for credit exposures: Driven by the retail segment. *Economic Systems* 100763.
- Ou, S., Jiang, J., Liu, Y., Kanthan, K., Sheth, A., Van Praagh, A., Gates, D., Chen, K.-H., He, K., 2020. Annual default study: Defaults will edge higher in 2020. Technical Report. Moody's Investor Service.
- Royston, J., 1982. Algorithm AS 181: the W test for normality. *Journal of the Royal Statistical Society. Series C (Applied Statistics)* 31 (2), 176–180.
- Royston, J., 1983. Some techniques for assessing multivariate normality based on the Shapiro-Wilk W. *Journal of the Royal Statistical Society: Series C (Applied Statistics)* 32 (2), 121–133.
- Shapiro, S.S., Wilk, M.B., 1965. An analysis of variance test for normality (complete samples). *Biometrika* 52 (3/4), 591–611.
- Tang, Q., Tang, Z., Yang, Y., 2019. Sharp asymptotics for large portfolio losses under extreme risks. *European Journal of Operational Research* 276 (2), 710–722.
- Tarashev, N., 2010. Measuring portfolio credit risk correctly: Why parameter uncertainty matters. *Journal of Banking & Finance* 34 (9), 2065–2076.
- Wernz, J., 2020. Bank management and control: Strategy, Capital and Risk Management. Springer, 2nd Ed.