POLITECNICO
MILANO 1863

Financial Engineering Final Project

Model Risk on Regulatory Capital

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Academic year 2021/2022

Abstract

The purpose of our project is to investigate the Credit capital requirements in the Internal Rating Based approach considering the uncertainty linked to the presence of two different parameters which are: the probability of default (**PD**) and the loss given default (**LGD**).
In the first section we introduce the problem and the main reasons of this type of analysis.
In the second section our goal is to consider the distributional assumption on LGD and PD and to statistically test it using a Shapiro-Wilk test, both univariate and bivariate. Then, we also compute the Pearson correlation between the two parameters and its 95% Confidence Interval.
In the third section we consider the measure of model risk in capital requirement: first of all we compute the capital requirements in the Nominal approach, then we value the Add-on, introducing uncertainty on one parameter at a time and on both parameters together. Finally we do some Robustness checks of the results.
In the last section we consider the case in which LGD and k are modelled via a Double t-student with the same parameters calibrated in the Gaussian case and the number of d.o.f. ν.

Keywords: IRB Capital requirements, Estimation Error, VaR

Contents

1 Introduction

First of all, we exploit the 2007-2009 Financial crisis to analyze how risk relates to bank business models and to understand the set of International Banking rules, based on a minimum capital as a reserve against future loss exposures. In the past few years, several new models for the measurement of portfolio credit risk have been proposed. They have the potential to effect major changes in the ways banks are managed and regulated.
Under the Basel II guidelines, banks are allowed to use their own estimated risk parameters for the purpose of calculating Regulatory Capital. This is known as the internal ratings-based (IRB) approach to capital requirements for Credit risk.
Moreover in the IRB approach the valuation of the capital requirement is based on the computation of the value at risk (VaR) of bank's credit portfolio, considering the Asymptotic Single Risk Factor model. The main goal is to focus our attention on the measure of model risk in credit capital requirements. Therefore we consider two important parameters that describe credit exposures: the obligor's probability to default (**PD**) and the Loss-Given-Default (**LGD**).
Then, we evaluate the impact of the uncertainty of each parameter of the model on the capital requirement. In our analysis we consider two models which are the **Vasicek** and **the Double t student** based on two different assumptions: **Large Homogeneous Portfolio** and **Homogeneous Portfolio**.

	Credit	Market
Time	1 year	10 days
alpha	99.9%	99%

2 Statistical analysis

In this section we consider a descriptive statistics for annual loss given default (LGD), annual corporate default (PD_{AR}) and speculative grade firms (PD_{SG}).

	min	max	mean	median	std	obs
LGD	36.25%	78.81%	55.26%	54.76%	10.25%	37
PD_{AR}	0.35%	5.00%	1.59%	1.25%	1.01	37
PD_{SG}	0.94%	12.09%	4.30%	3.54%	2.62%	37

Table 1: Descriptive statistics

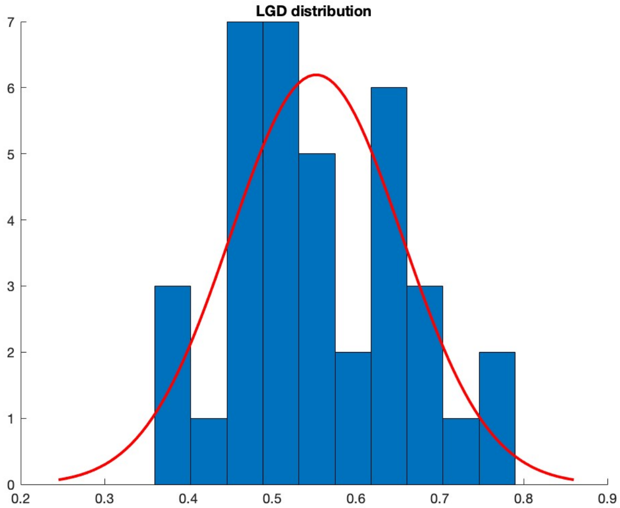

Figure 1: LGD qualitative distribution

2.1 Shapiro - Wilk test univariate

We test the distributional assumptions on PD and LGD using a univariate Shapiro-Wilk test. In fact, we perform the Shapiro-Wilk test to determine if the null hypothesis of composite normality is a reasonable assumption regarding the population distribution of a random sample. The desired significance level, α, is an optional scalar input (default = 0.05). We obtain the following results:

	W	p-value	Distribution
LGD	0.983	0.840	Normal
k_{AR}	0.987	0.941	Normal
k_{SG}	0.979	0.706	Normal

Table 2: Outputs of the Shapiro - Wilk test on LGD and k. We highlight W which is the Shapiro - Wilk test statistics and the corresponding p-value.

2.2 Shapiro - Wilk test bivariate : Royston test

The Shapiro-Wilk test (1965), is generally considered to be an excellent test of univariate normality. It is only natural to extend it to the multivariate case, as done by Royston (1982). In order to do this test we use a function implemented in Matlab[4], based on the paper of Royston[2].

	R	DoF	p-value	alpha	Joint Distribution
LGD - k_{AR}	0.045	1.959	0.976	0.05	Normal
LGD - k_{SG}	0.185	2.022	0.914	0.05	Normal

Table 3: Outputs of the Royston test on the joint distributions. We notice that R is the Royston statistics.

From the previous table we can observe that we never reject the null hypothesis of normality.

2.3 QQ - plot

Another way to verify the Normality assumption is to use the Matlab command `qqplot` that makes an empirical QQ-plot of the quantiles of the data in the vector versus the quantiles of a standard Normal distribution.

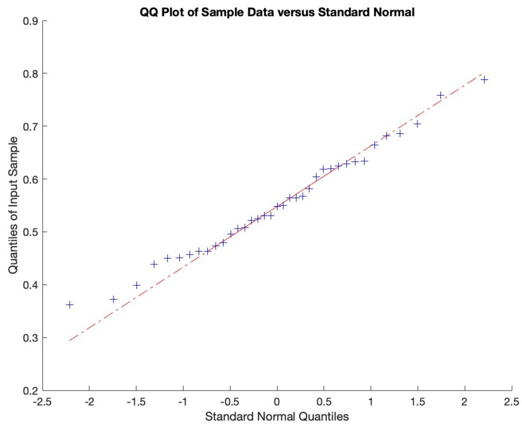

Figure 2: LGD

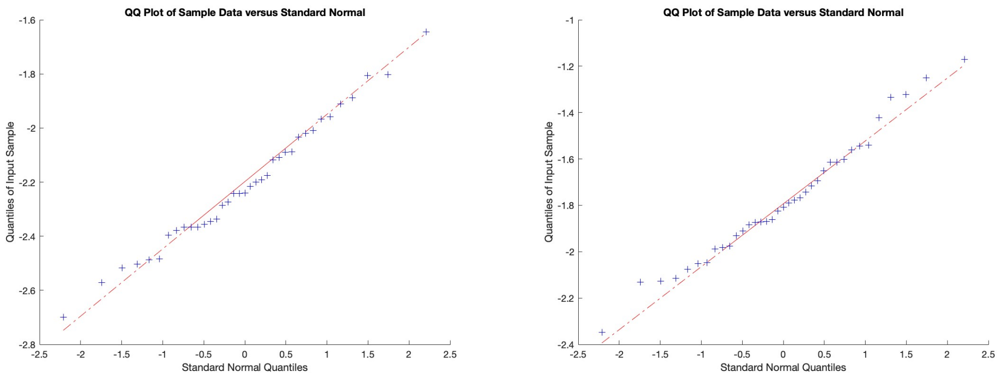

Figure 3: k_{AR} - k_{SG}

This is a more qualitative approach to determine the normality of a distribution, nevertheless it is quite reliable and easy to be interpreted. To have normality it is enough that the line is a good fitter of the distribution. From the previous graphs we highlight that the assumptions regarding the normal distribution hold.

2.4 Pearson correlation

In our analysis we are interested in the case in which holds a correlation between the parameters we are estimating. Aiming to study this case we have to determine how our variables are correlated together. An easy choice is the Pearson correlation.
The Pearson correlation measures the strength of the linear relationship between two variables. It has a value between -1 to 1, with a value of -1 meaning a total negative linear correlation, 0 being no correlation, and +1 meaning a total positive correlation.
Computing this kind of correlation and its 95% confidence interval we obtain the following results:

	ρ_{LGD-k}	CI
All Ratings (AR)	0.716	(0.511,0.844)
Speculative (SG)	0.599	(0.342,0.773)

Table 4: Pearson Correlation between the variables of interest: LGD, k. We highlight the value of the estimator ρ and its 95% confidence interval.

We can notice that there is a positive correlation both between LGD - k_{AR} and LGD - k_{SG}.

As before, we prefer to have a graphical representation of our results. In order to obtain this we do the scatter plot, a graph in which the values of two variables are plotted along two axes and the pattern of the resulting points reveals if any correlation is present.

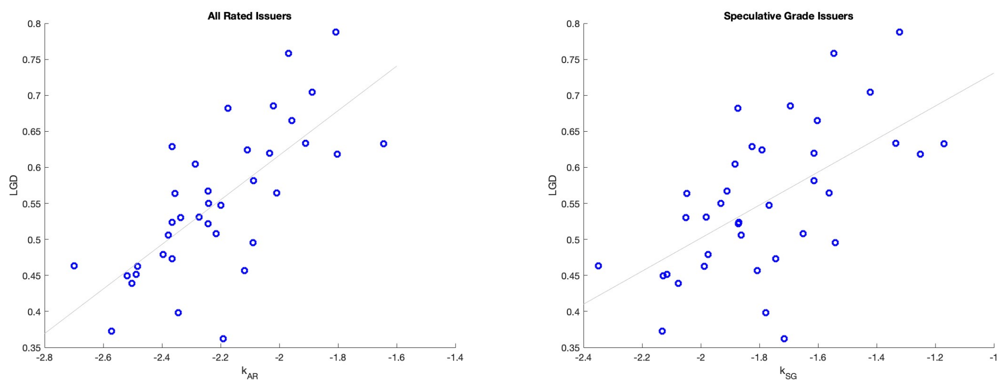

Figure 4: Scatter plots of LGD-k_{AR} and LGD-k_{SG}

Since the cloud of points plotted is distributed alongside the bisector of the first quadrant we can conclude that there is a positive correlation between parameters.

Usually the quantitative approaches are really good and reliable. Although we prefer to also take some qualitative checks to be sure there are no extreme cases or some implementation errors.

3 Measure of model risk in capital requirement

3.1 Capital requirements in the nominal model

In this section we compute the Regulatory capital (RC) as a fraction of the total exposure at default in the Naive case for a homogeneous credit portfolio with All Ratings, and a credit portfolio with only Speculative Grade firms.
The capital adequacy per unit exposure at default (hereinafter regulatory capital or RC) is

$$RC = VaR_{\alpha}[L] - \mathbb{E}[L] \tag{1}$$

where L is the portfolio loss rate. Now we assume α equal to 99.9% as established by the Basel Committee for credit risk, and after we try a stress test using a different value.
In this first section we study the nominal model, the one actually used. In this case the relation above is expressed as

$$RC^{naive} = L\hat{G}D \cdot \Phi\left(\frac{\Phi^{-1}(P\hat{D}) - \sqrt{\rho(P\hat{D})} \cdot \Phi^{-1}(1-\alpha)}{\sqrt{1-\rho(P\hat{D})}}\right) - EL^{naive} \tag{2}$$

where

$$EL^{naive} = L\hat{G}D \cdot P\hat{D} \tag{3}$$

	AR	SG
RC^{naive}	0.0866	0.1224

Table 5: Regulatory Capital (RC) with the Naive approach

Remark: It is important to compute the values of RC with the Naive approach because it is a popular solution of the IRB approach due to the presence of an analytical closed formula.

3.2 Regulatory Capital Add - on

The model presented before has the great advantage to be described through a closed formula. From now on we lose this advantage and we need to perform some Monte Carlo simulations. In this section our aim is to evaluate the impact of the uncertainty of each parameter on the capital requirement and in particular of PD-LGD dependency.
We consider :

$$k \sim \mathcal{N}(\hat{k}, \sigma_k^2) \ , \tag{4}$$

$$LGD \sim \mathcal{N}(L\hat{G}D, \sigma_{LGD}^2) \tag{5}$$

where

$$P\hat{D} = \mathbb{E}[PD]$$

$$L\hat{G}D = \mathbb{E}[LGD]$$

and σ_k^2 and σ_{LGD}^2 are the variances of k and of LGD.

In order to determine $\hat{k}$ we invert the following equation:

$$\hat{PD} = \int_{-\infty}^{\infty} \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}} \cdot \Phi(\hat{k} + \sigma_k \cdot x)dx \quad \simeq \quad \Phi(\hat{k}) - \frac{\sigma_k^2}{2} \cdot \frac{\hat{k}}{\sqrt{2\pi}} \cdot e^{\frac{-\hat{k}^2}{2}} \tag{6}$$

	AR	SG
$\hat{k}$	-2.206	-1.778
σ_k	0.237	0.268

Table 6: Parameters $\hat{k}$ and σ_k^2, while the values for $L\hat{G}D$ and σ_{LGD}^2 are in Table 1

To compute the values of the capital requirements we perform a Monte Carlo simulation. First of all, we simulate N_{sim} values for the common risk factor M and the unknown parameters. Depending on which case we are considering there are different parameters to be estimated and in different ways. Then, we compute the Loss for each simulation, we determine the VaR and the mean of the Loss distribution, so we have all the elements to compute the new Regulatory Capital.
We consider the following cases for Speculative Grade and All Ratings:

- **Case 1 - LGD only**: LGD simulated, k fixed.
- **Case 2 - k only**: LGD fixed, k simulated.
- **Case 3 - LGD,k independent**: LGD, k simulated.
- **Case 4 - LGD,k correlated**: LGD, k simulated as bivariate.

3.3 Case 1: Simulation

In this section we consider the case in which we have **LGD simulated** and $\hat{k}$ **fixed**.

$$LGD_{sim} \quad = \quad L\hat{G}D + randn(N_{sim}, 1) \cdot \sigma_{LGD}^2$$

Moreover we determine $\hat{k}$ by inverting equation 6 for $\hat{PD}$.

3.4 Case 2: Simulation

In this section we consider the case in which we have **k simulated** and $L\hat{G}D$ **fixed**.

$$k_{sim} \quad = \quad \hat{k} + randn(N_{sim}, 1) \cdot \sigma_k^2$$

Then we compute $L\hat{G}D$ as:

$$L\hat{G}D = \mathbb{E}[LGD]$$

3.5 Case 3: Simulation

In this section we consider the case in which we have both **LGD** and **k simulated** and **independent**.

$$LGD_{sim} = \hat{LGD} + randn(N_{sim}, 1) \cdot \sigma^2_{LGD} \tag{7}$$

$$k_{sim} = \hat{k} + randn(N_{sim}, 1) \cdot \sigma^2_{k} \tag{8}$$

3.6 Case 4: Simulation

In this section we consider the case in which we have both **LGD** and **k simulated** but **correlated** with Pearson correlation ρ_{LGD-k} estimated in the previous section. In order to do that we use the Matlab command $\texttt{mvnrnd(}\mu\texttt{,}\Sigma\texttt{,}N_{sim}\texttt{)}$ which returns a matrix R of random vectors, chosen from the multivariate normal distribution with mean vector μ, and covariance matrix Σ.
In fact, we compute the mean vector and the covariance matrix as following:

$$\mu = [\hat{LGD} \quad \hat{k}]$$

$$\Sigma = \begin{bmatrix} \sigma^2_{LGD} & Cov(LGD, k) \\ Cov(k, LGD) & \sigma^2_{k} \end{bmatrix}$$

where

$$Cov(LGD, k) = Cov(k, LGD) = \rho \cdot \sigma_{LGD} \cdot \sigma_k$$

3.7 Results: Confidence Interval 99.9%

The aim of the study is to find the impact of the estimation on the Regulatory Capital. Indeed, we make a bar-plot in which we highlight the new regulatory capital with respect to the Notional case.

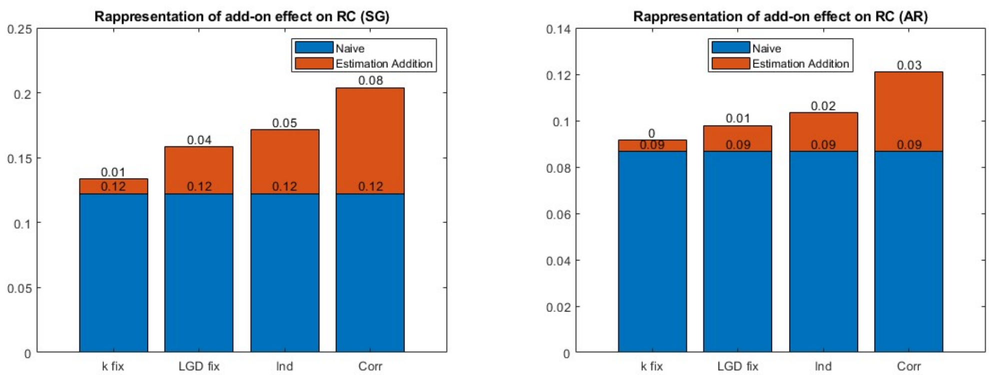

Figure 5: Bar-plots of the Add-On effect on RC both for AR and SG

To have a more complete analysis of the estimation effect we define the Add-On as a quantity based on the difference in RC and EL with respect to the Naive approach.

$$Add - On = \frac{(RC - RC_{naive}) + (EL - EL_{naive})}{RC_{naive}} \tag{9}$$

Using the standard convention of an α=99.9% for the computation of the VaR we obtain the following values:

		AR	SG
LGD	(only)	5.788%	9.031%
k	(only)	12.924%	29.409%
LGD,k	(independent)	19.372%	39.923%
LGD,k	(correlated)	39.553%	66.800%

Table 7: Add-On due to parameter uncertainties considering $\alpha = 99.9\%$

We can observe that the estimation effect of simulating both the parameters in an indipendent way is approximately the sum of the two single estimation errors. On the other hand, when we estimate both parameters as a bivariate distribution, we can observe that the estimation effect is quite strong. In this framework we notice that RC are higher with respect to the Naive approach. In particular we highlight that in the Case 4 the value of RC is much greater than the Naive one. Indeed, as we can see in the Figure 5, the value of RC in the correlated case (SG) has to be corrected with an increment which is more than half of the previous value.

In order to understand our results we recall the role of the Regulatory Capital in the bank's system. From the point of view of the bank we would have a low RC so we are induced to take more risk. On the other side, the regulator wants a higher RC and, then, the bank will have a more prudential behaviour in its investments. Our analysis highlights that the RC computed with Nominal framework is lower. A possible explanation of this approach is the presence of a closed formula which implies an easy implementation both for big and small companies. As a consequence there is a reduction in the operational risk, factor considered in Basel II.

4 Robustness Check

4.1 Homogeneus Portfolio (not Large)

We lose the assumption on the Large Homogeneous Portfolio and we consider a portfolio composed by a small amount of obligors. In particular we consider 50 obligors. This is a typical robustness check to do, indeed it is also done by Löffler[1] and Tarashev[3] in their respective papers.
Each obligor would be described, considering the Gordy model usually accepted, by the expression:

$$\begin{aligned} X_i &= \sqrt{\rho} M + \sqrt{1-\rho}\epsilon_i \\ &\text{with i = 1,...,n} \\ M, &\ \epsilon_i \text{ i.i.d. st.n. rvs.} \end{aligned} \tag{10}$$

M is a common credit risk factor and it is common for all the obligors. As we said, in our case, we have 50 obligors and so we will have that ϵ is a vector of 50 elements. Moreover, as done in Löffler[1] we took 20 000 simulations of the market situations, i.e. M is a vector of 20 000 elements. Then, technically speaking X will be a matrix.
The second step is to define the default point k, that usually is given by $k = \phi^{-1}(PD)$.
As usual, the default condition is given by $X_i < k$. Applying this condition to our matrix and summing up in the dimension of the number of obligors, we can find for each market simulation how many obligors default. This step is important because now we can compute the probability to default, simply counting the occurrences and dividing it by the total number of simulations. Logically speaking we are searching how many times we observed n defaults over all the simulations (e.g. over 20 000 simulations we observe that only 5 times there are 35 defaults, then the probability of 35 defaults is 5/20 000). We can now determine the Loss, that will be given by the product between the number of observed defaults and the Loss Given Default (LGD). Finally to compute the Regulatory Capital we just need to value the Expected Loss and the VaR, computed as the percentile of the Conditional Expected Loss.

As discussed above, in section 3.2, we have four cases of interest. Approaching case 2 (and consequentially 3 and 4) we faced a problem. Our default point parameter, discriminant to find the defaults, is described by a vector, which we simulate. Since it would have had no sense to use a vector as such constraint we decided to take the average of the simulated one. To be coherent with that choice, we took the average of the LGD simulated vector. This second choice is driven also from the reading of the paper of Löffler[1] in which we understand he does the same thing.

		AR	SG
LGD	(only)	0.1227	0.1727
k	(only)	0.1165	0.1684
LGD,k	(independent)	0.1168	0.1687
LGD,k	(correlated)	0.1166	0.1689

Table 8: Regulatory capital due to parameter uncertainties considering $\alpha = 99.9\%$

As we can observe the Regulatory Capitals in the case of an HP portfolio are higher than the one in the LHP case. In general, since our sample has been drastically reduced we would have expected a difference from the previous case. We observe that reducing the sample, we obtain, in general, higher RC values.
We think that having higher results in the HP case is a good conclusion from the point of view of the regulator. Our deduction is that having a few number of obligors implies an increase of the risk, since the default of one obligor would impact with greater intensity. An higher Regulatory Capital implies a greater exposure of the bank, and this is a plausible request since we have a few number of obligors.

Now we study our results more in detail, using some graphs to highlight their peculiarities. We can observe a direct comparison between the Regulatory Capital in the LHP and the HP cases using a bar-plot.

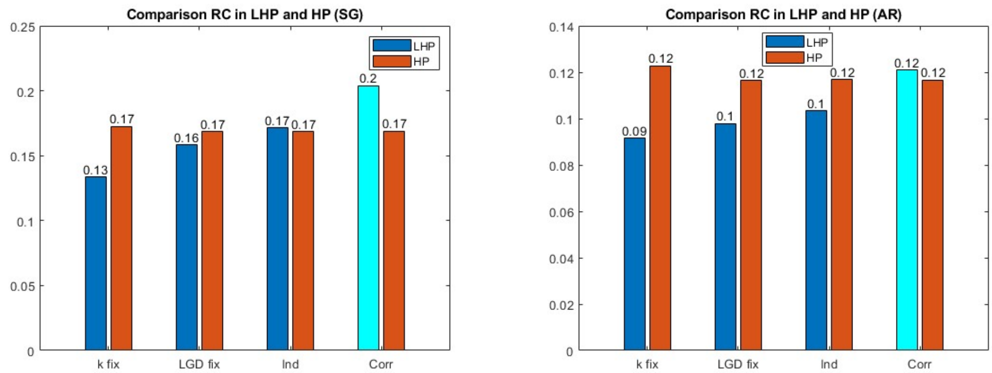

Figure 6: Bar plots to compare RC from the HP and LHP case. The value for the HP case are in Table 8, while the value for the LHP case are omitted to avoid repetitions

In the bar-plot of Figure 6 we can observe that usually the regulatory capital is higher in the case of few obligors. When both parameters are estimated as a bivariate gaussian distribution we have an exception, indeed in this case the regulatory capital seems to be greater in the HP case. If this event will turn out to be true, it could mean that estimating the variables instead of using the Nominal approach would not work well for small banks with a little number of obligors. Or, at least, that the correlated case doesn't work well. Also the add-ons have a strange behavior:

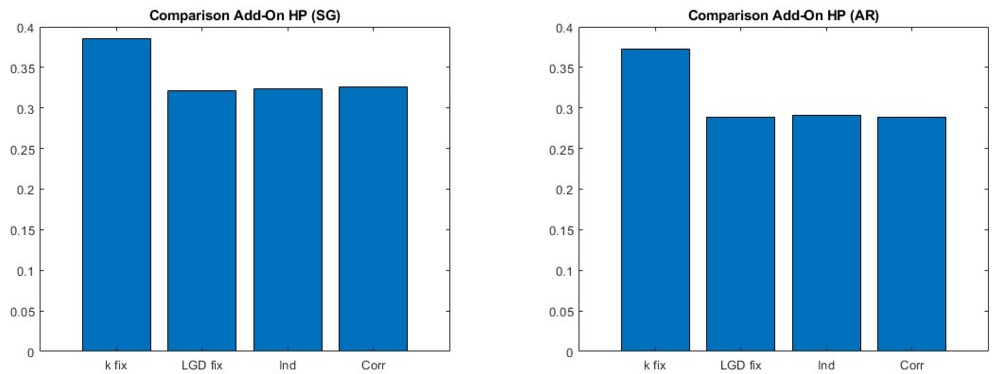

Figure 7: Comparison of Add-On through Bar-plots in HP case both for AR and SG.

From the graph of Figure 7 we notice that the estimation of only LGD has a great impact on the Add-on. Instead the estimation of the k variable seems to have a leveling effect, i.e. when we estimate both LGD and k the result is the same of estimating only k. This behavior is quite unexpected and to check that our procedure is correct we decide to plot the distribution of our computed loss, the expected loss and the VaR.

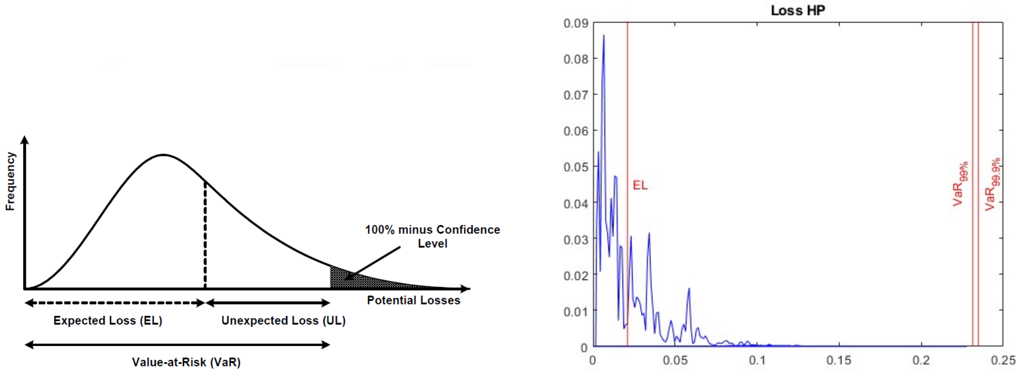

Figure 8: Plots of the Loss computed, Expected Loss and VaR considering 500 obligors, $N_{sim} = 2 \cdot 10^5$

We think that the plot we obtain is comparable with the one expected from the theory. This fact leads us to conclude that our procedure could be correct and we confirm the conclusion above. In the graph we plot also the VaR for $\alpha = 99\%$ and now we are going to explain it.

4.2 Confidence Interval 99%

Up to now we used α=99.9% because it is the standard value in analyzing Credit Capital Requirement. Another situation considered in Basel II is the Market Capital Requirement, and it is evaluated for α=99%. The Credit RC presents always a worst situation and requires higher capital than the Market RC, since by convention we consider a longer time horizon (10 days in the Market case against 1 day in the Credit case) and a larger confidence interval (CI= 1% in the Market case against CI= 0.1% in the Credit case).
So in this section we are assuming that our portfolio is not only exposed to Credit risk but also (or only) to Market risk.

Results: Confidence Interval 99%

		AR	SG
LGD	(only)	3.299%	5.834%
k	(only)	14.952%	28.434%
LGD,k	(independent)	18.440%	34.542%
LGD,k	(correlated)	36.908%	56.807%

Table 9: Add - on due to parameter uncertainties considering $\alpha = 99\%$

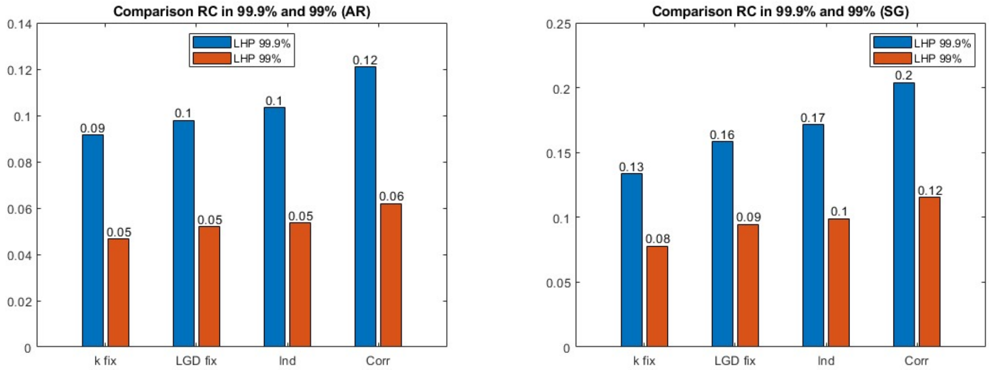

Figure 9: Bar-plots for RC in the HP approach considering $\alpha = 99.9\%$ and $\alpha = 99\%$ both for AR and SG.

From the previous graphs we observe that the values of RC considering $\alpha = 99.9\%$ are greater that the ones computed with $\alpha = 99\%$.

5 Capital Stress test: Double t- Student

5.1 Ks - test2

First of all, we perform a `ks - test2` which applies a Kolmogorov-Smirnov (K-S) test to determine if two random samples, X1 and X2, are drawn from the same underlying continuous population. In our case, we compare the dataset with its estimation obtained through a t-Student distribution.

```
kstest2(LGD,  LGD_hat+std(LGD)*trnd(nu,length(LGD),1))
kstest2(k_SG,  k_SG_hat+std(k_SG)*trnd(nu,length(k_SG),1))
kstest2(k_AR,  k_AR_hat+std(k_AR)*trnd(nu,length(k_AR),1))
```

t_distribution_test.m

Therefore, we can notice that LGD, k_{AR} and k_{SG} can be modelled as t-Student distributions. In this section, we consider the case where LGD and k are modelled via a **Double t-student** with the same parameters calibrated in the Gaussian case and a number of d.o.f. ν.
In fact we consider the following framework:

$$LGD_{sim} = \sigma_{LGD} * trnd(\nu, N_{sim}, 1) + \hat{LGD};$$

$$k_{sim} = \sigma_k * trnd(\nu, N_{sim}, 1) + \hat{k};$$

which are just a modification of equations 7 and 8.
In order to consider the case in which we have both LGD and k simulated but correlated with Pearson correlation ρ_{LGD-k} we create a function `my_mvtrand` which returns random numbers of Multivariate t-distribution.
We obtain the Multivariate t- distribution as the ratio between the Normal distribution and the square root of a χ^2 corrected by its degrees of freedom:

$$Y = \mu + \sqrt{\frac{\nu}{\chi^2}} \cdot Z$$

In the code the Chi-square distribution is obtained through a Gamma distribution.

```matlab
%   - N_sim: number of random values you want to simulate
%   - mu: pX1 vector of bivariate t distribution
%   - sigma: pXp Sigma matrix
%   - nu: Degree of Freedom

p = length(mu);
Y=gamrnd(nu/2,1,[N_sim 1])/(nu/2);
G=1./Y;
Z=mvnrnd(zeros(1,p),sigma,N_sim);
distribution=mu'+sqrt(G).*Z;
```

my_mvtrand.m

To decide what is the best ν to estimate a Gaussian distribution through a t-student we plot the following graphs:

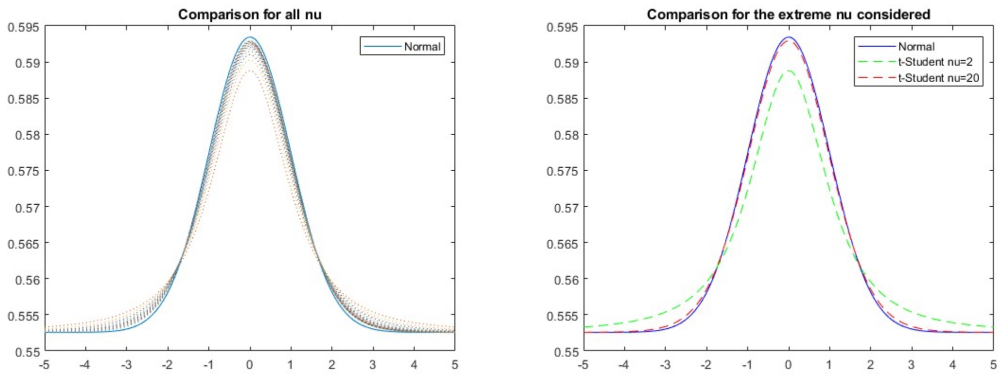

Figure 10: t-student distribution varying ν from 2 to 20

A qualitative observation leads us think that $\nu = 20$ would be enough to estimate a normal distribution correctly.

We repeat the previous computations for LHP approach varying ν from 2 to 20.

AR		ν=2	ν=3	ν=6	ν=15	ν=20
LGD	(only)	32.799%	6.920%	2.721%	4.750%	5.224%
k	(only)	0.237%	1.251%	2.669%	6.476%	7.598%
LGD,k	(ind)	40.967%	10.556%	6.619%	11.994%	13.410%
LGD,k	(corr)	322.431%	63.900%	24.620%	29.818%	31.672%

Table 10: Add - on due to parameter uncertainties considering $\alpha = 0.999$ with Double t-student approach

SG		ν=2	ν=3	ν= 6	ν =15	ν=20
LGD	(only)	38.488%	9.436%	5.199%	7.455%	7.795%
k	(only)	0.468%	2.755%	12.639%	20.269%	22.292%
LGD,k	(ind)	53.618%	17.959%	19.793%	29.818 %	31.926%
LGD,k	(corr)	332.202%	104.259%	49.534%	56.139%	58.048%

Table 11: Add - on due to parameter uncertainties considering α = 99% with Double t-student approach

5.2 Homogeneous Portfolio via Double t Student approach

In order to perform the Homogeneous portfolio approach we use the same procedure that we have described in section 4.1, except for the modification of `trnd` and `my_mvtrand` as described in section 5.

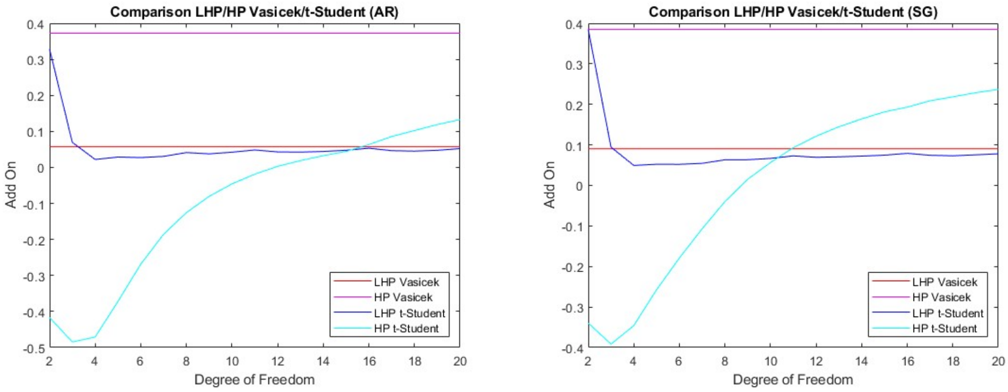

Figure 11: Case 1 k fixed, LGD simulated: comparison of the Add-On considering α = 99.9%

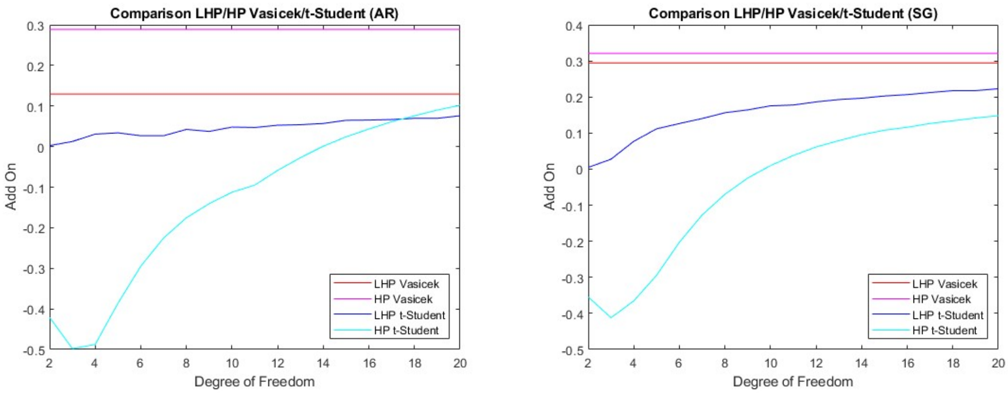

Figure 12: Case 2 LGD fixed, k simulated: comparison of the Add-On considering α = 99.9%

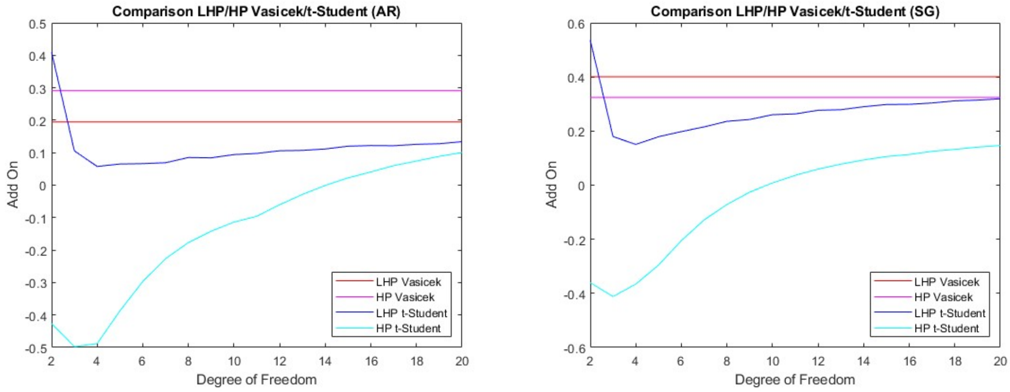

Figure 13: Case 3 k simulated, LGD simulated, independent: comparison of the Add-On considering $\alpha = 99.9\%$

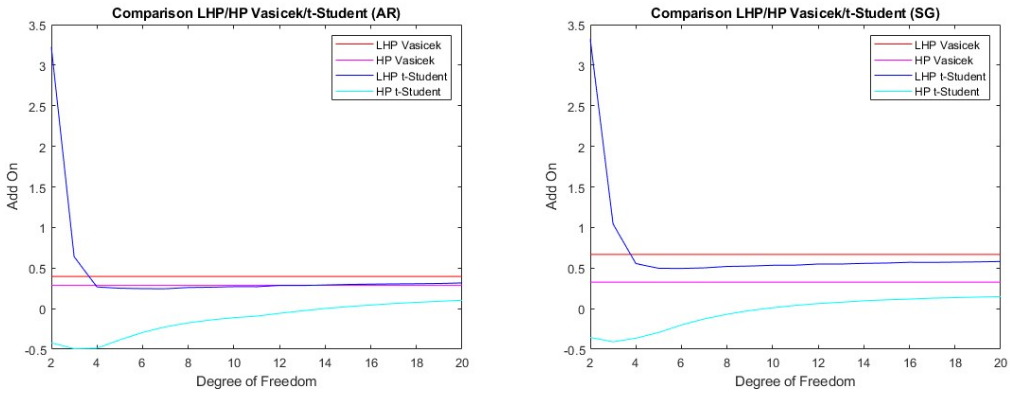

Figure 14: Case 4 k simulated, LGD simulated, correlated: comparison of the Add-On considering $\alpha = 99.9\%$

As we can see from the previous graphs, if we increase the number of degrees of freedom (ν), the results obtained with the Double t-Student approach are quite similar to the results obtained with the Vasicek model. This fact is known because a t-Student distribution tends to a Gaussian one if $\nu \to +\infty$.
On the other hand, from the graphs we observe that there is still not equivalence. For an additional check, we compute the values with $\nu = 100$.

	ν=100	AR	$Vasicek_{AR}$	SG	$Vasicek_{SG}$
LGD	(only)	5.662%	5.788%	8.875%	9.031%
k	(only)	11.739%	12.924%	27.819%	29.409%
LGD,k	(independent)	17.960%	19.372%	38.154%	39.923%
LGD,k	(correlated)	37.332%	39.553%	64.821%	66.800%

Table 12: Comparison between the Add - on computed via Double t- student approach with $\nu = 100$ and $\alpha = 99.9\%$ and the ones computed with the Vasicek approach.

As it is shown in the previous table the results computed with Double t-student approach ($\nu = 100$) are very similar to the ones obtained via Vasicek approach.

6 Conclusions

To sum up, in this paper we prove how to incorporate the uncertainty about the parameters in a large portfolio and we deduce that the most important contribution to the Add-On is due to the dependency between the two parameters (LGD and k). In fact the Add-On for this last case ranges from 39% up to 69%. On the other hand, in the HP case we lose this effect.
In order to forecast our parameters we do some statistical analysis supporting the Normality assumptions requested by the Vasicek model. This procedure is shown in section 2.
After that, we consider the Nominal approach, presented in section 3, which is a biased estimate of the correct Regulatory Capital because it neglects the estimation noise.
The most significant results of section 4 about Robustness check are summarized in the following graphs:

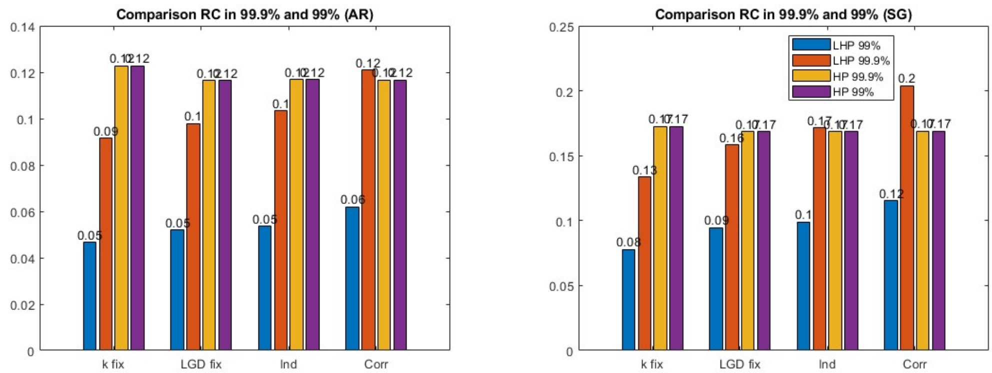

Figure 15: Bar-plots for RC both for HP and LHP approaches considering $\alpha = 99.9\%$ and $\alpha = 99\%$

Here we observe that in the HP case (brown in the Figure) the values are higher than the ones computed in the LHP case (orange in the Figure), apart for the correlated case exception. Moreover the estimations of the LHP case, considering $\alpha = 99.9\%$ (orange in the Figure), are greater than the LHP case with $\alpha = 99\%$ (blue in the Figure).
Regarding the Capital stress check in section 5, we report that, with ν sufficiently large, a Double t-student model tends to the Vasicek one.

Now we try to explain our results from the point of view of the regulator. Taking into account the estimation errors, the Regulatory Capital increases with respect to the Nominal approach in the case of large banks and increases much more for small banks. Indeed, considering a few number of obligors, as it could be for a small bank, the impact of just one obligor defaulting would be higher. This fact leads us to think that asking for higher Capital requirements could be reliable. Nevertheless we point out that an explanation of using the Nominal approach is the presence of a closed formula which implies an easy implementation both for big and small companies. Moreover, the costs of transition for a small bank to use a model that takes into account the estimation error are quite big. To conclude we think that a good solution could be to introduce the *estimation error model* for large banks and to maintain the actual procedure for small banks.

$$PD = \frac{\#Defaulted\ Obligors}{\#Total\ Obligors}$$

$$Loss = LGD \cdot \#Defaulted$$

6.1 Addition after revision

It would have been better to always use the same number of simulations, in particular in the passage from LHP to HP, and the same for the t-student case.
The code has been adjourned in that sense, however due to lack of time, results and plot have not been reloaded.

References

[1] Löffler G., *The effects of estimation error on measures of portfolio credit risk.*, Journal of Banking & Finance, 27 (2003), p. 1427–1453.

[2] Royston J.P., *An Extension of Shapiro and Wilk's W Test for Normality to Large Sample.*, Journal of the Royal Statistical Society. Series C (Applied Statistics), 32 (1983), pp. 121–133.

[3] Tarashev N., *Measuring portfolio credit risk correctly: Why parameter uncertainty matters.*, Journal of Banking & Finance, 34 (2010), p. 2065–2076.

[4] Trujillo-Ortiz A., Hernandez-Walls R., Barba-Rojo K. and Cupul-Magana L., *Roystest:royston's multivariate normality test. a matlab file.* `https://it.mathworks.com/matlabcentral/fileexchange/17811-roystest`, 2007.