POLITECNICO
MILANO 1863

Geoinformatics Engineering

Positioning and Location Based Services Laboratories Report

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Content

1 Description of the laboratory 2

2 Workflow 2

2.1 Convert the coordinates of the origin from ITRF Geodetic to Global Cartesian........ 2

2.2 Convert from body frame (Local Level LL) to Local Cartesian 3

2.3 Convert from Local Cartesian to ITRF Global Cartesian........ 3

2.4 Convert ITRF GC coordinates to ETRF GC coordinates through EPN website........ 3

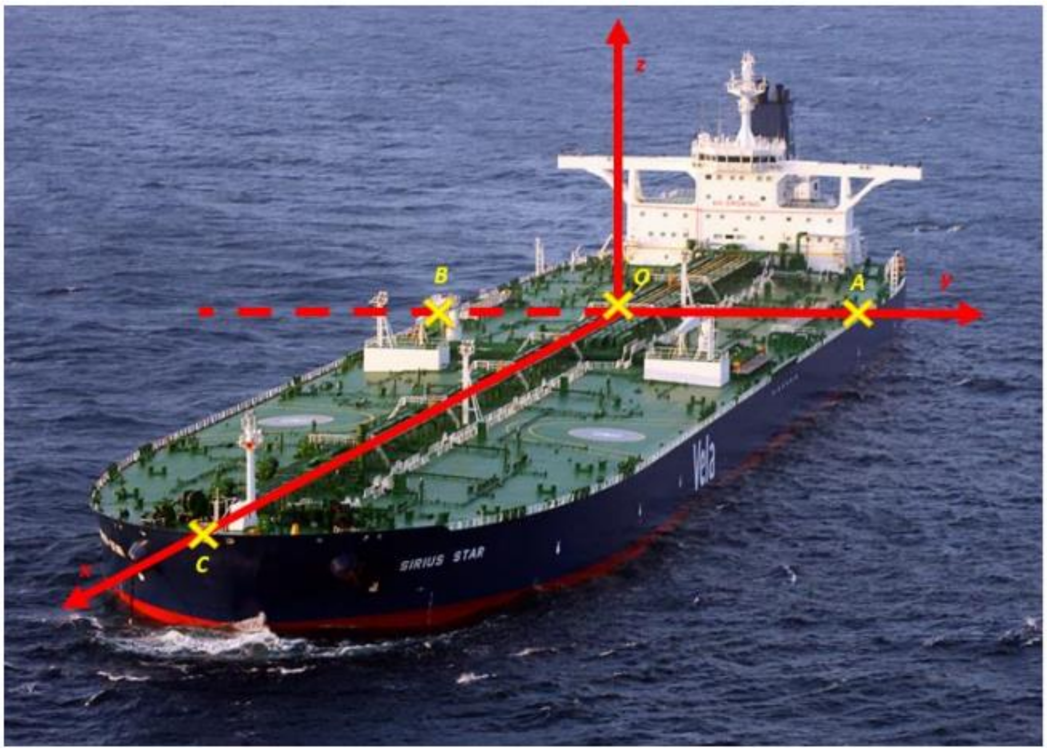
2.5 Convert ETRF GC to Geodetic for A,B,C 4

2.6 Variance propagation........ 4

3.Results 5

1 Description of the laboratory

A container ship is sailing in Genova's port. The ship is equipped with a GNSS receiver, which position is known in ITRF with a certain level of accuracy. The ship has a length of 300 meters and a width of 60. Known the considerable dimensions, a body reference frame is available for navigation purposes: it's origin O is known and placed in correspondence of the GNSS receiver. The X axis is oriented in the motion direction, Z axis is perpendicular to the ship plane and in the up direction, Y axis is oriented to complete the right-handed triad. The position of three points A, B, C of the ship is provided in the body frame.



2 Workflow

2.1 Convert the coordinates of the origin from ITRF Geodetic to Global Cartesian

First of all, we need to convert the coordinates of origin from sexagesimal to decimal,

and then to radian. Then, applying the formula of from geodetic to cartesian.

$$X_P = (R_N + h_P)\cos\varphi_P \cos\lambda_P$$
$$Y_P = (R_N + h_P)\cos\varphi_P \sin\lambda_P$$
$$Z_P = [R_N(1-e^2) + h_P]\sin\varphi_P$$

Where R_N is the curvature radius:

$$R_N = \frac{a}{\sqrt{(1-e^2\sin^2\varphi_P)}}$$

2.2 Convert from body frame (Local Level LL) to Local Cartesian

In order to convert form Local Level to Local Cartesian we need to compute the rotational matrix with the deflection angle with respect to the origin O and the alignment angle between Local East in O and C: Rx(-ξ), Ry(η) and Rx(α). These matrices are needed to define the rotational matrix RLC→LL that transposed provides the required RLC→LL matrix.

$$\boldsymbol{x}_{LL}(P_i) = R_{LC\to LL}\boldsymbol{x}_{LC}(P_i)$$

$$R_{LC\to LL} = R_z(\alpha)\, R_y(\eta) R_x(-\xi)$$

$$R_{LL\to LC} = (R_{LC\to LL})^T$$

$$\boldsymbol{x}_{LC}(P_i) = R_{LL\to LC}\boldsymbol{x}_{LL}(P_i)$$

$$\mathbf{R}_x(-\xi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\xi) & -\sin(\xi) \\ 0 & \sin(\xi) & \cos(\xi) \end{bmatrix}$$

$$\mathbf{R}_y(\eta) = \begin{bmatrix} \cos(\eta) & 0 & -\sin(\eta) \\ 0 & 1 & 0 \\ \sin(\eta) & 0 & \cos(\eta) \end{bmatrix}$$

$$\mathbf{R}_z(\alpha) = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.3 Convert from Local Cartesian to ITRF Global Cartesian

To get the coordinates in Global Cartesian we need to evaluate the rotational matrix beside the origin O.

$$\mathbf{R}_0 = \begin{bmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\sin\varphi_0\cos\lambda_0 & -\sin\varphi_0\sin\lambda_0 & \cos\varphi_0 \\ \cos\varphi_0\cos\lambda_0 & \cos\varphi_0\sin\lambda_0 & \sin\varphi_0 \end{bmatrix}$$

Where:
$\lambda_0 = geodetic\ longitude\ of\ O$
$\varphi_0 = geodetic\ latitude\ of\ O$

And then apply the formula from Local to Global Cartesian.

$$\boldsymbol{x}_{LC}(P_i) = R_0 \Delta\boldsymbol{x}(P_i, P_0)$$

$$\Delta\boldsymbol{x} = (\boldsymbol{x}_{GC}(P_i) - (\boldsymbol{x}_{GC}(P_i))$$

$$R_{GC\to LC} = R_0$$

$$\boldsymbol{x}_{GC}(P_i) = \boldsymbol{x}_{GC}(P_0) + {R_0}^T \boldsymbol{x}_{LC}(P_i)$$

2.4 Convert ITRF GC coordinates to ETRF GC coordinates through EPN website

Using previous results to convert ITRF coordinates in ETRF, choose from ITRF2014 to ETRF2014 on September 1st 2022.

2.5 Convert ETRF GC to Geodetic for A,B,C

First, we should compute the following parameters:

$$e_b^2 = \frac{a^2 - b^2}{b^2}$$

$$r = \sqrt{X^2 + Y^2}$$

$$\psi = \arctan\left(\frac{Z}{r\sqrt{1-e^2}}\right)$$

$$R_N = \frac{a}{\sqrt{(1 - e^2 \sin^2 \varphi_P)}}$$

And then, we can obtain the new angles in geodetic cartesian:

$$\lambda = \arctan\left(\frac{Y}{X}\right)$$

$$\varphi = \arctan\frac{(Z + e_b^2 b \sin^3 \psi)}{(r - e^2 a \cos^3 \psi)}$$

$$h = \frac{r}{\cos\varphi} - R_N$$

where a=major axis=6378137 and e=eccentricity.

2.6 Variance propagation

The estimation of the error is quantified by the variance. The given data are affected by an initial error that propagates through the computation of each transformation that occurs between:

- Local Level and Local Cartesian
- Local Cartesian and Global Cartesian

These are required to obtain the accuracy of the final coordinates.

Starting point:

$$\boldsymbol{C_{LL}}(P_i) = \begin{bmatrix} \sigma_E^2 & 0 & 0 \\ 0 & \sigma_N^2 & 0 \\ 0 & 0 & \sigma_U^2 \end{bmatrix}$$

Covariances in LC:

$$\boldsymbol{C_{LC}}(P_i) = R_{LL\to LC}\ \boldsymbol{C_{LL}}(P_i)\ \ R_{LL\to LC}{}^{T}$$

Covariances in GC:

$$\boldsymbol{C_{GC}}(\Delta\boldsymbol{x}) = R_0{}^{T}\ \boldsymbol{C_{LC}}\ R_0$$
$$\boldsymbol{C_{GC}}(P_i) = \boldsymbol{C_{GC}}(P_O) + \boldsymbol{C_{GC}}(\Delta\boldsymbol{x})$$

Final covariance matrices in $[\varphi, \lambda, h]$

$$C_{[\varphi,\lambda,h]} = R_0(\varphi,\lambda,h)\boldsymbol{C_{GC}}R_0(\varphi,\lambda,h)^{\boldsymbol{T}}$$

Finally, we can get the standard deviation in Latitude, Longitude, h in cm for each point

3 Results

Local cartesian coordinates of points A, B and C in meters:

Point A:

E: -15.206

N: 25.861

U: -0.001

Point B:

E: 15.206

N: -25.861

U: 0.001

Point C:

E: 172.406

N: 101.372

U: -0.013

ITRF global cartesian coordinates of points A, B, C

Point A:

X: 4509854.339

Y: 709345.362

Z: 4439229.142

Point B:

X: 4509885.357

Y: 709381.026

Z: 4439192.183

Point C:

X: 4509772.998

Y: 709522.485

Z: 4439283.094

ETRF global geodetic coordinates of points A, B, C

Point A:

Lat: 44.000 23.000 24.821

Long: 8.000 56.000 19.282

h: 69.998

Point B:

Lat: 44.000 23.000 23.145

Long: 8.000 56.000 20.656

h: 69.999

Point C:

Lat: 44.000 23.000 27.267

Long: 8.000 56.000 27.758

h: 69.988

Standard deviations of points A, B, C in East, North, Up in cm

Point A:

East: 10.2

North: 10.2

Up: 10.2

Point B:

East: 10.2

North: 10.2

Up: 10.2

Point C:

East: 14.1

North: 14.1

Up: 14.1