Corso di Laurea in Fisica

Tutorato di Analisi 1

Integrali indefiniti

Esercizio 1.
Calcolare i seguenti integrali indefiniti immediati e per sostituzione:

1.a) $\int (7-2x)^3\,dx$ $\qquad \left[-\frac{(7-2x)^4}{8}+c\right];$

1.b) $\int \frac{2}{(3-5x)^6}\,dx$ $\qquad \left[\frac{2}{25}(3-5x)^{-5}+c\right];$

1.c) $\int \frac{3}{1+9x^2}\,dx$ $\qquad [\operatorname{arctg}(3x)+c];$

1.d) $\int \frac{1}{\sqrt{1-4x^2}}\,dx$ $\qquad \left[\frac{1}{2}\arcsin(2x)+c\right];$

1.e) $\int \frac{1+\sin x}{(x-\cos x)^3}\,dx$ $\qquad \left[-\frac{1}{2(x-\cos x)^2}+c\right];$

1.f) $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$ $\qquad \left[\log(e^x+e^{-x})+c\right];$

1.g) $\int \sin^2 x\,\cos^3 x\,dx$ $\qquad \left[\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+c\right];$

1.h) $\int \frac{\sin(1-\sqrt{x})}{\sqrt{x}}\,dx$ $\qquad \left[2\cos(1-\sqrt{x})+c\right];$

1.i) $\int \frac{\cos x}{1+\sin^2 x}\,dx$ $\qquad [\operatorname{arctg}(\sin x)+c];$

1.l) $\int \frac{1}{1+\cos x}\,dx$ $\qquad \left[\operatorname{tg}\frac{x}{2}+c\right];$

1.m) $\int \frac{\sin\log x}{x}\,dx$ $\qquad [-\cos(\log x)+c];$

1.n) $\int \frac{3^{\frac{1}{x^3}}}{x^4}\,dx$ $\qquad \left[-\frac{3^{\frac{1}{x^3}}}{3\log 3}+c\right];$

1.o) $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x-1}}\,dx$ $\qquad \left[\frac{2}{3}(e^x-1)^{\frac{3}{2}}+2(e^x-1)^{\frac{1}{2}}+c\right];$

1.p) $\displaystyle\int \frac{2+\sqrt[5]{x}}{\sqrt[5]{x^2}}\,dx$ $\qquad \left[5\left(\frac{2}{3}\sqrt[5]{x^3}+\frac{1}{4}\sqrt[5]{x^4}\right)+c\right];$

1.q) $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx$ $\qquad \left[-\frac{1}{|x|}+c \quad \text{se } a=0\,,\right.$

$$\left. -\frac{1}{a}\arcsin\left(\frac{a}{|x|}\right)+c=\frac{2}{a}\text{arctg}\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)+c' \text{ se } a\neq 0\right].$$

Esercizio 2.
Calcolare i seguenti integrali ricordando il metodo d'integrazone per parti:

2.a) $\displaystyle\int x^2e^{3x}\,dx$ $\qquad \left[\frac{e^{3x}}{3}\left(x^2-\frac{2}{3}x+\frac{2}{9}\right)+c\right];$

2.b) $\displaystyle\int x^3\log\,x\,dx$ $\qquad \left[\frac{x^4}{4}\left(\log x-\frac{1}{4}\right)+c\right];$

2.c) $\displaystyle\int x^3\sin\,x\,dx$ $\qquad \left[-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+c\right];$

2.d) $\displaystyle\int \cos^2 x\,dx$ $\qquad \left[\frac{\sin(2x)}{4}+\frac{x}{2}+c\right];$

2.e) $\displaystyle\int e^{2x}\sin\,x\,dx$ $\qquad \left[\frac{2}{5}\,e^{2x}\sin x-\frac{1}{5}\,e^{2x}\cos x+c\right];$

2.f) $\displaystyle\int \log^2 x\,dx$ $\qquad \left[x\,\log^2 x-2x\log x+2x+c\right];$

2.g) $\displaystyle\int x\text{arctg}x\,dx$ $\qquad \left[\frac{\text{arctg}x}{2}(x^2+1)-\frac{x}{2}+c\right];$

2.h) $\displaystyle\int \arccos x\,dx$ $\qquad \left[x\arccos x-\sqrt{1-x^2}+c\right];$

2.i) $\displaystyle\int \sqrt{x}\log 2x\,dx$ $\qquad \left[\frac{2}{3}\,x^{\frac{3}{2}}\left(\log(2x)-\frac{2}{3}\right)+c\right];$

2.l) $\displaystyle\int (x+1)\log(x+2)\,dx$ $\qquad \left[(\frac{1}{2}x^2+x)\log(x+2)-\frac{x^2}{4}+c\right];$

2.m) $\displaystyle\int \cos\,\log\,x\,dx$ $\qquad \left[\frac{x}{2}\left(\cos(\log x)+\sin(\log x)\right)+c\right];$

2.n) $\int x^3 \log(x+1)\, dx$ $\left[\frac{1}{4}(x^4-1)\log(x+1) - \frac{1}{4}\left(\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x\right) + c\right]$.

Esercizio 3.
Calcolare i seguenti integrali ricordando il metodo della decomposizione in frazioni parziali:

3.a) $\int \frac{2x^2-1}{x^3-2x^2+x-2}\, dx$ $\left[\frac{7}{5}\log|x-2| + \frac{3}{10}\log(x^2+1) + \frac{6}{5}\text{arctg}x + c\right]$;

3.b) $\int \frac{x^2}{x+1}\, dx$ $\left[\frac{(x-1)^2}{2} + \log|x+1| + c\right]$;

3.c) $\int \frac{x+1}{x^3-1}\, dx$ $\left[\frac{1}{3}\log\left(\frac{(x-1)^2}{x^2+x+1}\right) + c\right]$;

3.d) $\int \frac{x^2-2x}{(2x-1)(x^2+1)}\, dx$ $\left[-\frac{3}{10}\log|2x-1| + \frac{2}{5}\log(x^2+1) - \frac{3}{5}\text{arctg}x + c\right]$;

3.e) $\int \frac{5x^2+11x-2}{(x+5)(x^2+9)}\, dx$ $\left[2\log|x+5| + \frac{3}{2}\log(x^2+9) - \frac{4}{3}\text{arctg}\left(\frac{x}{3}\right) + c\right]$;

3.f) $\int \frac{x^3+x+1}{x^4+x^2}\, dx$ $\left[\log|x| - \frac{1}{x} - \text{arctg}x + c\right]$;

3.g) $\int \frac{1}{(x^2-1)^2}\, dx$ $\left[\frac{1}{4}\log\left|\frac{x+1}{x-1}\right| + \frac{x}{2(1-x^2)} + c\right]$;

3.h) $\int \frac{1}{(x^2-1)^3}\, dx$ $\left[\frac{3}{16}\log\left|\frac{x-1}{x+1}\right| + \frac{3x}{8(x^2-1)} - \frac{x}{4(x^2-1)^2} + c\right]$;

3.i) $\int \frac{dx}{(1-x)\sqrt{1+x}}$ $\left[\frac{\sqrt{2}}{2}\log\left(\frac{\sqrt{1+x}+\sqrt{2}}{|\sqrt{1+x}-\sqrt{2}|}\right) + c\right]$.