

# KILL: Knowledge Injection via Lambda Layer

Matteo Magnini\*    Giovanni Ciatto\*    Andrea Omicini\*

\*Dipartimento di Informatica – Scienza e Ingegneria (DISI)  
Alma Mater Studiorum – Università di Bologna  
{matteo.magnini, giovanni.ciatto, andrea.omicini}@unibo.it

WOA 2022:  
Workshop “From Objects to Agents”  
September 1–2, 2022, Genova, Italy



Next in Line...

- 1 Premises on Symbolic Knowledge Injection
- 2 KILL: Knowledge Injection via Lambda Layer
- 3 Case study
- 4 Discussion and future works



# Symbolic Knowledge Injection I

**Definition** [Besold et al., 2017, Xie et al., 2019, Calegari et al., 2020]

Symbolic Knowledge Injection (SKI) can be defined as:

*any algorithmic procedure affecting how sub-symbolic predictors draw their inferences in such a way that predictions are either computed as a function of, or made consistent with, some given symbolic knowledge.*



# Symbolic Knowledge Injection II

## Symbolic knowledge

A symbolic representation consists of: [van Gelder, 1990]

- ① a set of symbols;
- ② a set of grammatical rules governing the combining of symbols;
- ③ elementary symbols and any admissible combination of them can be assigned with meaning.
  - ⇒ Symbolic knowledge is both human and machine interpretable,
  - first order logic (FOL) is an example of symbolic representation.



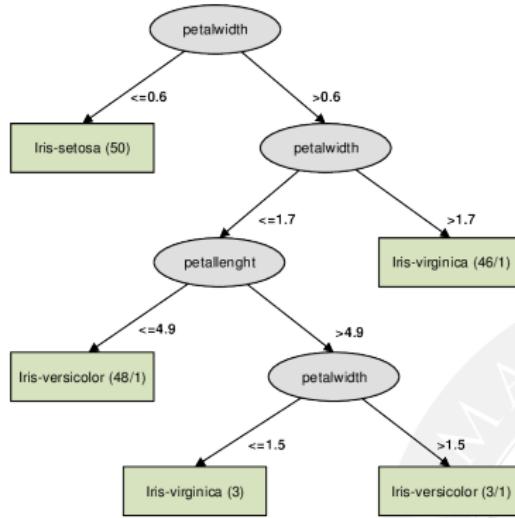
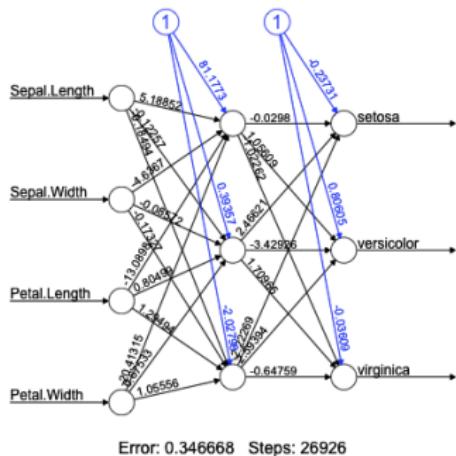
# Symbolic Knowledge Injection III

## Sub-symbolic predictors

- deep neural networks (DNN);
  - convolutional neural networks (CNN),
  - recurrent neural networks (RNN);
- kernel machines;
- basically everything that is sub-symbolic (models consisting of vectors, tensors, etc. of real numbers with no meaning for a human).



# Symbolic Knowledge Injection IV



# Symbolic Knowledge Injection V

Set of propositional logic rules built from the previous decision tree:

*iris(SepalLength, SepalWidth, PetalLength, PetalWidth, setosa)*:-  
    *PetalWidth* <= 0.6.

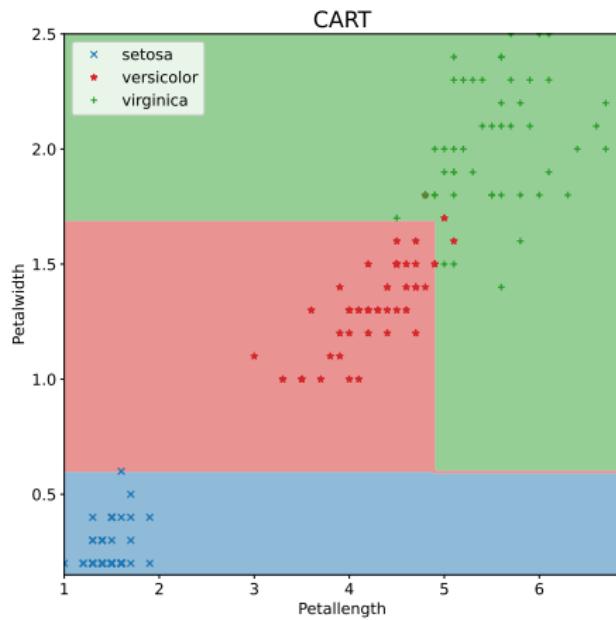
*iris(SepalLength, SepalWidth, PetalLength, PetalWidth, versicolor)*:-  
    *PetalWidth* > 0.6, *PetalWidth* <= 1.7, *PetalLength* <= 4.9.

*iris(SepalLength, SepalWidth, PetalLength, PetalWidth, virginica)*:-  
    *PetalWidth* > 0.6, *PetalWidth* <= 1.5, *PetalLength* > 4.9.

*iris(SepalLength, SepalWidth, PetalLength, PetalWidth, versicolor)*:-  
    *PetalWidth* > 1.5, *PetalWidth* <= 1.7, *PetalLength* > 4.9.

*iris(SepalLength, SepalWidth, PetalLength, PetalWidth, virginica)*:-  
    *PetalWidth* > 1.7.

# Symbolic Knowledge Injection VI



# Symbolic Knowledge Injection VII

## Why?

There are several benefits:

- prevent the predictor to become a black-box!;
- reduce learning time;
- reduce the data size needed for training;
- improve predictor's accuracy;
- build a predictor that behave as a logic engine.



# Symbolic Knowledge Injection VIII

Explainability can be achieved: [Gunning, 2016]

## Post-hoc explanation

- applying an algorithm of symbolic knowledge extraction on a trained predictor;
- output → logic rules that describe the predictor's behaviour.

## By design

- constraining the behaviour of predictors that are natively black-boxes with symbolic knowledge;
- structuring the predictor's architecture with symbolic knowledge;
- output → a predictor that does not violate the prior knowledge.

# Symbolic Knowledge Injection IX

## How?

There exist three major ways to perform knowledge injection on sub-symbolic predictors:

- **constraining**, a cost factor proportional to the violation of the knowledge is introduced during learning;
- **structuring**, the architecture of the predictor is built in such a way to mimic the knowledge;
- **embedding**, the symbolic knowledge is embedded into a tensor form and it is given in input as training data to the predictor.



# Next in Line...

- 1 Premises on Symbolic Knowledge Injection
- 2 KILL: Knowledge Injection via Lambda Layer
- 3 Case study
- 4 Discussion and future works



# Algorithm I

## KILL: Knowledge Injection via Lambda Layer

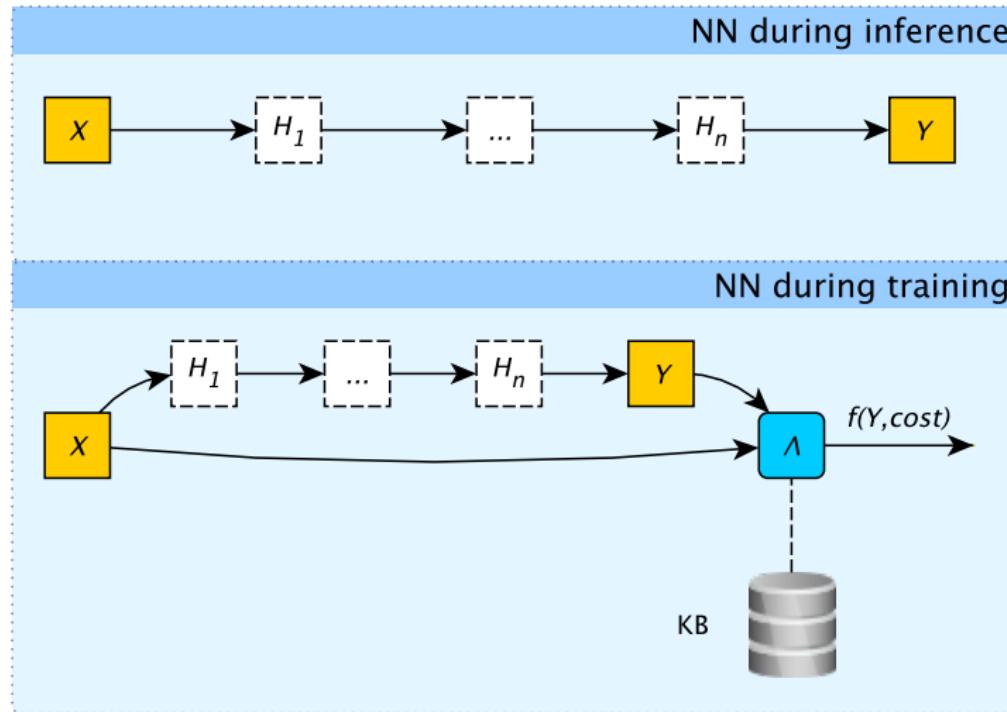
A general SKI algorithm that does not impose constraints on the sub-symbolic predictor to enrich, except being a neural network.

- aim → enrich;
- predictor → neural network;
- how → constraining;
- logic → stratified Datalog with negation.

Public implementation on PSyKI. [Magnini et al., 2022]



# Algorithm II



# Algorithm III

Formula	C. interpretation	Formula	C. interpretation
$\llbracket \neg \phi \rrbracket$	$\eta(1 - \llbracket \phi \rrbracket)$	$\llbracket \phi \leq \psi \rrbracket$	$\eta(\llbracket \phi \rrbracket - \llbracket \psi \rrbracket)$
$\llbracket \phi \wedge \psi \rrbracket$	$\eta(\max(\llbracket \phi \rrbracket, \llbracket \psi \rrbracket))$	$\llbracket \text{class}(\bar{X}, y_i) \leftarrow \psi \rrbracket$	$\llbracket \psi \rrbracket^*$
$\llbracket \phi \vee \psi \rrbracket$	$\eta(\min(\llbracket \phi \rrbracket, \llbracket \psi \rrbracket))$	$\llbracket \text{expr}(\bar{X}) \rrbracket$	$\text{expr}(\llbracket \bar{X} \rrbracket)$
$\llbracket \phi = \psi \rrbracket$	$\eta( \llbracket \phi \rrbracket - \llbracket \psi \rrbracket )$	$\llbracket \text{true} \rrbracket$	0
$\llbracket \phi \neq \psi \rrbracket$	$\llbracket \neg(\phi = \psi) \rrbracket$	$\llbracket \text{false} \rrbracket$	1
$\llbracket \phi > \psi \rrbracket$	$\eta(0.5 - \llbracket \phi \rrbracket + \llbracket \psi \rrbracket)$	$\llbracket X \rrbracket$	x
$\llbracket \phi \geq \psi \rrbracket$	$\eta(\llbracket \psi \rrbracket - \llbracket \phi \rrbracket)$	$\llbracket k \rrbracket$	k
$\llbracket \phi < \psi \rrbracket$	$\eta(0.5 + \llbracket \phi \rrbracket - \llbracket \psi \rrbracket)$	$\llbracket p(\bar{X}) \rrbracket^{**}$	$\llbracket \psi_1 \vee \dots \vee \psi_k \rrbracket$

\* encodes the penalty for the  $i^{th}$  neuron

\*\* assuming predicate  $p$  is defined by  $k$  clauses of the form:

$$p(\bar{X}) \leftarrow \psi_1, \dots, p(\bar{X}) \leftarrow \psi_k$$

# Algorithm IV

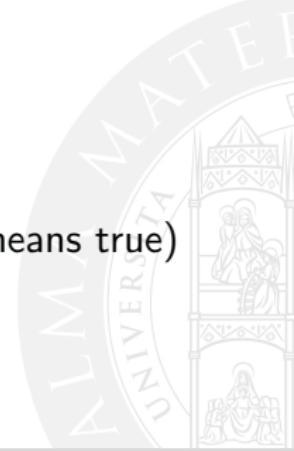
## Cost function

Whenever the neural network wrongly predicts a class and violates the prior knowledge a cost proportional to the violation is added. In this way the output of the network differs more from the expected one and this affects the back propagation step.

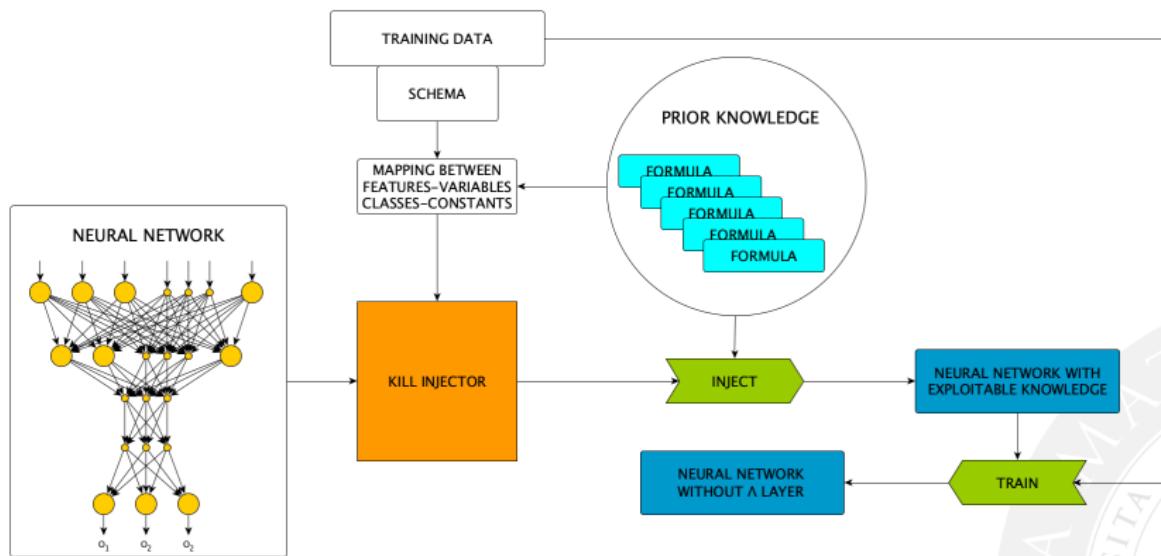
$$Y' = f(Y, \text{cost})$$

$$f = Y \times (1 + \text{cost})$$

$$\text{cost}(X, Y) = \eta(p(X) - (1 - Y)) \quad (1 - Y \text{ because } 0 \text{ means true})$$



# Algorithm V



# Next in Line...

- 1 Premises on Symbolic Knowledge Injection
- 2 KILL: Knowledge Injection via Lambda Layer
- 3 Case study
- 4 Discussion and future works



# Case study I

## PHDS: Poker Hand Data Set

Each record represents one poker hand. 5 cards identified by 2 values: suit and rank. Classes: 10. Training set: 25.010. Test set: 1.000.000.

<b>id</b>	<b>S1</b>	<b>R1</b>	<b>S2</b>	<b>R2</b>	<b>S3</b>	<b>R3</b>	<b>S4</b>	<b>R4</b>	<b>S5</b>	<b>R5</b>	<b>class</b>
1	1	10	1	11	1	13	1	12	1	1	9
2	2	11	2	13	2	10	2	12	2	1	9
3	3	12	3	11	3	13	3	10	3	1	9
4	4	10	4	11	4	1	4	13	4	12	9
5	4	1	4	13	4	12	4	11	4	10	9
6	1	2	1	4	1	5	1	3	1	6	8
7	1	9	1	12	1	10	1	11	1	13	8
8	2	1	2	2	2	3	2	4	2	5	8
9	3	5	3	6	3	9	3	7	3	8	8
10	4	1	4	4	4	2	4	3	4	5	8
11	1	1	2	1	3	9	1	5	2	3	1
12	2	6	2	1	4	13	2	4	4	9	0
13	1	10	4	6	1	2	1	1	3	8	0
14	2	13	2	1	4	4	1	5	2	11	0
15	3	8	4	12	3	9	4	2	3	2	1

# Case study II

Some injected rules.

Class	Logic Formulation
Pair	$\text{class}(R_1, \dots, S_5, \text{pair}) \leftarrow \text{pair}(R_1, \dots, S_5)$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_1 = R_2$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_1 = R_3$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_1 = R_4$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_1 = R_5$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_2 = R_3$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_2 = R_4$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_2 = R_5$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_3 = R_4$
	$\text{pair}(R_1, \dots, S_5) \leftarrow R_3 = R_5$

# Case study III

Two Pairs

```

----- class(R1, ..., S5, two) ← two(R1, ..., S5)
two(R1, ..., S5) ← R1 = R2 ∧ R3 = R4
two(R1, ..., S5) ← R1 = R3 ∧ R2 = R4
two(R1, ..., S5) ← R1 = R4 ∧ R2 = R3
two(R1, ..., S5) ← R1 = R2 ∧ R3 = R5
two(R1, ..., S5) ← R1 = R3 ∧ R3 = R5
two(R1, ..., S5) ← R1 = R5 ∧ R2 = R3
two(R1, ..., S5) ← R1 = R2 ∧ R4 = R5
two(R1, ..., S5) ← R1 = R4 ∧ R2 = R5
two(R1, ..., S5) ← R1 = R5 ∧ R2 = R4
two(R1, ..., S5) ← R1 = R3 ∧ R4 = R5
two(R1, ..., S5) ← R1 = R4 ∧ R3 = R5
two(R1, ..., S5) ← R1 = R5 ∧ R3 = R4
two(R1, ..., S5) ← R2 = R3 ∧ R4 = R5
two(R1, ..., S5) ← R2 = R4 ∧ R3 = R5
two(R1, ..., S5) ← R2 = R5 ∧ R3 = R4
-----
```

# Case study IV

	$\text{class}(R_1, \dots, S_5, \text{three}) \leftarrow \text{three}(R_1, \dots, S_5)$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_3$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_4$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_5$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_1 = R_3 \wedge R_1 = R_4$
Three of a Kind	$\text{three}(R_1, \dots, S_5) \leftarrow R_1 = R_3 \wedge R_1 = R_5$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_1 = R_4 \wedge R_1 = R_5$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_2 = R_3 \wedge R_2 = R_4$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_2 = R_3 \wedge R_2 = R_5$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_2 = R_4 \wedge R_2 = R_5$
	$\text{three}(R_1, \dots, S_5) \leftarrow R_3 = R_4 \wedge R_3 = R_5$
Flush	$\text{class}(R_1, \dots, S_5, \text{flush}) \leftarrow \text{flush}(R_1, \dots, S_5)$
	$\text{flush}(R_1, \dots, S_5) \leftarrow S_1 = S_2 \wedge S_1 = S_3 \wedge S_1 = S_4 \wedge S_1 = S_5$
	$\text{class}(R_1, \dots, S_5, \text{four}) \leftarrow \text{four}(R_1, \dots, S_5)$
	$\text{four}(R_1, \dots, S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_3 \wedge R_1 = R_4$
	$\text{four}(R_1, \dots, S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_3 \wedge R_1 = R_5$
	$\text{four}(R_1, \dots, S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_4 \wedge R_1 = R_5$
	$\text{four}(R_1, \dots, S_5) \leftarrow R_1 = R_3 \wedge R_1 = R_4 \wedge R_1 = R_5$
Four of a Kind	$\text{four}(R_1, \dots, S_5) \leftarrow R_2 = R_3 \wedge R_2 = R_4 \wedge R_2 = R_5$

# Case study V

## Setup

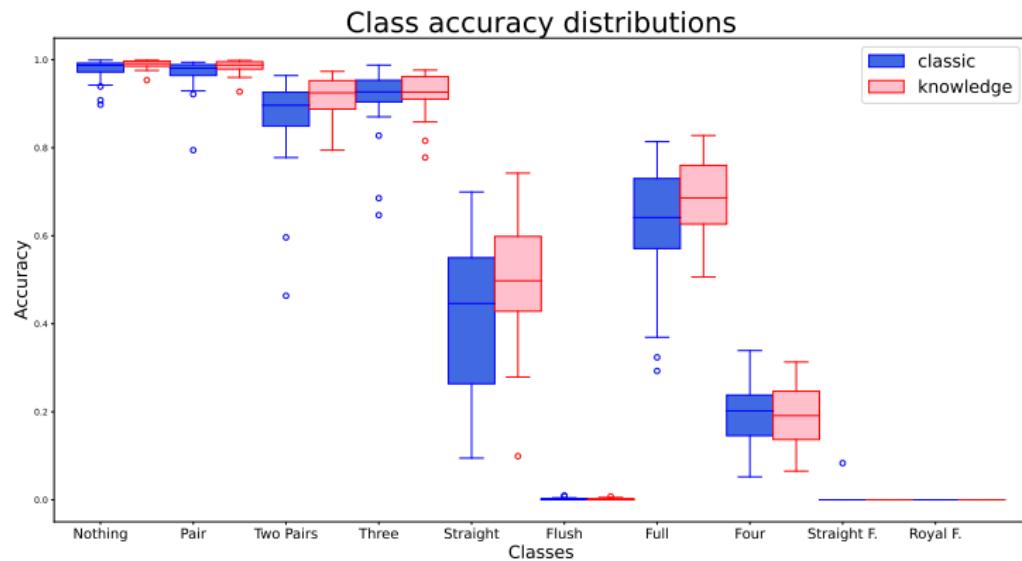
- neural network: 3-layers fully connected (128, 128, 10 neurons per layer respectively) with rectified linear unit (ReLU) as activation function, except for the last layer (softmax);
- knowledge: see previous slides;
- categorical cross-entropy as loss function
- training: Adams as optimiser for 100 epochs (with early stop conditions);
- experiment repeated 30 times to have a statistic significant population.



# Case study VI

Metric	Classic	KILL	Metric	Classic	KILL
Accuracy	0.962	0.978	Acc. Straight	0.415	0.509
Macro-F1	0.512	0.538	Acc. Flush	0.002	0.002
Weighted-F1	0.96	0.977	Acc. Full	0.628	0.69
Acc. Nothing	0.977	0.989	Acc. Four	0.186	0.19
Acc. Pair	0.968	0.985	Acc. Straight F.	0.003	0
Acc. Two Pairs	0.867	0.914	Acc. Royal F.	0	0
Acc. Three	0.913	0.922			

## Case study VII



# Next in Line...

- 1 Premises on Symbolic Knowledge Injection
- 2 KILL: Knowledge Injection via Lambda Layer
- 3 Case study
- 4 Discussion and future works



# Discussion and future works I

## Discussion

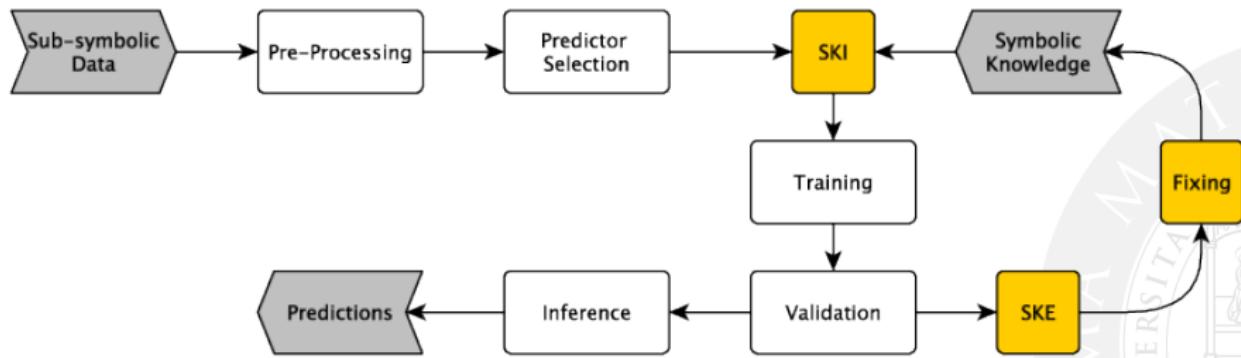
- the case study shows that KILL can be applied with success when the injected knowledge is correct;
- predictions for less represented classes are not improved as much as for more frequent ones (this is very likely a common feature for all SKI algorithms based on constraining).



# Discussion and future works II

## Future works

- explore scenarios where the injected knowledge is not perfectly correct;
- test the performances of KILL inside the train-extract-fix-inject workflow.



# KILL: Knowledge Injection via Lambda Layer

Matteo Magnini\*    Giovanni Ciatto\*    Andrea Omicini\*

\*Dipartimento di Informatica – Scienza e Ingegneria (DISI)  
Alma Mater Studiorum – Università di Bologna  
{matteo.magnini, giovanni.ciatto, andrea.omicini}@unibo.it

WOA 2022:  
Workshop “From Objects to Agents”  
September 1–2, 2022, Genova, Italy



# References |

[Besold et al., 2017] Besold, T. R., d'Avila Garcez, A. S., Bader, S., Bowman, H., Domingos, P. M., Hitzler, P., Kühnberger, K., Lamb, L. C., Lowd, D., Lima, P. M. V., de Penning, L., Pinkas, G., Poon, H., and Zaverucha, G. (2017).

Neural-symbolic learning and reasoning: A survey and interpretation.

*CoRR*, abs/1711.03902

<http://arxiv.org/abs/1711.03902>.

[Calegari et al., 2020] Calegari, R., Ciatto, G., and Omicini, A. (2020).

On the integration of symbolic and sub-symbolic techniques for XAI: A survey.

*Intelligenza Artificiale*, 14(1):7–32

DOI:10.3233/IA-190036.

[Gunning, 2016] Gunning, D. (2016).

Explainable artificial intelligence (XAI).

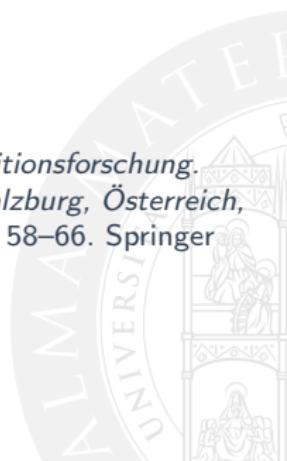
Funding Program DARPA-BAA-16-53, Defense Advanced Research Projects Agency (DARPA)

<http://www.darpa.mil/program/explainable-artificial-intelligence>.



# References II

- [Magnini et al., 2022] Magnini, M., Ciatto, G., and Omicini, A. (in press, 2022).  
On the design of psyki: A platform for symbolic knowledge injection into sub-symbolic predictors.  
In Calvaresi, D., Najjar, A., Winikoff, M., and Främling, K., editors, *Explainable and Transparent AI and Multi-Agent Systems - Fourth International Workshop, EXTRAAMAS 2022, Virtual Event, May 9-10, 2022, Revised Selected Papers*, Lecture Notes in Computer Science. Springer  
<https://apice.unibo.it/xwiki/bin/view/Publications/PsykiExtraamas2022>.
- [van Gelder, 1990] van Gelder, T. (1990).  
Why distributed representation is inherently non-symbolic.  
In Dorffner, G., editor, *Konnektionismus in Artificial Intelligence und Kognitionsforschung. Proceedings 6. Österreichische Artificial Intelligence-Tagung (KONNAI), Salzburg, Österreich, 18. bis 21. September 1990*, volume 252 of *Informatik-Fachberichte*, pages 58–66. Springer  
DOI:10.1007/978-3-642-76070-9\_6.



# References III

[Xie et al., 2019] Xie, Y., Xu, Z., Meel, K. S., Kankanhalli, M. S., and Soh, H. (2019). Embedding symbolic knowledge into deep networks.

In Wallach, H. M., Larochelle, H., Beygelzimer, A., d'Alché-Buc, F., Fox, E. B., and Garnett, R., editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 4235–4245

<https://proceedings.neurips.cc/paper/2019/hash/7b66b4fd401a271alc7224027ce111bc-Abstract.html>.

