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Microfluidics and diffusion

Introduction

The objectives of this practical is to analyze diffusion figures of two miscible liquids in a microfluidic channel.

The first part of this work consists in understanding the flow profile in microfluidic channels, based on typical Navier Stokes equations.

This will allow you to understand better what happens in the system you'll be working on in the laboratory. The setup will be focused on the flow inside a T-shape microfluidic channel to study liquid diffusion in each other. Here, the two liquids are clear water and dyed water to easily observe the diffusion phenomenon between both layers.

A whole experimental setup shall be built around it to flow the various liquids.

Basic Fluid Mechanics and Microfluidics

The first part of this work consists in setting up the equations for microfluidics flow in a rectangular shaped channel. This will allow a better understanding of the velocity and pressure profiles inside the chip.

The governing equations comprise the Navier-Stokes and continuity equations:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_i \partial x_i} + \frac{1}{\rho} f_i \qquad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho U_i}{\partial x_i} = 0 \qquad (2)$$

Under some simplifying assumptions, we can deduce important theoretical results for microfluidic flow, e.g., the flow can be assumed steady and two-dimensional (e.g., $(\vec{x}, \vec{z})$ plane).

Question

Assuming a rectangular channel of height 'H' and length L_c show that the shear terms $(U_j \frac{\partial U_i}{\partial x_i})$ in the Navier-Stokes equations vanish whenever L_c>>H.

Hint: Use the continuity equation coupled with a scaling analysis

Since without shear, turbulence cannot be sustained, we can assume the flow to be laminar. A dimensionless quantity, the Reynolds number (Re), i.e., the ratio of inertial to viscous forces, often helps determine whether a flow is turbulent or laminar as a function of the density, channel mean velocity, a characteristic length scale (for instance, the channel's height), and the molecular viscosity:

$$Re = \frac{\rho U l}{\nu} \qquad (3)$$

Question

When do you expect the flow to be turbulent, at high or low Re? Give an order of magnitude of Re for water flowing in microfluidic channels.

Hint: Think about honey versus water. Which one is more viscous? Which one do you expect to be more or less turbulent for a given velocity?

Using the Navier-Stokes equations, we can determine an analytical expression for the velocity profile within the microfluidic channel. Simplifying assumptions typically include ideal no-slip conditions at the liquid-channel interface, i.e., the fluid velocity at the channel's wall vanishes to zero.

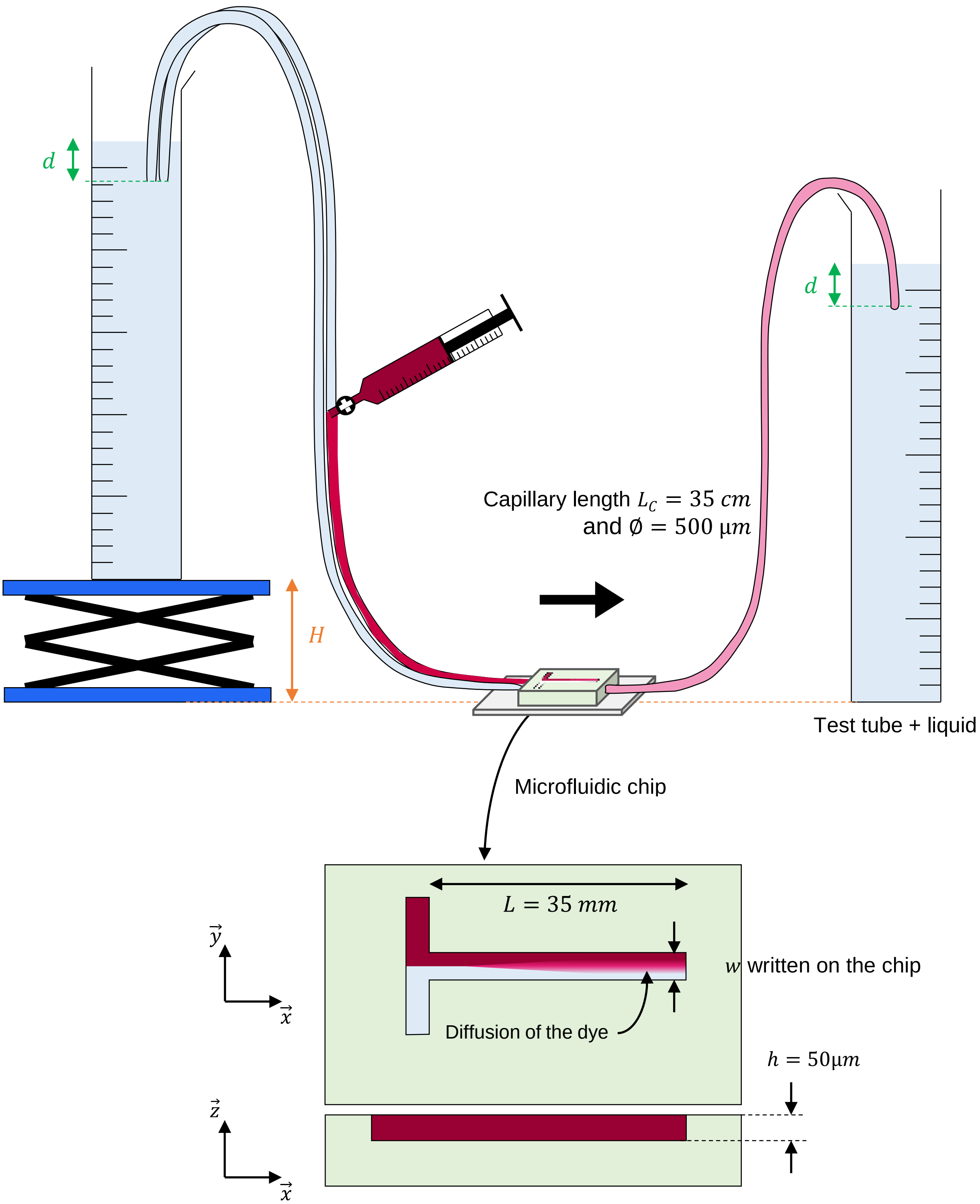
Question

Derive the velocity profile in a rectangular microfluidic channel of half-height 'h_m'. How would the expression change for a cylindrical channel of radius 'R'?

Hint: For the rectangular channel, start by writing the momentum balance in the x-direction.

Experimental setup

Fluidic setup



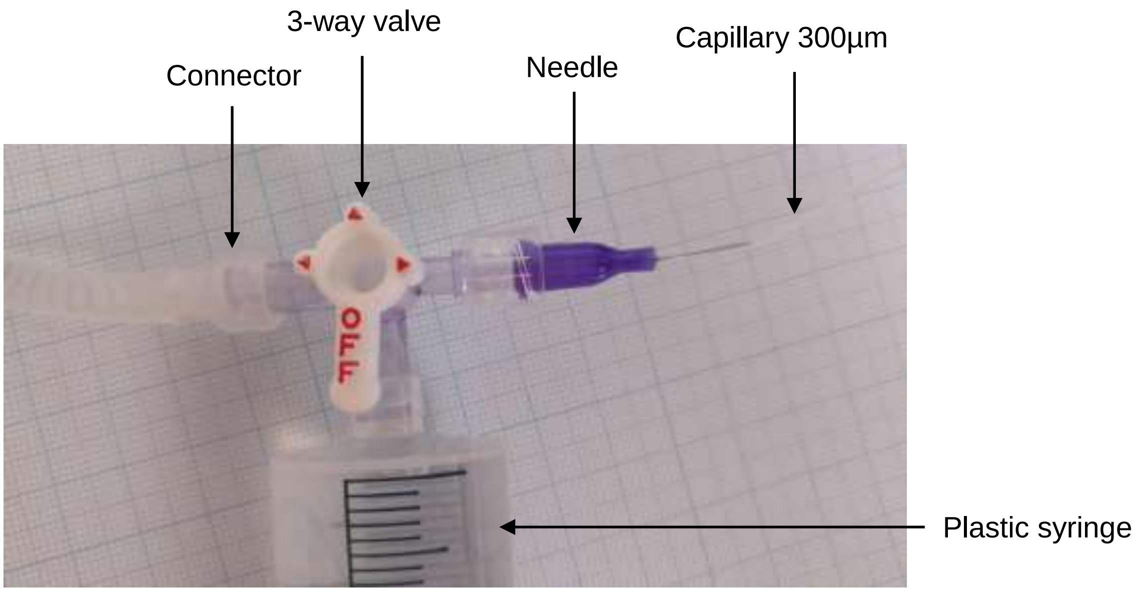


Image recording

You have a camera to record movies and/or take pictures from your functioning device.

<u>Station 1</u>: turn the computer on. Plug both USB connectors, first the lamp and only then the camera. Open the Digital Viewer software and run the live acquisition mode.

<u>Station 2</u>: plug the camera and lamp, and open the Labview acquisition software. Press the green button for live acquisition.

<u>Station 3</u>: turn the computer on. Plug both USB connectors, first the lamp and only then the camera. Open the DinoCapture software and run the live acquisition mode.

<u>Station 4</u>: open the uEye software. Chose the “Monochrome” program and run the live acquisition mode. Set up the number of frames per second at the minimum.

Preliminary questions

These questions are about the setup introduced in page 3.

Question

What is the pressure drop along the system (chip + capillaries)?

Help: the pressure drop happening in the capillaries is negligible as compared to the chip.

The liquid injection will be driven by the pressure drop between the inlets and the outlet of the chip.

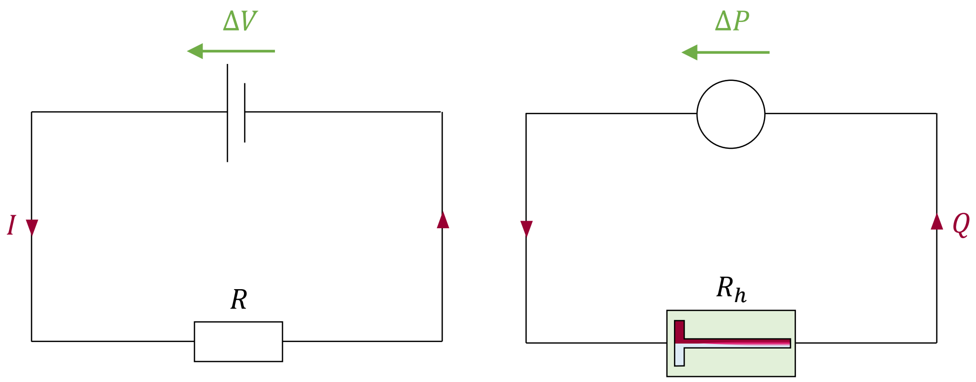
Question

What happens if you lift the test tube? If you lower it? How can you explain it?

Pressure drop and hydraulic resistance

Use the tools at your disposal to develop the experimental setup. Use an additional syringe to force water into the capillaries and air out of them. Fill the second syringe with 10 mL of dyed water.

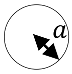
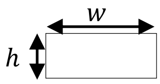
The hydraulic resistance R_h of a channel is its capacity to let liquid flow through it; it depends on the liquid section (h, w) and the fluid viscosity. This is analogous to electric circuits; in the case of a typical "Poiseuille" flow, one can compare these two simple setups:



Question

Based on this analogy, how can you express the flowrate Q as a function of the pressure drop and the hydraulic resistance?

The hydraulic resistance depends on the dynamic viscosity μ of the liquid and the dimensions of the channel. Its expression is also different depending on the channel's shape:

CHANNEL SHAPE		EXPRESSION OF R_h
Round		$R_h = \frac{8\,\mu\,L_C}{\pi a^4}$
Rectangle		$R_h = \frac{12\,\mu\,L_C}{wh^3(1-0.63\,h/w)}$

It is possible to find again these expression using the expression of the velocity profile you calculated earlier.

Question

Using the equations you wrote in the first part of this work, explain how you obtain the expression for the hydraulic resistance, as presented in the table above, for a cylindrical channel.

<u>Hint</u>: Find the flow rate from the velocity profile previously derived. Then, by inspection, find R_h.

Question

Express and give an approximate value of your microfluidic chip hydraulic resistance R_h.

<u>NB</u>: water dynamic viscosity at 20°C is $\mu = 10^{-3} Pa.s$

Question

In a linear Poiseuille flow, what is the expression of the mean fluid speed V as a function of Q?

Express V as a function of ΔP and the channels dimensions.

Analysis of the diffusion process

If you have a closer look at the interface between the two liquids, you see that it gets more and more blurred along the channel. This is due to a diffusion phenomenon from dyed and clear water solutions in each other.

The equation governing the diffusion phenomenon, the second Fick's law, focuses on the partial derivative functions of the diffusing species concentration C:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

Question

Starting from Fick's law equation, give an order of magnitude of the diffusion coefficient D as a function of time and diffusion length L_D.

Question

Knowing the mean speed V of the fluid, express L_D as a function of the position x of the analyzed section and of the applied pressure ΔP and the channel dimensions.

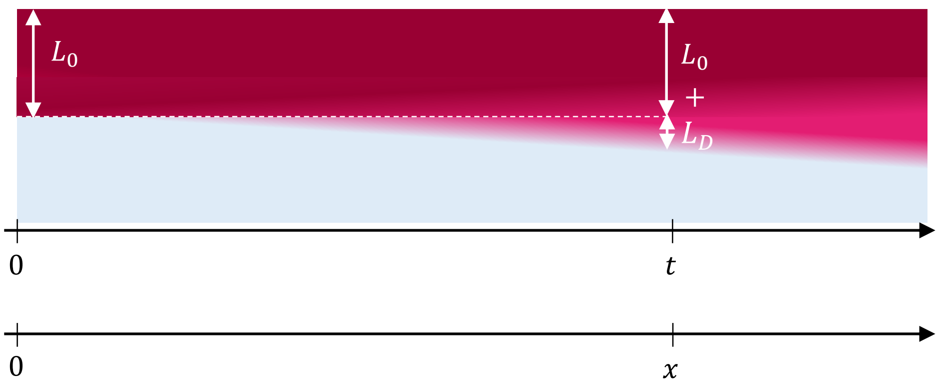
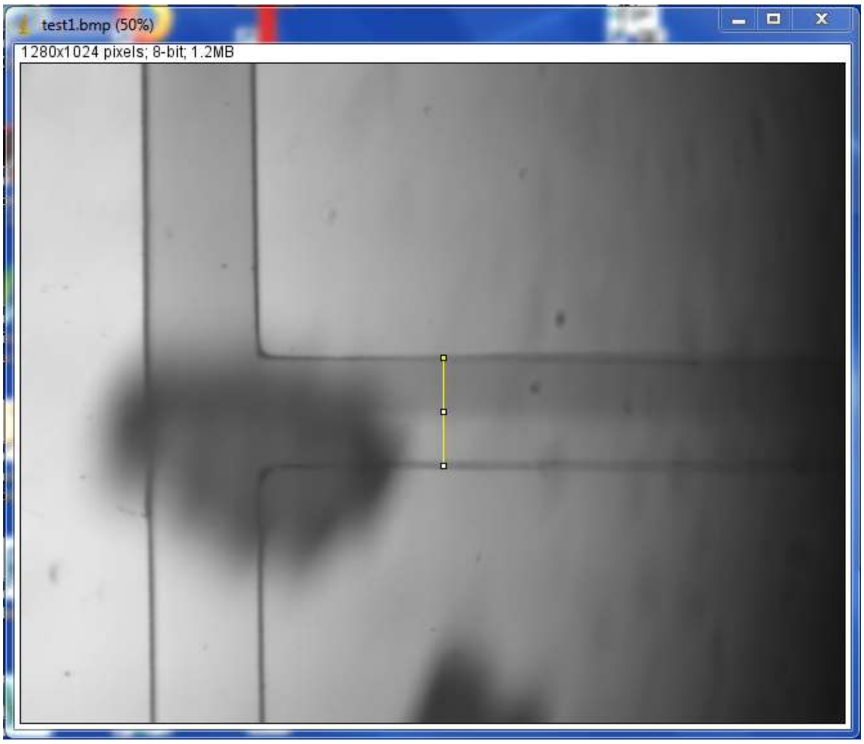
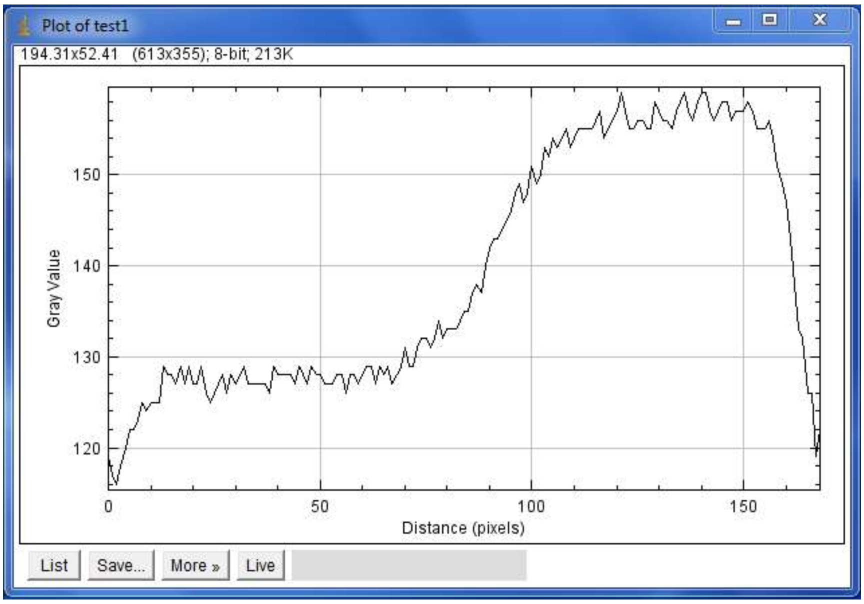


Image analysis

Use the ImageJ / Fiji software to analyze the snapshots you recorded from the camera. By turning your image to a grayscale picture, and tracing the line across your channel and selecting Analyze > Plot profile, you can measure the value of the section pixels.





Question

Use the camera to take pictures of the channel and the diffusion figure for different x values.

Use the ImageJ / Fiji software to analyze the snapshots / video.

Plot the $L = f(x)$ graph you got from the experiments.

Give an approximate value for the diffusion coefficient of dyed water in clear water. Comment your result.

End of practical work