MLE Estimation for Bitcoin Processes

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1 Introduction

In the last years, a rise in the volume of trade in cryptocurrencies has been observed as part of the wider phenomenon of the technological revolution of global finance. This determined interest in new portfolio opportunities that include assets in cryptocurrencies. The high volatility that cryptocurrencies typically possess leads many to question their stability and to associate them with potential bubbles. The collapse of the DAO in a matter of months after its launch in 2016 epitomises their fragility.

The first decentralised digital currency was Bitcoin (BTC), and it has remained hugely popular since its inception in 2009. Numerous studies have been undertaken concerning the modelling of BTC prices. In 2013, Kristoufek [3] proposed a bivariate Vector-AutoRegression (VAR) model to fit weekly log returns of Bitcoin prices. In 2017, Hayes [2] used a cross-section dataset consisting of 66 digital currencies to perform a regression to understand the driver behind the prices of cryptocurrencies. Most recently, in 2020, Hou et al. [1] successfully explored a SVCJ model for daily bitcoin prices.

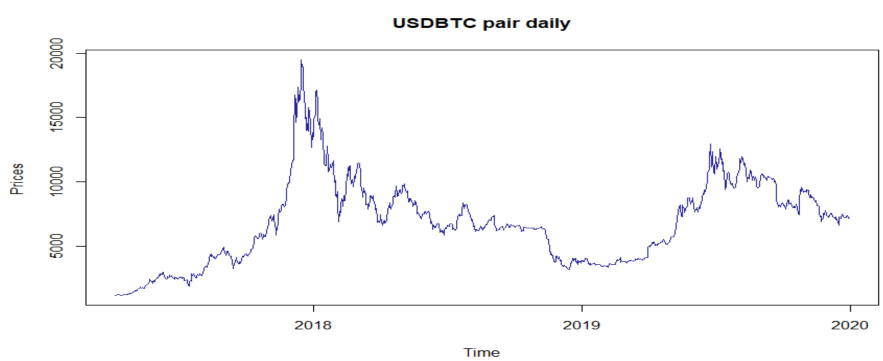
However, it has yet to be determined in the literature how such models perform over differing time frequencies. This is exactly what we seek to do in this paper. The models we shall compare include popular diffusion and jump-diffusion models. The frequencies we research are minute, hourly, and daily data. Our results show that general diffusion models fit better than jump models for minute-by-minute prices. This is consistent with stock pricing models at minute frequencies. The converse is true for lower frequency data, with jump models such as Jump CIR and Pienaar [4] better explaining the data.

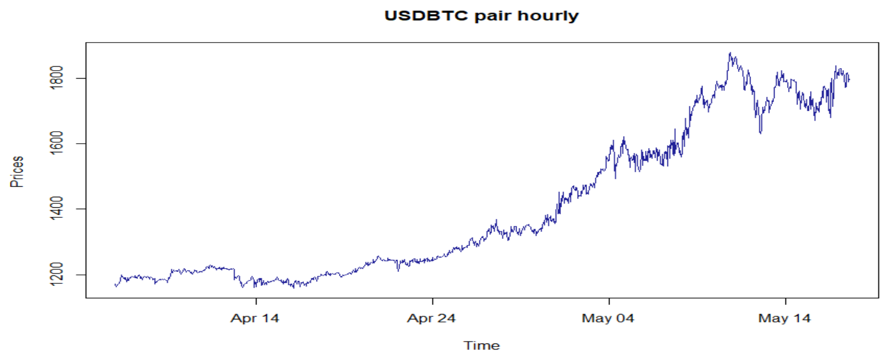
2 Data

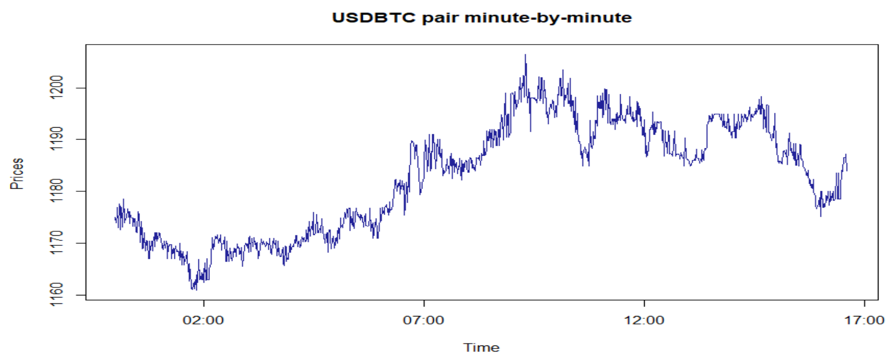
We import daily, hourly and minute-by-minute prices for USD-Bitcoin pair from SFOX (San Francisco Open Exchange), a cryptocurrency prime dealer. The data is structured as OHLC candlesticks. For our analysis we take into consideration the closing price at the end of the period. These are aggregated from their network of liquidity providers which comprises both public and OTC desks. The prices span 1000 datapoints whether days, hours or minutes after 5 April 2017 24:00.

Table 1: Data summary

Data period	End	Min	1st Quarter	Median	Mean	3rd Quarter	Max
day	31/12/2019 24:00	1162	3988	6633	6795	8698	19500
hour	17/5/2017 16:00	1158	1208	1309	1404	1587	1879
minute	6/4/2017 16:40	1161	1172	1185	1183	1192	1206







3 Methodology

In our analysis we compare five different selcted types of financial models from two broad categories: pure diffusion and jump-diffusion models. Using CRAN package Diffusion6Rjgqd [4], we run our estimation by maximising the likelihood via the random walk Metropolis-Hastings algorithm (RWMH). In the case of Jump-Diffusion models the function employed will attempt to decode and estimate the probability of each transition containing a jump before proceeding with the estimate.

3.1 Diffusion Models

Diffusion models due to their almost-surely-continuous nature have been widely applied in financial modelling. Here we compare different models from the general model CKLS (Chan–Karolyi–Longstaff–Sanders). Diffusion models selected comprise both Cox-Ingersoll-Ross (CIR) and Ornstein-Uhlenbeck process.

'Model 1 (Vasicek)'

$$dX_t = \theta_1(\theta_2 - X_t)dt + \theta_3^2 dW_t$$

'Model 2 (Cox-Ingersoll-Ross)'

$$dX_t = \theta_1(\theta_2 - X_t)dt + \theta_3\sqrt{X_t}dW_t$$

'Model 3'

$$dX_t = \theta_1(\theta_2 - X_t)dt + \theta_3 X_t dW_t$$

where W_t denotes the Wiener process.

3.2 Jump-diffusion models

Jump-diffusion models instead offer the possibility to take into account more abrupt changes in the stock prices through a jump term. In the models considered dP_t describes a Poisson Process with normal distributed jumps of form $dP_t = z_t X_t dN_t$ that arrive with intensity $\lambda(X_t, t) = \lambda_0(t) + \lambda_1(t)X_t$.

'Model 4 (Jump CIR)'

$$dX_t = \theta_1(\theta_2 - X_t)dt + \theta_3 X_t dW_t + dP_t$$

'Model 5 (Pienaar)'

$$dX_t = \theta_1(\theta_2 + \theta_7 \sin(8\pi t + (\theta_8 - 0.5)2\pi) - X_t)dt + \theta_3 X_t^2 dW_t + dP_t$$

with dP_t a Poisson process with frequency θ_4 in Model 4 and $\theta_4 X_t$ in Model 5. Both are subject to normal distributed jumps ($\mu_j = \theta_5$,$\sigma_j = \theta_6$).

Model 5 was developed by Pienaar for DiffusionRjgqd package which we employ for our numerical solutions[4]. The particularity is that it contains a cyclical drift component corresponding to a volatility cycle, state-dependent volatility (i.e., the volatility of volatility varies in accordance with the level of the process) and state-dependent jump intensity. In practice it assumes that the rate of arrivals for jumps changes accordingly with volatility. Consequently when volatility is high, jumps are more likely.

4 Table of results

Table 2: Comparing the fit of various models to Minute Data

Model number DIC	**Theta 1 PD**	**Theta 2**	**Theta 3**	**Theta 4**	**Theta 5**	**Theta 6**	**Theta 7**	**Theta 8**
1	-11.97	24.69	3157.07	-	-	-	-	-
5744.12	**2.69**							
2	-0.093	24.82	5.94	-	-	-	-	-
4398.90	**2.35**							
3	-2.099	24.04	1.343	-	-	-	-	-
4393.99	**1.09**							
4	63.90	29.961	4.91	38759	0.001	0.000	-	-
9649.99	**3.9**							
5	-4.5	17.73	2.68	2.65	3.96	12.10	0.002	0.843
8464.81	**2.18**							

Table 3: Comparing the fit of various models to Hourly Data

Model number DIC	**Theta 1 PD**	**Theta 2**	**Theta 3**	**Theta 4**	**Theta 5**	**Theta 6**	**Theta 7**	**Theta 8**
1	-2.93	23.01	1528.5	-	-	-	-	-
8409	**2.12**							
2	-4.17	24.483	38.108	-	-	-	-	-
8277.31	**2.12**							
3	-4.38	23.459	0.967	-	-	-	-	-
8157.97	**1.96**							
4	0.345	0.024	0.830	28.54	0.005	0.047	-	-
8078.02	**2.53**							
5	-3.13	23.866	0.474	1.894	1.297	33.130	0.059	0.006
7947.04	**6.53**							

Table 4: Comparing the fit of various models to Daily Data

Model number DIC	**Theta 1 PD**	**Theta 2**	**Theta 3**	**Theta 4**	**Theta 5**	**Theta 6**	**Theta 7**	**Theta 8**
1	0.417	-0.49	1.436	-	-	-	-	-
14722.29	**2.262**							
2	-0.36	23.56	74.48	-	-	-	-	-
14226.91	**2.803**							
3	-1.08	26.19	0.842	-	-	-	-	-
13972.27	**2.414**							
4	-0.15	3.77	0.53	86.8	0.003	0.068	-	-
13837.87	**3.91**							

5 Conclusion

The results from table 2 are clear. The DIC values for the 3 SV models without jumps were significantly lower than those of model 4 and 5 , which included jumps. Model 4 and 5 have relatively higher PDs, which suggest that these models are worse at describing the high frequency data. Interestingly, in tables 3 and 4 (hourly and daily data), it is the simpler jump-free models that have comparatively high DIC and worse pd estimates.

Minute data is so frequent that shocks and jumps are rarely visible. Prices are essentially continuous, and so a jump model simply is not an accurate portrayal of the data. Indeed, the MLE estimates for model 4 show that the best fit with this model has very high intensity ($\theta_4 = 38{,}759$) small ($\theta_5 = 0.001$) jumps.

However, through speculation or otherwise, an hour is enough for BTC prices to change significantly. Due to this fact, a model including infrequent but large jumps can better capture the underlying causes between BTC price movements.

6 Bibliography

References

[1] Cathy Chen, Wolfgang Karl Härdle, Ai Hou, and Weining Wang. Pricing cryptocurrency options: The case of crix and bitcoin. *SSRN Electronic Journal*, 01 2018.

[2] Adam S. Hayes. Cryptocurrency value formation: An empirical study leading to a cost of production model for valuing bitcoin. *Telematics and Informatics*, 34(7):1308 – 1321, 2017.

[3] Ladislav Kristoufek. Bitcoin meets google trends and wikipedia: Quantifying the relationship between phenomena of the internet era. *Scientific reports*, 3:3415, 12 2013.

[4] Etienne A D Pienaar and Melvin M Varughese. *DiffusionRgqd: An R Package for Performing Inference and Analysis on Time-Inhomogeneous Quadratic Diffusion Processes*, 2015. R package version 0.1.3.