NI & AC

A NS Agent for Pacman Maze

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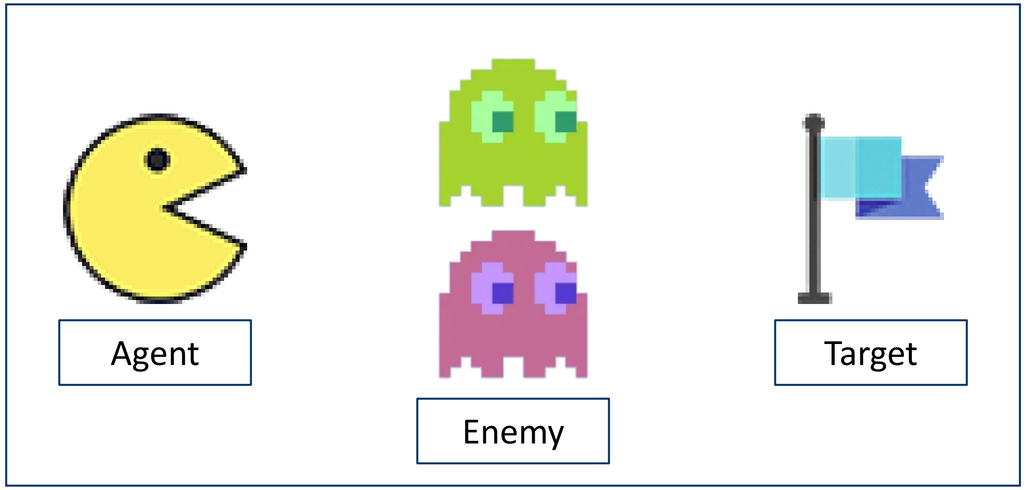
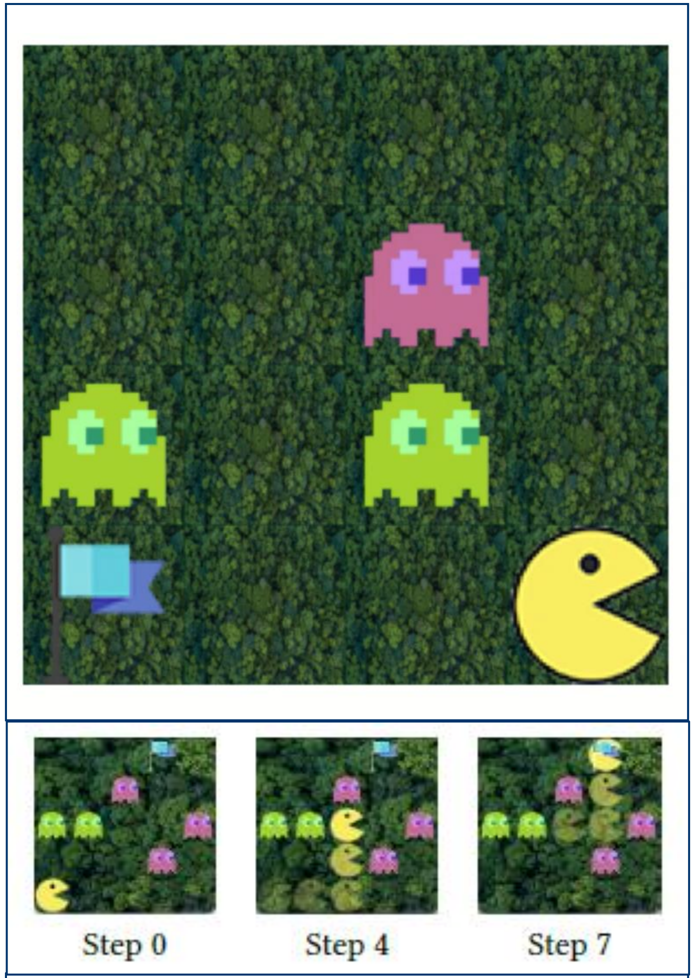
Pacman Maze

- Pacman Maze:
  - Grid: *M* x *N* cells.
  - 4 possible moves: ↑ ↓ ← →.
  - *K* enemies, all initial positions are randomly chosen.
  - **Goal**: the agent must reach the flag within *T* moves without stepping on a cell occupied by an enemy.

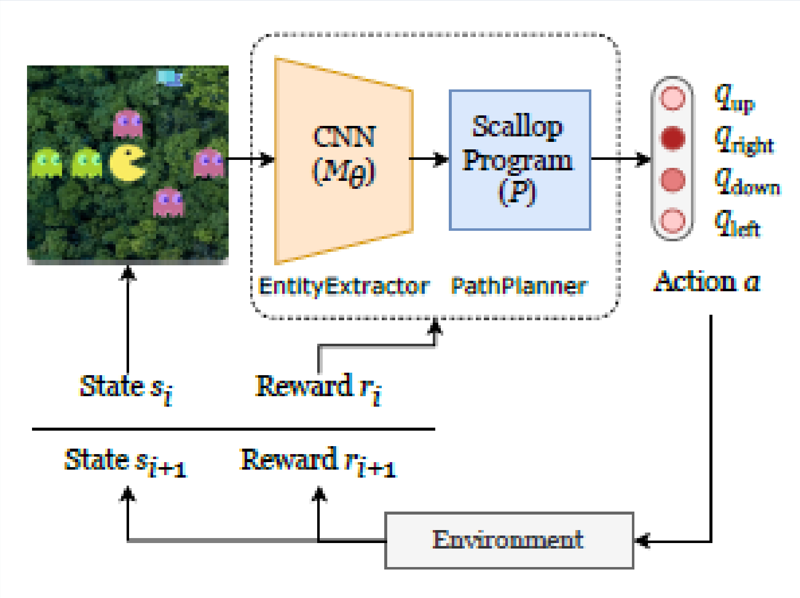




Example of games with a random agent.

DQNS Agent

- Neural-symbolic agent:
  - **Neural component**:
    - CNN-like to extract entities (PyTorch).
    - From image to distribution over *[agent, empty, enemy, target]* for each cell.
  - **Logical component**:
    - Predict next move (Scallopy).
    - Probabilistic path search on a graph.
- Agent trained using (Deep) **Q-learning**:
  - ε-greedy policy network = neural comp. + logical comp.
  - Soft update of the target network.



Structure of the agent. [source]

$$\theta' \leftarrow \tau \cdot \theta + (1 - \tau) \cdot \theta$$

Soft update of the target network's parameters.

Loss & Backpropagation

- Loss based on three factors:
  - **Expected vs predicted state-action value** → Huber loss.
  - **Constraints violation** → Smooth L1 loss:
    - Number (probability) of cells identified as targets (should be *1*).
    - Number (probability) of cells identified as enemies (should be *K*).

$$l_n = \begin{cases} 0.5(x_n - y_n)^2, & if\ |x_n - y_n| < \delta \\ \delta(|x_n - y_x| - 0.5\delta), & otherwise \end{cases}$$

Huber loss. [source]

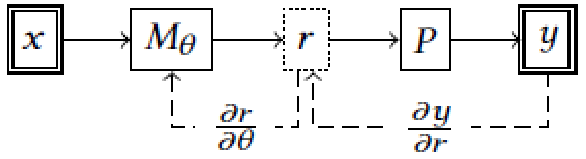
$$l_n = \begin{cases} 0.5(x_n - y_n)^2/\beta\,, & if\ |x_n - y_n| < \beta \\ |x_n - y_x| - 0.5\beta, & otherwise \end{cases}$$

Smooth L1 loss. [source]

$$loss(X,Y) = Humber(X,Y) + SmoothL1(X,Y)$$

- **Single backpropagation step:**
  - Neural component: inherently differentiable.
  - Logical component: logical equations are written using **differentiable provenance**, i.e. logical operators are mapped to differentiable ones.

$AND, OR, \ldots \rightarrow \times, +, \ldots$



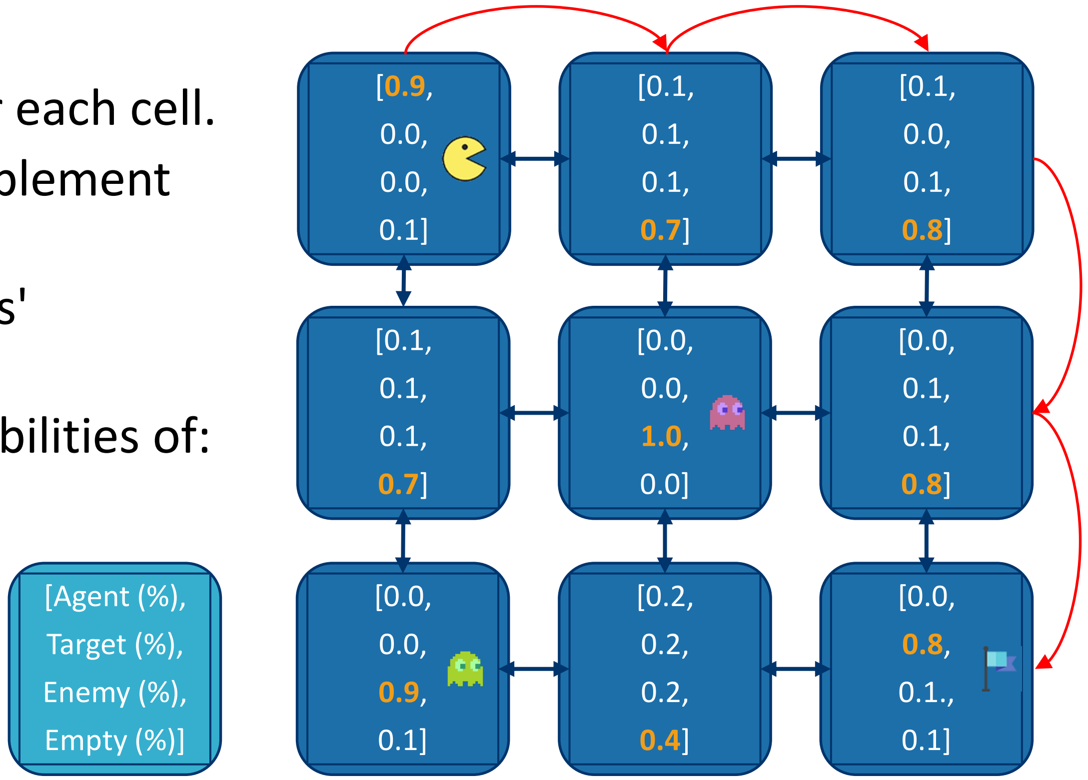
Backpropagation in a NS model. [source]

Logical Component

- **Logical component**:
  1. Read the prediction of the neural component for each cell.
  2. Assign each node a probability equal to the complement of the enemy presence probability.
  3. Path probability = conjunction of traversed nodes' probabilities.
  4. Compute the next move by conjoining the probabilities of:
     1. The current position;
     2. The goal position;
     3. And the path to follow.

```
next_position(xp, yp, a) = agent(x, y)
      and edge(x, y, xp, yp, a)
next_action(a) = next_position(x, y, a) and
      path(x, y, gx, gy) and
      target(gx, gy)
```

Formula used to compute the next action.



Probabilistic path search on a graph.

Q-Learning

- In Q-Learning:
  - **Policy network's output** is an **approximation of the state-action value function** Q.
  - At each step, this estimate is used, together with the reward obtained, to improve the quality of the next predictions.

- But here the **logical component returns a distribution over the action space**. It only works thanks to the chosen rewards.

```
class AvoidingArena(gym.Env):
  def __init__(
    self,
    grid_dim: Tuple[int, int] =(5, 5),
    cell_size: float = 0.5,
    dpi: int = 80,
    num_enemies: int = 5,
    easy: bool = False,
    default_reward: float = 0.00,
    on_success_reward: float = 1.0,
    on_failure_reward: float = 0.0,
    remain_unchanged_reward: float = 0.0,
  ):
    """
```

$$Q^{\pi}(s,a) = r(s,a) + \gamma \sum_{s' \in S} V^{\pi}(s')\mathbb{P}(s'|s,a)$$

Q function: π is the policy, s the state, a the action, r the reward and V is the state value function.

$$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha_t(r_t + \gamma \max_{a_{t+1}} Q(s_{t+1},a_{t+1}) - Q(s_t,a_t))$$

Q-Learning update rule.

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Extending the Rewards

Custom Provenance

```python
# ---- Custom provenance ----
class MyProvenance(scallopy.provenance.ScallopProvenance):
    def __init__(self, min :float, max :float):
        super(MyProvenance, self).__init__()
        self.min = min
        self.max = max
        return

    def name(self):
        return "custom-provenance"

    def zero(self):
        return self.min

    def one(self):
        return self.max

    def add(self, t1, t2):
        raise Exception("Not implemented")

    def mult(self, t1, t2):
        return torch.clip(t1 + t2, min=self.min, max=self.max)

    def negate(self, t):
        return torch.clip(-t, min=self.min, max=self.max)

    def saturated(self, t1, t2):
        return bool(t1 > t2)
```

Reward as Tuple Element

```
# connectivity: path(node_i, node_j, num_moves, reward)
# {L} is the maximum number of moves allowed
# {R} is a lower bound for the reward
path(x, y, x, y, 1, r) = node(x, y, r)

path(x, y, xp, yp, 1, r) =
      edge(x, y, xp, yp, _) and
      node(xp, yp, r)

path(x, y, xpp, ypp, {L}, {R}) =
      node(x, y, _) and
      node(xpp, ypp, _) and
      $abs(x - xpp) + $abs(y - ypp) > 1 # lower bound

path(x, y, xpp, ypp, l + 1, rp + rn) =
      path(x, y, xp, yp, l, rp) and
      edge(xp, yp, xpp, ypp, _) and
      node(xpp, ypp, rn) and
      path(x, y, xpp, ypp, _, r) and
      r < rp + rn and
      l + 1 <= {L}
```

Limitations of Scallop

- Main Scallop limitations:
  A. **Lack of documentation** for advanced operations.
  B. **Lack of support for filtering** tuples at generation time.
  C. **Limited** provenance **customizability**.
  D. **GPU unused** by logic component → High training time even for small neural component. [Git Issue]

- In practice, **proposed solutions** for handling different rewards are **ineffective**:
  - Reward as tuple element → Huge memory footprint even for small grids (due to point B).
  - Custom provenance → Incorrect rewards due to inability to implement the saturation criterion correctly (due to point C).

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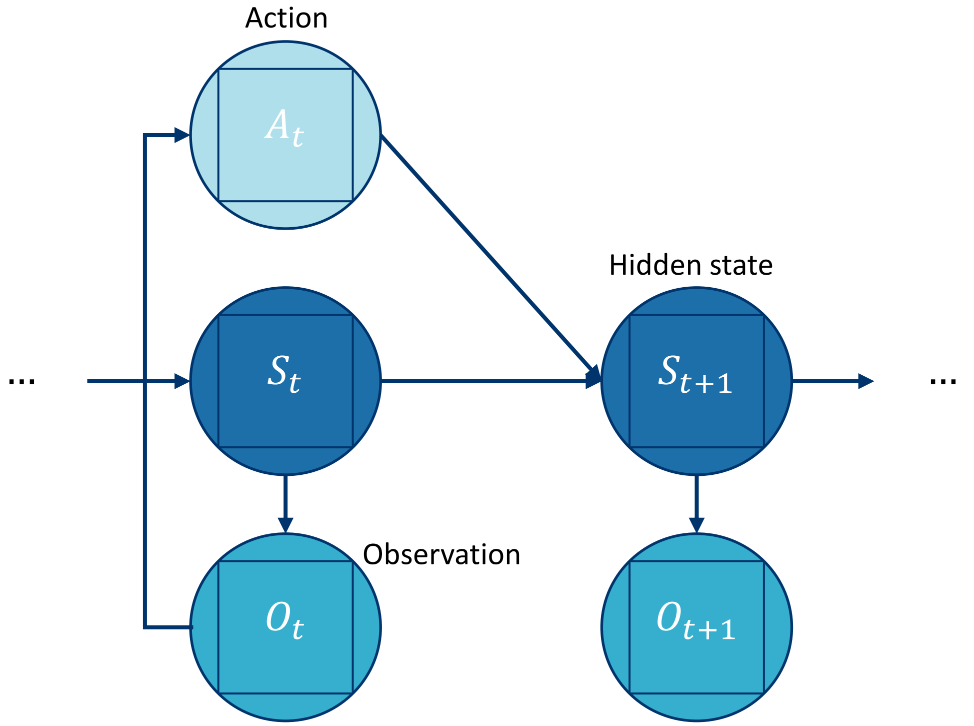
Neural Component

- How can the neural component be improved, considering the limitations of Scallop?

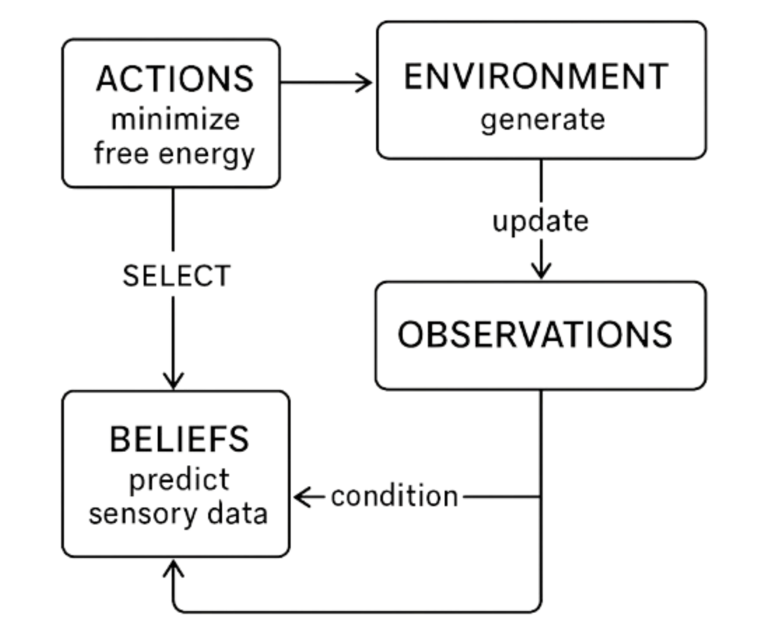
- Original neural component → CNN:
  - 2x 2D-Convolutional layers;
  - 2x Dense layers;
  - ReLu as activation function;
  - Softmax as last activation function.



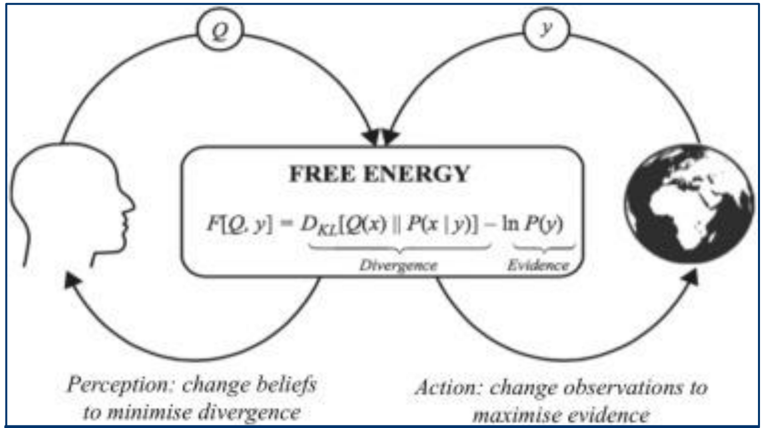
PGM of the game: in dark blue the hidden states.

Active Inference

- A different approach to the problem:
  - **Minimize uncertainty** instead of maximizing the reward.



Generic AI schema. [source]



Perception and action play a dual role. [source]

- **Challenges**:
  - A detailed generative model is needed.
  - Simulating and managing uncertainty.
  - Scalability.

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Possible Implementations

- Deep Active Inference with Variation Policy Gradient (**DAI$_{VPG}$**):
  - Gradient descent to **minimize Expected Free Energy** (EFE).

  → Not full AI
  → Inaccurate

- Deep Active Inference Free Action Model (**DAI$_{FA}$**):
  - **Minimizes** the **Variational Free Energy**.

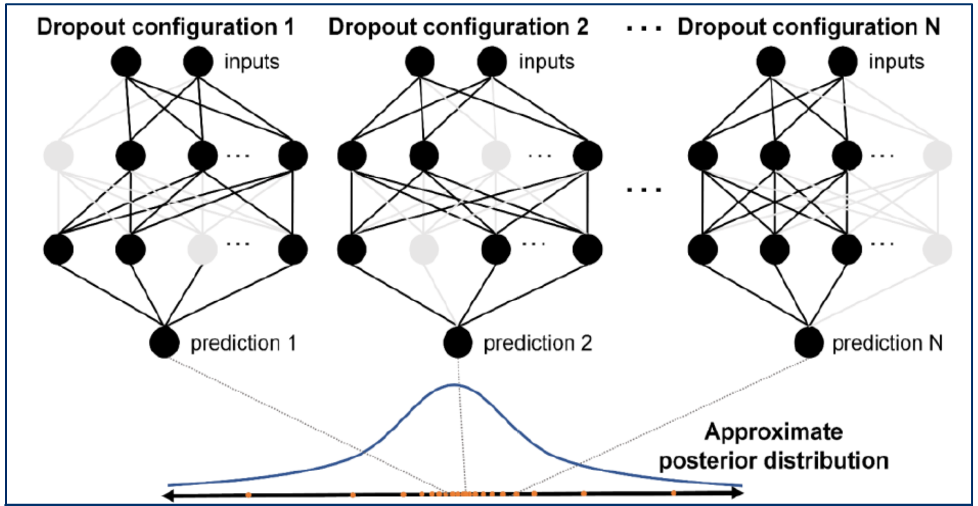
  → No scalability
  → Unclear preferences

- Deep Active Inference with Monte Carlo Tree (**DAI$_{MC}$**):
  - Monte Carlo Tree Search to explore action sequences.

  → Proper AI
  → Heavy and complex to train

Our Approach: MC Dropout

- **Monte Carlo (MC) Dropout**:
  - **Dropout** layers kept active also **during evaluation.**
  - So multiple passes with different dropout masks for each input → multiple predictions.
  - This allows **uncertainty estimation** in predictions, and it **approximates variational inference** (Gal et al.).



MC Dropout illustrative example. [source]

- Pros:
  - Training time unaffected.
  - Parallel evaluation → time unchanged.
- Cons:
  - Poorly approximates complex posterior distributions.
  - Sensitive to hyperparameters (e.g., dropout rate).

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Experimental Results

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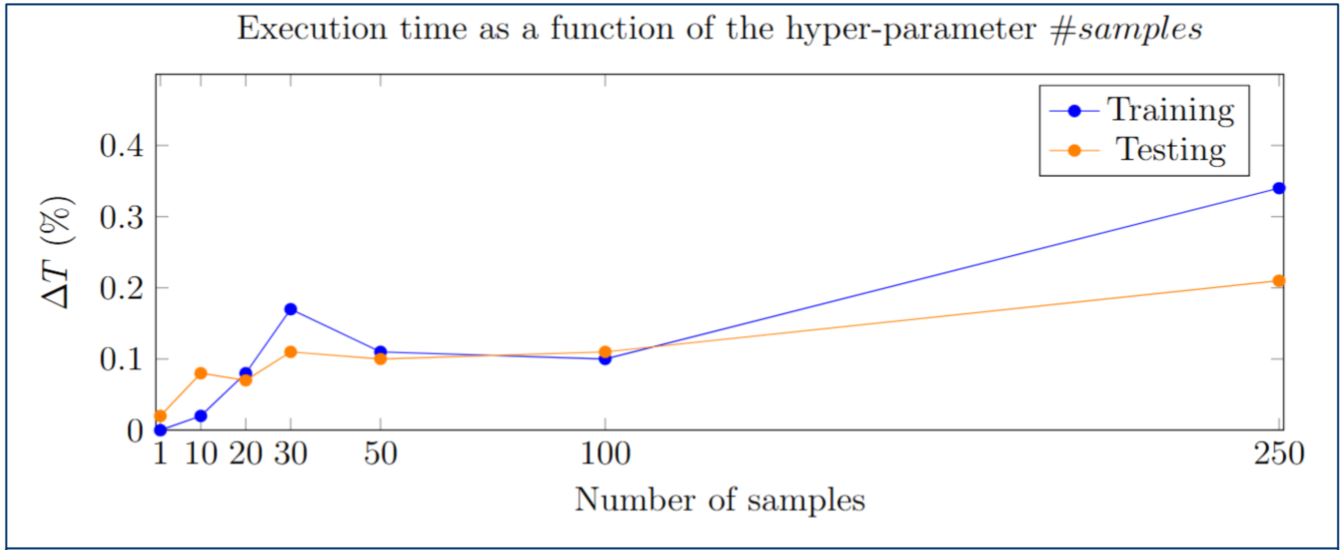
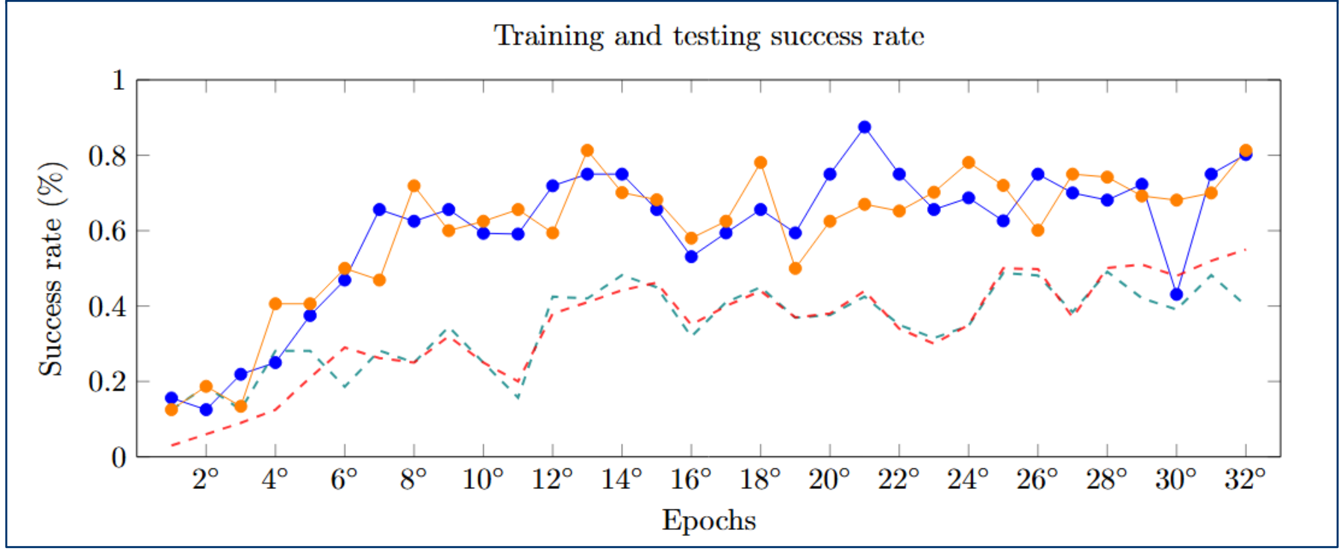
Test 1

Execution Time (sec)		
#Samples	T_{train}	T_{test}
1	2,43	0,54
10	2,49	0,55
20	2,69	0,55
30	2,88	0,56
50	3,21	0,56
100	3,53	0,67
250	4,74	0,81



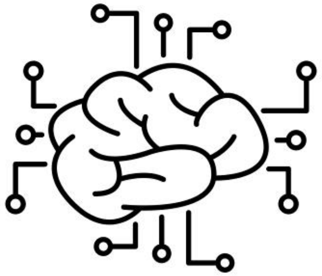
Test 2

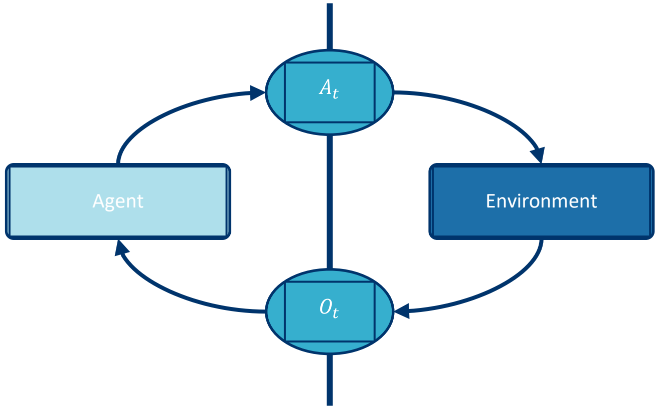
Legend		
	DQNS Agent	Original Agent
Train		
Test		



Conclusions

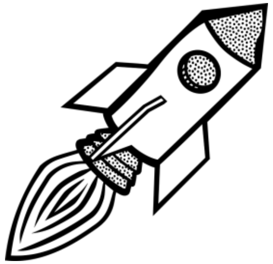
- Scallop:
  - A framework for neuro-symbolic programming .
  - With its strengths and potential limitations.





- MC Dropout:
  - A trade-off between efficiency and Bayesian inference.
  - Good accuracy and less prone to overfitting.

- Further developments:
  - More training & testing → more reliable conclusions.
  - More complex neural and/or logical components.



Grazie