



POLITECNICO  
MILANO 1863



**NAIK**  
**JUST LEARN IT.**

Online Learning Applications Project  
Pricing and Advertising

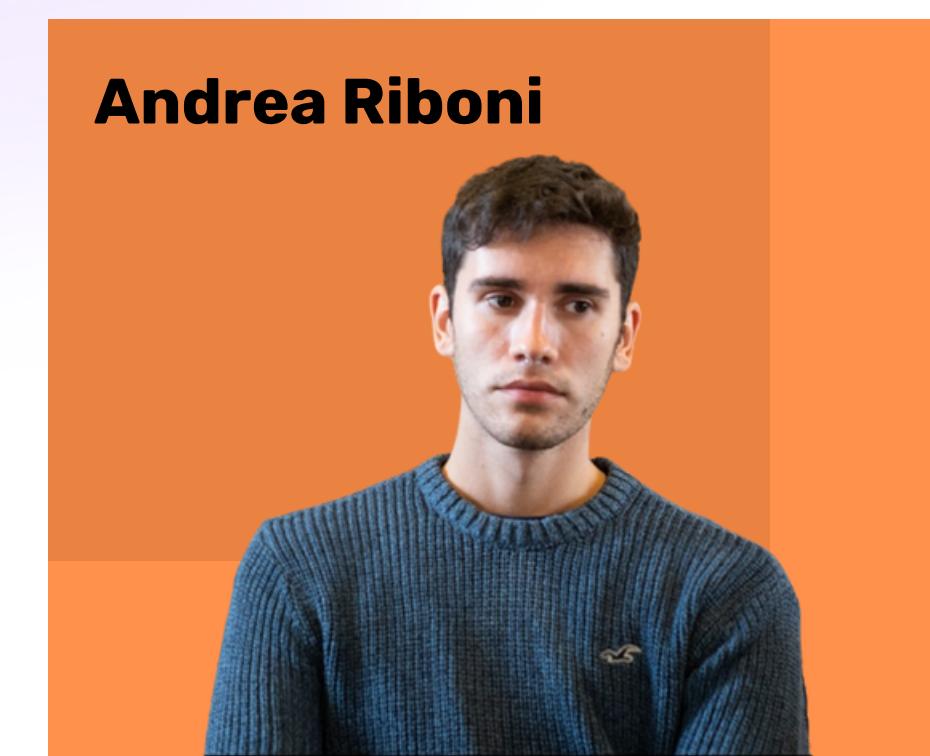
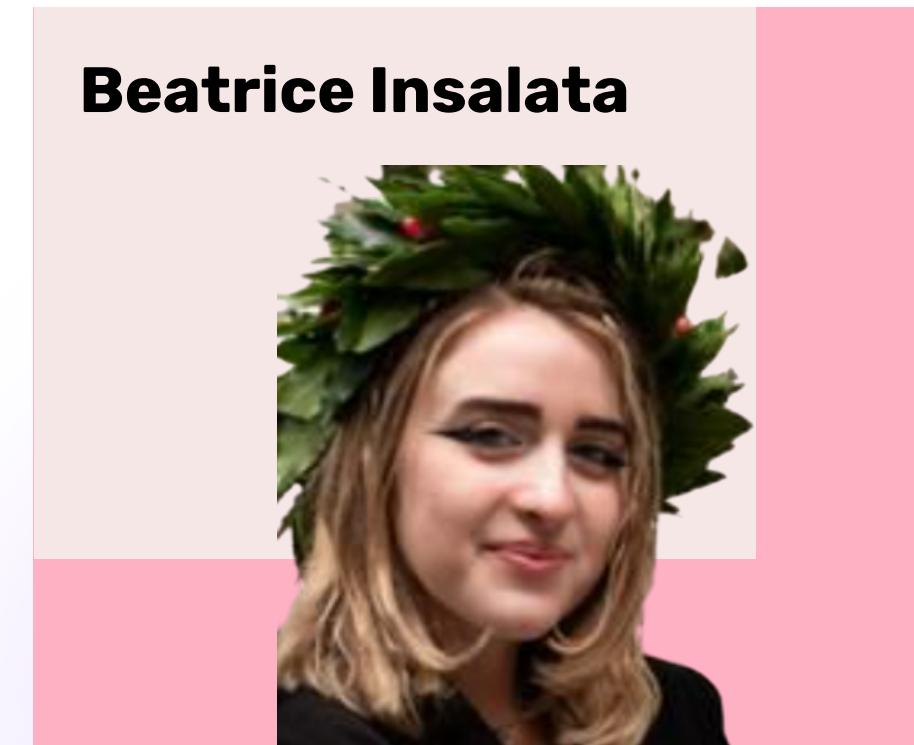
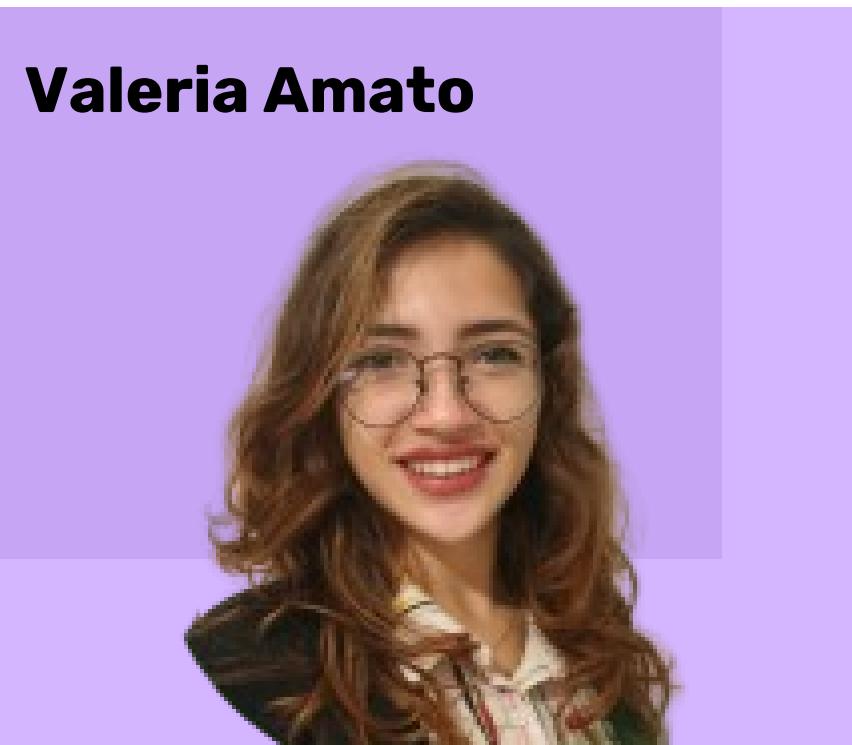
A.Y. 2022/2023

Valeria Amato - 10641790  
Beatrice Insalata - 10708628  
Emanuele Paci - 10681377  
Matteo Pancini - 10656944  
Andrea Riboni - 10699906



# MEET OUR AMAZING TEAM

Five EIT Digital Data Science students



# PROJECT OVERVIEW

INNOVATION MEETS YOUR COMFORT

The choice of the product was made considering its relevance in online sales, the potential for seasonality, and the possibility to appeal to different kinds of users. Mountain trekking shoes are a valuable product that stays on a relatively stable price range.

Medium-quality, fashionable design shoes were considered to be sold in the interval:

[50,100,150,200,250]



→  
Users' reserve price can be related to different factors



SEASONALITY



ONLINE MARKET



PERSONAL INTEREST

# USER CLASSES

The user classes were defined considering relevant characteristics that can influence the level of attractiveness of the product.  
They were divided in three main categories considering two main characteristics:

## AGE

Younger users tend to have a higher level of activity

## PERSONAL INTEREST

Users who have an interest in outdoor activities

### YOUNG & SPORTY



The optimal target for our product.

### ADULT & SPORTY



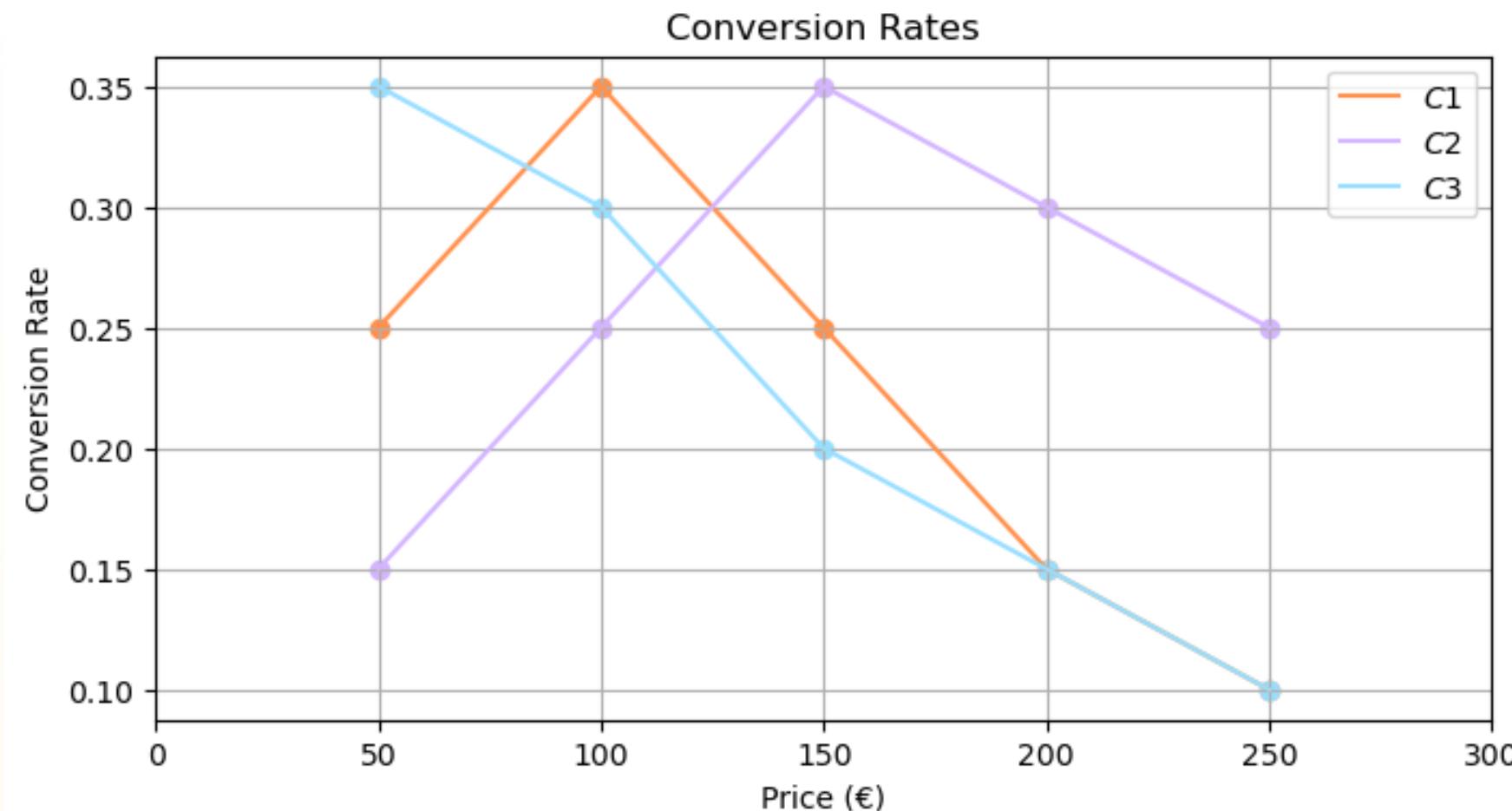
A valuable customer to address.

### YOUNG AND NON-SPORTY



A great potential public for our offer.

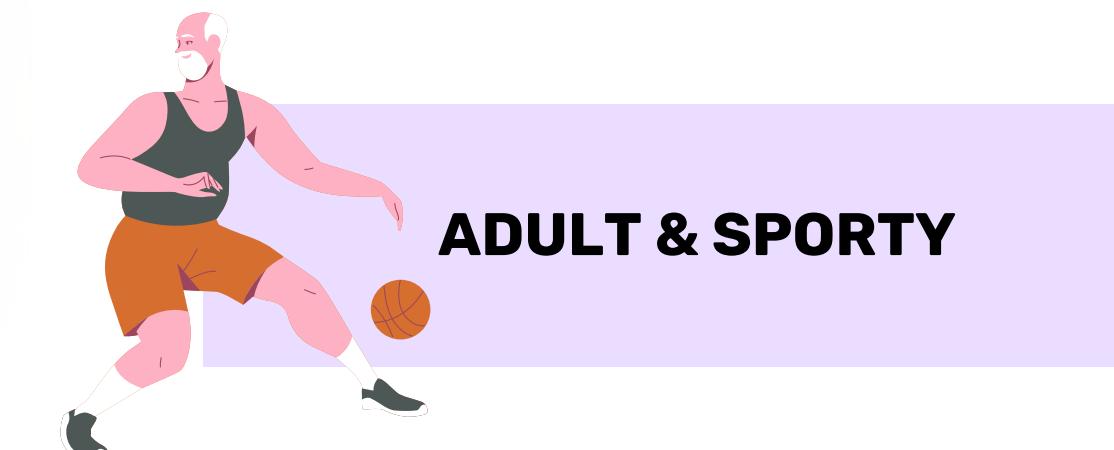
# CONVERSION RATES



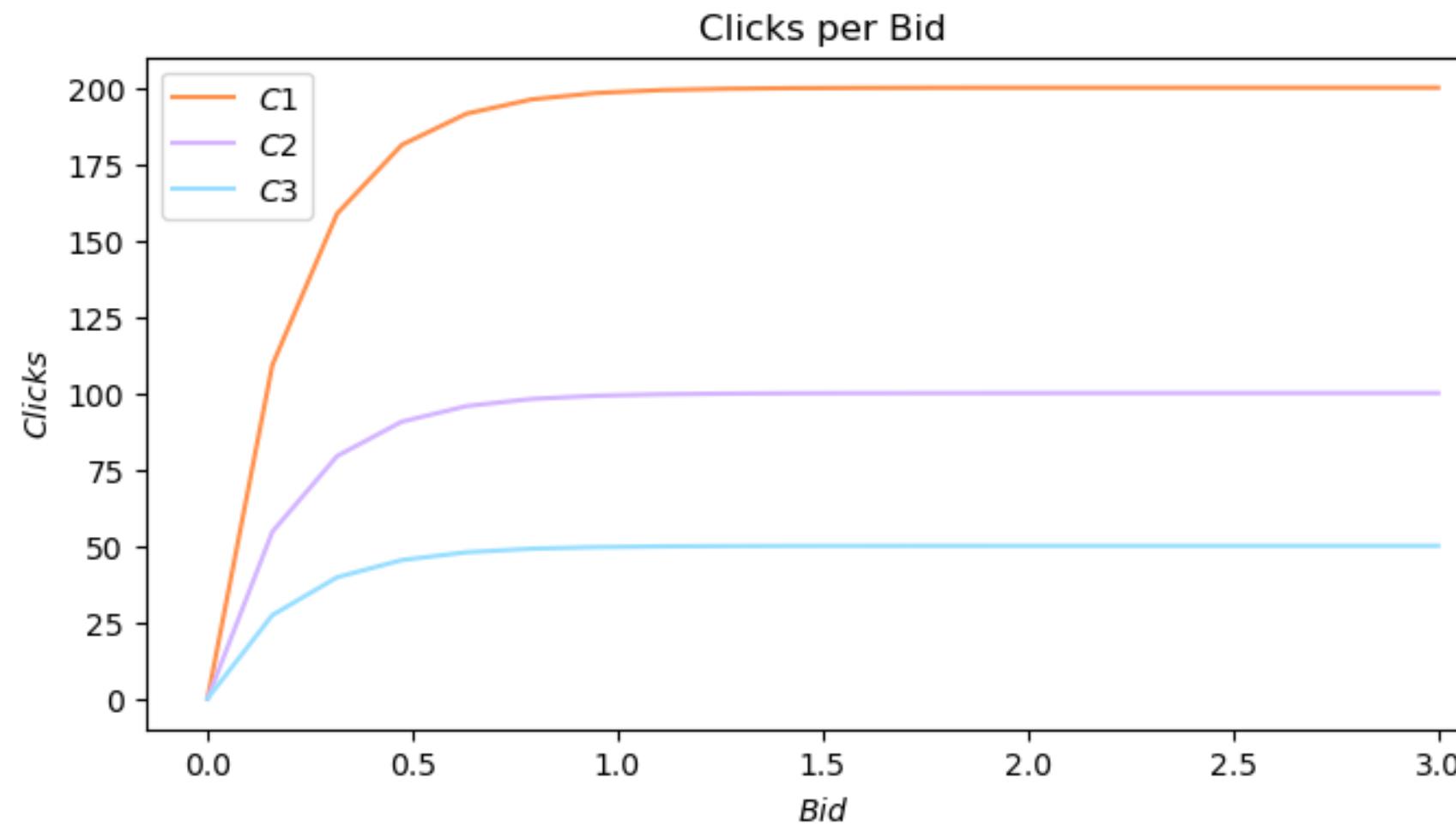
## A PRICE-QUALITY TRADE-OFF

Sporty users are skeptical and will less likely buy a product if its price is too low.

Non-sporty users are less interested in the quality of the product and are more willing to buy the shoes even when the price is *too low*.

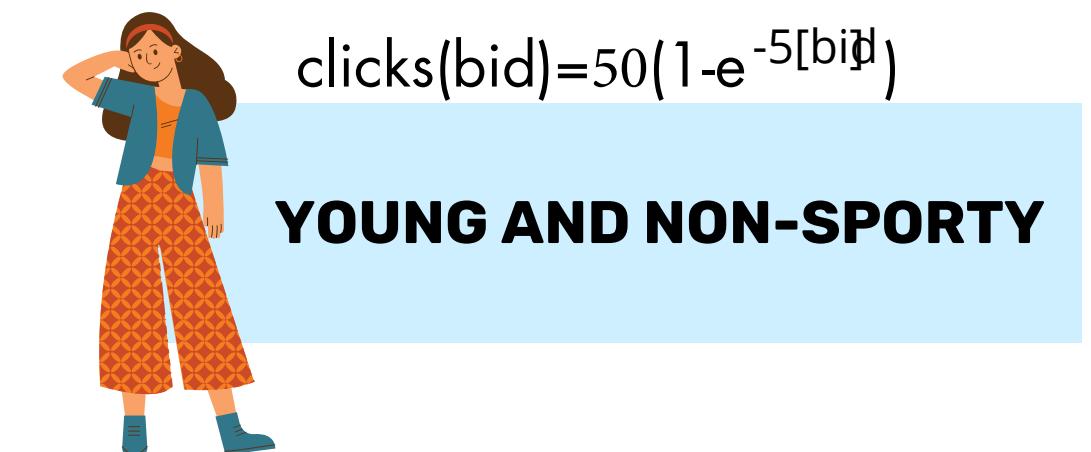
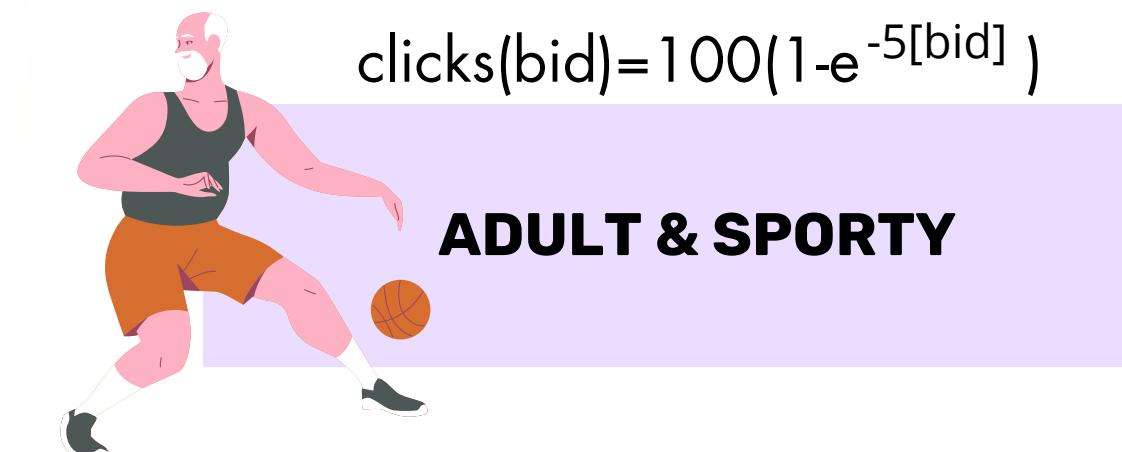
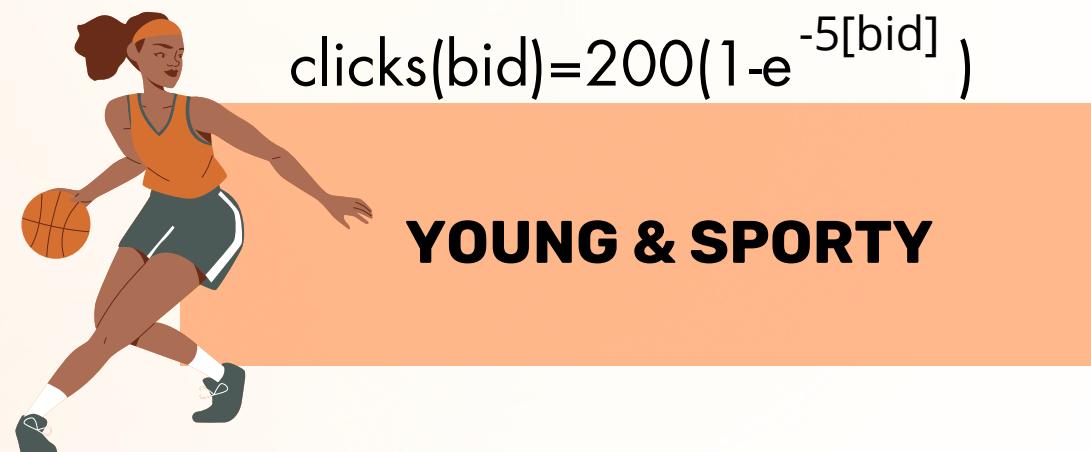


# AMOUNT OF CLICKS

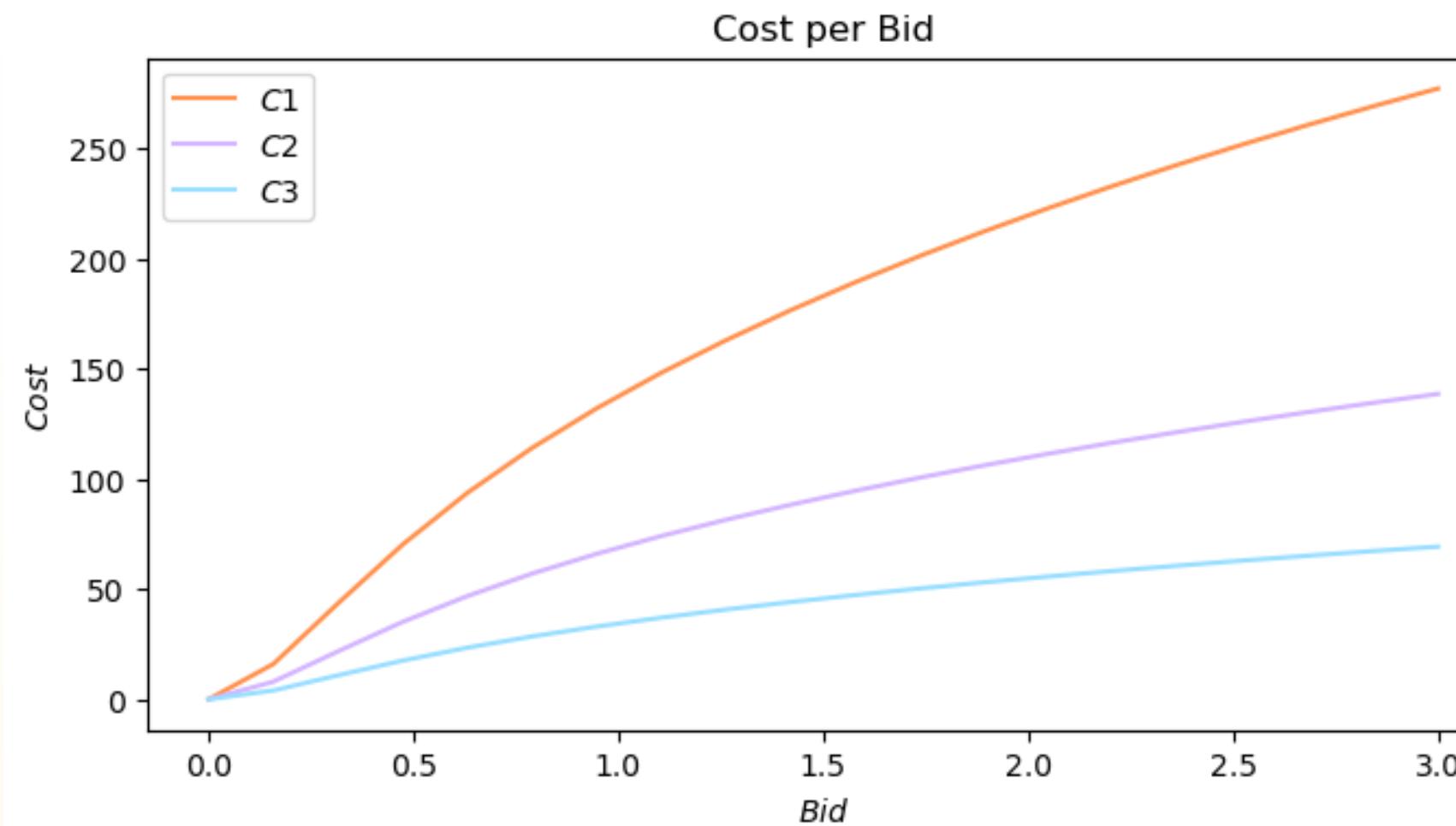


## AN EXPONENTIAL APPROACH

The three classes are represented by similar behaviors for this curve, where **sporty-users** are more likely willing to click an ad. Being tech-savvy, **youngsters** are the ones with the highest clicks-per-bid ratio.



# COST PER BID



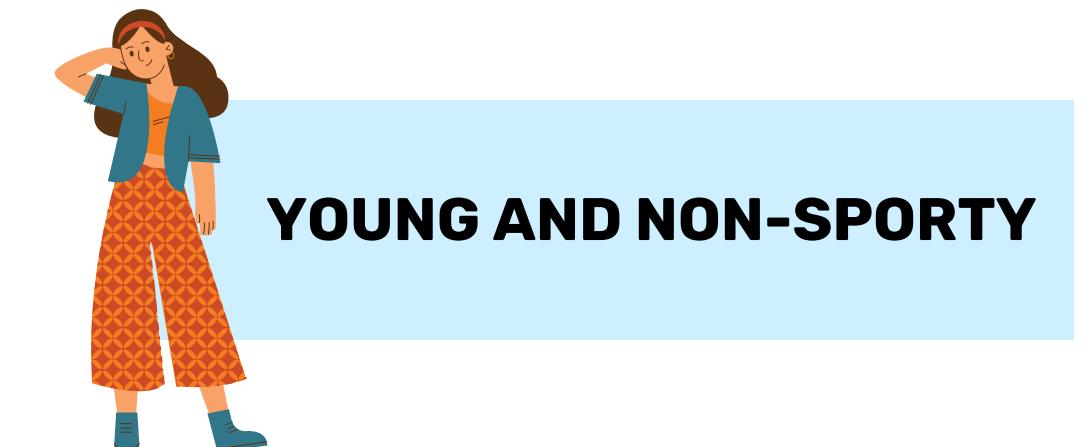
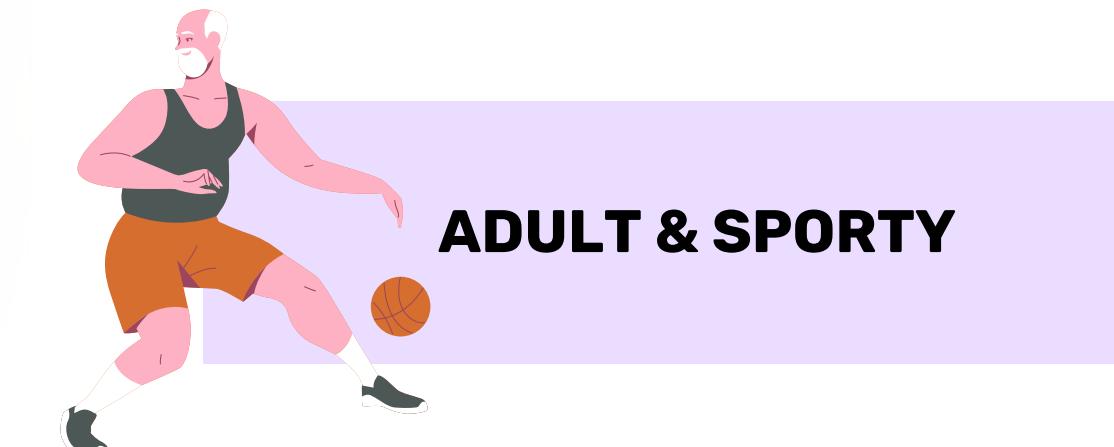
## A LOGARITHMIC APPROACH

The cost per bid is expressed by a product.

The total cost for a bid  $b$  is given by

$$\text{cost}(b) = \log(b+1) * \text{clicks}(b)$$

Therefore the cost curve is based on the click curve,  
but scales also logarithmically with the bid value.



# THE CLAIRVOYANT



## COMPUTING THE OPTIMAL PARAMETERS

To determine the optimal bid and the optimal price, we resort to the exact method, which is the exhaustive search over all possible combinations.

This is repeated for each class of users.

**3**

**User Classes**

**5**

**Possible Prices**

**100**

**Possible Bids**

[50, 250]

(0, 3]

50-step

100-step

We decided that the manufacturing cost for the NAIK shoes is 30% of the selling price  
Therefore, the margin is constant: 70% of the selling price

# STEP 1: LEARNING FOR PRICING

## SCENARIO



ALL USERS BELONG  
TO CLASS C1



PRICING CURVES ARE  
UNKNOWN



ADVERTISING CURVES  
ARE KNOWN

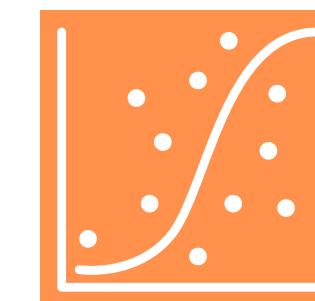
## REQUEST



APPLY TS  
ALGORITHM

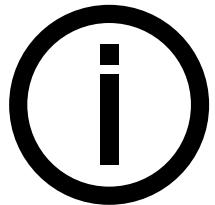


APPLY UCB1  
ALGORITHM

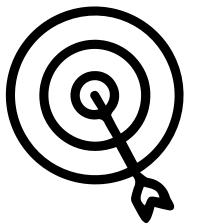


PLOT CUMULATIVE &  
INSTANTANEOUS REWARD /  
CUMULATIVE & INSTANTANEOUS  
REGRET

# ***Introducing: UCB1 and TS***



- For this project, we are going to use algorithms designed to work in MAB settings
- Mainly, UCB1 and TS



- We use both algorithms to estimate the conversion rates
- They present some key differences

## ***UCB1***

- Deterministic
- Pulls arms based on their upper confidence bound

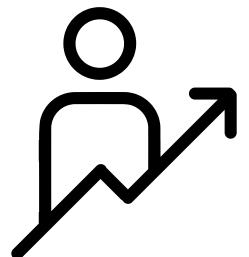
## ***TS***

- Probabilistic, Bayesian
- Pulls arms based on a posterior probability

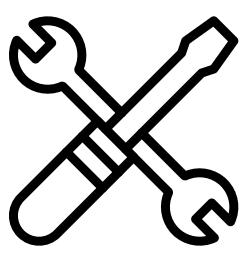
# STEP 1: SETUP



- The advertising environment is known.
- We consider the bid value that maximizes the reward, according to the **clairvoyant algorithm**

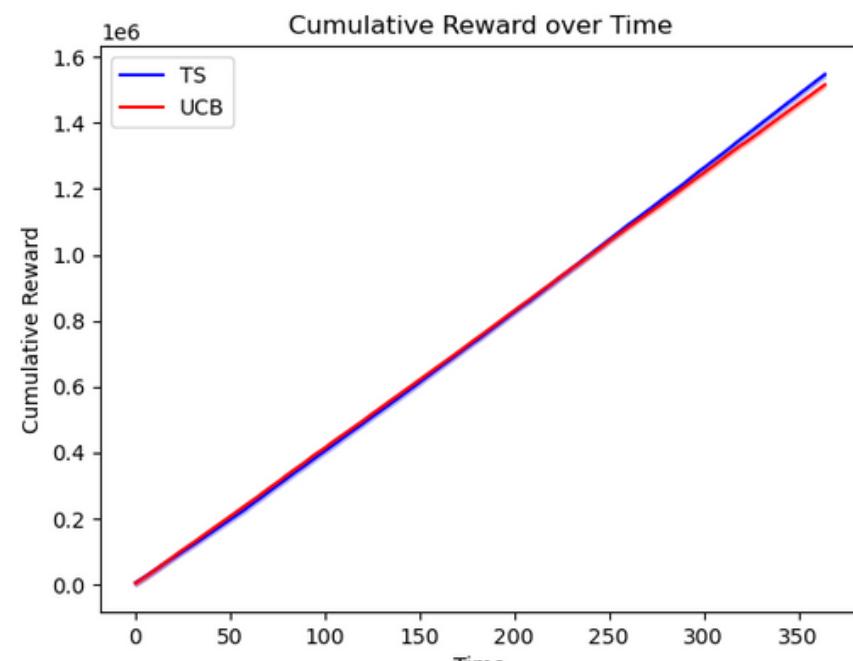


- To average the results, 1000 experiments are ran over the 365 days.
- Two learners are deployed: **UCB1** and **TS**
- In each round, the learners choose the bid that maximizes the product between the estimated conversion rate and the margin, for this class of users.

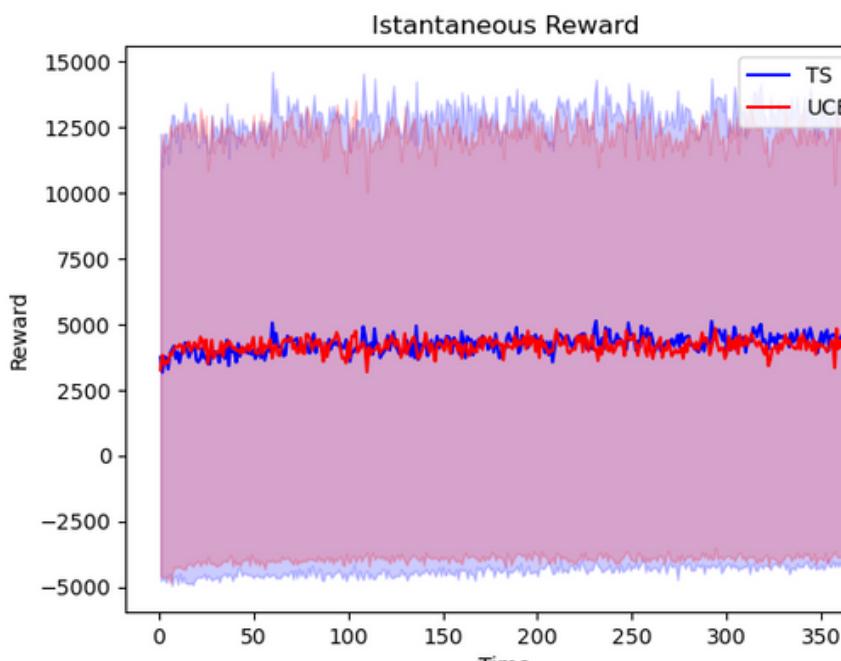


- The **advertising environment** provides methods for accessing the functions related to the bid value (generating observation rounds)
- The **pricing environment** gives access to conversion rates and rewards per round
- The **TS Learner** contains the logic of the algorithm, pulling arms and updating the distribution parameters.

# STEP 1: RESULTS

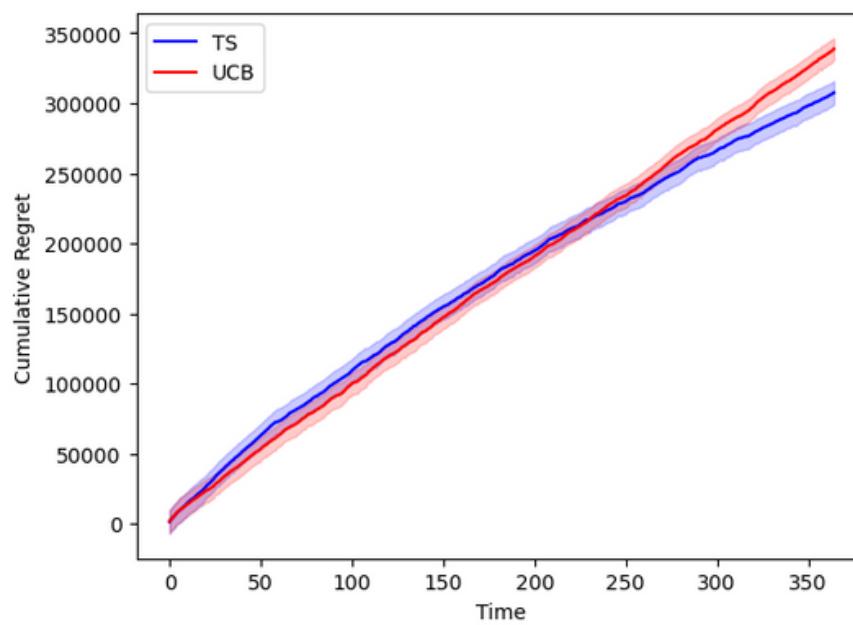


**Cumulative reward**

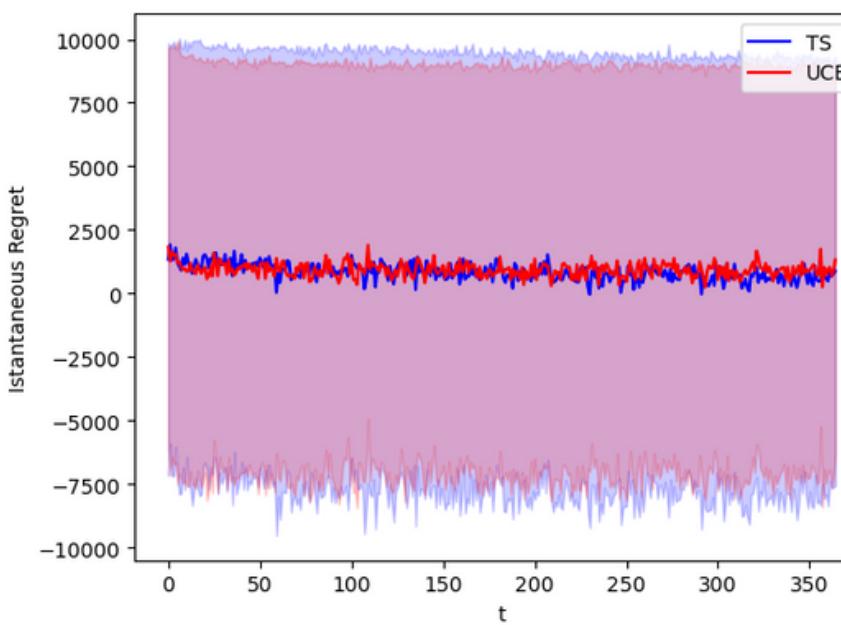


**Instantaneous reward**

- Both **TS** and **UCB1** achieve a sublinear regret
- **TS** has a slightly higher reward than **UCB1**'s, which is also more stable



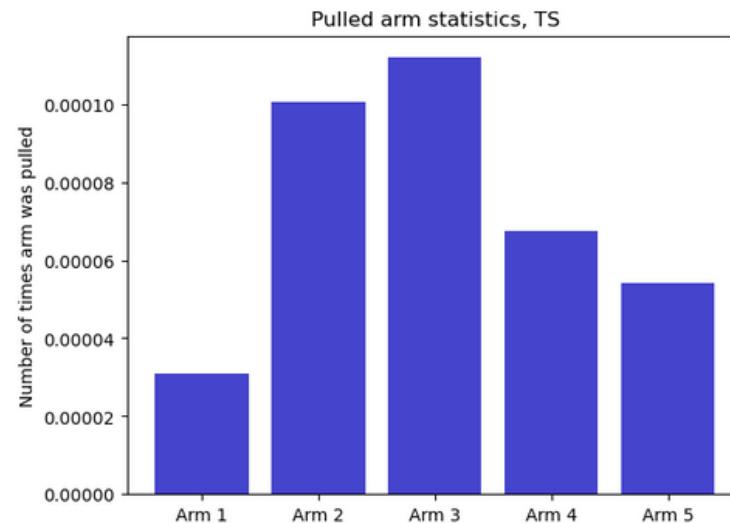
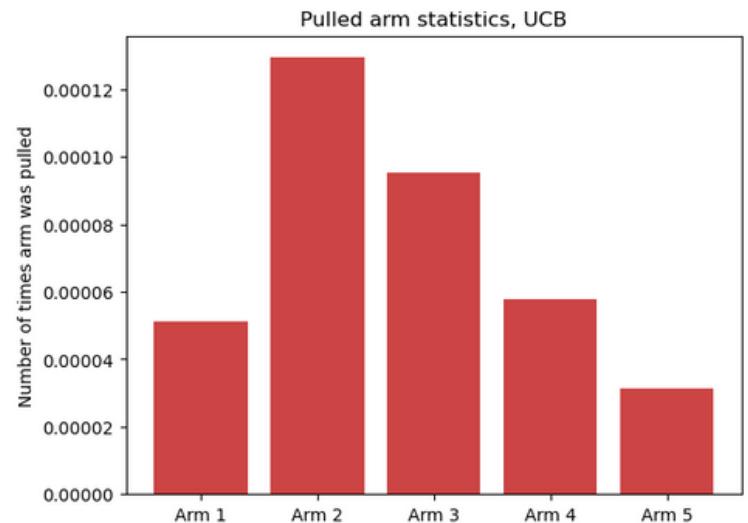
**Cumulative regret**



**Instantaneous regret**

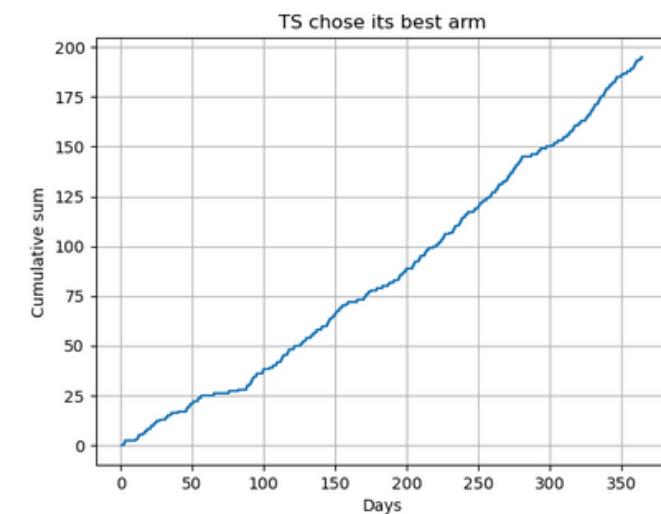
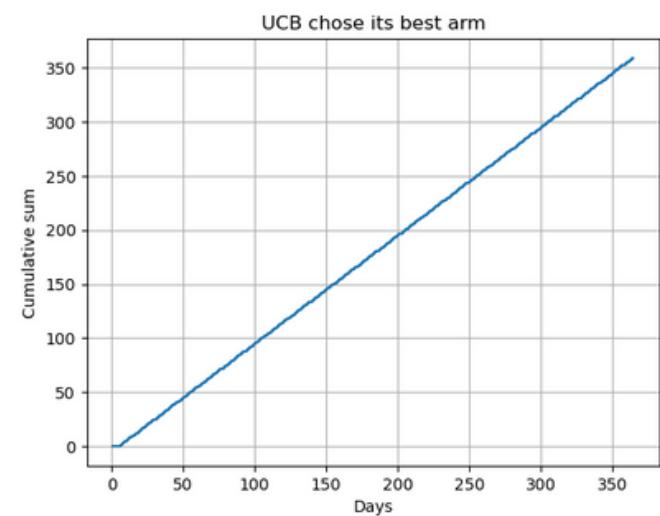
- Given that, **TS** also has a lower cumulative regret, performing better, in the long run, thanks to the exploration-exploitation tradeoff.

# STEP 1: IN DEPTH ANALYSIS



If we consider the number of times each arm has been pulled, we see that the two learners have converged towards different arms.

- **UCB1** towards arm 2
- **TS** towards arm 3



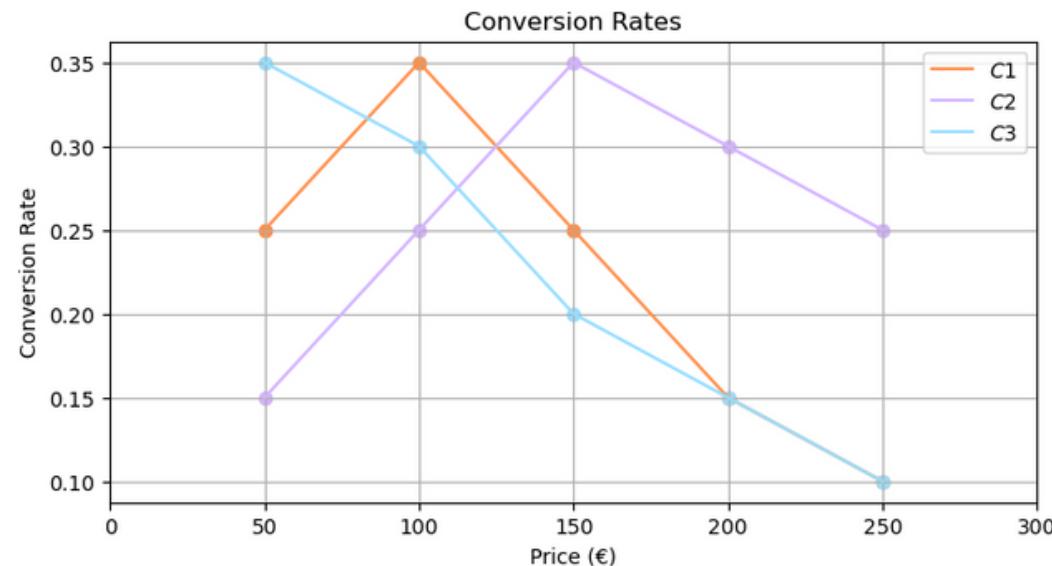
By plotting all the times **UCB1** has pulled arm 2 and **TS** has pulled arm 3 over the time horizon, we can truly observe that **TS** explores the other arms much more

# STEP 1: IN DEPTH ANALYSIS

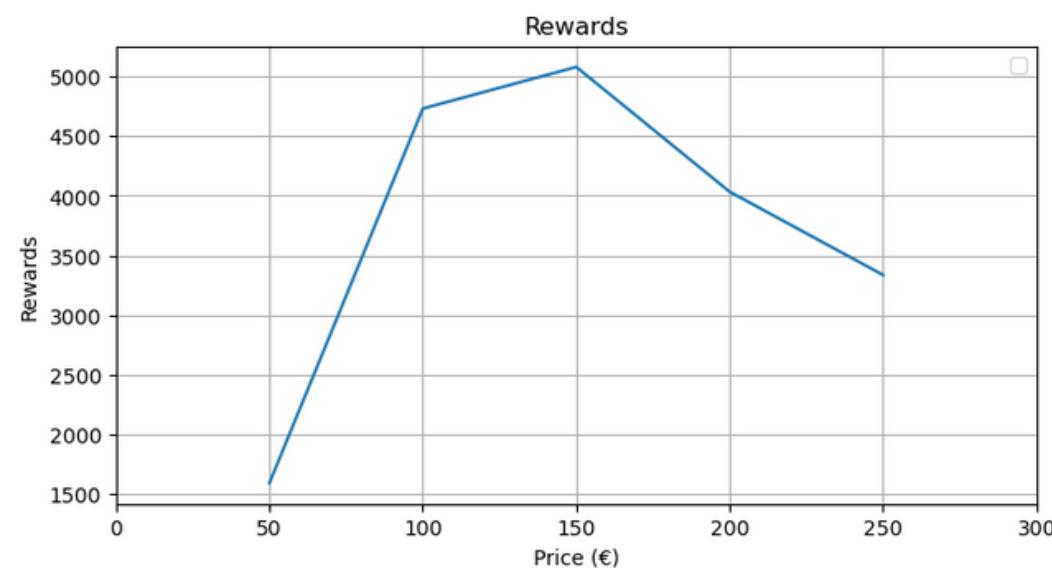


As we have seen, TS achieves slightly better result in this scenario.

This makes sense: the **clairvoyant algorithm** identified arm 3 to be the best. In fact,



Even though arm 2 has the greatest class C1 conversion rate,



Arm 3 has the highest reward.

# STEP 2: LEARNING FOR ADVERTISING

## SCENARIO



ALL USERS BELONG  
TO CLASS C1



PRICING CURVES  
ARE KNOWN



ADVERTISING CURVES  
ARE UNKNOWN

## REQUEST



APPLY GP-TS  
ALGORITHM

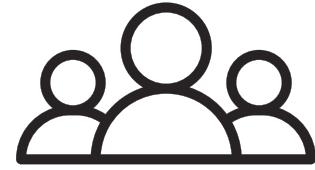


APPLY GP-UCB  
ALGORITHM

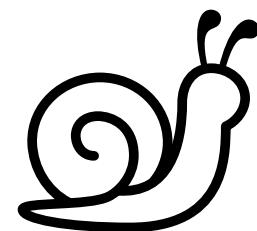


PLOT CUMULATIVE &  
INSTANTANEOUS REWARD /  
CUMULATIVE &  
INSTANTANEOUS REGRET

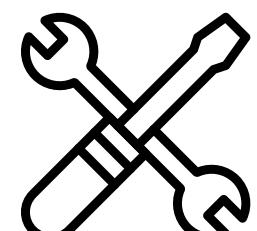
## STEP 2: SETUP



- At each round, a price is pulled and the advertising problem is solved with the suggested price

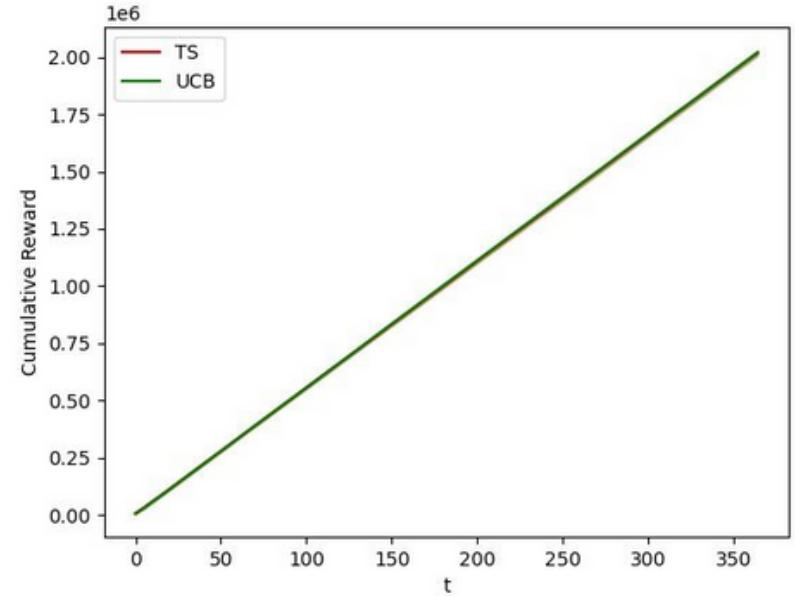


- To average the results, 15 experiments (computationally heavy) were run over 365 days.
- UCB and TS are the two algorithms to compare for the advertising problem

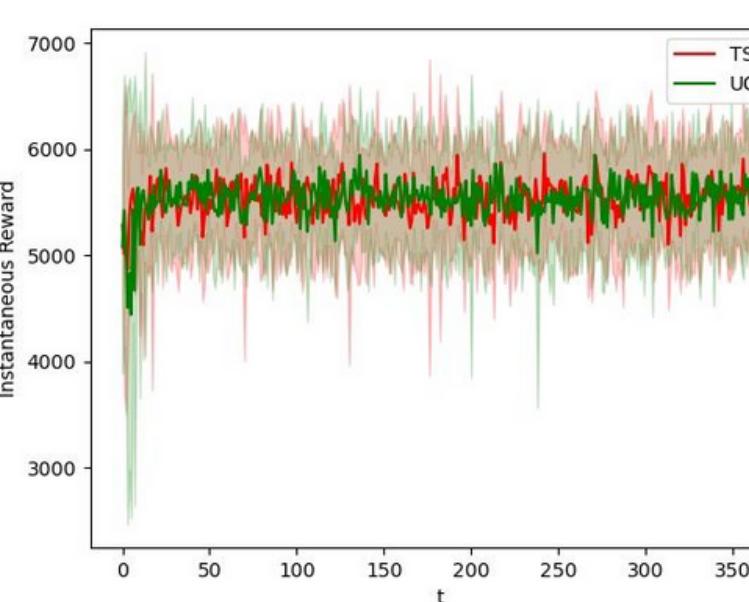


- The **Bidding Environment**, during each round, generates the number of clicks, the cumulative cost, and the gained reward.

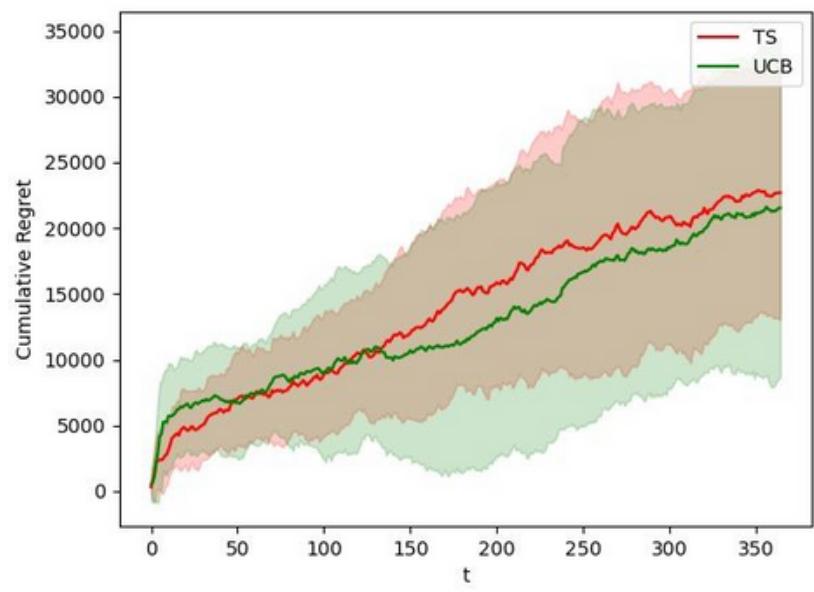
# STEP 2: RESULTS



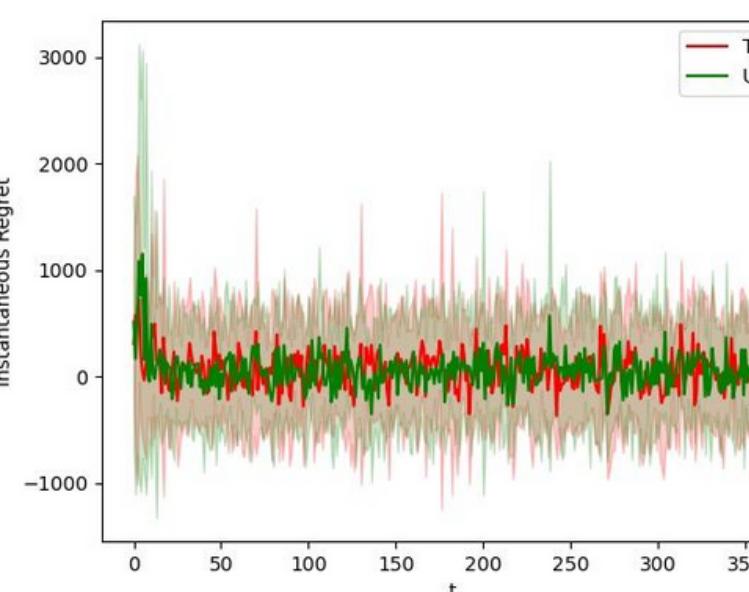
Cumulative reward



Instantaneous reward



Cumulative regret



Instantaneous regret

- A sublinear regret has been achieved
- Both **TS** and **UCB1** get very similar results, but TS performed a little better, maybe because:
  - The optimistic nature of UCB1
  - An unbalanced trade-off between exploration and exploitation: a small number of arms, so no need for an extensive search

# STEP 3: LEARNING FOR JOINT PRICING AND ADVERTISING

## SCENARIO



ALL USERS BELONG  
TO CLASS C1



PRICING CURVES  
ARE UNKNOWN



ADVERTISING CURVES  
ARE UNKNOWN

## REQUEST



APPLY GP-TS  
ALGORITHM

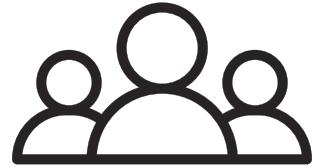


APPLY GP-UCB  
ALGORITHM

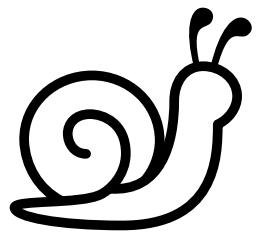


PLOT CUMULATIVE &  
INSTANTANEOUS REWARD /  
CUMULATIVE &  
INSTANTANEOUS REGRET

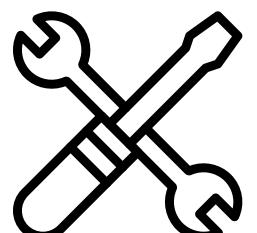
## STEP 3: SETUP



- Two phases of optimization: price and advertising
- At each round, a price is pulled and the advertising problem is solved with the suggested price

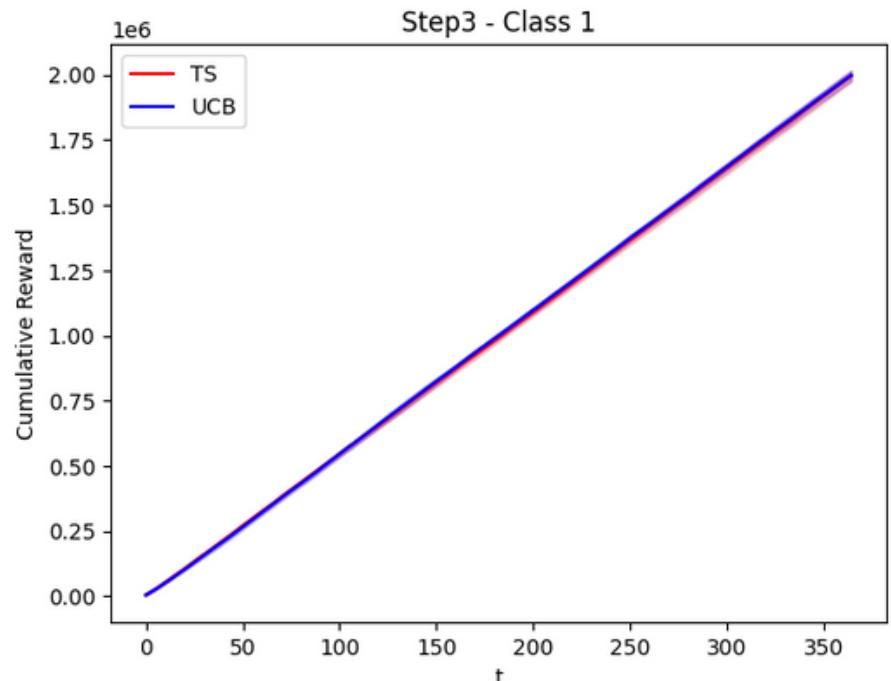


- To average the results, 50 experiments are run over 365 days.
- No request has been done over the pricing problem, so the algorithm used is TS because it was the one that performed better in the first step
- UCB1 and TS are the two algorithms to compare for the advertising problem

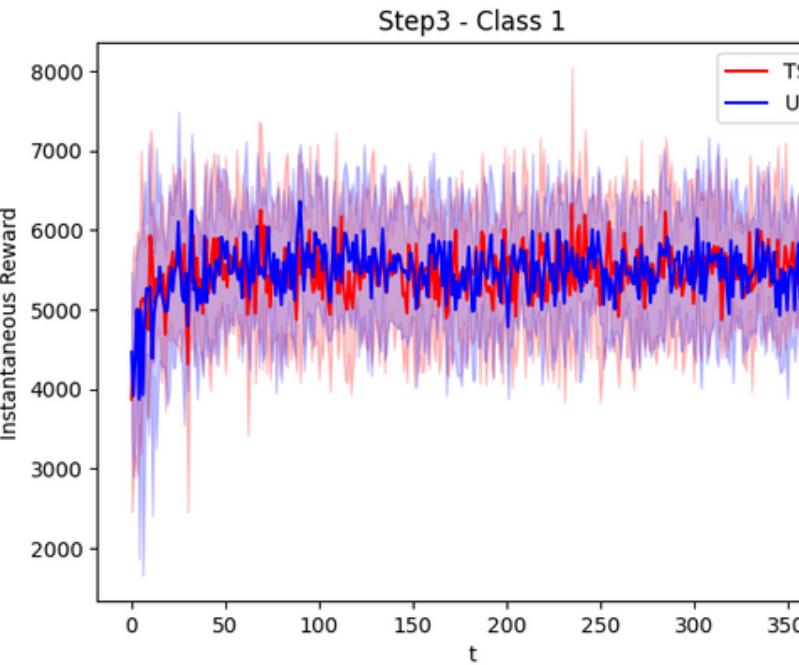


- The **Bidding Environment**, during each round, generates the number of clicks and the cost of the bid.
- The **Pricing Environment**, during each round, generates the number of clicks and the cost of the bid.

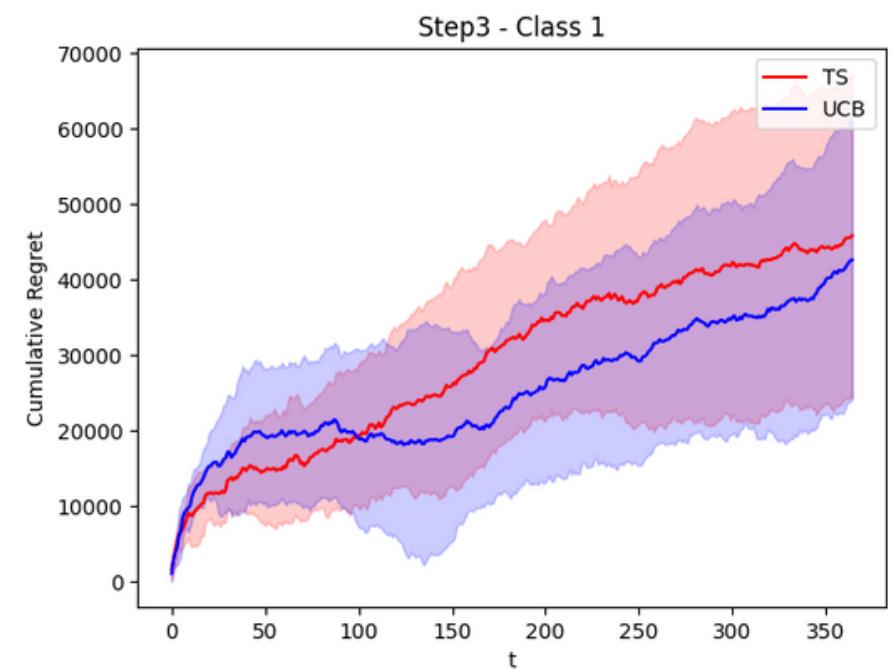
# STEP 3: RESULTS



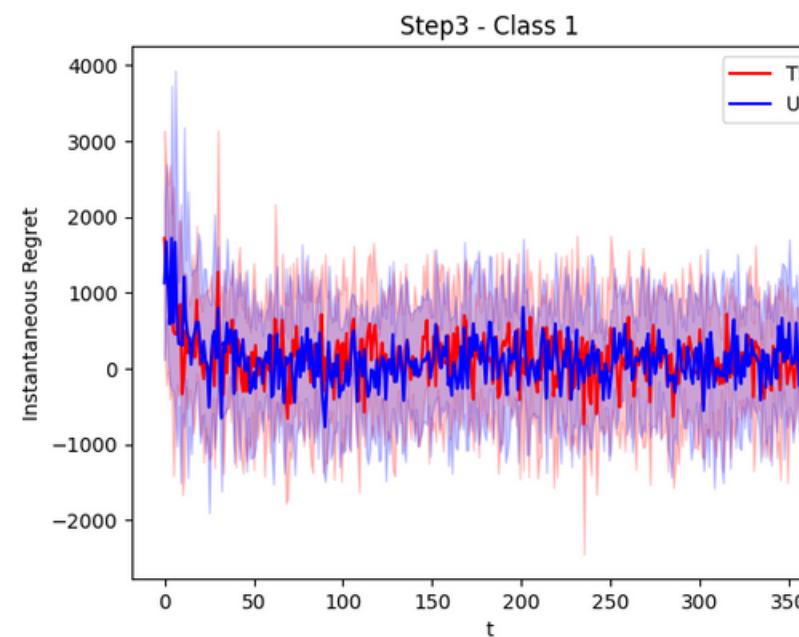
Cumulative reward



Instantaneous reward



Cumulative regret



Instantaneous regret

- A slightly sublinear regret has been achieved
- Both **TS** and **UCB1** get very similar results
- In particular, **TS** has a lower regret for the first 100 days, but then **UCB1** takes the lead
- The reward is a bit higher the reward we had in STEP1

# STEP 4: CONTEXTS AND THEIR GENERATION

## SETTING



ALL CLASSES OF  
USERS: C1, C2, C3



PRICING CURVES  
ARE UNKNOWN



ADVERTISING CURVES  
ARE UNKNOWN

## SCENARIO 1



CONTEXT STRUCTURE  
IS KNOWN



APPLY GP-TS & GP-UCB  
ALGORITHMS  
  
PLOT CUMULATIVE &  
INSTANTANEOUS REWARD /  
CUMULATIVE &  
INSTANTANEOUS REGRET

## SCENARIO 2

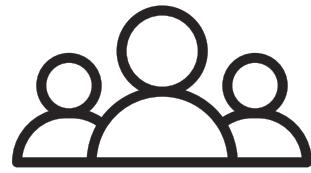


CONTEXT STRUCTURE  
IS UNKNOWN

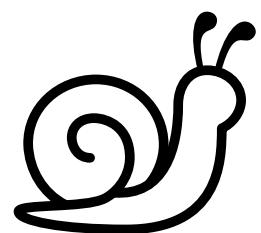


APPLY GP-TS & GP-UCB  
ALGORITHMS WITH /  
WITHOUT CONTEXT  
  
PLOT CUMULATIVE &  
INSTANTANEOUS REWARD /  
CUMULATIVE &  
INSTANTANEOUS REGRET

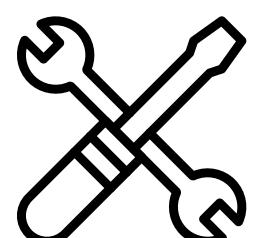
## STEP 4.1: SETUP



- We treat each class independently, assigning a different Learner to each of them
- At each round, for each class, we pull both a price and a bid
- Being a disaggregate scenario, we consider the regret as the sum of each class' regret

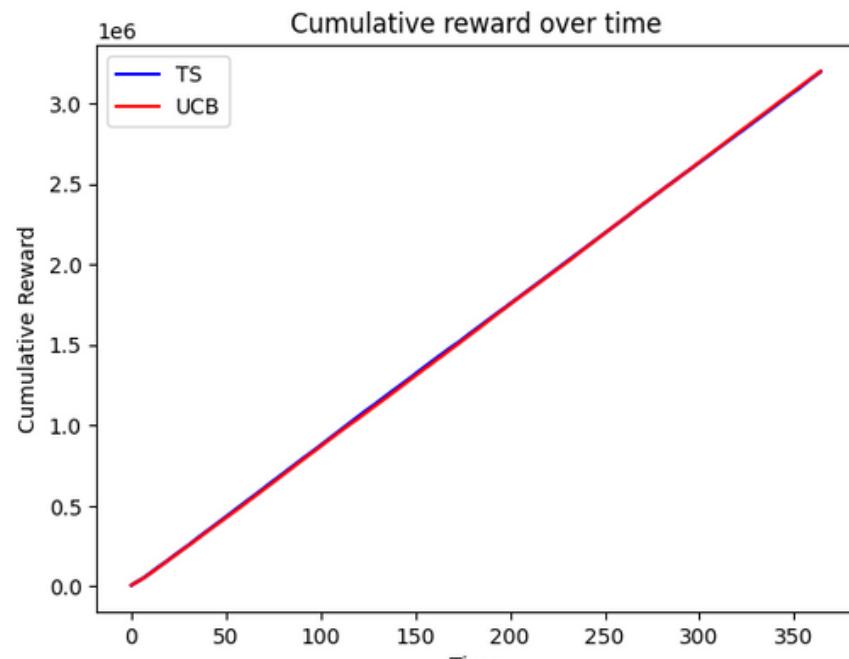


- The execution takes a lot of time
- To average the results feasibly, only 5 experiments are run over 365 days.
- UCB1 and TS are the two algorithms we are comparing

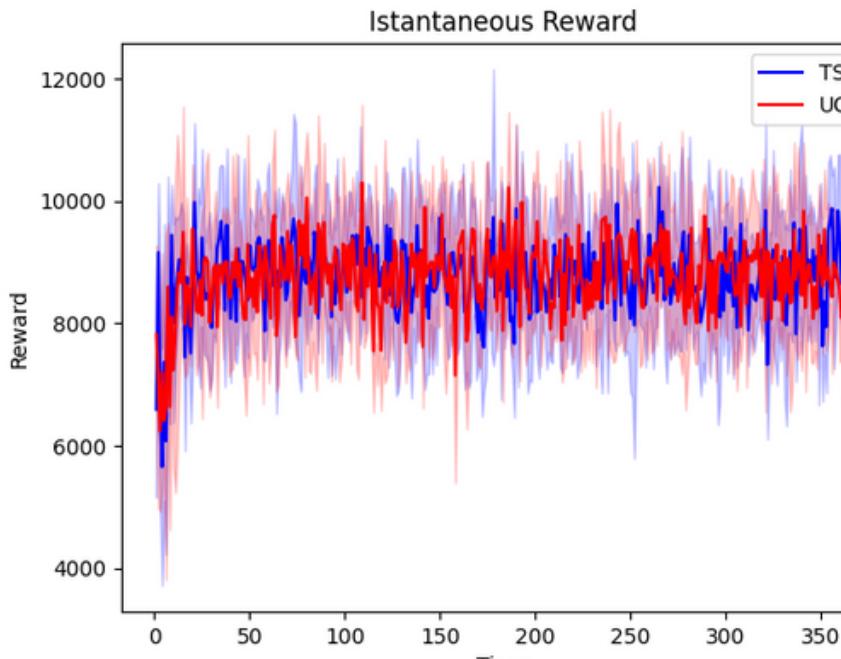


- The **Environment**, during each round, generates the reward based on both the price and the bid that have been pulled by the Learner
- The Learner has to learn both the optimal price and the two advertising curves  
*fate un check se è stato interpretato bene?*

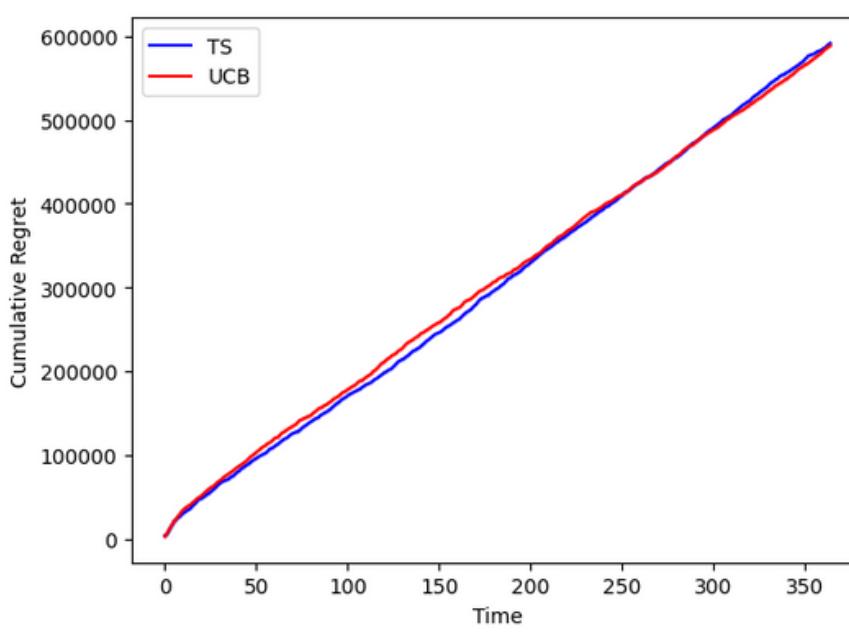
# STEP 4.1: RESULTS



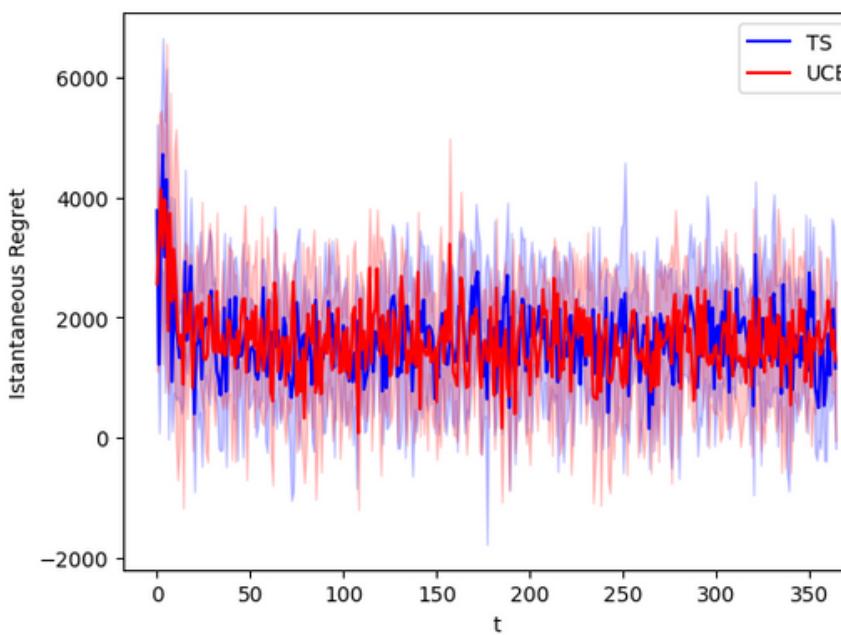
**Cumulative reward**



**Instantaneous reward**



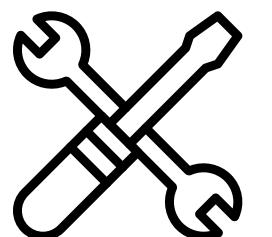
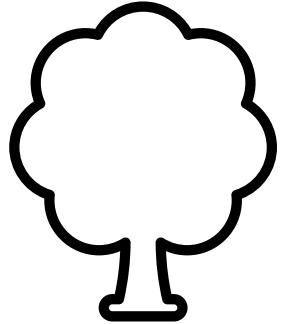
**Cumulative regret**



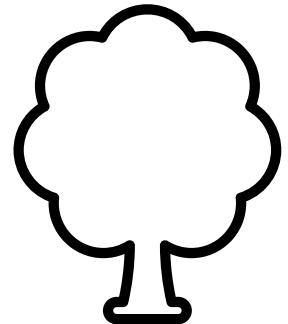
- A slightly sublinear regret has been achieved
- Both **TS** and **UCB1** get very similar results
- In particular, **TS** has a lower regret for the first 250 days, but then **UCB1** takes the lead
- The reward is twice the reward we had in STEP1

## STEP 4.2: SETUP

- In this scenario, we adopt the context generation algorithm
  - In the beginning, we have a general aggregated model
  - Each two weeks, the context generation algorithm is performed: new contexts will be introduced and some existing contexts could be removed
- 
- Every possible combination of features is explored
  - For each combination, we compute a probabilistic lower bound for the reward that we would get from that context
  - For each context, we determine the best price and the best bid
- 
- The **Context Manager** is responsible for creating and optimizing the context



## STEP 4.2: SETUP



- The probabilistic lower bound we use is the Hoeffding's bound, with a confidence of 95%

$$\bar{x} - \sqrt{-\frac{\log(\delta)}{2|Z|}}$$

- When a feature is chosen, we have to decide whether it is worth splitting. We split when the lower bound of the expected reward is not smaller than the lower bound of the expected reward when we do not split.
- Therefore, the splitting condition is

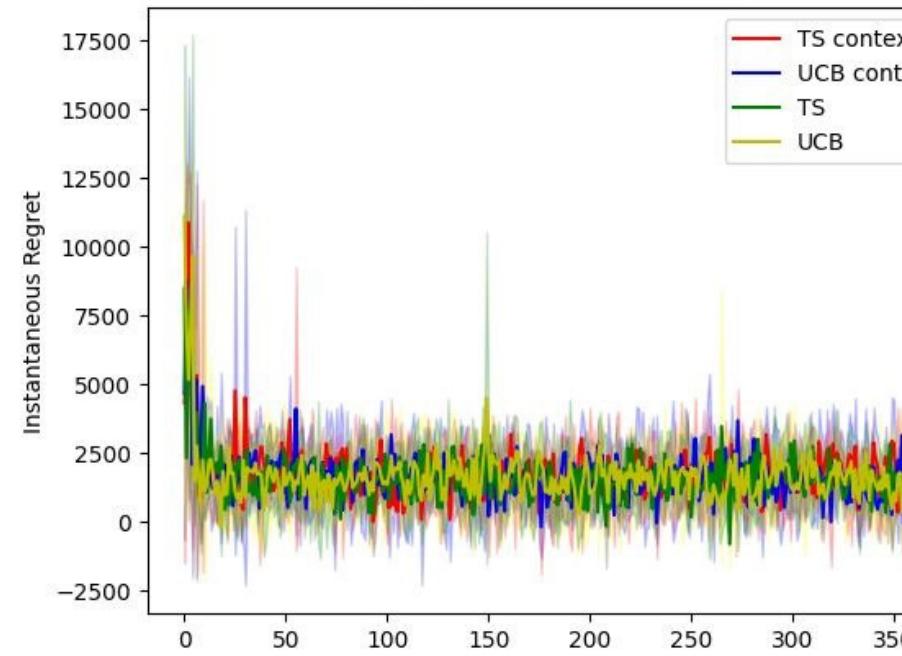
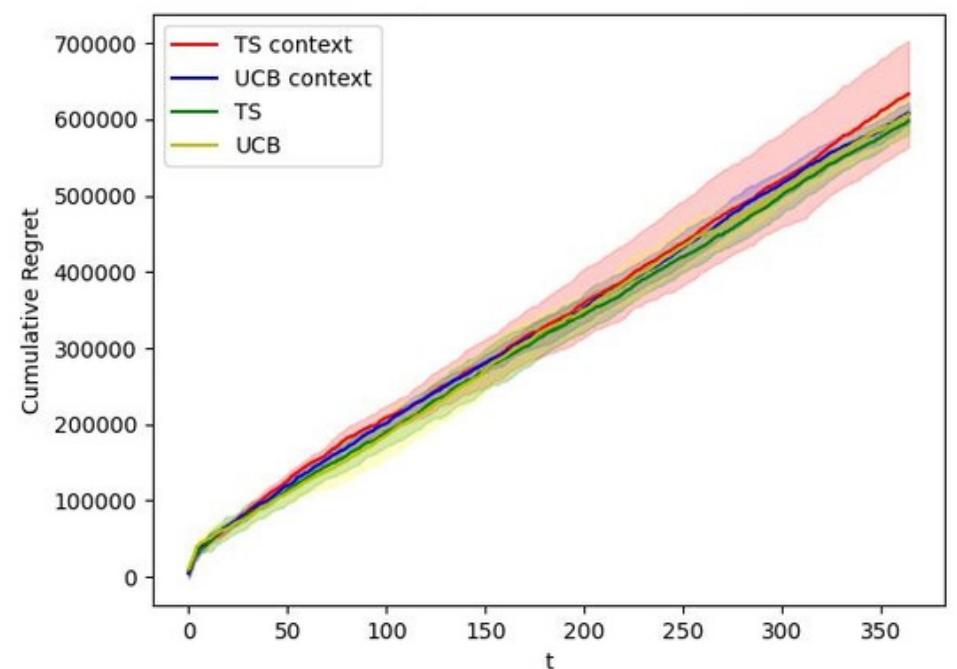
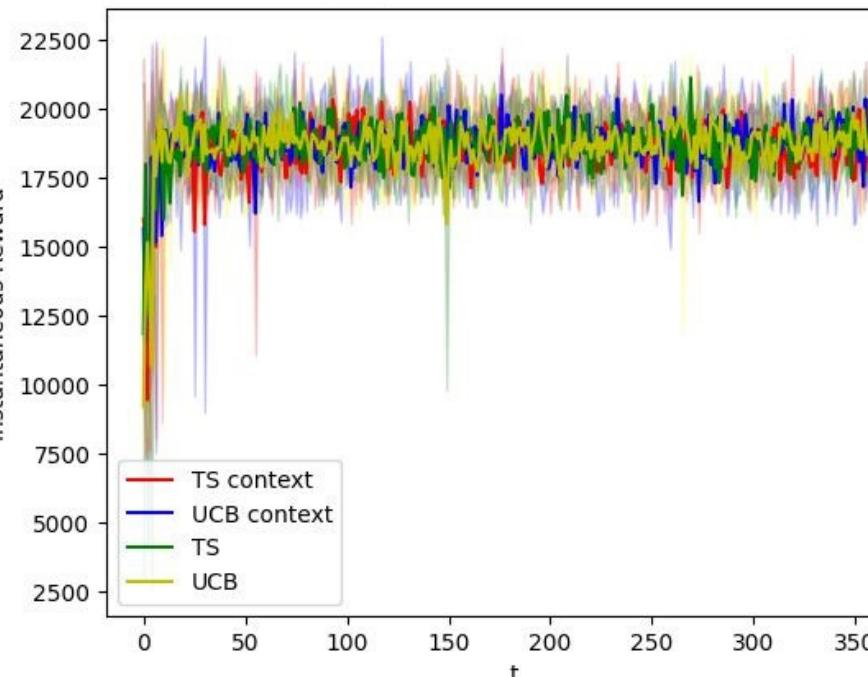
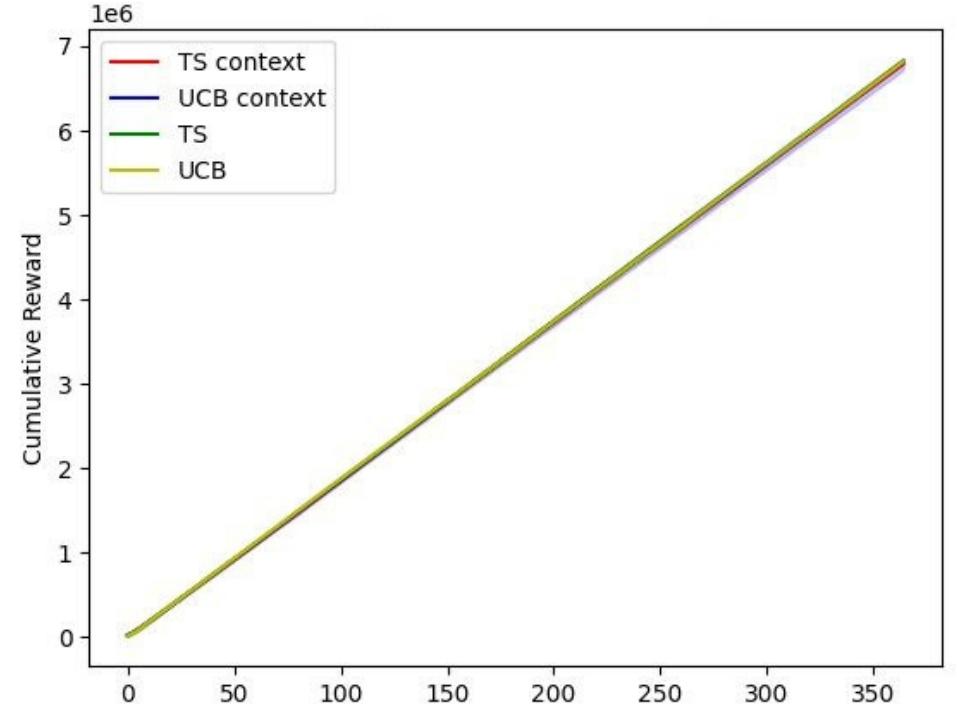
$$\underline{p}_{c_1} \underline{\mu}_{a_{c_1}^*, c_1} + \underline{p}_{c_2} \underline{\mu}_{a_{c_2}^*, c_2} \geq \underline{\mu}_{a_{c_0}^*, c_0}$$

C: the context structure. C0 is the original one, C1 and C2 are the newly generated ones.

p: the lower bound of the probability of occurrence of a user that belongs to the context C

mu: the lower bound of the reward of the optimal arm of context C

# STEP 4.2: RESULTS



- A sublinear regret has been achieved
- The cumulative reward for the deployed algorithms is similar
- Surprisingly, the contextual version of **TS** achieves the highest cumulative regret

- The reward is 5 times the reward we had in STEP1
- However, the regret is larger than in scenario 1. This makes sense because the context structure is unknown
- We expected to see more differences between the contextual and the non-contextual version, but the results are very similar

# STEP 5: DEALING WITH NON-STATIONARY ENVIRONMENTS WITH TWO ABRUPT CHANGES

## SCENARIO



ALL USERS BELONG  
TO CLASS C1



PRICING CURVES ARE UNKNOWN,  
NON-STATIONARY AND WITH  
THREE DIFFERENT PHASES



ADVERTISING CURVES  
ARE KNOWN

## REQUEST



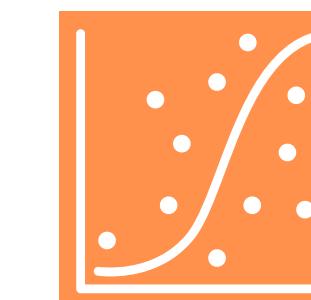
APPLY UCB1  
ALGORITHM WITH  
SLIDING WINDOWS



APPLY UCB1  
ALGORITHM WITH  
CHANGE DETECTION



SENSITIVITY ANALYSIS

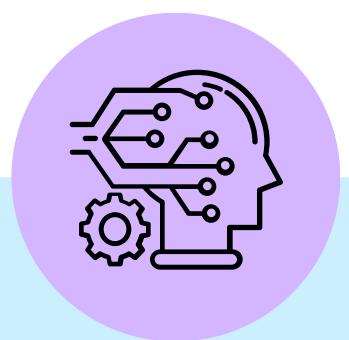


PLOT CUMULATIVE &  
INSTANTANEOUS REWARD /  
CUMULATIVE &  
INSTANTANEOUS REGRET

# 3 PHASES

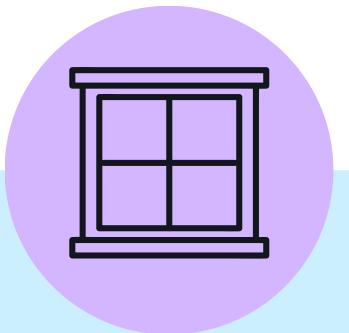


# 3 LEARNERS



## ***Normal UCB***

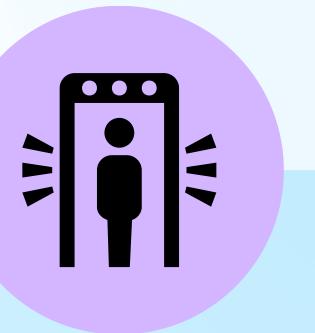
Normal Upper Confidence Bound algorithm



## ***Sliding Window UCB***

Parameter proportional to  $\sqrt{T}$

More weight to recent observations while still considering a reasonable amount of historical data



## ***Change Detection UCB (CUSUM)***

$M, \epsilon$  constant

consistent and reliable change detection behaviour

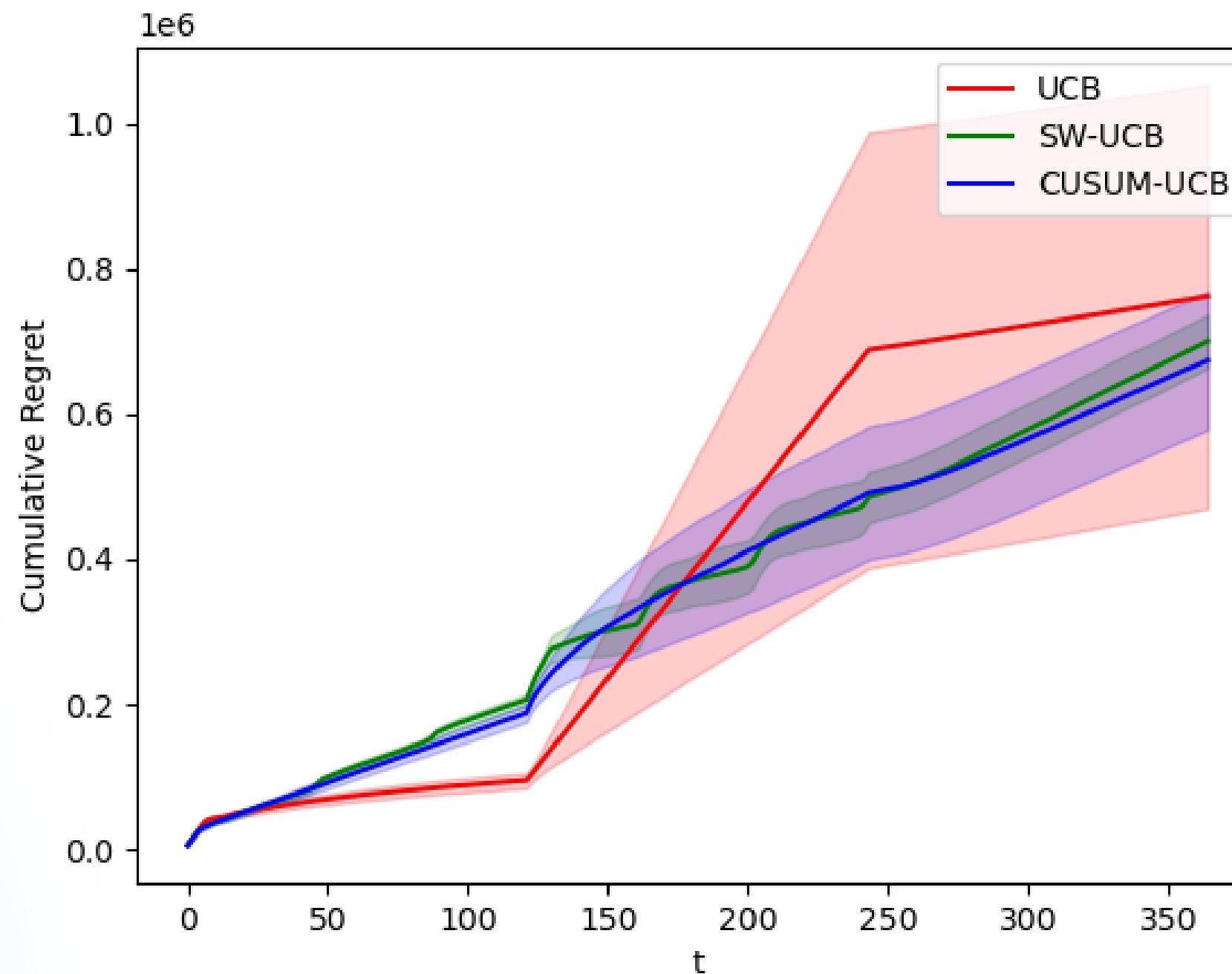
$$h \propto \log T$$

balance between detecting changes promptly and avoiding false positives

$$h \propto \frac{\sqrt{\log T}}{T}$$

balance between exploration and exploitation

# RESULTS



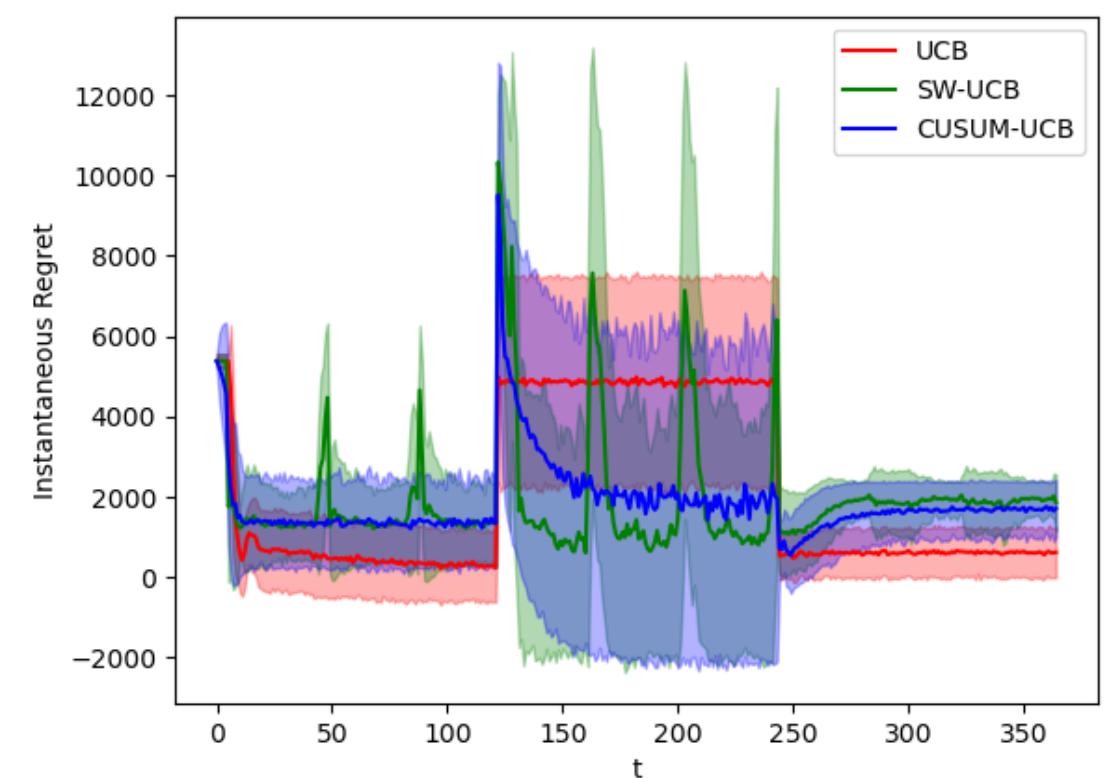
SW-UCB shows peaks that follow the flow of the window

1. Sample exit window -> exploration (infinite confidence)
2. Worst arm pulled once
3. Poor reward
4. Never pulled again per window

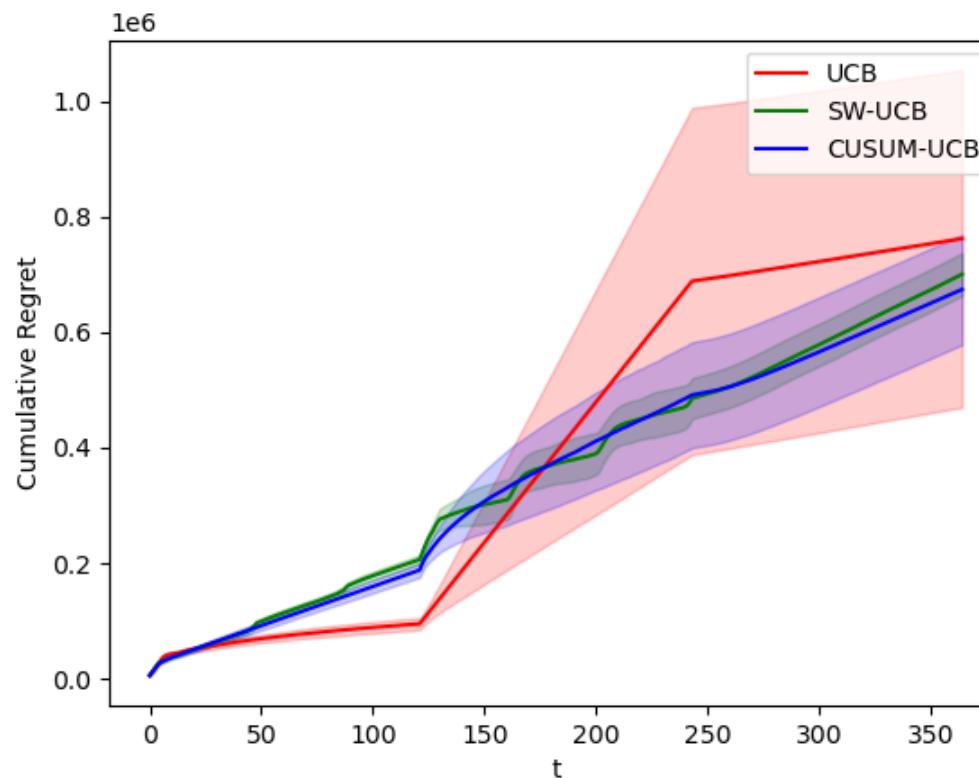
# RESULTS

From the considerations about the regret follow the plots about rewards

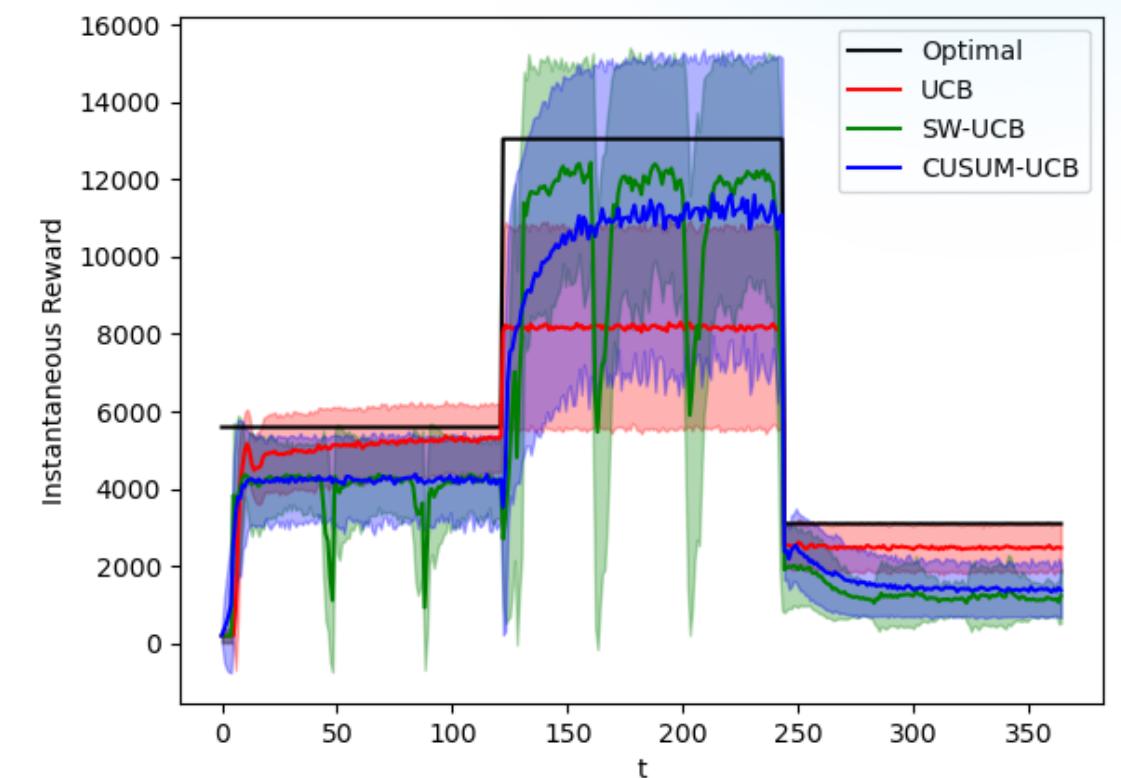
**Instantaneous Regret**



**Cumulative Reward**



**Instantaneous Reward**



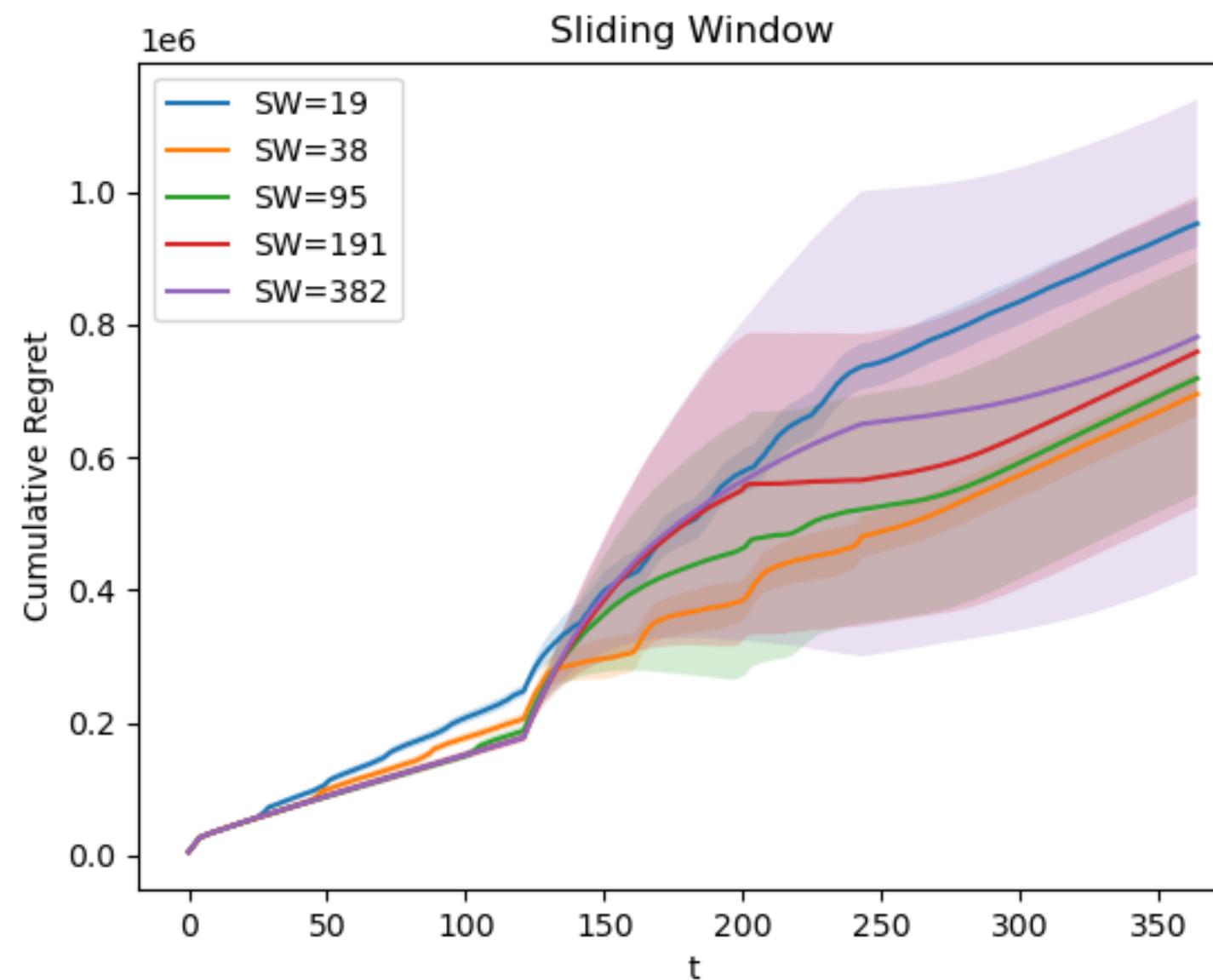
# ***SENSITIVITY ANALYSIS - SOFTWARE PARAMETERS***

## **Sliding Window Size**

It determines the number of recent observations or time steps considered. It is critical to the analysis's responsiveness, stability, noise sensitivity, and computing complexity.

# SENSITIVITY ANALYSIS - SLIDING WINDOW

$$\text{window\_sizes} = \{1, 2, 5, 10, 20\} * \sqrt{T}$$



- responds quickly to short-term changes
- more sensitive to abrupt shifts or anomalies
- more subject to noise and random oscillations
- potentially less trustworthy estimations
- smoothed and steady representation of the data
- limiting the impact of short-term volatility
- removal of random noise and outliers
- response to immediate changes is diminished

Medium sliding window sizes perform better than the largest and smallest ones in terms of regrets

# SENSITIVITY ANALYSIS - CUSUM PARAMETERS

**M**

## Sensitivity Parameter

The magnitude threshold used for change detection. It determines the number of consecutive observations needed to trigger a change detection event.

**eps**

## Threshold for Change Detection

The threshold for detecting a change in the observed data. It determines the minimum magnitude of change required to trigger a change detection event.

**h**

## Confirmation Threshold

The decision threshold for change detection. It determines the level of deviation required to trigger a change detection event.

**alpha**

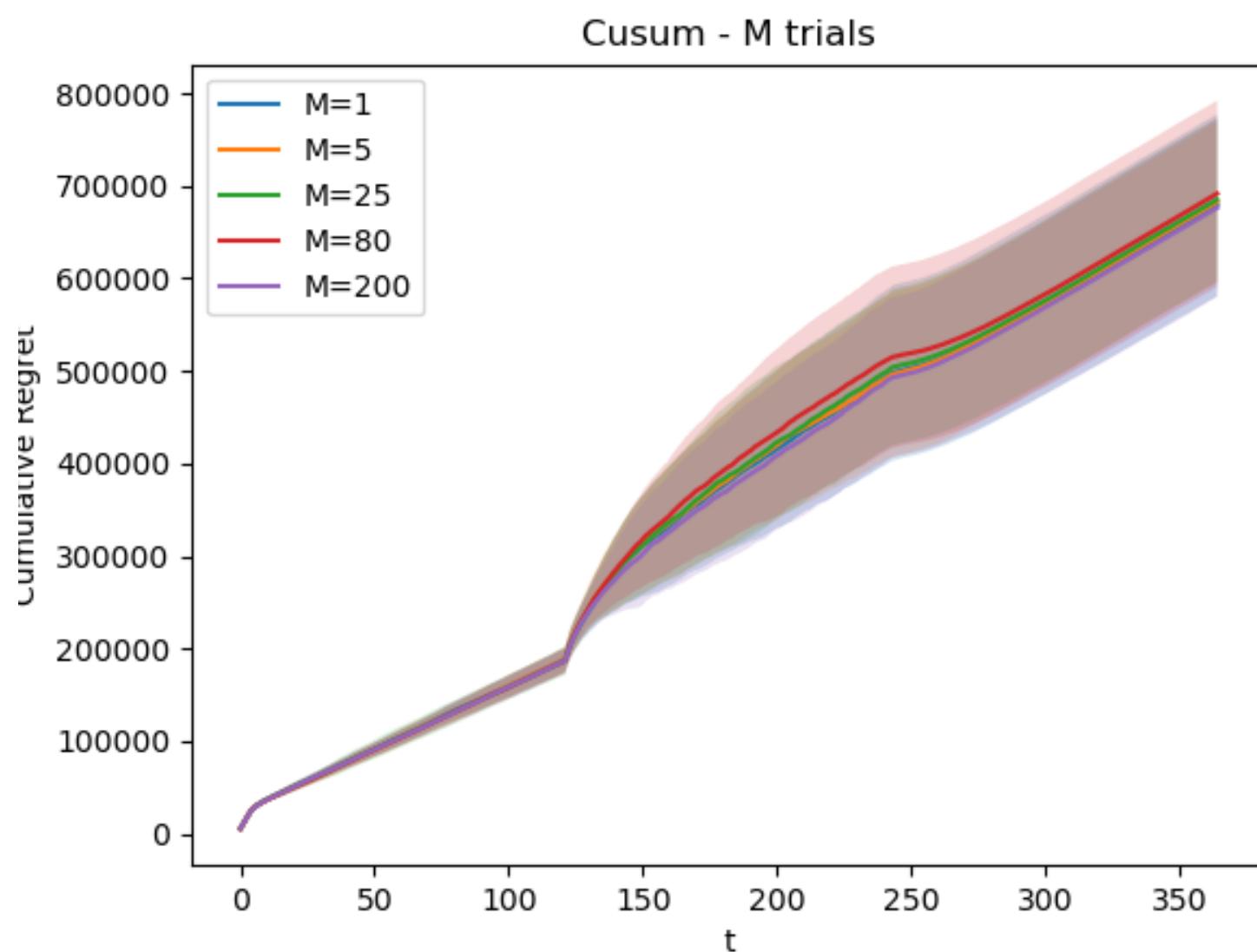
## Exploration Probability

The significance level or the type I error rate. It determines the threshold for considering a change as statistically significant.

# SENSITIVITY ANALYSIS - CUSUM ( $M$ )

$$M = \{1, 5, 25, 80, 200\}$$

*Sensitivity Parameter*



- more sensitive to small changes
- detect subtle variations quickly
- rapid adaptation to changes
- increase the possibility of false-positive detections



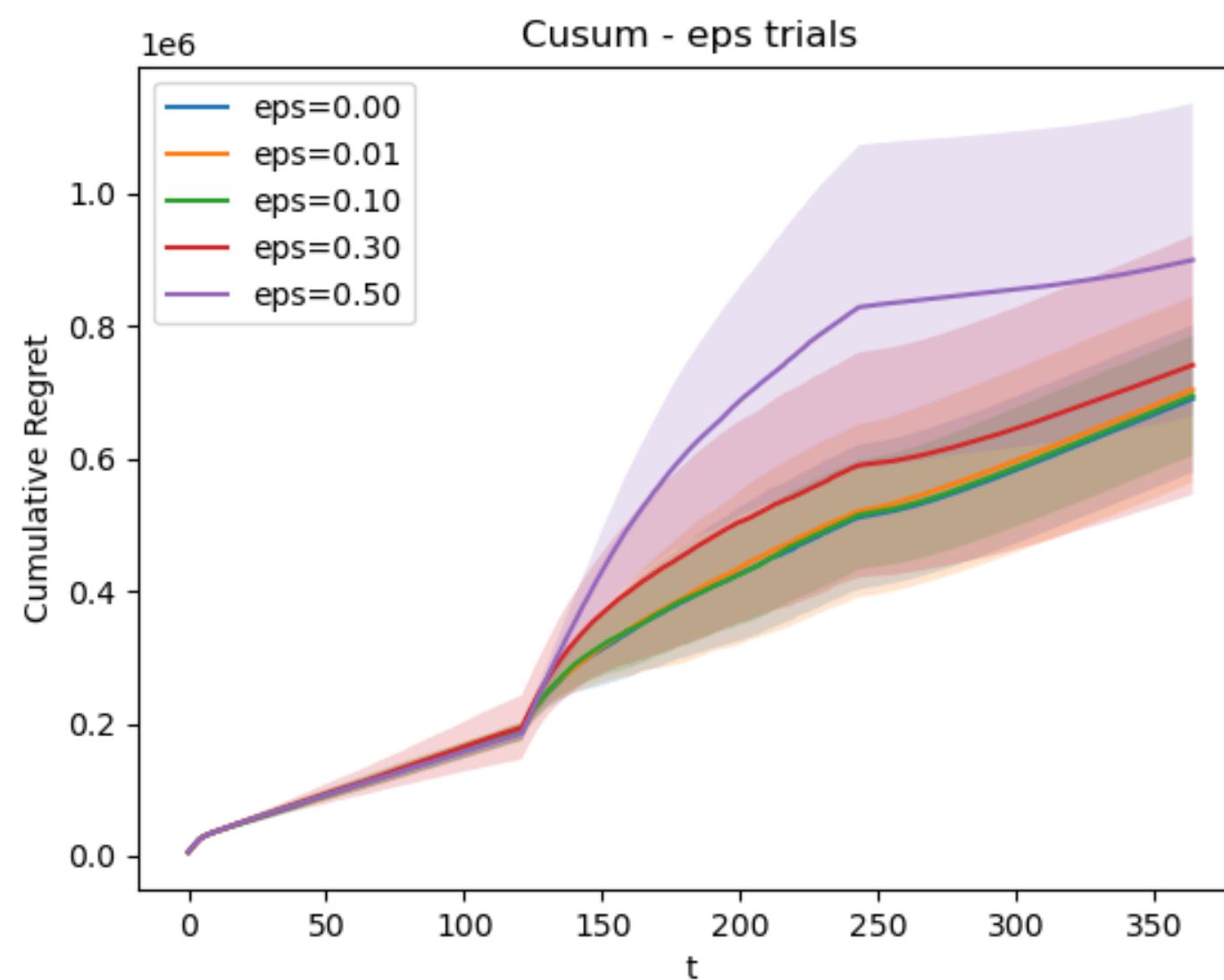
- needing a greater magnitude of change to trigger a detection
- minimises the likelihood of false-positive detections
- slower adaptability
- potentially ignoring smaller changes

No much difference between the different values.  $M$  as a parameter has less relevance and importance than others

# ***SENSITIVITY ANALYSIS - CUSUM (eps)***

$$\epsilon = \{0.001, 0.01, 0.1, 0.3, 0.5\}$$

*Threshold for Change Detection*



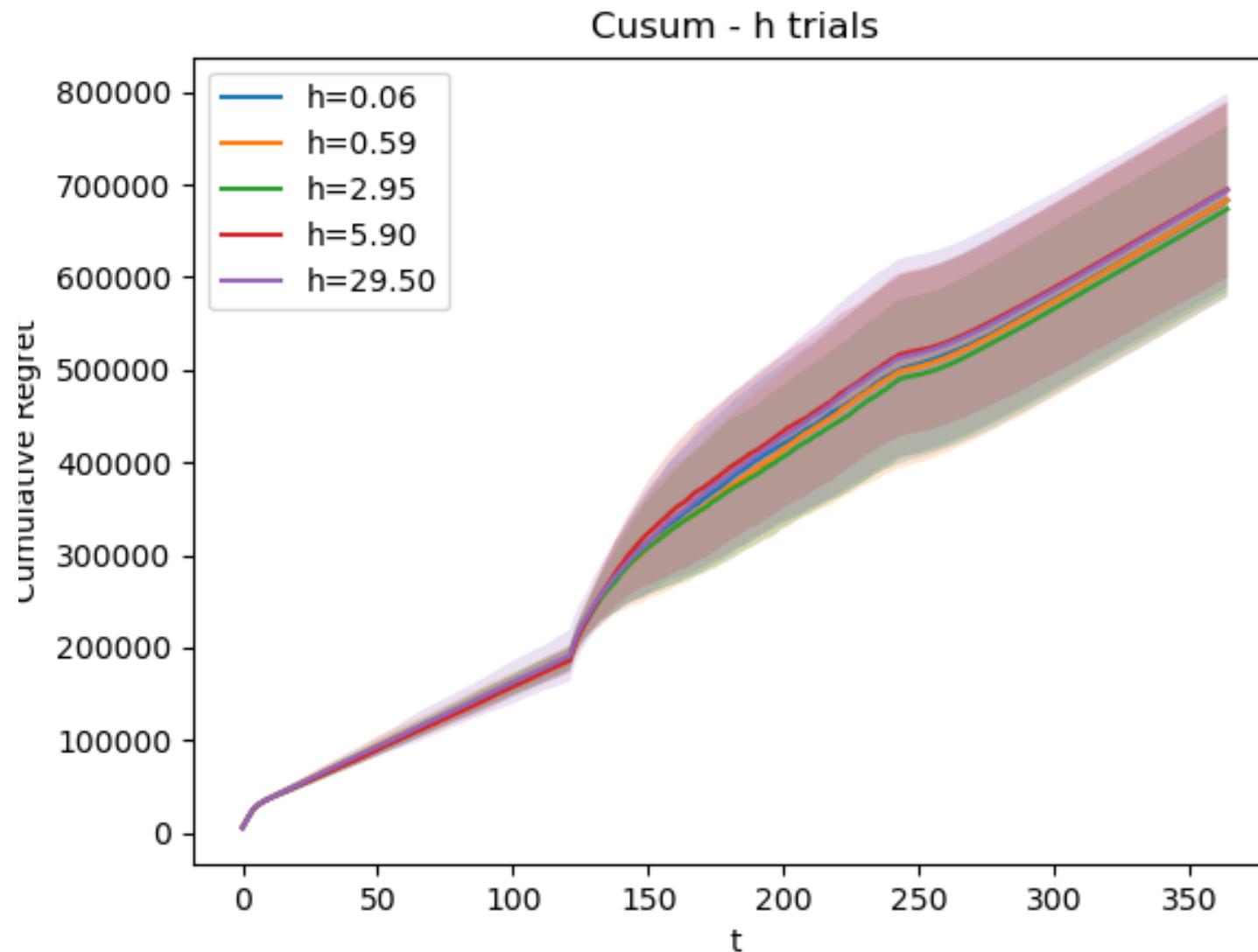
- sensitive to even minor changes in the data
- high responsiveness to identify tiny shifts
- may increase the risk of false-positive detections
- minor variations may be misidentified as substantial changes
- needing a greater magnitude of change to trigger a detection
- minimises the likelihood of false-positive detections
- reduce the sensitivity to minor changes
- potentially overlooking smaller changes

An higher regret is obtained from higher epsilon values, with a decreasing regret for smaller values.

# SENSITIVITY ANALYSIS - CUSUM ( $h$ )

$$h = \{0.01, 0.1, 0.5, 1, 5\} * \log T$$

**Confirmation Threshold**



- extremely sensitive to departures from expected behaviour
- detects tiny changes in observed data fast
- increase the likelihood of false-positive detections
- random fluctuations may be detected as substantial changes

- needing a higher amount of deviation to trigger a detection
- minimises the likelihood of false-positive detections
- reduce the sensitivity to minor changes
- detecting significant and noteworthy deviations is more critical

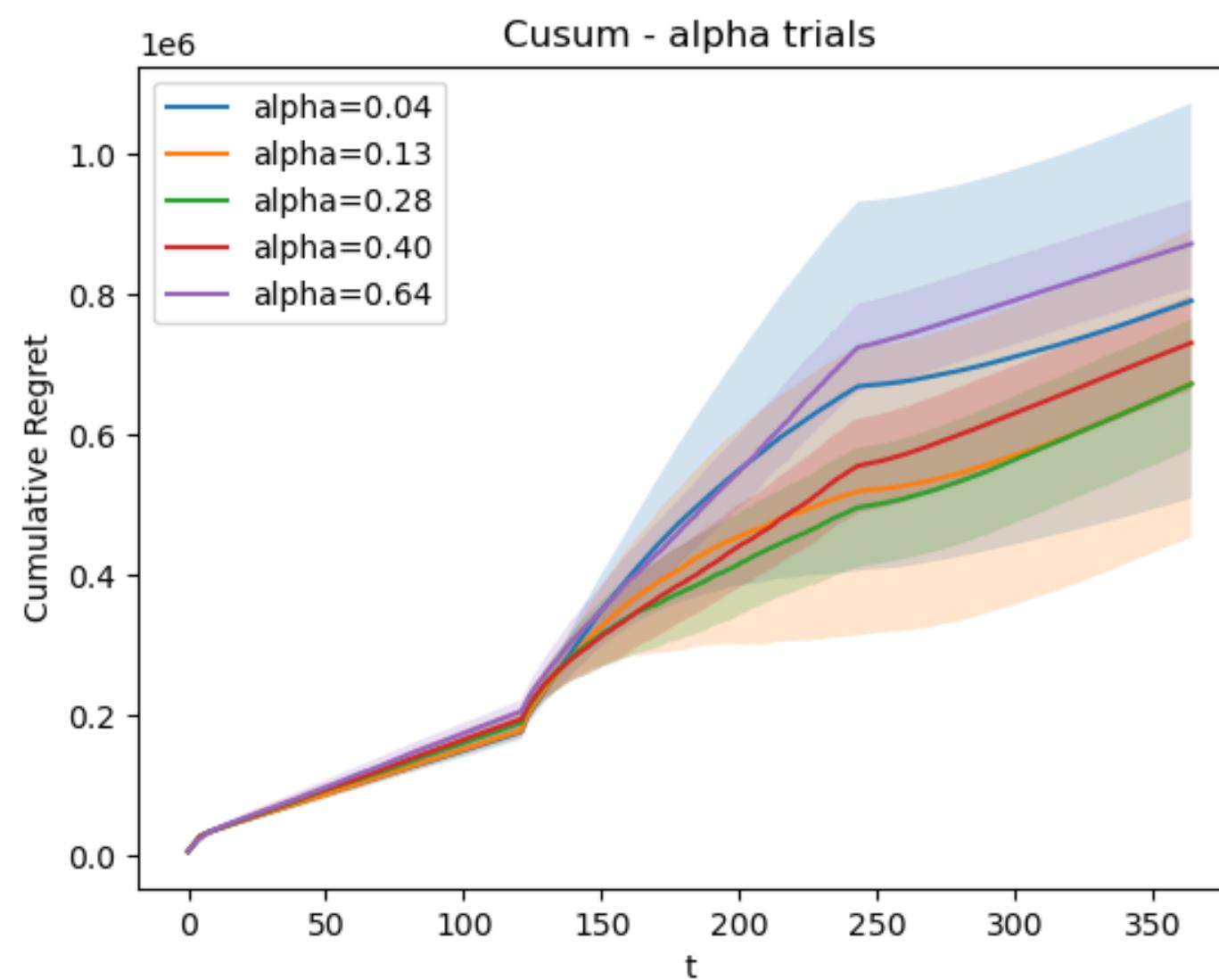


The values of  $h$  are all almost similar, underlining a rather marginal relevance of the parameter.

# ***SENSITIVITY ANALYSIS - CUSUM ( $\alpha$ )***

$$\alpha = \{0.1, 1, 5, 10, 25\} * \sqrt{\frac{\log(T)}{T}}$$

***Exploration Probability***



- stronger proof is required before a change
- minimising the possibility of false-positive detections
- reduce the sensitivity
- potentially causing to overlook certain true changes in the data
- more sensitive at identifying possible changes
- increases the possibility of false-positive detections
- trade-off between sensitivity and specificity

The best values are those in the middle. The exploration part works well when balanced, so if you explore too little, this leads to less convergence, as well as if you explore too much at random.

# STEP 6: DEALING WITH NON-STATIONARY ENVIRONMENTS WITH MANY ABRUPT CHANGES

## SETTING

ALL USERS BELONG TO CLASS C1



PRICING CURVES ARE UNKNOWN, NON-STATIONARY AND WITH THREE DIFFERENT PHASES



ADVERTISING CURVES ARE KNOWN



## SCENARIO 1

ENVIRONMENT IS NON-STATIONARY + FIXED BID



DEVELOP & APPLY EXP3 ALGORITHM



VERIFY THAT EXP3 PERFORMS WORSE THAN UCB1 NON-STATIONARY

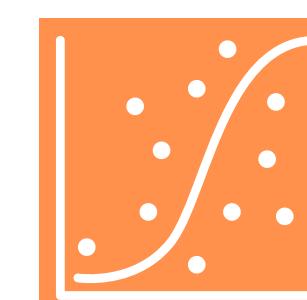


## SCENARIO 2

ENVIRONMENT HAS AN HIGHER DEGREE OF NON-STATIONARITY



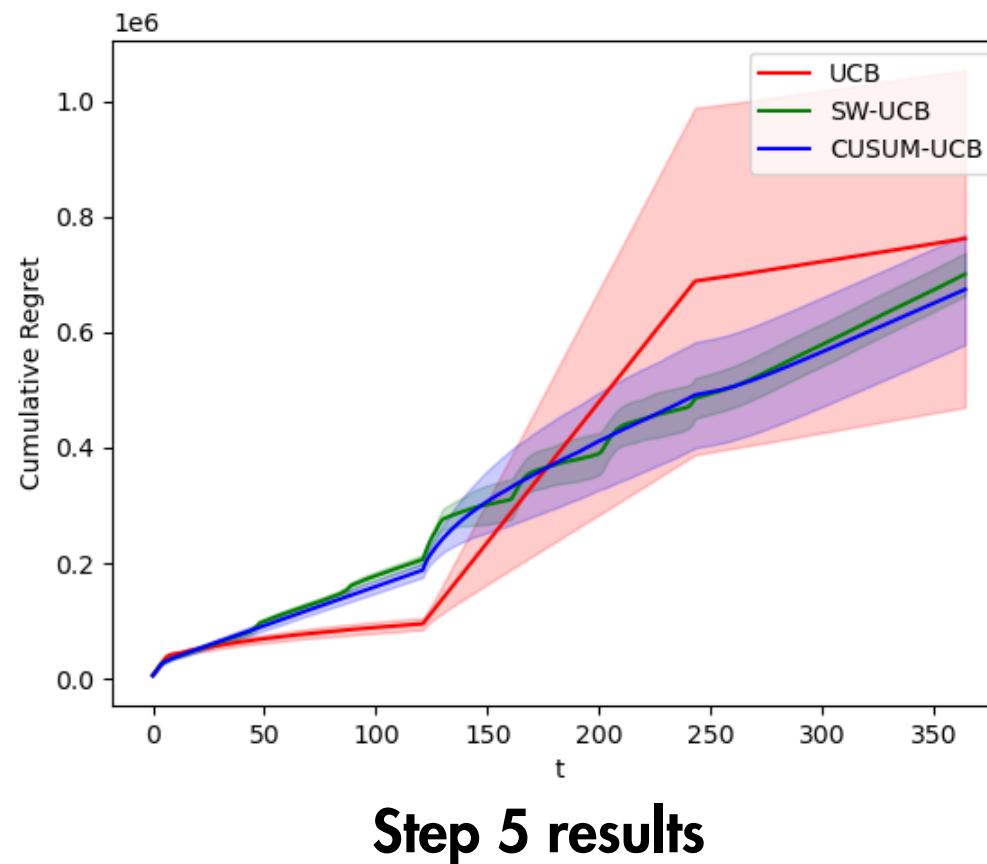
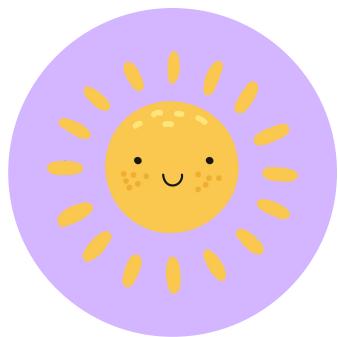
EXP3 UCB



APPLY EXP3, UCB1 & THE TWO NON-STATIONARY FLAVORS OF UCB1

VERIFY THAT EXP3 OUTPERFORMS THE NON-STATIONARY VERSION OF UCB1

# 3 PHASES: GETTING BETTER



- We are now going to simplify the previous scenario while trying to improve our results
- In this new setting, the bid is fixed. We choose the optimal bid value (1.25), computed using the Clairvoyant Algorithm
- Moreover, we introduce a new algorithm that should perform better in non-stationary environments

# Introducing: EXP3

- EXP3 is designed to work for MAB problems
- It actively explores the arms, to improve its knowledge about them
- The exploration is based on a probability distribution over the arms ("action probability")
- At each time step, the pulled arm is chosen upon the action probability

$$p_i(t) = (1 - \gamma) \frac{w_i(t)}{\sum_{j=1}^k w_j(t)} + \frac{\gamma}{N}$$

N: number of arms

Gamma: exploration (hyper)parameter [0, 1]

w(t): weights associated to each arm at a given time step

- The weight is computed as

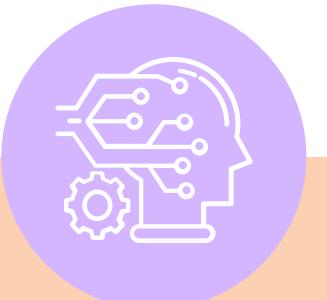
$$w_{j_t}(t+1) = w_{j_t}(t) \cdot e^{\frac{\gamma}{N} \hat{x}_{i_t}}$$

j: arm

x: expected reward from that arm

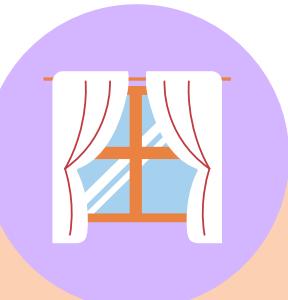
- Due to its exploratory nature, it should be able to adapt better to abrupt changes

# 4 LEARNERS



## **Normal UCB**

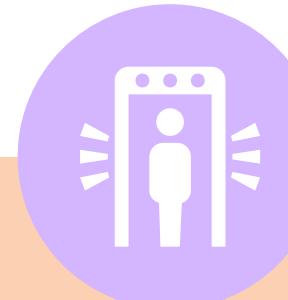
Normal Upper Confidence Bound algorithm



## **Sliding Window UCB**

Parameter proportional to  $\sqrt{T}$

More weight to recent observations while still considering a reasonable amount of historical data



## **Change Detection UCB (CUSUM)**

$M, \epsilon$  constant  
consistent and reliable change detection behaviour

$$h \propto \log T$$

balance between detecting changes promptly and avoiding false positives

$$h \propto \frac{\sqrt{\log T}}{T}$$

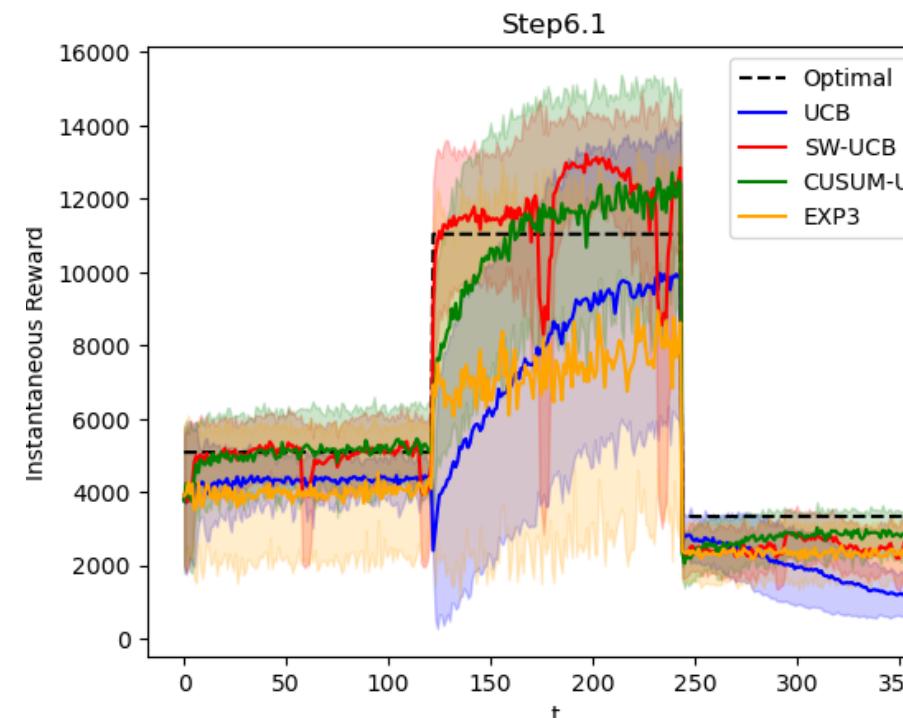
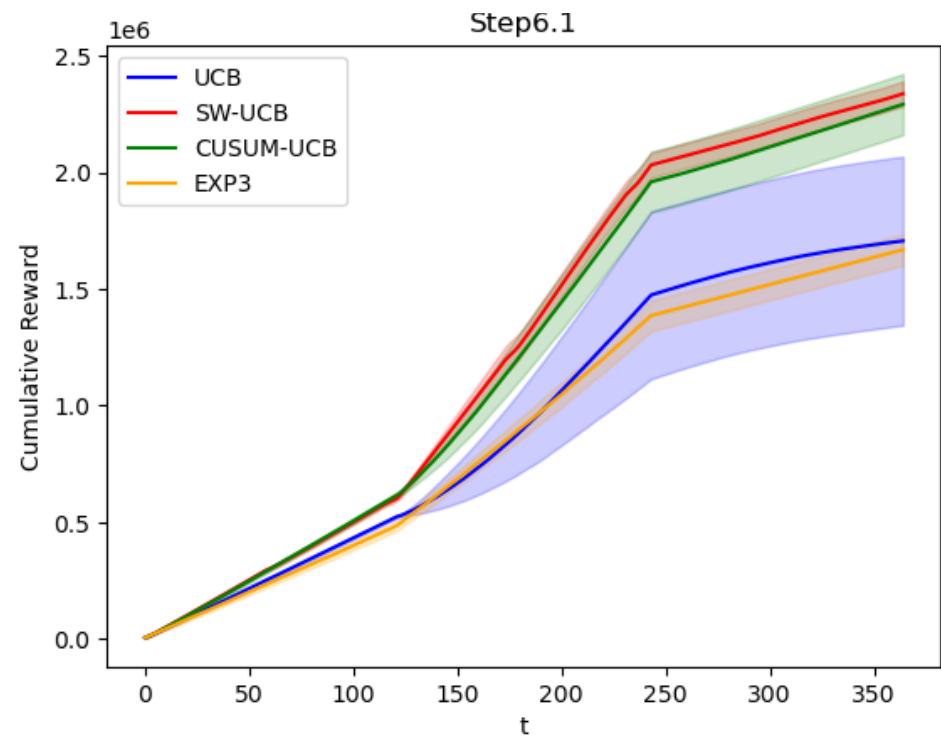
balance between exploration and exploitation



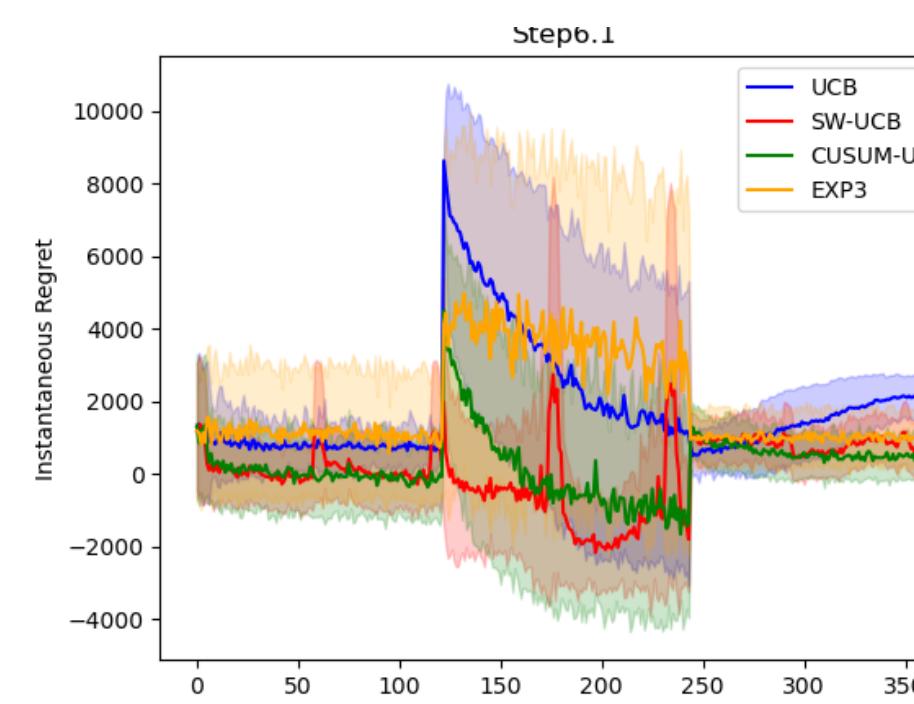
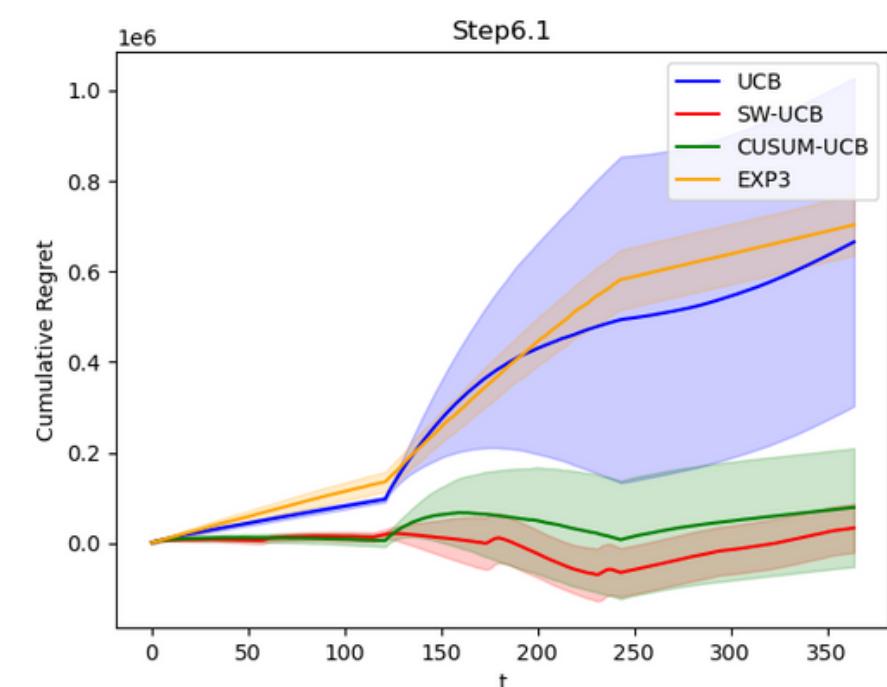
## **EXP3**

Exponential-weight algorithm for Exploration and Exploitation

# STEP 6.1: RESULTS

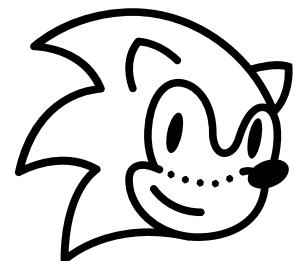


**Cumulative reward**



- EXP3 got the worst performance out of these algorithms, even by tuning its gamma parameter.
- If we compare the instantaneous rewards for UCB1 and EXP3 it is evident that UCB1 learns way faster
- EXP3's weights have been initialized uniformly. Other initializations could help improve the algorithm's performance
- This behavior can be explained by the fact that EXP3 explores the environment a lot and improvements can be seen when the changes are more frequent
- SW-UCB imposes itself as the winner of Step 6.1

# 3 PHASES: GETTING BETTER



- We said that EXP3 is really about exploration
- What if the environment changes much faster?

# 5 PHASES



## NEW YEAR

Regular behaviours

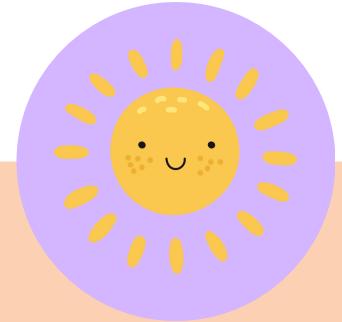
Best price: 150€



## SPRING

Lower conversion rates

Best price: 250€



## SUMMER

Way too hot

Best price: 150€



## AUTUMN

Sales!

Best price: 100€



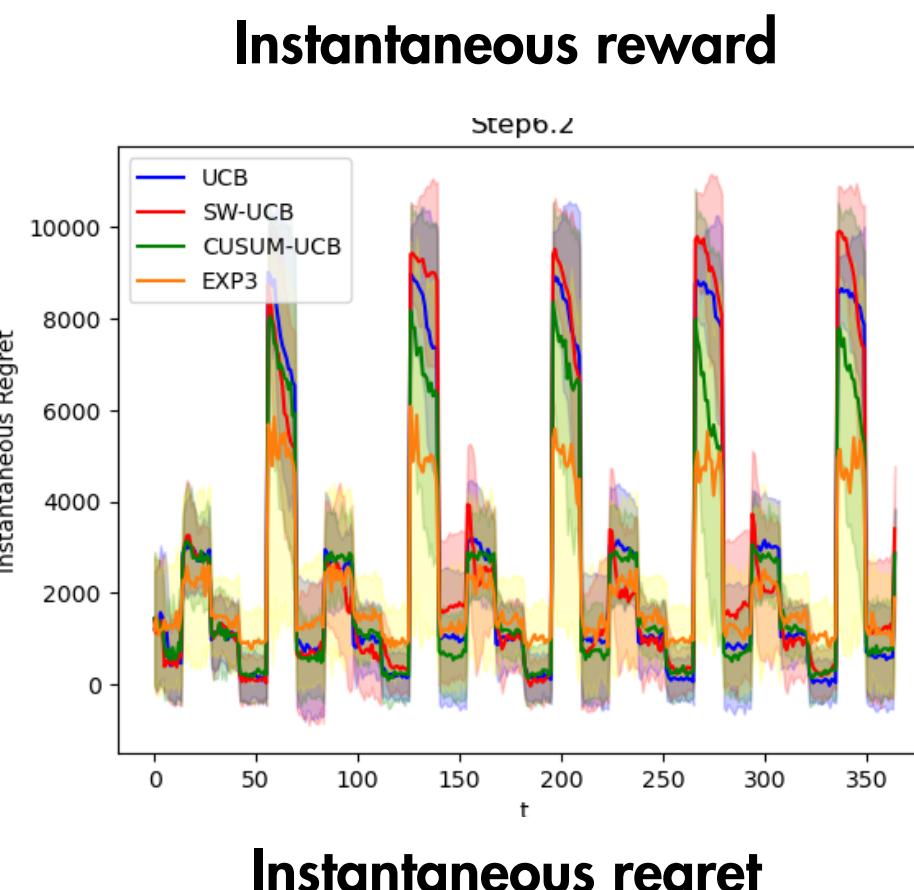
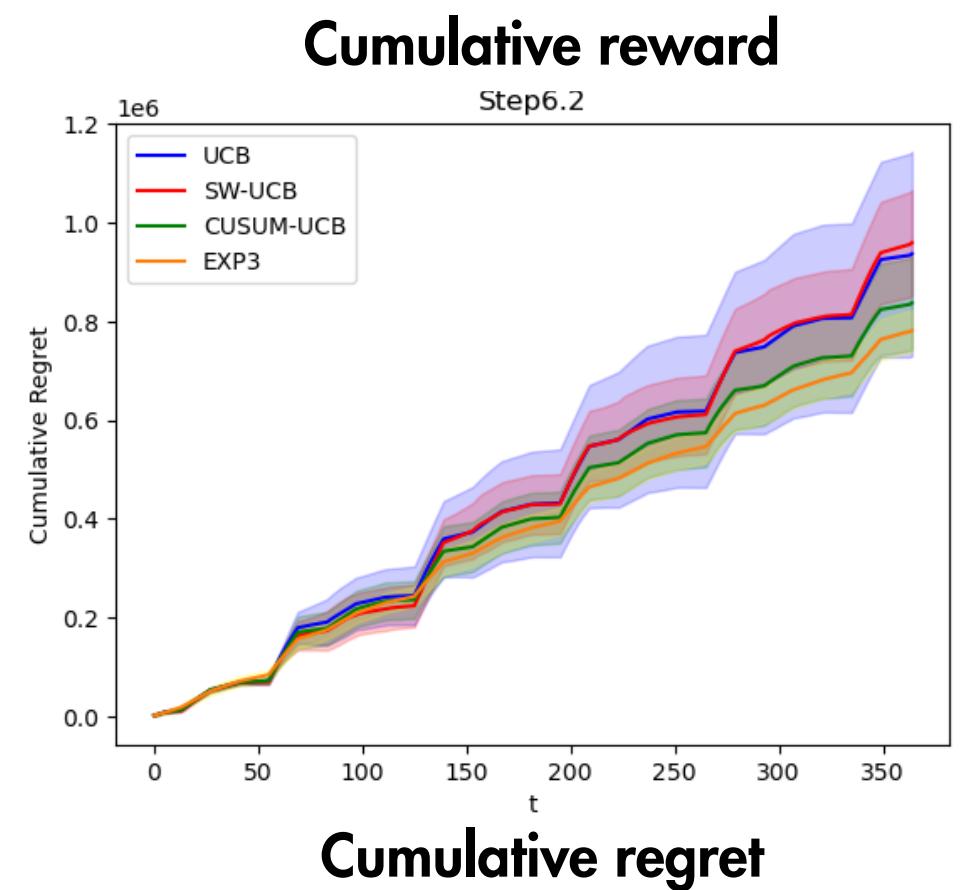
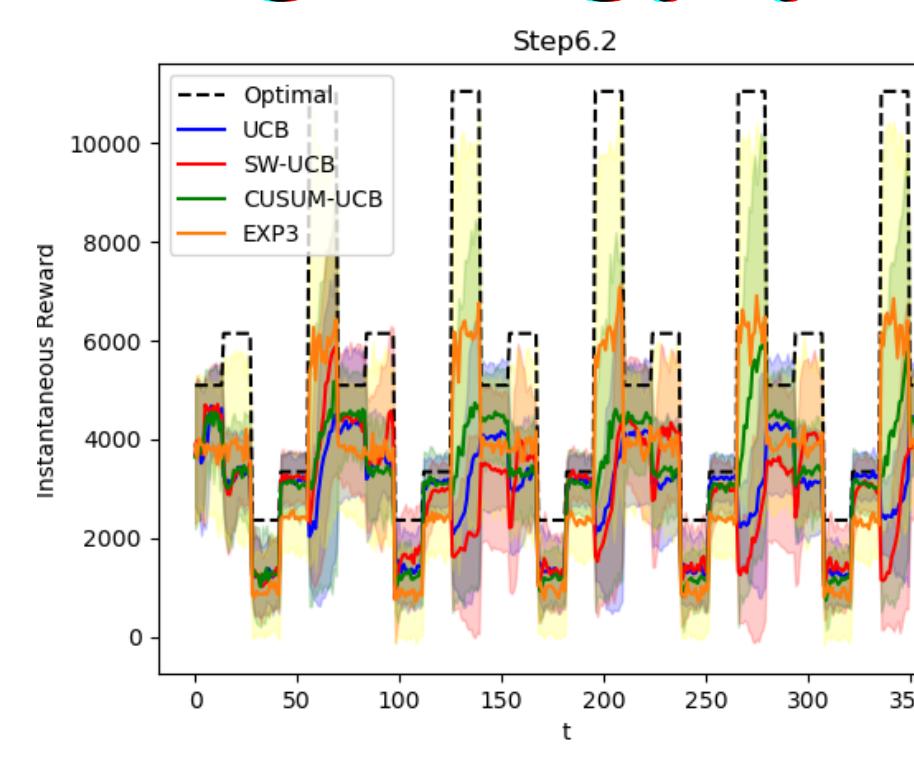
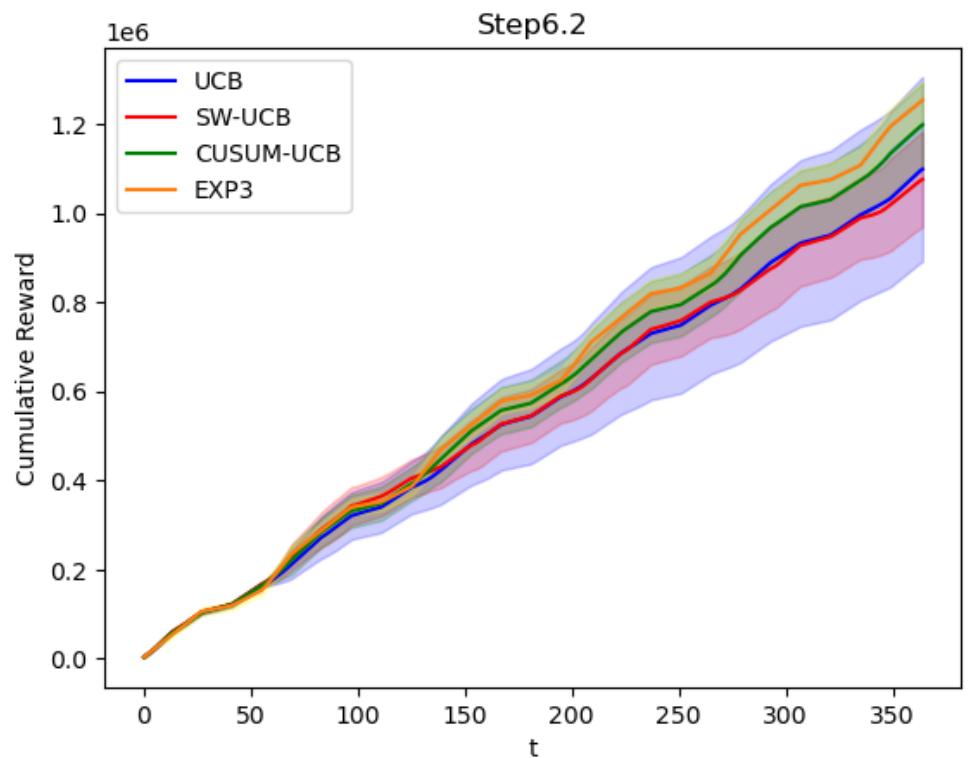
## CHRISTMAS

Higher prices due to gifts

Best price: 200€

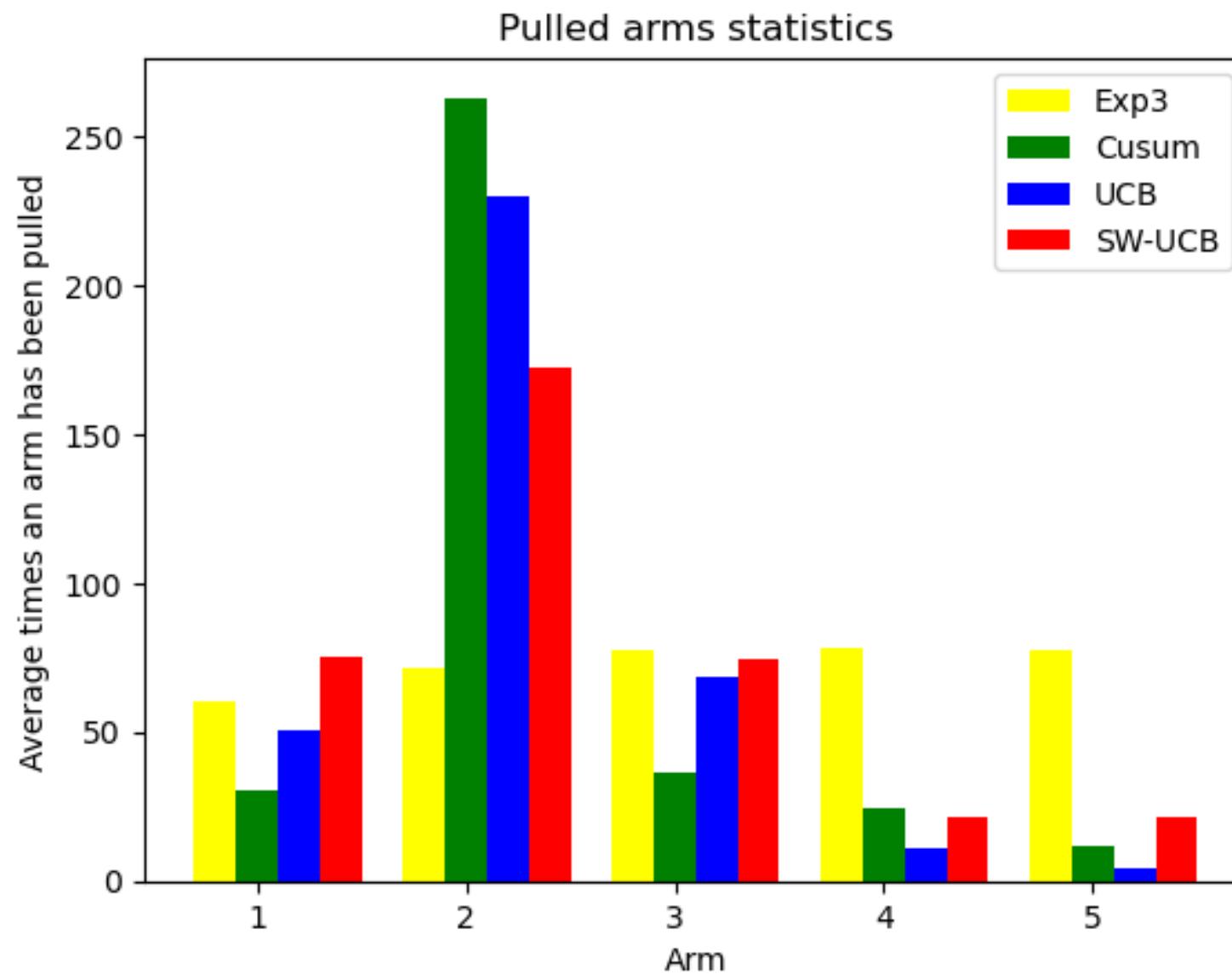
New Year, Sales-Season and Christmas use the same conversion rates used in Step5.

# STEP 6.2: RESULTS



- EXP3 brought us the highest cumulative reward and the lowest cumulative regret, implying that it correctly balanced exploration and exploitation.
- EXP3's gamma has been manually tuned very few times: there is a margin for improvement
- CumSumUCB has similar performances w.r.t. EXP3, while the other two algorithms could not achieve optimal results
- The various abrupt changes have decreased the overall cumulative reward w.r.t to the previous scenario

## STEP 6.2: ABOUT THE EXPLORATION



- EXP3 has pulled the arms almost equally
- The opposite happened for CumSumUCB, for which arm 2 is definitely the most pulled one

# THANK YOU!

Valeria Amato - 10641790  
Beatrice Insalata - 10708628  
Emanuele Paci - 10681377  
Matteo Pancini - 10656944  
Andrea Riboni - 10699906



POLITECNICO  
MILANO 1863