

# Mr.CAS— A Minimalistic (pure) *Ruby* CAS for Fast Prototyping and Code Generation

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## Abstract

There are complete **Computer Algebra System** (CAS) systems on the market with complete solutions for manipulation of analytical models. But exporting a model [that implements specific algorithms on specific platforms](#), for target language [or for particular numerical library](#), is often a rigid procedure that requires manual post-processing, even with a good software. This work presents a *Ruby* library that exposes core CAS capabilities—i.e. simplification, substitution, evaluation, etc. [The library aims at programmers that need to rapidly prototype and generate numerical code](#) for different target languages, [while keeping separated mathematical expression from the code generation rules, where best practices for numerical conditioning are implemented](#). The library is written in pure *Ruby* language and is compatible with most *Ruby* interpreters.

*Keywords:* CAS, code-generation, Ruby

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## 1. Motivation and significance

*Ruby* [1] is a purely object-oriented scripting language designed in the mid-1990s by Yukihiro Matsumoto, internationally standardized since 2012 as ISO/IEC 30170.

With the advent of the *Internet of Things*, a compact version of the *Ruby* interpreter called *mRuby* (*eMbedded Ruby*) [2] has been published on *GitHub* by Matsumoto, in 2014. The new interpreter is a lightweight implementation, aimed at both low power devices and PC, and complies with the standard[3]. *mRuby* has a completely new API, and it is designed to be embedded in complex projects as a front-end interface—e.g., a numerical optimization suite may use *mRuby* to for problem definition.

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12 The *Ruby* code-base exposes a large set of utilities in core and standard  
 13 libraries, that can be furthermore expanded through third party libraries,  
 14 or *gems*. Among the large number of available gems, *Ruby* still lacks an  
 15 **automatic symbolic differentiation** (ASD) [4] engine that handles basic  
 16 computer algebra routines, compatible with all different *Ruby* interpreters.

17 Nowadays *Ruby* is mainly known thanks to the web-oriented *Rails* frame-  
 18 work. Its expressiveness and elegance though make it intriguing for use in  
 19 the scientific/technical field. An ASD-capable gem would prove a fundamen-  
 20 tal step in this direction, including the support for flexible code generation  
 21 for high-level software—e.g., IPOPT [5, 6].

22 *Mr.CAS*<sup>1</sup> is a gem implemented in pure *Ruby* that supports symbolic  
 23 differentiation (SD) and some computer algebra operations [7]. The library  
 24 aims at:

- 25 • support [programmers in](#) rapid prototyping of numerical algorithms and  
 26 in code generation, even for different target languages;
- 27 • when dealing with [implementation of](#) mathematical models [in numeri-](#)  
 28 [cal algorithms](#), allow a clean separation between conditioning rules for  
 29 [numerical instabilities and actual mathematical formulations](#), in order  
 30 to support [generation of code that is more robust with respect to issue](#)  
 31 [that can be introduced by specific applications](#);
- 32 • create a complete open-source CAS system for the standard *Ruby* lan-  
 33 guage, as a long-term effort.

34 Other CAS libraries for *Ruby* are available:

35 ***Rucas*** [8], ***Symbolic*** [9] : milestone gems, yet at early stage and with dis-  
 36 continued development status. Both offer basic simplification routines,  
 37 although they lack differentiation.

38 ***Symengine*** [10] : is a wrapper of the *symengine* C++ library. The back-  
 39 end library is very complete, but it is compatible only with the *vanilla*  
 40 *C Ruby* interpreter and has several dependencies. At best of Author  
 41 knowledge, at the moment it seems not working using the *Ruby 2.x*  
 42 interpreter.

43 In Section 2, *Mr.CAS* containers and tree structure are explained in detail  
 44 and applied to basic CAS tasks. In Section 3, two examples on how to  
 45 use the library as code generator or as interface are described. Finally, the

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<sup>1</sup>Minimalistic Ruby Computer Algebra System

46 reasons behind the implementation and the long term desired impact are  
 47 depicted in Section 4. All code listings are available at [http://bit.ly/Mr\\_](http://bit.ly/Mr_CAS_examples)  
 48 [CAS\\_examples](http://bit.ly/Mr_CAS_examples).

## 49 2. Software description

### 50 2.1. Software Architecture

51 *Mr.CAS* is an object oriented ASD gem that supports some computer  
 52 algebra routines such as *simplifications* and *substitutions*. When gem is re-  
 53 quired, it overloads methods of `Fixnum` and `Float` classes, making them  
 54 compatible with fundamental symbolic classes.

55 Each symbolic expression (or operation) is the instance of an object, that  
 56 inherits from a common virtual ancestor: `CAS::Op`. An operation encaps-  
 57 ulates sub-operations recursively, building a tree, that is the mathematical  
 58 equivalent of function composition:

$$(f \circ g) \tag{1}$$

59 When a new operation is created, it is appended to the tree. The num-  
 60 ber of branches are determined by the parent container class of the current  
 61 symbolic function. There are three possible containers:

62 **`CAS::Op`** single sub-tree operation—e.g.  $\sin(\cdot)$ .

63 **`CAS::BinaryOp`** dual sub-tree operation—e.g. exponent  $x^y$ —that inherits  
 64 from `CAS::Op`.

65 **`CAS::NaryOp`** operation with arbitrary number of sub-tree—e.g. sum  $x_1 +$   
 66  $\dots + x_N$ —that inherits from `CAS::Op`.

67 Fig. 1 contains a graphical representation. The different kind of containers  
 68 allows to introduce some properties—i.e. *associativity* and *commutativity* for  
 69 sums and multiplications [11]. Each container exposes the sub-tree as in-  
 70 stance properties. Basic containers interfaces and inheritances are shown in  
 71 Fig. 2. [For a complete overview of all classes and inheritance, please refer to](#)  
 72 [software documentation](#).

73 Terminal leaves of the graph are the classes `CAS::Constant`, `CAS::Va-`  
 74 `riable` and `CAS::Function`. The first models a simple numerical value,  
 75 while the second represents an independent variable, that can be used to  
 76 perform derivatives and evaluations, and the latter is a prototype of implicit  
 77 functions. As for now, those leaves exemplify only real scalar expressions,  
 78 with definition of complex, vectorial and matricial extensions as milestones  
 79 for the next major release.

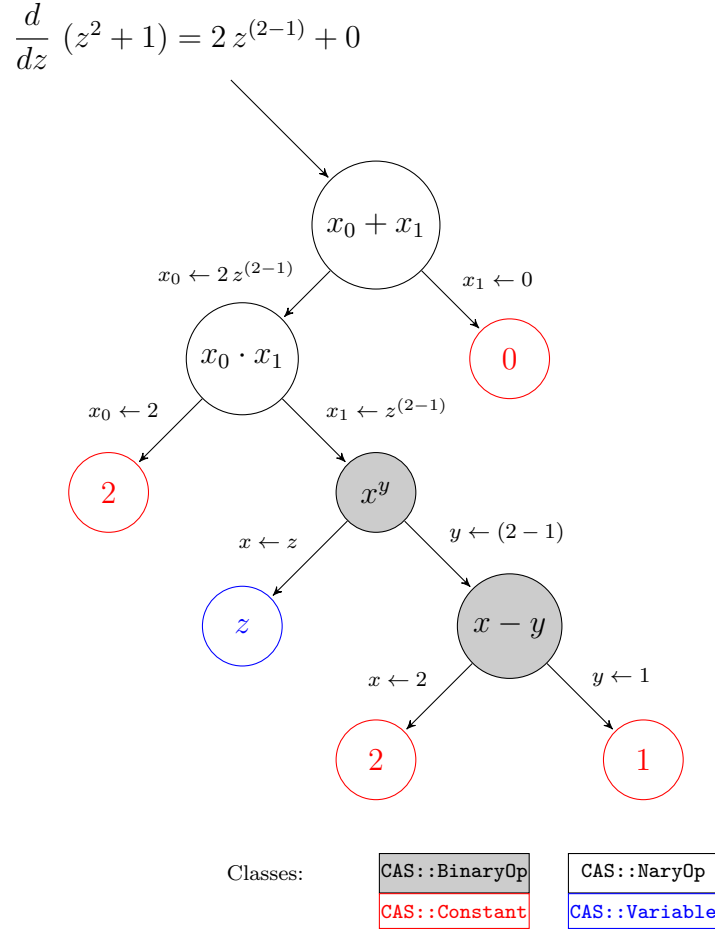


Figure 1: Tree of the expression derived in Listing 1

80 SD (`CAS::Op#diff`) crosses the graph until it reaches ending nodes. A  
 81 terminal node is the starting point for derivatives accumulation, the mathe-  
 82 matical equivalent of the chain rule:

$$(f \circ g)' = (f' \circ g) g' \quad (2)$$

83 The recursiveness is used also for simplifications (`CAS::Op#simplify`), sub-  
 84 stitutions (`CAS::Op#subs`), evaluations (`CAS::Op#call`) and code genera-  
 85 tion.

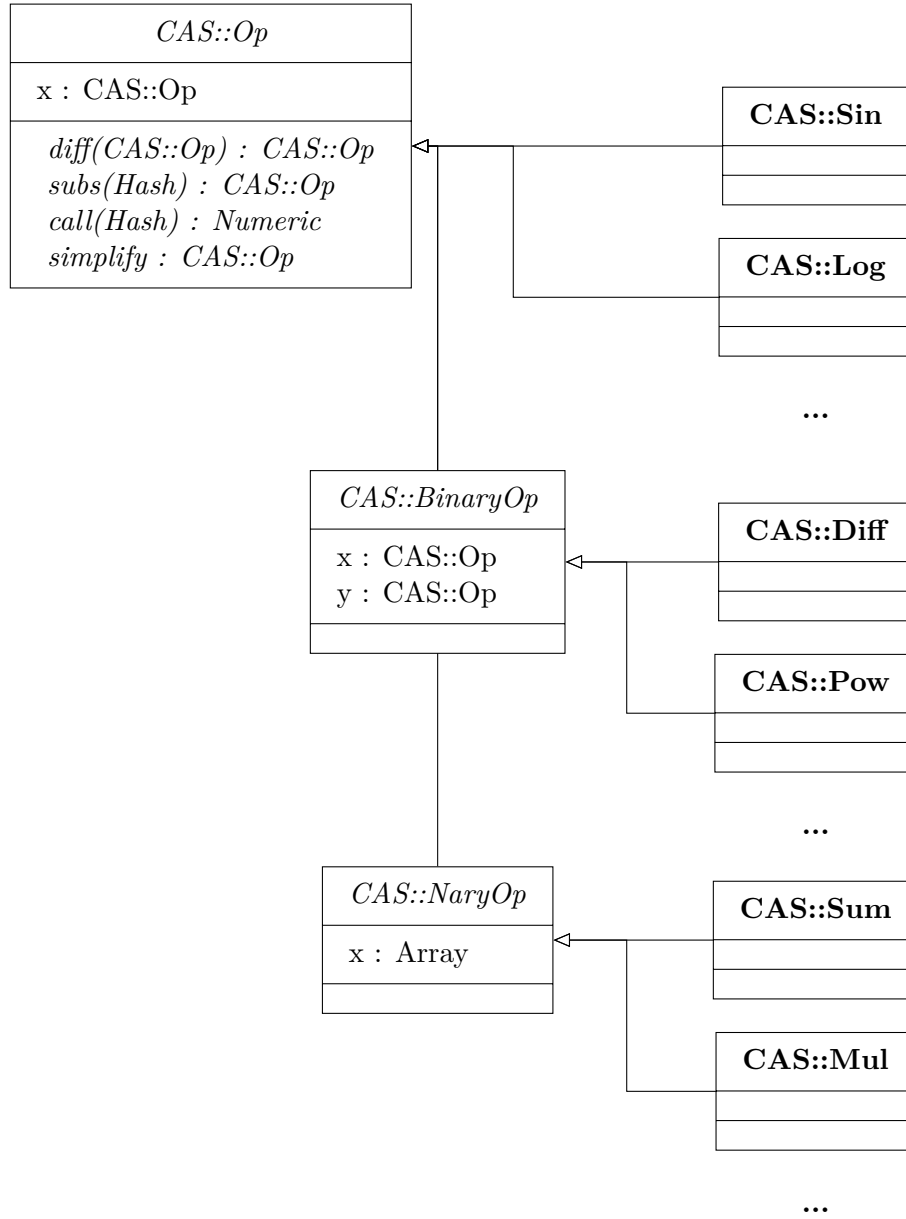


Figure 2: Reduced version of classes interface and inheritance. The figure depicts the basic abstract class **CAS::Op**, from which the *single argument* operations inherit. **CAS::Op** is also the ancestor for other kind of containers, namely the **CAS::BinaryOp** and **CAS::NaryOp**, models of container with *two* and *more arguments*

## 86 2.2. Software Functionalities

### 87 2.2.1. Basic Functionalities

88 *No additional dependencies are required.* The gem can be installed through  
89 *rubygems.org* provider<sup>2</sup>. Functionalities must be required run-time using the  
90 Kernel method: `require r.CAS`. All methods and classes are encapsulated  
91 in the module `CAS`.

92 **SD** is performed with respect to independent variables (`CAS::Variable`  
93 `ble`) through forward accumulation, even for implicit functions. The dif-  
94 ferentiation is done by the method `CAS::Op#diff`, having a `CAS::Variable`  
95 `ble` as argument:

Listing 1: Differentiation example

```
96  
97 z = CAS.vars 'z'           # creates a variable  
98 f = z ** 2 + 1             # define a symbolic expression  
99 f.diff(z)                  # derivative w.r.t. z  
100 # => (((z)^((2 - 1)) * 2 * 1) + 0)  
101 g = CAS.declare :g, f      # creates implicit expression  
102 g.diff(z)                  # derivative w.r.t. z  
103 # => (((z)^((2 - 1)) * 2 * 1) + 0) * Dg[0](((z)^(2) + 1)))  
104
```

105 **Automatic differentiation** (AD) is included as plugin and exploits  
106 properties of dual numbers to efficiently perform differentiation, see [12] de-  
107 tails. This differentiation strategy is useful in case of complex expressions,  
108 when explicit derivative's tree may exceed the call stack depth, that is plat-  
109 form dependent.

110 **Simplifications** are not executed automatically, after differentiation.  
111 Each node of the tree knows rules for simplify itself, and rules are called  
112 recursively, exactly like ASD. Simplifications that require an *heuristic expan-*  
113 *sion* of the sub-graph—i.e. some trigonometric identities—are not defined for  
114 now, but can be easily achieved through **substitutions**:

Listing 2: Simplification example

```
115  
116 x, y = CAS::vars 'x', 'y'   # creates two variables  
117 f = CAS.log( CAS.sin( y ) )  # symbolic expression  
118 f.subs y => CAS.asin(CAS.exp(x)) # performs substitution  
119 f.simplify                  # simplifies expression  
120 # => x  
121
```

122 The tree is numerically **evaluated** when independent variables values are  
123 provided in a feed dictionary. The graph is reduced recursively to a single  
124 numeric value:

---

<sup>2</sup>gem install Mr.CAS

Listing 3: Tree evaluation example

---

```

125
126 x = CAS.vars 'x' # creates a variable
127 f = x ** 2 + 1 # defines a symbolic expression
128 f.call x => 2 # evaluates for x = 2
129 # => 5.0
130

```

---

131 Symbolic expressions can be used to create comparative expressions, that  
 132 are stored in special container classes, modeled by the ancestor `CAS::Con-`  
 133 `dition`—e.g.  $f(\cdot) \geq g(\cdot)$ . This allow the definition of piecewise functions,  
 134 in `CAS::Piecewise`. Internally, `max(·)` and `min(·)` functions are declared as  
 135 operations that inherits from `CAS::Piecewise`—e.g.  $\max(f(\cdot), g(\cdot))$ .

Listing 4: Expressions and Piecewise functions

---

```

136
137 x, y = CAS.vars 'x', 'y'
138 f = CAS.declare :f, x
139 g = CAS.declare :g, x, y
140 h = CAS.declare :h, y
141
142 f.greater_equal g
143 # => (f(x) >= g(x, y))
144 pw = CAS::Piecewise.new(f,
145     CAS::Piecewise.new(g, h, y.equal(0)),
146     x.greater(0))
147 # => ((x > 0) ? f(x) : ((y == 0) ? g(x, y) : h(y)))
148 CAS::max f, g
149 # => ((f(x) >= g(x, y)) ? f(x) : g(x, y))
150

```

---

### 151 2.2.2. Meta-programming and Code-Generation

152 *Mr.CAS* is developed explicitly for **metaprogramming** and **code gen-**  
 153 **eration**. Expressions can be exported as source code or used as prototypes  
 154 for callable *closures* (`Proc` objects):

Listing 5: Graph evaluation example

---

```

155
156 x = CAS::vars 'x' # creates a variable
157 f = CAS::log(CAS::sin(x)) # define a symbolic function
158
159 proc = f.as_proc # exports callable lambda
160 proc.call 'x' => Math::PI/2
161 # => 0.0
162

```

---

163 Compiling a closure of a tree is like making its snapshot, thus any fur-  
 164 ther manipulation of the expression do not update the callable object. This  
 165 drawback is balanced by the faster execution time of a `Proc`: when a graph  
 166 needs *only to be evaluated* in a iterative algorithm, transforming it in a *clo-*  
 167 *sure* reduces the execution time per iteration.

Code generation should be flexible enough to export expressions' trees in a user's target language. Generation methods for common languages are included in specific **plugins**. Users can furthermore expand exporting capabilities by writing specific exportation rules, overriding method for existing plugin, or designing their own exporter:

Listing 6: Example of Ruby code generation plugin

---

```

173 # Rules definition for Fortran Language
174 module CAS
175   {
176     # . . .
177     CAS::Variable => Proc.new { "#{name}" }
178     CAS::Sin      => Proc.new { "sin(#{x.to_fortran})" },
179     # . . .
180   }.each do |cls, prc|
181     cls.send(:define_method, :to_fortran, &prc)
182   end
183 end
184
185 # Usage
186 x = CAS.vars 'x'
187 code = (CAS.sin(x)).to_fortran
188 # => sin(x)
189

```

---

### 3. Illustrative Examples

#### 3.1. Code Generation as C Library

In this example it is shown how a *user of Mr.CAS* can export a mathematical model as a C library. `c-opt` plugin implements advanced features such as code optimization and generation of libraries.

The library `example` implements the model:

$$f(x, y) = x^y + g(x) \log(\sin(x^y)) \quad (3)$$

Expression  $g(x)$  belongs to a external object, declared as `g_impl`, and its interface is described in `g_impl.h` header. **What should be noted is the form of the code exported:** the intermediate operation  $x^y$  is evaluated once, even if appears twice in our model. The C function that implements our model  $f(x, y)$  is declared with the token `f_impl`. The exporter uses as default type `double` for variables and function returned values.

Listing 7: Calling optimized-C exporter for library generation

---

```

203 # Model
204 x, y = CAS.vars :x, :y
205

```

---



```

206     g = CAS.declare :g, x
207
208     f = x ** y + g * CAS.log(CAS.sin(x ** y))
209
210     # Code Generation
211     g.c_name = 'g_impl'          # g token
212
213     CAS::CLib.create "example" do
214         include_local "g_impl"    # g header
215         implements_as "f_impl", f # token for f
216     end
217

```

218 Library created by CLib contains the following code:

Listing 8: C Header

Listing 9: C Source

<pre> // Header file for library: example.c  #ifndef example_H #define example_H  // Standard Libraries #include &lt;math.h&gt;  219 // Local Libraries #include "g_impl"  // Definitions  // Functions double f_impl(double x, double y);  #endif // example_H </pre>	<pre> // Source file for library: example.c  #include "example.h"  double f_impl(double x, double y) {     double __t_0 = pow(x, y);     double __t_1 = g_impl(x);     double __t_2 = sin(__t_0);     double __t_3 = log(__t_2);     double __t_4 = (__t_1 + __t_3);     double __t_5 = (__t_0 + __t_4);      return __t_5; }  // end of example.c </pre>
--	---

220 The function  $g(x)$  models the following operation:

$$g(x) = (\sqrt{x+a} - \sqrt{x}) + \sqrt{\pi+x} \quad (4)$$

221 and may suffer from *catastrophic numerical cancellation* [13] when  $x$  value  
222 is considerably greater than  $a$ . User may decide to specialize code genera-  
223 tion rules for this particular expression, *stabilizing it* through rationalization.  
224 Without modifying the actual model  $g(x)$  in Listing 10 the rationalization  
225 is inserted into exportation rules for differences of square roots<sup>3</sup>. This rule  
226 is valid only for the current user script. For more insight about `__to_c` and  
227 `__to_c_impl`, refer to the software manual.

<sup>3</sup>i.e.:  $\sqrt{\phi(\cdot)} - \sqrt{\psi(\cdot)} = \frac{\phi(\cdot) - \psi(\cdot)}{\sqrt{\phi(\cdot)} + \sqrt{\psi(\cdot)}}$

Listing 10: Conditioning in exporting function

---

```

228
229 # Model
230 a = CAS.declare "PARAM_A"
231
232 g = (CAS.sqrt(x + a) — CAS.sqrt(x)) + CAS.sqrt(CAS::Pi + x)
233
234 # Particular Code Generation for difference between square roots.
235 module CAS
236   class Diff
237     alias :__to_c_impl_old :__to_c_impl
238
239     def __to_c_impl(v)
240       if @x.is_a? CAS::Sqrt and @y.is_a? CAS::Sqrt
241         "(#{@x.x.__to_c(v)} + #{@y.x.__to_c(v)}) / " +
242         "( #{@x.__to_c(v)} + #{@y.__to_c(v)} )"
243       else
244         self.__to_c_impl_old(v)
245       end
246     end
247   end
248 end
249
250 CAS::CLib.create "g_impl" do
251   define "PARAM_A()", 1.0 # Arbitrary value for PARAM_A
252   define "M_PI", Math::Pi
253   implements_as "g_impl", g
254 end
255
256 puts g
257 # => ((sqrt((x + PARAM_A())) — sqrt(x)) + sqrtπ(( + x)))
258

```

---

259 It should be noted the **separation between the model**—that does not  
 260 contain **stabilization**—**and the code generation rule**—that overloads, for  
 261 this particular case and this particular language, the predefined code gener-  
 262 ation rule [to obtain the conditioned code](#). Obviously, a user can decide to  
 263 apply directly the conditioning on the model, [but this may change the cal-](#)  
 264 [culus behavior in further manipulation](#). The result of Listing 10 is reported:

Listing 11: g\_impl Header

```

// Header file for library: g_impl.c

#ifndef g_impl_H
#define g_impl_H

// Standard Libraries
#include <math.h>

// Local Libraries
265

// Definitions
#define PARAM_A() 1.0
#define M_PI 3.141592653589793

// Functions
double g_impl(double x);

#endif // g_impl_H

```

Listing 12: g\_impl Source

```

// Source file for library: g_impl.c

#include "g_impl.h"

double g_impl(double x) {
    double __t_0 = PARAM_A();
    double __t_1 = (x + __t_0);
    double __t_2 = sqrt(__t_1);
    double __t_3 = sqrt(x);
    double __t_4 = (__t_1 + x) / ( __t_2 +
        __t_3 );
    double __t_5 = (M_PI + x);
    double __t_6 = sqrt(__t_5);
    double __t_7 = (__t_4 + __t_6);

    return __t_7;
}

// end of g_impl.c

```

### 266 3.2. Using the module as interface

267 As example, an implementation of an algorithm that estimates the *order*  
 268 *of convergence* for trapezoidal integration scheme [14] is provided, using the  
 269 symbolic differentiation as interface.

270 Given a function  $f(x)$ , the trapezoidal rule for primitive estimation in the  
 271 interval  $[a, b]$  is:

$$I_n(a, b) = h \left( \frac{f(a) + f(b)}{2} + \sum_{k=1}^{n-1} f(a + kh) \right) \quad (5)$$

272 with  $h = (b - a)/n$ , where  $n$  mediates the integration's step size. When exact  
 273 primitive  $F(x)$  is known, approximation error is:

$$E[n] = F(b) - F(a) - I_n(a, b) \quad (6)$$

274 The error has an asymptotic expansion of the form:

$$E[n] \propto C n^{-p} \quad (7)$$

275 where  $p$  is the convergence order. Using a different value for  $n$ , for example  
 276  $2n$ , the ratio 8 takes the approximate vale:

$$\frac{E[n]}{E[2n]} \approx 2^p \quad \rightarrow \quad p \approx \log_2 \left( \frac{E[n]}{E[2n]} \right) \quad (8)$$

277 Following Listings contain the implementation of the described procedure  
 278 using the proposed gem and the well known *Python* [15] library *SymPy* [16].

Listing 13: Ruby version

Listing 14: Python version

```
require 'Mr.CAS'

def integrate(f, a, b, n)
  h = (b - a) / n

  func = f.as_proc

  sum = ((func.call 'x' => a) +
        (func.call 'x' => b)) / 2.0

  for i in (1..n)
    sum += (func.call 'x' => (a + i*h))
  end
  return sum * h
end

279 def order(f, a, b, n)
  x = CAS.vars 'x'

  f_ab = (f.call x => b) -
    (f.call x => a)
  df = f.diff(x).simplify
  f_1n = integrate(df, a, b, n)
  f_2n = integrate(df, a, b, 2 * n)

  return Math.log(
    (f_ab - f_1n) /
    (f_ab - f_2n),
    2)
end

x = CAS.vars 'x'
f = CAS.arctan x

puts(order f, -1.0, 1.0, 100)
# => 1.9999999974244451
```

```
import sympy
import math

def integrate(f, a, b, n):
  h = (b - a)/n
  x = sympy.symbols('x')
  func = sympy.lambdify((x), f)

  sums = (func(a) +
        func(b)) / 2.0

  for i in range(1, n):
    sums += func(a + i*h)

  return sums * h

def order(f, a, b, n):
  x = sympy.symbols('x')

  f_ab = sympy.Subs(f, (x), (b)).n() - \
    sympy.Subs(f, (x), (a)).n()
  df = f.diff(x)
  f_1n = integrate(df, a, b, n)
  f_2n = integrate(df, a, b, 2 * n)

  return math.log(
    (f_ab - f_1n) /
    (f_ab - f_2n),
    2)

x = sympy.symbols('x')
f = sympy.atan(x)

print(order(f, -1.0, 1.0, 100))
# => 1.9999999974244451
```

### 280 3.3. ODE Solver with Taylor's series

281 In this example we assume a user needs to generate a solving step for  
 282 specific ODE problems, using Taylor's series method [17]. Given an ODE in  
 283 the form:

$$y'(x) = f(x, y(x)) \quad (9)$$

284 the integration step with order  $n$  has the form:

$$y(x+h) = y(x) + h y'(x) + \cdots + \frac{h^n}{n!} y^{(n)}(x) + E_n(x) \quad (10)$$

285 where, obviously, it is possible to use equation 9, which brings to the following  
286 recurrent identity:

$$y^{(i)}(x) = \frac{\partial y^{(i-1)}(x)}{\partial x} + \frac{\partial y^{(i-1)}(x)}{\partial y} y'(x) \quad (11)$$

287 For this algorithm, three methods are defined. The first evaluates the facto-  
288 rial, the second evaluates the list of required derivatives, and the third returns  
289 the integration step in a symbolic form. The result of the third method is  
290 transformed in a C function. In this particular case, the ODE is  $y' = xy$ .

Listing 15: Generator for ODE integration step

---

```

291 $x, $y, $h = CAS::vars :x, :y, :h
292 # Evaluates n!
293 def fact(n); (n < 2 ? 1 : n * fact(n-1)); end
294 # Evaluates all derivatives required by the order
295 def coeff(f, n)
296   df = [f]
297   for _ in 2..n
298     df << df[-1].diff($x).simplify + (df[-1].diff($y).simplify * df[0])
299   end
300   return df
301 end
302 # Generates the symbolic form for a Taylor step
303 def taylor(f, n)
304   df = coeff(f, n)
305   y = $y
306   for i in 0...df.size
307     y = y + (($h ** (i + 1))/(fact(i + 1)) * df[i])
308   end
309   return y.simplify
310 end
311
312 # Example function for the integrator
313 f = $x * $y
314 # Exporting a C function
315 clib = CAS::CLib.create "taylor" do
316   implements_as "taylor_step", taylor(f, 4)
317 end
318

```

---

320 For the resulting C code, please check online version of the examples.

321 Other usage examples are available online<sup>4</sup>—i.e. adding a user defined

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<sup>4</sup>[http://bit.ly/Mr\\_CAS\\_examples](http://bit.ly/Mr_CAS_examples)

322 `CAS::Op` that implements the `sign(.)` function with the appropriate opti-  
323 mized C generation rule; exporting this operation as a continuous function  
324 through overloading or substitutions; performing a symbolic Taylor’s series;  
325 writing an exporter for the  $\text{\LaTeX}$  language; a Newton-Raphson algorithm  
326 using automatic differentiation plugin.

## 327 4. Impact

328 *Mr.CAS* is a midpoint between a CAS and an ASD library. It allows  
329 to manipulate expressions while maintaining the complete control on how  
330 the code is exported. Each rule is overloaded and applied run-time, without  
331 the need of compilation. Each user’s model may include the mathematical  
332 description, code generation rules and high level logic that should be intrinsic  
333 to such a rule—e.g. exporting a `Hessian` as `pattern` instead of `matrix`.

334 Our research group is including `Mr.CAS` in a solver for optimal control  
335 problem with indirect methods, as interface for problems’ description [18].

336 As a long term ambitious impact, this library will become a complete CAS  
337 for *Ruby* language, filling the empty space reported by *SciRuby* for symbolic  
338 math engines.

## 339 5. Conclusions

340 This work presents a pure *Ruby* library that implements a minimalis-  
341 tics CAS with automatic and symbolic differentiation that is aimed at code  
342 generation and meta-programming. Although at an early developing stage,  
343 *Mr.CAS* has promising feature, some of them shown in Section 3. Also, this  
344 is the only gem that implements symbolic manipulation for this language.

345 Language features and lack of dependencies simplify the use of the module  
346 as interface, extending model definition capabilities for numerical algorithms.  
347 All core functionalities and basic mathematics are defined, with the plan to  
348 include more features in next releases. Reopening a class guarantees a *liquid*  
349 behaviour, in which users are free to modify core methods and their needs.

350 Library is published in *rubygems.org* repository and versioned on *github.com*,  
351 under MIT license. It can be included easily in projects and in inline inter-  
352 preter, or installed as a standalone gem.

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#### 400 Current code version

Nr.	Code metadata description	Please fill in this column
C1	Current code version	0.2.7
C2	Permanent link to code/repository used for this code version	github.com/MatteoRagni/cas-rb & rubygems.org/gems/Mr.CAS
C3	Legal Code License	MIT
C4	Code versioning system used	<i>git</i> (GitHub)
C5	Software code languages, tools, and services used	<i>Ruby</i> language
C6	Compilation requirements, operating environments	<i>Ruby</i> $\geq 2.x$
C7	If available Link to developer documentation/manual	rubydoc.info/gems/Mr.CAS
C8	Support email for questions	info@ragni.me

Table 1: Code metadata (mandatory)