Real-time quantum error mitigation in training VQAs

Based on: arXiv:2311.05680

Matteo Robbiati, Alejandro Sopena, Andrea Papaluca, Stefano Carrazza

Prologue

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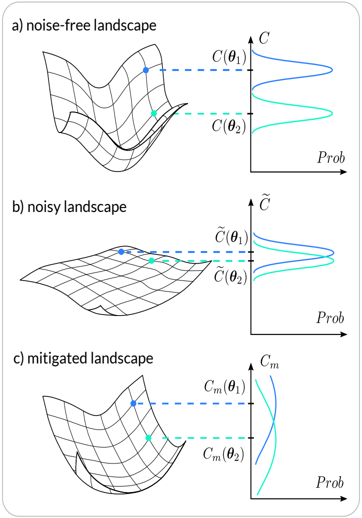

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Credits to arXiv:2109.01051

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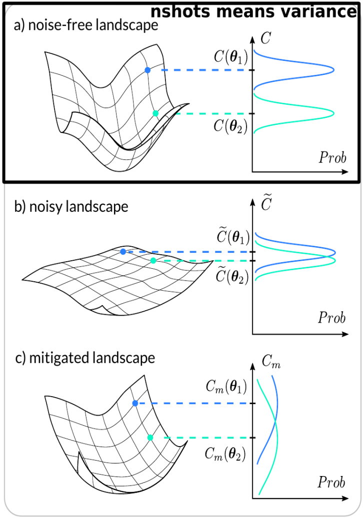

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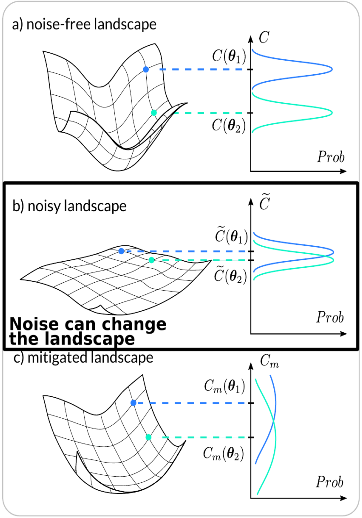

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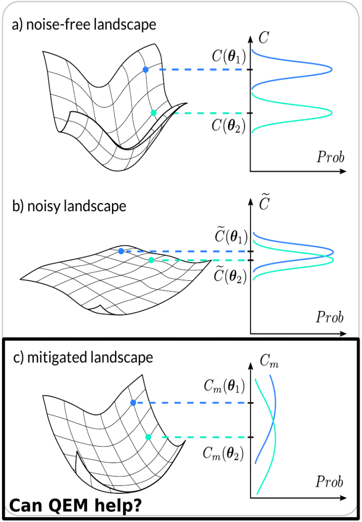

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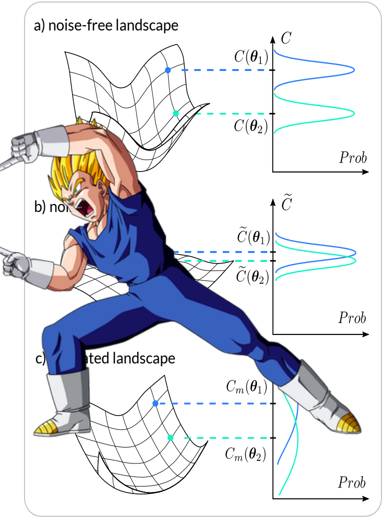
How did we start? 📘 arXiv:2109.01051



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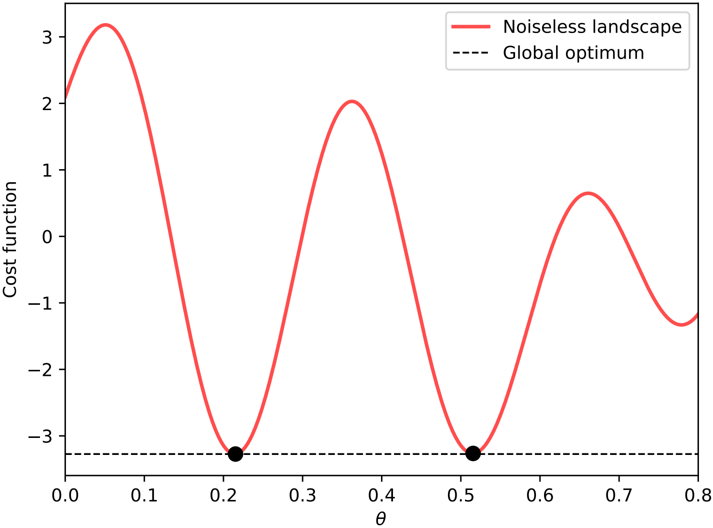
Step 1: about noise



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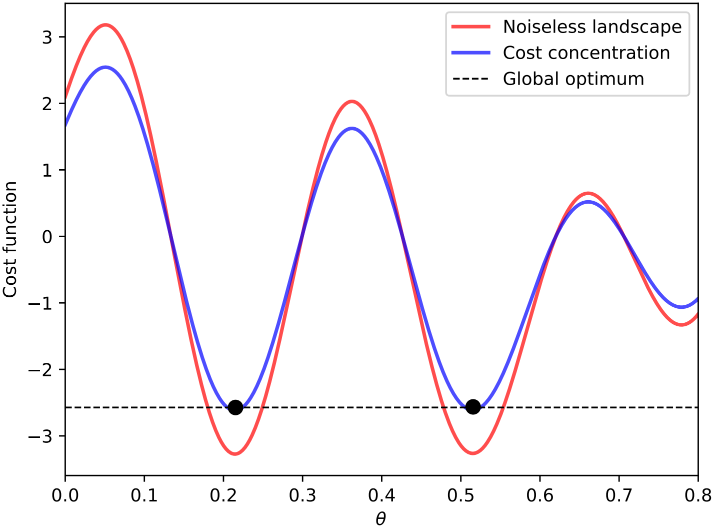

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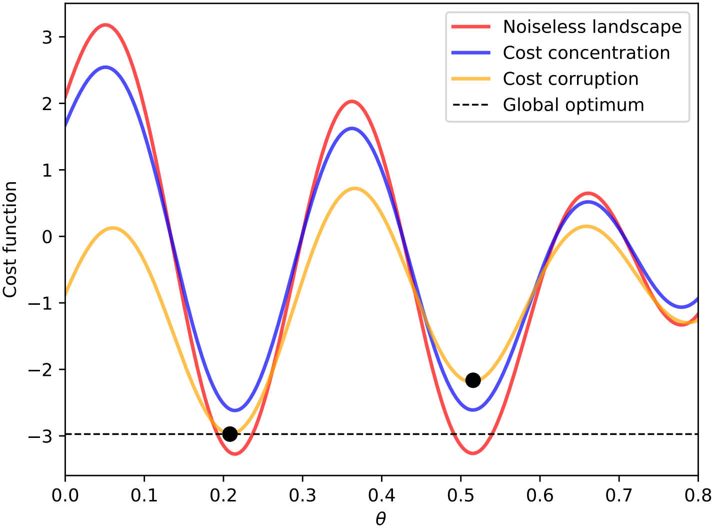

A case study

Step 2: a proper target

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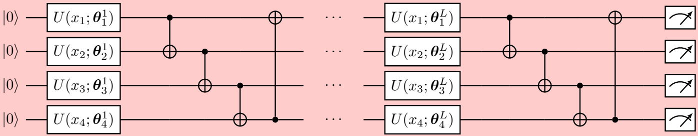
N-dim fit: $y = g(\boldsymbol{x})$

We build an N qubits circuit $\mathcal{U}(\boldsymbol{x};\boldsymbol{\theta})$:



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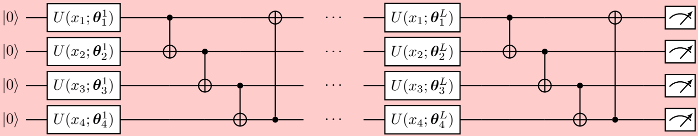

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Cost function

We want a cost function C which suffers of **cost corruption** and **concentration**:

$$C_{\rm mse}(\boldsymbol{x}|\boldsymbol{\theta}) = \frac{1}{N_{\rm data}} \sum_{i}^{N_{\rm data}} \left[f(\boldsymbol{x}^i, \boldsymbol{\theta})_{\mathcal{O}} - g(\boldsymbol{x}^i) \right]^2$$

with $\quad f(\boldsymbol{x}^i, \boldsymbol{\theta})_{\mathcal{O}} = \langle \psi_0 | \mathcal{U}^\dagger(\boldsymbol{x}^i; \boldsymbol{\theta}) \, \mathcal{O} \, \mathcal{U}(\boldsymbol{x}^i; \boldsymbol{\theta}) | \psi_0 \rangle \quad$ and $\quad \mathcal{O} = \sigma_z^{\otimes N}$.

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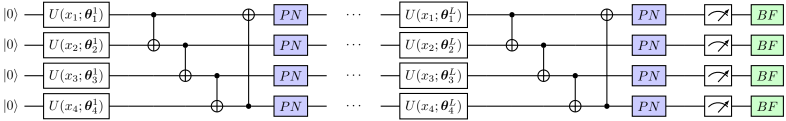

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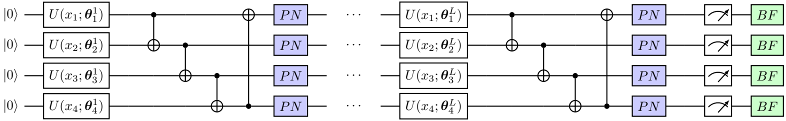

In particular:

- PN channel with probs. $-1 < q_x, q_y, q_z < 1$ on each qubit after each layer;
- symmetric readout noise $\mathcal{M}$ of single-qubit bit-flip (BF) with prob. $(1 - q_M)/2$.

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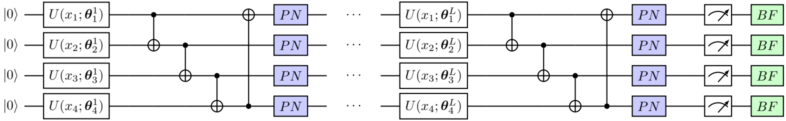

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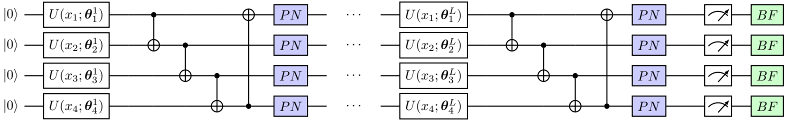

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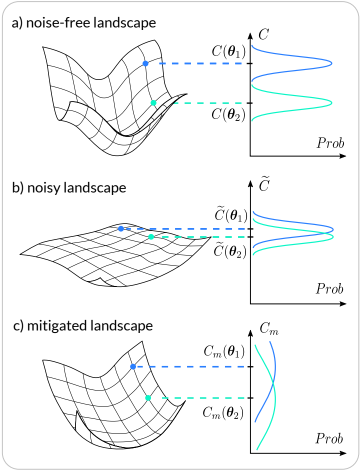
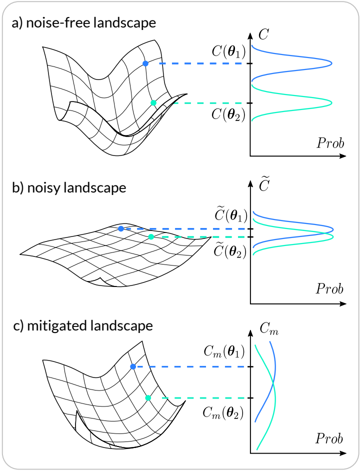
$$|f_{\mathrm{noisy}}| < 2q_M^N q^{2l+2}\left(1 - \frac{1}{2^N}\right)$$

Can quantum error mitigation help? arXiv:2109.01051





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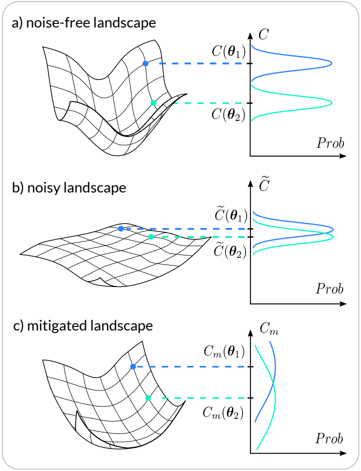

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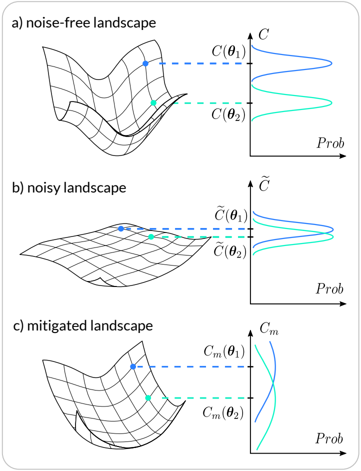

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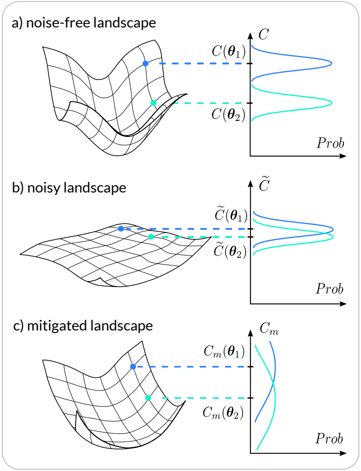

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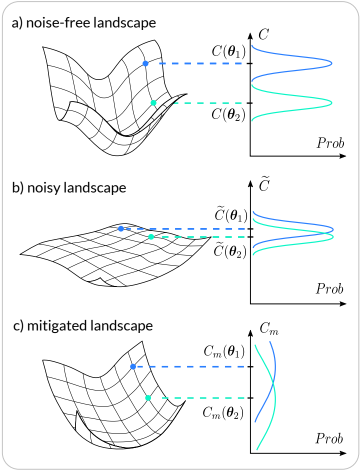

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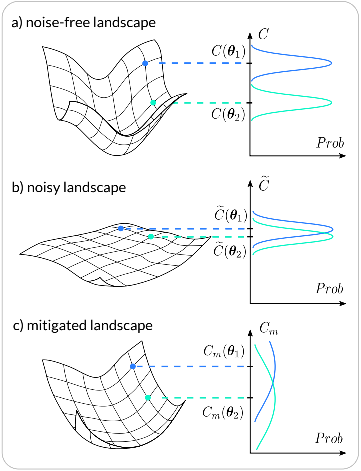

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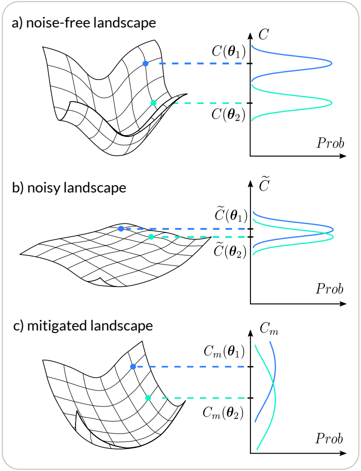

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Good news!

It can help with cost corruption while remaining neutral to cost concentration!

can we use it efficiently in VQAs?

RTQEM pipeline

We define a Real-Time Quantum Error Mitigation (RTQEM) procedure.

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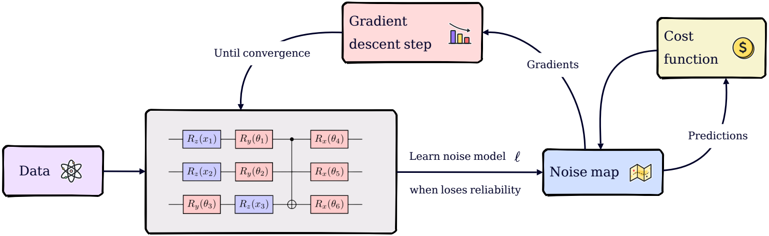

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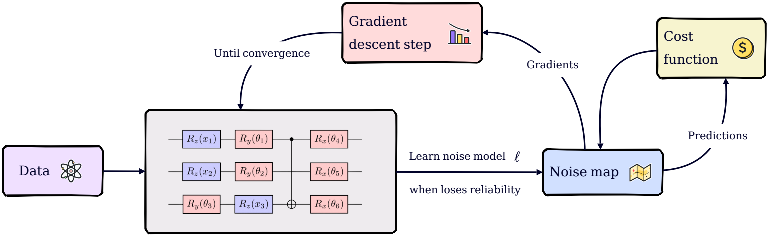

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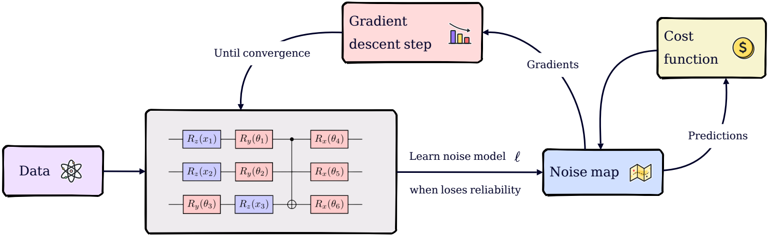

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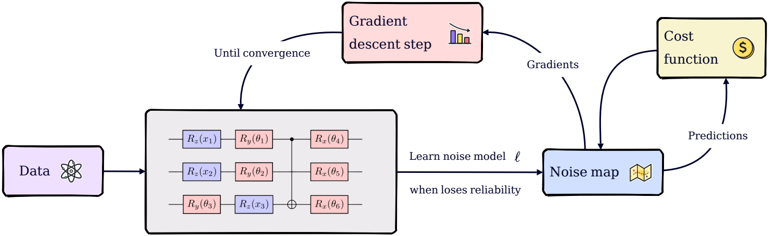

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3. use ℓ to clean up both predictions and gradients.

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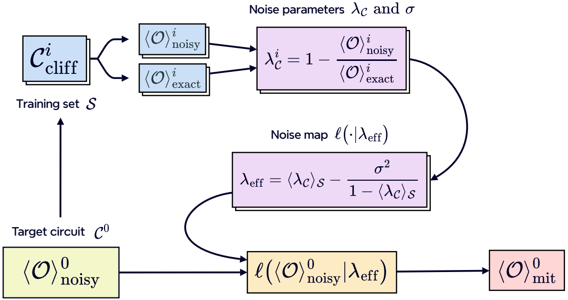

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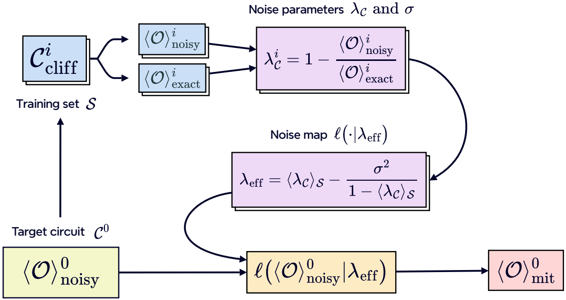

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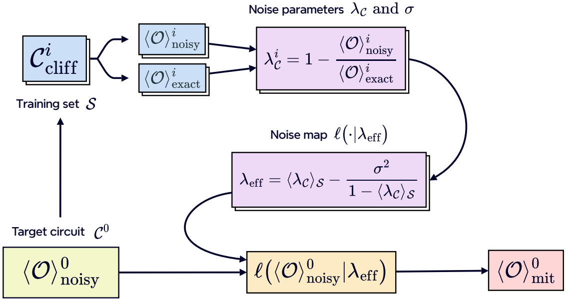

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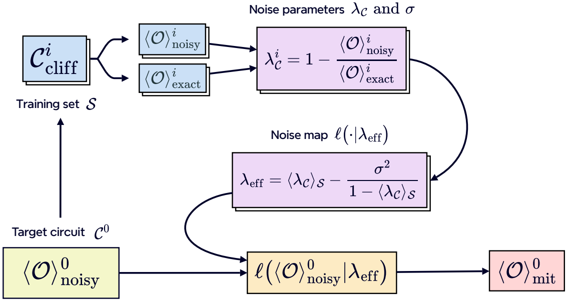

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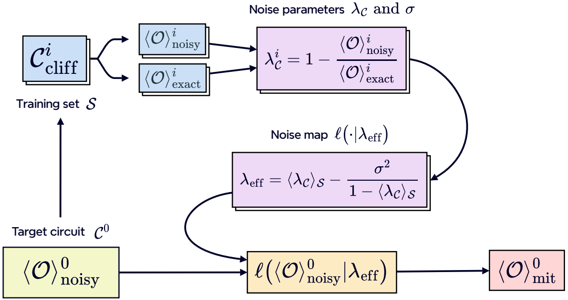

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2. process them so that their expectation values on Pauli strings is $+1$ or -1;
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4. build a phenomenological noise map:

$$\ell(\langle O\rangle|\lambda_{\text{eff}}) = \frac{(1-\langle\lambda_{\mathcal{C}}\rangle_{\mathcal{S}})}{(1-\langle\lambda_{\mathcal{C}}\rangle_{\mathcal{S}})^2+\sigma^2}\langle O\rangle_{\text{noisy}}.$$

We don't need to recompute QEM at each iteration!

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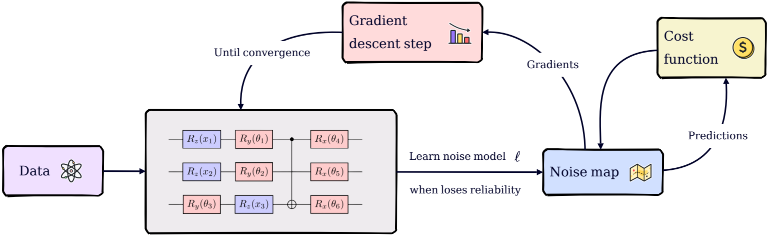

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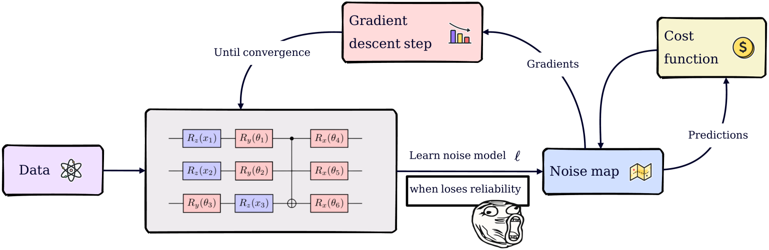

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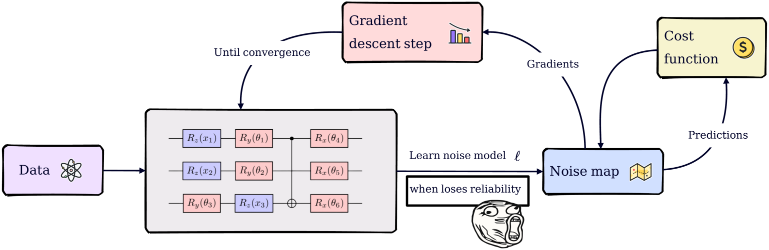

✎ we defined a metric $D(z, \ell(z)) = |z - \ell(z)|$ to quantify the distance between a well known expval z and its mitigated value.

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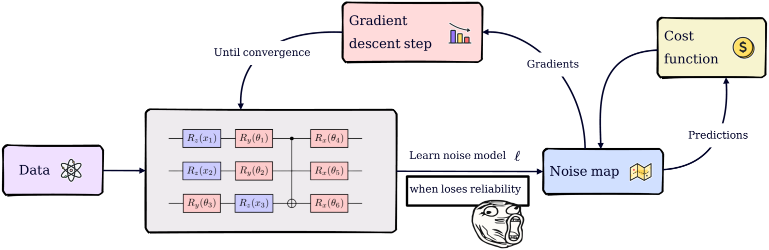


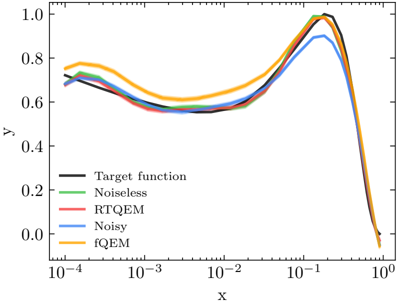
we defined a metric $D(z, \ell(z)) = |z - \ell(z)|$ to quantify the distance between a well known expval z and its mitigated value.

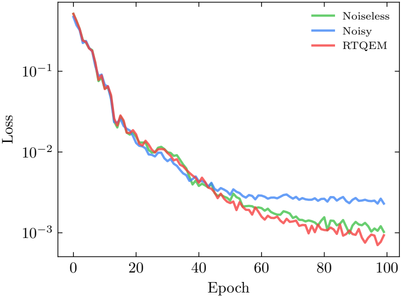
if D exceeds some arbitrary threshold ε, then the map ℓ is recomputed.

Static noise scenario

One dimensional HEP target: the u-quark PDF

Parameter	N_{train}	N_{params}	N_{shots}	$\mathrm{MSE}_{\mathrm{rtqem}}$	$\mathrm{MSE}_{\mathrm{nomit}}$	Noise
Value	30	16	10^4	0.008	0.018	local Pauli



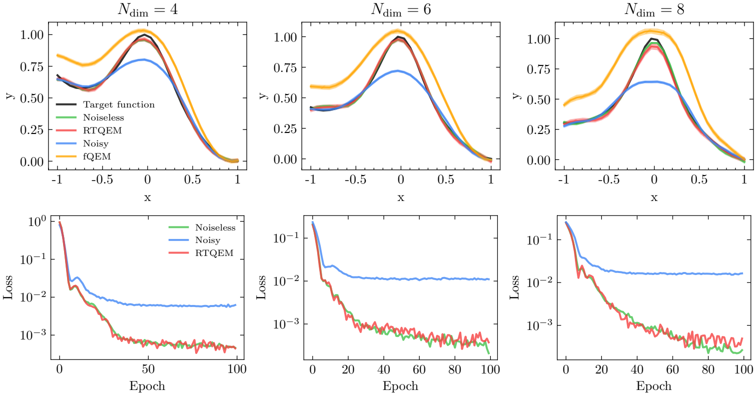


1. thanks to the RTQEM procedure, we reach a good minimum of the cost function;
2. the QEM is not effective is applied to a corrupted scenario (orange curve).

Multidimensional target

Dummy N-dim function: $f_{\mathrm{ndim}}(\boldsymbol{x};\boldsymbol{\beta}) = \sum_{i=1}^{N_{\mathrm{dim}}} \left[\cos(\beta_i x_i)^i + (-1)^{i-1}\beta_i x_i\right]$.

Job ID	N_{train}	N_{params}	N_{shots}	$\mathrm{MSE}_{\mathrm{rtqem}}$	$\mathrm{MSE}_{\mathrm{nomit}}$	Noise
$N_{\mathrm{dim}} = 4$	30	48	10^4	0.003	0.043	local Pauli
$N_{\mathrm{dim}} = 6$	30	72	10^4	0.002	0.083	local Pauli
$N_{\mathrm{dim}} = 8$	30	96	10^4	0.004	0.118	local Pauli



Evolving noise scenario

RTQEM on a superconducting qubit

We move the *PN* vector with a Random Walk-like procedure. Namely, each component q_j is evolved from epoch k to epoch $k+1$ as

$$q_j^{(k+1)} = q_j^k + r\delta,$$

where $r \sim \{-1, +1\}$ and the step is sampled from a normal distribution $\delta \sim \mathcal{N}(0, \sigma_\delta)$.

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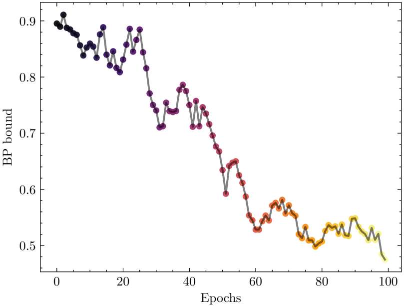

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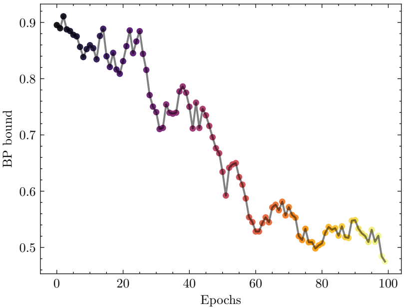

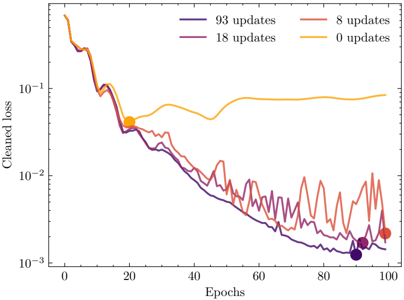

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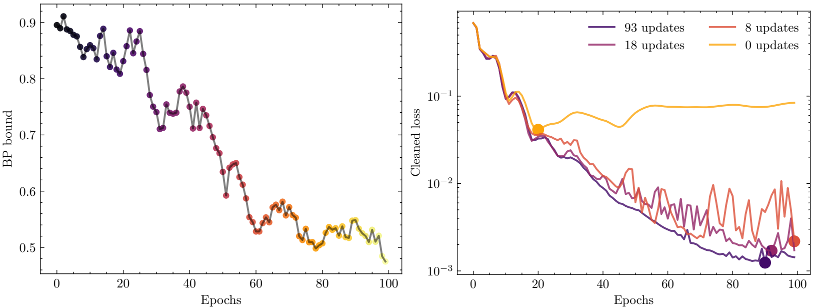
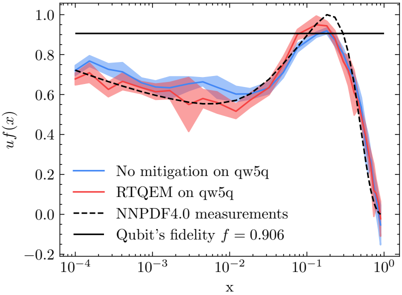
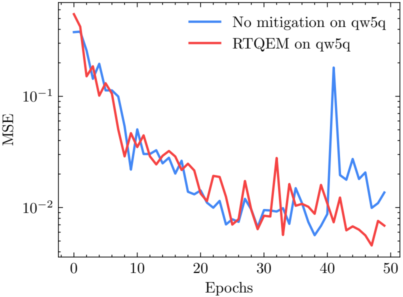


With a limited number of updates we have a considerable advantage!

RTQEM on a superconducting qubit

Parameter	N_{train}	N_{params}	N_{shots}	$\mathrm{MSE}_{\mathrm{rtqem}}$	$\mathrm{MSE}_{\mathrm{nomit}}$	Noise
Value	15	16	500	0.0042	0.0055	real noise





RTQEM allows exceeding the natural bound imposed by noise.

We perform a longer training on two different devices (and noises!) using the same initial conditions of the previous slide but $N_{\mathrm{epochs}} = 100$.

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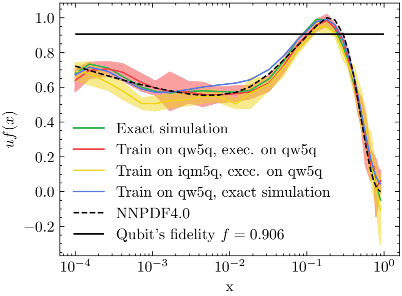

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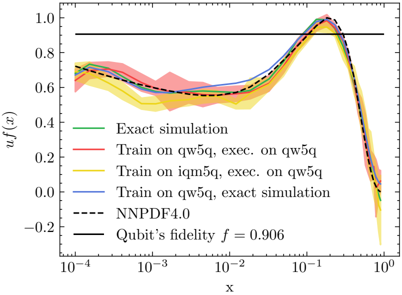

- `qw5q` from QuantWare and controlled using Qblox instruments;
- `iqm5q` from IQM and controlled using Zurich Instruments.

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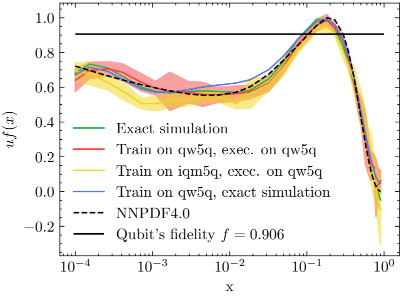

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Train.	Epochs	Pred.	Config.	MSE
`qw5q`	50	`qw5q`	noisy	0.0055
`qw5q`	50	`qw5q`	RTQEM	0.0042
`qw5q`	100	`qw5q`	RTQEM	0.0013
`iqm5q`	100	`qw5q`	RTQEM	0.0037
`qw5q`	100	`sim`	RTQEM	0.0016

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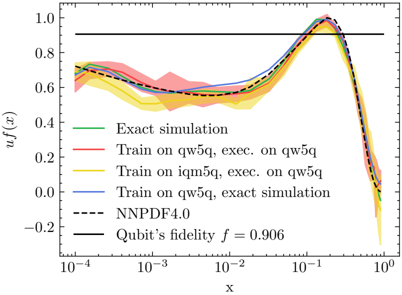

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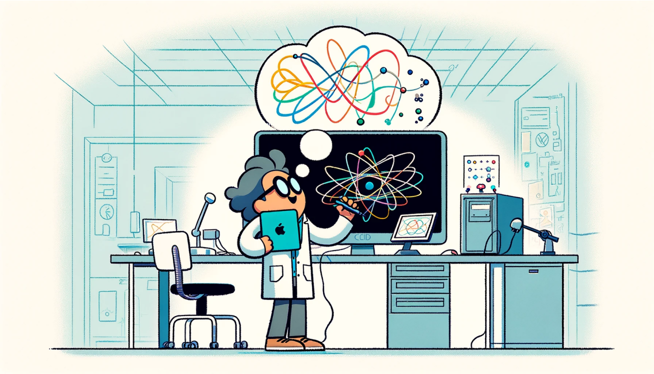
All the hardware results are obtained deploying the $\boldsymbol{\theta}_{\text{best}}$ on qw5q.

Outlook

- Can improve RTQEM robustness? Is ICS enough to face any scenario?
- how about combining more techniques?
- how about adding some memory (inertia) to the noise model? Can this be robust against sudden (and temporary) system's fluctuations?
- how about moving the mitigation routine on chip to boost the process?



DALLE, please, draw a PhD student trying hard to fit Parton Distribution Functions on a quantum computer at CERN