Quantum computing Challenges

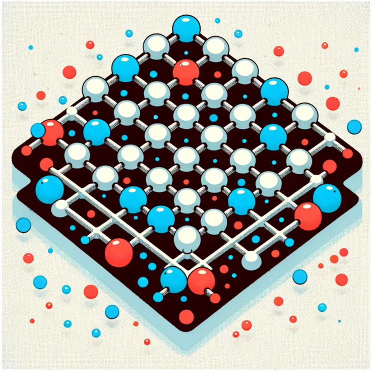
Pre-colloquium

Matteo Robbiati

February 2024

Compute quantum mechanics

- Representing N particles is difficult;



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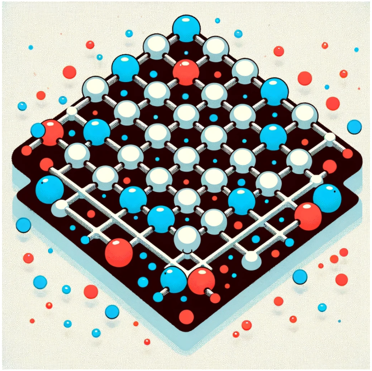
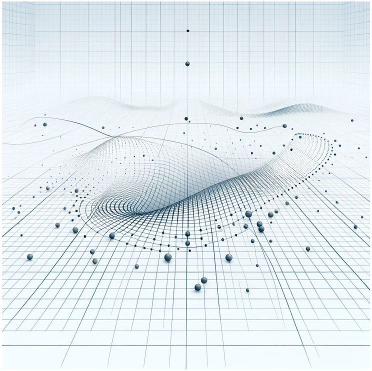
- Representing N particles is difficult;
- considering N spins $(\uparrow, \downarrow)$, we deal with a 2^N dimensional Hilbert space!





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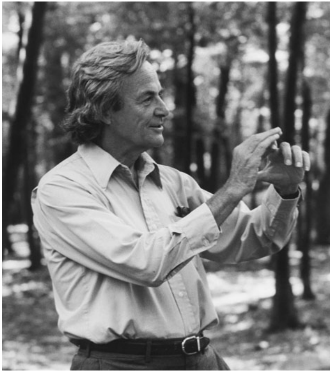

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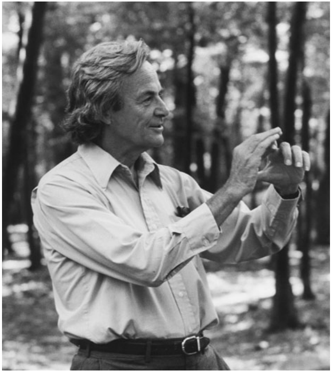

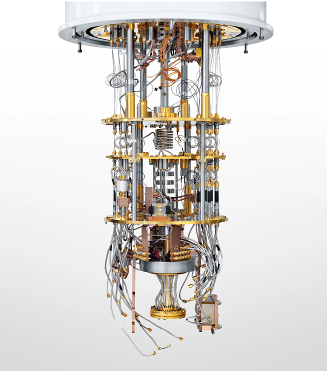

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Richard Feynman, 1982, Simulating Physics with Computers

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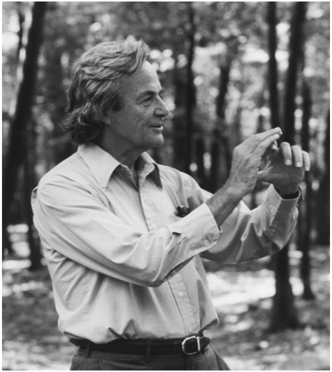

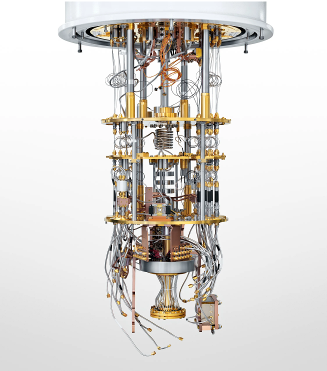

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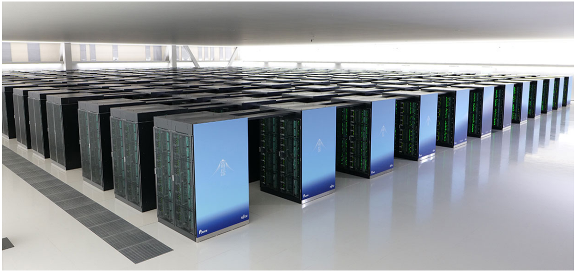
6. a 50 qubits state cannot even be represented by Fugaku supercomputer!



Some smart strategies

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1. Variational Monte Carlo (VMC): given a wave function $\Psi(\boldsymbol{x}|\boldsymbol{\theta})$ and a target H, MC methods are used to minimize:

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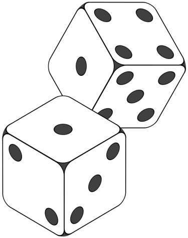

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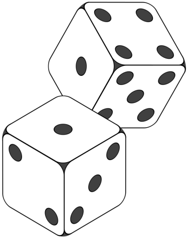

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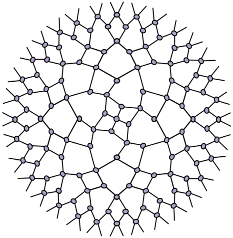

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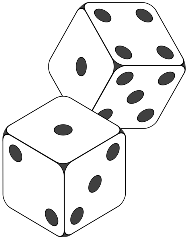
3. Neural Network Quantum States: use complex ANNs to represent the state.



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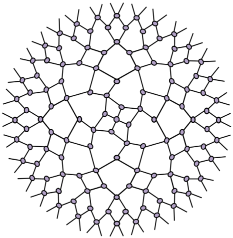

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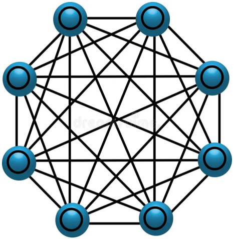

arXiv:1606.02318

A snapshot of quantum computing

Qubits

Qubits

1. classical bits are replaced by **qubits** $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.

Superconducting loops
A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.

Trapped ions
Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.

Silicon quantum dots
These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.

Topological qubits
Quasiparticles can be seen in the behavior of electrons channeled through semi-conductor structures. Their braided paths can encode quantum information.

Diamond vacancies
A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.

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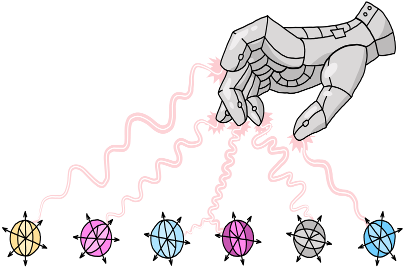

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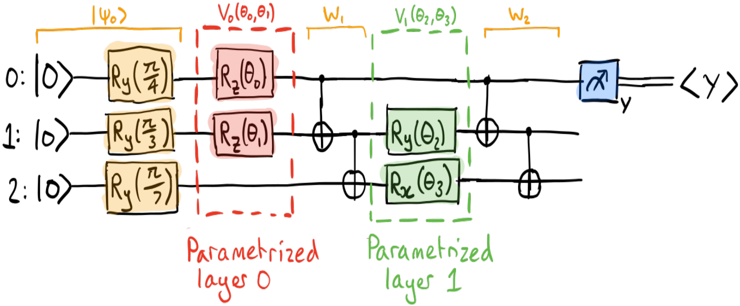

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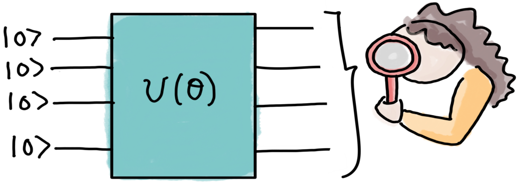
4. to access the information we need to measure the system.



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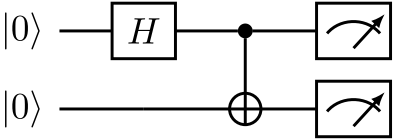

Quantum Machine Learning

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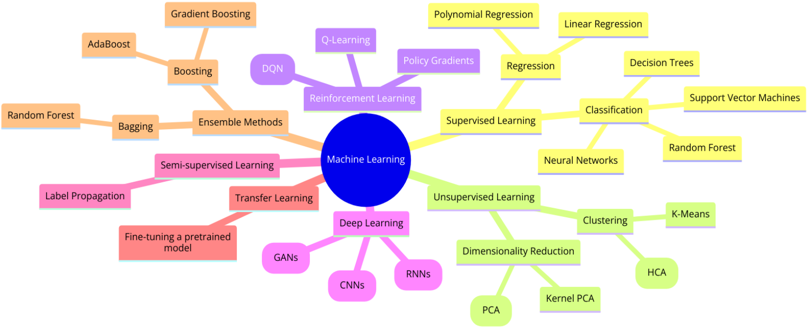

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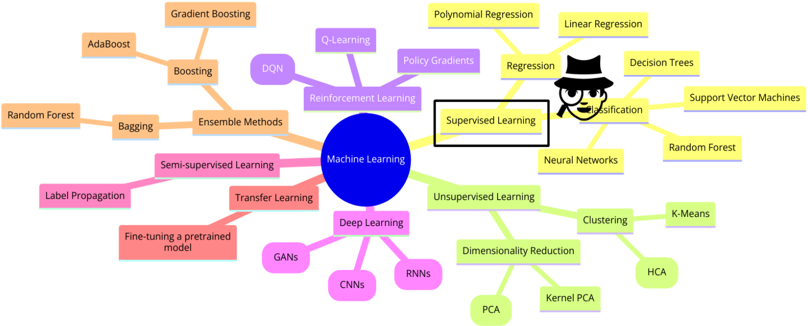

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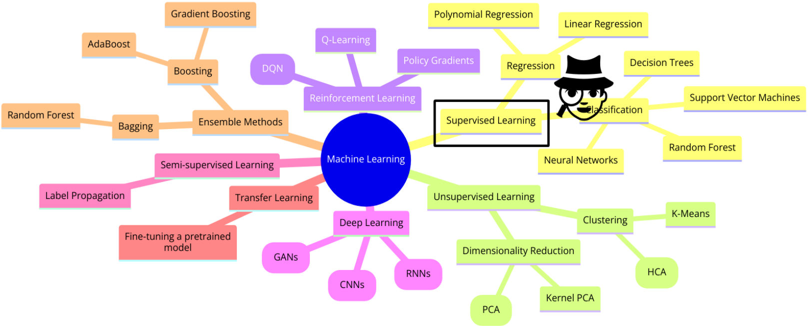

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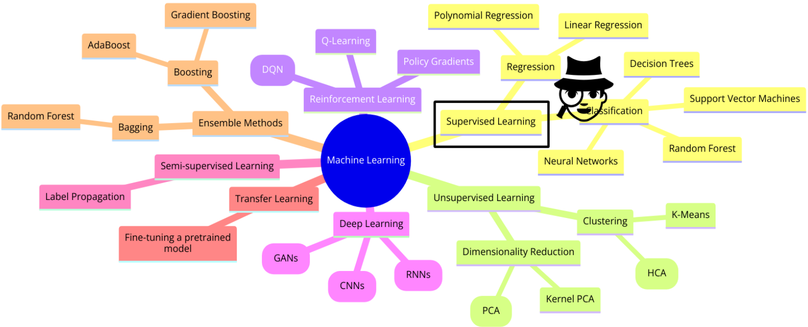

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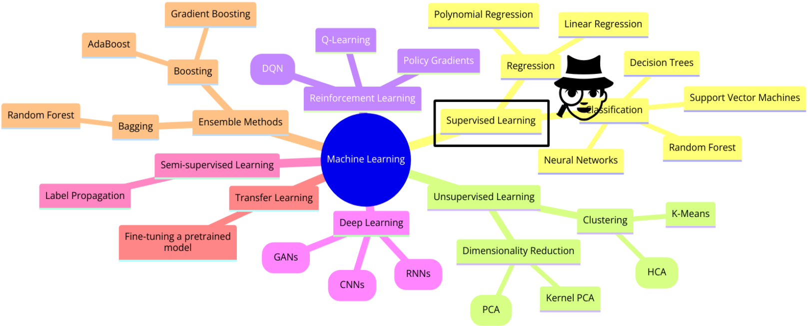

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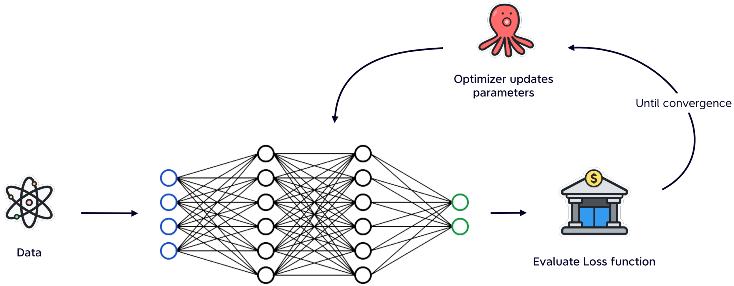

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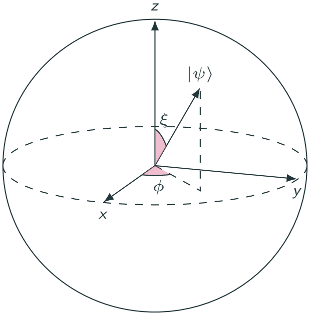

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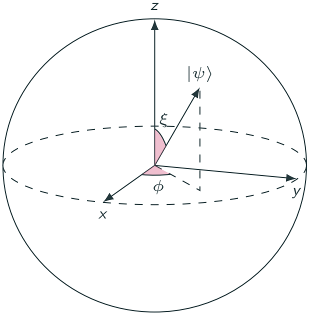

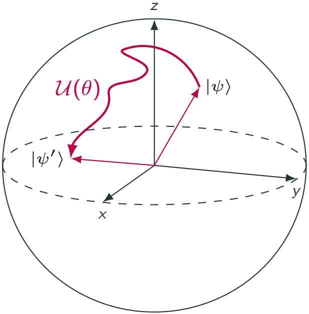

We can use as parametric gates the rotation around the axis of the block sphere:

$$R_k(\theta) = \exp\left[-i\theta\sigma_k\right], \qquad \text{with} \qquad \sigma_k \in \{I, \sigma_x, \sigma_y, \sigma_z\}.$$

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Machine Learning

$\mathcal{M}$: model;

$\mathcal{O}$: optimizer;

$\mathcal{J}$: loss function.

(x, y): data

Expected values

$y_{est} \equiv \langle\psi'|\hat{O}|\psi'\rangle$

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$\mathcal{Q}$: qubits;

$\mathcal{S}$: superposition;

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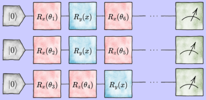
Circuit execution



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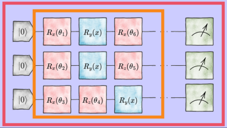

Optimizer $\mathcal{O}$
Hybrid-strategy

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loss function $\mathcal{J}$
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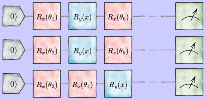
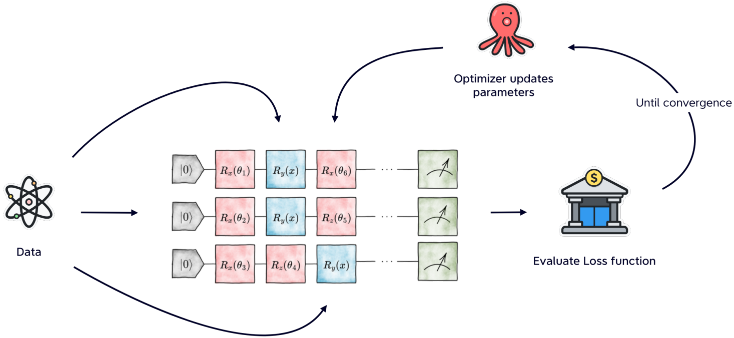


From ML to QML



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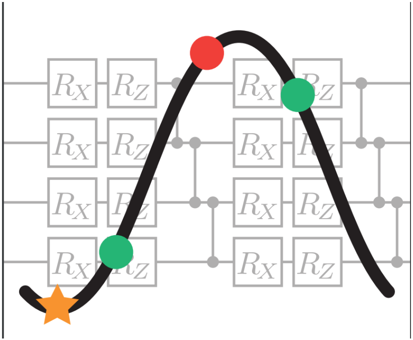

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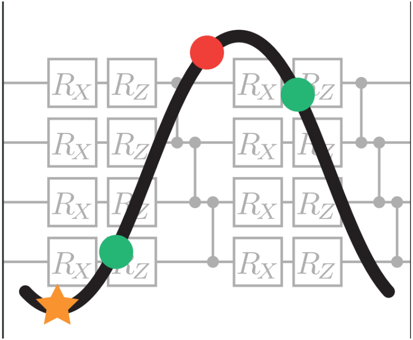

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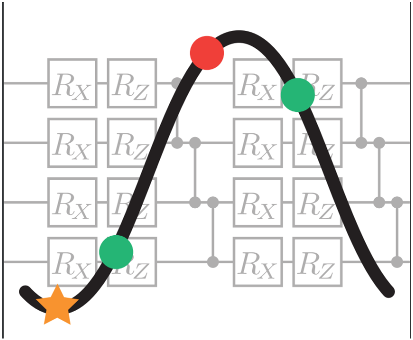

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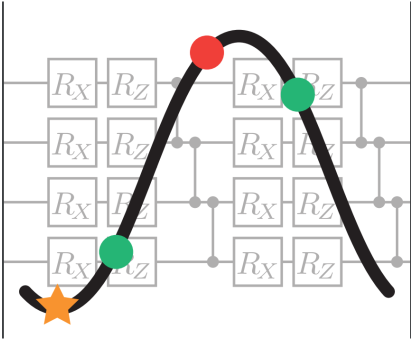
3. **Solovay-Kitaev theorem**: the number of gates needed by $\mathcal{U}$ to represent V with precision δ is $\mathcal{O}(\log^c \delta^{-1})$, where $c < 4$.



A final example

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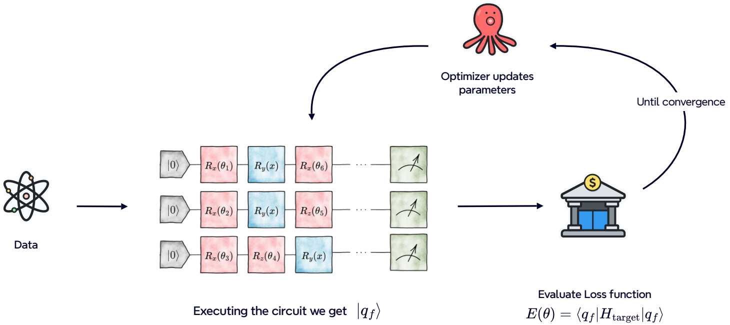

Let's code it!