# Real-time error mitigation for variational optimization on quantum hardware


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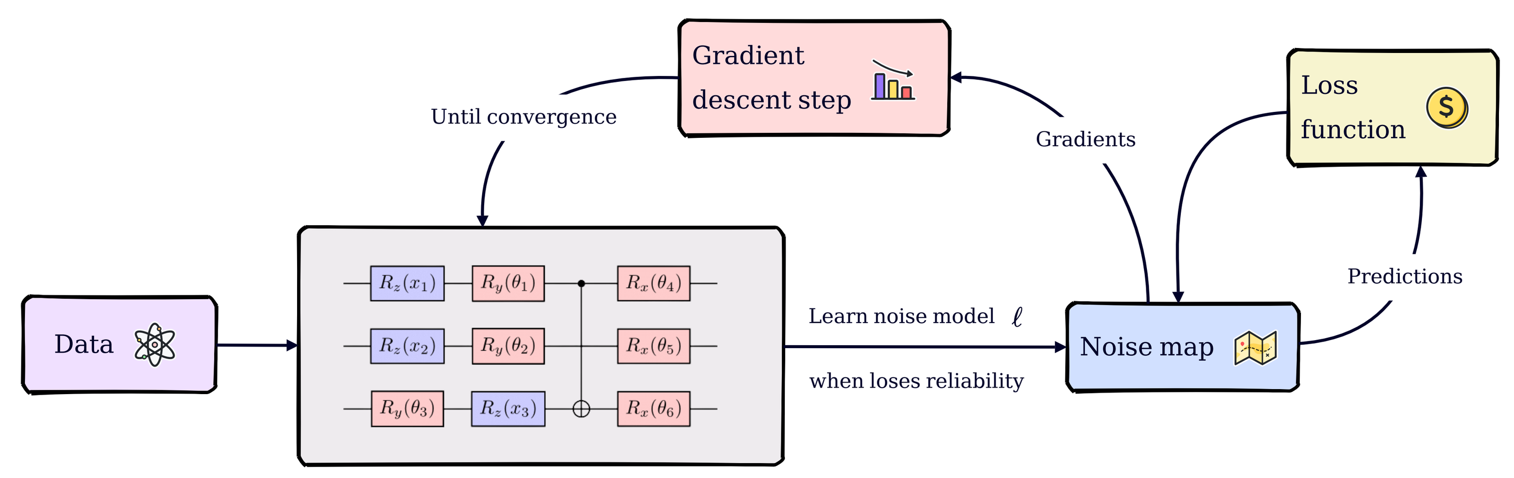
## Aim

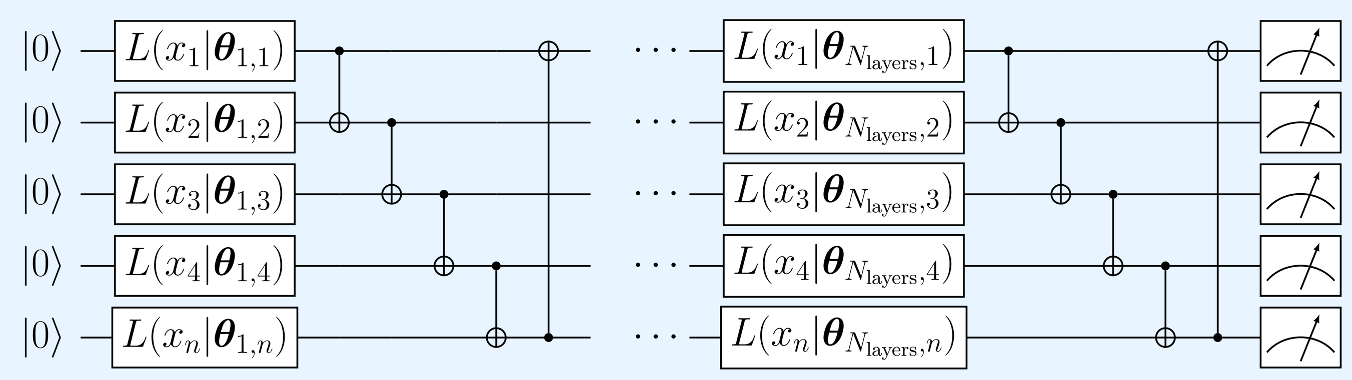
we put forward the inclusion of error mitigation routines in the process of training Variational Quantum Circuit (VQC) models. In detail, we define a Real Time Quantum Error Mitigation (RTQEM) algorithm to coadiuvate the task of fitting functions on quantum chips with VQCs.

## Schematic pipeline of the RTQEM algorithm




## Ansatz

We tackle multi-dimensional regression problems using a VQC as Quantum Machine Learning (QML) model. The data $\boldsymbol{x}$ are encoded into the circuit via Data Reuploading [1]:



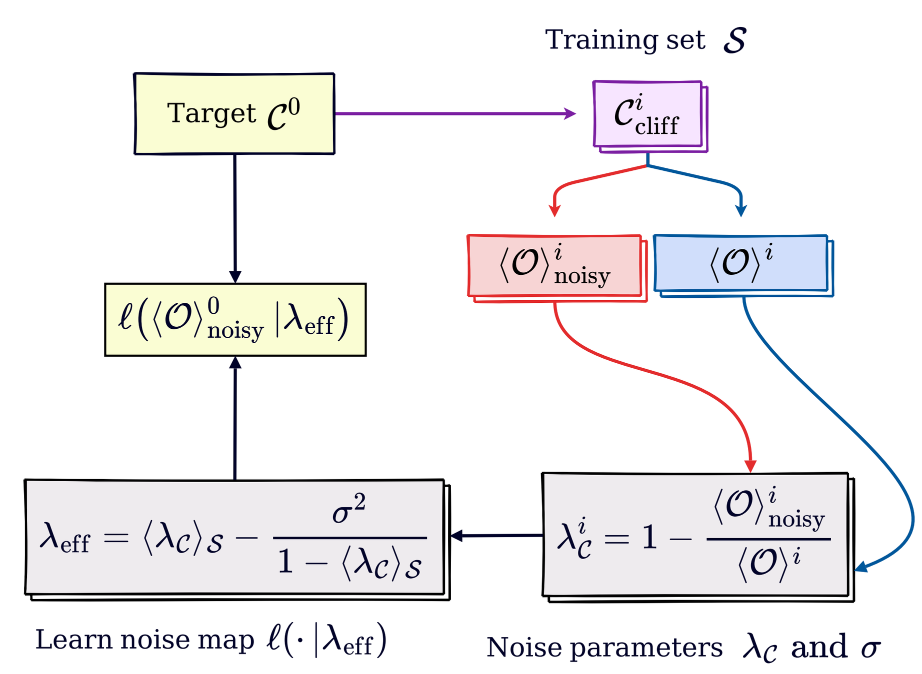
where we use the following definition of the uploading channel:

$$L(x_j|\boldsymbol{\theta}_{l,j}) = R_z(\theta_3 x_j + \theta_4) R_y(\theta_1 \kappa(x_j) + \theta_2)\,, \tag{1}$$

which uploads the $j$-th component of $\boldsymbol{x}$ at the circuit layer $l$.

## Noise of a quantum hardware

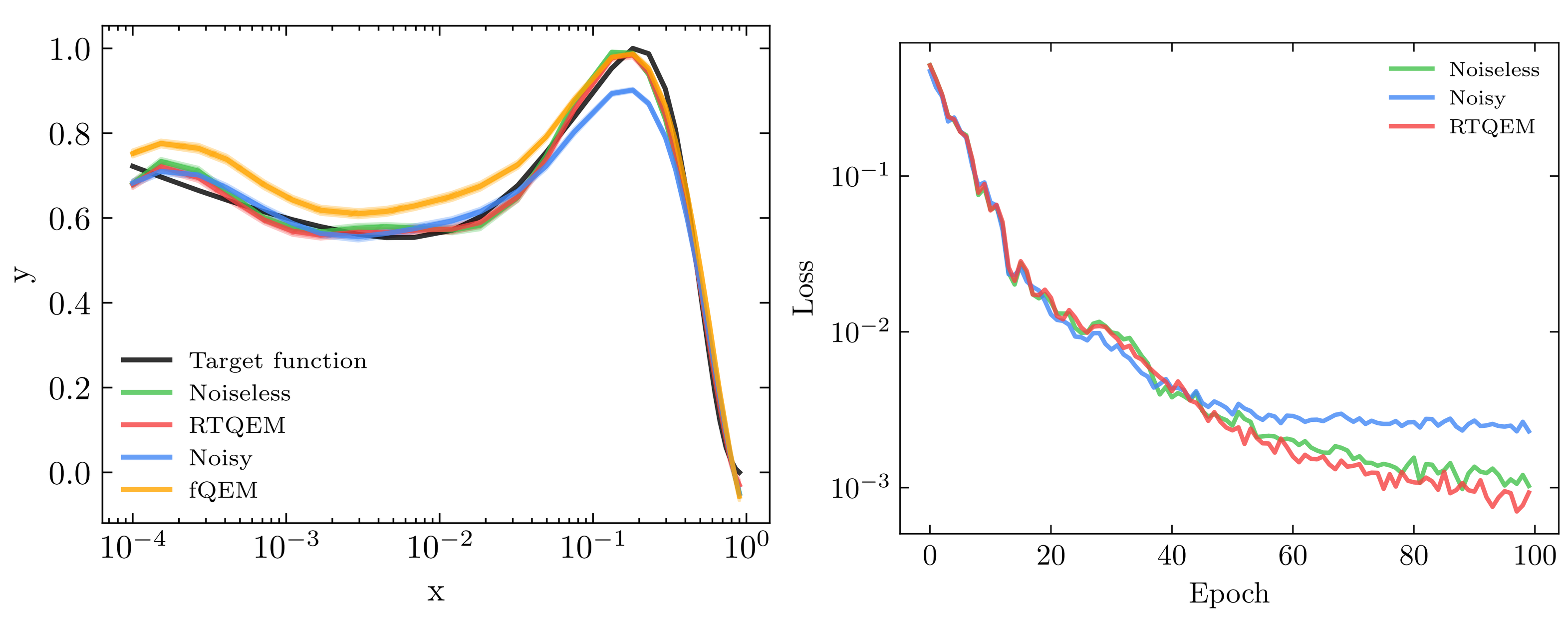
We consider a quantum system affected by local pauli noise with parameters $-1 \leq q_X, q_Y, q_Z \leq +1$ and readout noise parametrized by bit-flip probability $(1-q_M)/2$. This setup gives rise to Noise-Induced Barren Plateaus (NIBP) [2], which tend to concentrate the expectation value around 0.




To mitigate the effect of the noise, we use the Importance Clifford Sampling (ICS) [3] technique, which is a learning-based method which can be used to learn a noise map $\ell$ using a training set of Clifford circuits $\mathcal{S} = \{\mathcal{C}^i_{\rm cliff}\}$ built on top of the target circuit $\mathcal{C}^0$.

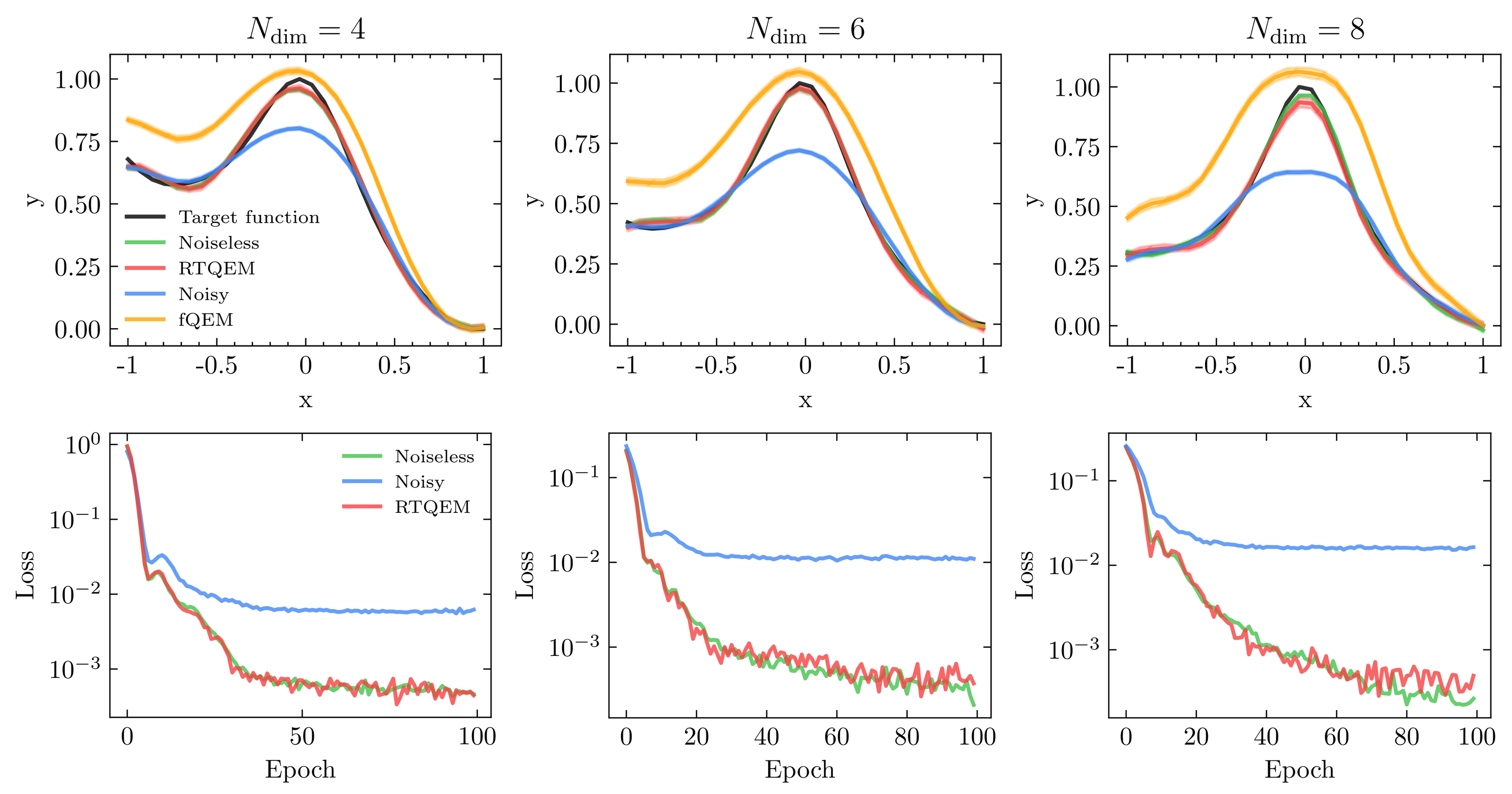
## Simulation 1-dim: $u$-quark PDF

We firstly use a single-qubit circuit to fit the $u$-quark Parton Distribution Function (PDF). We set $q_M = 0.005$, $q_X = 0.007$, $q_Y = 0.003$ and $q_Z = 0.002$. We compare four configurations: noiseless, noisy unmitigated, noisy with mitigation on the final predictions (fQEM) and noisy trained with RTQEM.




## Simulation $n$-dim

We then tackle a simple multi dimensional target to scale up with the number of qubits.



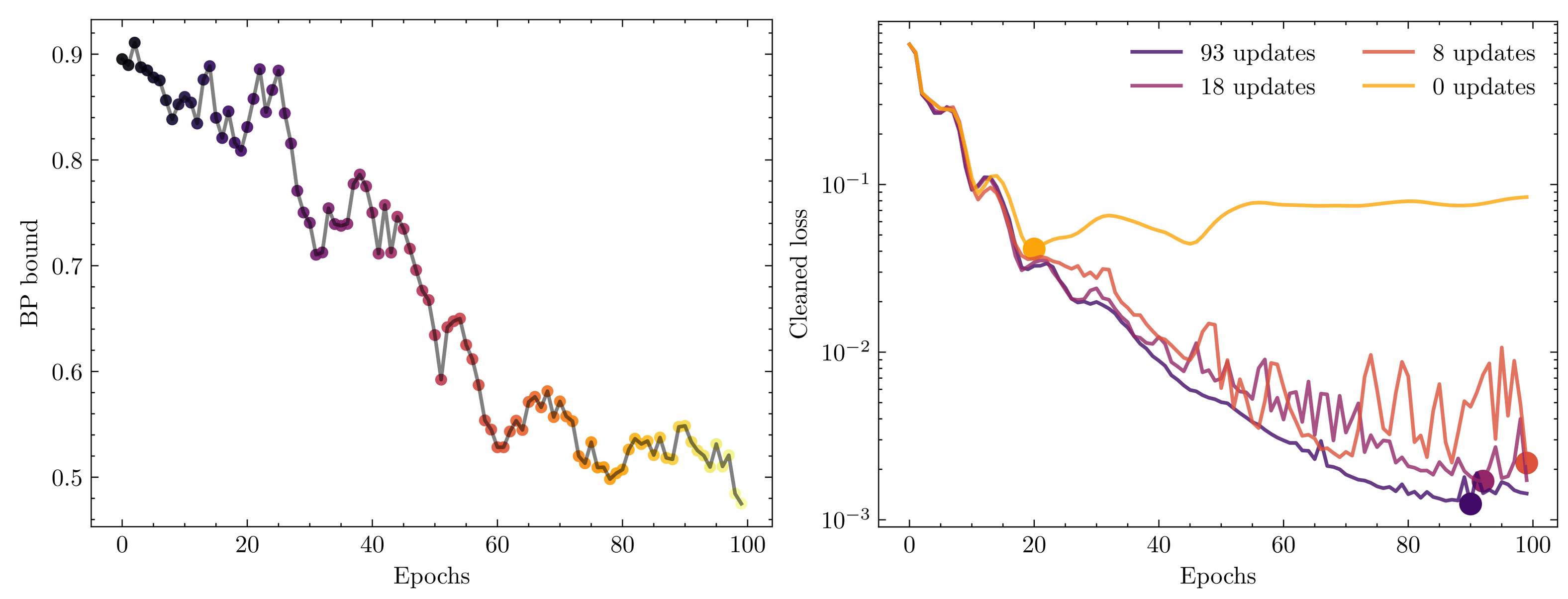

## Simulation results

Mean squared error between the target labels and the predicted values.

| Target | $\mathrm{MSE}_{\rm noiseless}$ | $\mathrm{MSE}_{\rm noisy}$ | $\mathrm{MSE}_{\rm fqem}$ | $\mathrm{MSE}_{\rm rtqem}$ |
|---|---|---|---|---|
| $u$ PDF | 0.008 | 0.018 | 0.023 | 0.008 |
| $\cos 4$d | 0.003 | 0.043 | 0.140 | 0.003 |
| $\cos 6$d | 0.002 | 0.083 | 0.214 | 0.002 |
| $\cos 8$d | 0.001 | 0.118 | 0.360 | 0.004 |

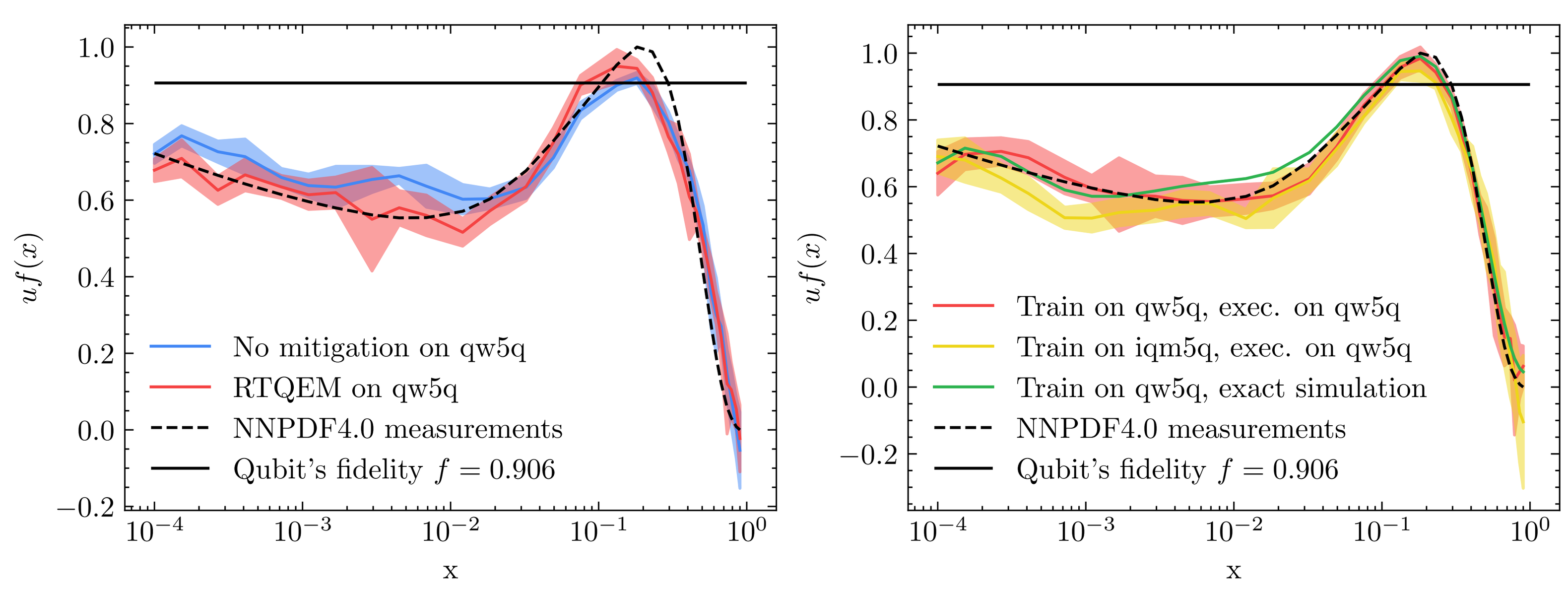
## Evolving noise scenario

To study how the RTQEM procedure behave in a realistic scenario, we let the noise parameters vary following a Random Walk-like evolution.




## $u$-quark PDF fit on superconducting devices

We finally test the RTQEM algorithm on two superconducting devices.




## Hardware results

We benchmark the MSE values of various prediction configurations.

| Training | Predictions | Config. | $N_{\rm epochs}$ | MSE |
|---|---|---|---|---|
| qw5q | qw5q | Noisy | 50 | 0.0055 |
| qw5q | qw5q | RTQEM | 50 | 0.0042 |
| qw5q | qw5q | RTQEM | 100 | 0.0013 |
| iqm5q | qw5q | RTQEM | 100 | 0.0037 |
| qw5q | sim | RTQEM | 100 | 0.0016 |

## References


[1] A. Pérez-Salinas, A. Cervera-Lierta, E. Gil-Fuster, and J. I. Latorre, "Data re-uploading for a universal quantum classifier," *Quantum*, vol. 4, p. 226, feb 2020.

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[3] D. Qin, Y. Chen, and Y. Li, "Error statistics and scalability of quantum error mitigation formulas," *npj Quantum Information*, vol. 9, apr 2023.