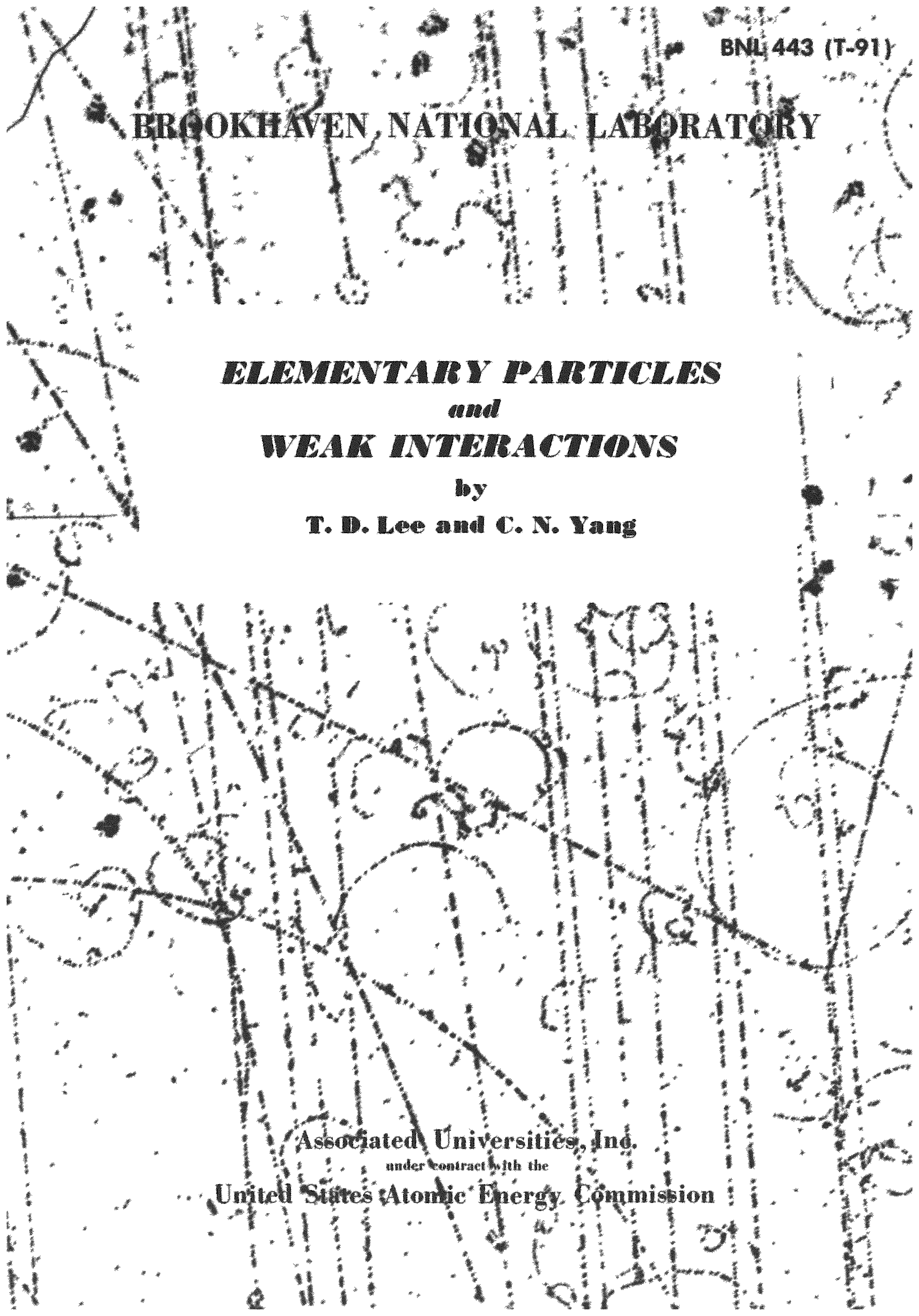

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BROOKHAVEN NATIONAL LABORATORY

ELEMENTARY PARTICLES *and* *WEAK INTERACTIONS*

by

T. D. Lee and C. N. Yang

Associated Universities, Inc.
under contract with the
United States Atomic Energy Commission

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Foreword

The substance of these notes comes from published and unpublished research of the undersigned. In form and presentation these notes follow largely a series of six lectures given by one of us (T.D. Lee) at Brookhaven National Laboratory during January 1957. The lecture notes were originally edited by Drs. L.C.L. Yuan, B.H. McCormick, W. Chinowsky, and R.K. Adair, to all of whom grateful acknowledgment is hereby made. Dr. Yuan is especially to be thanked for the time and advice he generously contributed in the process of the later expansion and changes in the notes.

T.D. Lee
C.N. Yang

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I. GENERAL REVIEW

1. Introduction

In these discussions we shall try to review some general patterns of the interactions between various elementary particles and to study some general questions concerning the symmetry properties of these particles. The first natural question that one would like to ask is, what precisely constitutes an elementary particle? Suppose a new particle is observed, how do we know that it is an elementary particle and not merely a composite system consisting of some already known elementary particles? The answer is that we do not know. Nevertheless, the term "elementary particle" is well defined in a negative sense. We believe we understand what is meant by an atom, a molecule, and a nucleus. Any small particle that is not an atom, not a molecule, and not a nucleus (except the hydrogen nucleus) is called an elementary particle.

At first sight these so-called elementary particles form a very inhomogeneous group. The mass of a particle may be large (as $M_{\Xi} \cong 1321$ Mev) or may be very small (as the mass of $\gamma = 0$). The lifetime may be very short (as the lifetime of $\Sigma^0 \cong 10^{-20}$ sec) or may be very long (as in the case of a proton which has infinite lifetime). The particle may be electrically charged or may be neutral. Except for a very few of them, we do not know the spin and parity of these particles. Again except for those between a very few of these particles, we do not know their interactions. In fact there is still confusion as to the identity of the particles that have already been observed. Some of these particles that look very different may turn out to be the same particles. Some of them that look very similar may indeed be several different particles. In spite of this almost complete lack of understanding, some general patterns and general rules have been found. These properties will be our main topics of discussion.

2. Review of the Classifications of Interactions

We find that the interactions between these elementary particles fall into four distinct groups:

(i) Strong interactions. This group includes the forces responsible for the production and the scattering of nucleons, pions, hyperons, and K-mesons. It is characterized by a coupling constant of the order of magnitude 1 ($f^2/\hbar c \cong 1$).

(ii) Electromagnetic interactions. These are characterized by the coupling constant, $e^2/\hbar c = 1/137$.

(iii) Weak decay interactions. These are characterized by a dimensionless coupling constant, $\mathcal{G}^2/\hbar c \cong 10^{14}$.

[(iv) Gravitational interactions. The gravitational interaction can be characterized by a dimensionless coupling constant $Gm^2/\hbar c \cong 2 \times 10^{39}$ with G as Newton's gravitational constant and m chosen to be the mass of the proton. The gravitational interaction will not be further discussed.]

Among these three interactions only the electromagnetic interaction is well understood. About the other two groups, the strong interactions and the weak decay interactions, we know really very little. Nevertheless, we believe they possess certain symmetry properties.

3. Invariance Properties and Conservation Laws that are USUALLY Accepted as Exact

We shall now list those symmetry properties and conservation laws that were, before the end of 1956, generally accepted to be valid for all three groups of interactions. These are:

(i) Conservation of energy and momentum. This follows from the invariance under translations in the four-dimensional space. The infinitesimal translations in space are represented by the energy-momentum operators P_μ. Thus, the homogeneity of space implies the conservation of energy and momentum.

(ii) Invariance under the proper Lorentz transformation. A proper Lorentz transformation is a Lorentz transformation without either space inversion or time reversal. This invariance implies, among other things, the conservation of angular momentum.

(iii) (?) Invariance under space inversion P (or, the conservation of parity). P is a transformation which changes $\mathbf{r} \to -\mathbf{r}$; $t \to +t$; and particles $\to$ particles.

(iv) (?) Invariance under time reversal T (i.e., $\mathbf{r} \to +\mathbf{r}$; $t \to -t$; and particle $\to$ particle).

(v) (?) Invariance under charge conjugation C. The charge conjugation operator C changes a particle to its antiparticle, but leaves $\mathbf{r} \to +\mathbf{r}$ and $t \to +t$.

(vi) Conservation of charge Q. This conservation law is related to the invariance under gauge transformation.

(vii) Conservation of heavy particle number N.

Because of our belief that these invariance principles and conservation laws are valid for all interactions, strong, electromagnetic, and weak interactions, the most important characteristics of an elementary particle are its mass, spin, parity, charge, and heavy particle number. The question of its detailed dynamical behavior, such as scattering cross sections, production cross sections, decay modes, and lifetimes, is usually studied under specific assumptions about these intrinsic characteristics of the particle.

However, recent experiments[1,2] have led to a different picture of these symmetry principles. In particular, they show that the invariance under space inversion and charge conjugation are not valid in certain weak interactions. We shall return to these questions in detail in our later discussions.

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[1] C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes, and R.P. Hudson, *Phys. Rev.* **105**, 1413 (1957).

[2] R.L. Garwin, L.M. Lederman, and M. Weinrich, *Phys. Rev.* **105**, 1415 (1957); J.I. Friedman and V.L. Telegdi, *Phys. Rev.* **105**, 1681 (1957).

4. Invariance Properties That Are Approximately True

It has been found that in addition to the above symmetry principles there are some further approximate invariance principles. These are the conservation of isotopic spin I and the conservation of I_z (or strangeness S).[3, 4] We shall discuss the properties of these approximate conservation laws.

A. PION-NUCLEON SYSTEM

We review briefly the concept and the validity of isotopic spin in a system consisting of pions and nucleons. We describe the proton and the neutron as a spinor in the isotopic spin space:

	I	I_z	
p	½	½	(1.1)
n	½	$-$½ .	

The pions are considered as forming a vector in the isotopic spin space according to the assignments:

	I	I_z	
π^+	1	1	
π^0	1	0	(1.2)
π	1	-1 .	

The total isotopic spin of a pion-nucleon system is given by the quantum mechanical sum of the isotopic spin vectors of all particles. If the total isotopic spin **I** is conserved, then all physical observables are left invariant under a rotation in the isotopic spin space. We find that for the *strong interactions* this is indeed true.

Role of the Electromagnetic Field

From the assignments (1.1) and (1.2) we find the following empirical relation between the charge Q and I_z *for a pion-nucleon system:*

$$Q=I\ +(N/2) \tag{1.3}$$

where N is the heavy particle number ($N=1$ for p and n; $N=0$ for π). Because of this relationship the electromagnetic field destroys the invariance under arbitrary isotopic rotations except along the z axis. Thus the total isotopic spin quantum number I is not conserved in the electromagnetic interaction, while I_z is still conserved. To see this in detail let us represent the field of the nucleon by a two-component spinor function

$$\psi=\begin{pmatrix}\psi_p\\ \psi_n\end{pmatrix}.$$

---

[3] M Gell-Mann, *Phys Rev* **93**, 933 (1953)

[4] K Nishijima, *Progr Theoret Phys (Japan)* **12**, 107 (1954)

The corresponding isotopic spin operator for a nucleon is then given by

$$\mathbf{I}_{\text{nucleon}}=(1/2)\int \psi^{\dagger}\gamma_4 \boldsymbol{\tau}\psi d^3r \tag{1.4}$$

with $\boldsymbol{\tau}$ representing the 2×2 Pauli matrices. The electromagnetic field is described by the Hamiltonian

$$H_{\gamma}=-\int j_{\mu}A_{\mu}d^3r \tag{1.5}$$

where A_{μ} is the electromagnetic potential and j_{μ} is the electric current given by

$$j_{\mu}=(1/2)ie[\psi^{\dagger}\gamma_4\gamma_{\mu}(1+\tau_3)\psi]\,. \tag{1.6}$$

We therefore immediately find that I_z commutes with H_{γ},

$$[I_z,H_{\gamma}]=0\,, \tag{1.7}$$

which means that I_z is conserved in the electromagnetic interactions. On the other hand, I_x, I_y, and $I^2\,(=\,I_x{}^2+I_y{}^2+I_z{}^2)$ all do not commute with H_{γ}. In fact, under a rotation in the isotopic spin space the noncommuting part in H_{γ} behaves like the third component of an isotopic spin vector. Consequently, we find that the electromagnetic interactions can violate conservation of I by

$$\Delta I=0,\ \pm 1\,, \tag{1.8}$$

whereas I_z is still conserved, i.e.,

$$\Delta I_z=0\,. \tag{1.9}$$

Similarly, we can apply the above arguments to any pion-nucleon system and obtain selection rules identical to Equations (1.8) and (1.9). The breakdown of the total isotopic spin by the electromagnetic fields accounts, at least in a qualitative sense, for the mass difference between the neutron and the proton ($\Delta m\cong 2.5m_e$) and the mass difference between $\pi^{\pm}$ and π^0 ($\Delta m\cong 9m_e$).

B. CONSERVATION OF I SPIN IN SYSTEMS INVOLVING OTHER ELEMENTARY PARTICLES

It is natural (in fact, it is almost necessary) to assume that I spin is conserved for *all strong interactions.* Let us, as an example, consider the following strong interaction:

$$\pi^- + p \rightarrow \Lambda^0 + K^0\,. \tag{1.10}$$

Suppose, for the sake of argument, that this reaction does not conserve the isotopic spin vector **I**. By going through the virtual processes of emission and absorption of Λ^0+K^0, we would find that isotopic spin is not a good quantum number for the π^-+p system. Since the reaction (1.10) is a strong interaction, the violation of isotopic spin conservation in the pion-nucleon system will be strong. From our previous discussions we know that this is not the case. Consequently, reaction (1.10) is expected to conserve the isotopic spin. Similarly, we expect that the conservation law of isotopic spin should be valid for all strong interactions.

Accepting, then, that isotopic spin is conserved in all strong interactions, what must be the isotopic spin assignment of each of the other elementary particles in the various strong reactions, and what will be the relationship between I_z and Q for these new particles? Consider a particle A which is involved in the strong interactions. As the strong interactions conserve isotopic spin, we must assign to the particle A an isotopic spin quantum number I and its z component I_z. By a rotation in the I spin space, we generate $2I + 1$ states that are degenerate with respect to the strong interactions. We shall show that these isotopic spin multiplets are expected to be states of charges differing from each other by unity.

Let us consider the charge Q of one of these states. It is easy to see that if Q is a function of I_z then it must be a linear function of I_z. This follows from the fact that in a system of these particles A, the total charge Q and the total I_z of the system are given, respectively, by the linear sum of the Q and I_z of the individual particles. Consequently Q can at most be a linear function of I_z, i.e.,

$$Q = \alpha_A I_z + (Y_A/2) \tag{1.11}$$

where α_A and Y_A are quantities independent of I_z.

We want to demonstrate that the constant α_A must be 1. Let us consider a strong interaction of the form

$$\text{pions} + \text{nucleons} \rightarrow A + B\,. \tag{1.12}$$

From Equation (1.3) and the assumption that I_z is conserved in reaction (1.12), we have

$$(Q)_{\text{total}} = (I_z)_{\text{total}} + [(\mathcal{N})_{\text{total}}/2] = Q_A + Q_B. \tag{1.13}$$

For simplicity, suppose that

$$I_A = I_B = ½\,,$$

and consider first the state where the z components of both I spins are up $(\uparrow_A)(\uparrow_B)$. We have then $(I_z)_{\text{total}} = 1$. The total charge Q is

$$Q = Q(\uparrow_A) + Q(\uparrow_B) \tag{1.14}$$

where $Q(\uparrow_A)$ and $Q(\uparrow_B)$ are the charges corresponding to the states $\uparrow_A$ and $\uparrow_B$. Since the total I spin is conserved, we must have also the corresponding state with $(I_z)_{\text{total}} = 0$, which is $(1/\sqrt{2})(\uparrow_A\downarrow_B + \downarrow_A\uparrow_B)$. Furthermore, by considering the left-hand side of the reaction (1.12) and by using Equation (1.13) we conclude that the corresponding total charge for this state, $(I_z)_{\text{total}} = 0$, must be $Q - 1$ with Q given by Equation (1.14). Hence we have

$$Q(\uparrow_A) + (Q(\downarrow_B) = Q(\downarrow_A) + Q(\uparrow_B) = Q - 1\,.$$

By comparing with (1.11), we have

$$\alpha_A = \alpha_B = 1\,.$$

In a similar way, this result $\alpha_A = \alpha_B = 1$ can be generalized to particles of I values other than ½. It therefore follows that for all the strange particles[5] which interact strongly with the pion-nucleon system, we must have the relation,

[5]The term "strange particle" is again defined in a negative sense. It applies to any elementary particle which participates in the strong interactions but is not a nucleon or a pion.

$$Q = I_z + (Y/2) \tag{1 15}$$

where Y is a quantity independent of I_z

From this relation, Equation (1 15), there follow a number of consequences which we list below

(i) In all these discussions we shall assume that *there does not exist a doubly charged particle* Consequently from (1 15), I is limited to the values

$$I = 0,\ \tfrac{1}{2},\ 1 \tag{1 16}$$

for a single particle

(ii) If we assume as consistent with experimental evidence that there are no charged particles of about the same mass as the Λ^0, then we must have

$$I_\Lambda = 0 \tag{1 17}$$

(iii) Consider again the reaction

$$\pi \ + p \rightleftarrows \Lambda^0 + K^0 \tag{1 10}$$

The total isotopic spin of the left-hand side is $(I\ )_{\text{total}} = \tfrac{1}{2}$ or $3/2$ By using the above properties (i) and (ii) we conclude

$$I_K = \tfrac{1}{2}\,, \tag{1 18}$$

and for the I_z component

$$(I_z)_K \ = -\tfrac{1}{2} \tag{1 19}$$

From Equation (1 15) the corresponding particle with $I_z = +\tfrac{1}{2}$ must be a positively charged particle K By applying the charge conjugation operator $\mathcal{C}$ to K , K^0, we generate two other K-particles, K and $\bar{K}^0$, with

$$I_{\bar{K}} = \tfrac{1}{2}\,,\ (I_z)_{\bar{K}^0} = \tfrac{1}{2}, \quad \text{and} \quad (I_z)_K \ = -\tfrac{1}{2} \tag{1 20}$$

Thus we uniquely determine the I spin of the K-particles and deduce that there must be at least four such particles

(iv) In a manner entirely analogous to the above, we can use the reaction, say,

$$\pi \ + p \rightarrow \Sigma \ + K \tag{1 21}$$

By comparing the I_z value on both sides we conclude that

$$(I_z)_\Sigma = -1$$

Assuming that there is no doubly charged particle we have

$$(I\ )_\Sigma = 1\,, \tag{1 22}$$

which implies a triplet Σ and Σ^0

Role of Electromagnetic Interactions

Now since we have determined the relationship between charge and I_z, Equation (1 15), for all the strange particles, we can apply the same argument concerning the role of electromagnetic interaction as that used in the pion-neucleon system In an entirely similar way it can be shown that the electromagnetic interaction conserves I_z, but not the total isotopic spin I The selection rule is

$$\Delta I=0, \ \pm 1 \tag{1 23}$$

and

$$\Delta I_z=0 \tag{1 24}$$

Consequently, for example, Σ^0 is unstable against γ emission

$$\Sigma \rightarrow \Lambda^0+\gamma \tag{1 25}$$

Since I_z is conserved in the electromagnetic interaction as well as in the strong interactions and since the charge Q and the heavy particle number N are known to be conserved in all these interactions, any linear combinations of these three quantities will be conserved in both the strong and the electromagnetic interactions In particular, it is useful to define

$$S/2\equiv Q-I_z-(N/2), \quad \text{and} \quad Y\equiv S+N \tag{1 26}$$

Because of the conservation of I , it follows that both S and Y are conserved in the strong as well as the electromagnetic interactions The quantity S is called the strangeness quantum number [3 4] From the assignment of I_z for various particles we find

$$\begin{aligned} &(S)_\pi=(S)_p=(S)_n=0\,, \\ &(S)_K=(S)_K=+1\,, \\ &(S)_K=(S)_{\bar{K}}=-1\,, \\ &(S)_\Lambda=(S)_\Sigma=(S)_\Sigma=-1 \end{aligned} \tag{1 27}$$

Both S and Y do not vary with respect to different I components in the same I multiplets

Let us consider now the assignment of I to the cascade particle Ξ The production of the Ξ with two neutral K-mesons has been observed [6] However, we do not know whether these K-mesons are K 's or $\bar{K}^0$'s Since S is conserved in production processes, it follows that the S of Ξ is either zero or ± 2 depending on the nature of these K-mesons For Ξ , the charge Q is -1 and the heavy particle number N is $+1$ From Equation (1 26) we have

$$(I_z)_\Xi=-(3/2)-(½)(S)$$

In order that there be no doubly charged particle observed, we must have $|I_z|\leqslant 1$ Consequently, the isotopic spin assignment to Ξ is

$$(I_z)_\Xi=-½ \quad \text{and} \quad I_\Xi=½ \tag{1 28}$$

Thus there should exist another Ξ^0 with $(I_z)_{\Xi^0}=+½$

In Table 1 are listed various quantum numbers together with the masses and lifetimes for these particles

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[6]J D Sorrels R B Leighton and C D Anderson *Phys Rev* **100**, 1457 (1955)

Table 1

Particle	Lifetime (sec)	Mass (Mev)	Spin, Parity	I	I_z	(S)
Ξ	$(4\ 6<\tau<200)\times 10^{\ 10}$	1321 $(\pm 3\ 5)$		½	−½	−2
Σ	$1\ 6\ \times 10^{\ 10}$	1196 65$(\pm 0\ 35)$		1	−1	−1
Σ^+	$0\ 69\times 10^{\ 10}$	1189 7 $(\pm 0\ 25)$			+1	−1
Σ^0	$(<1)\times 10^{\ 11}$	1188 65$(^{+3}_{\ 2})$			0	−1
Λ^0	$3\ \ \times 10^{\ 10}$	1115		0	0	−1
p		938	½	½	½	0
n	10^3	939	½		−½	0
K	$1\ 2\ \times 10^{\ 8}$	494		½	½	1
K^0	$0\ 95\times 10^{\ 10}$	494			−½	1
$\overline{K}^0$	$0\ 95\times 10^{\ 10}$	494		½	½	−1
K	$1\ 2\ \times 10^{-8}$	494			−½	−1
π^+	$2\ 6\ \times 10^{\ 8}$	140	0	1	±1	0
π^0	$10^{\ 15}$	135	0		0	0
μ^+	$2\ 2\ \times 10^{\ 6}$	106	½			
$e^{\pm}$		0 51	½			
$\nu, \bar{\nu}$		0	½			
γ		0	1			

C. WEAK INTERACTIONS

If we examine the weak interactions in detail, we find that they are divided distinctly into two groups:

(i) Those which are characterized by a nonconservation of I_z with

$$\Delta I_z = \pm \tfrac{1}{2}\,. \tag{1.29}$$

The neutrinos are not involved in these reactions. As examples of these reactions we have the decay of all hyperons, the $K_{\pi 2}$, the $K_{\pi 3}$, etc.

(ii) Those which involve neutrinos, such as the β decay, the μ decay, the π decay, the $K_{\mu 2}$, etc.

These two groups seem to have completely different characteristics. Yet they share a remarkable common feature which is that the strengths of the coupling constants for these two groups seem to be about the same. Of course, we do not really know how to calculate these coupling constants, because only for very few of these weak interactions, like β decay and μ decay, do we know something about the detailed dynamics of the decay reaction. In most of the decays we do not know how the various fields are coupled. Consequently, we can only estimate these coupling constants in a very crude way. For cases where the detailed interaction is unknown, we use the formula (with $\hbar = c = 1$)

$$1/\tau = 2\pi \mathcal{G}^2 (1/R)^2 \rho_E \tag{1.30}$$

where ρ_E is the number of final states per unit energy,

$$\rho_E = \frac{8\pi^3}{\Omega} \int \prod_{i=1}^{n} \left(\frac{1}{8\pi^3} d^3p_i d^3r_i \right) \delta^3(\textstyle\sum \mathbf{p}_i)\, \delta(\textstyle\sum E_i - E)\ ;$$

Table 2

	Lifetime (sec)	$\mathcal{G}^2 \times 10^{14}$
$\Lambda^0 \to p+\pi$	3×10^{10}	1 6
$\Sigma \to n+\pi$	$1\ 6 \times 10^{10}$	1 2
$K^0 \to 2\pi$	$0\ 95 \times 10^{10}$	2 8
$\pi^+ \to \mu^+ + \nu$	$2\ 6 \times 10^{8}$	0 3
$\mu^+ \to e^+ + \nu + \bar{\nu}$	$2\ 2 \times 10^{6}$	2
$K^+ \to \mu^+ + \nu$	$1\ 2 \times 10^{8}$	0.02

R represents a characteristic length of these decays; and $\Omega=(4\pi/3)R^3$. The total number of the particles in the decay product is n, and $\mathbf{p}_i$, E_i are their corresponding momenta and energies. In Table 2 are listed the various lifetimes and the corresponding coupling constants for several of the decay interactions in both group (i) and group (ii). In all these reactions we use

$$R=\hbar/m_\pi c \tag{1.31}$$

for simplicity. These results of course have significance only in their crude order of magnitude. For the purpose of comparison we include in Table 2 also μ decay calculated in the same way, even though we do know a great deal about its coupling.[7]

We observe from Table 2 the remarkable fact that although the lifetimes of these particles vary over a wide range from 10^{-10} sec to 10^{6} sec the corresponding coupling constants $\mathcal{G}^2$ seem to be much more stable.

On the other hand, as remarked before, these decay interactions are separated into two distinct groups. In the first group, (i), the neutrinos are not involved; instead there is a nonconservation of I_z. In the other group, (ii), every reaction contains some neutrinos. Furthermore, these reactions are between many particles for which isotopic spin seems to play no role. The fact that they share approximately the same strength in coupling constants does suggest strongly a deep common origin for all weak interactions. As we shall see later, these weak interactions *may* share another significant feature, namely the violation of invariance under space inversion and charge conjugation.

[7]The coupling constants for β decay and μ capture are not included in Table 1 It is well known that they have the same order of magnitude as that for μ decay See, e g., E. Fermi, *Elementary Particles*, Yale University Press, 1951

II. THE θ-τ PUZZLE

Among the various interesting phenomena concerning elementary particles, we would like to discuss specifically first the θ-τ puzzle, because it was due to this puzzle that a re-examination of the experimental basis of our various supposedly exact conservation laws was made In Table 3 are listed the recently measured values of the mass, abundance, and lifetime of the various decay modes of charged K-mesons [8]

Table 3

Type	Abundance	Mass of K from primary particle	Mass of K from secondary particles	Lifetime
τ	5 56±0 41	966 3±2 1	966 1±0 7	$(1\ 19\pm0\ 05)\times10^{-8}$
τ'	2 15±0 47	967 7±4		
K_{μ}	58 2 ±3 0	967 2±2 2	965 8±2 4	$(1\ 24\pm0\ 02)\times10^{-8}$
K_{π}	28 9 ±2 7	966 7±2 0	962 8±1 8	$(1\ 21\pm0\ 02)\times10^{-8}$
$K_{\mu 3}$	2 83±0 95	969 ±5		$(0\ 88\pm0\ 23)\times10^{-8}$
K_{e3}	3 23±1 30			$(1\ 44\pm0\ 46)\times10^{-8}$

We see that the masses are extremely close to each other and the lifetimes agree within the experimental error of $\sim\pm5\%$ About three years ago, Dalitz pointed out that by plotting the angular and energy distribution of the three π-mesons from the decay of the τ-meson ($\equiv K_{\pi 3}$), it is possible to determine the spin and parity of the τ *In the following discussion of the θ-τ puzzle, we shall assume that both spin and parity are absolutely conserved*

1. Review of the Spin-Parity Determination of θ and τ (Dalitz's Analysis[9])

Let us consider first a θ-meson The θ-meson is defined to be a K-particle which can decay, among other modes, into two π-mesons, e g ,

$$\theta^{+}\rightarrow\pi^{+}+\pi^{0} \tag{2 1}$$

Assuming that both the spin and parity are conserved in reaction (2 1), the parity

[8] R W Birge, D H Perkins, J R Peterson, D H Stork, and M N Whitehead, *Nuovo cimento* **4**, 834 (1956), V Fitch and R Motley, *Phys Rev* **101**, 496 (1956), L W Alvarez, F S Crawford, M L Good, and M L Stevenson, *Phys Rev* **101**, 503 (1956), J Orear, G Harris, and S Taylor, *Phys Rev* **104**, 1463 (1956)

[9] R Dalitz, *Phil Mag* **44**, 1068 (1953), *Phys Rev* **94**, 1046 (1954), E Fabri, *Nuovo cimento* **11**, 479 (1954)

of the θ is uniquely determined by its spin value. Let J be the spin of the θ-particle. Because each π-meson is a pseudoscalar, we have

$$P_\theta = (-1)^J \tag{2.2}$$

where $P_\theta =$ parity of the θ-meson. Thus, the spin-parity assignment of θ can only be (0+), (1−), (2+), etc.

On the other hand, the possible spin-parity assignments for a τ-meson are quite different. The τ-meson is defined to be a K-meson which can decay, among other modes, into three π-mesons, e.g.,

$$\tau^- \rightarrow \pi^+ + \pi^+ + \pi^- \, . \tag{2.3}$$

The final decay state of a τ-meson is characterized by two momenta in its center-of-mass system. We may choose these two momenta to be:

(i) The relative momentum, $\mathbf{p}$, between the two π^+ mesons.

(ii) The momentum, $\mathbf{k}$, of the π^- in the center-of-mass system of the τ-meson. (It may differ by a factor 3/2, if the momentum is chosen to be that of the π^- with respect to the center-of-mass system of the two π^+ system.)

Let $\mathbf{J}$ be the spin of the τ-meson; then

$$\mathbf{J} = \mathbf{L}_p + \mathbf{L}_k \tag{2.4}$$

where $\mathbf{L}_k$ and $\mathbf{L}_p$ are respectively the orbital angular momentum of the π^- and the relative angular momentum of the two π^+. If the spin of the τ is zero, then the final state of the three pions must consist of states with $L_p^2 = L_k^2$. Hence the parity of the final state must be −1. However, if the spin of τ is not zero then the parity of the final state can be either +1 or −1. Consequently the spin-parity assignments for τ are (0−), (1±), (2±), etc.

It is easy to see the following simple conclusions:

(i) If the spin of τ is zero and if parity is conserved in the decay, then $\theta \neq \tau$.

(ii) If there exists a zero energy pion (π^+ or π^-) in the 3π state of τ decay and if parity is conserved in the decay, then $\theta \neq \tau$.

(iii) If there exists a zero energy π^- in the 3π state of the τ decay then the spin of τ must be even. If, further, parity is conserved, the parity of τ must be odd.

Of course, in reality it is not possible to observe a zero energy pion. But quite a few low energy (~⅓ Mev) π^- and π^+ have been observed. Thus, even without detailed statistical analysis, it is to be expected that *most probably* the spin of τ is even, and that if parity is conserved in the decay process then τ and θ are two different particles. To evaluate the exact meaning of the likelihood it is necessary to perform a detailed statistical analysis.

Let us characterize the system by an angle θ and a parameter ε defined to be

$$\theta = \angle(\mathbf{p},\mathbf{k}) \quad \text{and} \quad \varepsilon = (k/k_{\max})^2 \, . \tag{2.5}$$

If $\psi(\mathbf{p},\mathbf{k})$ represents the final state wave function of the 3π in the τ decay then the probability distribution function $P(\theta,\varepsilon)$ is

$$P(\theta,\varepsilon) \propto |\psi(\mathbf{p},\mathbf{k})|^2 \sqrt{\varepsilon(1-\varepsilon)}\, d\varepsilon \, d(\cos\theta) \, . \tag{2.6}$$

Experimentally with an energy value ranging from ~⅓ Mev to ~50 Mev and with a total of ~1000 events, Dalitz and others find that

$$|\psi|^2=1 \tag{2.7}$$

is an extremely good fit to these data. From the above conclusion (ii) we see that if τ is the same as θ, then $|\psi|^2$ must be zero if, say, $\varepsilon=0$. In fact, it is possible to prove the following rigorous statements concerning the behavior of ψ at various limiting regions.

(iv) If τ is the same as θ and if parity is conserved in the τ decay, then

$$\text{as } \varepsilon\to 0, \ |\psi|^2\to \varepsilon^n \quad (n\geqslant 1)\,;$$

$$\text{as } \varepsilon\to 1, \ |\psi|^2\to (1-\varepsilon)^m \quad (m\geqslant 2)\,;$$

$$\text{and as } \theta\to 0, \ |\psi|^2\to \sin^2\theta\,. \tag{2.8}$$

To prove (iv), we consider first the simple case that the spin of τ is 1. From Equation (2.2) we see that if τ is the same as θ then the parity of τ is -1. Consequently the orbital parity due to **k** and **p** must be $+1$. Thus, ψ should be an axial vector. Furthermore, ψ must be an even function of **p** because of the Bose-statistical property of the two π^+-mesons. Hence it is easy to deduce that

$$\boldsymbol{\psi}=(\mathbf{k}\times\mathbf{p})(\mathbf{k}\cdot\mathbf{p})f[k^2, (\mathbf{k}\cdot\mathbf{p})^2, p^2]\,.$$

Or,

$$|\psi|^2\propto\varepsilon^2(1-\varepsilon)^2\sin^2\theta\cos^2\theta|f|^2\,. \tag{2.9}$$

The function f is expected to be a regular function of its arguments. (This is the case, e.g., if the decay of τ can be represented by a local field theory with decay interaction involving derivative couplings to any high, but finite, orders.) Hence we see that the wave function ψ satisfies the conditions given by Equation (2.8). Similarly, by going over the same type of arguments of forming various tensor functions with 2 vectors, **k** and **p**, one can easily prove (2.8) for any other spin values.

From (2.9) we see that it is very difficult to pick a function f such that in all observed regions in (θ,ε) space $|\psi|^2\cong 1$ so as to be compatible with experimental results. In fact, there are good arguments to expect the function f to be near 1. The reason is, as pointed out by Dalitz, that these pions are very low in energy, and an expansion of f into powers of k^2 and p^2 may be legitimate,

$$f=1+0(k^2R^2)+0(p^2R^2)+\ \dots\ . \tag{2.10}$$

From dimensional grounds we expect R to be some length that characterizes the τ-meson. For low energy pions we may neglect the terms $0(k^2R^2)$ and $0(p^2R^2)$ and we have

$$f\cong 1\,. \tag{2.11}$$

By using the condition $f\cong 1$ in the distribution function ψ, we can calculate the probability that one obtains $|\psi|^2=1$ from experimental events if ψ is actually given by Equation (2 9) This probability will be extremely small ($\lesssim 10^{-30}$)[10] Similar conclusions can be obtained for other spin values (except for very high ones) Thus, we conclude that *it is extremely improbable that τ and θ are the same particle* (under, of course, the framework that parity is conserved) The most probable spin-parity assignments of τ are $0-$ and $2-$

One may question how good is the approximation $f \cong 1$ The average kinetic energy of π in the τ decay is $\sim$26 Mev If R is taken to be $\hbar/m_\pi c$ where m_π is the pion mass, then

$$(k^2R^2) \cong 1/3 ,$$

which is not very small However the region where the distribution function $|\psi|^2$ [Equation (2 8)] deviates most from the experimental situation is precisely the region (e g , $k \to 0$ or $p \to 0$) where the relevant expansion parameter is small Consequently, one expects that the conclusions of Dalitz are statistically very significant Thus, there seem to be two particles of different spin-parity values The difficulty is, then, why should they have, within experimental error, the same mass and the same lifetime? This is the famous θ-τ puzzle

2. Previous Attempts to Solve the θ-τ Puzzle

Mainly for historical reasons we shall review some attempts that have been made to solve the θ-τ puzzle Although the Dalitz analysis was made more than three years ago, at that time various different masses of K-mesons were reported In fact there were indications that there might be a big mass difference between the various K-particles Also, the statistics of Dalitz's analysis at that time were not very good No one was greatly alarmed that there probably existed a K-particle (τ-meson) which was different from the θ-particle There were too many K-particles anyway However, about the spring of 1955, the situation was changed The experimental mass values gradually converged, though still with large probable error The Dalitz plot had more points and seemed to indicate convincingly that the τ is a $(0-)$-particle Also, at about the same time people started to measure the lifetime Before the lifetime measurements were done, some physicists speculated about what would happen if the lifetimes turned out to be the same At first sight it seemed that, if the lifetimes of $K_{\pi 2}$ and $K_{\pi 3}$ are measured to be the same, then this evidence would be used to argue that θ must be the same particle as τ and that the conclusion of Dalitz's analysis is probably a manifestation of some statistical fluctuations Later the experiments on lifetimes indeed showed that the observed decay constants for $K_{\pi 2}$ and $K_{\pi 3}$ are about the same [8]

Nevertheless, with all the available evidence at that time (1955) it was not difficult to find schemes which could be made compatible with (i) the results of Dalitz's analysis, (ii) the aparent identity of lifetimes, and (iii) the approximate equality of masses In describing these attempts we shall assume that the conclusions concerning Dalitz's analysis are correct and that τ and θ are two different particles

[10] See, e g , *Proc Sixth Ann Rochester Conf*, Interscience, New York, 1956

A. APPARENT LIFETIME EQUALITY

One such hypothesis, proposed to explain the apparent identity in lifetimes, is the so-called cascade process[11] The idea is that τ and θ are two different particles with two different lifetimes, say, 10^{-8} sec and 10^{-10} sec (On phase space arguments, one expects $K_{\pi 3}{}^{+}$ to live longer than $K_{\pi 2}{}^{+}$) The long-lived one, say τ, is assumed to have a heavier mass In addition to its other decay modes, τ decays into the light one, θ, by γ radiations Under the experimental conditions in these lifetime measurements, only the long-lived K-mesons would be observed Thus this cascade process can account for their apparent equality of lifetimes If the spin-parities of τ and θ are $0-$ and $0+$, then the cascade process is

$$\tau \rightarrow \theta + 2\gamma \tag{2 11}$$

In order to make the branching ratio correct a mass difference $m_\tau - m_\theta \cong 10$ Mev is required

B. MASS DEGENERACY[1]

By taking analogy with isotopic spin invariance, a mass degeneracy means possibly a new symmetry property This, for example, is the case of mass degeneracy between a proton and a neutron The only difference is that we now have degeneracy between states with different parities, instead of between states with different charges If one regards this mass degeneracy between τ and θ to be not an accident, then it means, just as in the case for isotopic spin, that the strong interaction must be invariant under a new symmetry operator This operator, denoted by C_p, when acting on a θ-particle changes it into a τ, and when acting on τ converts it into a θ The operator C_p is called "parity conjugation" by analogy with "charge conjugation "

$$C_p|\theta\rangle = |\tau\rangle, \quad C_p|\tau\rangle = |\theta\rangle \tag{2 12}$$

The approximate mass degeneracy now follows from the assumption that C_p commutes with the strong part of the Hamiltonian

$$[C_p, H_{\text{strong}}] = 0 \tag{2 13}$$

Because of Equation (2 13) it follows that all strong interactions should be invariant under the operation of C_p In particular, let us take an example, say,

$$\pi^- + p \rightarrow \Lambda_1{}^0 + \theta^0 \tag{2 14}$$

Under the operation C_p, there is no change in $\pi^- + p$ but θ^0 becomes τ^0 Therefore the Λ^0 must be a parity doublet, which we shall denote by $\Lambda_1{}^0$ and $\Lambda_2{}^0$ Equation (2 14) becomes

$$\pi^- + p \rightarrow \Lambda_2{}^0 + \tau^0 \tag{2 15}$$

under C_p

We conclude therefore that there must be a parity doubling not only of the K-mesons but also of hyperons There will be two Λ's of opposite parity and two Σ 's

[11] T D Lee and J Orear, *Phys Rev* **100**, 932 (1956)

[1] T D Lee and C N Yang, *Phys Rev* **102**, 290 (1956)

of opposite parity, etc In fact, there must be a parity doublet for all strange particles with odd strangeness quantum number The commutation relation between C_p and the H_γ (electromagnetic interaction) is not known A natural choice, of course, is to take advantage of the above explanation for the lifetimes and to assume that C_p does not commute with H_γ This can introduce a large mass difference and make the cascade process possible Combining these two possibilities, it seemed that one could have an explanation for the θ-τ puzzle However, within half a year,[8] the mass measurements have been greatly improved, with the result that the mass difference can at most be ~ 1 or 2 Mev, which makes process (2 11) very unlikely A direct search for such γ-rays was performed[13] and it also led to negative results

In the early part of 1956 it seemed that the true sloution of the θ-τ puzzle might in fact lie in something quite different Thus, an investigation of the experimental basis of the law of conservation of parity was made [14] We shall discuss now in some detail the conclusions reached through such an investigation

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[13]L Alvarez, *Proc Sixth Ann Rochester Conf*, Interscience, New York, 1956

[14]T D Lee and C N Yang, *Phys Rev* **104**, 254 (1956)

III. EXPERIMENTAL LIMITS ON THE VALIDITY OF PARITY CONSERVATION

In this section we shall try to discuss the limit on the validity of parity conservation in various fields of physics If parity is not a rigorously conserved quantum number then eigenstates ψ of the entire Hamiltonian are, in general, not eigenstates of the parity operator Thus we expect

$$\psi = \psi_p + F\psi_{\ p} \qquad (3\ 1)$$

where ψ_p and $\psi_{\ p}$ are of opposite parity and

$$F = \text{probability amplitude for parity mixing} \qquad (3\ 2)$$

It is useful to know from the various evidence in atomic and nuclear physics exactly what is the upper limit one can impose on the magnitude of F

1. Atomic Spectroscopy

From the various parity selection rules concerning the radiative transitions for an atomic system, we find an upper limit for F

$$|F|^2_{\text{atom}} < \left(\frac{r}{\lambda}\right)_{\text{atom}} \cong 10^{\ 6} \qquad (3\ 3)$$

for a typical atomic transition In (3 3), r is the radius of the atom and λ the wavelength of the radiation In principle, by studying transitions involving photons of long wavelengths it is possible to make this upper limit much smaller than $10^{\ 6}$ (It may not be impossible to reach the limit $|F|^2_{\text{atom}} < 10^{\ 12}$)

2. Nuclear Spectroscopy

While the above condition sets a limit on parity nonconservation in atomic interactions, the same limit cannot be applied directly to nuclear interactions Nevertheless, by using the various parity selection rules in nuclear spectroscopy, say β decay, it is possible to put a corresponding limit for the nuclear system,

$$|F|^2_{\text{nucleus}} < \left(\frac{r}{\lambda}\right)^2_{\text{nucleus}} \cong 10^{\ 4} \qquad (3\ 4)$$

3. Nuclear Reactions

The measurement by Chamberlain et al [15] on the double scattering of protons offers a very direct test of parity conservation In this experiment, a beam of incom-

[15] O Chamberlain, E Segrè, R Tripp, C Wiegand, and T Ypsilantis, *Phys Rev* **93**, 1430 (1954)

ing protons with momentum $\mathbf{p}_1$ is scattered by a first target into momentum $\mathbf{p}_2$ and is further scattered by a second target into momentum $\mathbf{p}_3$. If parity is conserved, the cross section should be independent of such a quantity as $(\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{p}_3$. The measurement shows the absence of such terms, giving directly an upper limit on F. From the measurements one can conclude

$$|F|^2 < 10^{-4}\,. \tag{3.5}$$

4. Static Electric Dipole Moment

If parity is conserved then the static electric dipole moment of any eigenstate of the Hamiltonian must be zero. Thus a measurement on the absence of such electric dipole moment for elementary particles gives also an upper limit of F. Smith, Purcell, and Ramsey[16] have measured the electric dipole moment of the neutron and found it to be smaller than 5×10^{-20} cm $\times$ electronic charge. If one takes the natural size of the neutron to be 10^{-13} cm, this gives a very severe limit on F:

$$|F|^2 < 3 \times 10^{-13}\,. \tag{3.6}$$

This limit applies directly to the structure of the neutron.

However, it is possible to show that *if time-reversal invariance holds then the static electric dipole moment must still be zero even though parity may not be conserved.* This is so because if parity is not conserved then the wave function ψ is a mixture of states with opposite parities ψ_p and ψ_{-p} as indicated by Equation (3.1). But if time reversal is invariant then ψ_p and ψ_{-p} will be 90° out of phase. Thus they cannot contribute to the diagonal element of the electric dipole moment which is a real quantity. We shall give a formal proof of the impossibility of having a static electric dipole if time reversal is invariant.

Consider a particle A with spin $\mathbf{J}$. The state function $|A\rangle_m$ describes the particle A at rest with $J_z = m$. The time reversal operator T is represented by

$$T = U_T \times \text{complex conjugation} \tag{3.7}$$

where U_T is a unitary operator. If invariance under time reversal is assumed, then

$$T|A\rangle_m = U_T|A^*\rangle_m = e^{i\delta_m}|A\rangle_{-m} \tag{3.8}$$

where * represents a pure complex conjugation, and $e^{i\delta_m}$ is a possible phase factor. Let $\mathbf{D}$ be the electric dipole moment,

$$\mathbf{D} = \sum e_i \mathbf{r}_i\,. \tag{3.9}$$

The average value of $\mathbf{D}$ must be proportional to that of $\mathbf{J}$.

$$\langle A|\mathbf{D}|A\rangle_m = K\langle A|\mathbf{J}|A\rangle_m \tag{3.10}$$

where K is a real numerical constant. If we take the complex conjugation of both sides in Equation (3.10) and replace 1 by $U_T^\dagger U_T$, Equation (3.10) becomes

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[16] E. M. Purcell and N. F. Ramsey, *Phys. Rev.* **78**, 807 (1950); Smith et al. as quoted in N. F. Ramsey, *Molecular Beams*, Oxford University Press, 1956.

$$\langle A^*|U_T^\dagger U_T \mathbf{D}^* U_T^\dagger U_T|A^*\rangle_m = K^*\langle A^*|U_T^\dagger U_T \mathbf{J}^* U_T^\dagger U_T|A^*\rangle_m .$$

From (3.8) and the properties that

$$T\mathbf{J}T^{-1} = U_T \mathbf{J}^* U_T^\dagger = -\mathbf{J} \quad \text{and} \quad T\mathbf{r}T^{-1} = U_T \mathbf{r}^* U_T^\dagger = +\mathbf{r}, \tag{3.11}$$

we have

$$\langle A|\mathbf{D}|A\rangle_{-m} = -K\langle A|\mathbf{J}|A\rangle_{m} . \tag{3.12}$$

Comparison of (3.10) and (3.12) shows that

$$\langle A|\mathbf{D}|A\rangle_m = 0 . \tag{3.13}$$

Thus, the static electric dipole moment must be zero if time reversal is invariant.[17] However, the matrix elements of **r** between two *different* states of dominantly the *same parity* are, in general, not zero if time reversal invariance holds but parity is not conserved. (Here the term "same parity" refers to that part determined by the strong interactions.)

Thus, if time reversal invariance holds, the most severe limit on parity nonconservation is given by spectroscopic evidence and experiments of the nuclear double scattering type. These limits are already quite strong and thus demand that the strong interactions and the electromagnetic interactions both must conserve parity. However, these limits throw no light on the invariance properties of the weak interactions.

5. β Decay Experiments

Prior to the recent experiments on β angular distribution from polarized nuclei and on the longitudinal polarization of β-rays, there already existed an immense body of knowledge in the field of β decay. These previous experiments[18] consist of (i) spectra (allowed, forbidden, etc.) and ft values, (ii) β-neutrino correlation, (iii) β-γ correlation, (iv) polarized nuclei and the angular distribution of secondary γ-rays, and (v) β-γ-γ' angular correlation. We shall show that *these experiments* (i) *to* (v) *are not relevant so far as the question of parity conservation in weak interactions is concerned.* They neither prove nor disprove the conservation of parity in β decay.

The most general form of the interaction Hamiltonian for nonconservation of parity is

$$H_{\text{int}} = \sum_i (\psi_p^\dagger O_i \psi_n)(C_i \psi_e^\dagger O_i \psi_\nu + C_i' \psi_e^\dagger O_i \gamma_5 \psi_\nu), \quad i = S, T, V, A, P, \tag{3.14}$$

where $O_S = \gamma_4$, $O_V = \gamma_4\gamma_\mu$, $O_T = -[i/(2\sqrt{2})]\gamma_4(\gamma_\lambda\gamma_\mu - \gamma_\mu\gamma_\lambda)$, $O_A = -i\gamma_4\gamma_\mu\gamma_5$, and $O_P = \gamma_4\gamma_5$. In Equation (3.14) derivative couplings are not included. The conclusions [Equations (3.18) and (3.20) below] are correct even if there are such derivative coupling terms. We have now ten complex coupling constants. Any observed quantity will be related to the sum of the absolute squares of certain matrix elements.

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[17] By the *CPT* theorem (see Chapter IV) the same is true if $C\cdot P$ is invariant. That the static dipole moment must be zero if $C\cdot P$ is invariant was first pointed out by L. Landau [*Nuclear Phys* 3, 127 (1957)]

[18] See, e.g., K Siegbahn, *Beta- and Gamma-Ray Spectroscopy,* Interscience, New York, 1955.

$$\sum|M|^2=(\sum f_{ij}C_i^*C_j+\text{c.c.})+(\sum f_{ij}'C_i'^*C_j'+\text{c.c.})+(\sum g_{ij}C_i^*C_j'+\text{c.c.}) \tag{3.15}$$

where f_{ij}, f_{ij}' and g_{ij} are certain functions of the measured momenta and spins. It is well known that, as the neutrino has zero mass, it satisfies not only the familiar Dirac equation

$$\left(\gamma_\mu\frac{\partial}{\partial x_\mu}\right)\psi_\nu=0\,, \tag{3.16}$$

but also the equation

$$\gamma_\mu\left(\frac{\partial}{\partial x_\mu}\right)\gamma_5\psi_\nu=0\,. \tag{3.17}$$

Thus, one has[19]

$$f_{ij}=f_{ij}'\,. \tag{3.18}$$

Furthermore, we can show that the g_{ij} must be pseudoscalar quantities. To see this, let us consider the following *formal* transformation:

$$C_i\rightarrow C_i\,,\quad C_i'\rightarrow -C_i' \tag{3.19}$$

together with $\mathbf{r}\rightarrow-\mathbf{r}$; $\mathbf{p}\rightarrow-\mathbf{p}$; and spin $\mathbf{s}\rightarrow+\mathbf{s}$. This formal mathematical transformation leaves the Hamiltonian H_{int} invariant. Thus it must also leave Equation (3.15) invariant. It then follows that under this formal transformation the interference terms g_{ij} must transform as

$$g_{ij}(\mathbf{p},\mathbf{s},\ \ldots\ )\rightarrow g_{ij}(-\mathbf{p},+\mathbf{s},\ \ldots\ )=\ -g_{ij}(\mathbf{p},\mathbf{s},\ \ldots\ ). \tag{3.20}$$

Consequently g_{ij} transform like pseudoscalar quantities. This means that in order to answer the question of parity conservation it is necessary to observe, experimentally, a pseudoscalar quantity. To observe a pseudoscalar quantity one must measure at least three linear momenta $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ or a spin $\mathbf{s}$ and a momentum $\mathbf{p}$ so as to form quantities like $(\mathbf{p}_1\times\mathbf{p}_2)\cdot\mathbf{p}_3$ or $\mathbf{s}\cdot\mathbf{p}$, etc. In the experiments on the spectra, β-ν correlation and β-γ correlation, it is clear that no such pseudoscalar quantity can be formed. With the parity nonconserving Hamiltonian Equation (3.14), the theoretical results for these experiments are identical with that of the conventional parity conserving Hamiltonian provided one replaces $C_i^*C_j$ in the old formulas:

$$C_i^*\,C_j\rightarrow(C_i^*\,C_j+C_i'^*\,C_j')\,. \tag{3.21}$$

In the measurement of polarized nuclei and the angular distribution of the secondary γ-ray it is possible to form pseudoscalar terms like

$$(\mathbf{s}\cdot\mathbf{p}_\gamma)\,. \tag{3.22}$$

However, since γ interaction conserves parity and since the multipole γ radiation in nuclear transitions has very accurately defined parities, the observed angular distribution must be invariant under the transformation

[19]It is important to note that in general (3.15) is invariant under the mathematical transformation $C_i\rightarrow C_i'$ and $C_i'\rightarrow C_i$. This property can serve as a good check for the correctness of various expressions [cf. Equations (5.3), (5.11), etc.]. We wish to thank Dr. Pauli for a communication on the usefulness of this transformation.

$$\mathbf{p}_\gamma \to -\mathbf{p}_\gamma \tag{3 23}$$

Thus terms of the form (3 22) cannot exist [20]

In the case of β-γ-γ' angular correlation measurements, one can easily form pseudoscalar quantities that are also invariant under transformation (3 23), such as

$$\mathbf{P}_e \cdot (\mathbf{p}_\gamma \times \mathbf{p}_{\gamma'})(\mathbf{p}_\gamma \cdot \mathbf{p}_{\gamma'}) \tag{3 24}$$

These terms cannot be ruled out by using the parity conservation property of the γ radiation However, if time reversal is invariant for the strong interactions (including γ interactions), then such terms must all vanish [This follows immediately from Equation (4 29)] Consequently the absence of such terms can be used to prove the invariance of time reversal for strong interactions and electromagnetic interactions but not for the invariance properties of the weak interactions We can summarize as follows The previous accurate measurements of β decay (i) to (v) do not throw any light whatsoever upon a possible nonconservation of parity in the decay process In order to detect possible parity nonconservation terms we must try to perform other experiments such as to measure $\mathbf{s} \cdot \mathbf{p}_e$, etc

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[20] The nonexistence of terms like $\mathbf{s} \cdot \mathbf{p}_\gamma$ and $\mathbf{p}_e \cdot (\mathbf{p}_\gamma \times \mathbf{p}_{\gamma'})$ can also serve as evidence for the parity conservation for the strong interactions (including γ interactions) in a nuclear system, but not for weak interactions

IV. SOME GENERAL DISCUSSIONS ON THE CONSEQUENCES DUE TO POSSIBLE NON-INVARIANCE UNDER *P*, *C*, AND *T*

1. The *CPT* Theorem.[21] Equality of Mass and Lifetime Between a Particle and its Antiparticle[22]

Before we discuss in detail the various tests on the conservation of parity *P*, charge conjugation *C*, and time reversal *T* in weak interactions, it is useful to recall a general theorem concerning the interrelationship between these operators *C*, *P*, *T* and proper Lorentz invariance.

The *CPT* Theorem: If a local Lagrangian theory (which may contain derivative couplings to any high but finite orders) is invariant under the proper Lorentz transformations, it is invariant under the product of *CPT* (and its permutation *PCT*, etc.) although the theory may not be separately invariant under each one of these three operators *C*, *P*, and *T*.

It follows from this theorem that if *P* is not conserved in the weak interactions, then at least one of the other invariances *C* or *T* should not be conserved. In the following discussions we shall assume that the general framework of field theory, under which the *CPT* theorem is proved, is valid. At first sight it seems that the observed equality of lifetimes in the decay of π^+, π^- and the similar equality for the μ^+, μ^- may already form a proof that *C* is conserved in weak decays. As we shall see, the equality of the masses and lifetimes for a particle and its antiparticle follows directly from proper Lorentz invariance and the *CPT* theorem. It does not prove at all that *C* is invariant. We shall state these consequences of the *CPT* theorem in the form of two theorems. Let *H* be the complete Hamiltonian which may be separated into two parts

$$H = H_s + H_w \tag{4.1}$$

where H_s represents the strong interactions together with the γ interactions, and H_w the weak interactions. We assume that both H_s and H_w are invariant under the proper Lorentz transformation. Consequently, H_s and H_w are both invariant under the compound operation of *PCT*, i.e.,

$$PCTHT^{-1}C^{-1}P^{-1} = PCU_T H^* U_T^{\dagger} C^{\dagger} P^{\dagger} = H \tag{4.2}$$

where *P*, *C*, U_T are all unitary operators [cf. Equation (3.7)]. We shall further assume that H_s is invariant under the separate operation of each one of these three operators *C*, *P*, and *T* while H_w may or may not be invariant under *C*, *P*, and *T* separately. (The operators *C*, *P*, *T* are defined by using the invariance properties of H_s.)

[21] See W. Pauli's article in *Niels Bohr and the Development of Physics*, Pergamon, London, 1955, J. Schwinger, *Phys. Rev.* **91**, 720, 723 (1953), **94**, 1366 (1953), G. Luders, *Kgl. Danske Videnskab Selskab, Mat.-fys. Medd.* **28**, No. 5 (1954)

[22] T. D. Lee, R. Oehme, and C. N. Yang, *Phys. Rev.* **106**, 340 (1957)

Theorem 1:[23] If α is a stable particle, then

$$M_\alpha = M_{\bar{\alpha}} \tag{4 3}$$

where M_α and $M_{\bar{\alpha}}$ are the masses of α and its antiparticle $\bar{\alpha}$ Equation (4 3) is valid to all orders in H_u

Proof: Consider a particle α at rest

$$H|\alpha\rangle = M_\alpha|\alpha\rangle \quad \text{or} \quad H^*|\alpha^*\rangle = M_\alpha|\alpha^*\rangle \tag{4 4}$$

and

$$PCU_T H^*|\alpha^*\rangle = M_\alpha \cdot PCU_T|\alpha^*\rangle \tag{4 5}$$

Let

$$|\bar{\alpha}\rangle \equiv PCU_T|\alpha^*\rangle \tag{4 6}$$

We then have

$$H|\bar{\alpha}\rangle = M_\alpha|\bar{\alpha}\rangle \tag{4 7}$$

by using Equation (4 2) From the definitions of P, C, T [see Chapter I, section 3, and Equation (3 8)] we know that if $|\alpha\rangle$ represents a particle at rest with spin J and its z component J_z, then $|\alpha\rangle$, defined by Equation (4 6), represents its antiparticle state also at rest and with its spin along z component $-J_z$ Theorem 1 follows immediately from Equation (4 7)

Remarks: What we have proved is actually more than just the equality of masses By taking $|\alpha\rangle$ to be any eigenstate of the total Hamiltonian H we can generate another state $|\bar{\alpha}\rangle$ by (4 6) which has the same eigenvalue Thus the complete energy spectrum for a group of particles is identical with that of a corresponding group of antiparticles By considering the energy spectrum of a particle, say a proton p, in a magnetic field, one can prove the equality of the magnitude of magnetic moment of p and $\bar{p}$ (again to all orders of H_u)

Theorem 2: Consider the decay of A via H_w,

$$A \to B \quad \text{and} \quad \bar{A} \to \bar{B} \tag{4 8}$$

(with the states $B \neq \bar{B}$), [1] then, to the lowest order in H_u,

$$(\text{lifetime of } A) = (\text{lifetime of } \bar{A}) \tag{4 9}$$

Proof: Since we are only interested in the lowest order in H_u, the states $|A\rangle_m$, $|B\rangle_m$, $|\bar{A}\rangle_m$, $|\bar{B}\rangle_m$ can be taken to be eigenstates of H_s with z component spin $J_z = m$ Furthermore, since H_s is invariant under T and C, we have [cf Equation (3 8)]

$$|\bar{A}\rangle_m = C|A\rangle_m\,, \qquad |\bar{B}\rangle_m = C|B\rangle_m\,, \tag{4 10}$$

$$T|A\rangle_m = e^{i\delta_m(A)}|A\rangle_{-m}\,, \qquad T|B\rangle_m = e^{i\delta_m(B)}|B\rangle_{-m}\,, \tag{4 11}$$

and similar equations for $|\bar{A}\rangle_m$ and $|\bar{B}\rangle_m$ under time reversal The states $|B\rangle_m$ and $|\bar{B}\rangle_m$ are taken to be stationary states consisting of standing waves (including all the strong interactions)

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[23] T D Lee and C N Yang, *Phys Rev* **105**, 1671 (1957)

[1] This condition $B \neq \bar{B}$ applies to cases, e g , where A and $\bar{A}$ have opposite charges $\pm Q$ or opposite heavy particle number $\pm N$, etc General discussions concerning cases where B may be the same as $\bar{B}$ (e g , in the decay of K and $\bar{K}$) will be found in Chapter V, section 9

We separate

$$H_w = H_+ + H_- \quad \text{with} \quad H_\pm \equiv \frac{1}{2}[H_w \pm P H_w P^\dagger]\,. \tag{4.12}$$

Hence

$$P H_\pm P^\dagger = \pm H_\pm\,. \tag{4.13}$$

The matrix elements $\langle B|H_\pm|A\rangle$ are related to $\langle \bar{B}|H_\pm|\bar{A}\rangle$ by the CPT theorem,

$$\langle B|H_\pm|A\rangle_m^* = \langle B|T^{-1}TH_\pm T^{-1}T|A\rangle_m$$

$$= \langle B|TH_\pm T^{-1}|A\rangle\,{}_m e^{i\theta} = \langle B|C^\dagger P^\dagger H_\pm PC|A\rangle\,{}_m e^{i\theta} = \pm\,\langle \bar{B}|H_\pm|\bar{A}\rangle\,{}_m e^{i\theta} \tag{4.14}$$

where $\theta = \delta_m(A) - \delta_m(B)$. In the expression for the lifetime of A, no pseudoscalar quantities can be formed. From arguments similar to those used in the previous chapter, we conclude that there can be no interference term between H_+ and H_-. Thus we have

$$(\text{lifetime})_A = \sum_B (|\langle B|H_+|A\rangle_m|^2 + |\langle B|H_-|A\rangle_m|^2)\cdot(\text{phase space})_B\,. \tag{4.15}$$

Equation (4.15) is clearly independent of the m value. From (4.14) we have

$$(\text{lifetime of } A) = (\text{lifetime of } \bar{A}).$$

From theorems 1 and 2, we conclude that *the equality of the masses and lifetimes of a particle and its antiparticle cannot be used as evidence for the invariance under the charge conjugation operator C. Rather, it may serve as evidence for the validity of the CPT theorem.* Indeed, as we shall see later, the operator C is not conserved, at least in some of the weak interactions.

2. General Remarks Concerning Invariance or Non-invariance Under Time Reversal

Consider the weak decay of a particle

$$A \to B \tag{4.16}$$

through the weak interaction H_w. Let H_w be written as a sum of many terms (with C_i as coupling constants),

$$H_w = \sum C_i H_i\,, \tag{4.17}$$

such that under a time reversal operation T

$$T H_w T^{-1} = \sum C_i^* H_i \quad \text{and} \quad T H_i T^{-1} = H_i\,. \tag{4.18}$$

Thus, if T is invariant then C_i are all real and vice versa. The proof or disproof concerning the invariance under time reversal, then, rests completely on the possibility of measuring the relative phases between these C_i. We consider first case A.

A. CASE IN WHICH THERE ARE NO STRONG INTERACTIONS BETWEEN VARIOUS DECAY PRODUCTS IN THE FINAL STATES

In this case the final state is given as

$$\psi = \sum_{\mathbf{p},\mathbf{s}} \langle \mathbf{p}, \mathbf{s} \,|\, H_w \,|\, A \rangle \cdot |\, \mathbf{p}, \mathbf{s} \rangle \tag{4.19}$$

where $|\,\mathbf{p},\mathbf{s}\rangle$ represents free particle states of momenta $\mathbf{p}_1$, $\mathbf{p}_2$ ... and spins $\mathbf{s}_1$, $\mathbf{s}_2$ ... in the final state B. By assumption $|\mathbf{p}, \mathbf{s}\rangle$ is an eigenstate of H_s. From (4.17) we can write

$$\psi = \sum C_i M_i(\mathbf{p}, \mathbf{s}) \cdot |\mathbf{p}, \mathbf{s}\rangle \tag{4.20}$$

where

$$M_i(\mathbf{p}, \mathbf{s}) \equiv \langle \mathbf{p}, \mathbf{s}|H_i|A\rangle \,. \tag{4.21}$$

Using (4.18) and the property [cf. Equation (3.11)]

$$T|\mathbf{p}, \mathbf{s}\rangle = |-\mathbf{p}, -\mathbf{s}\rangle \,, \tag{4.22}$$

we have

$$M_i^*(\mathbf{p}, \mathbf{s}) = M_i(-\mathbf{p}, -\mathbf{s}) \,. \tag{4.23}$$

Now let us consider the measurement of an observable O which is a function of the momenta $\mathbf{p}_i$ and spins $\mathbf{s}_i$ of some of the particles in the final state. Using (4.20), we obtain

$$\langle\psi|O|\psi\rangle = \sum_{i,j} C_i^* C_j O_{ij} \tag{4.24}$$

where

$$O_{ij} \equiv \sum_{\mathbf{p},\mathbf{s}} M_i^*(\mathbf{p}, \mathbf{s}) M_j(\mathbf{p}, \mathbf{s}) \langle\mathbf{p}, \mathbf{s}|O|\mathbf{p}, \mathbf{s}\rangle. \tag{4.25}$$

It is clear that

$$O_{ij}^* = O_{ji} \,.$$

Let us separate various observables O into even and odd functions $\mathbf{p}$ and $\mathbf{s}$ (see Table 4). We denote by O_+ the even functions and O_- the odd functions. Hence

$$\langle\mathbf{p}, \mathbf{s}|O_\pm|\mathbf{p}, \mathbf{s}\rangle = \pm \langle -\mathbf{p}, -\mathbf{s}|O_\pm|-\mathbf{p}, -\mathbf{s}\rangle \,, \tag{4.26}$$

and

$$T O_\pm T^{-1} = \pm O_\pm \,. \tag{4.27}$$

Using (4.25), (4.22), and (4.27) we have

$$(O_+)_{ij} = (O_+)_{ji} = \text{real} \quad \text{and} \quad (O_-)_{ij} = -(O_-)_{ji} = \text{imaginary}. \tag{4.28}$$

Table 4

Some Examples of O_+ and O_-

O_+	O_-
$\mathbf{p}_1 \cdot \mathbf{p}_2$	$\mathbf{s} \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$
$\mathbf{s} \cdot \mathbf{p}$	$\mathbf{p}_1 \cdot (\mathbf{p}_2 \times \mathbf{p}_3)$
$\mathbf{s}_1 \cdot \mathbf{s}_2$	$\mathbf{s}_1 \cdot (\mathbf{s}_2 \times \mathbf{s}_3)$

Thus, the diagonal elements of $O_\pm$ are related to the real and imaginary parts of $C_i^* C_j$, i.e.,

$$\langle\psi|O_+|\psi\rangle=\sum(O_+)_{ij}(C_i^*C_j+C_iC_j^*) \quad \text{and} \quad \langle\psi|O_-|\psi\rangle=\sum(O_-)_{ij}(C_i^*C_j-C_iC_j^*)\,. \tag{4.29}$$

Consequently, we reach the conclusion that *if in the final state there are no strong interactions between the various decay products then the existence of any observables of the form O_- serves as a proof that H_w is not invariant under time reversal.*

Next we consider case B.

B. CASE IN WHICH THERE ARE STRONG INTERACTIONS BETWEEN VARIOUS DECAY PRODUCTS IN THE FINAL STATE

In this case $|\mathbf{p,s}\rangle$ is not an eigenstate of the strong Hamiltonian H_s. Let $|B\rangle$ be an eigenstate of H_s. Furthermore state $|B\rangle$ is chosen to be a stationary state. Since H_s is invariant under T, we have [cf. Equation (3.8)]

$$T\,|B\rangle_m=|B\rangle_{-m} \tag{4.30}$$

where $J_z=m$. For simplicity we have absorbed the phase $e^{i\delta_m}$ in the definition of $|B\rangle_{-m}$. The final state in the momentum representation is now

$$|\psi\rangle=\sum_{B,\mathbf{p},\mathbf{s}}\langle\mathbf{p},\mathbf{s}|B^{\text{out}}\rangle\,\langle B|H_w|A\rangle\cdot|\mathbf{p},\mathbf{s}\rangle \tag{4.31}$$

where the sum also extends over various final states B. The state $|B^{\text{out}}\rangle$ is the outgoing wave part of the stationary states $|B\rangle$. The state $|B^{\text{out}}\rangle$ contains a phase factor $e^{i\eta_B}$ with η_B as the phase shift due to the final state interactions. Thus, by using (4.30), (4.22), and (4.18) we can write Equation (4.31) as

$$|\psi\rangle=\sum C_i e^{i\eta_B}\,M_{iB}(\mathbf{p},\mathbf{s})\cdot|\mathbf{p},\mathbf{s}\rangle \tag{4.32}$$

with

$$M_{iB}^*(\mathbf{p},\mathbf{s})=M_{iB}(-\mathbf{p},-\mathbf{s})\,. \tag{4.33}$$

Similar to (4.24), we have

$$\langle\psi|O|\psi\rangle=\sum C_i^*C_j e^{-i\eta_B+i\eta_B}\,O_{iB,jB'} \tag{4.34}$$

and

$$O_{iB,jB'}=\sum_{\mathbf{p}\,\mathbf{s}}M_{iB}^*(\mathbf{p},\mathbf{s})M_{jB'}(\mathbf{p},\mathbf{s})\,\langle\mathbf{p},\mathbf{s}|O|\mathbf{p},\mathbf{s}\rangle\,. \tag{4.35}$$

Again, as in case A, we separate O into two different types, O_+ and O_-. Similar to (4.29), their matrix elements are respectively

$$\langle\psi|O_+|\psi\rangle=\sum(O_+)_{iB,jB'}[(C_i^*C_j+C_iC_j^*)\cos(\eta_B-\eta_{B'})+ \\ i(C_i^*C_j-C_iC_j^*)\sin(\eta_B-\eta_{B'})] \tag{4.36}$$

and

$$\langle\psi|O_-|\psi\rangle=\sum(O_-)_{iB,jB'}[(C_i^*C_j-C_iC_j^*)\cos(\eta_B-\eta_B\ )+ \\ i(C_i^*C_j+C_iC_j^*)\sin(\eta_B-\eta_{B'})]\,. \tag{4.37}$$

Also, as in the case of (4.28) we can prove

$$(O_+)_{iB,jB} = \text{real} \quad \text{and} \quad (O_-)_{iB,jB'} = \text{imaginary}\,. \tag{4.38}$$

It is easy to see that if there are no final state interactions ($\eta_B = \eta_{B'} = \ldots = 0$) then Equation (4.37) reduces to (4.29). In the present case *measurements on both O_+ and O_- may serve as tests on time reversal.* The terms $(C_i^* C_j - C_j^* C_i) \sin(\eta_B - \eta_{B'})$ in O_+ and the terms $(C_i^* C_j - C_j^* C_i) \cos(\eta_B - \eta_{B'})$ in O_- can be used to detect possible violation of time reversal invariance. As examples we list, for β decay, terms of the form $(\mathbf{p}_e \cdot \mathbf{p}_\nu)Z$, $(\mathbf{s} \cdot \mathbf{p}_e)Z$, and $\mathbf{s} \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$. (For detailed discussions see Chapter V, sections 1 to 3.)

3. Remark on Invariance Under Charge Conjugation

In this section we shall make only a general remark concerning invariance under charge conjugation. If we separate H_w into the parity conserving part H_1 and the parity nonconserving part H_2 [cf. Equation (4.12)],

$$H_w = H_1 + H_2 \tag{4.39}$$

with

$$PH_1P^\dagger = H_1 \quad \text{and} \quad PH_2P^\dagger = -H_2\,. \tag{4.40}$$

Furthermore, we decompose H_1 and H_2 into the sum of various terms as in Equation (4.17),

$$H_1 = \sum_i (C_1)_i (H_1)_i, \quad H_2 = \sum_i (C_2)_i (H_2)_i \tag{4.41}$$

with

$$T(H_\alpha)_i T^{-1} = (H_\alpha)_i \quad (\alpha = 1, 2)\,. \tag{4.42}$$

Thus, *if T is invariant, $(C_1)_i$ and $(C_2)_i$ must all be real.* From the CPT theorem it follows now that *if charge conjugation C is conserved then $(C_1)_i$ must be real while $(C_2)_i$ must be imaginary.*

4. Summary

We summarize the above discussions [cf. Equations (4.36) and (4.37)].

(i) Let O_+ be a scalar and be even in $\mathbf{p}$ and $\mathbf{s}$ (e.g., $\mathbf{p}_1 \cdot \mathbf{p}_2$). The observation of such a term in the form

$$\langle O_+ \rangle_{\text{scalar}} = \sum O_{iB,jB'} (C_i^* C_j + C_i C_j^*) \cos(\eta_B - \eta_{B'}) \tag{4.43}$$

does not violate the invariance under P, nor under C, nor under T; while the presence of a term in the form

$$\langle O_+ \rangle_{\text{scalar}} = \sum i O_{iB,jB'} (C_i^* C_j - C_i C_j^*) \sin(\eta_B - \eta_{B'}) \tag{4.44}$$

violates the invariance under C and T, but not under P [e.g., the $\mathbf{p}_e \cdot \mathbf{p}_\nu)Z$ term in Equation (5.3)].

(ii)[22] Let O_+ be a pseudoscalar but still be even in $\mathbf{p}$ and $\mathbf{s}$ (e.g., $\mathbf{s} \cdot \mathbf{p}$). The observation of such a term in the form

$$\langle O_+\rangle_{ps}=\sum O_{iB,jB'}(C_i^*C_j+C_iC_j^*)\cos(\eta_B-\eta_{B'}) \tag{4.45}$$

violates the invariance under P and C, but not under T [e.g., the Z independent term of $\mathbf{s}\cdot\mathbf{p}_e$ in Equation (5.11)]; while the presence of a term in the form

$$\langle O_+\rangle_{ps}=\sum iO_{iB,jB'}(C_i^*C_j-C_iC_j^*)\sin(\eta_B-\eta_{B'}) \tag{4.46}$$

violates the invariance under P and T, but not under C [e.g., the $(\mathbf{s}\cdot\mathbf{p}_e)Z$ term in Equation (5.11)].

(iii) Let O_- be a scalar and be odd in $\mathbf{p}$ and $\mathbf{s}$ [e.g., $\mathbf{s}\cdot(\mathbf{p}_1\times\mathbf{p}_2)$]. The observation of such a term in the form

$$\langle O_-\rangle_{\text{scalar}}=\sum O_{iB,jB'}(C_i^*C_j-C_iC_j^*)\cos(\eta_B-\eta_{B'}) \tag{4.47}$$

violates the invariance under T and C, but not under P; while the existence of a term in the form

$$\langle O_-\rangle_{\text{scalar}}=\sum iO_{iB,jB'}(C_i^*C_j+C_iC_j^*)\sin(\eta_B-\eta_{B'}) \tag{4.48}$$

does not violate the invariance under P, not under C, nor under T. A particular example of (4.48) is the existence of the usual production of polarized nuclei by a single scattering. Because of time reversal invariance it follows that such polarization is zero in the Born approximation.

(iv) Let O_- be a pseudoscalar and be odd in $\mathbf{p}$ and $\mathbf{s}$ [e.g., $\mathbf{p}_1\cdot(\mathbf{p}_2\times\mathbf{p}_3)$]. The observation of such a term in the form

$$\langle O_-\rangle_{ps}=\sum O_{iB,jB'}(C_i^*C_j-C_iC_j^*)\cos(\eta_B-\eta_{B'}) \tag{4.49}$$

violates the invariance under P and T, but not under C; while the existence of a term like

$$\langle O_-\rangle_{ps}=\sum iO_{iB,jB'}(C_i^*C_j+C_iC_j^*)\sin(\eta_B-\eta_{B'}) \tag{4.50}$$

violates the invariance under P and C, but not under T.

V. VARIOUS POSSIBLE EXPERIMENTAL TESTS[25] ON INVARIANCE UNDER P, C, AND T IN WEAK INTERACTIONS

1. Electron-Neutrino Correlation in β Decay

We take for the β decay Hamiltonian

$$H=\sum(\psi_p^{\dagger}O_i\psi_n)[C_i(\psi_e^{\dagger}O_i\psi_\nu)+C_i'(\psi_e^{\dagger}O_i\gamma_5\psi_\nu)] \tag{5.1}$$

where i runs over the usual S, V, T, A, and P types of interactions with

$$O_S=\gamma_4,\quad O_V=\gamma_4\gamma_\mu,\quad O_T=-[i/(2\sqrt{2})]\,\gamma_4(\gamma_\lambda\gamma_\mu-\gamma_\mu\gamma_\lambda),$$
$$O_A=-i\gamma_4\gamma_\mu\gamma_5,\quad \text{and}\quad O_P=\gamma_4\gamma_5\,. \tag{5.2}$$

The angular energy distribution of the electron for the allowed transition is given by[26]

$$N(W,\theta)dW\sin\theta\,d\theta=\frac{\xi}{4\pi^3}F(Z,W)\,pW(W_0-W)^2$$
$$\times\left[1+\frac{p}{W}\cos\theta\,(a+a'\frac{Ze^2}{\hbar cp})+\frac{b}{W}\right]\sin\theta\,d\theta \tag{5.3}$$

where

$$\xi=(|C_S|^2+|C_V|^2+|C_S'|^2+|C_V'|^2)|M_F|^2+$$
$$(|C_T|^2+|C_A|^2+|C_T'|^2+|C_A'|^2)|M_{GT}|^2\,, \tag{5.4}$$

$$a\xi=(1/3)(|C_T|^2-|C_A|^2+|C_T'|^2-|C_A'|^2)|M_{GT}|^2-$$
$$(|C_S|^2-|C_V|^2+|C_S'|^2-|C_V'|^2)|M_F|^2\,, \tag{5.6}$$

$$a'\xi=(i/3)(C_AC_T^*-C_TC_A^*+C_A'C_T'^*-C_T'C_A'^*)|M_{GT}|^2+$$
$$i(C_V^*C_S-C_S^*C_V+C_V'^*C_S'-C_S'^*C_V')|M_F|^2\,, \tag{5.7}$$

$$b\xi=\gamma(C_S^*C_V+C_SC_V^*+C_S'^*C_V'+C_S'C_V'^*)|M_F|^2+$$
$$\gamma(C_T^*C_A+C_A^*C_T+C_T'^*C_A'+C_A'^*C_T')|M_{GT}|^2\,. \tag{5.8}$$

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[25] Since January 1957 a large number of experiments have been performed to test the non-conservation properties of P and C in weak decays. For a review of these numerous experimental works, see, e.g., *Proc. Seventh Ann. Rochester Conf.*, Interscience, New York, 1957.

[26] Cf. Equation (A.4) of reference 14. The existence of the term $a'Ze^2/\hbar cp$ was pointed out by M. Morita and R. Curtis.

In the above expressions, all unexplained notations are identical with the standard notations.[27] Equations (5.4) to (5.8) can be obtained directly by applying the general rules, Equation (3.21), to the corresponding "old" expressions calculated previously under the assumption that parity is conserved.

We notice that the term containing

$$a'Z(\mathbf{p}_e \cdot \mathbf{p}_\nu) \tag{5.9}$$

in (5.3) is of the form described by Equation (4.44) with the phase shifts due to the Coulomb effect ($\mathbf{p}_e$, $\mathbf{p}_\nu$ are respectively the momenta of e and ν). Thus, the presence of this term would violate the invariance under C and T.

2. Experiments with β Decay from Polarized Nuclei[14]

We consider first the experiment on angular distribution for allowed β transition from polarized nuclei. Let θ be the angle between $\mathbf{p}_e$ and direction of the spin $\mathbf{J}$ of the polarized nuclei. The angular distribution is, in general, of the form

$$1 + \alpha \cos\theta\,. \tag{5.10}$$

The corresponding expression α for $\mathbf{J} \to \mathbf{J} - 1$(no) transition is

$$\alpha = \beta \frac{\langle J_z \rangle}{J}$$

with

$$\beta = Re\left[\pm(C_T C_T'^* - C_A C_A'^*) + i\frac{Ze^2}{\hbar c p}(C_A C_T'^* + C_A' C_T^*)\right]\frac{v_e}{c}\,\frac{2}{\xi + (\xi b/W)}\,|M_{GT}|^2\,. \tag{5.11}$$

For $J \to J+1$(no), α is given by

$$\alpha = -\beta J_z/(J+1)\,. \tag{5.12}$$

For $J \to J$(no), the corresponding α is[28]

$$\alpha = \beta \frac{J_z}{J(J+1)} + \beta' \frac{J_z}{\sqrt{J(J+1)}} \tag{5.13}$$

where

$$\beta' = Re\Big[(C_S^* C_T' + C_S'^* C_T - C_V^* C_A' - C_V'^* C_A) \pm$$

$$i\frac{Ze^2}{\hbar c p}(C_S^* C_A' + C_S'^* C_A - C_V^* C_T' - C_V'^* C_T)\Big]\frac{v_e}{c}\,\frac{2M_F M_{GT}}{\xi + (\xi b/W)}\,.$$

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[27] See, e g , the article by M E Rose in *Beta- and Gamma-Ray Spectroscopy*, Interscience, New York, 1955, pp 271-91

[28] Equation (5 13) has been independently calculated by many authors M Morita (private communication), R Curtis and R Lewis (private communication), J D Jackson, S B Treiman, and H W Wyld (*Phys Rev* , in press) In deriving Equation (5 13) we assume that the strong interactions are invariant under time reversal Consequently the nuclear matrix elements M_F and M_{GT} are real quantities If the strong interactions are not invariant under time reversal then the product $(M_F M_{GT})$ should be replaced by $(M_F^* M_{GT})$ and set inside the square brackets together with the coupling constants It may be emphasized that the condition $\beta' = 0$ can be used as evidence of invariance under time reversal for both the weak and the strong interactions

In Equations (5.11) to (5.13) the upper signs are for e^- emission and the lower signs for e^+ emission.

The detection of $\alpha \neq 0$ gives definite proof that P is not conserved. We notice further that in the expression for β [Equation (5.11)] there are two different terms of the forms of $\mathbf{J}\cdot\mathbf{p}_e$ and $Z\mathbf{J}\cdot\mathbf{p}_e$. Comparing with Equations (4.45) and (4.46) we see that the presence of the first term violates the invariance of P and C while the presence of the second term violates the invariance of P and T.

The first conclusive experimental evidence on the nonconservation of parity in weak interactions was done by Wu, Ambler, Hayward, Hoppes, and Hudson[1] using polarized Co^{60}. The decay scheme for Co^{60} is

$$\mathrm{Co}^{60} \rightarrow \mathrm{Ni}^{60} + e^- + \bar{\nu} \tag{5.14}$$

with $J=5 \rightarrow J=4$(no) and $v_e/c \cong 0.65$. Wu et al. obtained for the asymmetry parameter β,

$$\beta \cong -0.7\,. \tag{5.15}$$

Since from the He^6 recoil experiment[29] we know that

$$\frac{|C_A|^2+|C_A'|^2}{|C_T|^2+|C_T'|^2} < \frac{1}{3} \tag{5.16}$$

and since for Ni^{60}

$$Ze^2/\hbar c = 0.2\,,$$

the second term in the expression for β [Equation (5.11)] has an upper limit

$$\left| \frac{2(v_e/c)Re[\imath(Ze^2/\hbar c p)(C_A C_T'^* + C_A' C_T^*)]}{|C_A|^2+|C_A'|^2+|C_T|^2+|C_T'|^2} \right| < 0.23\,. \tag{5.17}$$

Thus from the observed magnitude [Equation (5.15)] we conclude that *in β decay both C and P are not conserved.* By more accurate measurement to study the Z dependence or p dependence of the asymmetry parameter β and β' one can obtain information concerning time reversal invariance.

3. Other β Decay Experiments

There are other experiments in β decay which can serve as tests for possible non-invariance under P, C, and T. We list the following:

A. β-γ CORRELATION AND THE CIRCULAR POLARIZATION OF THE γ-RAY[14]

Consider a successive β,γ decay scheme

$$A \rightarrow B^* + e^+ + \nu \quad \text{and} \quad B^* \rightarrow B + \gamma \tag{5.18}$$

[29] B.M. Rustad and S.L. Ruby, *Phys Rev.* **97**, 991 (1955).

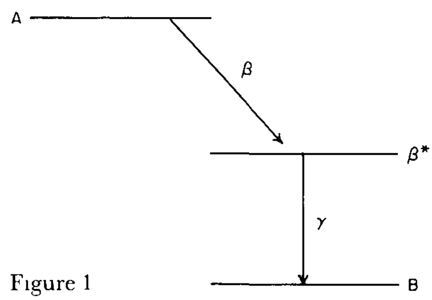

Figure 1

in which the initial nucleus A is not polarized (see Figure 1) Because of the nonconservation of parity, the intermediate nucleus B^* will be polarized along the momentum $\mathbf{p}_e$ of the β-ray The polarization of the B^* can then be detected by measuring the direction of the γ-ray with respect to $\mathbf{p}_e$ and the state of circular polarization of the γ-ray The magnitude of the polarization of B^* can be easily deduced from Equation (5 10)

B. POLARIZATION OF e

By measuring the longitudinal polarization of the electron $(\boldsymbol{\sigma}\cdot\mathbf{p}_e)$ from unpolarized nuclei one can also obtain a test of parity conservation [30] By a detailed study of its possible Z dependence it is also possible to obtain information on time reversal invariance (See Chapter VI, section 2, for a more detailed discussion)

C. POLARIZED NUCLEI AND RECOIL EXPERIMENTS[31]

From the experiments of Wu et al , it is proven that parity and charge conjugation are not conserved in β decay The next important question is, of course, on the invariance under time reversal T (By CPT theorem this is equivalent to the invariance under $C\cdot P$) In the above mentioned experiments, by a careful study of Z dependent terms such as

$$(\mathbf{J}\cdot\mathbf{p}_e)Z \quad \text{and} \quad (\mathbf{p}_e\cdot\mathbf{p}_\nu)Z, \tag{5 19}$$

information on T invariance can be obtained While their presence would prove that we do not have invariance under time reversal, their absence may be due to other reasons For example, in Equations (5 7) and (5 11), if

$$C_4 = C_4' = 0 \quad \text{and} \quad C_5 = C_5' = 0$$

T D Lee and C N Yang, *Phys Rev* **105**, 1671 (1957), L Landau, *Nuclear Phys* **3**, 127 (1957), J D Jackson, S B Treiman, and H W Wyld, *Phys Rev* **106**, 517 (1957) R B Curtis and R R Lewis, *Phys Rev* **107**, 543 (1957) For a summary of the various measurements on the longitudinal polarization of β particles see, e g , *Proc Seventh Ann Rochester Conf*, Interscience, New York, 1957

[31] J D Jackson, S B Treiman, and H W Wyld, *Phys Rev* **106**, 517 (1957)

then these terms [Equation (5.19)] would also vanish. A critical test is then to measure possible time reversal nonconservation terms that are due to the interference terms between C_S and C_T. One such possibility has been pointed out by Jackson, Trieman, and Wyld.[31]

Let us consider, for example, the neutron decay from polarized neutrons

$$n \rightarrow p + e + \bar{\nu} . \tag{5.20}$$

A simultaneous measurement of $\mathbf{p}_e$ and the recoil proton can give a measurement of

$$\sigma_n \cdot (\mathbf{p}_e \times \mathbf{p}_\nu) . \tag{5.21}$$

From the previous general argument, Equation (4.47), we see that *the Z independent term of this quantity violates the invariance under T and C.* The coefficient of this term is proportional to

$$Im(C_S C_T{}^* - C_V C_A{}^* + C_S' C_T'^* - C_V' C_A'^*) . \tag{5.22}$$

4. π Decay[14,23]

Consider the reaction for π^+ decay

$$\pi^+ \rightarrow \mu^+ + \nu . \tag{5.23}$$

The parity nonconservation for this reaction can be established by measuring the longitudinal polarization of the μ-meson. If the μ-meson has spin ½, its polarization state is described by a density matrix

$$1 + A_+ \boldsymbol{\sigma}_\mu \cdot \hat{\mathbf{p}}_\mu \tag{5.24}$$

where $\hat{\mathbf{p}}_\mu$ is a unit vector along the momentum of μ. Assuming that μ and ν have no strong interactions between them, from Equation (4.45) we see that *the presence of A_+ violates the conservation laws of P and C.*

The density matrix for the polarization of the μ from the corresponding π decay,

$$\pi \rightarrow \mu + \bar{\nu} , \tag{5.25}$$

is

$$1 + A_- \boldsymbol{\sigma}_\mu \cdot \hat{\mathbf{p}}_\mu . \tag{5.26}$$

By *CPT* theorem, the final result must be invariant under the operation of $C{\cdot}P{\cdot}T$. Since there are no final state interactions, we see that

$$A_+ = -A \; . \tag{5.27}$$

5. μ Decay[14,23]

The easiest way to analyze the spin state of the μ^+-meson from π decay is to use the possible parity nonconservation terms in μ decay

$$\mu^+ \rightarrow e^+ + \nu + \bar{\nu}. \tag{5.28}$$

If the μ-meson is polarized with spin $\boldsymbol{\sigma}_\mu$, the angular distribution of e would be of the form

$$1+B_+\boldsymbol{\sigma}_\mu\cdot\hat{\mathbf{p}}_e \tag{5 29}$$

where $\hat{\mathbf{p}}_e$ is a unit vector along the momentum of e Similar to (5 27), we have [3]

$$B_+=-B \tag{5 30}$$

Combining (5 24), (5 26), and (5 29), in the decays

$$\pi^+\rightarrow\mu^+\rightarrow e$$

the angular correlation between $\mathbf{p}_\mu$ and $\mathbf{p}_e$ is of the form

$$1+\alpha\ \hat{\mathbf{p}}_\mu\cdot\hat{\mathbf{p}}_e \tag{5 31}$$

with $\mathbf{p}_\mu$ measured in the rest system of π^+ and $\mathbf{p}_e$ in the rest system of μ Furthermore, from (5 27) and (5 30), we have

$$\alpha_+=\alpha \tag{5 32}$$

The observation of α_+ *proves that P and C are not conserved in both π decay and μ decay*

Experiments[2] on π-μ-e decays give for α_+

$$\alpha\ =-0\ 26\pm0\ 02 \tag{5 33}$$

with μ^+ stopped in carbon and

$$\alpha\ \cong-0\ 16 \tag{5 34}$$

with μ^+ stopped in emulsion The difference between these two numbers is, of course, due to the large depolarization effect in emulsion

The longitudinal polarization of the μ-meson from π decay offers a natural possibility for measuring the magnetic moment of the μ-meson This was first measured by Garwin et al [2] Their result gives a g value[33] for μ^+,

$$g=+2\ 00\pm0\ 10 \tag{5 35}$$

This value strongly indicates that the spin of μ is ½ A more accurate measurement of this is of particular interest because it may give a very severe test of validity electrodynamics at a much smaller distance ($\sim10^{-13}$ cm) than that tested by previous experiments with electrons

6. $K^\pm$ Decay

We consider first the decay of $K_{\mu 2}$

$$K^+\rightarrow\mu\ +\nu\ (\text{or}\ \bar{\nu}) \tag{5 36}$$

As in the π-μ-e decay, one would expect that here again the μ-meson could be polarized along its direction of motion, and then the μ-e decay would be an analyzer

[3] If in the μ^- decay two neutrinos are emitted, $\mu^-\rightarrow e^-+2\nu$, then the corresponding μ^+ decay is $\mu^+\rightarrow e^+ +2\bar{\nu}$ In this case Equation (5 30) is still correct

[33] More recently T Coffin et al [*Phys Rev* **106** ,1108 (1957)] measured the g value for μ^+ using the magnetic resonance technique They found $g_\mu=+2\ 0064\pm0\ 0048$

of the polarization through measurements of the distribution of $\mathbf{p}_\mu \cdot \mathbf{p}_e$. We will show later, in a special two-component theory of the neutrino, that this electron distribution is identical with the distribution from the π-μ-e decay if the K-meson spin is zero. Therefore in this special theory the K-μ-e decay distribution may offer a way to obtain some information about the spin of the K-meson[23] (cf. Chapter VII, section 1).

Next we consider the decays of $K_{\pi 2}$ and $K_{\pi 3}$:

$$K^+ \to \pi^+ + \pi^0 \tag{5.37}$$

and

$$K^+ \to \pi \;\; + 2\pi^+ . \tag{5.38}$$

In the decay of $K_{\pi 2}$ only one independent momentum can be measured in the rest system of K, and in the decay of $K_{\pi 3}$ only two independent momenta can be measured. In neither case can one form a pseudoscalar quantity out of these observed momenta. Thus, if the K-meson has zero spin (or if it is unpolarized) it is impossible to observe any interference term between the even and odd states in the decay of $K_{\pi 2}$ and $K_{\pi 3}$. *The strongest evidence for nonconservation of parity in this case is precisely the present θ-τ problem*, namely, *the K-meson can decay into a 2π system and a 3π system with the same lifetime and same mass*

The determination of the spin-parity of the K-meson through the angular energy distribution of 3π mode (Dalitz's analysis) can still be used. The distribution function is now given by

$$[|\psi_{J+}(\mathbf{k},\mathbf{p})|^2 + \alpha|\psi_{J} \;\; (\mathbf{k},\mathbf{p})|^2]\sqrt{\varepsilon(1-\varepsilon)}\; d\varepsilon \; d(\cos\theta) \tag{5.39}$$

where ψ_J and ψ_J are the wave functions for a 3π system with total angular momentum J and parity $+1$ and -1 respectively [cf. Equation (2.6)]. The constant α is a real positive number. It is to be expected that $J=0$ or 2 would still be favorable.

7. Λ^0 Decay and Σ Decay

Information concerning parity nonconservation can be obtained by studying the decay of hyperons.[14] We consider as an example the following reactions:

$$\pi \;\; + p \to \Sigma \;\; + K^+ \tag{5.40}$$

and

$$\Sigma \;\; \to n + \pi \;\; . \tag{5.41}$$

The first reaction (5.40) can be thought of as the polarizer of Σ . Since it is a strong interaction, in order to produce a polarized Σ it is necessary to measure the direction of two momenta, say $\mathbf{p}_{in}$, the momentum of the incoming pion, and $\mathbf{p}_\Sigma$, the momentum of Σ . The spin $\mathbf{s}_\Sigma$ of the Σ then will be polarized along the direction $(\mathbf{p}_{in} \times \mathbf{p}_\Sigma)$. In the subsequent decay (5.41), if in the angular distribution of $\mathbf{p}_{out}$ (the momentum of the decay pion in the rest system of Σ) a term of the form $\mathbf{s}_\Sigma \cdot \mathbf{p}_{out}$ is observed, then parity is not conserved. Because what one measures is a quantity of the form $\mathbf{p}_{out} \cdot (\mathbf{p}_{in} \times \mathbf{p}_\Sigma)$, it is necessary to exclude cases where $\mathbf{p}_{in}$ is parallel to $\mathbf{p}_\Sigma$.

We shall give a more detailed consideration of processes (5 40) and (5 41) by further assuming that (i) the spin of Σ is ½, (ii) the spin of K is zero, and (iii) in the production process only s and p waves are important [34] Under these assumptions, the differential production cross section per unit solid angle $d\Omega$ (in the center-of-mass system of production) of the Σ produced is given by

$$I(\theta)=|a+b\cos\theta|^2+|c|^2\sin^2\theta \tag{5 42}$$

where $\theta=\angle(\mathbf{p}_{in}, \mathbf{p}_\Sigma)$

The corresponding polarization $P(\theta)$ is

$$P(\theta)=[I(\theta)]^{-1}2\sin\theta\cdot Im[c^*(a+b\cos\theta)] \tag{5 43}$$

where $P(\theta)$ is defined to be the average spin of the Σ in units of $(½)\hbar$ It is important to notice that if $I(\theta)=(1\pm\cos\theta)$ then $P(\theta)=0$ at all angles

Because of the possible nonconservation of parity in the decay process (5 41) the polarization $P(\theta)$ can be measured Let R be the projection of $\mathbf{p}_{out}$ in the direction of $\mathbf{p}_{in}\times\mathbf{p}_\Sigma$ The distribution function for R at an angle θ of production is given by

$$W(\theta,\xi)d\Omega d\xi=I(\theta)d\Omega\cdot(½)\,[1+\alpha p(\theta)\xi]\alpha\xi \tag{5 44}$$

where

$$\xi=R/(\text{maximum value of } R)\cong R/(100\text{ Mev/c})$$

In terms of the coefficients a, b, and c, defined in (5 42), $W(\theta,\xi)$ can be written as

$$W(\theta,\xi)d\xi d\Omega=(|a+b\cos\theta|^2+|c|^2\sin^2\theta)(½)d\xi d\Omega+$$
$$\alpha\sin\theta\, Im[c^*(a+b\cos\theta)]\xi d\xi d\Omega \tag{5 45}$$

The existence of a nonvanishing α constitutes an unambiguous proof of parity nonconservation in Σ decay In such a case the final state of $(n+\pi)$ in process (5 41) is a mixture of $s_{1/2}$ and $p_{1/2}$ states with amplitudes, say, A and B respectively The asymmetry parameter α is related to these amplitudes by

$$\alpha=\frac{2Re(A^*B)}{|A|^2+|B|^2} \tag{5 46}$$

If time reversal leaves invariant the decay process of Σ then (cf Chapter IV, section 2)

$$\alpha=\pm\frac{2|A|\cdot|B|}{|A|^2+|B|^2}\cos(\delta_p-\delta_s), \tag{5 47}$$

where δ_p and δ_s are, respectively, the phase shifts of $(n+\pi)$ scattering in the $p_{1/2}$ and $s_{1/2}$ states at about 117 Mev in their center-of-mass system If the decay interaction is invariant under charge conjugation then (cf Chapter IV, section 3)

$$\alpha=\pm\frac{2|A|\cdot|B|}{|A|^2+|B|^2}\sin(\delta_p-\delta_s) \tag{5 48}$$

[34] T D Lee, J Steinberger, G Feinberg, P K Kabir, and C N Yang, *Phys Rev* **106**, 1367 (1957)

Thus a large observed value of α means that both C and P are not conserved in the decay of Σ . Similar considerations can easily be applied to the productions and decays of other hyperons.

8. Ξ Decay

Another example of a possible test of parity nonconservation is through the decay of the Ξ-particle.[14]

$$\Xi \rightarrow \Lambda^0 + \pi \tag{5.49}$$

and

$$\Lambda^0 \rightarrow p + \pi \ . \tag{5.50}$$

As in the case of π-μ-e decay, if parity is not conserved in both (5.49) and (5.50), then the distribution of the momentum of the Λ^0, $\mathbf{p}_\Lambda$, and that of the pion, $\mathbf{p}_\pi$, may contain odd powers of

$$(\mathbf{p}_\Lambda \cdot \mathbf{p}_\pi) \tag{5.51}$$

where $\mathbf{p}_\pi$ is measured in the rest system of Λ^0 and $\mathbf{p}_\Lambda$ in the rest system of Ξ .

We shall illustrate the calculation of such a distribution by considering the special case that the spins of Λ^0 and Ξ^- are both ½. Let ϕ_Ξ be the initial spin state of Ξ at rest. The wave function of the corresponding Λ^0 in the decay (5.49) can be written as

$$\phi_\Lambda = M_\Lambda \cdot \phi_\Xi \tag{5.52}$$

where

$$M_\Lambda = A + B\boldsymbol{\sigma} \cdot \hat{\mathbf{p}}_\Lambda$$

with $\hat{\mathbf{p}}_\Lambda$ a unit vector along $\mathbf{p}_\Lambda$ and A, B the relative probability amplitudes of the two final states of opposite parity ($|A|^2 + |B|^2 = 1$). The subsequent wave function of p in (5.50) is

$$\phi_p = M_p \cdot \phi_\Lambda \tag{5.53}$$

where

$$M_p = a + b\boldsymbol{\sigma} \cdot \hat{\mathbf{p}}_\pi \,, \quad (|a|^2 + |b|^2 = 1)$$

with $\hat{\mathbf{p}}_\pi$ a unit vector along $\mathbf{p}_\pi$.

The final distribution of $\mathbf{p}_\pi$ and $\mathbf{p}_\Lambda$ for an unpolarized Ξ is

$$(½)\sum |\phi_p|^2 = (½)\,\mathrm{tr.}\,[M_\Lambda^\dagger M_p^\dagger M_p M_\Lambda] = 1 + \alpha\cos\theta \tag{5.54}$$

where

$$\theta = \angle\,(\mathbf{p}_\Lambda, \mathbf{p}_\pi) \quad \text{and} \quad \alpha = (a^* b + b^* a)\,(A^* B + B^* A)\,.$$

In Equation (5.54), the sum extends over the two initial spin states of ϕ_Ξ $(=\uparrow,\downarrow)$. Thus the observation of a nonvanishing value for α shows that P is not conserved in both Ξ decay and Λ^0 decay.

9. K^0, $\bar{K}^0$ Decay[22]

As a final example we consider the decay of neutral K-mesons. The existence of two neutral K-particles of different lifetimes and many of their properties were predicted and discussed[35] under the assumption that the decay of the K-meson is strictly invariant under charge conjugation C. Although there is as yet no explicit proof that C is not conserved in the decay of the K-particle, the various recent experiments[25] on parity and charge conjugation nonconservations in other weak interactions do give a strong indication that C is probably not invariant in K decay. As we shall see, the curious behavior of K^0 and $\bar{K}^0$ turns out to be remarkably insensitive to any possible nonconservation of P, C, or T. From the strong production processes we know that there must exist two different states, K^0 and $\bar{K}^0$, of opposite strangeness quantum number [Equation (1.27)]. Thus, independent of any assumption about the invariance or non-invariance properties, there should exist, in general, two lifetimes for their decay. We shall discuss the K-$\bar{K}$ decay in some detail under the following two possibilities.

A. THE DECAY PROCESSES ARE INVARIANT UNDER T

First we consider the various consequences if under time reversal T the decay processes are invariant. It follows, then, from the CPT theorem that $C{\cdot}P$ is conserved, although C may not be conserved. We define the states $|K_1\rangle$ and $|K_2\rangle$ by

$$|K_1\rangle=(1/\sqrt{2})\,(|K^0\rangle+C{\cdot}P|K^0\rangle) \quad \text{and} \quad |K_2\rangle=(1/\sqrt{2})\,(|K^0\rangle-C{\cdot}P|K^0\rangle)\,. \tag{5.55}$$

Thus, $|K_1\rangle$ and $|K_2\rangle$ are eigenstates of $C{\cdot}P$ with eigenvalues $+1$ and -1 respectively. If $C{\cdot}P$ is strictly invariant, $|K_1\rangle$ and $|K_2\rangle$ each will decay exponentially in time. The lifetime of $|K_1\rangle$ is, in general, different from that of $|K_2\rangle$. Furthermore

$$K_1{}^0 \rightarrow \pi^+ + \pi^- \tag{5.56}$$

in addition to its other possible modes of decay. [For example, if $(\text{spin})_{K^0}=0$, 2, 4 . . . , then $K_1{}^0$ also $\rightarrow 2\pi^0$.] On the other hand $K_2{}^0$ cannot decay into a 2π system. This follows from the fact that a 2π system with total spin J is *always* an eigenstate of $C{\cdot}P$ with eigenvalue $+1$. From phase space arguments, it is expected that $K_1{}^0$ has a shorter lifetime.

In order to detect any difference between the present case and the case when C is conserved it is necessary to measure a pseudoscalar quantity. For example, in the decay of the long-lived K-particle, $K_2{}^0$,

$$K_2{}^0 \rightarrow \pi^+ + e^- + \bar{\nu}\,, \tag{5.57}$$

the electron may be longitudinally polarized and its polarization $(\boldsymbol{\sigma}\cdot\mathbf{p})_{e^-}$ may be measured. The corresponding quantity $(\boldsymbol{\sigma}\cdot\mathbf{p})_{e^+}$ for the positron in the decay,

$$K_2{}^0 \rightarrow \pi^- + e^+ + \nu\,, \tag{5.58}$$

is expected to be of opposite sign from that of $(\boldsymbol{\sigma}\cdot\mathbf{p})_{e^-}$. However in a measurement which does not involve a pseudoscalar quantity, the nonconservation of C can not be tested. For example, let r be the branching ratio

[35] M. Gell-Mann and A. Pais, *Phys. Rev.* 97, 1387 (1955).

$$r \equiv \frac{\text{rate of } K_2^0 \to \pi^+ + e^- + \bar{\nu}}{\text{rate of } K_2^0 \to \pi^- + e^+ + \nu}. \tag{5.59}$$

We should expect $r=1$ in the present case.

B. K^0 AND $\bar{K}^0$ DECAY PROCESSES MAY NOT BE INVARIANT UNDER T

Next we consider the more general case that under time reversal T the decay processes may not be invariant. We define $|\bar{K}^0\rangle$ to be

$$|\bar{K}^0\rangle = C \cdot |K^0\rangle \tag{5.60}$$

where the charge conjugation operator C is defined through the strong interaction. Suppose at $t=0$ a K-particle is produced. At a later time its wave function can be described as

$$\psi(t)=a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle , \tag{5.61}$$

or simply as

$$\psi(t)=\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}. \tag{5.62}$$

The differential equation for $\psi(t)$ can be written as

$$-d\psi/dt=(\Gamma+iM)\psi , \tag{5.63}$$

where

$$\Gamma_{11}=\Gamma_{22}=\sum_j \Gamma_{aj}=\sum_j \Gamma_{bj} , \quad \Gamma_{12}=\Gamma_{21}^*=\sum_j (\Gamma_{aj}\Gamma_{bj})^{1/2} e^{i\delta_j} \tag{5.64}$$

with

$\Gamma_{aj}=2\pi|H_{aj}|^2 \times$(density of states per unit energy);

$\Gamma_{bj}=2\pi|H_{bj}|^2 \times$(density of states per unit energy); and

$e^{i\delta_j}=$ phase of (H_{aj}/H_{bj});

where H_{aj} and H_{bj} are the matrix elements of the Hamiltonian for the decay processes, a and b refer to states $|K^0\rangle$ and $|\bar{K}^0\rangle$ respectively, and j refers to any possible decay state (consisting of standing waves) that is an eigenstate of H_{strong}. Thus state $|j\rangle$ has a definite spin, charge, and parity. That $\Gamma_{11}=\Gamma_{22}$ follows from

$$H_{aj}^* = \pm H_{b\bar{j}}, \tag{5.65}$$

with $|\bar{j}\rangle = C|j\rangle$. Equation (5.65) is an immediate consequence of the CPT theorem [cf. Equation (4.14)].

Similarly, we have for the mass operator M,

$$M^\dagger = M \quad \text{and} \quad M_{11}=M_{22}. \tag{5.66}$$

Equation (5.63) can now be readily solved. Its eigenstates, defined by

$$(\Gamma+iM)\psi_\pm=\lambda_\pm\psi_\pm ,$$

are

$$\psi_\pm=\begin{pmatrix} p \\ \pm q \end{pmatrix}(|p|^2+|q|^2)^{-1/2} , \tag{5.67}$$

with the corresponding time constants

$$\lambda_{\pm}=\Gamma_{11}+iM_{11}\pm(pq)\,, \tag{5.68}$$

where p and q are two complex numbers given by

$$p^2=\Gamma_{12}+iM_{12} \quad \text{and} \quad q^2=\Gamma_{21}+iM_{21}=\Gamma_{12}{}^*+iM_{12}{}^*\,. \tag{5.69}$$

If at $t=0$ a K^0-particle is produced, then at a later time the state function ψ can be expressed in terms of these two eigenstates $\psi_\pm$ as

$$\psi(t)=[1/(2p)](|p|^2+|q|^2)^{1/2}(\psi_+e^{-\lambda_+t/2}+\psi_-e^{-\lambda_-t/2})\,. \tag{5.70}$$

It is convenient to separate the real and imaginary parts of $\lambda_\pm$. Without loss of generality we may write

$$\lambda_+=\gamma_+ \quad \text{and} \quad \lambda_-=\gamma_-+2i\Delta\,, \tag{5.71}$$

where γ_+, γ_- are two real numbers representing the reciprocal lifetimes of the short-lived ones and the long-lived ones respectively, and Δ is the mass difference between these two eigenstates. One notices that *these two eigenstates ψ_+ and ψ_- do not in general represent the states K_1 and K_2* introduced in (5.55). *In fact they may not be orthogonal to each other.*

The fractional number of K-mesons that decay at time t after production is given by

$$\mathcal{N}(t)dt=-d(\psi^\dagger\psi)\,. \tag{5.72}$$

Using (5.63) one easily shows that

$$-\frac{d}{dt}(\psi^\dagger\psi)=\psi^\dagger\Gamma\psi\,.$$

By using Equations (5.67) to (5.70), Equation (5.72) becomes

$$\mathcal{N}(t)=(1/2)(1+\alpha)^{-1}\{\gamma_+e^{-\gamma_+t}+\gamma_-e^{-\gamma_-t}+\alpha e^{-(\gamma_++\gamma_-)t/2}[(\gamma_++\gamma_-)\cos\Delta t-2\Delta\sin\Delta t]\}\,, \tag{5.73}$$

where

$$\alpha=\psi_+{}^\dagger\psi_-=(|p|^2-|q|^2)(|p|^2+|q|^2)^{-1} \tag{5.74}$$

is a real number representing the non-orthogonality of these two eigenstates. The four real numbers γ_+, γ_-, Δ, and α characterize the decay of the K-particle. They satisfy the inequalities

$$\gamma_\pm\geqslant 0 \quad \text{and} \quad \alpha^2\leqslant\frac{4\gamma_+\gamma_-}{(\gamma_++\gamma_-)^2+4\Delta^2} \tag{5.75}$$

which follow from the fact that Γ is a positive Hermitian matrix. These conditions also insure that $\mathcal{N}(t)\geqslant 0$ for all t.

Experimentally $\mathcal{N}(t)$ is measurable. From $\mathcal{N}(t)$ one can in principle determine all four constants γ_+, γ_-, Δ, and α. Indications from existing experiments[36] show that probably $(\gamma_+/\gamma_-)>100$. Equation (5.75) then shows that $\alpha^2<4(\gamma_-/\gamma_+)<0.04$

The above discussion also leads easily to a determination of the branching ratio of the long-lived component (and the short-lived component) into the various decay

[36] K. Lande, E.T. Booth, J. Impeduglia, and L.M. Lederman, *Phys. Rev.* **103**, 1901 (1956).

modes. If time reversal is invariant the long-lived component is an eigenstate of $C{\cdot}P$. As discussed in part A above, its decay into charge conjugate channels such as $\pi^+ + e^- + \bar{\nu}$ and $\pi\ + e^+ + \nu$ must be equally probable. If time reversal is not strictly invariant, decays into $\pi^+ + e\ + \bar{\nu}$ and $\pi^- + e^+ + \nu$ may have different probabilities for the long-lived component.

We consider first the following decay channel of the K-particle:

$$K^0 \rightarrow e\ + \pi^+ + \bar{\nu}\,. \tag{5.76}$$

The final product may be in states with either parity $= +1$ or parity $= -1$. Let us denote the matrix elements for the decay process into these two types of states by f_1 and f_2. Similarly, we denote the matrix elements for

$$K^0 \rightarrow e^+ + \pi\ + \nu \tag{5.77}$$

with the final state having parity $= +1$ and parity $= -1$ by g_1 and g_2.

By using the CPT theorem and Equation (4.14), the corresponding matrix elements for the decay of $\bar{K}$,

$$\bar{K}^0 \rightarrow e^+ + \pi\ + \nu\,, \tag{5.78}$$

are related to that of (5.76). These elements are f_1^* and $-f_2^*$. Similarly the matrix elements for

$$\bar{K}^0 \rightarrow e\ + \pi^+ + \bar{\nu} \tag{5.79}$$

are g_1^* and $-g_2^*$. Let ψ_+ represent the long-lived component K_+ of the K-particle. The matrix elements for the decay of K_+,

$$K_+ \rightarrow e\ + \pi^+ + \bar{\nu}\,, \tag{5.80}$$

into the two different final parity states are proportional to $pf_1 + qg_1^*$ and $pf_2 - qg_2^*$, respectively, while the corresponding elements for

$$K_+ \rightarrow e^+ + \pi\ + \nu \tag{5.81}$$

are proportional to $pg_1 + qf_1^*$ and $pg_2 - qf_2^*$. The branching ratio r for the decay of K_+ into $e\ + \pi^+ + \bar{\nu}$ and $e^+ + \pi\ + \nu$ is therefore,

$$r = \frac{|pf_1 + qg_1^*|^2 + |pf_2 - qg_2^*|^2}{|pg_1 + qf_1^*|^2 + |pg_2 - qf_2^*|^2}. \tag{5.82}$$

A detection of $\gamma \neq 0$ would establish the non-invariance of K^0 decay under time reversal. However, from (5.82), we see that if $|p| = |q|$ then $r = 0$. Also, if $\alpha = 0$, the two eigenstates ψ_+, ψ are orthogonal and $|p| = |q|$. (This is the case if the mass operator M is negligible.) In this case the decays of long-lived components into charge conjugate channels such as $\pi^+ + e\ + \nu$ and $\pi\ + e^+ + \bar{\nu}$ are equally probable. Furthermore we recall that because the strong interaction conserves I_z, the behavior of $|K^0\rangle$ under a charge conjugation operator cannot be determined within a phase factor $e^{iI_z\theta}$. If ψ_+ is orthogonal to ψ, by choosing this phase factor to be that of (p/q) these two eigenstates ψ_+ and ψ can be made to be identical with $|K_1\rangle$ and $|K_2\rangle$, Equation (5.55). From experimental results on the two lifetimes of ψ_+ and ψ we know that $\alpha^2 < 0.04$. Thus the branching ratio r may be quite small even though time reversal may not be conserved.

As a final remark we note that because of the largeness of phase space volume for 2π decay it might be expected that in the Γ matrix [Equation (5.63)] only the matrix elements due to the decay into a certain 2π mode are of dominant importance. If in the calculation of the mass operator M the virtual processes via the same 2π mode give also the dominant contributions, then we should expect without further assumptions that the lifetime ratio γ_+/γ_- should be large; that the short-lived one ψ_- and the long-lived one ψ_+ should be almost identical with K_1 and K_2 respectively; and that ψ_- should decay mostly into that 2π mode while ψ_+ should decay dominantly to other modes such as $\pi^+ + e^- + \bar{\nu}$, etc.

VI. A TWO-COMPONENT THEORY OF THE NEUTRINO[37]

The various experimental results on nonconservation of parity and charge conjugation in β decay, π decay, and μ decay can be expressed in a particularly simple and appealing way by using a two-component theory of the neutrino

1. The Neutrino Field

Consider first the Dirac equation for a free spin ½ particle with zero mass Because of the absence of the mass term one needs only three anticommuting Hermitian matrices Thus the neutrino can be represented by a spinor function ϕ_ν which has only two components The Dirac equation for ϕ_ν can be written as ($\hbar=c=1$)

$$(\boldsymbol{\sigma}\cdot\mathbf{p})\phi_\nu=i\dot{\phi}_\nu \tag{6 1}$$

where σ_1, σ_2, σ_3 are the usual 2×2 Pauli matrices The relativistic invariance of this equation for proper Lorentz transformations (i e , Lorentz transformations without space inversion and time inversion) is well known In particular for the space rotations through an angle θ around, say, the z axis, the wave function transforms in the following way

$$\phi_\nu \rightarrow \exp(-i\sigma_3\theta/2)\,\phi_\nu$$

The σ matrices are therefore the spin matrices for the neutrino For a state with a definite momentum $\mathbf{p}$, the energy and the spin along $\mathbf{p}$ (defined to be the helicity $\mathcal{H}$) are given respectively by

$$H=(\boldsymbol{\sigma}\cdot\mathbf{p}) \quad \text{and} \quad \mathcal{H}=(\boldsymbol{\sigma}\cdot\mathbf{p})/|\mathbf{p}| \tag{6 2}$$

They are therfore related by

$$H=|\mathbf{p}|\,\sigma_p \tag{6 3}$$

In the c number theory, for a given momentum the particle therefore has two states a state with positive energy, and with ½ as the spin component along $\mathbf{p}$, and a state with negative energy and with $-$½ as the spin component along $\mathbf{p}$

It is easy to see that in a hole theory of such particles, *the spin of a neutrino* (defined to be a particle in the positive energy state) *is always parallel to its momentum*

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[37] The possibility of a two-component relativistic theory of a spin ½ particle was first discussed by H Weyl [*Z Physik* **56**, 330 (1929)] However, in such a theory parity is not manifestly con served, therefore, in the past it was always rejected (Cf W Pauli, *Handbuch der Physik*, Verlag Julius Springer, Berlin, 1933, Vol 24, pp 226-7) The possible use of this two component theory for expressing the nonconservation property of parity in neutrino processes was independently proposed and considered by T D Lee and C N Yang, *Phys Rev* **105**, 1671 (1957), A Salam, *Nuovo cimento* **5**, 299 (1957), and L Landau, *Nuclear Phys* **3**, 127 (1957)

while the spin of an antineutrino (defined to be a hole in the negative energy state) *is always antiparallel to its momentum* (i.e., the momentum of the antineutrino). Many of the experimental implications on nonconservation of parity and charge conjugation can be directly related to this correlation between the spin and the momentum of a neutrino. With the usual (right-handed) conventions which we adopt throughout this paper, the spin and the velocity of the neutrino represent the spiral motion of a right-handed screw ($\mathcal{H}_\nu = +1$), while the spin and the velocity of the antineutrino represent the spiral motion of a left-handed screw ($\mathcal{H}_{\bar{\nu}} = -1$).

We shall now discuss some general properties of this neutrino field under the further assumption that the law of conservation of leptons is valid.[18]

A. The mass of the neutrino and the antineutrino in this theory is necessarily zero. This is true for the physical mass even with the inclusion of all interactions. To see this, one need only observe that all the one-particle physical states consisting of one neutrino (or one antineutrino) must belong to a representation of the inhomogeneous proper Lorentz group identical with the representation to which the free neutrino states discussed above belong. For such a representation to exist at all the mass must be zero.

B. That the use of such a theory fits naturally into the nonconservation property of parity is well known. We see it also in the following way: Under a space inversion P, one inverts the momentum of a neutrino but not its spin direction. Since in this theory the two are always parallel, the operator P applied to a neutrino state leads to a nonexisting state. Consequently the theory is not invariant under space inversion.

C. By the same reasoning one concludes that the theory is also not invariant under charge conjugation C which changes a particle into its antiparticle but does not change its spin direction or momentum.

D. It is possible, however, for the theory to be invariant under the operation CP, as this operation changes a neutrino into an antineutrino and simultaneously reverses its momentum while keeping the spin direction fixed. By the CPT theorem it follows that the theory can be invariant under time reversal T.

For the free neutrino field, as described by (6.1), one can prove that the theory is indeed invariant under time reversal and under CP.

2. β Decay

To discuss the β decay phenomena with the two-component neutrino theory and to compare the present results with those calculated previously, it is convenient to indicate how one can use the conventional four-component formalism of the neutrino (with nonconservation of parity) and obtain the same results as the present theory.

We start from Equation (6.1) and enlarge the matrices by the following definitions: ($1 \equiv 2 \times 2$ unit matrix)

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[18]If the law of conservation of leptons is not valid then the mass of a two-component neutrino is, in general, not zero In a special case even parity may still be conserved See footnote 39 and K M Case, *Phys Rev* **107**, 307 (1957) Cf also A Salam, *Nuovo cimento* **5**, 299 (1957)

$$\boldsymbol{\alpha} \equiv \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \quad \beta \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{6.4}$$

$$\psi_\nu = \begin{pmatrix} \phi_\nu \\ 0 \end{pmatrix}, \tag{6.5a}$$

$$\boldsymbol{\gamma} \equiv -i\beta\boldsymbol{\alpha}, \quad \gamma_4 \equiv \beta, \quad \gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{6.6}$$

An immediate consequence of these definitions is

$$\gamma_5 \psi_\nu = -\psi_\nu . \tag{6.7a}$$

The free neutrino part of the Lagrangian is, as usual,

$$L_\nu = \psi_\nu{}^\dagger \gamma_4 \left(\gamma_\mu \frac{\partial}{\partial x_\mu} \right) \psi_\nu , \tag{6.8}$$

where $\psi_\nu{}^\dagger$ = Hermitian conjugate of ψ_ν. The most general interaction Lagrangian with no derivative couplings for the process

$$n \rightarrow p + e^- + \bar{\nu} \tag{6.9a}$$

is exactly as usual; namely, it is the sum of the usual S, V, T, A, and P couplings:

$$+L_{\text{int}} = -H_{\text{int}} = \sum - \mathfrak{G}_i (\psi_p{}^\dagger O_i \psi_n)(\psi_e{}^\dagger O_i \psi_\nu) \tag{6.10a}$$

where i runs over S, V, T, A, and P; and O_S, O_V, etc. are given in Equation (5.2).

It is not difficult to prove that Equations (6.5a) and (6.7a) are consistent with a relativistical theory even in the presence of the interaction (6.10a). Another way of proving this is to start from the conventional theory of the neutrino with the interaction Hamiltonian given in Equation (5.1) and observe that when

$$C_S = -C_S' = (1/2)\mathfrak{G}_S \quad \text{and} \quad C_V = -C_V' = (1/2)\mathfrak{G}_V, \quad \text{etc.}, \tag{6.11a}$$

the neutrino field ψ_ν always appears in interactions in the combinations $(1-\gamma_5)\psi_\nu$. In the explicit representation we have adopted above this means that only the first two components of ψ_ν contribute to the interaction. *All calculations using the conventional theory of the neutrino with the Hamiltonian Equation* (5.1) *concerning β decay therefore give the same result as the present theory if we take the choice of constants* (6.11a). There exists, however, the possibility that in the decay of the neutron a neutrino (defined to be a right-handed screw) is emitted,

$$n \rightarrow p + e + \nu . \tag{6.9b}$$

The corresponding general form (not including derivatives of the fields) of the Hamiltonian is

$$H_{\text{int}} = \sum \mathfrak{G}_i{}' (\psi_p{}^\dagger O_i \psi_n)(\psi_e{}^\dagger O_i \psi_\nu{}') \tag{6.10b}$$

where O_i has been defined in Equation (5.2). The field $\psi_\nu{}'$ is a four-component spinor defined in terms of the two-component neutrino field ϕ_ν by

$$\psi_\nu{}' = \begin{pmatrix} 0 \\ \sigma_2 \phi_\nu{}^\dagger \end{pmatrix}. \tag{6.5b}$$

From Equation (6.6) we see that

$$\gamma_5 \psi_\nu' = + \psi_\nu' . \tag{6.7b}$$

It can be shown that (6.5b) and (6.7b) are consistent with a relativistic theory even in the presence of interaction (6.10b). *It can also be proved that one can use again the Hamiltonian Equation* (5.1) *for the conventional theory of the neutrino with the choice of the coupling constants*

$$(\tfrac{1}{2})\, \mathfrak{G}_S' = C_S = C_S', \quad (\tfrac{1}{2})\, \mathfrak{G}_V' = C_V = C_V', \quad \text{etc.} \tag{6.11b}$$

and obtain the same result as the present theory

The two possible choices (6.11a) and (6.11b) depend on whether, in the β decay of the neutron process, (6.9a) or (6 9b) prevails, i.e., whether an antineutrino ($\mathfrak{H}_{\bar\nu} = -1$, a left-handed screw) or a neutrino ($\mathfrak{H}_\nu = +1$, a right-handed screw) is emitted. We shall see that experimentally it will be easy to decide which of the two choices is appropriate. [In this section we do not consider the possibility of the simultaneous presence of both (6.9a) and (6.9b).][39]

A. ANGULAR DISTRIBUTION OF β-RAYS FROM POLARIZED NUCLEI

The experimental results of Wu et al. show that in the decay of

$$Co^{60} \rightarrow Ni^{60} + e + \bar\nu, \quad J=5 \underset{(no)}{\rightarrow} J=4 , \tag{6.12}$$

the angular distribution of e is

$$1 + \alpha \cos\theta \tag{6.13}$$

with

$$\alpha \cong -0.7 \frac{\langle J_z \rangle}{J} \tag{6.14}$$

and

$$\theta = \angle\, (\mathbf{J}, \mathbf{p}_e) .$$

This result is consistent with the coupling constants assignment

$$C_T' = -C_T, \quad C_A' = -C_A \cong 0 . \tag{6.15}$$

By comparing (6.15) with (6.11a) and (6.11b) we see that in the β decay a $\bar\nu$ (with a left-hand helicity, $\mathfrak{H}_{\bar\nu} = -1$) is emitted, while in the β^+ decay a ν (with a right-hand helicity, $\mathfrak{H}_\nu = +1$) is emitted

[39] If the conservation law of leptons is valid then (6 9a) and (6 9b) cannot be simultaneously present It is of interest to note that if reactions (6 9a) and (6 9b) were simultaneously present with exactly equal amplitude, the angular distribution of β-rays from polarized nuclei, for example, would appear to be symmetrical In such a case the theory is identical with the conventional Majorana theory of the neutrino with a parity conserving Hamiltonian This, of course, is not the case that occurs in nature The general relationship between the present two-component theory and the Majorana theory with a parity nonconserving Hamiltonian and, possibly, nonzero mass has recently been investigated by K Case, *Phys Rev* **107**, 307 (1957)

B. LONGITUDINAL POLARIZATION OF β-RAYS

As remarked before, if parity is not conserved the electrons can be longitudinally polarized.[30] We shall see that with the two-component theory of the neutrino all β emitters can be used as almost perfect polarizers for the β-rays. The degree of polarization depends on v/c of the β-rays. Let us consider a coupling term between e^- and ν,

$$\psi_e^+ O_i \psi_\nu ,$$

with O_i given in Equation (5.2). By using Equation (6.7a), together with the commutation relations

$$O_i \gamma_5 = \pm \gamma_5 O_i$$

with the plus sign for $i = V, A$, and the minus sign for $i = S, T, P$, we obtain

$$\begin{aligned} \psi_e^+ O_i \psi_\nu &= \psi_R^+ O_i \psi_\nu \quad \text{if } i = V, A \,; \\ &= \psi_L^+ O_i \psi_\nu \quad \text{if } i = S, T, P \,; \end{aligned} \tag{6.16}$$

where

$$\psi_R = (\tfrac{1}{2})(1 - \gamma_5)\psi_\nu \quad \text{and} \quad \psi_L = (\tfrac{1}{2})(1 + \gamma_5)\psi_\nu \,. \tag{6.17}$$

If the electron is extremely relativistic, $v/c = 1$, then ψ_R and ψ_L are both eigenstates of the free Hamiltonian of the electron, provided the mass term is neglected. In this case if the coupling (6.16) is S and/or T and/or P the electron will be 100% polarized with its spin antiparallel to its momentum (i.e., with a left-handed helicity), while if the coupling is V and/or A the electron is also 100% polarized but with its spin parallel to its momentum (i.e., with a right-handed helicity).

In general for relativistic or nonrelativistic electrons the term $(1 \mp \gamma_5)/2$ acts as a projection operator for the longitudinal polarization. The helicity $\mathcal{H}$ can be calculated as

$$\mathcal{H}_e = \langle \boldsymbol{\sigma} \cdot \hat{\mathbf{p}}_e \rangle_{\text{Av}} = \pm v/c \tag{6.18}$$

where $\hat{\mathbf{p}}_e$ is a unit vector along the momentum of e . In Equation (6.18) the plus sign is for V, A couplings and the minus sign for S, T, P couplings. Thus we see that *in*

$$n \rightarrow p + e \;\; + \bar{\nu}$$

if vector and axial vector couplings are absent the electrons will be longitudinally polarized with

$$\mathcal{H}_{e^-} = -v/c \,. \tag{6.19}$$

Equation (6.19) is true for any β emitter independent of whether the nuclei are polarized or not, and is independent of whether the β decay is allowed or forbidden. The possible deviation from (6.19) can then be used as a measure for the strengths of vector and axial vector coupling constants. In deriving Equations (6.18) and (6.19) we have not included possible depolarization effects due to a Coulomb field. However,

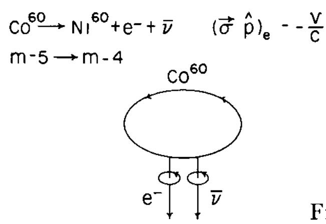

Figure 2

if $v/c = 1$ then it is easy to see that these results of longitudinal polarization cannot be effected by any Coulomb field [40]

Similarly for any β emission,

$$p \to n + e + \nu ,$$

the positions are longitudinally polarized with

$$\mathcal{H}_e = +v/c \tag{6 20}$$

provided V and A couplings are zero

Because of these properties it is possible to understand in a simple way that in the decay of Co^{60}, Equation (6 12), if $\bar{\nu}$ is emitted the electrons are emitted predominantly antiparallel to the spin direction of Co^{60} [i e , $\alpha < 0$ in Equation (6 14)] To understand the sign of α, let us neglect the A coupling, and consider the special case with

$$(J_z)_{Co^{60}} = 5 \to (J_z)_{Ni} = 4$$

in the Co^{60} decay The e and $\bar{\nu}$ emitted are both left-handed particles (i e , spin antiparallel to momentum) Take the particular case that $\mathbf{p}_e$, $\mathbf{p}_\nu$ are all parallel to the $\pm z$ axes In this case the orbital angular momentum along the z axis is zero In order to conserve J_z, both e and $\bar{\nu}$ must be emitted predominantly parallel to each other with both of their momenta $\mathbf{p}_e$ and $\mathbf{p}_\nu$ along the $-z$ axis (cf Figure 2) By using (6 19), it is easy to see that in this case with $C_A = 0$ the relative probabilities are

$$1-(v/c) \quad \text{for} \cos\theta = +1$$

and

$$1+(v/c) \quad \text{for} \cos\theta = -1$$

According to Equations (5 11) and (6 11a) the complete formula of α for

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[40] This may be proved in the following way Consider the motion of a charged particle in a coulomb field U,

$$(\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m + U)\psi = E\psi$$

Except for the βm term the Hamiltonian commutes with $(1 \pm \gamma\ )$ Thus if the final expression for $\langle\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\rangle$ is expanded in a power series of (m/E) the term with zeroth power in (m/E) is independent of the presence of U i e

$$\langle\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\rangle_e = +1 + o(m/E)$$

for V, A couplings and

$$\langle\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\rangle = -1 + o(m/E)$$

for S, A, P couplings

$$J+1 \rightarrow J(\text{no}) \tag{6.21}$$

is

$$\alpha = +\frac{J_z}{J}\beta ,$$

$$\beta = -\frac{v}{c}\,\frac{|C_T|^2-|C_A|^2+(2Ze^2/\hbar cp)Im(C_A C_T^*)}{|C_T|^2+|C_A|^2}\,. \tag{6.22}$$

In (6.22) the Fierz term is put to be zero. The minus sign in the coefficient of $|C_A|^2$ is of course due to the fact that e is emitted with a right-hand helicity via the axial vector coupling.

By a similar argument, for the corresponding simple case of β^- decay in

$$J \rightarrow J+1(\text{no}) \tag{6.23}$$

the $(e\ ,\bar{\nu})$ system should carry a $(\Delta J_z) = -1$. Thus the electrons are expected to be emitted predominantlyparallel to $\mathbf{p}_{\bar{\nu}}$ and both of these momenta are parallel to the spin direction of the nuclei. The general formula for (6.23) is

$$\alpha = -\frac{\langle J_z \rangle}{(J+1)}\beta \tag{6.24}$$

with β given by (6.22). The difference of sign in (6.24) and (6.21) is completely expected.

In the decay

$$J \rightarrow J(\text{no})\,, \tag{6.25}$$

let us first consider the contribution due to the tensor coupling in the Gamow-Teller matrix element alone. Even if one takes the special case of the completely polarized nuclei $J_z = J$ the change of z component nuclear angular momentum can be 0 or 1. In the first case of $\Delta J_z = 0$ no asymmetry is present. In the second case, the e and $\bar{\nu}$ should carry a $\Delta J_z = +1$ which causes the asymmetry parameter α to be negative, $\alpha < 0$. Thus we expect in the transition (6.25) if $M_F = 0$ and $C_A = 0$ the sign of α should be negative but with its magnitude greatly reduced. The general formula for $J \rightarrow J(\text{no})$ is

$$\alpha = +\frac{\langle J_z \rangle}{J(J+1)}\beta + \frac{\langle J_z \rangle}{\sqrt{J(J+1)}}\beta' ,$$

$$\beta' = -Re\left[C_S^* C_T - C_V^* C_A + i\frac{Ze^2}{\hbar cp}(C_S^* C_A - C_V^* C_T)\right]\frac{v_e}{c}\,\frac{4M_F \cdot M_{GT}}{\xi + (\xi b/W)} \tag{6.26}$$

with ξ and b given in Equations (5.4) and (5.8).

Similarly it is easy to see that for β^+ decay the asymmetry parameter due to tensor coupling (or due to the axial vector coupling term alone) must change its sign. The complete formulas can be easily obtained from Equations (5.11) to (5.13).

3. Capture Cross Sections for the Neutrino

An experiment such as the one being carried out by Cowan et al [41] measures the cross section for neutrino absorption, which can be calculated in both the present theory and the usual theory Now one determines the magnitude of the β coupling constants to give the observed lifetimes of nuclei against β decay The calculated value of the cross section turns out then to be *twice as great* in the present theory as in the usual theory using the four-component theory and conservation of parity This follows from the following simple reasoning The neutrino flux is an experimental quantity independent of the theory If the neutrinos in a given direction have only one spin state instead of the usual two, by a detailed balancing argument they must have a cross section for absorption twice as great as the usual ones Actually from the experiments of Wu et al [1] one expects the neutrino emitted to be longitudinally polarized Thus an increment of cross section is expected if we use the hermiticity property of the H_{weak} This effect should be present even if the neutrino is described by a four-component theory with parity nonconservation

4. π Decay

In the decay of π-mesons at rest let us consider the component of angular momentum along the direction $\mathbf{p}_\mu$, the momentum of the μ-meson The orbital angular momentum contributes nothing to this component The μ spin component is therefore completely determined (irrespective of its total spin) by the spin component of the ν or $\bar{\nu}$ There are then two possibilities (i)

$$\pi^+ \to \mu^+ + \nu, \quad \mu^+ \text{ spin along } \mathbf{p}_\mu = +\tfrac{1}{2},$$

$$\pi^- \to \mu^- + \bar{\nu}, \quad \mu^- \text{ spin along } \mathbf{p}_\mu = -\tfrac{1}{2}, \tag{6 27}$$

or (ii)

$$\pi^+ \to \mu^+ + \nu, \quad \mu^+ \text{ spin along } \mathbf{p}_\mu = -\tfrac{1}{2},$$

$$\pi^- \to \mu^- + \nu, \quad \mu^- \text{ spin along } \mathbf{p}_\mu = +\tfrac{1}{2} \tag{6 28}$$

In each case the μ-mesons with fixed p_μ form a polarized beam Furthermore, the polarization is complete (i e , in a pure state) In this theory of the neutrino the π-μ decay is then a perfect polarizer of the μ-meson, and offers a natural way to measure the spin and the magnetic moment of the μ-meson (It turns out that the μ-e decay can serve as a good analyser, as we shall discuss in the next section)

The choice of the two possibilities (6 27) and (6 28) will be further discussed in Chapter VII

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[41] C L Cowan Jr , F Reines, F B Harrison, H W Kruse, and A D McGuire, *Science* **124**, 103 (1956)

5. μ Decay

For the μ -e decay the process can be

$$\mu \rightarrow e + \nu + \bar{\nu} \,, \tag{6 29}$$

or

$$\mu \rightarrow e + 2\nu \,, \tag{6 30}$$

or

$$\mu \rightarrow e + 2\nu \tag{6 31}$$

Consider process (6 29) first The decay interaction (without derivative coupling) can be written with the notations defined in Equation (5 2) [42]

$$H_{\text{int}} = \sum_i G_i(\psi_e{}^{\dagger} O_i \psi_\nu)(\psi_\nu{}^{\dagger} O_i \psi_\mu) \tag{6 32}$$

We have assumed in writing (6 32) that the spin of the μ-meson is ½

Becuse of the subsidiary condition (6 7a) satisfied by the neutrino field ψ_ν, the S-coupling term in the Hamiltonian (6 32) gives a result identical to that of the P-coupling term, the V-coupling term is the same as the A-coupling term, and the T-coupling term is identically zero Thus, in (6 32) there are only two independent constants,

$$g_1 \equiv G_S - G_P \quad \text{and} \quad g_2 \equiv G_V + G_A \tag{6 33}$$

The electron (or positron) emitted from a μ-meson decay at rest will be longitundinally polarized with a helicity $\mathcal{H}$ given by

$$\mathcal{H}_e = -\xi \quad \text{and} \quad \mathcal{H}_e = +\xi \tag{6 34}$$

where

$$\xi = (|g_1|^2 - 4|g_2|^2)(|g_1|^2 + 4|g_2|^2)^{1} \tag{6 35}$$

and the helicity $\mathcal{H}$ is defined by

$$\mathcal{H} = (\boldsymbol{\sigma} \cdot \mathbf{p})/|\mathbf{p}| \tag{6 2}$$

with $\boldsymbol{\sigma}$ and $\mathbf{p}$ the spin and momentum vectors of the electron (or positron) Equation (6 34) is independent of the polarization state of the μ-meson

For a μ at rest with spin completely polarized, the normalized electron distribution is given by

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[42] In reference 23 the Hamiltonian for μ decay is written as

$$H = \sum_{i\ V\ A} f_i(\psi_e{}^{\dagger} O_i \psi_\mu)(\psi_\nu{}^{\dagger} O_i \psi_\nu)$$

The f_S and f_A are related to G_i of (6 32) by

$$(½)(G_S - G_P) = f_A + f_V \quad \text{and} \quad G_V + G_A = f_A - f_V$$

It is also possible to write the Hamiltonian for μ decay in the form

$$H = \sum_i G_i'(\psi_e{}^{\dagger} O_i \psi_\nu')(\psi_\nu'{}^{\dagger} O_i \psi_\mu)$$

where ψ_ν' is given by (6 56) The G_i' are related to G_i by

$$(½)(G_S - G_P) = G_V' + G_A \quad \text{and} \quad G_V + G_A = -(½)(G_S' - G_P')$$

$$(dN)_e = 2x^2[(3-2x)+\xi\cos\theta(1-2x)]\,dx\,d\Omega(4\pi)^{-1} \tag{6.36}$$

where

$x = p/$(maximum electron momentum),

$\theta =$ angle between the electron momentum and the spin direction,

$d\Omega =$ solid angle of electron momentum,

and ξ is given by (6.35). The corresponding distribution of e^+ from a completely polarized μ^+ at rest is

$$(dN)_{e^+} = 2x^2[(3-2x)-\xi\cos\theta(1-2x)]dx\,d\Omega(4\pi)^{-1} \tag{6.37}$$

The decay probability per unit time is ($\hbar = c = 1$)

$$\lambda = m_\mu^5(|g_1|^2+4|g_2|^2)(3\times2^{11}\pi^3)\,. \tag{6.38}$$

The energy spectrum of the electron (or positron) is

$$dN = 2x^2(3-2x)dx\,. \tag{6.39}$$

The spectrum (6.39) is characterized by a Michel parameter[43] $\rho = 3/4$ which is not inconsistent with but *seems* to be slightly higher than the present experimental value[44] $\rho = 0.68$.

Integration of (6.36) and (6.37) over the energy of e^+ gives for the over-all angular distribution

$$(dN)_{e^{\mp}} = [1\pm(1/3)\xi]d\Omega(4\pi)^{-1}\,. \tag{6.40}$$

The mass of the electron (or positron) is neglected in all the above formulas (6.34) to (6.40).

That in (6.37) the angular distribution depends sensitively on the energy of the electron can be understood in a simple way. By using (6.16) to (6.18), we expect for the helicity of e [cf. (6.35)]

$$\mathcal{H}_e = -1 \quad \text{if only } g_1\neq0$$

and

$$\mathcal{H}_{e^+} = +1 \quad \text{if only } g_2\neq0. \tag{6.41}$$

Consequently, there is no interference term between g_1 and g_2.

Let us first discuss the case that only $g_1\neq0$ ($g_2=0$). The e emitted is of left-hand helicity. Consider the special case that $\mathbf{p}_e$, $\mathbf{p}_\nu$, $\mathbf{p}_{\bar\nu}$, and $\boldsymbol{\sigma}_\mu$ are all along the $+z$ or $-z$ axis. From conservation of angular momentum along the z axis it is easy to show that if $\boldsymbol{\sigma}_\mu // +z$ axis then for $x=1$,

$$\mathbf{p}_e // -z \text{ axis} \tag{6.42}$$

and for $x<1/2$,

$$\mathbf{p}_e // +z \text{ axis} \quad (g_1\neq0,\ g_2=0). \tag{6.43}$$

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[43] L Michel, *Proc Phys Soc (London)* **A63**, 514 (1950)

[44] C P Sargent, M Rinehart, L M Lederman, and K C Rogers, *Phys Rev* **99**, 885 (1955) These authors give $\rho = 0.68\pm0.10$ More recent measurements by L Rosenson (*Phys Rev*, in press) and by K Crowe, *Bull Am Phys Soc* **2**, 206 (1957), give the same value for ρ but with a smaller error A slight deviation of ρ value from 0.75 may indicate that there is a possible "non-local" effect in the μ decay Lagrangian

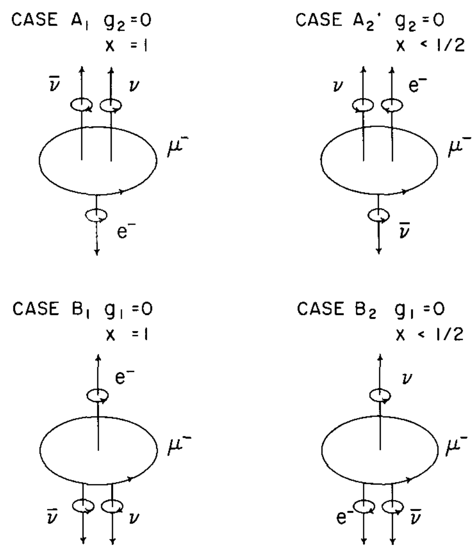

Figure 3

This can be seen directly by inspection of Figure 3 (case A_1 and case A_2). Furthermore for $x=1$ all three momenta $\mathbf{p}_e$, $\mathbf{p}_\nu$, and $\mathbf{p}_{\bar{\nu}}$ must be colinear. Thus, from (6.42), the corresponding angular distribution must be of the form $1-\cos\theta$ (with $x=1$ and $g_2=0$), which means that the asymmetry is maximum. For other values of $x(<1)$ these three momenta may not be colinear and the asymmetry does not attain its maximum value. A comparison between (6.42) and (6.43) explains the energy dependence $(1-2x)$ in the $\cos\theta$ term in (6.37). In an entirely similar way one can apply the above considerations to the case that $g_1=0$ and $g_2\neq 0$ (cases B_1 and B_2 of Figure 3).

Experimentally, the angular distribution of e^+ has been measured[2, 25] with respect to the momentum of the μ-meson from π decay. The experimental results seem to agree quite well with the distribution function (6.37). The results for μ^+ stopped in carbon were discussed in Chapter V, section 5. From Equation (5.33), it can be concluded that the parameter ξ, (6.35), must lie within the limits

$$1 \geqslant |\xi| > 0.78\,. \tag{6.44}$$

The algebraic sign of ξ depends on whether in π decay the helicity of μ is $+1$ or -1. Equation (6.44) gives only a lower limit of ξ. The actual value depends on the degree of depolarization of μ^+. The precise value and sign of ξ can be obtained more directly by a measurement of the helicity for e^+ from μ decay, Equation (6.34). This point will be further discussed in connection with the law of conservation of leptons in Chapter VII.

If in the μ^+ decay process

$$\mu \rightarrow e + 2\nu \tag{6.30}$$

or

$$\mu \rightarrow e + 2\bar{\nu} \tag{6.31}$$

prevails, the corresponding energy spectrum is characterized by a Michel parameter[43] $\rho=0$ which is not consistent with experiments.[44]

6. Remarks

In the above sections we have seen that the various effects due to nonconservation of parity and charge conjugation in β decay, π decay, and μ decay can be conveniently described by the use of a two-component theory of the neutrino. Nevertheless, as we have noticed before (cf. footnote 39) the mere use of a two-component theory does not preclude, for example, the possibility that in addition to

$$n \rightarrow p+e^{-}+\bar{\nu} \tag{6 9a}$$

we may also have

$$n \rightarrow p+e^{-}+\nu\,. \tag{6.9b}$$

From the experimental results on the slowness of the rate for double β decay processes and the largeness of asymmetry in the β angular distribution from polarized nuclei, we know that reaction (6.9b), if it exists at all, must be described by a much smaller coupling constant than that of reaction (6.9a). Recently K. Case[45] was able to show that by using the Majorana theory with a Hamiltonian which does not conserve parity it is possible to generalize the equation of motion [Equation (6.1)] for the neutrino field and to construct a two-component theory with, possibly,

$$m_\nu \neq 0\,.$$

In the special case that $m_\nu=0$, his generalization reduces to the present two-component theory discussed in this chapter. However, in the general case, if the mass of the neutrino $m_\nu \neq 0$ then the rate of double β decay process cannot attain its minimum value and the asymmetry of the β angular distribution from polarized nuclei cannot reach its maximum value. Yet experimentally, the mass of the neutrino $m_\nu \cong 0$, the rate of double β decay process $\cong 0$, and the asymmetry of the β angular distribution $\cong$ its maximum value. These three facts seem to be strongly suggestive of the possible existence of a law of conservation of leptons.[46] It can be shown easily that *the existence of a conservation law of leptons together with the use of a two-component theory of the neutrino necessitates* (i) $m_\nu=0$, (ii) *rate of double β decay = its minimum value,* and (iii) *parity must be nonconserved and the observed asymmetry due to such nonconservation can attain its maximum value.* In the next chapter we shall analyze in some detail the various consequences of such a conservation law of leptons.

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[45] K Case, *Phys Rev* **107**, 307 (1957)

[46] The concept of a possible conservation law of leptons was first considered by E Konopinski and H M Mahmond, *Phys Rev* **92**, 1045 (1953) Some discussions on analyzing the conservation law of leptons together with the use of a two-component theory of the neutrino are given in T D Lee and C N Yang, *Phys Rev* **105**, 1671 (1957)

VII. POSSIBLE LAW OF CONSERVATION OF LEPTONS AND THE UNIVERSAL FERMI INTERACTIONS

1. Law of Conservation of Leptons[46]

The law of conservation of leptons states that to each particle it is possible to assign a leptonic number l ($l \neq 0$ for leptons) and the sum of leptonic numbers must be conserved in all reactions From our previous discussions we know that β decay and μ decay are represented by[47]

$$n \rightarrow p+e\ +\bar{\nu} \tag{7 1}$$

and

$$\mu\ \rightarrow e\ +\nu+\bar{\nu} \tag{7 2}$$

Consequently, the assignments of l must be chosen as

$$\begin{array}{lll} l=\text{same (say, } l=-1) & \text{for } e\ , \nu, \text{ and } \mu\ , & \\ l=+1 & \text{for } e^{+}, \nu, \text{ and } \mu^{+}, \text{ and} & (7\ 3) \\ l=0 & \text{for } \pi, \gamma, K, \text{ and all heavy particles}^{48} & \end{array}$$

The conservation law of leptons then necessitates for the π decay

$$\pi\ \rightarrow \mu\ +\nu \quad \text{and} \quad \pi^{+} \rightarrow \mu^{+}+\nu \tag{7 4}$$

Equation (7 4) implies that the helicities $\mathcal{H}$ of μ in the rest system of π-mesons are

$$\mathcal{H}_{\mu}\ =-1 \quad \text{and} \quad \mathcal{H}_{\mu}\ =+1 \tag{7 5}$$

where $\mathcal{H}$ is defined to be

$$\mathcal{H}=(\boldsymbol{\sigma}\cdot\mathbf{p})/|\mathbf{p}| \tag{7 6}$$

with $\boldsymbol{\sigma}$, $\mathbf{p}$ the spin and momentum vectors The measurement of the helicities of μ and μ^{+} from π^{+} decay can serve as a test for the validity of the conservation law of leptons

Next we consider the helicities of e^{+} in the μ decay As we have discussed before, the helicities of e measured in the rest system of the μ-meson are given by [see (6 34) and (6 35)]

$$\mathcal{H}_{e}\ =-\xi \quad \text{and} \quad \mathcal{H}_{e}\ =+\xi \tag{7 7}$$

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[47] Throughout this chapter we use the two component theory for the neutrino

[48] That the leptonic number l for all heavy particles can be chosen as zero is evident Both pion and photon can be created singly, thus their leptonic numbers must both be zero If the number of leptons is absolutely conserved, then, because of the existence of decay modes $K_{\pi 2}$ and $K_{\tau 3}$, the leptonic number l for K mesons must also be zero

where ξ is related to the coupling constants for μ decay by (6.35) and (6.33). By comparing the theoretical angular distribution for e with the observed value we have already found that

$$1 \geqslant |\xi| > 0.78 . \tag{6.44}$$

Now if the law of conservation of leptons is correct, by using (7.5) we conclude that

$$-1 \leqslant \xi < -0.78 . \tag{7.8}$$

The minus sign for ξ is a consequence of the conservation law of leptons. Thus the measurement of the helicities of e^- and e^+ from μ decay can also serve as a test of the validity of the conservation law of leptons.

The conservation law of leptons also has a direct effect on the helicities of $\mu^{\mp}$ from $K_{\mu 2}{}^{\mp}$ decay. From the assignments of l in (7.3), the $K_{\mu 2}{}^{\mp}$ decay should be described by the reactions

$$K_{\mu 2}{}^- \to \mu^- + \bar{\nu} \quad \text{and} \quad K_{\mu 2}{}^+ \to \mu^+ + \nu . \tag{7.9}$$

Thus if the spin of K is zero the helicities of $\mu^{\mp}$ in the rest system of K are respectively[49]

$$\mathcal{H}_{\mu^-} = -1 \quad \text{and} \quad \mathcal{H}_{\mu^+} = +1 . \tag{7.10}$$

2. Universal Fermi Interactions

In this section we shall analyze the possibility of the so-called universal Fermi interactions by comparing the Hamiltonians for μ decay, β decay, and μ capture processes.[50] We represent these three processes by[51]

$$H_1 = \sum G_i (\psi_e{}^\dagger O_i \psi_\nu)(\psi_\nu{}^\dagger O_i \psi_\mu) + \text{hermitian conjugate},$$

$$H_2 = \sum \mathfrak{G}_i (\psi_p{}^\dagger O_i \psi_n)(\psi_e{}^\dagger O_i \psi_\nu) + \text{hermitian conjugate},$$

and

$$H_3 = \sum \mathcal{G}_i (\psi_\nu{}^\dagger O_i \psi_\mu)(\psi_n{}^\dagger O_i \psi_p) + \text{hermitian conjugate}, \tag{7.11}$$

respectively.

From (7.8) and the lifetime of the μ-meson we have for the coupling constants G_i of μ decay (cf. Chapter VI, section 5)

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[49]The application of the conservation law of leptons to $K_{\mu 2}$ decay is of interest because it gives a result opposite to that obtained by using the "attribute rule" proposed by R. G. Sachs [*Phys. Rev.* **99**, 1573 (1955)]. Cf. also W. G. Holladay, *Phys. Rev.*, in press.

[50]For references to previous works on the various Fermi interactions see, e.g., E. Fermi, *Elementary Particles,* Yale University Press, 1951; L. Michel, *Revs. Mod. Phys.* **29**, 159 (1957).

[51]In the present discussion of universal Fermi interactions we group the spinor fields ψ_e, ψ_ν, and ψ_μ in a particular form which is compatible with the idea of the conservation law of leptons. Had we grouped, for example, the μ decay interaction H in a different way (cf. footnote 42), many of the following conclusions about the similarity between the β decay coupling constants and the μ decay coupling constants would be changed. These changes can be easily obtained by using the relationship between G_i and G_i' given in footnote 42. (Compare especially G_i' with $\mathfrak{G}_i$.)

$$\eta \equiv \frac{|G_V+G_A|^2}{|G_S-G_P|^2} > 2.3 \tag{7.12}$$

and

$$|G_S-G_P|^2+4|G_V+G_A|^2=(7.5\times 10^{-49}\text{ erg-cm}^3)^2 . \tag{7.13}$$

The right-hand side of (7.12) is only a lower limit. The actual value of η is probably much larger than this limit. For example, $\eta \cong 5$ if in the measurement of π-μ-e decay 90% of the μ-mesons stopped in carbon are polarized and $\eta \to \infty$ if only 80% of the stopped μ-mesons are polarized.

The corresponding values for the various coupling constants for β decay have been extensively studied.[52] These results may be summarized[53] as follows:[52]

(i) From the absence of a Fierz term it can be concluded that

$$\frac{|\mathfrak{G}_S^* \mathfrak{G}_V+\mathfrak{G}_V^* \mathfrak{G}_S|}{|\mathfrak{G}_S^* \mathfrak{G}_S+\mathfrak{G}_V^* \mathfrak{G}_V|}=0.00\pm 0.10 \tag{7.14}$$

and

$$\frac{|\mathfrak{G}_A^* \mathfrak{G}_T+\mathfrak{G}_T^* \mathfrak{G}_A|}{|\mathfrak{G}_A^* \mathfrak{G}_A+\mathfrak{G}_T^* \mathfrak{G}_T|}=0.00\pm 0.04 . \tag{7.15}$$

(ii) From the β-ν angular correlation experiment on He^6 it can be concluded that

$$\left|\frac{\mathfrak{G}_A}{\mathfrak{G}_T}\right|^2<\frac{1}{3} . \tag{7.16}$$

(iii) From the β-ν angular correlation experiment on Ne^{19} it can be concluded that

$$\left|\frac{\mathfrak{G}_V}{\mathfrak{G}_S}\right|^2<1.4 \quad \text{if} \quad \mathfrak{G}_A \cong 0 \tag{7.17}$$

and

$$\left|\frac{\mathfrak{G}_V}{\mathfrak{G}_S}\right|^2<3 \quad \text{if} \quad \left|\frac{\mathfrak{G}_A}{\mathfrak{G}_T}\right|^2=\frac{1}{3} . \tag{7.18}$$

(iv) From the β-X diagram it can be concluded that

$$\frac{|\mathfrak{G}_S|^2+|\mathfrak{G}_V|^2}{|\mathfrak{G}_A|^2+|\mathfrak{G}_T|^2}=0.79 . \tag{7.19}$$

(v) From the ft value of O^{14} and (7.19), the absolute magnitudes of $|\mathfrak{G}_S|^2+|\mathfrak{G}_V|^2$ and $|\mathfrak{G}_A|^2+|\mathfrak{G}_T|^2$ are

$$|\mathfrak{G}_S|^2+|\mathfrak{G}_V|^2=(2.0\times 10^{-49}\text{ erg-cm}^3)^2 \tag{7.20}$$

and

$$|\mathfrak{G}_A|^2+|\mathfrak{G}_T|^2=(2.5\times 10^{-49}\text{ erg-cm}^3)^2 . \tag{7.21}$$

It should be noted that $\mathfrak{G}_i$ is related to C_i and C_i' in (5.1) by [cf. (6.11a)]

$$\mathfrak{G}_i=2C_i=-2C_i' . \tag{7.22}$$

[52]For detailed references on these experimental works see, e.g., K. Siegbahn, *Beta- and Gamma-Ray Spectroscopy*, Interscience, New York, 1955; L. Michel, *Revs. Mod. Phys.* **29**, 159 (1957).

[53]These results, (7.14) to (7.21), on various $\mathfrak{G}_i$ for β decay were compiled by C.S. Wu. We are grateful to Professor Wu for her permission to quote these results here.

(vi) Thus, if the β decay interaction is invariant under time reversal T, then

$$\mathfrak{G}_V \cong 0 \quad \text{and} \quad \mathfrak{G}_A \cong 0 . \tag{7.23}$$

By comparing $\mathfrak{G}_i$ [(7.20), (7.21), and (7.23)] with G_i [(7.12) and (7.13)] we see that the β decay coupling constants $\mathfrak{G}_i$ are very different from the μ decay coupling constants G_i. In this case the idea of universal Fermi interactions seems to be difficult to maintain.[51]

(vii) If β decay interaction is not invariant under time reversal then the limits on $\mathfrak{G}_i$ are those given by (7.20), (7.21), and the inequalities (7.17) and (7.18). In this case the β decay coupling constants may not be incompatible with the μ decay coupling constants.

In conclusion we wish to remark that to give a definitive statement on the universal Fermi interaction it is necessary to obtain a much sharper limit on $|\mathfrak{G}_V/\mathfrak{G}_S|^2$ than that in (7.17) and (7.18). Because of the nonconservation of parity this can now be obtained by a direct measurement on the helicities of $e^{\pm}$ from any β transition that has a large Fermi matrix element M_F [cf. (6.18)]. As we shall see in the next section, because of the nonconservation of parity it is also possible to measure the various $\mathcal{G}$ for μ^- capture processes.

3. μ^- Capture Process[54]

In this section we shall study in detail the capture of μ^- by nuclei,

$$\mu^- + p \rightarrow n + \nu . \tag{7.24}$$

The Hamiltonian for this process is given by

$$H_3 = \sum_i \mathcal{G}_i (\psi_n^\dagger O_i \psi_p)(\psi_\nu^\dagger O_i \psi_\mu) \tag{7.25}$$

where ψ_ν is the two-component neutrino field. We list the following results for capture of μ^- in hydrogen.

(i) The rate for process (7.24) in hydrogen is

$$(1/\tau)_{\text{cap}} = p_\nu^2 \xi / 2\pi^2 a^3 \tag{7.26}$$

where

$$\xi = |\mathcal{G}_S + \mathcal{G}_V|^2 + 3|\mathcal{G}_A + \mathcal{G}_T|^2 , \tag{7.27}$$

p_ν is the momentum of the neutrino and a is the Bohr radius of the μ^--mesic atom.

(ii) If in the capture process (7.24) the μ^- is completely polarized, the angular distribution of the neutron is of the form

$$1 + \alpha \cos \theta_1 \tag{7.28}$$

where

$$\theta_1 = \angle (\sigma_\mu, \mathbf{p}_n) \tag{7.29}$$

representing the angle between the spin of the μ^--meson and the momentum of the neutron. The asymmetry parameter α is given by

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[54] K. Huang, C.N. Yang, and T.D. Lee, *Phys. Rev.*, in press.

$$\alpha\xi = -|\mathcal{G}_S + \mathcal{G}_V|^2 + |\mathcal{G}_A + \mathcal{G}_T|^2 . \tag{7.30}$$

(iii) The transition probability $(1/\tau)_{\text{rad}}$ for the radiative capture process of μ^- in hydrogen,

$$\mu^- + p \to n + \nu + \gamma , \tag{7.31}$$

is

$$(1/\tau)_{\text{rad}} = \frac{e^2\eta}{6\pi\hbar c\xi}(1/\tau)_{\text{cap}} \tag{7.32}$$

where

$$\eta = |\mathcal{G}_S|^2 + |\mathcal{G}_V|^2 + 3|\mathcal{G}_A|^2 + 3|\mathcal{G}_T|^2 . \tag{7.33}$$

(iv) In process (7.31) the γ-ray is circularly polarized. The polarization parameter β may be defined as

$$\beta = \frac{N_R - N_L}{N_R + N_L} \tag{7.34}$$

where N_R and N_L are respectively the number of right-handed and left-handed γ-rays. The parameter β is given by

$$\beta\eta = |\mathcal{G}_S|^2 - |\mathcal{G}_V|^2 - 3|\mathcal{G}_A|^2 + 3|\mathcal{G}_T|^2 . \tag{7.35}$$

Equation (7.35) is independent of either the state of polarization of the captured μ^--meson or the energy of the γ-rays.

(v) For the radiative capture of a 100% polarized μ^- the angular distribution of the γ-ray is of the form

$$1 + \beta \cos\theta_2 \tag{7.36}$$

where β is given by Equation (7.34) and

$$\theta_2 = \angle\,(\boldsymbol{\sigma}_\mu, \mathbf{p}_\gamma) \tag{7.37}$$

where $\mathbf{p}_\gamma$ is the momentum of the γ-ray.

(vi) In the radiative capture process (7.31) the angular distribution of the γ-ray with respect to the momentum of the neutrino $\mathbf{p}_\nu$ is

$$1 + \gamma \cos\theta_3 \tag{7.38}$$

where

$$\theta_3 = \angle\,(\mathbf{p}_\nu, \mathbf{p}_\gamma)$$

and

$$\gamma\eta = -|\mathcal{G}_S|^2 + |\mathcal{G}_V|^2 - |\mathcal{G}_A|^2 + |\mathcal{G}_T|^2 . \tag{7.39}$$

In all the above expressions we neglect v/c terms for the nucleon wave function and we replace ψ_μ by its value at the origin.

(vii) In (7.24) we assume that the law of conservation of leptons is valid. Otherwise instead of (7.24) and (7.31) we may have

$$\mu^- + p \to n + \bar{\nu} \tag{7.40}$$

and

$$\mu^- + p \to n + \bar{\nu} + \gamma . \tag{7.41}$$

In the measurements of $(1/\tau)_{\text{cap}}$, $(1/\tau)_{\text{rad}}$, θ_1, θ_2, and θ_3 for reactions (7.40) and (7.41) the corresponding parameters ξ, η, α, β, γ are replaced by ξ' η', α', β', and γ' where

$$\xi'=\xi, \quad \eta'=\eta, \quad \alpha'=-\alpha, \quad \beta'=-\beta, \quad \text{and} \quad \gamma'=-\gamma. \tag{7.42}$$

(viii) If the μ^- capture process is invariant under time reversal then all $\mathcal{G}_i$ are real. The measurement of α, β, γ, η, ξ affords a complete determination of the four coupling constants $\mathcal{G}_V$, $\mathcal{G}_S$, $\mathcal{G}_A$, $\mathcal{G}_T$ plus a check on the validity of the conservation law of leptons. (It may also give a test on time reversal invariance.)

The above considerations can be extended to μ^- capture in heavy nuclei. However, the uncertainty in the nuclear matrix elements would make the corresponding formulas less definite.

VIII. TIME REVERSAL INVARIANCE AND MACH'S PRINCIPLE

From various recent measurements we know that P as well as C are not conserved, at least in some of the weak interactions There remains unanswered a very fundamental question which is whether $C \cdot P$ is invariant or not Or, by the CPT theorem we may ask whether T (time reversal) is invariant or not If it turns out that T may or may not be conserved, what will then be the implication? This leads us naturally to a discussion of Mach's principle

Should we follow the spirit of Mach's principle, we would believe that the laws of physics cannot depend on the geometrical coordinate system that we happen to choose There should exist no absolute system The present asymmetry may then be made compatible with this interpretation of Mach's principle in two ways

(i) If T is invariant, then $C \cdot P$ is invariant The right-left symmetry in space is retained by changing particle to antiparticle as we change from a right-handed system to a left-handed system[55]

(ii) If, experimentally, the weak interactions are found to be not invariant under T, the over-all symmetry may still be maintained by conjecturing the existence of two different kinds of elementary particles with the same masses, charges, and spins, but exhibiting opposite asymmetries under a space inversion In such a picture, the observed right-left asymmetry is ascribed not to a basic non-invariance under space inversion but to a cosmologically local preponderance of one kind of the elementary particles over the other kind Consequently, in this broader sense P is still conserved By the CPT theorem, all interactions are also invariant under $C \cdot T$ Thus, the time reversal symmetry can also be retained by changing particles to antiparticles as we reverse the chronological order of any sequence of events

If this is the case, then there must exist two types of protons, p_R and p_L, the right-handed one and the left-handed one At the present time, the protons in the laboratory must be predominantly of only one kind, say p_R, which accounts for the observed asymmetry and the observed Fermi-Dirac statistical characters of the protons This means that the free oscillation period between p_R and p_L must be longer than the age of the universe They could, therefore, both be regarded as stable particles It is reasonable to assume that there exists only one kind of electromagnetic field Thus, in an experiment on pair productions by γ radiation we expect the same cross sections for

$$\gamma \rightarrow p_R + \bar{p}_R \tag{8 1}$$

[55]C N Yang, International Congress on Theoretical Physics, Seattle, Sept 1956 [*Revs Mod Phys* **29**, 231 (1957)], T D Lee and C N Yang, *Phys Rev* **105**, 1671 (1957) This possibility was also independently considered and particularly emphasized by L Landau, *Nuclear Phys* **3**, 127 (1957), and by E P Wigner, *Bull Am Phys Soc* **2**, 36 (1957)

and for

$$\gamma \rightarrow p_L + \bar{p}_L \, . \tag{8.2}$$

The $\bar{p}_L$ would appear to be a stable negative particle with a mass equal to that of a proton. The detection of such particles, if produced, may not be difficult.