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GPS Estimation Algorithms for Precise Velocity, Slip and Race-track Position Measurements

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ABSTRACT

This paper investigates the use of carrier-phase differential GPS (CDGPS) for race car applications. In particular, experimental results are presented to demonstrate the use of CDGPS to accurately measure several key parameters of a test vehicle, including the inertial velocity, side-slip, and its precise location. This data is useful as a driver's coaching tool because it can be used to determine what the driver is doing and when, and also show precisely where these actions are being performed on the track. While CDGPS offers the potential of very precise position estimation, even a temporary blockage to the NAVSTAR constellation (*e.g.,* by trees/bridges) means that the measurement biases must be re-acquired before a good solution can be obtained. Various solutions to this problem have been investigated, but each presents new difficulties and/or requires more expensive equipment. However, this paper demonstrates that tracking the GPS Doppler frequency information provides a precise measure of the vehicle velocity that can be integrated to obtain very precise position estimates without having to solve for the CDGPS biases. The approach does, however, require additional external infrastructure to initialize the velocity integration.

INTRODUCTION

The Global Positioning System has been fully active for world-wide, all weather navigation since the early 1990's. A single L1 (1575.42 MHz, wavelength of the carrier signal is ≈19 cm) receiver is capable of tracking GPS code phase and determining the absolute position to within 1–5 m. Greater precision can be achieved using code (DCP) or carrier phase measurements (CDGPS) and differencing with a base station receiver to estimate relative position. Relative position estimation using differential code is accurate to approximately 20–50 cm and differential carrier is accurate to approximately 2–5 cm. While more accurate, CDGPS introduces the additional difficulty that an unknown number of integer value wavelengths must be removed from each GPS measurement. Researchers have suggested many methods for this bias estimation, both for post-processed and real-time kinematic (RTK) applications. A key point in this work is that the relative velocities are much easier to estimate because there is no associated biases to determine, thereby eliminating potential error introduced by the bias estimation algorithms. These accurate GPS Doppler values (mean and standard deviation of approximately 1–2 mm/s) suggest a new and relatively inexpensive measurement method that can be used to obtain position, velocity, and side slip data for both drivers and manufacturers.

The largest benefit for this work would be a new way to measure and present coaching information to a race car driver. By accurately locating the vehicle on the track, a comparison of the car state can be performed as a function of the location on the track rather than focusing on time information. For less experienced drivers, lap times will not be equivalent, so comparing the car state as a function of time provides an inaccurate means of comparison. However, knowing the location of each action taken would provide a systematic means of providing feedback to student drivers to show *where* to make a particular correction, as opposed to just *when*.

Additionally, methods are being explored for using a more robust scheme of Doppler measurements, by reducing the reliance on continuous communication to the base station receiver. Researchers have already investigated using the absolute velocity vector measurement available from relatively inexpensive GPS receivers to estimate aircraft attitude while in flight [7]. The innovation described in this paper is to investigate the integration of this accurate velocity measurement to estimate *absolute position* of the vehicle. While this method has a bias due to the uncertainty in the initial position, an external instigator (*i.e.,* the start/finish line) could be used to initialize the position integration. Other researchers have attempted speed and heading integration with encoders and lateral accelerometers [8, 14], however unmodeled biases in these measurements typically result in **significant** (*i.e.,* on the order of 50–100 m) errors after a lap, even though they provide an accurate estimate of the lap-times. The new method presented in this paper is similar, but we will experimentally demonstrate that GPS velocity estimate is of a much greater precision and accuracy.

This paper presents a series of experimental results conducted at multiple test locations. Local driving data was taken in order to validate procedures to be used at actual test facilities. CDGPS position, velocity, and attitude estimation were completed at the New Hampshire Inter-

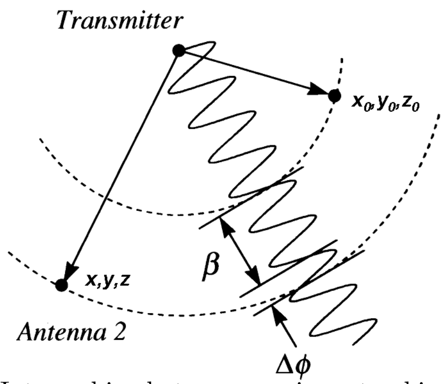

Fig. 1: Integer bias between receivers tracking the same transmitter signal

national Speedway (NHIS) while at the Skip Barber Racing School. With the GPS equipment mounted to a single vehicle, multiple drivers were recorded and compared. Other tests show the capability of track re-construction from CDGPS and integrating the precise relative velocity measurements. Test results are presented to show that the expected errors due to bias integration still exist when using the Doppler measurement. However, significant distances can be driven (≈500 m) while only incurring a 10–20 cm positioning error without any of the difficulties associated with determining the measurement biases.

MEASUREMENTS

A standard, inexpensive (∼ \$200–400) GPS receiver will track the code signal of the NAVSTAR constellation at the nominal L1 frequency. The transmission time from the known satellite locations provides a pseudorange measurement that is a function of the receiver location, the clock biases of the satellite and of the tracking receiver, and atmospheric delay errors. The actual measurement provided by the GPS receiver is an integrated phase from the first epoch when the receiver began tracking the signal. The phase measurement from the i^{th} GPS satellite is expressed as

$$\begin{aligned}\phi^i \;=\; & \frac{1}{\lambda}\sqrt{(x^i-x)^2+(y^i-y)^2+(z^i-z)^2} \\ & +\frac{c}{\lambda}(\tau^S-\tau_R)-\beta_0+\epsilon_{\text{atmospheric}}+\nu\end{aligned} \tag{1}$$

where:

x^i, y^i, z^i	= Position coordinates of i^{th} GPS satellite
x, y, z	= Position coordinates of the receiver
c	= Speed of light
τ^S	= Clock bias of the GPS satellite
τ_R	= Clock bias of the receiver
β_0	= Integer number of wavelengths between GPS satellite and user at start-up
ϵ	= Pseudorange transmission delays due to atmospheric disturbances
ν	= Noise on the carrier phase measurement

A single difference of phase measurements taken on two receivers can be used to eliminate common mode errors (*e.g.,* satellite clock bias and atmospheric delays). The difference $\Delta\phi^i$ provides a measure of the relative range between the two receivers and can be written as a function of the relative position $(\Delta x, \Delta y, \Delta z)$ and relative receiver clock biases $(\Delta\tau_R)$. The single difference equation is

$$\Delta\phi^i = \frac{1}{\lambda}\sqrt{\Delta x^2+\Delta y^2+\Delta z^2}+\frac{1}{\lambda}\Delta\tau_R+\beta^i+\nu \tag{2}$$

Much work has been done to estimate the measurement integer ambiguity, β, shown in Figure 1 (see for example Ref. [9] and the references therein). If the relative position is known exactly, the biases can be determined immediately, but any error introduced by the initial position will cause a slow drift of the position estimation error due to the motion of the GPS constellation. One method to find the ambiguity precisely is to keep all surveying antennas stationary and record the raw tracking data. During post-processing, the ambiguity could be solved by finding the integer combination that best matches all of the measurements. However, a time delay of 20–40 minutes might be required in order to obtain a sufficient rotation in the line-of-sight vectors to the NAVSTAR satellites. Other methods used for surveying applications are to rotate the antennas on a known baseline or simply switch locations of the base and remote receivers [13]. Since most dynamic applications require faster start-up, instantaneous on-the-fly (OTF) procedures have also been developed.

Two means for OTF initialization are augmenting the GPS system with a local ranging device or having the receiver track the L2 GPS frequency. Note that both methods typically result in an increase in the equipment costs. Pseudolites that generate exact replicas of the GPS code can be mounted around the area of use, thereby giving local range measurements without having to add any additional sensing hardware [2, 11]. But this method requires access to the receiver tracking algorithm so that the receiver knows that another satellite exists locally. Quick changes in the local line-of-sight to the nearby pseudolite can be used to rapidly determine the integer biases. However, adjustments must be made for the nonlinear effects of the local range measurements, if the initialization path is to be sufficiently short for quick resolution [1].

Dual frequency receivers that track L2 at 1227.60MHz as well as L1 can use *widelaning* that produce RTK relative position solutions. By tracking L1 and L2, the phase of the beat frequency (347.82MHz) can be measured. This new measurement has a wavelength of ≈ 86 cm, which significantly decreases the required integer search space, thus the ambiguity can be solved almost instantaneously [12]. While this method proves to be quick and robust, it typically requires a significantly more expensive GPS receiver.

The Doppler measurement used in this experiment is the instantaneous state in the correlator tracking loop filter. The accurate range rate is a function of the difference in absolute velocities between the receiver and GPS satellite. Again, differential measurements can be used to eliminate common mode errors, giving

$$\Delta\dot{\phi}^i = los_1^i \bullet \dot{\mathbf{x}}_1 - los_2^i \bullet \dot{\mathbf{x}}_2 + \frac{1}{\lambda}\Delta\dot{\tau}_R + \nu_d \tag{3}$$

where $\dot{\mathbf{x}}_j$ is the absolute velocity of the j^{th} receiver, and $\Delta\dot{\tau}_R$ is the relative clock drift between receivers and los_j^i is the unit line-of-sight vector from the j^{th} receiver to the i^{th} tracked GPS satellite. It is important to note that this velocity measurement does not have the equivalent of the constant bias terms, β^i, in Eq. 2.

With far transmitters, a planar wavefront assumption can be made, as shown in Figure 2. In this case, the line-of-sight vectors can be assumed parallel and the nonlinear Eqs. 2 and 3 simplify to

$$\Delta\phi^i = los^{i^T} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \frac{1}{\lambda}\Delta\tau + \hat{\beta}^i \tag{4}$$

$$\Delta\dot{\phi}^i = los^{i^T} \begin{bmatrix} \Delta\dot{x} \\ \Delta\dot{y} \\ \Delta\dot{z} \end{bmatrix} + \frac{1}{\lambda}\Delta\dot{\tau} \tag{5}$$

where $\hat{\beta}$ is the estimated value of the integer bias. With at least four tracked signals, a WLS solution can be obtained of the following equation

$$\begin{bmatrix} \Delta\Phi \\ \Delta\dot{\Phi} \end{bmatrix} = \begin{bmatrix} G & \mathbf{1} & 0 & 0 \\ 0 & 0 & G & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \hat{\beta} \\ \mathbf{0} \end{bmatrix} + \nu \tag{6}$$

where $\Delta\Phi$ and $\Delta\dot{\Phi}$ are the stacked measurements using the N satellites common to the two receivers. Similarly, G is a stack of los vectors for each satellite. The vectors $\mathbf{x}$ and $\dot{\mathbf{x}}$ are the relative position/clock bias $(\Delta x, \Delta y, \Delta z, \Delta\tau/\lambda)$ and relative velocity/clock drift $(\Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}, \Delta\dot{\tau}/\lambda)$, respectively. Given an estimate of the biases, the resulting WLS solution is

$$\begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} = (H^T R^{-1} H)^{-1} H^T R^{-1} \begin{bmatrix} \Delta\Phi \\ \Delta\dot{\Phi} \end{bmatrix} \tag{7}$$

where H is the measurement matrix in Eq. 6.

HEADING MEASUREMENT – CDGPS can also be used to measure the vehicle attitude in an inertial frame. The relative position of two antennas separated by a known baseline in the body frame can be measured by CDGPS to obtain a measure of the rotation of the vehicle with respect to the inertial frame [3]. Note that differencing across separate receivers can introduce errors in the time synchro-

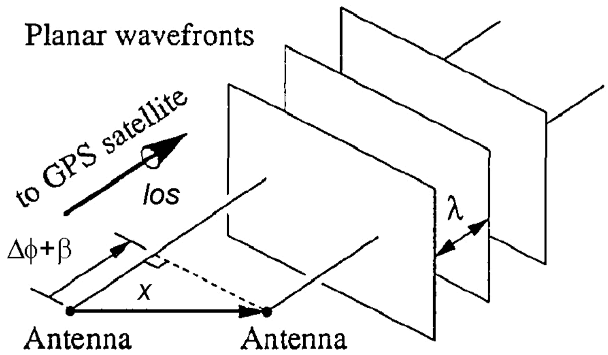

Fig. 2: CDGPS measurements and integer biases assuming a planar incoming wave-front.

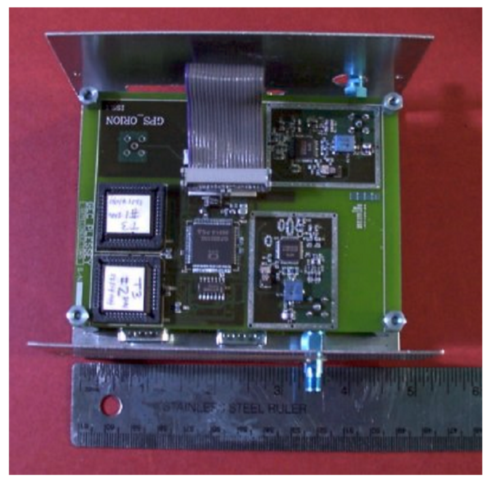

Fig. 3: Dual RF, single correlator/processor GPS receiver used for the CDGPS positioning and attitude experiments.

nization of measured data, which can corrupt the attitude accuracy. However, for our experiments, a single receiver with dual RF front-ends was used (see Figure 3) so the same clock was available to the processor and correlator. In this case the single difference eliminates the common mode clock error between all of the measurements.

Similar to the relative positioning Eq. 2, there is a carrier phase bias term in the attitude single difference. But a key advantage of the attitude estimation problem is that the length of the baseline is fixed, which significantly limits the possible extent of the integer search space. In this case, batch algorithms using multiple measurements as the baseline is rotated normal to vector direction [13] can resolve the ambiguity or it is possible to do an integer combination search [4]. In this experiment, it was assumed that the initial acceleration of the vehicle would be in a straight line with no induced side slip. Once a velocity threshold was reached the equation

$$\hat{\psi}_0 = \arctan\left(\frac{\dot{y_0}}{\dot{x_0}}\right) \tag{8}$$

was solved to estimate the heading direction. With this heading estimate available, we could then solve for the attitude biases, which were then fixed for the remaining measurements. All future heading measurements were found by solving Wahba's problem [10] of determining the rotation matrix necessary to match the vector measured in two separate frames. Since there is a single antenna pair, there is an ambiguity of the vehicle orientation about the baseline. However, the race-car used for the tests had a very stiff suspension, so the car roll motion was minimal. Note that the vertical measurements of position and velocity are the worst of the Cartesian directions, so the initialized pitch information of the vehicle was not accurate enough to provide useful results.

SIDE SLIP – The angular difference between the absolute heading vector and the direction of the velocity vector can be used to determine the side slip, α, of the test vehicle

$$\alpha(t) = \psi_{\text{vel}}(t) - \psi(t) \tag{9}$$

where $\psi_{\text{vel}}(t)$ is the result of Eq. 8 using the instantaneous values of the velocity and $\psi(t)$ is the yaw solution to Wahba's problem. This provides a very good measure of side slip because it does not rely on inertial sensors that typically have significant biases. Also note that these quantities can be obtained without having to solve for any measurement biases or increasing the noise by differentiating measurements [5].

TRACK CONSTRUCTION – For the tests using CDGPS, an estimate of the initial position was used to set the measurement biases. This initialization approach only works for the current and continuously tracked satellites. Thus no carrier phase measurements could be used following an occlusion and re-acquisition of the NAVSTAR satellites (since the carrier biases would then not be known). Pseudolites and other local ranging devices could be used to perform a bias estimation, but full track coverage with these devices is typically not possible. Also, with limited coverage and fast moving vehicles, the visibility time will be very short. Experience at NHIS showed that this time is typically too short for the receiver to determine that a new measurement is available and accurately track it before the car leaves the field-of-view.

A novel method of velocity integration proves to be more useful because, when a NAVSTAR signal is re-acquired following a loss, it can be included in the position estimation. Note that integrating measurements from sensors such as velocity encoders and lateral accelerometers has been used by other researchers to construct race track geometries [8]. However, these sensors typically have a significant measurement bias and cannot observe phenomena such as tire slip, vertical undulations of the track, and roll of the car because of banked roadway. When these biased measurements are integrated, the position estimate quickly grows to large errors on the order of 50 m [8]. In order to

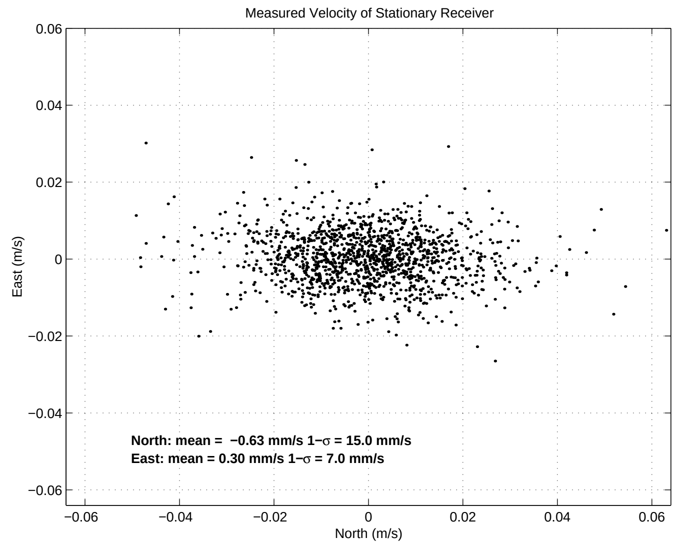

Fig. 4: DGPS Velocity Measurements of a Stationary Antenna

match the initial and final points of the integration, usually at the start/finish line, stretching and rotating errors are back propagated throughout the measured data, thereby introducing significant position errors around the track. For the GPS velocity integration method, measurement errors (biases and standard deviation) are quite small (see Figure 4), so the integration can be performed for much longer periods of time. Also, since it is measuring the true velocity of the vehicle, errors in side-slip or car modeling are completely eliminated. There is still the need to provide a single point (or for greater accuracy, multiple points around the track) to initialize the integration scheme. Of course, breaking the start/finish line is an obvious choice, but the lateral position on the track would also need to be known (as it would be needed with any approach of this type). Ref. [16] investigates using a pair of lasers to measure the exact location and time of the car as it passes certain points on the track.

Of course, to further improve robustness of position and velocity information, complementary inertial sensors could be included on the vehicle. During times of full signal loss, the INS measurements can continue the estimator propagation. Rather than simply integrating the biased measurements, the errors of the INS can be calculated when both GPS and INS are available. The corrected measurements will then be of much greater accuracy when running solely with INS. When the receiver resumes tracking satellites, the GPS integration can again take over.

Another possible improvement to add robustness to the velocity estimation would be to model the satellite clock drift locally and receive periodic updates from a base station receiver. The primary use of the base station is to difference out the satellite velocities and common mode errors (satellite clock and atmosphere) between the two receivers. The GPS data message includes ephemeris information of the satellite orbital parameters and expected

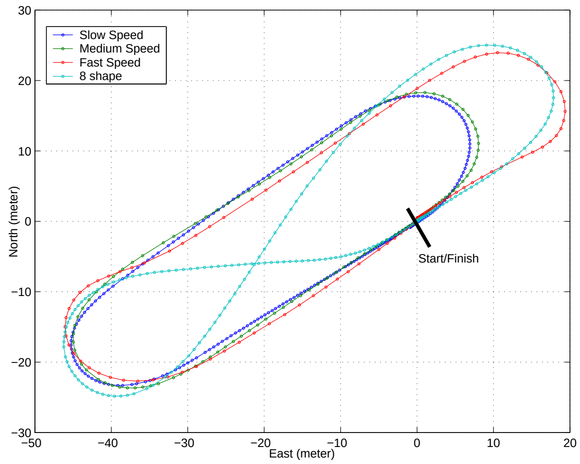

Fig. 5: Track patterns from first CDGPS experiments

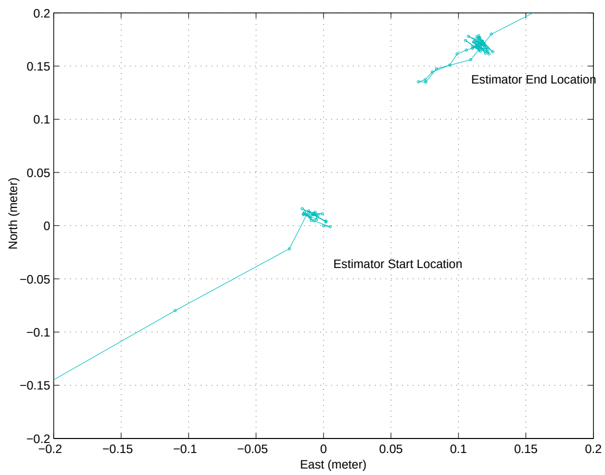

Fig. 6: Accuracy of start/end of CDGPS experiments

clock errors. Thus rather than differencing two measurements to eliminate the clock terms, it might be possible to estimate the expected clock bias/drift values for a short period of time (*e.g.,* several seconds), and then adjust the actual measurements received by the unit on the car. This would add robustness to the velocity estimation because continuous communication with the base station would not be necessary – only low frequency updates of the satellite clock biases would be required. Weighted least-squares absolute velocity measurements have noise values of 3–5 cm/s, which are too large, but with a properly formulated Kalman filter and periodic clock error updates, it might be possible to reduce these noise values. While this will increase the velocity error, it would significantly decrease the operational and computational costs while creating a more robust system overall.

EXPERIMENTAL RESULTS

LOCAL TRACK WITH CDGPS – The first tests consisted of using the carrier phase measurement for position and velocity estimation. A GPS antenna pair was mounted on a BMW M-3 so that the relative position/velocity and absolute attitude could be estimated. A modified Zarlink dual

Fig. 7: Antenna locations and body frame vector.

RF receiver (figure 3) was used as the unit on the car, with a standard Orion Zarlink receiver used for the base station. Raw carrier phase and Doppler measurements were transmitted at 10Hz to a processing computer to run a real-time WLS point solution estimation. The vehicle started from rest and the measurement biases were set by initializing the estimator with a known position. After the maneuver, the driver attempted to return the vehicle to the original location with cones as visual guides for driving lane and longitudinal position. With these inaccurate placement tools, the error in returning the car to the starting position is included in the estimate of the final position.

Figure 5 shows the multiple paths that were traversed on the top-level of a parking garage. Note that the separation of the data points provides a coarse measure of the speeds used in the various tests and that the driver was not attempting to follow a specific course. The primary focus was actual track construction and start/finish error. The slip angle could also be measured because the attitude was being estimated, however the speeds were too slow to induce significant differences between the attitude and velocity vectors.

Figure 6 shows a closeup of the data for a single run. The accuracy of the WLS solution is shown by the scatter data at the beginning and end of the run. The CDGPS estimation of the start/finish location shows a 20 cm displacement of the car's final position. These tests showed the repeatability and accuracy achievable with CDGPS when full and continuous visibility condition exist.

NEW HAMPSHIRE INTERNATIONAL SPEEDWAY – The experimental results presented were taken during a session of the Skip Barber Racing School, at the New Hampshire International Speedway. The test equipment consisted of the same dual RF GPS receivers with antennas and a Data-Linc Group SRM6000 radio modem. The two GPS antennas necessary for attitude measurement were mounted on top of the race car rollbar and on the nose cone of the vehicle as shown in Figure 7. The black arrow shows the full body frame vector used to solve Wahba's vector rotation problem. The raw GPS measurements from the receiver were transmitted in real-time to the base station computer that ran WLS position/velocity and attitude algorithms.

An initial data set was acquired with a slow moving vehicle to determine the shape and layout of the test track. Figure 8 shows the driving line created by an instructor from

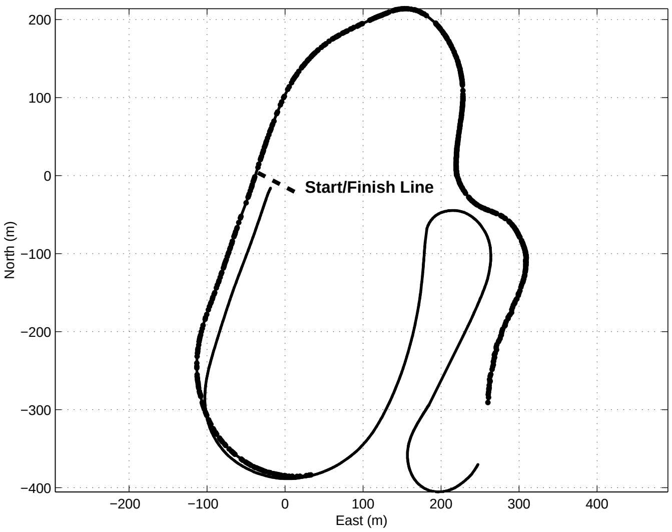

Fig. 8: CDGPS Position of vehicle traveling the expected driving line

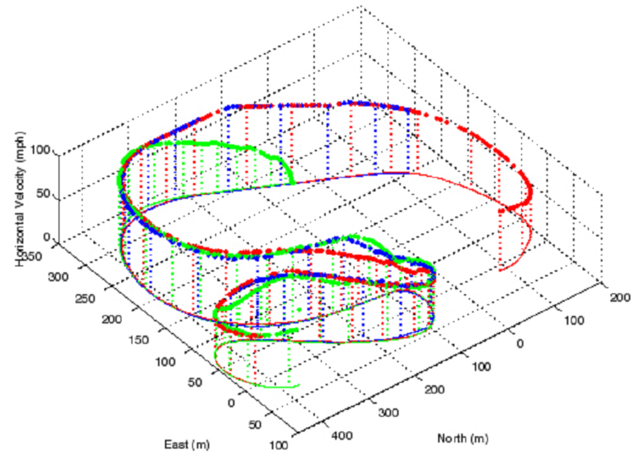

Fig. 9: Velocity comparison of three test runs of the GPS

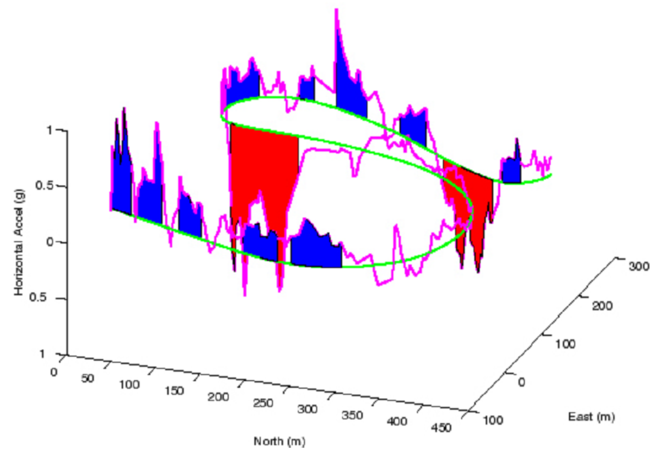

Fig. 10: Braking profile: Red indicates acceleration; Blue for braking

the racing school. The lateral difference at the start/finish line was a result of starting the estimation while in the pit lane but continuing a second time on the actual track. Data drop-outs are visible for a short period of time in the lower right corner of the plot. Unfortunately, there is a large tree on the inside of the track coupled with a small grove of trees on the outside that occluded a large number of the GPS signals in that part of the track. The local ranging pseudolite could not be used in that area because it could not be simultaneously visible to both the remote and base station receivers. Also, the fast moving car would only have short term visibility and would not guarantee tracking of the signal.

With the standard driving line created, all runs by the student drivers could be compared to show where they deviated from the "proper" path. Also, the position information is important because it should provide an easier way to interpret where driving performance can be enhanced by improving actions (*e.g.,* throttle and braking) at specific locations. Figure 9 shows the same track position layout in the x, y coordinates with the vertical component corresponding to the vehicle velocity. The velocity is no longer a function of time, which is standard for most driver teaching software, but rather a function of the track position. This provides a unique perspective in that comparison of three separate instances are superimposed on the track. It is interesting to note the velocity each driver uses while entering the hairpin turn #3. The "green" driver entered with a high speed, but had to brake heavily to safely maneuver the curve. On the other hand, the "blue" driver took too conservative an approach to the turn, slowing down too gradually well before the turn. A well trained driver would have comparable results from one test lap to another, but for novice drivers the velocity differences would be significant enough to identify the areas of potential improvement. Of course, a key problem is to determine effective ways to present this data to novice drivers in such a way that it can be easily understood and used during the next practice session. The information could be enhanced by coupling the state estimate with driver input (*e.g.,* throttle position, steering angle) for a better feel of position control rather than attempting to associate all measurements and training with time.

Given the accuracy of the velocity data, it is also possible to differentiate and filter the velocity data to interpret the acceleration and braking profiles of a driver traversing the track. The positive accelerations at the beginning of the track are representative of the car starting from rest and accelerating with short decreases due to gear shifting. Improvements could be suggested on how to get the most power out of the vehicle when starting from a stopped position. The other interesting aspect of Figure 10 is the two peaks in deceleration as the driver was entering the hair pin turn. Hitting the throttle during gear shifting with the braking foot has clearly interfered with the braking process entering this turn. A better profile would have a steadier deceleration and acceleration entering and exiting

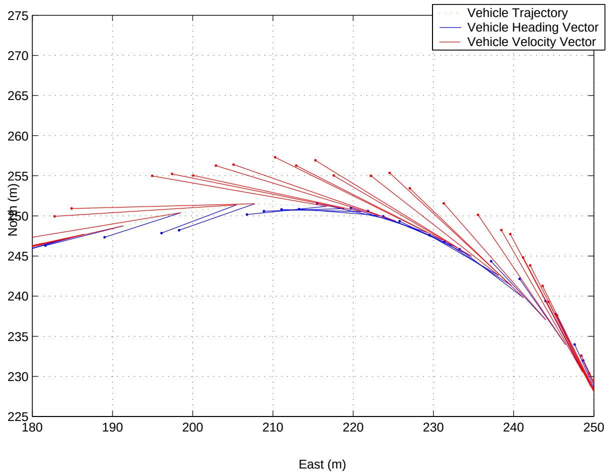

Fig. 11: Track position with corresponding velocity and heading vectors for a very localized region rather than the entire track.

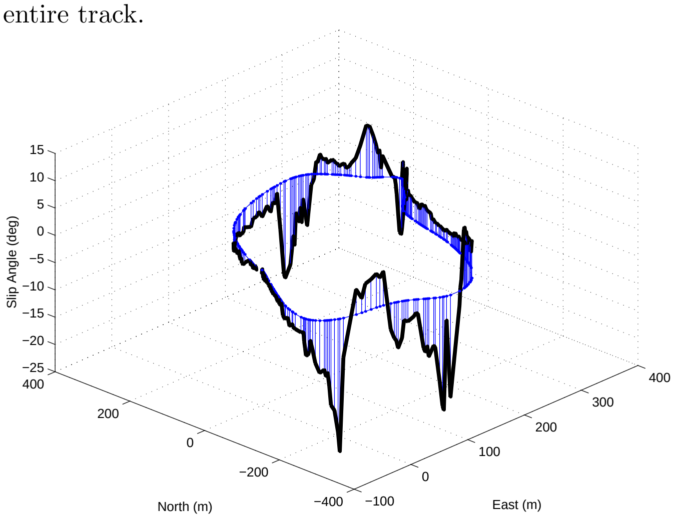

Fig. 12: The measured side-slip of the test vehicle from only GPS

each turn, respectively. This data could be used to replace or complement any onboard accelerometer measurements, which would be useful since a typical accelerometer has biases due to car pitch and the grade of the road.

Finally, some tests were conducted while traversing a limited portion of the track to avoid the occlusion caused by the trees in the south-eastern corner. A professional driver drove the car equipped with GPS receivers to measure the side slip of the vehicle. Figure 11 shows a short set of the results in two-dimensional plot where at each position we indicate the instantaneous heading vector as well as the instantaneous velocity vector. Figure 12 shows the calculated side slip as the vertical dimension on a 3D plot. As shown, during some very tight turns, the instructor pushed the slip angle to 20°–24°. Regular cornering typically resulted in side slip values of 5°–10°, with near zero slip measured on straight stretches of track.

The results of the experiments at the NHIS showed the repeatability of using CDGPS to precisely measure the position and velocity of a vehicle. Experiments also showed the viability of measuring car side-slip when an attitude ready receiver is available. However, the results also confirmed the previously discussed problem with CDGPS when there are occlusions that do not allow continuous visibility to the GPS signals. These results showed the importance of having other methods, such as widelaning [15], to resolve the integer ambiguity problem, otherwise it would be impossible to obtain continuous CDGPS position data around the track.

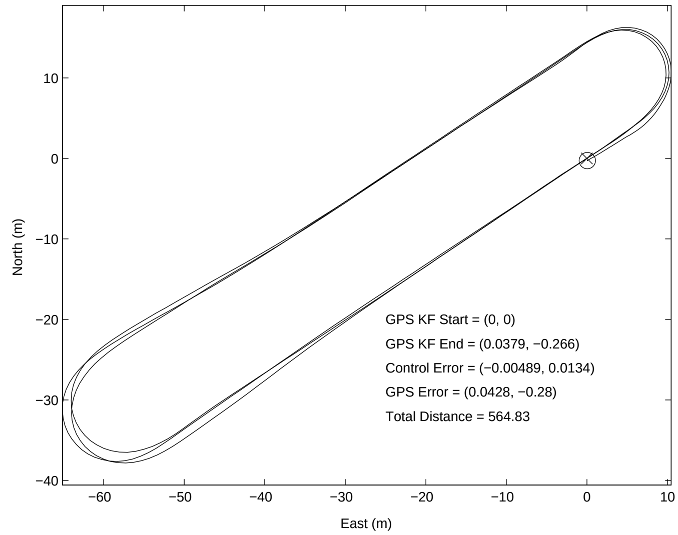

Fig. 13: Integrated GPS velocity from differential Doppler measurements

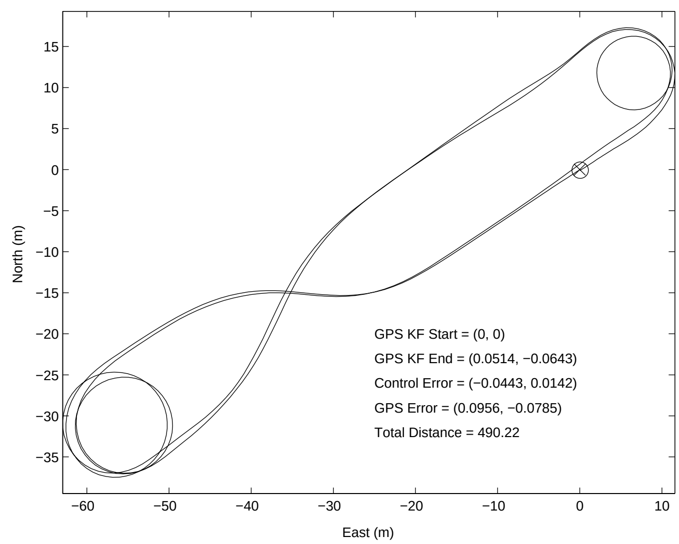

Fig. 14: Integrated GPS velocity from differential Doppler measurements

LOCAL TRACK WITH INTEGRATED VELOCITY – More recent testing has been conducted of the GPS velocity integration scheme introduced in the Track Construction section. A SuperStar^{TM} GPS receiver from CMC Electronics was used for differential Doppler GPS. The average rate of change of the Integrated Carrier Phase was used from the receiver output tracking data at a 5 Hz sample rate. A single antenna was attached to a car, so no side-slip measurements were available. The primary focus was to determine whether the GPS velocity vector could be integrated over large distances and still provide precise position information. Similar to the first CDGPS experiments, the vehicle was started and returned to a known location so that position values could be compared.

The first run is presented in Figure 13 where the X indicates the starting location and the O shows the final position. Three laps were completed around the top of a parking garage at approximately 15-20 mph. The final errors in the actual forward position were partially removed by having a laser point to a specific location on the vehicle both at the start and end of the maneuver. The lateral position and heading errors were measured and recorded so that they could be compared with the estimated values. The total time for the three laps was 140 seconds with the final position accuracy showing centimeter-level precision. The GPS Error in Figure 13 is the difference between the final position estimate and the actual antenna displacement measured after the maneuver. While not as precise as the CDGPS approach, this demonstrates that integrating the accurate velocity data can be used to obtain very good position estimates.

A second test was conducted where an arbitrary path was followed that included periods of zero velocity in order to investigate if there was any significant drift due to time. The random maneuvers is shown in Figure 14 with an elapsed time of 134 seconds, still the GPS estimation error was less than 10 cm in both the north and east directions. The total estimation error divided by the total time of each maneuver indicates that the position drifts are 0.7–2.0mm per second of integration. While these values are relatively small, they are slightly larger than expected from the mean error of the velocity estimate shown in Figure 4, and the reasons for this increase are under investigation.

CONCLUSIONS

This paper investigates using GPS Doppler measurements to produce an accurate (low bias) indication of the absolute velocity of a test vehicle that can be integrated to obtain relatively precise position estimates without the difficulty of having to resolve the carrier phase biases. CDGPS will have better position accuracy than integrated velocity, but continuous estimation is very difficult at a track because objects often block the GPS signals. Whichever method is used, the accuracy of GPS far exceeds the capabilities of inertial sensors. With a system that is more accurate in both position and velocity, direct comparison is possible between multiple runs of a test vehicle. Knowledge of the entire state of the vehicle as well as location can be used by coaches to improve a student's driving performance.

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