Part II: Models

Nicola Conci
nicola.conci@unitn.it

Why do we need models?

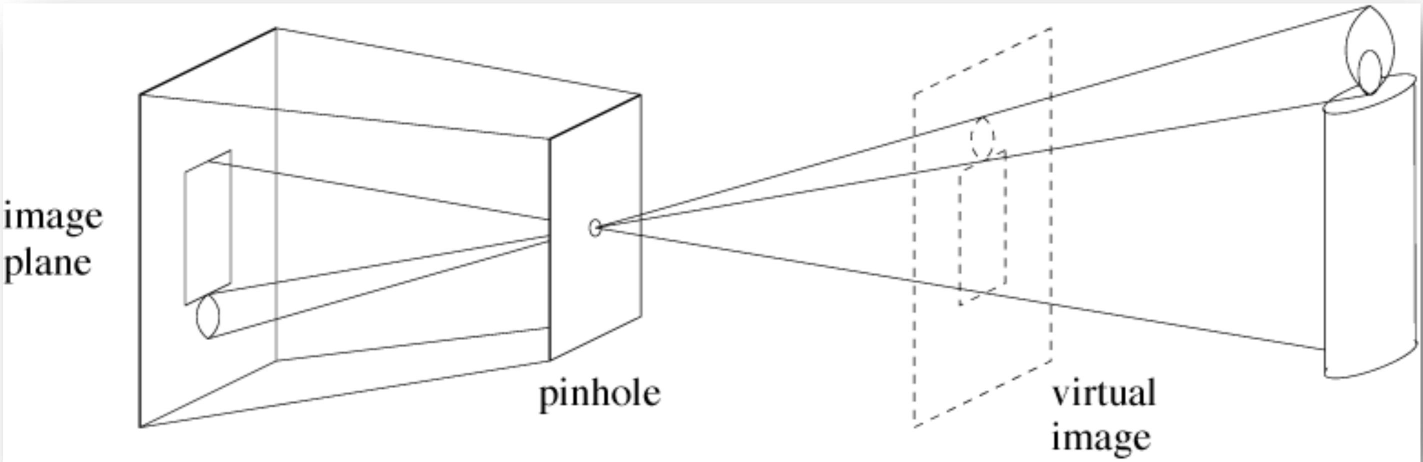
- They represent a good approximation of the real world

- They allow describing events through a parametric representation

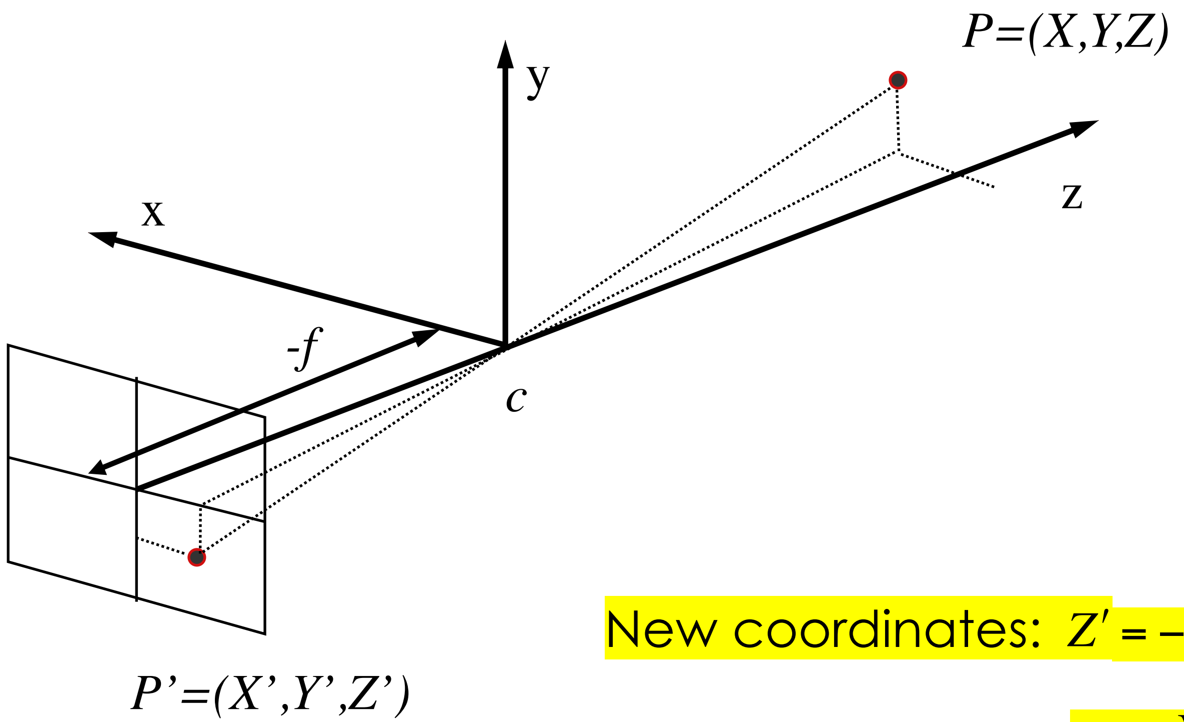
- Parameters can be extracted and used for processing

The pinhole camera model

- It's an abstract model
- It's the most common model



The pinhole camera model



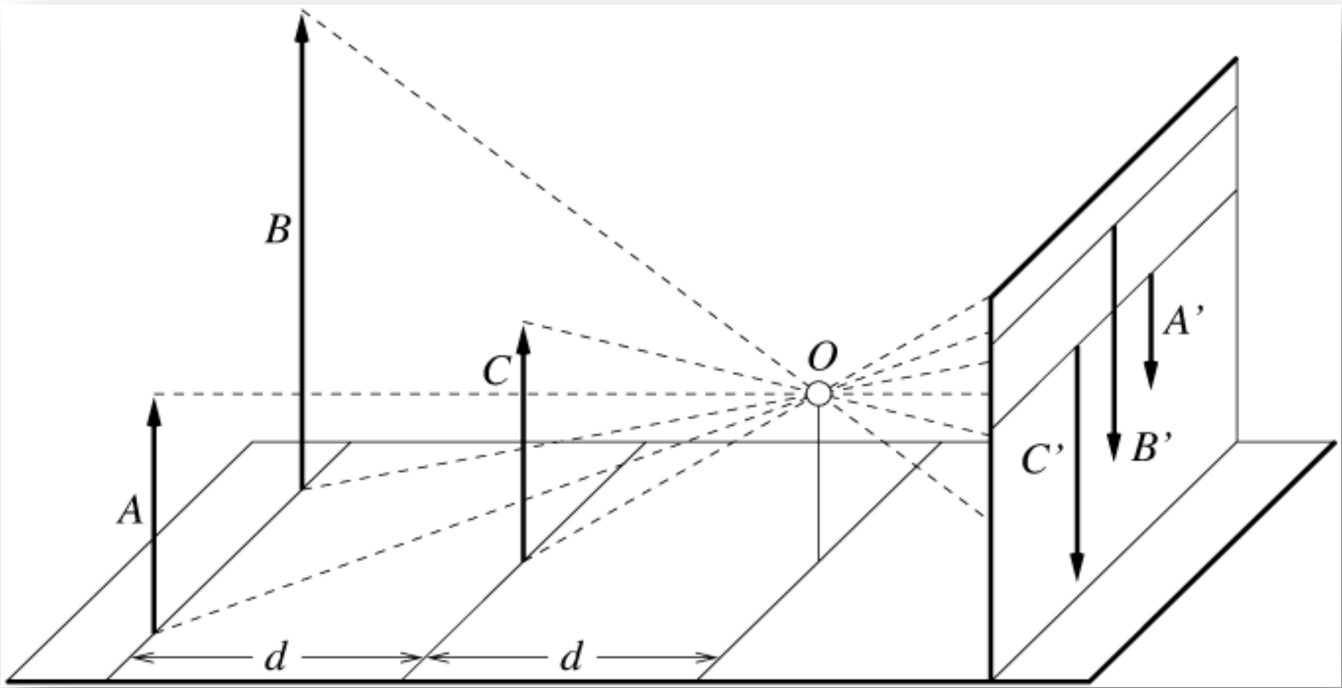
New coordinates: $Z' = -f, \quad X' = -f\,\frac{X}{Z}, \quad Y' = -f\,\frac{Y}{Z}$

$x = -X', \quad y = -Y'$

From the camera: $(X,Y,Z) \rightarrow (x,y,f) = (f\,\frac{X}{Z}, f\,\frac{Y}{Z}, f)$

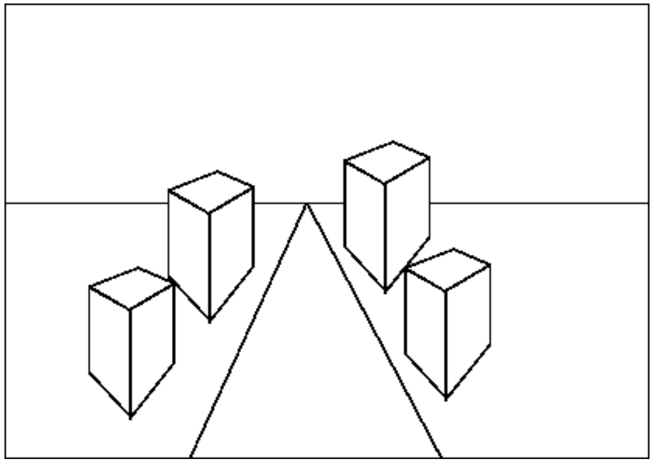
Features of the pinhole model

- If the object is far, it appears smaller



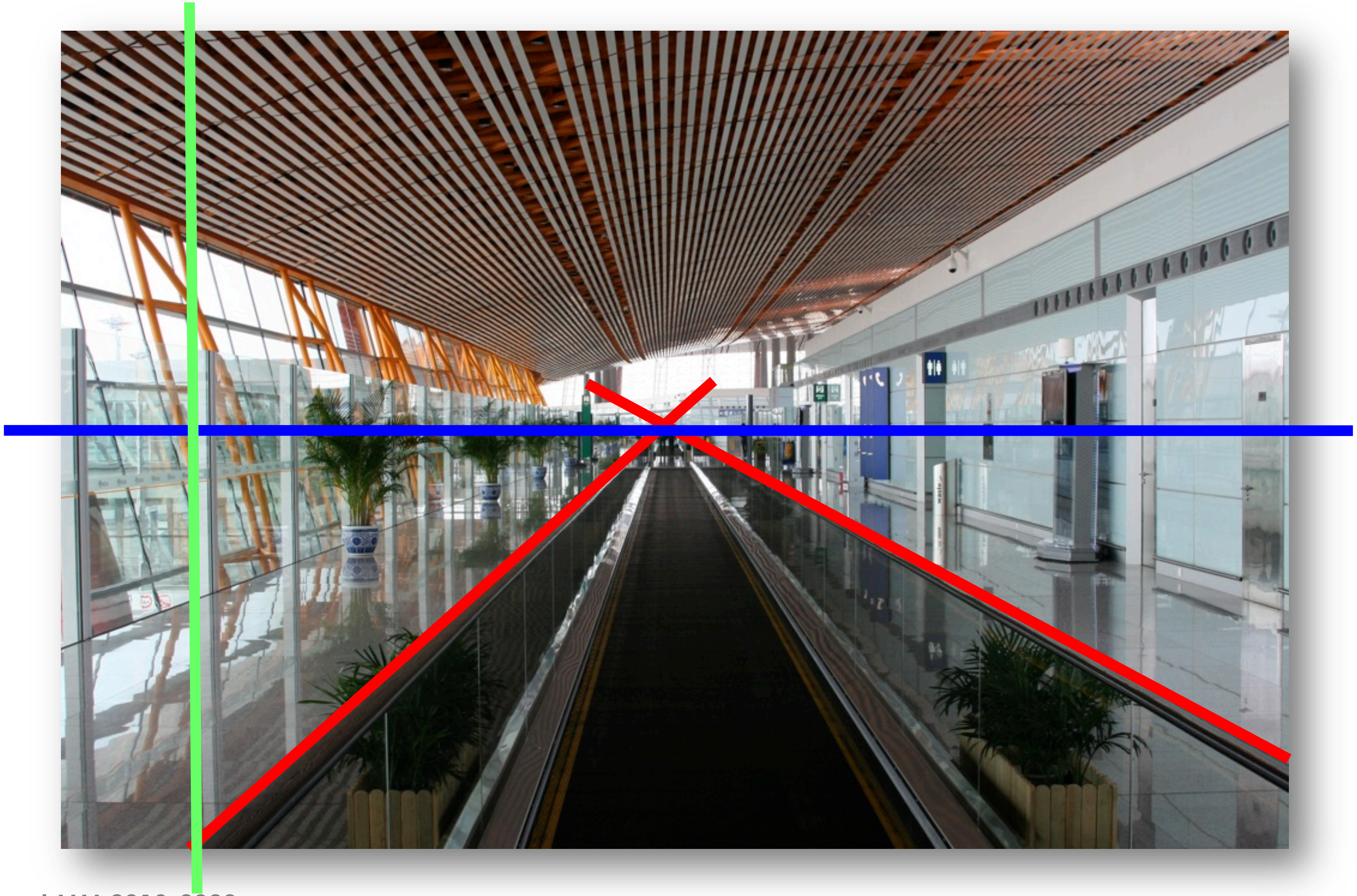
Features of the pinhole model

- Parallel lines converge to a single point
- Parallel lines on the same plane lead to *collinear* vanishing points.
- The line is called the *horizon* for that plane
- Vertical lines are perpendicular to the horizon



Vanishing Points and Horizon

Vanishing Points and Horizon



Projections

- Capturing devices record 2D projections of 3D (+ time) scenes:

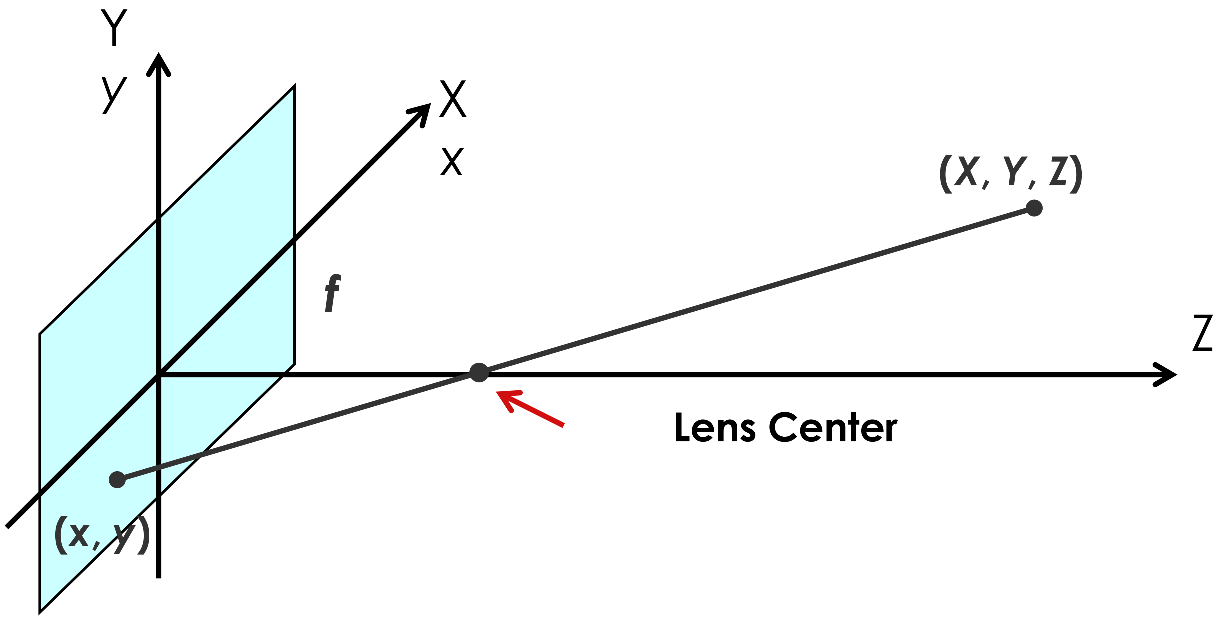
$$f : \Re^4 \rightarrow \Re^3$$

$$(X, Y, Z, t) \rightarrow (x, y, t)$$

- (X, Y, Z), (x, y), and t, are continuous variables.
- Two types of projection:
    - Perspective (central)
    - Ortographic (parallel)

Perspective projection

- Using the pinhole model, all rays pass through the center of projection, which corresponds to the lens.
- In the picture, the center is between the object and the image plane
- The image plane corresponds to the (X, Y) plane of the 3-D space.



Perspective projection

- From the pairs of triangles:

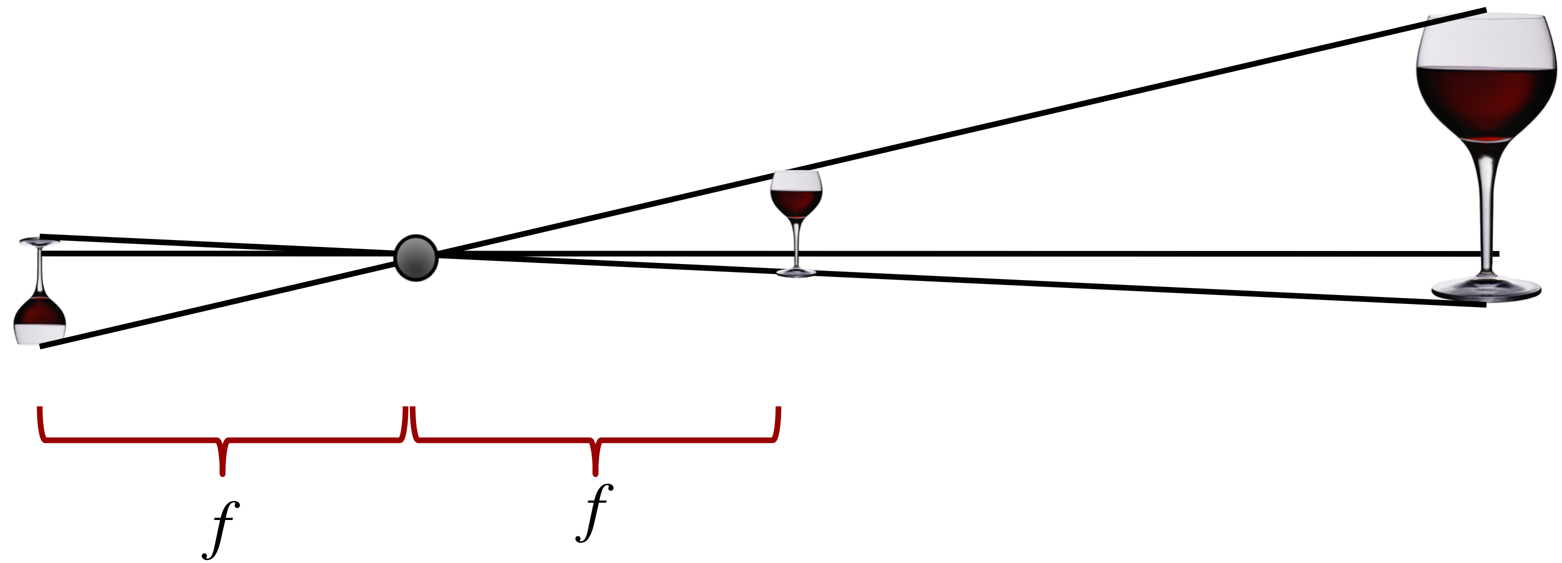
$$(x,0,0),(0,0,f),(0,0,0); \quad \text{and} \quad (X,0,Z),(0,0,f),(0,0,Z)$$
$$(y,0,0),(0,0,f),(0,0,0); \quad \text{and} \quad (Y,0,Z),(0,0,f),(0,0,Z)$$

- We obtain the following formulation:

$$\frac{x}{f} = \frac{X}{Z-f} \quad \text{and} \quad \frac{y}{f} = \frac{Y}{Z-f} \Rightarrow$$
$$\Rightarrow x = \frac{fX}{Z-f} \quad \text{and} \quad y = \frac{fY}{Z-f}$$

Perspective projection and the pinhole model

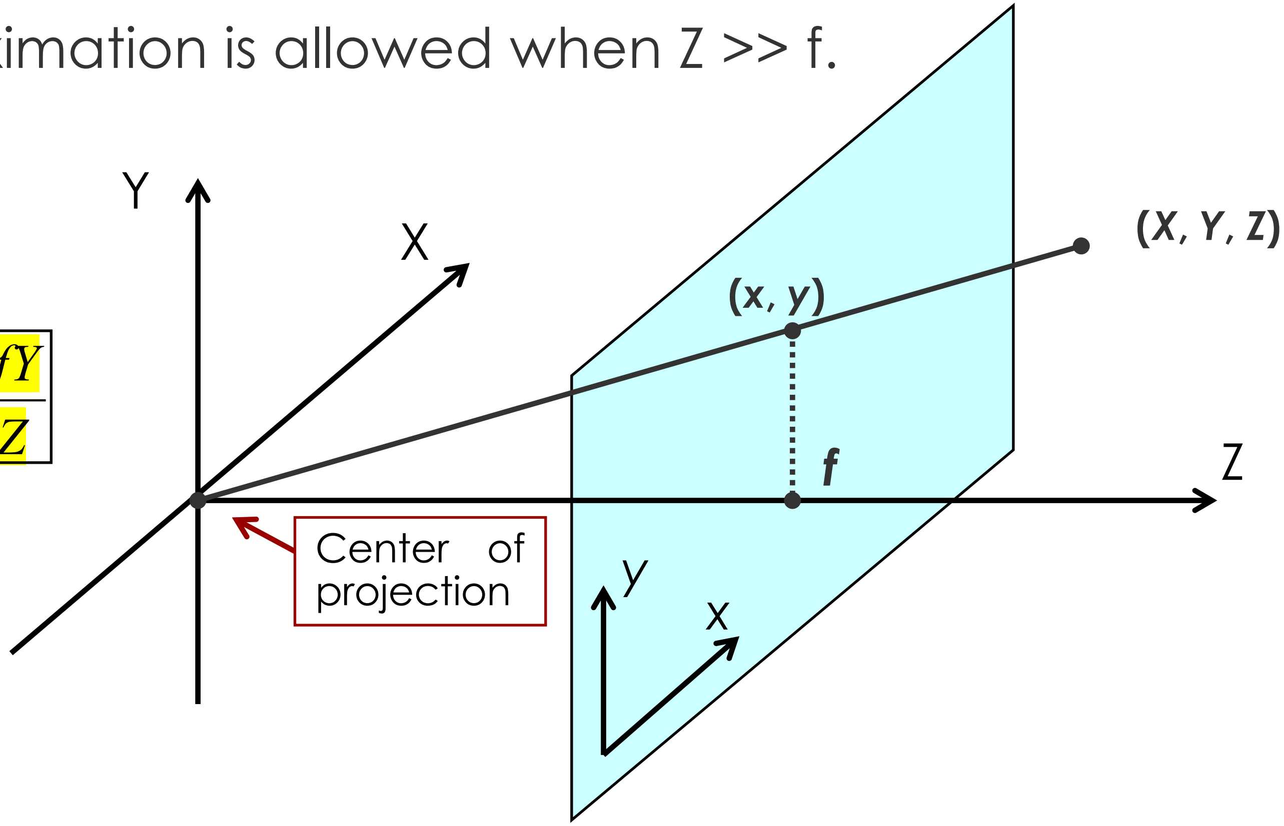
- For simplicity we usually consider the image plane on the same side of the "real world", to avoid the picture flip



Simplified perspective projection

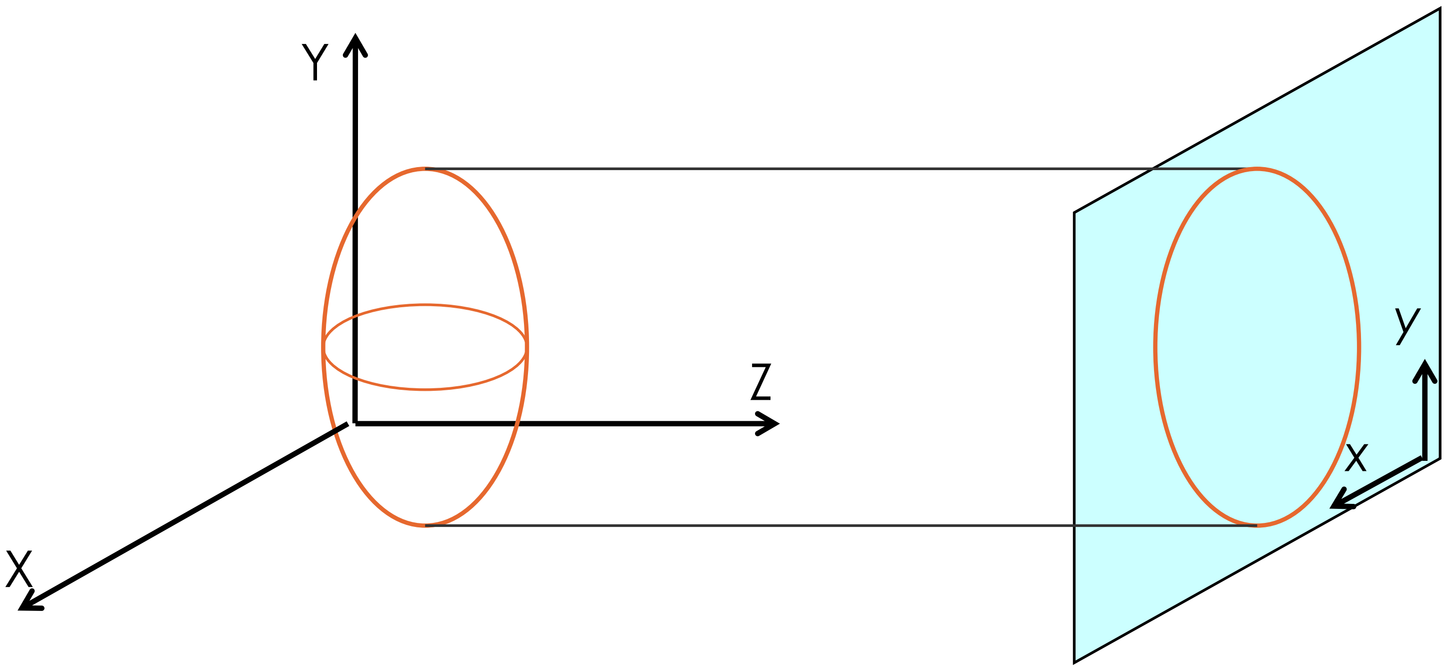
- The center of projection corresponds to the origin of the 3D space.
- The plane (X,Y) is parallel to (x,y)
- This approximation is allowed when Z >> f.

$$x = \frac{fX}{Z} \quad \text{e} \quad y = \frac{fY}{Z}$$



Orthographic projection

- It is assumed that all rays originated from the 3D object, and from the scene in general, are parallel among each other.
- In the drawing, the image plane is parallel to (X, Y)



Orthographic projection

- Assuming that the image plane is parallel to (X,Y), the orthographic projection can be simply described in Cartesian coordinates:

$$x = X \quad \text{and} \quad y = Y$$

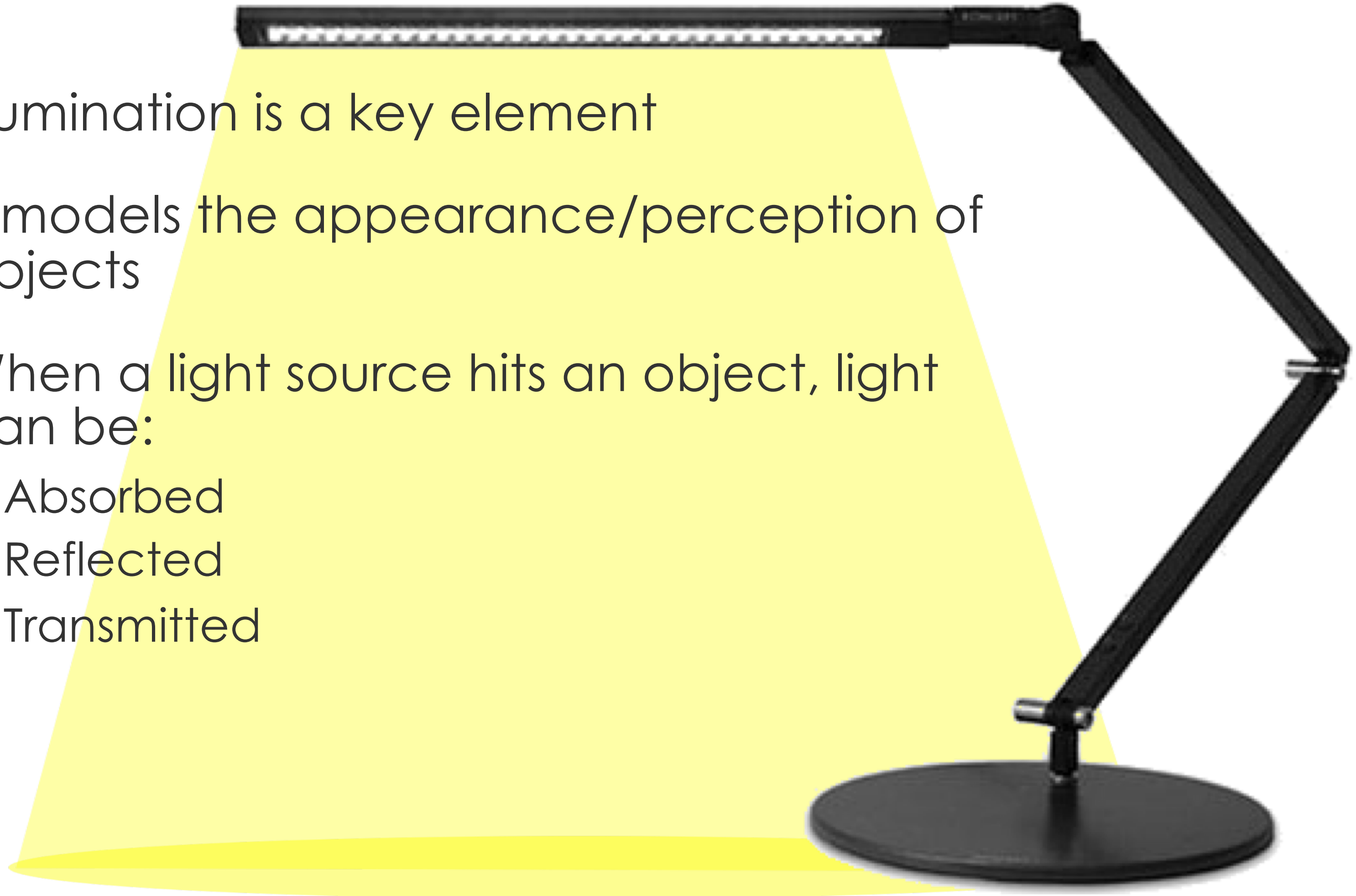
- Or in form of a matrix:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- Notes:
  - The **distance** of the object from the camera **does not affect** the intensity of the image projected onto the 2D plane.
  - It is a **good approximation** when the distance of the object is much bigger than the **depth** of the object itself

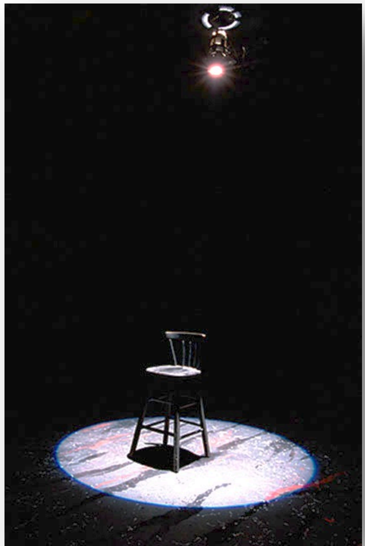
Illumination models

- Illumination is a key element
- It models the appearance/perception of objects
- When a light source hits an object, light can be:
  - Absorbed
  - Reflected
  - Transmitted



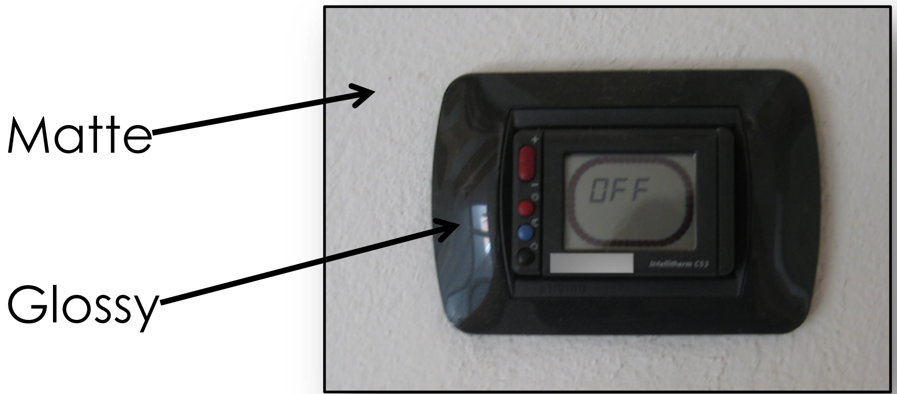
Illumination models

- It's complex to model
  - We perceive objects because they reflect light in specific wavelengths
  - Reflection can be:
    - Specular (more energy is concentrated in the light source direction)
    - Diffuse (constant in all directions, and the position of the observer is irrelevant)



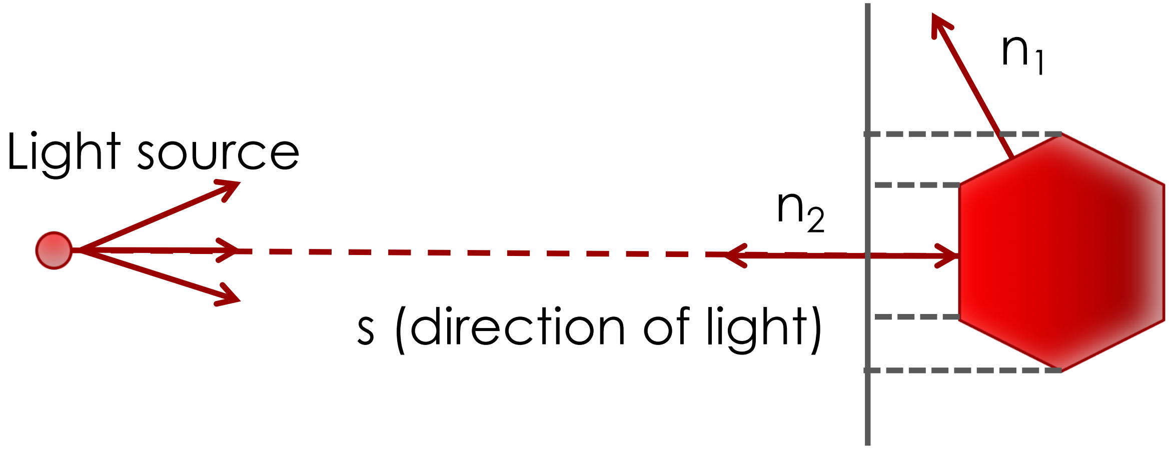
Illumination

- Surfaces vary in *specularity*
- Some of them (matte) reflect light uniformly in all directions
- Glossy objects reflect light in specific directions
- It also depends on the distance and the inclination of the light source



Illumination from one light source

- Problem: determine how the surface is irradiated by the light source
- Assumption: light is far, we can assume all rays can be represented by a single unit vector s (orthographic projection)
- For each surface element (btw dashed lines) the light is irradiated considering the cosine of the angle between the surface normal and the light direction
- i= n•s

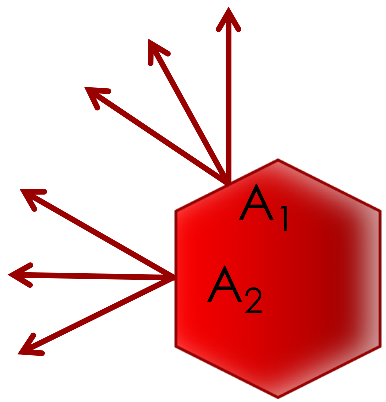


Lambertian surface

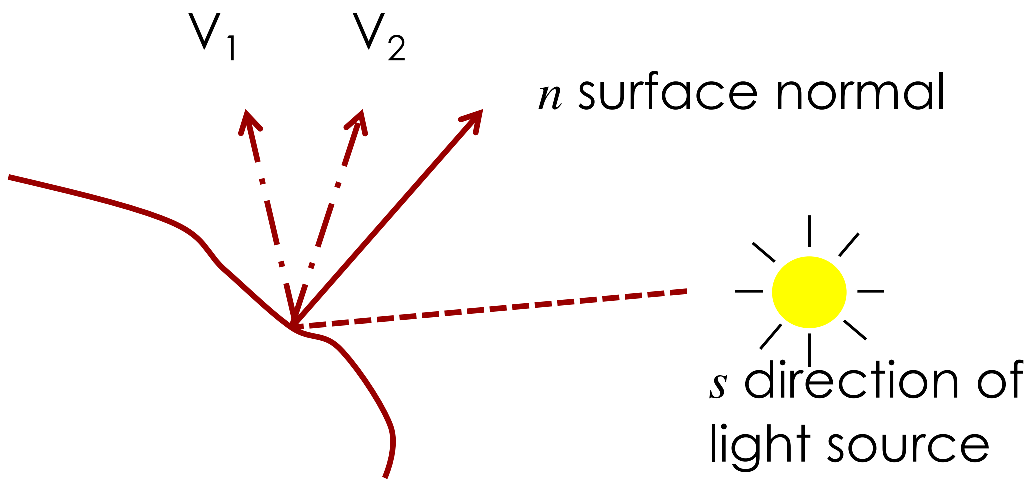
- Model for diffuse reflection
- The specular component is neglected
- The luminance of the surface is the same regardless of the viewing angle
- Possible when the surface if rough enough w.r.t. the light wavelength
- → each surface element reflects light evenly in all directions

Light source

s (direction of light)



Reflectance model



$I = \rho \mathbf{n} \bullet \mathbf{s} = |n||s| \cos\alpha$

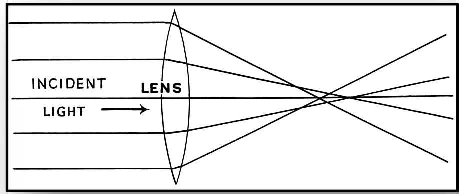
- ***n, s*** unit vectors
- ρ is the albedo
  - Ratio of the reflected illumination to the total illumination
  - In general intrinsic property of the surface
  - Not true in all cases, since some surfaces may reflect light differently depending on the view angle
  - An element is not visible if $n \bullet v < 0$, *with v the angle of the viewer*
- The pixel in image $I(r,c)$ depends on:
  - The light source direction ***s***
  - The normal of the element direction ***n***

Remarks: Cameras and Lenses

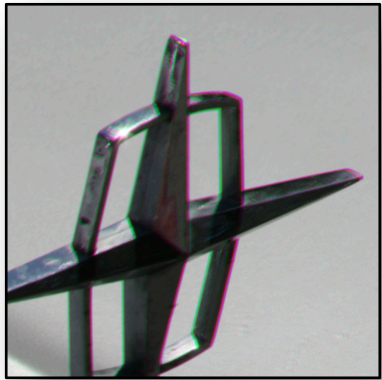
- Cameras are equipped with lenses (no pinhole in real life!)
- This means that the object is **on focus** if the distance from the center of the camera and the image plane obeys to the *thin lens* equation
- If not, we have aberrations
- We typically assume the object is on focus, but let's see what this means first!

Typical issues with lenses

- Spherical aberration (causes blurring)



- Chromatic aberrations

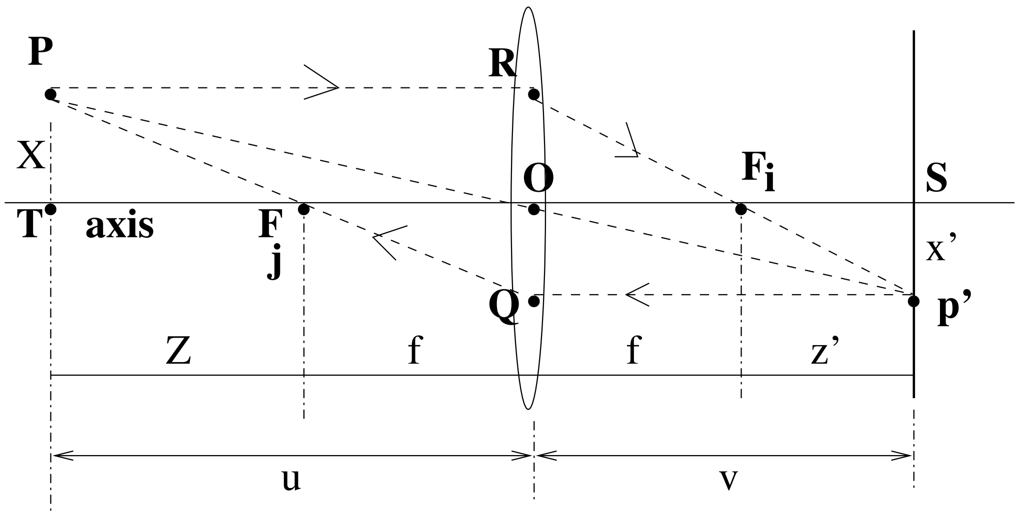


- Vignetting (two lenses, dirt, cheap cameras)

- Barrel distortion (short focal length – wide angle lenses)

Using lenses

- Pinhole is an abstract model
- Thin lens



$$\frac{X}{f} = \frac{x'}{z'} \qquad \frac{X}{Z+f} = \frac{x'}{z'+f}$$

Drawing taken from Shapiro/Stockman

Substituting X: $f^2 = Zz'$

where: $Z = u - f, \quad z' = v - f$

leads to: $uv = f(u+v)$

dividing by (uvf): $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

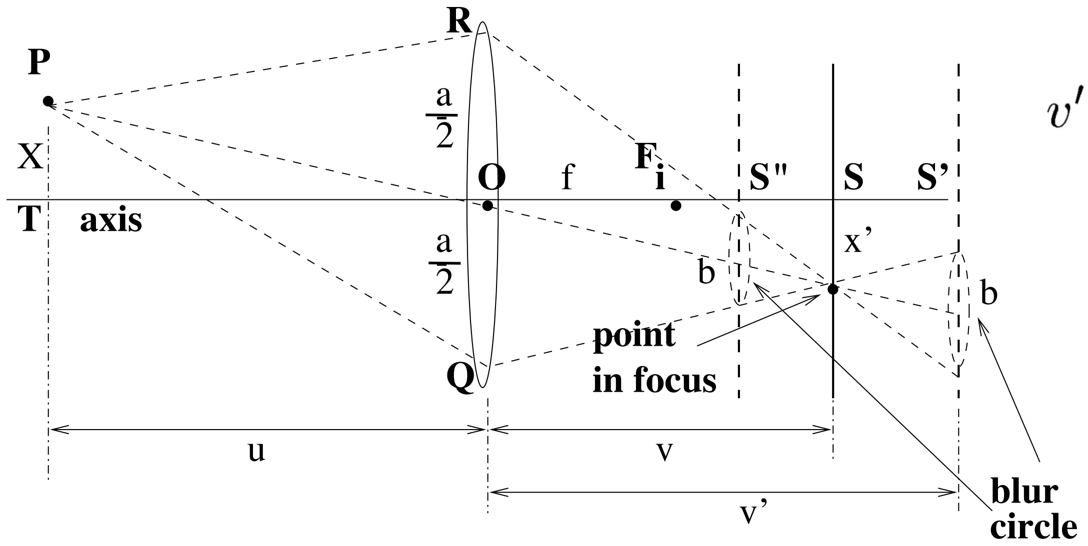
Notice: $u \to \infty \Rightarrow \frac{1}{f} = \frac{1}{v}$

Focus

- If we move the image plane, point $\mathbf{p'}$ is out of focus → v changes → v'
- If $\mathbf{P}$ is moved → u changes → u'
- Result: the image is blurred on the image plane.
- Instead of a point I see a circle!

Focus

- Considering that the blur can be acceptable if the circle is within b, what is the range of u?



$$v' = \frac{a+b}{a}v \quad : in \quad case \quad v' > v$$

$$v' = \frac{a-b}{a}v \quad : in \quad case \quad v' < v$$

$$u_n = \frac{u(a+b)}{a + \frac{bu}{f}}$$

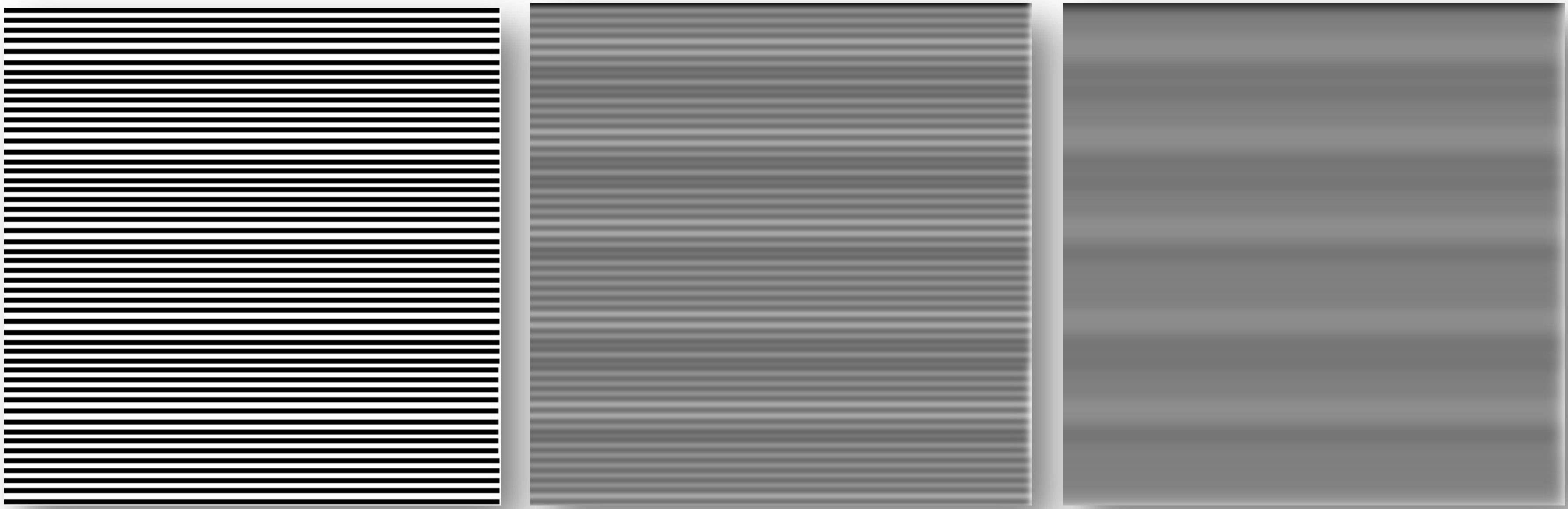
$$u_r = \frac{u(a-b)}{a - \frac{bu}{f}}$$

Notice that:

- In general u>f => u_n<u
- If f becomes smaller, u_n is closer to the camera
- $u_r > u$
- If f becomes smaller, u_r is farther from the camera
- If $u \rightarrow \infty$ rays are parallel and converge to the camera center
- Depth of field: difference between the far and near planes limiting b

Resolution and blur

- A camera with a NxM CCD can detect N/2 horizontal lines (one pixel left between two lines)
- If blur is larger than 1 pixel, the image will be grey
- Depending on the problem you have to solve, the appropriate lens must be chosen



Resolving power

- $R_p = 1/(2\Delta)$ [lines/mm]
- Δ = pixel spacing (inches or mm)
- Example:
  - CCD_{size} = 10mm square, 500x500px
  - $R_p = 1/[2x(2x10^{-2})] = 25$ [lines/mm]
- Example:
  - Spacing of cones in the fovea = $\Delta = 10^{-4}$ inches
  - $R_p = 5x10^3$
  - f = 20mm=0.8in. (diameter of the eye)
  - Subtended angle $\theta = 2.5x10^{-4}$ rad = 1 min of arc = 1/60 deg = 0.016 deg
  - Human eye can see a pencil stroke 0.5mm wide at 2m distance.

Autofocus

- What is it?
  - Capability of focusing a specific portion of the image
- How?
  - Active
  - Passive
  - Combination of the two

Active autofocus

- Mainly used for point-and-shoot cameras
- Polaroid in 1986: SONAR
- Today: Infrared, up to 6m
- Example:
  - an infrared signal is sent and the time between sending and receiving is computed
- Issues
  - Obstacles
  - Glossy and bright surfaces

Passive autofocus

- More expensive SLR (single-lens reflex) cameras
- Distance computed using image analysis
  - Take a strip of pixels and analyze the distribution
  - If values are too similar object is out of focus
  - If contrast is high, object is on focus
  - Problems with flat surfaces
  - Good cameras compute the metric on the vertical and horizontal axes

What camera do I have?

- Is there an infrared emitter?
- Otherwise:
  - Go outdoors and point at an area of the sky with no clouds (or in general a flat surface).
  - Press the shutter button halfway down. If you get a "focus okay" indication, it's an active autofocus system.
  - If you get a "focus not okay" indication, it's a passive autofocus system. The CCD cannot find any contrast in a blue sky, so it gives up.

The whole system