Assignment 1 - Risk Management

1. First part

The derived hazard rate curves for IG and HY issuers assuming piece-wise constant hazard rate and the Z-spread derived from the four bonds are shown in the following tables:

	$h(t=0,t=1y)$	$h(t=1y,t=2y)$
IG Hazard rate (bp):	46.9	128.0
HY Hazard rate (bp):	396.8	253.1

IG 1y Z-spread:	28.6 bp
IG 2y Z-spread:	52.8 bp
HY 1y Z-spread:	239.6 bp
HY 2y Z-spread:	197.0 bp

The hazard rates obtained are positive, as we expected from the theory since $\bar{B}(t,T) < B(t,T)$, and the instantaneous hazard rates computed are small ($h(t,T) << 1$). We observe that the Hazard rate of the HY class is always bigger than the one of the IG class: this numerical result confirms that the rates given to the bonds issued by the firm Alpha and by the firm Beta are correct.
We can notice also that for the IG class, the Hazard rate between 1 year and 2 years is greater than the Hazard rate in the first year: this could be justified by the fact that as time goes on, the conditions of the firm could change and the default of the company could become more likely in the future. On the other hand, for the HY class the hazard rate decreases as time increases: in this case the bonds issued by the firm beta are in the category of "junk" bonds, therefore the risk of default is sensibly higher; the hazard rate between 1 year and 2 years is lower than the hazard rate in the first year, since the fact that the company survived up to 1 year reduces the default risk in the next year.
A similar trend can be observed in the Z-spread by passing from the bond with one year maturity to the bond with two years maturity, and in general the results obtained for the Z-spread are coherent with the Hazard rates computed. We can highlight that the Z-spread obtained are always lower than the corresponding Hazard rates, in according to the theoretical result $0 < z(t,T) < h(t,T)$.

2. Second part

Assuming that the rating migration process is described by a time homogeneous Markov Chain, we compute its associated transition matrix Q.
With the values of the Hazard rate previously obtained, we can compute in $t=0$ the probabilities of default in $t=1y$ and $t=2y$ of the two classes; then by using the Chapman Kolmogorov Equation in the homogeneous case (i.e. $Q(t=0,T=2y) = Q(t=0,T=1y)^2$), we can determine the other transition probabilities.
By imposing this condition, it is enough to solve the following linear equations:

$$D_{IG} = x d_{IG} + (1-x-d_{IG}) d_{HY} + d_{IG} \Rightarrow x = \frac{D_{IG}-d_{IG}-d_{HY}+d_{IG}d_{HY}}{d_{IG}-d_{HY}}$$

$$D_{HY} = (1-y-d_{HY}) d_{IG} + y d_{HY} + d_{HY} \Rightarrow y = \frac{D_{HY}-d_{HY}-d_{IG}+d_{IG}d_{HY}}{d_{HY}-d_{IG}}$$

where:

- d_{IG}/d_{HY}: probability of default in the first year for an IG/HY Bond computed in $t=0$
- D_{IG}/D_{HY}: probability of default in two years for an IG/HY Bond computed in $t=0$
- x/y: probability of a firm with rating IG/HY in $t=0$ to still have rating IG/HY in $t=1$ year

We obtain the following transition matrix:

Q=

	IG	HY	Def
IG	0.76153	0.23379	0.00468
HY	0.39087	0.57023	0.03891
Def	0	0	1

We observe that for the probability of remaining in the same class beween the initial instant and the final instant is higher than the probability of changing class both for IG and HY rated firms, even if for the HY firm is slightly lower.

This result can be linked to the results obtained in the first part: if a firm rated HY does not default in one year, the hazard rate and the Z-spread will decrease the following year, therefore we expect that the probability of passing to IG class is quite high, as the numerical results confirm.