Assignment 2 - Risk Management

1. Questions, First Part

To evaluate the Present Value of the IG bond in a years' time according to the CreditMetrics approach we used the function `FV_risky_bond`: considering the cash flows at $t = 1.5y$ and $t = 2y$, we found the forward discounts $B(0, 1y, t)$ at the corresponding dates; from the hypothesis of a time-homogeneous Markov chain, we retrieved the hazard rates, which were used to compute the survival probabilities in the cases where the bond remained in the IG class or migrated to the HY class. Finally, we computed the present value in a years' time by discounting the future cash flows for the survival probabilities and for the forward discounts and adding the recovery term (in the case of default in the first year, only the recovery term obtained in t=1y was considered). To obtain the Present Value in a years time, the output vector of the function `FV_risky_bond` was multiplied by the vector of transition probabilities from the IG class. The obtained results are shown in Table 1.

	IG	HY	Def
1y forwards:	100.51	98.43	40.00
Present Value in a years time:		99.76	

Table 1: Present Value in a years' time of the IG bond

Then, we computed the barriers to default and to the down migration as

Barrier Def $= \mathcal{N}^{-1}(\mathbb{Q}_{IG->Def})$ and Barrier downgrade $= \mathcal{N}^{-1}(\mathbb{Q}_{IG->Def} + \mathbb{Q}_{IG->HY})$

After that, we proceeded by executing the $N = 10\,000$ Monte Carlo simulation for the 200 issuers, and we positively observed that the simulated probabilities were very close to the transition matrix's probabilities. By taking into account defaults only, we evaluated the one-year and 99.9% VaR by finding the total loss in each simulation and by computing the corresponding quantile with the Matlab function `quantile`. We repeated the same procedure by taking into account the cases of defaults, of migration to the HY class and of unchanged rating. The obtained results are shown in Table 2.

AVR correlation	VaR - default only	VaR - default and migration
$\rho = 0$	1.49	1.27

Table 2: VaR with AVR correlation $\rho = 0$ for 200 names

2. Questions, Second Part

By augmenting the number of simulations to $N = 1\,000\,000$ (to match the simulated and the transition probabilities), we repeated the aforementioned procedure to evaluate the one-year and 99.9% VaR by taking into account defaults and migrations for different values of the AVR correlation ρ. Results are shown in Table 3.

AVR correlation	VaR - default only	VaR - default and migration
$\rho = \sqrt{0.12}$	3.88	4.21
$\rho = \sqrt{0.24}$	7.17	7.80
$\rho = \sqrt{0.213}$	6.27	6.94

Table 3: Credit VaR with different AVR correlations for 200 names

3. Questions, Third Part

To estimate the relevance of the concentration of the portfolio, we repeated the computations above by considering only 20 bonds. The results with $N = 1\,000\,000$ simulations are shown in Table 4.

AVR correlation	VaR - default only	VaR - default and migration
$\rho = 0$	5.98	5.82
$\rho = \sqrt{0.12}$	8.96	8.74
$\rho = \sqrt{0.24}$	8.96	9.99
$\rho = \sqrt{0.213}$	8.96	9.68

Table 4: Credit VaR with different AVR correlations for 20 names

4. Discussion

From the obtained results, we can notice the following arguments:

- It is FALSE that the inclusion of migration risk at very high confidence level (e.g. 99.9% in our case) has no material impact on VaR measurement if the portfolio is well diversified, since the biggest variation observed in the case of $\rho = \sqrt{0.213}$ is about 10.7%.
  In the case in which $\rho = 0$, the VaR computed by taking into account defaults only is slightly lower than the VaR considering default and migration scenarios: the profit made in case of unchanged rating compensates the losses that occur in the other situations.
  In all the other cases, the inclusion of the migration risk increases the $\text{VaR}_{99\%}$: by considering also the probability of downgrading the rating of the bond, the loss of the bond could be higher. However, we can observe that all the variations in the results are "small" variations of some decimal points (the biggest variation is 0.67€), therefore these are not enormous variations in absolute value.
- It is TRUE that the portfolio VaR is very sensitive to AVR correlations: as the correlation among the Asset Value Returns increases, the VaR of the portfolio increases too. This result can be graphically noted by observing Figure 1, where the results are plotted for different levels of correlation: all the lines representing the VaR are monotone increasing, therefore as the correlation increases, the VaR increases as well. This result could be related to default and/or migration clustered events that can happen as the correlation increases.
- It is FALSE that inclusion of migration risk causes the increase of VaR under any correlation assumptions: as it was previously mentioned, in the case in which the issuers are independent ($\rho = 0$), the VaR computed by taking into account defaults only is lower than the VaR considering default an migration scenarios (either considering 20 or 200 names): we can assume that the losses are compensated by the profit made in case of unchanged rating. In the other cases, the correlation tends to create clustered migration events which result to have a negative impact on the loss of the portfolio.
- It is FALSE that the credit Portfolio Model is not sensitive to concentration risk if it is based on a single systematic factor: indeed, the VaR computed by considering only 20 names - with the same significance level $\alpha = 99.9\%$ and the same correlation ρ - is always bigger than the VaR computed by considering 200 names. We can observe this also in Figure 1: the VaR computed by taking into account defaults only or defaluts and migrations for 20 names (yellow and purple dashed lines) are always above the VaR in the two cases for 200 names (blue and orange dashed lines). This result remarks the importance of having a diversified portfolio to limit the losses.

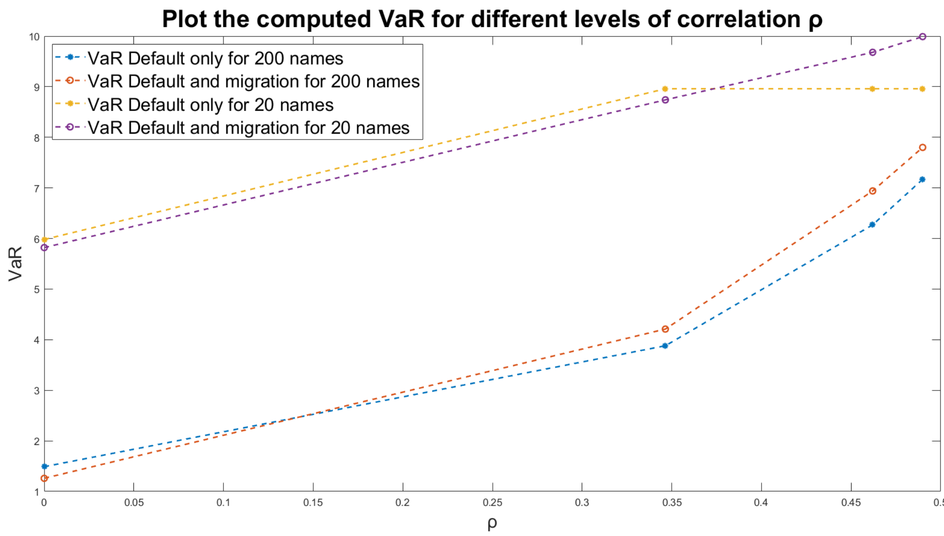

Figure 1: Graph representing the computed values of the VaR different correlations levels and different number of names