POLITECNICO
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# Assignment 3 - Group 17

Alessandro Torazzi, Matteo Torba, Giovanni Urso, Chiara Zucchelli

## Contents

# 1 Exercise: Asset Swap

We computed the Asset Swap Spread Over Euribor3m by reversing the formula of the Asset Swap. The annual coupons of the fixed leg where paid annually on the same swap dates, while the floating leg cash flows where corresponded every three months: we computed the payment dates considering only the business days thanks to the Matlab function `busdate`, by applying the modified follow convenction. We obtained the following result:

$$s^{asw} = -34.1027 \text{ bps}$$

We can notice that the spread computed has a negative sign: from this result we can suppose that the issuer YY of the bond has a very high rating. Indeed, we know from the theory that if the invoice price is around one then the Asset Swap Spread is equal to the Spread Over Libor: $s^{asw} = s^{spol}$. In this case this hypothesis is verified, since $\bar{C} = 1.01 \sim 1$, therefore for issuers belonging to the rating class $AA+$ or $AAA$, we expect to have a negative Spread Over Libor, and consequntly a negative Asset Swap Spread, as obtained from our computations.

# 2 Case Study: CDS Bootstrap

We computed the missing CDS through spline interpolation using the Matlab function `interp1`, obtaining the following result:

$$\bar{s}_{6y}^{ISP} = 39.5 \text{ bps}$$

To build the piecewise constant $\lambda(t)$ curve for the issuer ISP, we started by the approximated computation which neglects the accrual term: firstly, we equalized the values of the two legs of a CDS contract using the CDS spreads curve, and then we analytically inverted this equation to explicit the survival probabilities from the settlement date to each of the 7 years; from those probabilities we were able to obtain the intensities by simply reversing the formula. To evaluate the impact of the accrual term, we repeated the same computations by adding the accrual term to the fee leg of the CDS contract and we observed the following results:

| Intensities: | without accrual (bps) | with accrual (bps) |
|---|---|---|
| $\lambda(0,1)$ | 48.1 | 48.2 |
| $\lambda(1,2)$ | 58.4 | 58.5 |
| $\lambda(2,3)$ | 68.7 | 69.0 |
| $\lambda(3,4)$ | 86.7 | 87.1 |
| $\lambda(4,5)$ | 73.7 | 74.0 |
| $\lambda(5,6)$ | 61.2 | 61.4 |
| $\lambda(6,7)$ | 85.5 | 85.8 |

Table 1: Piecewise constant approximated and exact intensities

We can notice that the accrual term does not impact significantly our computations: the exact intensities are always bigger than the approximated ones, however they are really close to each other. To extimate the difference, we computed the maximum difference over the 7 years, obtaining the negligeable difference:

$$max_{i=1,...,7} \; |\lambda_{exact}(t_{i-1}, t_i) - \lambda_{approx}(t_{i-1}, t_i)| = 0.3512570055 \text{ bps}$$

Finally, we computed the intensity under the Jarrow-Turnbull approximation, assuming a constant $\lambda$ and continuously paid CDS spread. We directly computed it as

$$\lambda^{J-T} = \frac{\bar{s}_{7y}}{1-\pi} = 68.3 \text{ bps}$$

where $\pi$ is the recovery rate, and $\bar{s}_{7y}$ is the spread computed for the last maturity considered. We computed also the execution times for these three different approaches: the Jarrow-Turnbull approximation is the fastest method, while the first two methods need more or less the same time. The execution time of the exact intensities' computations is slightly lower than the time needed to compute the intensities by neglecting the accrual term: this depends by the fact that some repeated computations may be saved in the RAM of the computer.
To compare the results, we plotted the intensities in the graph shown in Figure 1.

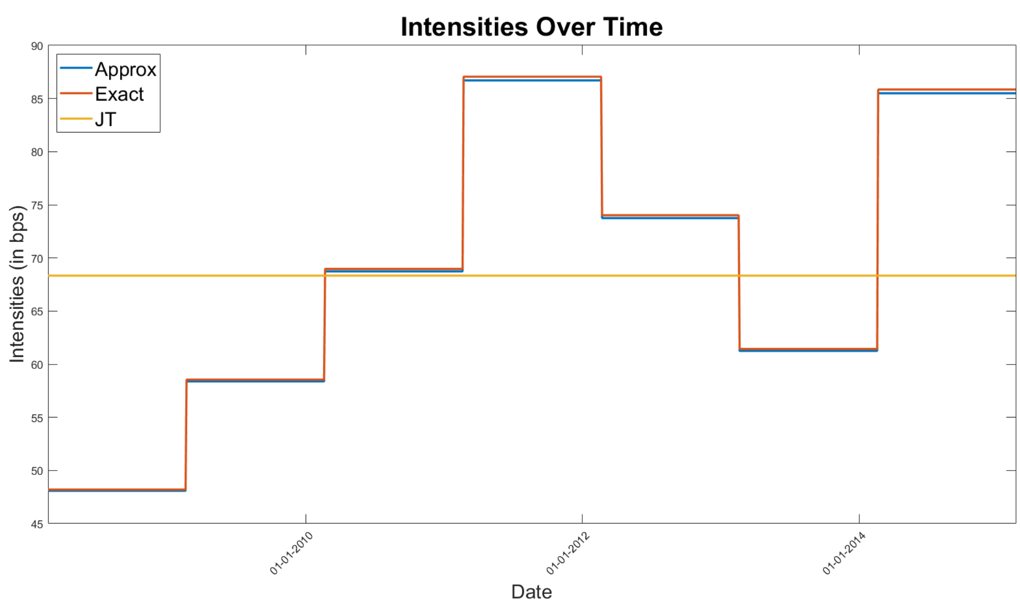


Figure 1: Graphical representation of the approximated intensities (blue line), exact intensities (orange line) and aprrocimeted intensities via Jarrow-Turnbull (yellow line)

We observe that the approximated and the exact intensities have a bump shape, which is one of the problems of the single-name models since usually this is not observed in the markets.

From the graph, we can also observe that the Jarrow-Turnbull intensity roughly seems to be a mean of the other intensities over the 7 years period. To further analyze this hypothesis, we chose to compute the constant intensity via JT for all the spreads from 1 to 7 years and compare them with the mean values of the exact intensities up to the maturity considered (both graphically and numerically). The graphical result is shown in Figure 2.

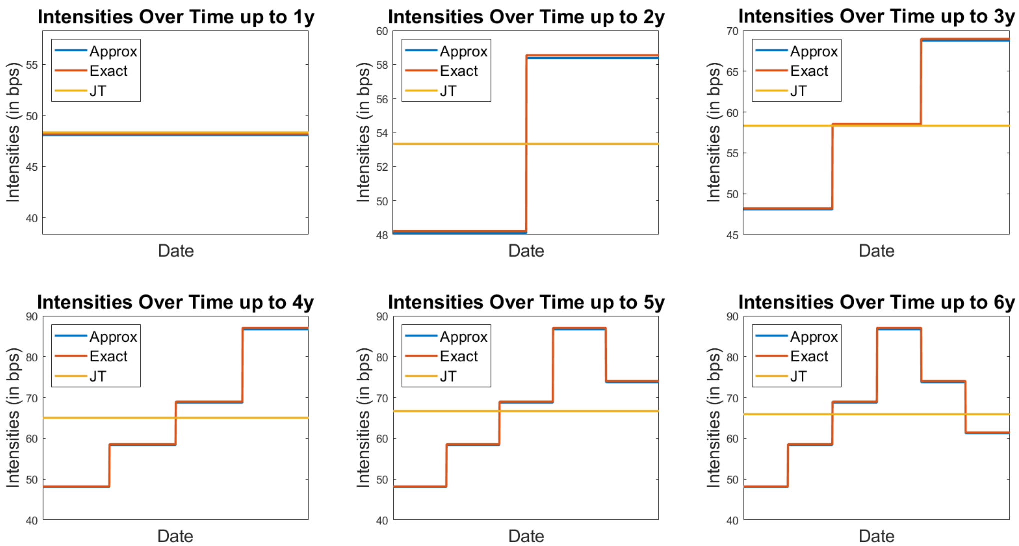


Figure 2: Graphical representation of the approximated intensities (blue line), exact intensities (orange line) and aprrocimeted intensities via Jarrow-Turnbell (yellow line) for different maturities

These graphs confim the hypothesis aforementioned, since the Jarrow-Turnbell approximation is always more or less the mean of the piece-wise constant intensities computed with the other two methods. This is confirmed also by the numerical computations: the maximum difference between the mean of the exact intensities and the corresponding JT-approximation is very small and is equal to 0.8 bps; however, we can observe that this difference tends to become bigger as the time interval grows. All the numerical results are shown in Table2.

| Maturity | $\bar{\lambda}_{exact}$ (bps) | $\lambda_{JT}$ (bps) | Difference (bps) |
|---|---|---|---|
| 1 year: | 48.2 | 48.3 | -0.1 |
| 2 years: | 53.4 | 53.3 | 0.0 |
| 3 years: | 58.6 | 58.3 | 0.2 |
| 4 years: | 65.7 | 65.0 | 0.7 |
| 5 years: | 67.4 | 66.7 | 0.7 |
| 6 years: | 66.4 | 65.9 | 0.5 |
| 7 years: | 69.2 | 68.3 | 0.8 |

Table 2: Mean values of the exact intenities VS JT approximation up to different maturities

# 3 Exercise: Price First to Default

We started by computing the missing CDS through spline interpolation with `interp1`, with the same procedure used before, obtaining the following result:

$$\bar{s}_{6y}^{UCG} = 48.0 \text{ bps}$$

We retrived the intensities and survival probabilities from the UCG CDS spread curve, by using the function presented above. We realized that the default probabilities for UCG are higher than ISP's one, as expected by observing the quoted spreads for the two firms.

It is asked to price a First to Default, written on ISP and UCG, with maturity of 4 years: in the Li model framework, the price of such derivative can be computed via Monte-Carlo simulations.

For each simulation, we sampled 2 independent standard normal variables and we induced the correlation between them, by applying a Gaussian Copula with a given $\rho = 0.20$. We then proceeded by using the PIT (Probability Integral Tranform) in order to sample the time to default of the two firms.

At that point, we selected the minimum among the two default times and we found the NPV for contingent and fee legs in function of the spread of the FTD, that we want to find. As we wanted the contract at par at settlement date, we imposed the expected value of the difference among the NPVs to be equal to zero, thus we obtained the spread just as the ratio between the the quantities simulated above. We then also calculated the 95% Fiellers's confidence interval[1] (1) (2) for the estimation obtaining the following results:

| Spread FTD | Lower Bound | Upper Bound |
|---|---|---|
| 81.9779 bps | 75.0475 bps | 88.9512 bps |

Table 3: Spread of FTD and Confidence interval with 10000 simulations

$$\text{Lower Limit} = \frac{(\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}}) - \sqrt{(\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}})^2 - (\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2)(\bar{Y}^2 - t_q^2\hat{\sigma}_{\bar{Y}}^2)}}{\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2} \tag{1}$$

$$\text{Upper Limit} = \frac{(\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}}) + \sqrt{(\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}})^2 - (\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2)(\bar{Y}^2 - t_q^2\hat{\sigma}_{\bar{Y}}^2)}}{\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2} \tag{2}$$

where:

$$\bar{R} = \frac{\bar{Y}}{\bar{X}} \tag{3}$$

and:

[1] see Franz (2007), "Ratios: A short guide to confidence limits and proper use"

$$\bar{X} = \text{mean of } X \tag{4}$$

$$\bar{Y} = \text{mean of } Y \tag{5}$$

$$\sigma^2_{\bar{X}} = \text{variance of } \bar{X} = \frac{1}{N(N-1)} \sum_{i=1}^{N} (X_i - \bar{X})^2 \tag{6}$$

$$\sigma^2_{\bar{Y}} = \text{variance of } \bar{Y} = \frac{1}{N(N-1)} \sum_{i=1}^{N} (Y_i - \bar{Y})^2 \tag{7}$$

$$\sigma^{X\bar{Y}} = \frac{1}{N(N-1)} \sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y}) \tag{8}$$

$$t_q = \text{t percent point value with } N-1 \text{ degrees of freedom} \tag{9}$$

Then we repeated the same procedure by neglecting the Accrual term and, as expected, we noticed that the results are very close to the previous ones (Spread FTD = 82.4090 bps, Lower Bound = 75.3904 bps and Upper Bound = 89.4796 bps).

In order to check the correct implementation of the Monte-Carlo procedure, we tested the function pricing the single name CDS on ISP and UCG. The following tables shows the results of this check:

| Name | CDS Quoted spread (bps) | CDS MC spread (bps) | CDS MC bid | CDS MC ask |
|---|---|---|---|---|
| ISP | 39.00 | 37.34 | 32.65 | 42.04 |
| UCG | 46.00 | 45.36 | 40.41 | 50.33 |

Table 4: Quoted VS Monte-Carlo simulated Spreads of CDS on ISP and UCG

We can observe that the Monte-Carlo simulation returns values of the spreads that are very close to the quoted values, and in both cases the quoted spread falls inside the confidence interval.

We are asked to describe the behaviour of the derivative for different correlations. Before running the simulation for a dense set of $\rho$, we observe the default probabilities for a smaller set:

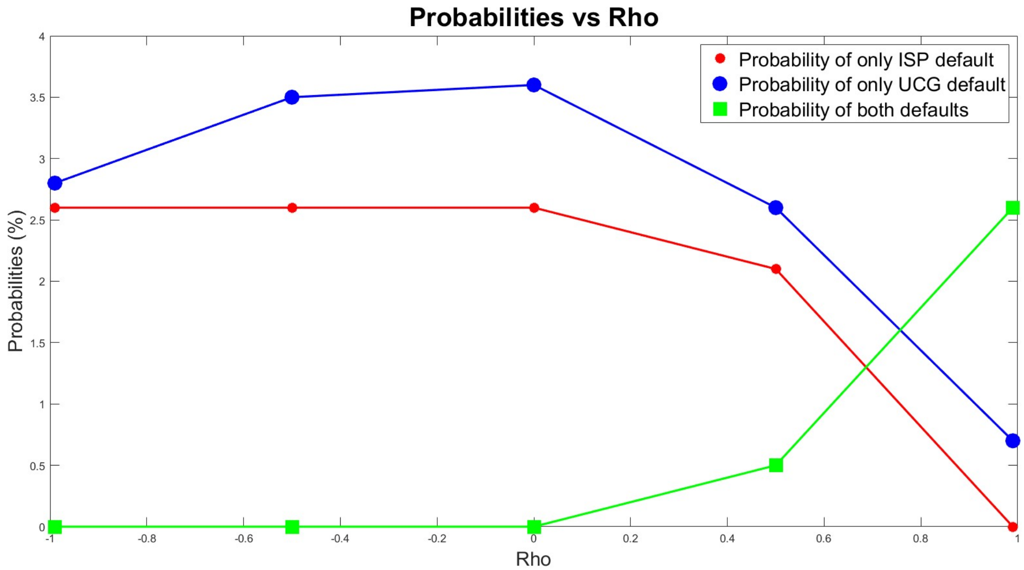


Figure 3: Graph representing the evolution of the default probabilities that only the obligor ISP defaults (red line), that only the obligor UCG defaults (blue line) and that both deafult (green line)

As we expected, the percentage of simulations in which the firm UCG is the first to default is always higher than the percentage of cases in which ISP is the first to default: the result of our simulations is coehrent with the uncorrelated default propabilities computed above (as we noticed, the default probabilities of UCG were higher than the default probabilities of ISP). Moreover, the percentage of simulations where both the firms default spikes for correlation values higher than 0.5.

In order to see the effect of the correlation on the spread of the FTD, we run the Monte-Carlo simulations for $\rho$ taking values between $-0.99$ and 0.99. We plotted the obtained results in Figure 4, alongside with the Spread of FtD bid and ask (computed as the extreme values of the confidence interval) and with the Spreads of ISP and UCG CDSs.

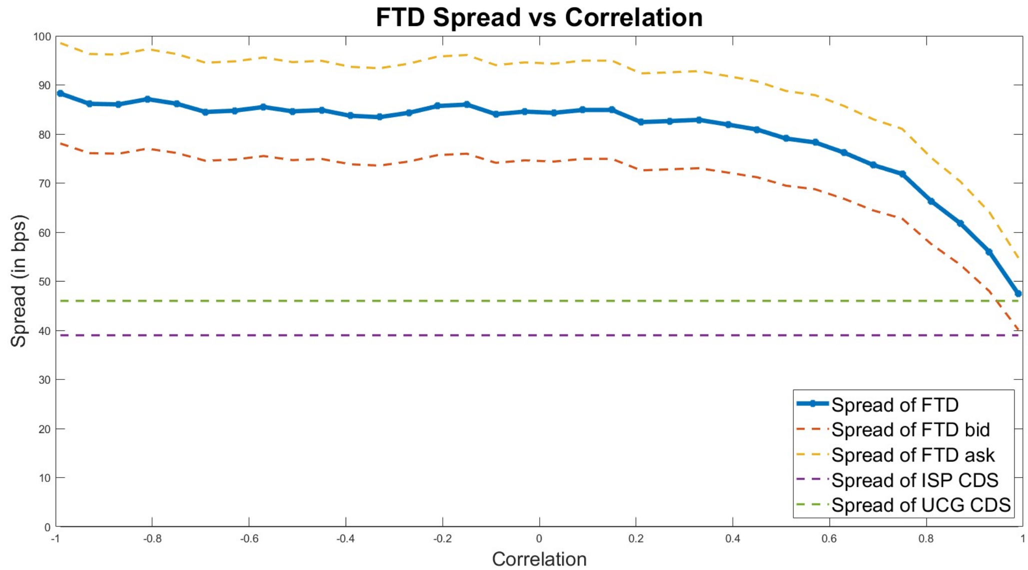


Figure 4: Graph representing the Spread of the first to Default for dirrerent correlations

We observe that for strong negative correlations, the mean value of the FTD Spread is more or less the sum of the two CDS Spreads: in those cases if one firm defaults the other does not, thus the FTD and a portfolio built with a protection buyer position on the two CDS will always have the same contingent leg payments. As the correlation increases, the value of the FTD Spread decreases and the mean value of the Confidence Interval takes values slightly lower than the sum of the two CDS spreads: we expected to obtain this result, otherwise we could have made an arbitrage by buying the two CDS separately. In the region of Figure3 where we observe a large increase of simultaneous defaults, the spread decreases significantly as mentioned in the article proposed [2]. The spread reaches its minimum when $\rho$ approaches 1 and its value becomes approximatelty the maximum between the single name CDS spreads of the names. This last result is reasonable and expected, since in the limit case of having the defaults exactly in the same istant, the FTD replicates the behaviour of the CDS on the most risky name.

[2] see David X. LI (1999), "On Default Correlation: A Copula Function Approach"