Lab Exercise #1: Evaluation of PVT

Laboratory on Least Mean Square positioning solution

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Least Mean Square Solution Computation

- In the most general case the PVT solution is given by the LMS equation

$$\boxed{\Delta\mathbf{x} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\Delta\rho}$$

- One possible way to solve for the PVT is to use the LMS algorithm not applied a single time, but iterated in order to provide a more precise solution
- At each iteration, the estimated position is expected to be closer to the real user location

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Recursive Least Mean Square

- For each time instant n, a number of iterations $k = 1, \dots, K$ is performed for the same measurements vector $\boldsymbol{\rho}_n$

INITIAL POSITION/APPROXIMATION POINT

RANGE VECTOR GEOMETRICALLY EVALUATED FROM $\hat{\mathbf{x}}_n^0$ AND EPHEMERIS

VECTOR OF MEASURED PSEUDORANGES

Initialization	$\hat{\mathbf{x}}_n^0$
Procedure for k^{th} iteration	$\Delta\widehat{\boldsymbol{\rho}}_n^k = \widehat{\boldsymbol{\rho}}_n^k - \boldsymbol{\rho}_n$
	$\Delta\hat{\mathbf{x}}_n^k = \left((\mathbf{H}_n^k)^{\mathrm{T}}\mathbf{H}_n^k\right)^{-1}(\mathbf{H}_n^k)^{\mathrm{T}}\Delta\widehat{\boldsymbol{\rho}}_n^k$
	$\hat{\mathbf{x}}_n^k = \hat{\mathbf{x}}_n^{k-1} + \Delta\hat{\mathbf{x}}_n^k$

- $\mathbf{H}$ is updated at each iteration k
- The initial approximation point, $\hat{\mathbf{x}}_n^0$, could be the same at each time n
- K has to be chosen in order to have “stable” solution (generally $K < 10$)

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Recursive Least Mean Square

- $\hat{\mathbf{x}}_n^k = [\hat{x}_n^k \quad \hat{y}_n^k \quad \hat{z}_n^k \quad \hat{b}_n^k]$ is the estimated position/state
- The number of visible satellites J is not constant over observation time n
- $\Delta\hat{\boldsymbol{\rho}}_n^k$ is the vector of the J_n pseudoranges differences (between measured and calculated) at the k-th iteration
- At each iteration k, $\mathbf{H}_n^k$ is the geometrical matrix obtained from the previous estimated position:

Satellite coordinates are fixed for any k

$$\mathbf{H}_n^k = \begin{bmatrix} a_{x,1} & a_{y,1} & a_{z,1} & 1 \\ a_{x,2} & a_{y,2} & a_{z,2} & 1 \\ a_{x,3} & a_{y,3} & a_{z,3} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{x,J_n} & a_{y,J_n} & a_{z,J_n} & 1 \end{bmatrix}$$

$$a_{x,j} = \left[\frac{x_{j,n} - \hat{x}_n^k}{\hat{\rho}_{j,n}}\right], \ a_{y,j} = \cdots, a_{z,j} = \cdots$$

$$\hat{\rho}_{j,n} = \sqrt{\left(x_{j,n} - \hat{x}_n^k\right)^2 + \left(y_{j,n} - \hat{y}_n^k\right)^2 + \left(z_{j,n} - \hat{z}_n^k\right)^2}$$

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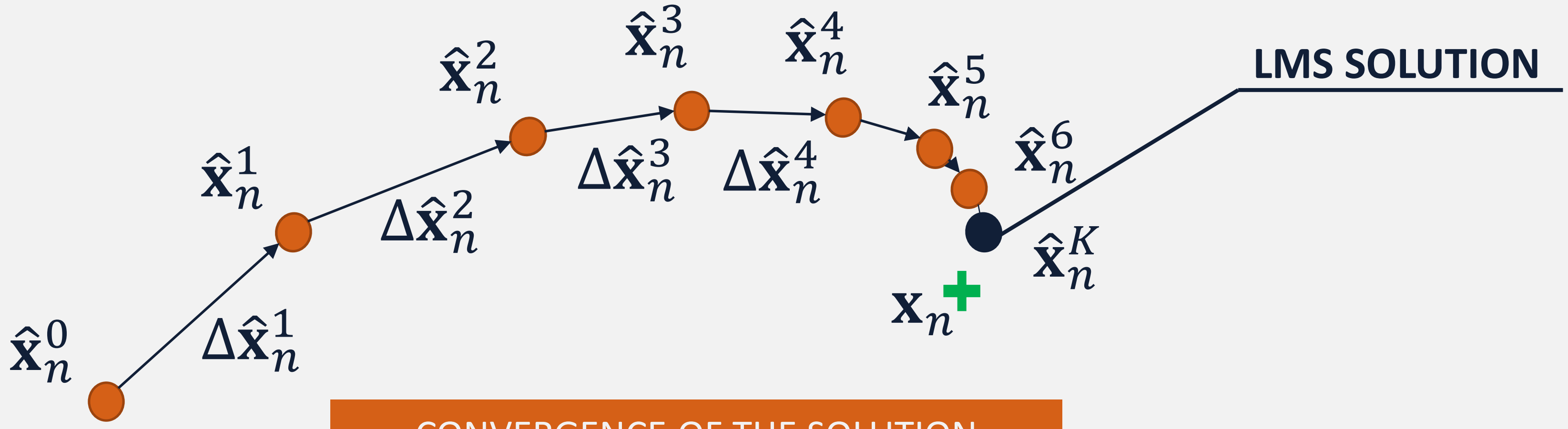
Recursive Least Mean Square

- Adding $\Delta\hat{\mathbf{x}}_n^k$ to the estimated position at the previous iteration, a new estimation of the position is obtained

$$\boxed{\hat{\mathbf{x}}_n^k = \hat{\mathbf{x}}_n^{k-1} + \Delta\hat{\mathbf{x}}_n^k}$$



CONVERGENCE OF THE SOLUTION

From raw to corrected pseudorange

- The measurement of the pseudoranges must be corrected of all the predictable contributions to the errors
- After the correction there will be still a random contribution that is modeled by the User Equivalent Range Error (UERE)

$$\boxed{UERE \sim \mathcal{N}(0, \sigma_{\mathrm{UERE}})}$$

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Different errors for different satellites

Basic LMS algorithm attributes the same relevance to all the measurements but actually pseudoranges are not equally precise.

- In real cases, each pseudorange may be characterized by a different value of standard deviation, $\sigma_{j,\mathrm{UERE}}$
- They can still be considered uncorrelated such that:

$$\mathbf{R} = \mathrm{Cov}(\boldsymbol{\rho}) = \begin{bmatrix} \sigma^2_{1,\mathrm{UERE}} & 0 & 0 & \dots & 0 \\ 0 & \sigma^2_{2,\mathrm{UERE}} & 0 & \dots & 0 \\ 0 & 0 & \sigma^2_{3,\mathrm{UERE}} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma^2_{\mathrm{J},\mathrm{UERE}} \end{bmatrix}$$

Different errors for different satellites

- The direct calculation of the covariance matrix is not possible since the $\rho_{j,n}$ are time dependent (multiple realizations for the same instant n are not available)
- We can estimate the $\sigma_{j,\mathrm{UERE}}$ for each pseudorange ρ_j analyzing the error on the pseudorange itself along the time (supposing it is ergodic)
- Dependency on time must be removed
- Since pseudorange measurement follows a quadratic trend, the second order derivative can be applied to remove the variation along the time

Use MATLAB function `diff(X,` *`order`*`)`

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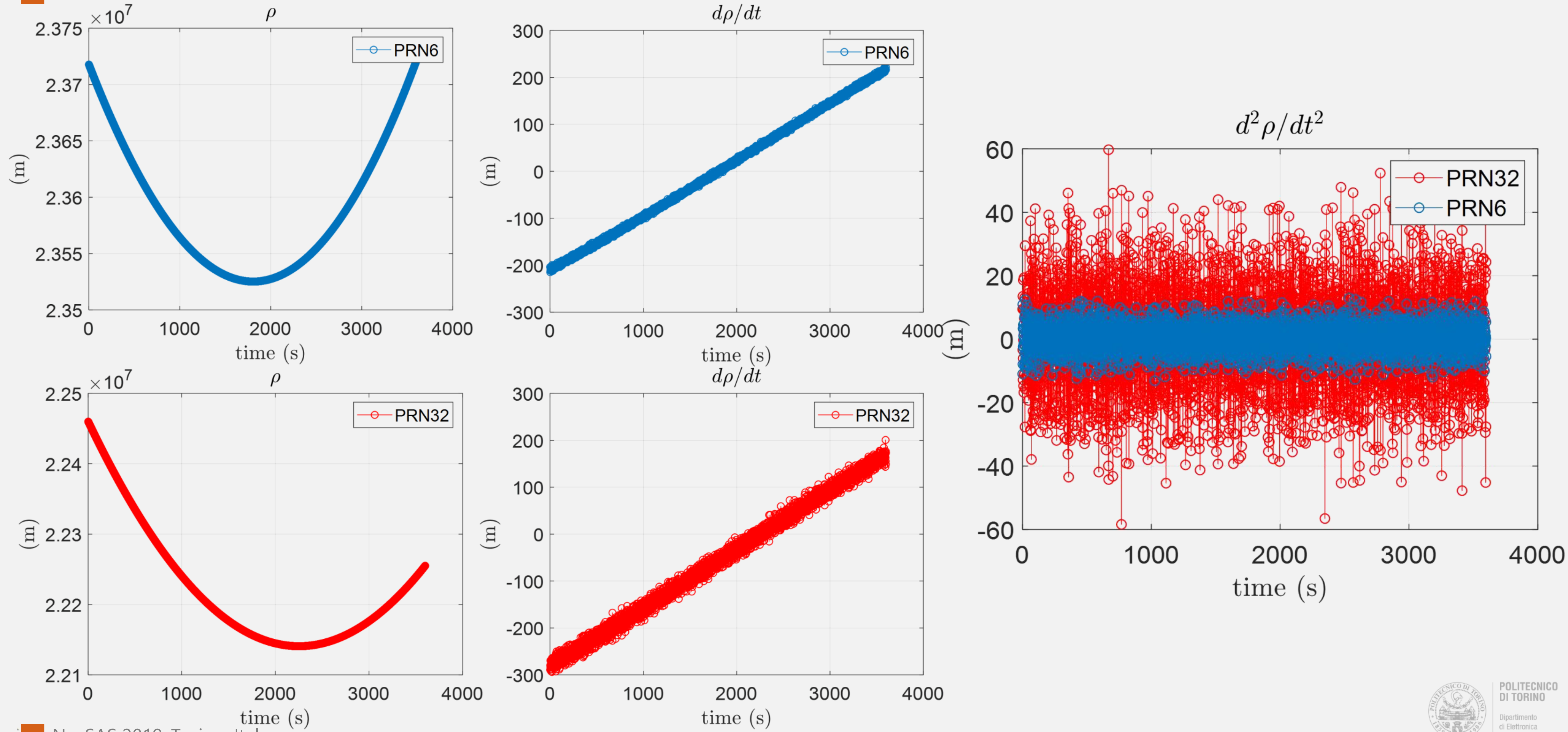
Pseudorange de-trending by differentiation



Weighted Least Mean Square

- A second approach foresees to solve the system by means of a **Weighted Least Squares** (WLS) algorithm, characterized by the introduction of the weighting matrix $\mathbf{W}$, which is a positive definite matrix
- Since some measurements may be known to be more accurate than others, the measurement accuracy is known to be characterized by the inverse of the measurement errors covariance matrix $\mathbf{R}$
- It is natural to select $\mathbf{W} = \mathbf{R}^{-1}$ to give the least weighting to the most uncertain measurements.

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Uncorrelated measurements

- The weight matrix can be estimated from the measurements, thus designing a new **weighted geometrical matrix** $\overline{\mathbf{H}}_n^k$
- For each time instant n, a number of iterations $k = 1, \dots, K$ is performed for the same measurements vector $\boldsymbol{\rho}_n$

Initialization	$\hat{\mathbf{x}}_n^0$
Procedure for k^{th} iteration	$\Delta\widehat{\boldsymbol{\rho}}_n^k = \widehat{\boldsymbol{\rho}}_n^k - \boldsymbol{\rho}_n$
	$\overline{\mathbf{H}}_n^k = \left((\mathbf{H}_n^k)^{\mathrm{T}}\mathbf{W}\mathbf{H}_n^k\right)^{-1}(\mathbf{H}_n^k)^{\mathrm{T}}\mathbf{W}$
	$\Delta\hat{\mathbf{x}}_n^k = \overline{\mathbf{H}}_n^k \, \Delta\widehat{\boldsymbol{\rho}}_n^k$
	$\hat{\mathbf{x}}_n^k = \hat{\mathbf{x}}_n^{k-1} + \Delta\hat{\mathbf{x}}_n^k$

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ICT4SS - Lab Exercise #1, 02 April 2019

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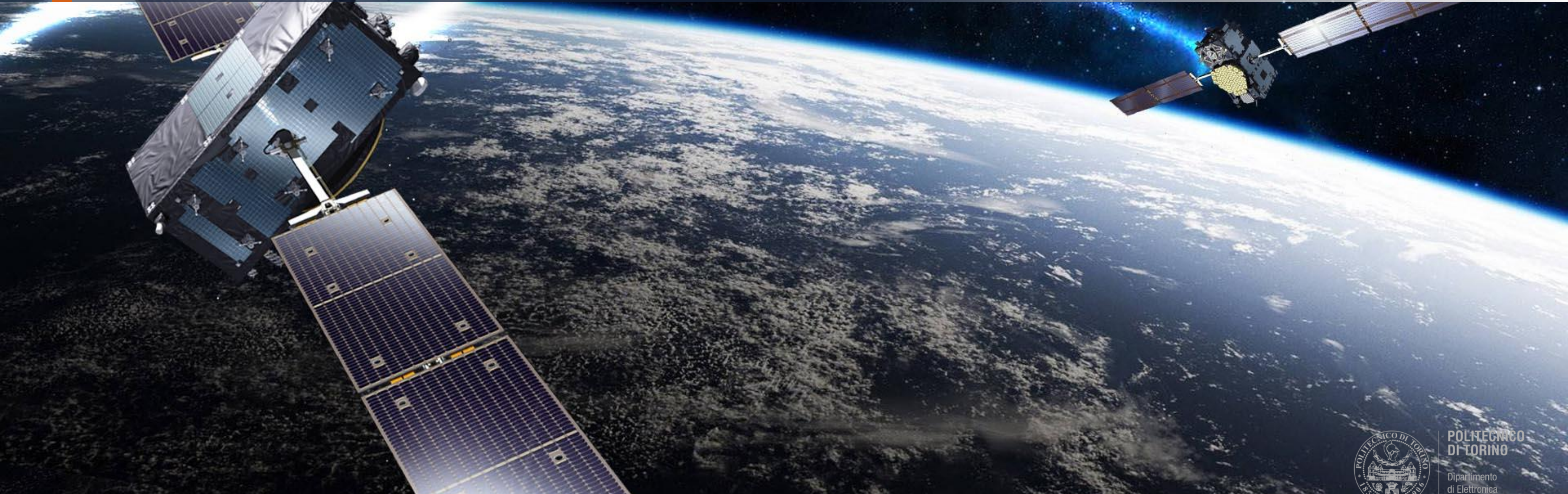

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Variables for GPS constellation

USER and SATELLITE STATE VECTORS (position and clock bias)

Unknown User Position

$$\mathbf{x}_n = [x_n \quad y_n \quad z_n \quad b_{n,GPS}]$$

Known Satellite Position

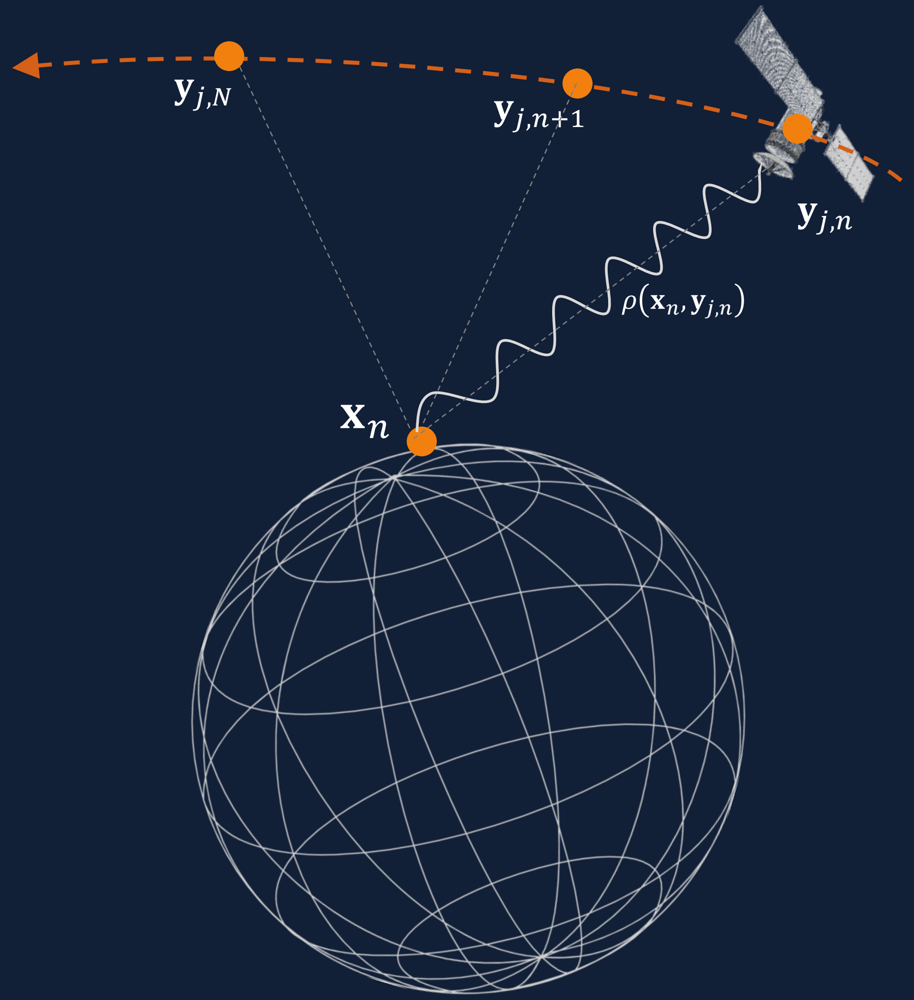
$$\mathbf{y}_{j,n} = [x_{j,n} \quad y_{j,n} \quad z_{j,n}\ ]$$

PSEUDORANGE EQUATION

Measured range distance from the satellite

$$\rho(\mathbf{x}_n, \mathbf{y}_{j,n}) = \sqrt{(x_n - x_{j,n})^2 + (y_n - y_{j,n})^2 + (z_n - z_{j,n})^2} + b_{n,GPS}$$

- $n \in (1,2, \dots, N)$ is the time index
- $j \in (1,2, \dots, J)$ is the satellite identifier
- Data collections include 3600 seconds of satellites observations from a static position $\mathbf{x}_n$: $\mathbf{x}_n = \mathbf{x}_{n+1} = \cdots = \mathbf{x}_N$, where $N = 3600$



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Observables Data Structures

PSEUDORANGE MEASUREMENTS (GPS)

RHO.GPS				
	$n = 1$	$n = 2$	..	$n = N$
GPS.PRN_1	$\rho(\mathbf{x}_1, \mathbf{y}_{1,1})$	$\rho(\mathbf{x}_2, \mathbf{y}_{1,2})$	…	$\rho(\mathbf{x}_N, \mathbf{y}_{1,N})$
GPS.PRN_2	$\rho(\mathbf{x}_1, \mathbf{y}_{2,1})$	$\rho(\mathbf{x}_2, \mathbf{y}_{2,2})$	…	$\rho(\mathbf{x}_N, \mathbf{y}_{2,N})$
GPS.PRN_3	$\rho(\mathbf{x}_1, \mathbf{y}_{3,1})$	$\rho(\mathbf{x}_2, \mathbf{y}_{3,2})$	…	$\rho(\mathbf{x}_N, \mathbf{y}_{3,N})$
…	…	…	…	…
GPS.PRN_J	$\rho(\mathbf{x}_1, \mathbf{y}_{J,1})$	$\rho(\mathbf{x}_2, \mathbf{y}_{J,2})$	…	$\rho(\mathbf{x}_N, \mathbf{y}_{J,N})$

SATELLITE POSITIONS FROM EPHEMERIS (GPS)

SV_ECEF.GPS	
SV ID	$\mathbf{Y}_j$
GPS.PRN_1	$\mathbf{Y}_1$
GPS.PRN_2	$\mathbf{Y}_2$
GPS.PRN_3	$\mathbf{Y}_3$
…	…
GPS.PRN_J	$\mathbf{Y}_J$

Ex: SV Position State Vector History			
$\mathbf{Y}_1$	x	y	z
$n = 1$	$x_{1,1}$	$y_{1,1}$	$z_{1,1}$
$n = 2$	$x_{1,2}$	$y_{1,2}$	$z_{1,2}$
$n = 3$	$x_{1,3}$	$y_{1,3}$	$z_{1,3}$
…	…	…	…
$n = N$	$x_{1,N}$	$y_{1,N}$	$z_{1,N}$

- The number of visible satellites is not constant over observation time n
- The folder named «NominalUERE» contains pseudorange measurements with the same σ_{UERE}
- The folder named «RealisticUERE» contains pseudorange measurements with satellite-dependent σ_{UERE}
- Pseudorange measurements can be considered an ergodic random process over short time periods
- It is possible to select different constellations using strings GPS, GLO, BEI, GAL in the data structure

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Lab session

TASK **1**

Load data from the *NominalUERE* folder. Check the satellites visibility at each time instant n for all the constellations. Plot the number of the visible satellites as well as the measured pseudoranges along the time.

TASK **2**

Choose a dataset and a constellation (e.g. GPS, Galileo) and develop a Least Mean Square (LMS) positioning algorithm and estimate the user state, $\hat{\mathbf{x}}_n^K$, at each time instant n.
Verify the position on Google Earth: convert from ECEF to LLA and use *writeKML_GoogleEarth.m*.

TASK **3**

Compute the standard deviation of the position error over time $n = 1,2,..,N$ and compare the quality of the obtained solution for different datasets and constellations. Motivate the results.

TASK **4**

Estimate the $\sigma_{j,\mathrm{UERE}}$ for all satellites using the dataset from the *realisticUERE* folder. Implement the **Weigthed Least Mean Square** (WLMS) repeating Tasks 2 and 3.
Compare the performance of LMS and WLMS on *realisticUERE* dataset.

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Simulation hint

For each epoch $n = 1:N$

- Find the available pseudorange measurements
- Build the measurement vector $\boldsymbol{\rho}_n = [\rho_{1,n} \quad \cdots \quad \rho_{J,n}]$
- Retrieve the corresponding satellites coordinates $\mathbf{y}_{j,n}$
- Compute PVT solution (K iterations)

end

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How to use refMap

- Install the Mapping toolbox
- `refMap(mean(latitude), mean(longitude), 'Here I am!');`