POLITECNICO DI TORINO
DET - DIATI

Geodesy Exercises

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Ellipsoid parameters

$$\alpha = \frac{a-c}{a}$$

$$e'^2 = \frac{a^2 - c^2}{c^2}$$

$$e^2 = \frac{a^2 - c^2}{a^2}$$

$$\boldsymbol{e'^2 = \frac{e^2}{1-e^2}}$$

$$c^2 = a^2(1-e^2)$$

BESSEL (1841)	a = 6.377.397 m	α = 1/299,2
CLARKE (1880)	a = 6.378.243 m	α = 1/293,5
HAYFORD (1909)	**a = 6.378.388 m**	**α = 1/297,0**
WGS84 (1984)	a = 6.378.137 m	α = 1/298,257223563

Ex.1 Ellipsoid's parameters

ELLISSOIDE	***a* [m]**	α	$c = a(1-\alpha)$	e^2	e'^2
Delambre	6.376.985	1/308.6			
Everest	6.377.276	1/300.8			
Bessel	6.377.397	1/299.1528128			
Fisher	6.378.338	1/288.5			
Clarke	6.378.249	1/293.5			
Helmert	6.378.140	1/298.3			
Hayford	6.378.388	1/297.0	6.356.911,9	0,0067226700	0,0067681702
Krassovsky	6.378.245	1/298.3			
WGS84	6.378.137	1/298.257223563			

To estimate the ellipsoid's parameters

Ex.2 coordinates transformation from (ϕ,λ,h) to (XYZ)

Parametric equation of the ellipsoid

$$\begin{cases} X = \left(\dfrac{a}{W} + h\right)\cos\varphi \cdot \cos\lambda \\ Y = \left(\dfrac{a}{W} + h\right)\cos\varphi \cdot \sin\lambda \\ Z = \left[\dfrac{a}{W}\left(1 - e^{2}\right) + h\right] \cdot \sin\varphi \end{cases}$$

where

$$W = \sqrt{1 - e^{2} \cdot \sin^{2}\varphi}$$

Ex.2 coordinates transformation from (φ,λ,h) to (XYZ)

points	Latitudine (φ)	Longitudine (λ)	h (m)
1	44° 45’ 01”,03930	7° 24’ 29”,20335	322,4909
2	44° 47’ 10”,90505	7° 30’ 26”,53939	305,7367

Considering the following ellipsoids:

- **WGS 84**
- **Hayford**

Try to change «h1», adding 2000 m. How is this value distributed in X, Y, Z?

Ex.3 coordinates transformation from (XYZ) to (ϕ,λ,h)

Longitude estimation

$$\lambda = \mathrm{arctg}\frac{Y}{X}$$

Iterative procedure to estimate latitude and h

Step 1

$$r = \sqrt{X^2 + Y^2}$$

And an approx latitude:

$$\varphi = \mathrm{arctg}\frac{Z}{r}$$

Step 2

$$W = \sqrt{1 - e^2 \cdot \sin^2 \varphi}$$

$$N = \frac{a}{W}$$

Step 3

$$\boxed{h = \frac{X}{\cos\varphi\cos\lambda} - N}$$

$$\boxed{\varphi = arctg\frac{Z}{r\left(1 - \frac{e^2 N}{N + h}\right)}}$$

Threshold convergence =10^-8 (in h and latitude)

Ex.3 coordinates transformation from (XYZ) to (ϕ,λ,h)

Points	X [m]	Y [m]	Z [m]
1	4,499,525.4271	585,034.1293	4,467,910.3596
2	4,495,694.2695	592,457.8605	4,470,744.7781
3	4,503,484.7172	578,160.7507	4,465,024.3002
4	4,498,329.3715	562,840.7651	4,472,537.6125

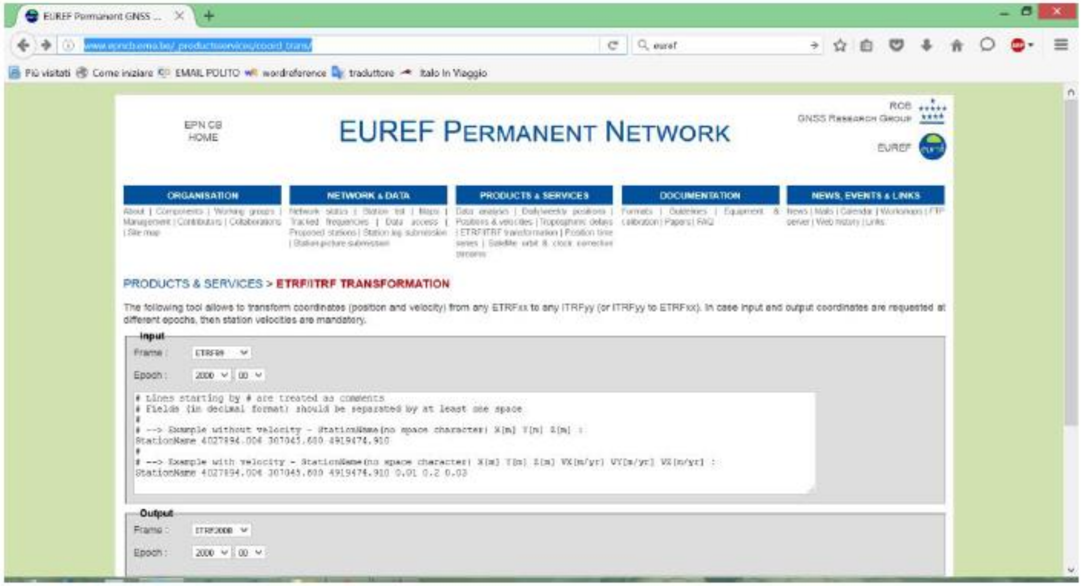
Considering these following ellipsoids:

- **WGS 84**
- **Hayford**

Ex.4 – RS transformation

http://www.epncb.oma.be/

http://www.epncb.oma.be/_productsservices/coord_trans/



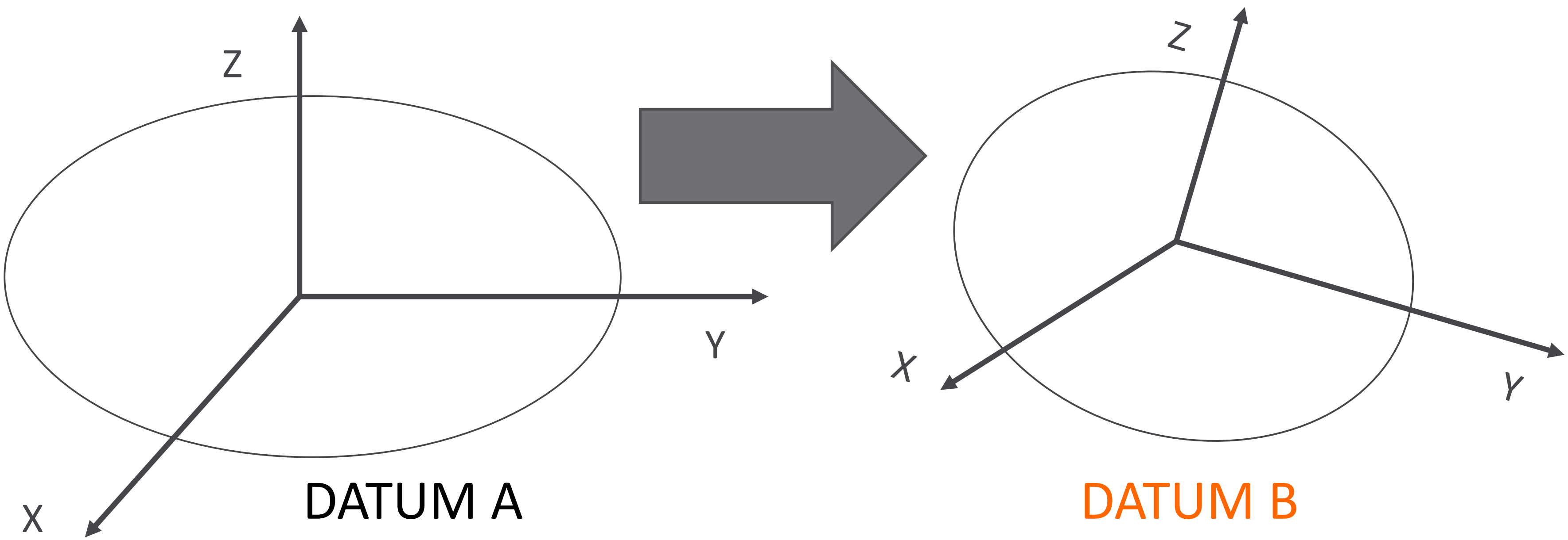
Using the XYZ of the EX. 2, to make these following transformations:

ETRF89 → ITRF89

ETRF2000 → ITRF2000

To analyze the differences.

Ex.5 – Helmert transformation



$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_B = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + \lambda \begin{bmatrix} 1 & Rz & -R_y \\ -Rz & 1 & R_x \\ Ry & -Rx & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_A$$

Ex.5 – Helmert transformation

Linearizing the equation around the approx. values:

$$\begin{bmatrix} X_B - X_A - T_X^{(o)} \\ Y_B - Y_A - T_Y^{(o)} \\ Z_B - Z_A - T_Z^{(o)} \end{bmatrix} = \begin{bmatrix} \Delta T_X \\ \Delta T_Y \\ \Delta T_Z \end{bmatrix} + \begin{bmatrix} \Delta\lambda & R_Z & -R_X \\ -R_Z & \Delta\lambda & R_X \\ R_Y & -R_X & \Delta\lambda \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix}$$

Solving the system with the least squares::

$$\begin{bmatrix} 1 & 0 & 0 & X_A & 0 & -Z_A & Y_A \\ 0 & 1 & 0 & Y_A & -Z_A & 0 & -X_A \\ 0 & 0 & 1 & Z_A & Y_A & X_A & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \cdot \begin{pmatrix} \Delta T_X \\ \Delta T_X \\ \Delta T_X \\ \Delta\lambda \\ R_X \\ R_Y \\ R_Z \end{pmatrix} - \begin{bmatrix} X_B - X_A \\ Y_B - Y_A \\ Z_B - Z_A \\ \cdots\cdot \end{bmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \cdots\cdot \end{pmatrix}$$

Ex.5 – Helmert transformation

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & X_A & 0 & -Z_A & Y_A \\ 0 & 1 & 0 & Y_A & -Z_A & 0 & -X_A \\ 0 & 0 & 1 & Z_A & Y_A & X_A & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{pmatrix} \Delta T_X \\ \Delta T_X \\ \Delta T_X \\ \Delta\lambda \\ R_X \\ R_Y \\ R_Z \end{pmatrix}}_{\mathbf{x}} - \underbrace{\begin{bmatrix} X_B - X_A \\ Y_B - Y_A \\ Z_B - Z_A \\ \cdots\cdot \end{bmatrix}}_{\mathbf{l_0}} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \cdots\cdot \end{pmatrix}$$

Residuals

$$x = (A^T A)^{-1}(A^T l_0)$$

Least squares solution

NB: In order to avoid singular matrix, it is better to divide the coordinates (X, Y, Z) by 10^6. At the end, It is necessary to multiply the translations by this value.

Ex.5 – Helmert transformation

To estimate the HELMERT PARAMETER using the points reported in «points_helmert.txt».

1) *Parameters estimation*
2) *Analysis of the residuals*

Residuals (v)= $Ax-l_0$

3) *Comments of the results*