

# Function Reference

Here is the documentation for all of SPART functions (the current list is incomplete).

- [Kinematics](#)
- [Dynamics](#)
- [Robot Model](#)
- [Attitude Transformations](#)
- [Utilities](#)

## Kinematics

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**Kinematics**(*R0, r0, qm, robot*)

Computes the kinematics – positions and orientations – of the multibody system.

`[RJ,RL,rJ,rL,e,g]=Kinematics(R0,r0,qm,robot)`

**Parameters:**

- R0 – Rotation matrix from the base-link CCS to the inertial CCS – [3x3].
- r0 – Position of the base-link center-of-mass with respect to the origin of the inertial frame, projected in the inertial CCS – [3x1].
- qm – Displacements of the active joints – [n\_qx1].
- robot – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- RJ – Joints CCS 3x3 rotation matrices with respect to the inertial CCS – as a [3x3xn] matrix.
- RL – Links CCS 3x3 rotation matrices with respect to the inertial CCS – as a [3x3xn] matrix.
- rJ – Positions of the joints, projected in the inertial CCS – as a [3xn] matrix.
- rL – Positions of the links, projected in the inertial CCS – as a [3xn] matrix.
- e – Joint rotation/sliding axes, projected in the inertial CCS – as a [3xn] matrix.
- g – Vector from the origin of the ith joint CCS to the origin of the ith link CCS, projected in the inertial CCS – as a [3xn] matrix.

Remember that all the output magnitudes are projected in the **inertial frame**.

Examples on how to retrieve the results from a specific link/joint:

To retrieve the position of the  $i$ th link: `rL(1:3,i)` .

To retrieve the rotation matrix of the  $i$ th joint: `RJ(1:3,1:3,i)` .

See also: `src.robot_model.urdf2robot()` and `src.robot_model.DH_Serial2robot()` .

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### **DiffKinematics**(*R0, r0, rL, e, g, robot*)

Computes the differential kinematics of the multibody system.

`[Bij,Bi0,P0,pm]=DiffKinematics(R0,r0,rL,e,g,robot)`

**Parameters:**

- *R0* – Rotation matrix from the base-link CCS to the inertial CCS – [3x3].
- *r0* – Position of the base-link center-of-mass with respect to the origin of the inertial frame, projected in the inertial CCS – [3x1].
- *rL* – Positions of the links, projected in the inertial CCS – as a [3xn] matrix.
- *e* – Joint rotation/sliding axes, projected in the inertial CCS – as a [3xn] matrix.
- *g* – Vector from the origin of the  $i$ th joint CCS to the origin of the  $i$ th link CCS, projected in the inertial CCS – as a [3xn] matrix.
- *robot* – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- *Bij* – Twist-propagation matrix (for manipulator  $i>0$  and  $j>0$ ) – as a [6x6xn] matrix.
- *Bi0* – Twist-propagation matrix (for  $i>0$  and  $j=0$ ) – as a [6x6xn] matrix.
- *P0* – Base-link twist-propagation “vector” – as a [6x6] matrix.
- *pm* – Manipulator twist-propagation “vector” – as a [6xn] matrix.

Use `src.kinematics_dynamics.Kinematics()` to compute `rL,e` , and `g` .

See also: `src.kinematics_dynamics.Kinematics()` and `src.kinematics_dynamics.Jacob()` .

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### **Velocities**(*Bij, Bi0, P0, pm, u0, um, robot*)

Computes the operational-space velocities of the multibody system.

`[t0,tL]=Velocities(Bij,Bi0,P0,pm,u0,um,robot)`

**Parameters:**

- $B_{ij}$  – Twist-propagation matrix (for manipulator  $i>0$  and  $j>0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $B_{i0}$  – Twist-propagation matrix (for  $i>0$  and  $j=0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $P_0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $p_m$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $u_0$  – Base-link velocities  $[\omega, \dot{r}]$ . The angular velocity is projected in the body-fixed CCS, while the linear velocity is projected in the inertial CCS –  $[6 \times 1]$ .
- $u_m$  – Joint velocities –  $[n \times 1]$ .
- `robot` – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $t_0$  – Base-link twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $t_L$  – Manipulator twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.

Use `src.kinematics_dynamics.DiffKinematics()` to compute  `$B_{ij}$` ,  `$B_{i0}$` ,  `$P_0$` , and  `$p_m$` .

See also: `src.kinematics_dynamics.Jacob()`

**Jacob(*rp, r0, rL, P0, pm, i, robot*)**

Computes the geometric Jacobian of a point  $p$ .

$[J_0, J_m] = \text{Jacob}(rp, r_0, r_L, P_0, p_m, i, \text{robot})$

**Parameters:**

- $r_p$  – Position of the point of interest, projected in the inertial CCS –  $[3 \times 1]$ .
- $r_0$  – Position of the base-link, projected in the inertial CCS –  $[3 \times 1]$ .
- $r_L$  – Positions of the links, projected in the inertial CCS – as a  $[3 \times n]$  matrix.
- $P_0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $p_m$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $i$  – Link id where the point  $p$  is located – int 0 to  $n$ .
- `robot` – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $J_0$  – Base-link geometric Jacobian –  $[6 \times 6]$ .
- $J_m$  – Manipulator geometric Jacobian –  $[6 \times n \times q]$ .

**Examples:**

To compute the velocity of the point  $p$  on the  $i$ th link:

```
%Compute Jacobians
[J0, Jm]=Jacob(rp,r0,rL,P0,pm,i,robot);
%Twist of that point
tp=J0*u0+Jm*um;
```

See also: `src.kinematics_dynamics.Kinematics()`, `src.kinematics_dynamics.DiffKinematics()`

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### Accelerations(*t0, tL, P0, pm, Bi0, Bij, u0, um, u0dot, umdot, robot*)

Computes the operational-space accelerations (twist-rate) of the multibody system.

`[t0dot,tLdot]=Accelerations(t0,tL,P0,pm,Bi0,Bij,u0,um,u0dot,umdot,robot)`

#### Parameters:

- *t0* – Base-link twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- *tL* – Manipulator twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- *Bij* – Twist-propagation matrix (for manipulator  $i > 0$  and  $j > 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- *Bi0* – Twist-propagation matrix (for  $i > 0$  and  $j = 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- *P0* – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- *pm* – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- *u0* – Base-link velocities  $[\omega, \dot{r}]$ . The angular velocity is projected in the body-fixed CCS, while the linear velocity is projected in the inertial CCS –  $[6 \times 1]$ .
- *um* – Joint velocities –  $[n_{qx} \times 1]$ .
- *u0dot* – Base-link accelerations  $[\dot{\omega}, \ddot{r}]$ . The angular acceleration is projected in a body-fixed CCS, while the linear acceleration is projected in the inertial CCS –  $[6 \times 1]$ .
- *umdot* – Manipulator joint accelerations –  $[n_{qx} \times 1]$ .
- *robot* – Robot model (see [SPART Tutorial – Robot Model](#)).

#### Returns:

- *t0dot* – Base-link twist-rate vector  $[\dot{\omega}, \ddot{r}]$ , projected in inertial frame – as a  $[6 \times 1]$  matrix.
- *tLdot* – Manipulator twist-rate vector  $[\dot{\omega}, \ddot{r}]$ , projected in inertial frame – as a  $[6 \times n]$  matrix.

See also: `src.kinematics_dynamics.Jacobdot()`.

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### Jacobdot(*rp, tp, r0, t0, rL, tL, P0, pm, i, robot*)

Computes the geometric Jacobian time-derivative of a point *p*.

$[J0dot, Jmdot]=\text{Jacobdot}(rp, tp, r0, t0, rL, tL, P0, pm, i, robot)$

- Parameters:**
- $rp$  – Position of the point of interest, projected in the inertial CCS –  $[3 \times 1]$ .
  - $tp$  – Twist of the point of interest  $[\omega, rdot]$ , projected in the inertial CCS –  $[6 \times 1]$ .
  - $r0$  – Position of the base-link center-of-mass with respect to the origin of the inertial frame, projected in the inertial CCS –  $[3 \times 1]$ .
  - $t0$  – Base-link twist  $[\omega, rdot]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
  - $rL$  – Positions of the links, projected in the inertial CCS – as a  $[3 \times n]$  matrix.
  - $tL$  – Manipulator twist  $[\omega, rdot]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
  - $P0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
  - $pm$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
  - $i$  – Link id where the point  $p$  is located – int 0 to  $n$ .
  - $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

- Returns:**
- $J0dot$  – Base-link Jacobian time-derivative – as a  $[6 \times 6]$  matrix.
  - $Jmdot$  – Manipulator Jacobian time-derivative – as a  $[6 \times n_q]$  matrix.

Examples:

To compute the acceleration of a point p on the  $i$ th link:

```
%Compute Jacobians
[J0, Jm]=Jacob(rp,r0,rL,P0,pm,i,robot);
Compute Jacobians time-derivatives
[J0dot, Jmdot]=Jacobdot(rp,tp,r0,t0,rL,tL,P0,pm,i,robot)
%Twist-rate of that point
tpdot=J0*u0dot+J0dot*u0+Jm*umdot+Jmdot*um;
```

See also: `src.kinematics_dynamics.Accelerations()` and `src.kinematics_dynamics.Jacob()`.

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### Center\_of\_Mass( $r0, rL, robot$ )

Computes the center-of-mass (CoM) of the system.

$r\_com = \text{Center\_of\_Mass}(r0, rL, robot)$

- Parameters:**
- $r0$  – Position of the base-link, projected in the inertial CCS –  $[3 \times 1]$ .
  - $rL$  – Positions of the links, projected in the inertial CCS – as a  $[3 \times n]$  matrix.
  - $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- `r_com` – Location of the center-of-mass, projected in the inertial CCS – [3x1].

Use `src.kinematics_dynamics.Kinematics()` to compute `rL`.

This function can also be used to compute the velocity/acceleration of the center-of-mass. To do it use as parameters the velocities `r0dot, rLdot` or acceleration `r0ddot, rLddot` and you will get the CoM velocity `rcomdot` or acceleration `rcomddot`.

See also: `src.kinematics_dynamics.Kinematics()`

**NOC(*r0, rL, P0, pm, robot*)**

Computes the Natural Orthogonal Complement (NOC) matrix (generalized Jacobian).

`[N] = NOC(r0,rL,P0,pm,robot)`

**Parameters:**

- `r0` – Position of the base-link, projected in the inertial CCS – [3x1].
- `rL` – Positions of the links, projected in the inertial CCS – as a [3xn] matrix.
- `P0` – Base-link twist-propagation “vector” – as a [6x6] matrix.
- `pm` – Manipulator twist-propagation “vector” – as a [6xn] matrix.
- `robot` – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- `N` – Natural Orthogonal Complement (NOC) matrix – a [(6+6\*n)x(6+n\_q)] matrix.

**Examples:**

To compute the velocities of all links:

```
%Compute NOC.
[N] = NOC(r0,rL,P0,pm,robot)
%Generalized twist (concatenation of the twist of all links).
t=N*[u0;um];
%Twist of the base-link
t0=t(1:6,1);
%Twist of the ith link
i=2;
ti=t(6*i:6+6*i,1);
```

See also: `src.kinematics_dynamics.Jacob()` and `src.kinematics_dynamics.NOCdot()`.

**NOCdot(*r0, t0, rL, tL, P0, pm, robot*)**

Computes the Natural Orthogonal Complement (NOC) matrix time-derivative.

$\dot{N}$  = NOCdot( $r_0, t_0, r_L, t_L, P_0, p_m, robot$ )

**Parameters:**

- $r_0$  – Position of the base-link center-of-mass with respect to the origin of the inertial frame, projected in the inertial CCS –  $[3 \times 1]$ .
- $t_0$  – Base-link twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $r_L$  – Positions of the links, projected in the inertial CCS – as a  $[3 \times n]$  matrix.
- $t_L$  – Manipulator twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $P_0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $p_m$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $\dot{N}$  – Natural Orthogonal Complement (NOC) matrix time-derivative – as a  $[(6+6 \times n) \times (6+n_q)]$  matrix.

**Examples:**

To compute the operational-space accelerations of all links:

```
%Compute NOC
[N] = NOC(r0,rL,P0,pm,robot)
%Compute NOC time-derivative
[Ndot] = NOCdot(r0,t0,rL,tL,P0,pm,robot)
%Twist time-derivatives of all the links
tdot=N*[u0dot;umdot]+Ndot*[u0;um];
%Twist time-derivative of the base-link
t0dot=tdot(1:6,1);
%Twist time-derivative of the ith link
i=2;
tidot=tdot(6*i:6+6*i,1);
```

See also: `src.kinematics_dynamics.Jacobdot()` and `src.kinematics_dynamics.NOC()` .

## Dynamics

**FD**(*tau0, taum, wF0, wFm, t0, tm, P0, pm, IO, Im, Bij, Bi0, u0, um, robot*)

This function solves the forward dynamics (FD) problem (it obtains the acceleration from forces).

$[u_0\dot{,}u_m\dot{}] = \text{FD}(\tau_0, \tau_m, w_{F0}, w_{Fm}, t_0, t_m, P_0, p_m, I_O, I_m, B_{ij}, B_{i0}, u_0, u_m, robot)$

**Parameters:**

- $\tau_0$  – Base-link forces  $[n, f]$ . The torque  $n$  is projected in the body-fixed CCS, while the force  $f$  is projected in the inertial CCS –  $[6 \times 1]$ .
- $\tau_{um}$  – Joint forces/torques – as a  $[n_{qx1}]$  matrix.
- $wF_0$  – Wrench acting on the base-link center-of-mass  $[n, f]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $wF_m$  – Wrench acting on the links center-of-mass  $[n, f]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $t_0$  – Base-link twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $t_L$  – Manipulator twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $P_0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $p_m$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $I_0$  – Base-link inertia matrix, projected in the inertial CCS – as a  $[3 \times 3]$  matrix.
- $I_m$  – Links inertia matrices, projected in the inertial CCS – as a  $[3 \times 3 \times n]$  matrix.
- $B_{ij}$  – Twist-propagation matrix (for manipulator  $i > 0$  and  $j > 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $B_{i0}$  – Twist-propagation matrix (for  $i > 0$  and  $j = 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $u_0$  – Base-link velocities  $[\omega, \dot{r}]$ . The angular velocity is projected in the body-fixed CCS, while the linear velocity is projected in the inertial CCS –  $[6 \times 1]$ .
- $u_m$  – Joint velocities –  $[n_{qx1}]$ .
- `robot` – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $u_0 \dot{\phantom{x}}$  – Base-link accelerations  $[\dot{\omega}, \ddot{r}]$ . The angular acceleration is projected in a body-fixed CCS, while the linear acceleration is projected in the inertial CCS –  $[6 \times 1]$ .
- $u_m \dot{\phantom{x}}$  – Manipulator joint accelerations –  $[n_{qx1}]$ .

See also: `src.kinematics_dynamics.ID()` and `src.kinematics_dynamics.I_I()`.

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**ID**( $wF_0, wF_m, t_0, t_L, t_0 \dot{\phantom{x}}, t_L \dot{\phantom{x}}, P_0, p_m, I_0, I_m, B_{ij}, B_{i0}, robot$ )

This function solves the inverse dynamics (ID) problem (it obtains the generalized forces from the accelerations) for a manipulator.

$[\tau_0, \tau_{um}] = \text{ID}(wF_0, wF_m, t_0, t_L, t_0 \dot{\phantom{x}}, t_L \dot{\phantom{x}}, P_0, p_m, I_0, I_m, B_{ij}, B_{i0}, robot)$



**Parameters:**

- $wF0$  – Wrench acting on the base-link center-of-mass  $[n,f]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $wFm$  – Wrench acting on the links center-of-mass  $[n,f]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $t0$  – Base-link twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $tL$  – Manipulator twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $t0\dot{}$  – Base-link twist-rate vector  $[\dot{\omega}, \ddot{r}]$ , projected in inertial frame – as a  $[6 \times 1]$  matrix.
- $tL\dot{}$  – Manipulator twist-rate vector  $[\dot{\omega}, \ddot{r}]$ , projected in inertial frame – as a  $[6 \times n]$  matrix.
- $P0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $pm$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $I0$  – Base-link inertia matrix, projected in the inertial CCS – as a  $[3 \times 3]$  matrix.
- $Im$  – Links inertia matrices, projected in the inertial CCS – as a  $[3 \times 3 \times n]$  matrix.
- $B_{ij}$  – Twist-propagation matrix (for manipulator  $i > 0$  and  $j > 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $Bi0$  – Twist-propagation matrix (for  $i > 0$  and  $j = 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- `robot` – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $\tau0$  – Base-link forces  $[n,f]$ . The torque  $n$  is projected in the body-fixed CCS, while the force  $f$  is projected in the inertial CCS –  $[6 \times 1]$ .
- $\tau_{um}$  – Joint forces/torques – as a  $[n_{qx1}]$  matrix.

See also: `src.kinematics_dynamics.Floating_ID()` and `src.kinematics_dynamics.FD()` .

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**Floating\_ID**(*wF0, wFm, Mm\_tilde, H0, t0, tm, P0, pm, I0, Im, Bij, Bi0, u0, um, umdot, robot*)

This function solves the inverse dynamics problem (it obtains the generalized forces from the accelerations) for a manipulator with a floating base.

$[\tau_{um}, u0\dot{}] =$

`Floating_ID(wF0,wFm,Mm_tilde,H0,t0,tm,P0,pm,I0,Im,Bij,Bi0,u0,um,umdot,robot)`

**Parameters:**

- $wF0$  – Wrench acting on the base-link center-of-mass  $[n,f]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $wFm$  – Wrench acting on the links center-of-mass  $[n,f]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $M0\_tilde$  – Base-link mass composite body matrix – as a  $[6 \times 6]$  matrix .
- $Mm\_tilde$  – Manipulator mass composite body matrix – as a  $[6 \times 6 \times n]$  matrix.
- $t0$  – Base-link twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $tL$  – Manipulator twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $P0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $pm$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $I0$  – Base-link inertia matrix, projected in the inertial CCS – as a  $[3 \times 3]$  matrix.
- $Im$  – Links inertia matrices, projected in the inertial CCS – as a  $[3 \times 3 \times n]$  matrix.
- $B_{ij}$  – Twist-propagation matrix (for manipulator  $i > 0$  and  $j > 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $Bi0$  – Twist-propagation matrix (for  $i > 0$  and  $j = 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $u0$  – Base-link velocities  $[\omega, \dot{r}]$ . The angular velocity is projected in the body-fixed CCS, while the linear velocity is projected in the inertial CCS –  $[6 \times 1]$ .
- $um$  – Joint velocities –  $[n\_qx1]$ .
- $umdot$  – Manipulator joint accelerations –  $[n\_qx1]$ .
- $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $\tau0$  – Base-link forces  $[n,f]$ . The torque  $n$  is projected in the body-fixed CCS, while the force  $f$  is projected in the inertial CCS –  $[6 \times 1]$ .
- $\tau_{aum}$  – Joint forces/torques – as a  $[n\_qx1]$  matrix.

See also: `src.kinematics_dynamics.sID()` and `src.kinematics_dynamics.FD()` .

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**`I_I(R0, RL, robot)`**

Projects the link inertias in the inertial CCS.

$[I0, Im] = I\_I(R0, RL, robot)$

**Parameters:**

- $R0$  – Rotation matrix from the base-link CCS to the inertial CCS –  $[3 \times 3]$ .
- $RL$  – Links CCS  $3 \times 3$  rotation matrices with respect to the inertial CCS – as a  $[3 \times 3 \times n]$  matrix.
- $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $I_0$  – Base-link inertia matrix, projected in the inertial CCS – as a  $[3 \times 3]$  matrix.
- $I_m$  – Links inertia matrices, projected in the inertial CCS – as a  $[3 \times 3 \times n]$  matrix.

See also: `src.kinematics_dynamics.MCB()` .

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**MCB( $I_0$ ,  $I_m$ ,  $B_{ij}$ ,  $B_{i0}$ ,  $robot$ )**

Computes the Mass Composite Body Matrix (MCB) of the multibody system.

$[M_0\_tilde, M_m\_tilde] = \text{MCB}(I_0, I_m, B_{ij}, B_{i0}, robot)$

**Parameters:**

- $I_0$  – Base-link inertia matrix, projected in the inertial CCS – as a  $[3 \times 3]$  matrix.
- $I_m$  – Links inertia matrices, projected in the inertial CCS – as a  $[3 \times 3 \times n]$  matrix.
- $B_{ij}$  – Twist-propagation matrix (for manipulator  $i > 0$  and  $j > 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $B_{i0}$  – Twist-propagation matrix (for  $i > 0$  and  $j = 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $M_0\_tilde$  – Base-link mass composite body matrix – as a  $[6 \times 6]$  matrix .
- $M_m\_tilde$  – Manipulator mass composite body matrix – as a  $[6 \times 6 \times n]$  matrix.

See also: `src.kinematics_dynamics.I_I()` .

---

**GIM( $M_0\_tilde$ ,  $M_m\_tilde$ ,  $B_{ij}$ ,  $B_{i0}$ ,  $P_0$ ,  $p_m$ ,  $robot$ )**

Computes the Generalized Inertia Matrix (GIM)  $H$  of the multibody vehicle.

This function uses a recursive algorithm.

$[H_0, H_{0m}, H_m] = \text{GIM}(M_0\_tilde, M_m\_tilde, B_{ij}, B_{i0}, P_0, p_m, robot)$

**Parameters:**

- $M_0\_tilde$  – Base-link mass composite body matrix – as a  $[6 \times 6]$  matrix .
- $M_m\_tilde$  – Manipulator mass composite body matrix – as a  $[6 \times 6 \times n]$  matrix.
- $B_{ij}$  – Twist-propagation matrix (for manipulator  $i > 0$  and  $j > 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $B_{i0}$  – Twist-propagation matrix (for  $i > 0$  and  $j = 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $P_0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $p_m$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $H_0$  – Base-link inertia matrix – as a  $[6 \times 6]$  matrix.
- $H_{0m}$  – Base-link – manipulator coupling inertia matrix – as a  $[6 \times n_q]$  matrix.
- $H_m$  – Manipulator inertia matrix – as a  $[n_q \times n_q]$  matrix.

To obtain the full generalized inertia matrix  $H$ :

```
%Compute H
[H0, H0m, Hm] = GIM(M0_tilde, Mm_tilde, Bij, Bi0, P0, pm, robot);
H=[H0,H0m;H0m';Hm];
```

See also: `src.kinematics_dynamics.CIM()` .

---

**`CIM(t0, tL, I0, Im, M0_tilde, Mm_tilde, Bij, Bi0, P0, pm, robot)`**

Computes the Generalized Convective Inertia Matrix  $C$  of the multibody system.

**Parameters:**

- $t_0$  – Base-link twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times 1]$  matrix.
- $t_L$  – Manipulator twist  $[\omega, \dot{r}]$ , projected in the inertial CCS – as a  $[6 \times n]$  matrix.
- $I_0$  – Base-link inertia matrix, projected in the inertial CCS – as a  $[3 \times 3]$  matrix.
- $I_m$  – Links inertia matrices, projected in the inertial CCS – as a  $[3 \times 3 \times n]$  matrix.
- $M_0\_tilde$  – Base-link mass composite body matrix – as a  $[6 \times 6]$  matrix .
- $M_m\_tilde$  – Manipulator mass composite body matrix – as a  $[6 \times 6 \times n]$  matrix.
- $B_{ij}$  – Twist-propagation matrix (for manipulator  $i > 0$  and  $j > 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $B_{i0}$  – Twist-propagation matrix (for  $i > 0$  and  $j = 0$ ) – as a  $[6 \times 6 \times n]$  matrix.
- $P_0$  – Base-link twist-propagation “vector” – as a  $[6 \times 6]$  matrix.
- $pm$  – Manipulator twist-propagation “vector” – as a  $[6 \times n]$  matrix.
- $robot$  – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- $C_0$  -> Base-link convective inertia matrix – as a  $[6 \times 6]$  matrix.
- $C_{0m}$  -> Base-link - manipulator coupling convective inertia matrix – as a  $[6 \times n_q]$  matrix.
- $C_{m0}$  -> Manipulator - base-link coupling convective inertia matrix – as a  $[n_q \times 6]$  matrix.
- $C_m$  -> Manipulator convective inertia matrix – as a  $[n_q \times n_q]$  matrix.

To obtain the full convective inertia matrix  $C$ :

```
%Compute the Convective Inertia Matrix C
[C0, C0m, Cm0, Cm] = CIM(t0,tL,I0,Im,M0_tilde,Mm_tilde,Bij,Bi0,P0,pm,robot)
C=[C0,C0m;Cm0,Cm];
```

See also: `src.kinematics_dynamics.GIM()` .

## Robot Model

---

### `urdf2robot(filename, verbose_flag)`

Creates a SPART robot model from a URDF file.

`[robot,robot_keys] = urdf2robot(filename,verbose_flag)`

- Parameters:**
- `filename` – Path to the URDF file.
  - `verbose_flag` – True for verbose output (default False).

- Returns:**
- `robot` – Robot model (see [SPART Tutorial – Robot Model](#)).
  - `robot_keys` – Links/Joints name map (see [SPART Tutorial – Robot Model](#)).

This function was inspired by: [https://github.com/jhu-lcsr/matlab\\_urdf/blob/master/load\\_ne\\_id.m](https://github.com/jhu-lcsr/matlab_urdf/blob/master/load_ne_id.m)

---

### `DH_Serial2robot(DH_data)`

Transforms a description of a multibody system, provided in Denavit-Hartenberg parameters, into the SPART robot model.

`[robot,T_Ln_EE] = DH_Serial2robot(DH_data)`

- Parameters:**
- `DH_data` – Structure containing the DH parameters. (see [Using the Denavit-Hartenberg convention with SPART](#)).

- Returns:**
- `robot` – Robot model (see [SPART Tutorial – Robot Model](#)).
  - `T_Ln_EE` – Homogeneous transformation matrix from last link to end-effector –[4x4].

DH descriptions are only supported for serial configurations.

---

### `ConnectivityMap(robot)`

Produces the connectivity map for a robot model.

[branch,child,child\_base]=ConnectivityMap(robot)

**Parameters:**

- robot – Robot model (see [SPART Tutorial – Robot Model](#)).

**Returns:**

- branch – Branch connectivity map. This is a [nxn] lower triangular matrix. If the i,j element is 1 it means that the ith and jth link are on the same branch.
- child – A [nxn] matrix. If the i,j element is 1, then the ith link is a child of the jth link.
- child\_base – A [nx1] matrix. If the ith element is 1, the ith link is connected to the base-link.

See also: `src.robot_model.urdf2robot()` and `src.robot_model.DH_Serial2robot()` .

## Attitude Transformations

---

### Angles321\_DCM([Angles](#))

Convert the Euler angles (321 sequence), x-phi, y-theta, z-psi to its DCM equivalent.

DCM = Angles321\_DCM(Angles)

**Parameters:**

- Angles – Euler angles [x-phi, y-theta, z-psi] – [3x1].

**Returns:**

- DCM – Direction Cosine Matrix – [3x3].

See also: `src.attitude_transformations.Angles123_DCM()` and `src.attitude_transformations.DCM_Angles321()` .

### Angles123\_DCM([Angles](#))

#codegen Convert the Euler angles (123 sequence), x-phi, y-theta, z-psi to DCM.

DCM = Angles123\_DCM(Angles)

**Parameters:**

- Angles – Euler angles [x-phi, y-theta, z-psi] – [3x1].

**Returns:**

- DCM – Direction Cosine Matrix – [3x3].

See also: `src.attitude_transformations.Angles321_DCM()` and `src.attitude_transformations.DCM_Angles321()` .

# Utilities

---

## SkewSym(*x*)

Computes the skew-symmetric matrix of a vector, which is also the left-hand-side matricial equivalent of the vector cross product

$[x\_skew] = \text{SkewSym}(x)$

**Parameters:**

- $x$  -  $[3 \times 1]$  column matrix (the vector).

**Returns:**

- $x\_skew$  -  $[3 \times 3]$  skew-symmetric matrix of  $x$ .