SPART Tutorial – Introduction

This tutorial covers the basic functionality of SPART and introduces some concepts of multibody systems. The tutorial is structured in three sections:

- Introduction – Covers the nomenclature and conventions used by SPART.
- SPART Tutorial – Robot Model – Covers the URDF description of a multibody system and the SPART `robot` structure.
- SPART Tutorial – Kinematics – Covers the kinematics – positions, orientations, velocities, and accelerations – of the system.
- SPART Tutorial – Dynamics – Covers how to obtain the inertia matrices, equations of motion, and solve the forward/inverse dynamic problem.

The code in this tutorial can be found in `examples/URDF_Tutorial/URDF_Tutorial.m`.

Note

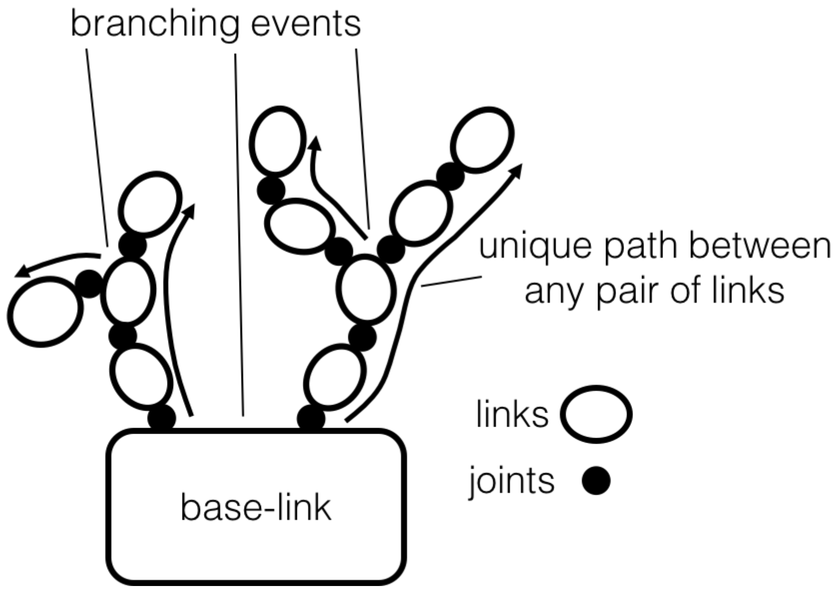
Before starting this tutorial make sure you have correctly installed and configured SPART. See Installing SPART for instructions.

Kinematic tree topology

A multibody system is defined as a collection of bodies coupled by massless **joints**. The bodies of the system – also known as **links** – are arranged in one of two basic types of kinematic chains:

- Kinematic trees, when the path between any pair of links is unique. This are also known as open-loop kinematic chains.
- Closed-loop kinematic chains, when the path between any pair of links is not unique.

SPART is only able to handle multibody systems composed of **rigid bodies** arranged in **kinematic trees**.



Kinematic tree.

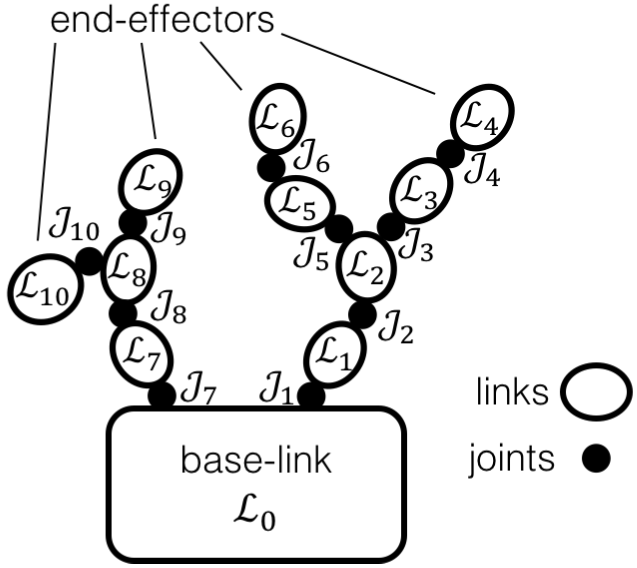
Joint/Link nomenclature and numbering scheme

A link is denoted by $\mathcal{L}_i$ and a joint by $\mathcal{J}_i$, with the number i used as a unique identifier of each link or joint. In SPART, the number i is used to access the results associated with a specific link or joint.

In a kinematic tree, one of the links is designated as the **base-link**, with $i=0$ and $\mathcal{L}_0$. The base-link can be selected arbitrarily among all the links, yet an obvious choice usually exists.

A link $\mathcal{L}_i$ can be connected with an arbitrary number of other links via an equal number of joints. Only one of these other links lies within the path connecting $\mathcal{L}_i$ and the base-link $\mathcal{L}_0$. This *previous/upstream* link $\mathcal{L}_{i-1}$ is known as the *parent link* of $\mathcal{L}_i$ and the joint connecting these two links is $\mathcal{J}_i$. The rest of links directly connected to link $\mathcal{L}_i$ are *child links* $\mathcal{L}_{i+1}$. A *branching event* occurs when a link has multiple children.

In the regular number scheme used by SPART, each children link is given a higher number i than its parent, with the base link given the number $i=0$. In a branching event multiple numbering options exist and they can be chosen arbitrarily among them. The notation $i+1$ and $i-1$ is here abused to denote the child and parent link or joint, even when they are not sequentially numbered. Additionally, the total number of joints n is also used to refer to the last joint and link of a branch. The last link of a branch is also commonly referred to as an **end-effector**.



Regular numbering scheme.

Joints types

Three type of joints primitives can be modeled with SPART:

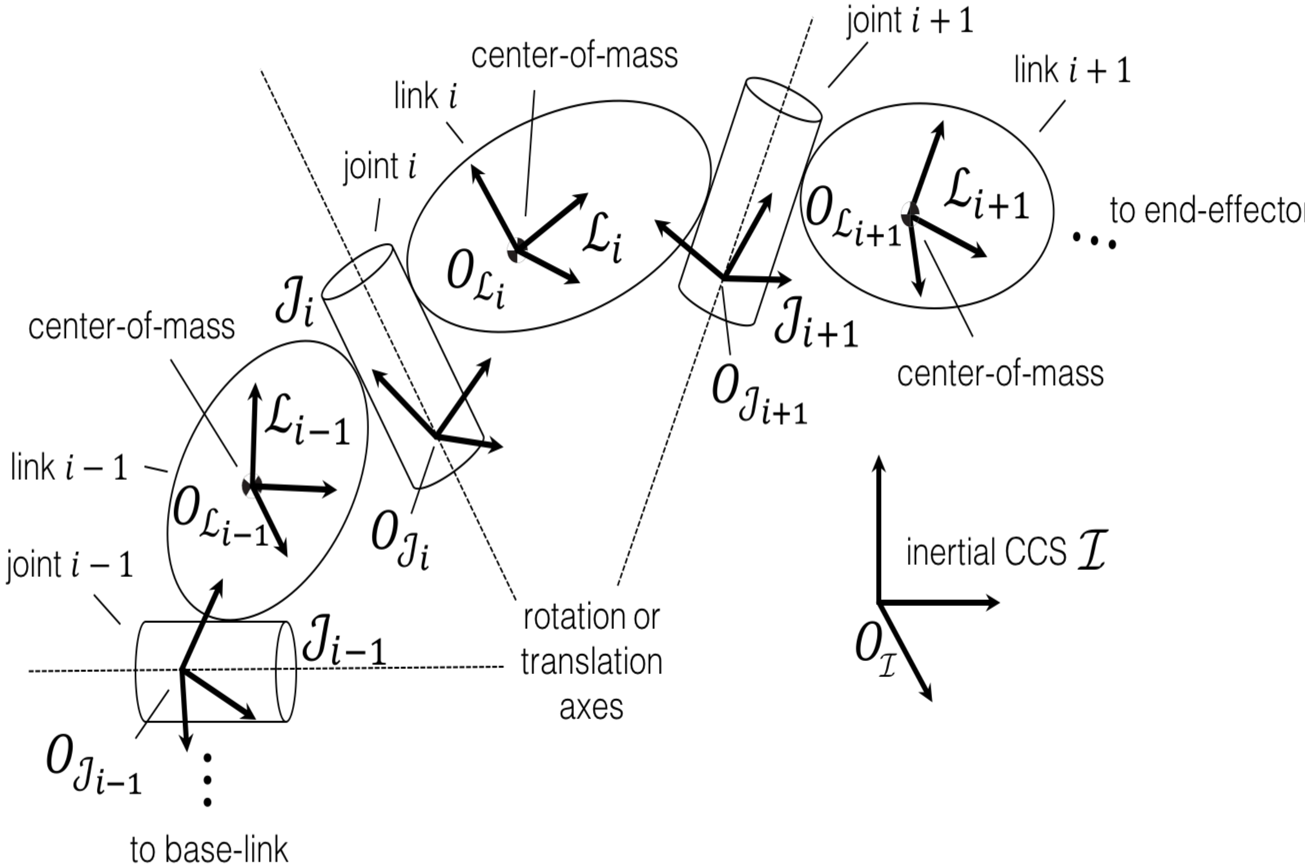
- Fixed – a joint rigidly connecting the pair of links (zero degrees-of-freedom).
- Revolute – a joint that allows a rotation around a common rotation axis $\hat{e}_i$ (one degree-of-freedom).
- Prismatic – a joint that allows a translation along a common sliding axis $\hat{e}_i$ (one degree-of-freedom).

More complex joints (*e.g.*, planar, spherical, helical, …) can be constructed as a combination of these primitive joints connected by massless and dimensionless links.

Only revolute and prismatic joints are active joints, with the rotation or translation **displacement** denoted by q_i.

Cartesian Coordinate Systems (CCS)

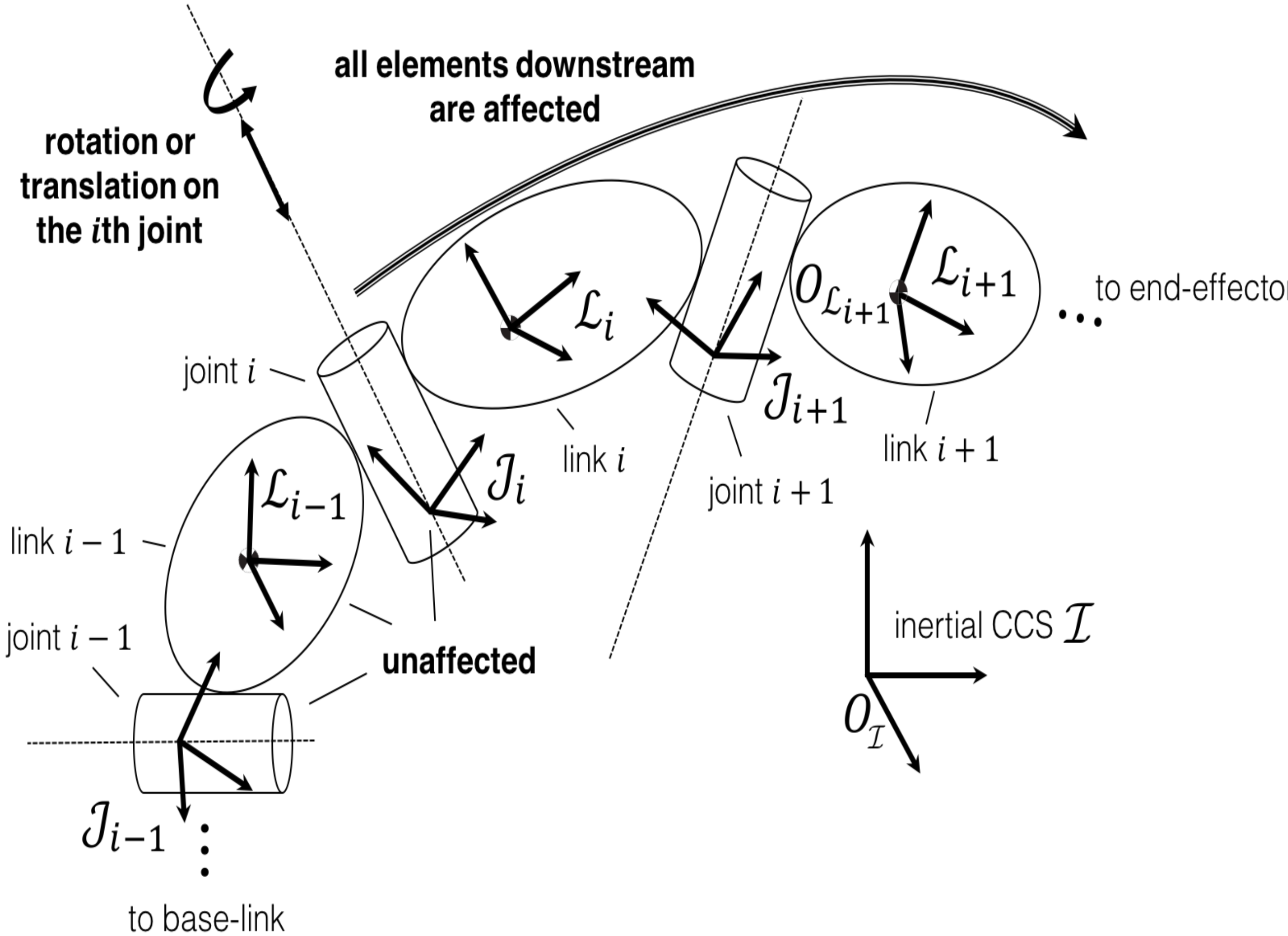
Each link and joint has an associated Cartesian Coordinate System (CCS). The origin of the link CCS is located at the link's center-of-mass, and the origin of the joint CSS is located on the rotation/sliding axis. The orientations of the CCS are arbitrary.



Cartesian Coordinate Systems.

Joint displacements

Another convention in SPART is that a displacement on a joint affects all the elements downstream, but it doesn't affect the orientation or position of that joint CCS.



Effects of a joint displacement.