Ricevimento 18-11-21

Esercizi sui sistemi da pag 26 a pag 70

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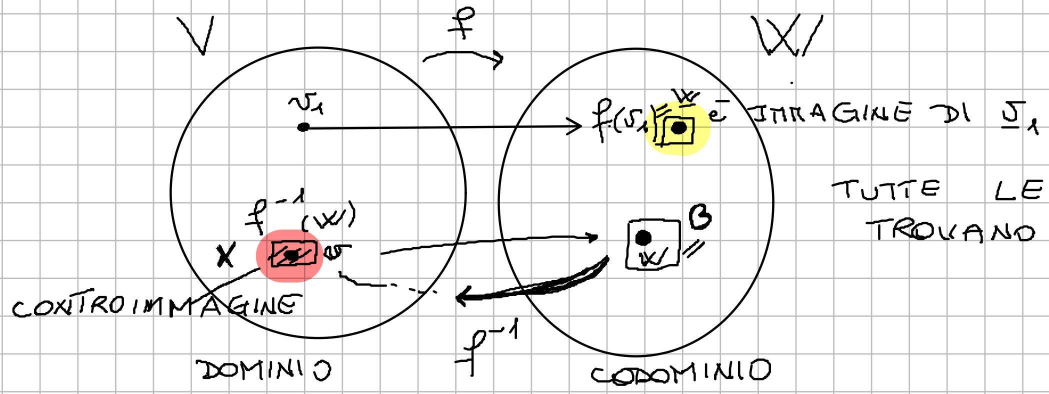
SISTEMI LINEARI SI TROVANO NELLA CONTROIMMAGINE $f^{-1}(\dots,\dots,\dots,\dots)$

ESEMPIO

1) CALCOLARE, AL VARIARE DI h, LA CONTROIMMAGINE $f^{-1}(1,0,1)$



$\underline{w} = f(\underline{v}_1)$

TUTTE LE IMMAGINI SI TROVANO NEL CODOMINIO

TUTTE LE CONTROIMMAGINI STANNO NEL DOMINIO

tale che

$$\boxed{f^{-1}(\underbrace{1,0,1}_{w})} = \{ v \in V \mid f(v) = w \}$$

cercare $\underline{X}$

$$\Downarrow$$

$$f(v) = \underline{w}$$

$$\underline{A \cdot X} = \underline{W}$$

CORRISPONDE

$$A\,X = B$$

$$(A,B) = \left(\quad A \quad , \quad B \quad \right)$$

$$\left(\quad A \quad , \quad \begin{matrix} 1 \\ 0 \\ 1 \end{matrix} \quad \right)$$

CALCOLARE $f^{-1}(\underbrace{\boxed{1,0,1}}_{B})$ AVRÒ SIST. LINEARE LA CUI COLONNA B DEI TERMINI NOTI È $\underbrace{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}_{B}$

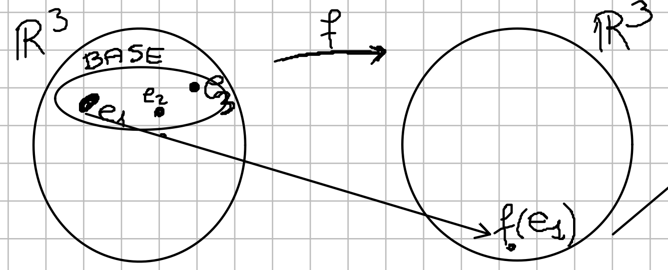
A È LA MATRICE RICAVATA DALL'APPLICAZ. LINEARE, DAI DATI ASSEGNATI NEL TESTO

$$\begin{cases} \cdot\, f(-1,0,0) = (-1-h,\, 0,\, -h) \\ \cdot\, f(1,0,1) = (-h+1,\, 0,\, 3h) \\ \cdot\, f(0,1,-1) = (3h+2,\, -1,\, 3h) \end{cases}$$

$f(0e_1+1e_2-1e_3)$

$$M(f) = A = \begin{array}{ccc} c_1 & c_2 & c_3 \\ \downarrow & \downarrow & \downarrow \end{array} \begin{pmatrix} f(e_1) & f(e_2) & f(e_3) \\ \vdots & \vdots & \vdots \end{pmatrix}$$

$e_1 = (1,0,0)$ $\quad e_2 = (0,1,0)$ $\quad e_3 = (0,0,1)$



$(-1,0,0) = -e_1 \Rightarrow f(-e_1) = (-1-h,\, 0,\, -h)$

LINEARITÀ N°2 DI STAMATTINA

$-f(e_1) = (-1-h,\, 0,\, -h)$

CAMBIO DI SEGNO:

$f(e_1) = (1+h,\, 0,\, h)$

$$A = \begin{array}{c} c_1 \\ \downarrow \end{array} \begin{pmatrix} 1+h & ? & ? \\ 0 & & \\ h & & \end{pmatrix}$$

$(1,0,1) = 1\,\hat{\imath} + 0\,\hat{\jmath} + 1\,\hat{k} = 1\,e_1 + 0\,e_2 + 1\,e_3 = e_1 + e_3$

$(1,0,1) = e_1 + e_3$ $\qquad \cdot\, \underbrace{f(1,0,1) = (-h+1,\, 0,\, 3h)}_{\text{SI TRASFORMA IN}}$

$f(e_1+e_3) = (-h+1,\, 0,\, 3h)$

LINEARITÀ N°1

$f(e_1) + f(e_3) = (-h+1,\, 0,\, 3h)$

$$\begin{cases} -f(e_1) = (-1-h,\, 0,\, -h) \\ f(e_1) + f(e_3) = (-h+1,\, 0,\, 3h) \\ f(e_2) - f(e_3) = (3h+2,\, -1,\, 3h) \end{cases}$$

DATI

$$\begin{cases} f(e_1) = (h+1,\, 0,\, h) \\ (h+1,0,h) + f(e_3) = (-h+1,\, 0,\, 3h) \\ \;\dots \end{cases} \Rightarrow \begin{cases} f(e_1) = \dots \\ f(e_3) = (-h+1,0,3h) - (h+1,0,h) \\ \dots \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} f(e_1) = \dots \\ f(e_3) = (-h+\cancel{1}-h-\cancel{1},\, 0,\, 3h-h) = (-2h,\, 0,\, 2h) \\ f(e_2) - (-2h,0,2h) = (3h+2,-1,3h) \end{cases} \Rightarrow f(e_2) = (3h+2,-1,3h) + (-2h,0,2h)$$

$$\Rightarrow \begin{cases} f(e_1) = \dots \\ f(e_3) = \dots \\ f(e_2) = (3h+2-2h,\, -1,\, 5h) \end{cases}$$

$$\begin{cases} f(e_1) = (h+1,\, 0,\, h) & 1° \text{ COLONNA} \\ f(e_3) = (-2h,\, 0,\, 2h) & 3° \text{ COLONNA} \\ f(e_2) = (h+2,\, -1,\, 5h) & 2° \text{ COLONNA} \end{cases}$$

$$M(f)=A=\begin{pmatrix} h+1 & h+2 & -2h \\ 0 & -1 & 0 \\ h & 5h & 2h \end{pmatrix}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ f(e_1) & f(e_2) & f(e_3) \end{array}$$

CONTROIMMAGINE DI $(1,0,1)$ ERA $A\underline{X}=B$ $\quad B=\begin{pmatrix}1\\0\\1\end{pmatrix}$

SIST. LINEARE

$$\begin{pmatrix} h+1 & h+2 & -2h \\ 0 & -1 & 0 \\ h & 5h & 2h \end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\0\\1\end{pmatrix} \iff \begin{cases}(h+1)x_1+(h+2)x_2-2hx_3=1\\ -x_2=0\\ hx_1+5hx_2+2hx_3=1\end{cases}$$

DISCUTIAMO E RISOLVIAMO IL SIST. LINEARE

$$(A,B)=\left(\begin{array}{ccc|c} h+1 & h+2 & -2h & 1 \\ 0 & -1 & 0 & 0 \\ h & 5h & 2h & 1 \end{array}\right)$$

$A_{3\times3}$ "SISTEMA QUADRATO"

3 equ. in 3 incognite

APPLICO CRAMER "SIST DETERMINATO $\iff$ DET $A\neq0$" E POI LA FORMULA

Calcoliamo $\text{DET}A=-2h(h+1)-2h^2=-2h^2-2h-2h^2=\boxed{-4h^2-2h}$

SE $\text{DET}A\neq0$ $\quad -4h^2-2h\neq0\to \cancel{4}h^2+\cancel{2}h\neq0\to 2h^2+h\neq0$

$\to \underline{h(2h+1)}\neq0 \to$ $h\neq0$, $h\neq-\frac{1}{2}$ ALLORA SISTEMA DETERMINATO (SOLO UNA SOLUZ)

Qual è? LA DARÀ LA FORMULA DI CRAMER

$$x_1=\frac{\text{DET}\,B_1}{\text{DET}\,A}=\frac{\text{DET}\begin{pmatrix}1 & h+2 & -2h\\ 0 & -1 & 0\\ 1 & 5h & 2h\end{pmatrix}}{-4h^2-2h}=\frac{-\cancel{4h}^{\,2}}{-\cancel{2h}(2h+1)}=\frac{2}{2h+1}$$

$$x_2=\frac{\text{DET}\,B_2}{\text{DET}\,A}=\frac{\text{DET}\begin{pmatrix}h+1 & 1 & -2h\\ 0 & 0 & 0\\ h & 1 & 2h\end{pmatrix}}{-4h^2-2h}=\frac{0}{--}=0$$

$$x_3=\frac{\text{DET}\,B_3}{\text{DET}\,A}=\frac{\text{DET}\begin{pmatrix}h+1 & h+2 & 1\\ 0 & -1 & 0\\ h & 5h & 1\end{pmatrix}}{-4h^2-2h}=\frac{-\cancel{h}-1+\cancel{h}}{-4h^2-2h}=\frac{1}{4h^2+2h}$$

$$S=\left\{\left(\frac{2}{2h+1},\ 0,\ \frac{1}{4h^2+2h}\right)\right\}\quad h\neq0,-\frac{1}{2}$$

LE VIAMO LE DUE CONDIZIONI:

- $h = 0$ RICOMINCIAMO

$$(A,B) = \left(\begin{array}{ccc|c} \boxed{1} & 2 & 0 & 1 \\ 0 & \boxed{-1} & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{array}\right)$$

$$\underbrace{\quad\quad\quad}_{\det A = 0} \qquad B$$

CRAMER NON È APPLICABILE → PASSO A R.C. N°1

$\rho(A) = ?$

$\rho(A) = 2$; $\rho(A,B) = ?$ HO SOLO UNA 3X3 $\rho(A,B) = 3$

SIST IMPOSSIBILE $S = \emptyset$

- $h = -\frac{1}{2}$

$$(A,B) = \left(\begin{array}{ccc|c} 1/2 & -\frac{1}{2}+2 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ -1/2 & -5/2 & -1 & 1 \end{array}\right)$$

$\neq 0$

$\det A = 0$

CRAMER NON È APPLICABILE ⇒ R.C. N°1

$\rho(A) < 3$ $\rho(A) = 2$ $\rho(A,B) = ?$

$$\begin{vmatrix} -1/2 & 1 & 1 \\ 0 & 0 & 0 \\ -1/2 & -1 & 1 \end{vmatrix} = 0 ;$$ NE PRENDO UN ALTRO

$$\begin{vmatrix} 1/2 & 3/2 & 1 \\ 0 & -1 & 0 \\ -1/2 & -5/2 & 1 \end{vmatrix} = -\frac{1}{2} - \frac{1}{2} = -1 \neq 0$$

$\rho(A,B) = 3$

$S = \emptyset$

R. Se $h \neq 0, -\frac{1}{2}$ $S = \{(\quad)\}$

Se $h = 0, -\frac{1}{2}$ $S = \emptyset$

Rivediamo i compiti del 2021

25-1-21 LEGGE $f: \mathbb{R}^3 \to \mathbb{R}^3$

$$f(x,y,z) = (hy,\ (h+1)x + hy - z,\ -y - hz)$$

COME TROVARE $M(f) = A$?

$$M(f) = A = \begin{pmatrix} C_1 & C_2 & C_3 \\ f(e_1) & f(e_2) & f(e_3) \\ \vdots & \vdots & \vdots \end{pmatrix}$$

$f(e_1) = f(1,0,0) = \text{coeff. della } x = (0,\ h+1,\ 0)$

$f(e_2) = f(0,1,0) = \text{coeff della } y = (h,\ h,\ -1)$

$f(e_3) = \text{~~~~~~~~} = (0,\ -1,\ -h)$

$$A = \begin{pmatrix} 0 & h & 0 \\ h+1 & h & -1 \\ 0 & -1 & -h \end{pmatrix}$$

21/6/21 $f: \mathbb{R}^3 \to \mathbb{R}^3$

NEI DATI COMPAIONO LE IMMAGINI

$$\begin{cases} \overbrace{f(2,0,0)}^{f(2e_1+0e_2+0e_3)} = (4,0,0) \\ f(1,0,-1) = (h,0,h) \\ \underbrace{f(0,-1,0)}_{f(-e_2)} = (0,-h,0) \end{cases}$$

$$\begin{cases} 2f(e_1) = (4,0,0) \\ f(e_1) - f(e_3) = (h,0,h) \\ -f(e_2) = (0,-h,0) \end{cases} \Rightarrow \begin{cases} f(e_1) = (2,0,0) \\ (2,0,0) - f(e_3) = (h,0,h) \to f(e_3) = (2-h,0,-h) \\ f(e_2) = (0,h,0) \end{cases}$$

$$A = M(f) = \begin{pmatrix} 2 & 0 & 2-h \\ 0 & h & 0 \\ 0 & 0 & -h \end{pmatrix}$$

12/7/21 $f: \mathbb{R}^4 \to \mathbb{R}^4$

DI NUOVO COMPAIONO LE IMMAGINI

$$\begin{cases} f(1,0,0,0) = (1,0,0,0) \\ f(1,0,1,0) = (h,0,2,0) \\ f(0,1,0,0) = (0,h-1,0,h) \\ f(0,0,0,1) = (0,0,0,h) \end{cases} \to \begin{cases} f(e_1) = (1,0,0,0) & C_1 \\ f(e_1) + f(e_3) = (h,0,2,0) & \\ f(e_2) = (0,h-1,0,h) & C_2 \\ f(e_4) = (0,0,0,h) & C_4 \end{cases}$$

$\to f(e_3) = (h,0,2,0) - f(e_1) = (h,0,2,0) - (1,0,0,0) = \underline{(h-1,0,2,0)}_{C_3}$

$$\underset{4\times 4}{A = M(f)} = \begin{pmatrix} 1 & 0 & h-1 & 0 \\ 0 & h-1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & h & 0 & h \end{pmatrix}$$

15/9/21 TORNA LA LEGGE

$f: \mathbb{R}^3 \to \mathbb{R}^4 \qquad f(x,y,z) = (x-y, -x+hz, x+z, -x+hy)$

$f(e_1) = f(1,0,0) =$ coeff. della x $= (1,-1,1,-1) \leftarrow C_1$

$f(e_2) = f(0,1,0) =$ coeff della y $= (-1,0,0,h) \leftarrow C_2$

$f(e_3) = \text{~~~~~~~~} = (0,h,1,0) \leftarrow C_3$

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & h \\ 1 & 0 & 1 \\ -1 & h & 0 \end{pmatrix}$$

4×3

30/9/21 TORNA LA LEGGE

$f: \mathbb{R}^3 \to \mathbb{R}^3 \qquad f(x,y,z) = (0, y+hz, hx+hy+z)$

DIRETTAMENTE

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & h \\ h & h & 1 \end{pmatrix}$$