Es. $m[1,5] = \min_{1 \le k < 5} \left(m[1,k] + m[k+1,5] + P_0 P_k P_5 \right)$

$$m[i,j] \begin{cases} 0 & i = j \\ i \le k < j \end{cases} \begin{cases} \min \left\{ m[i,k] + m[k,j] + P_{i-1} \cdot P_k \cdot P_j \right\} \end{cases}$$

Matrice dei valori di k che minimizzano l'espressione {}

Matrix chain multiply (A, S, i, j)

```
IF i = j
    RETURN A[i]
ELSE
    X ← MATRIX CHAIN MULTIPLY (A, S, i, S[i,j])
    Y ← MATRIX CHAIN MULTIPLY (A, S, S[i,j]+1, j)
    RETURN MATRIX CHAIN MULTIPLY (X, Y)
```

P = (30, 35, 15, 5, 10, 20 25)
0 1 2 3 4 5 6

	1	2	3	4	5	6
1	0	15750	7875	9375	11875	15125
2	/	0	2625	4375	7125	10500
3	/	/	0	750	2500	5750
4	/	/	/	0	1000	3500
5	/	/	/	/	0	5000
6	/	/	/	/	/	0

m

$m[1][6] =$
1 (10500 + 30·25·35) =
2 (15750 + 5750 + 30·25·15) =
3 (7875 + 3500 + 30·25·5) = 15.125
4 (9375 + 5000 + 30·25·10) =
5 (11875 + 30·25·20) =

	1	2	3	4	5	6
1	/	1	1	3	3	3
2	/	/	2	3	3	3
3	/	/	/	3	3	3
4	/	/	/	/	4	5
5	/	/	/	/	/	5
6	/	/	/	/	/	/

S

$A_1 \; A_2 \; A_3 \; A_4 \; A_5 \; A_6$?

$$m[i,j] \begin{cases} 0 & i=j \\ i \le k < j \left\{ \min \left\{ m[i,k] + m[k,j] + P_{i-1} \cdot P_k \cdot P_j \right\} \right. \end{cases}$$

$P = (30, 35, 15, 5, 10, 20, 25)$ (índices 0 1 2 3 4 5 6)

$m[1,2] = \min\left((1)\ m[1][1] + m[2][2] + 30\ 35\ 15\right)$

$= \min(0 + 0 + 15.750) = 15.750$

$m[2,3] = \min\left((2)\ m[2,2] + m[3,3] + P_1 \cdot P_2 \cdot P_3\right) = 2625$

$m[3,4] = \min\left((3)\ m[3,3] + m[4,4] + P_2 \cdot P_3 \cdot P_4\right) = 750$

$m[4,5] = \min\left((4)\ m[4,4] + m[5,5] + P_3 \cdot P_4 \cdot P_5\right) = 1.000$

$m[5,6] = \min\left((5)\ m[5,5] + m[6,6] + P_4 \cdot P_5 \cdot P_6\right) = 5.000$

$m[1,3] = \min\{(1)\ 0 + 2625 + 30 \cdot 35 \cdot 5) = 7875$

$(2)\ 15750 + 0 + 30 \cdot 15 \cdot 5) = 18.000\} = 5250$

$m[2][4] = \min\{2\ (0 + 750 + 35 \cdot 15 \cdot 10 = 6000$

$3\ (2625 + 0 + 35\ 5\ 10 = 4375\} = 4375$