19 dicembre 2022

lunedì 19 dicembre 2022 08:57

$$\int \frac{dx}{e^x+5} = \int \frac{e^x}{e^x(e^x+5)}\,dx = \left[\int \frac{dt}{t(t+5)}\right]_{t=e^x}$$

$$\frac{1}{t(t+5)} = \frac{A}{t} + \frac{B}{t+5}$$

$$\int \cos^7 x\,dx = \int \cos x\,(1-\sin^2 x)^3\,dx = \left[\int (1-t^2)^3\,dt\right]_{t=\sin x}$$

$$\int \cos^8 x\,dx = \int \left(\frac{1+\cos 2x}{2}\right)^4 dx$$

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$$

$$\int \sin^9 x\,dx = -\int (-\sin x)\,(1-\cos^2 x)^4\,dx = -\left[\int (1-t^2)^4\,dt\right]_{t=\cos x}$$

$$\int \sin^{12} x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^6 dx$$

$$\int \sin^5 x\,\cos^7 x\,dx = \int \cos x\cdot \sin^5 x\,(1-\sin^2 x)^3\,dx =$$
$$= \left[\int t^5(1-t^2)^3\,dt\right]_{t=\sin x}$$

opp. $-\int (-\sin x)(1-\cos^2 x)^2 \cos^7 x\,dx = -\left[\int (1-t^2)^2\, t^7\,dt\right]_{t=\cos x}$

FRATTE $\quad \dfrac{a(x)}{b(x)}$

- $b(x)=0 \qquad \alpha$ sol di quest'eq $\Rightarrow$ il denom. presenta un fattore $x-\alpha$

se $\alpha = b+ic$ anche $b-ic$ è sol con la stessa molteplicità

$$[x-(b+ic)]\,[x-(b-ic)] = [(x-b)-ic]\,[(x-b)+ic] =$$
$$= (x-b)^2 - (ic)^2 = (x-b)^2 + c^2 \quad c>0$$

se $b(x) = (x-3)^2\, x\,(x^2+5)\,[(x-2)^2+7]^3$ i fratti sempl. sono

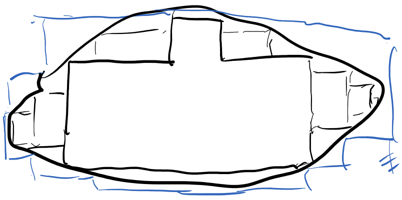
$$\frac{A_1}{x-3} \quad \frac{A_2}{(x-3)^2} \quad \frac{A_3}{x} \quad \frac{B_1x+C_1}{x^2+5} \quad \frac{B_2x+C_2}{(x-2)^2+7} \quad \frac{B_3x+C_3}{((x-2)^2+7)^2} \quad \frac{B_4x+C_4}{[(x-2)^2+7]^3}$$

$$\int \frac{dx}{(x-2)^2+7} = \frac{1}{\sqrt7}\,\operatorname{arctg}\frac{x-2}{\sqrt7} + k$$

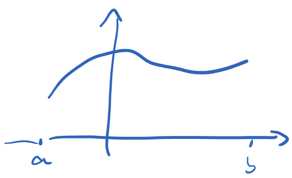
$$D\,\frac{1}{x^2+c^2} = \frac{-2x}{(x^2+c^2)^2}$$

$$\int \frac{dx}{(x^2+c^2)^2} = \frac{1}{c^2}\int \frac{c^2+x^2-x^2}{(\qquad)^2}\,dx = \frac{1}{c^2}\int\frac{dx}{(x^2+c^2)} + \frac{1}{c^2}\,\frac{1}{2}\int \frac{-2x}{(x^2+c^2)^2}\cdot x\,dx$$

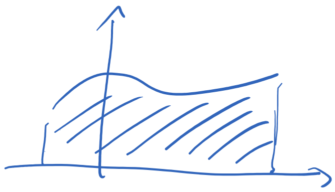
$$\int \frac{dx}{(x^2+c^2)^2} = \frac{1}{c^2}\int \frac{c^2+x^2-x^2}{(\quad)^2}\,dx = \frac{1}{c^2}\int\frac{dx}{(x^2+c^2)} + \frac{1}{c^2}\frac{1}{2}\int \underbrace{\frac{-2x}{(x^2+c^2)^2}}_{D\left(\frac{1}{x^2+c^2}\right)}\cdot x\,dx$$

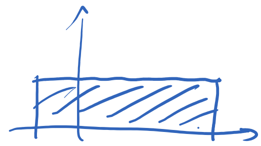


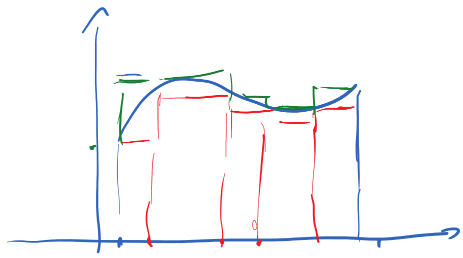
$\sup \underline{A} = \inf \overline{A}$



area $\mathrm{gr}(f) = 0$







DER. FUNZ. INTEGRALE

Se $c > x$

$$\frac{f(x)-f(c)}{x-c} = \frac{\int_c^x f(t)\,dt}{x-c} = \frac{-\int_x^c f(t)\,dt}{x-c} =$$

$$= \frac{\int_x^c f(t)\,dt}{c-x} \leftarrow \bar{x} \text{ di Riemann}$$

$$= f(\bar{x}) \qquad \bar{x} \in\,]x, c[$$

$$f(x) = \int_x^2 e^{t^2}\,dt = -\int_2^x e^{t^2}\,dt \qquad f'(x) = -e^{x^2}$$

$$f(x) = \int_{3x+1}^{\cos x} e^{t^2}\,dt = \int_{3x+1}^{0} \ldots + \int_0^{\cos x} = -\int_0^{3x+1} \ldots + \int_0^{\cos x}$$

$$f'(x) = -e^{(3x+1)^2}\,3 + e^{\cos^2 x}(-\sin x)$$

eq. tang al grafico di $f(x)=\int_6^x e^{t^2}dt$ nel p. di asc. $x=6$

$$y=f(c)+f'(c)(x-c)$$

$f(6)=0$

$f'(x)=e^{x^2}$ $\qquad f'(6)=e^{36}$ $\qquad y=e^{36}(x-6)$

f prim di e^{x^2} tale che $f(4)=2$

$$\int e^{x^2}dx=\int_4^x e^{t^2}dt+k$$

$f(4)=2$

$\int_4^4 e^{t^2}dt+k=2 \qquad k=2$

$$I=\int \underset{\substack{\uparrow\\ f\,D}}{(x-2)}\log\frac{x+3}{x}\,dx=\left(\frac{x^2}{2}-2x\right)\log\frac{x+3}{x}-$$

$$-\int\frac{x^2-4x}{2}\,\frac{\cancel{x}}{x+3}\,\frac{\cancel{x}-\cancel{x}-3}{x^{\cancel{2}}}=$$

$$=\left(\frac{x^2}{2}-2x\right)\log\frac{x+3}{x}+\frac{3}{2}\underbrace{\int\frac{x^2-4x}{x^2+3x}\,dx}_{J}$$

$$\frac{x^2-4x}{x^2+3x}=\frac{x^2+3x-7x}{x^2+3x}=1-\frac{7}{x+3}$$

$$J=\int\left(1-\frac{7}{x+3}\right)dx=x-7\log|x+3|+k$$

$$\int \underset{\substack{\uparrow\\ f\,D}}{x^2}\cos 4x\,dx=\frac{\sin 4x}{4}x^2-\int\frac{\sin 4x}{4}2x\,dx=$$

$$=\frac{\sin 4x}{4}x^2+\frac{1}{2}\int\underset{\substack{\uparrow\\ f\,D}}{(-\sin 4x)}\,x\,dx=$$

$$=\frac{\sin 4x}{4}x^2+\frac{\cos 4x}{8}-\frac{1}{2}\int\cos 4x\,dx=$$

$$=\frac{\sin 4x}{4}x^2+\frac{\cos 4x}{8}-\frac{\sin 4x}{8}+k$$

$$\int x^2\cos 4x\,dx=\frac{1}{4}\int 4x^2\cos(4x)\,dx$$

$$\int\frac{dx}{tg^2x}=\int\frac{1+tg^2x}{tg^2x(1+tg^2x)}\,dx=\left[\underbrace{\int\frac{dt}{t^2(1+t^2)}}_{J}\right]_{t=tg\,x}$$

$$\frac{1}{t^2(1+t^2)}=\frac{A}{t}+\frac{B}{t^2}+\frac{Ct+D}{t^2+1}$$

$$\int x^3 \ldots x^2\,dx=\frac{1}{2}\int 2x\;x^2\cos x^2\,dx=\frac{1}{2}\left[\int t\cos t\,dt\right]$$

$$\int x^3 \cos x^2\, dx = \frac{1}{2}\int 2x \; x^2 \cos x^2\, dx = \frac{1}{2}\left[\int t \cos t\, dt\right]_{t=x^2}$$

per parti

$f'(x) = f(x)$

y funzione incognita

$y' = 2y$

$(\cos x)\, y^{IV} - (\log x)\, y''' + x\, y'' - y' + 2y = 3x + 1$

eq diff del IV ordine

eq diff del I ord. Sia $f : X \subseteq \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y)$

$$y' = f(x,y) \qquad (1)$$

è la ricerca di una funz. $y(x)$ def in un interv. (a,b) derivabile e tale che $\forall x \in (a,b)$

$$(x, y(x)) \in X$$

$$y'(x) = f(x, y(x))$$

y soluzione o integrale della (1)

es. se $f(x,y) = f(x)$ $\quad y' = f(x)$ è il problema della ricerca delle primitive

int. generale = ins di tutte le sol.

" particolare = una sol.

$(x_0, y_0) \in X$
$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$$
PROBLEMA DI CAUCHY