3 ottobre 22

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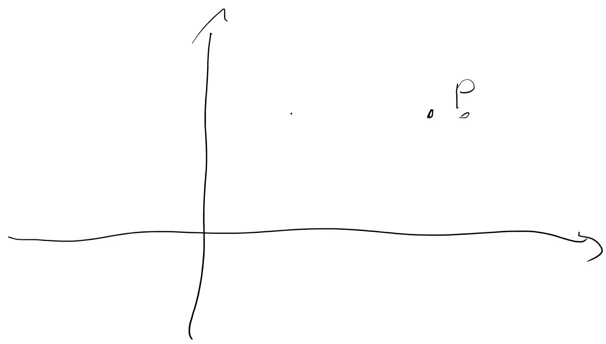
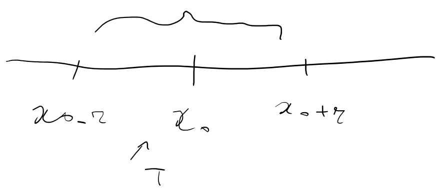


$P_0 \ (x_0, y_0)$

$x, y \in \mathbb{R}$

$X \subseteq \mathbb{R}^2$



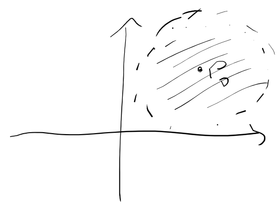
$a, b \in \mathbb{R} \quad d(a,b) = |a-b|$

$x \in I \iff d(x, x_0) < r \iff |x - x_0| < r$

$P_1 \ (x_1, y_1)$

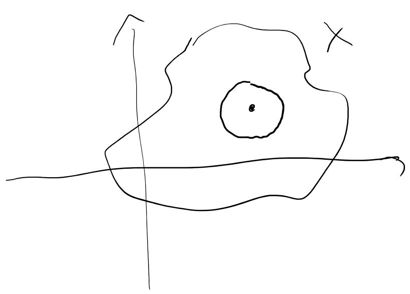
$P_2 \ (x_2, y_2)$

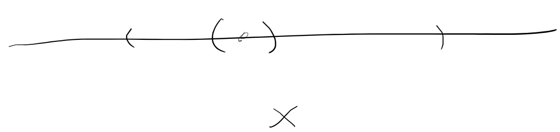
$$d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



$P_0 \ (x_0, y_0) \qquad r > 0$

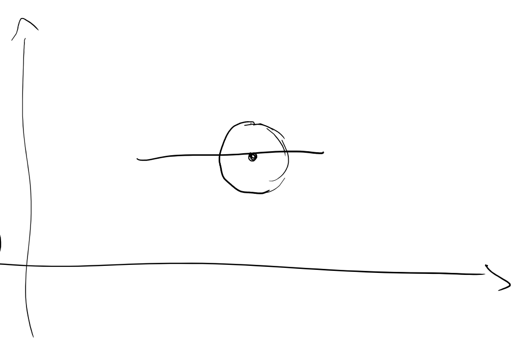
$$B(P_0, r) = I_r(P_0) = \{(x,y) \in \mathbb{R}^2 : \sqrt{(x-x_0)^2 + (y-y_0)^2} < r$$





P_0 interno ad X se $\exists \ r > 0 : B(P_0, r) \subseteq X$

$\text{int}(X) = \mathring{X} =$ interno di X = {P interni ad X}



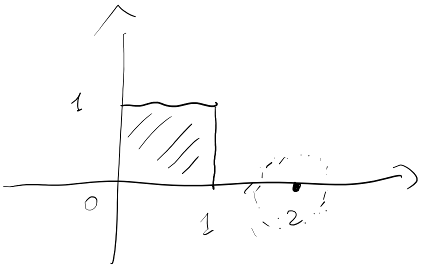
X aperto se $X = \emptyset$ opp. $X \neq \emptyset$ e $X = \text{int}(X)$

X aperto se $X = \emptyset$ opp. $X \neq \emptyset$ e $X = \mathrm{int}(X)$

X chiuso se $\mathbb{R}^2 \setminus X$ è aperto

$\emptyset$ e $\mathbb{R}^2$ sono sia ap. che chiusi

$P_0 \in \mathbb{R}^2$ p. di accumulazione per X se $\forall\, r>0$ $B(P_0, r) \cap \left(X \setminus \{P_0\}\right) \neq \emptyset$



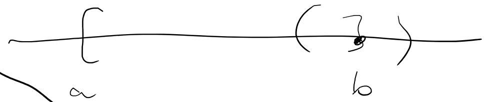
$X = \left([0,1] \times [0,1]\right) \cup \{ \underset{\substack{\uparrow \\ P}}{(2,0)} \}$

$P \in X$ ma non è di accum.

$P_0 \in \mathrm{int}(X) \Rightarrow P_0$ è di acc.

il viceversa no

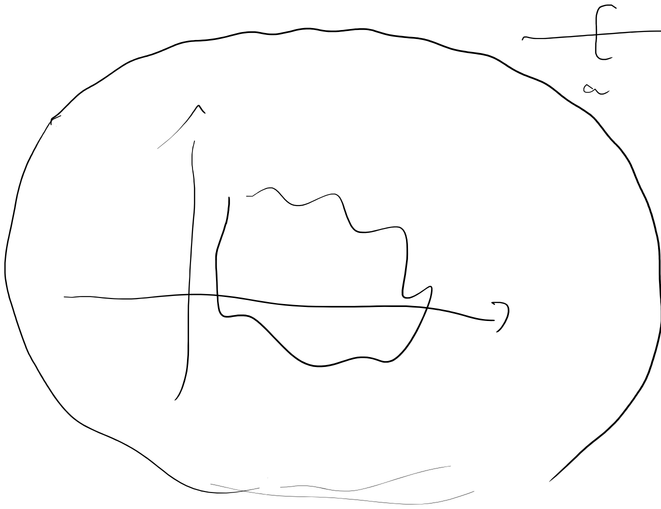
$D(X)$ = derivato di X = ins. dei p. di acc.



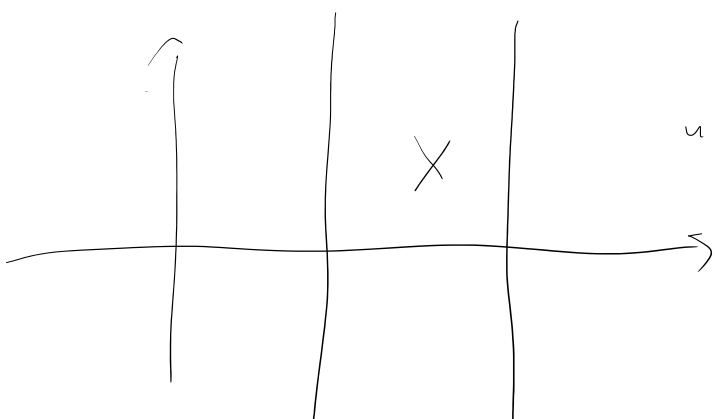
X chiuso

$\Updownarrow$

$D(X)$



X limitato se è contenuto in un cerchio

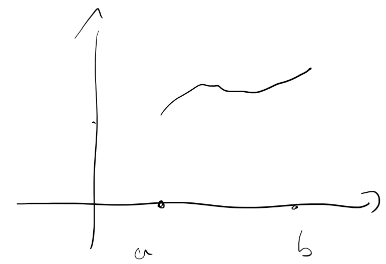


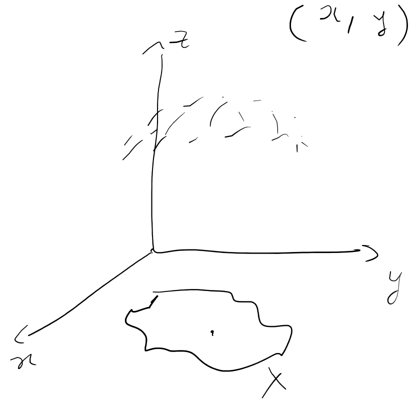
non è lim.

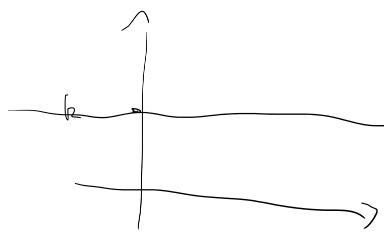
FUNZIONI REALI DI DUE VARIABILI REALI

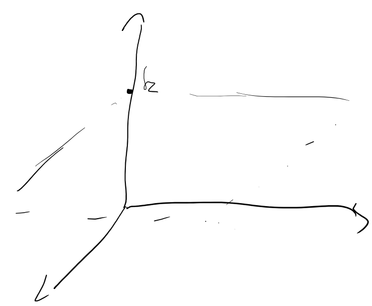
$X \subseteq \mathbb{R}^2 \qquad f : X \to \mathbb{R} \qquad f(x,y)$

$(x,y) \in X \longrightarrow f(x,y) \in \mathbb{R}$









$f(x,y) = k$

$\forall (x,y) \in \mathbb{R}^2$

$f(x,y) = x$

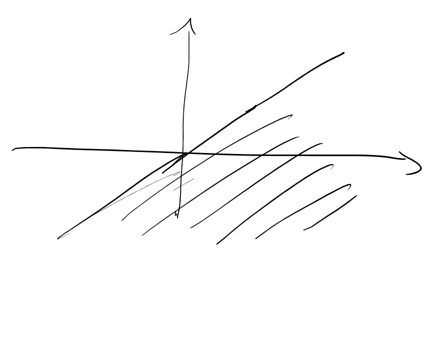
$\forall (x,y) \in \mathbb{R}^2$

$f(x,y) = y$

Esercizi

$f(x,y) = \dfrac{1}{\sqrt{x-y}}$

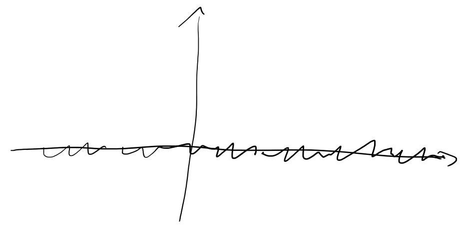
$$\begin{cases} x - y \geq 0 \\ \sqrt{x-y} \neq 0 \end{cases} \Rightarrow x - y > 0$$



$X = \{(x,y) \in \mathbb{R}^2 : x > y\}$

$f(x,y) = \text{arctg}\, \frac{x}{y} \qquad y \neq 0 \qquad X = \{(x,y) : \ldots$

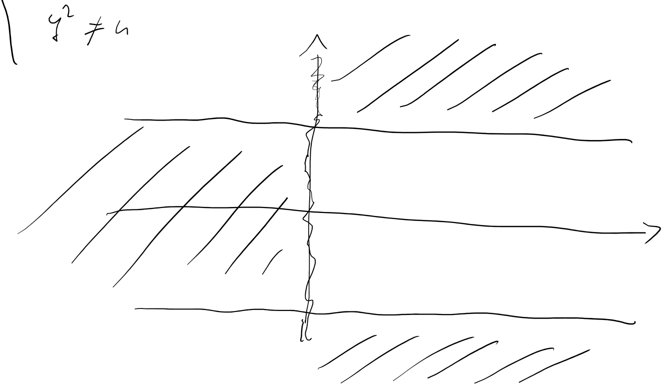
$f(x,y) = \operatorname{arctg} \frac{x}{y}$ $\qquad y \neq 0 \qquad X = \{(x,y) : y \neq 0\}$



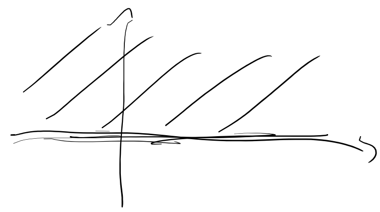
$f(x,y) = \log \frac{x}{y^2-4}$

$$\begin{cases} \frac{x}{y^2-4} > 0 \\ y^2 \neq 4 \end{cases}$$

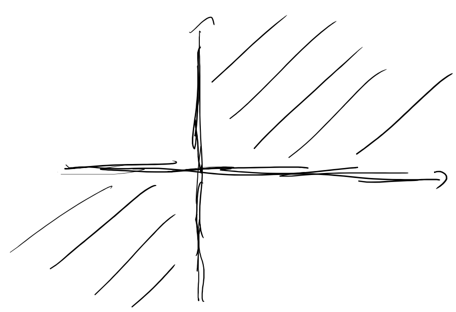
$\begin{matrix} x > 0 \\ y < -2 \vee y > 2 \end{matrix} \quad \vee \quad \begin{matrix} x < 0 \\ -2 < y < 2 \end{matrix}$



$f(x,y) = \sqrt[4]{x^2 y}$ $\qquad y \geq 0$



$f(x,y) = \sqrt[4]{xy}$ $\qquad xy \geq 0$



LIMITI

RIPASSO:

$X \subseteq \mathbb{R} \qquad x_0 \in DX \qquad \ell \in \mathbb{R}$

$\lim_{x \to x_0} f(x) = \ell$ se $\forall \varepsilon > 0 \ \exists \delta > 0$: se $x \in X$, $x \neq x_0$, $|x - x_0| < \delta$

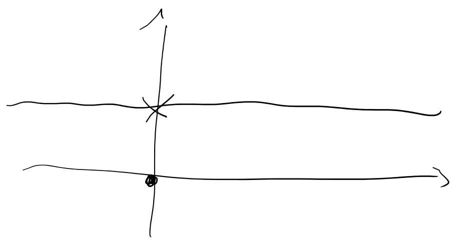
si ha $|f(x) - \ell| < \varepsilon$

$\Updownarrow$

$\Updownarrow$

$x_0 - \delta < x < x_0 + \delta$

$$l-\varepsilon < f(x) < l+\varepsilon$$



$$f(x) = \begin{cases} 1 & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = 1$$

DEF. NUOVA

$f: X \to \mathbb{R}$ $\quad X \subseteq \mathbb{R}^2$ $\quad P_0(x_0, y_0) \in D(X)$ $\quad l \in \mathbb{R}$

DEF. $\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x,y) = l$ se $\forall \varepsilon > 0 \ \exists \delta > 0$: se $\underset{(x,y)}{P} \in B(P_0, \delta)$, $P \neq P_0$ $(\cdot)$

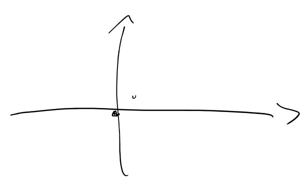
si ha $|f(x,y) - l| < \varepsilon$

$$(\cdot) \qquad 0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$$

DEF. $\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x,y) = \underset{-\infty}{+\infty}$ se $\forall k > 0 \quad \exists \delta > 0$:

se $(x,y) \in X$, $0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$ si ha $f(x,y) \underset{< -k}{> k}$

es. $$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{\sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = 1$$

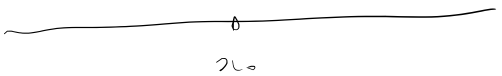


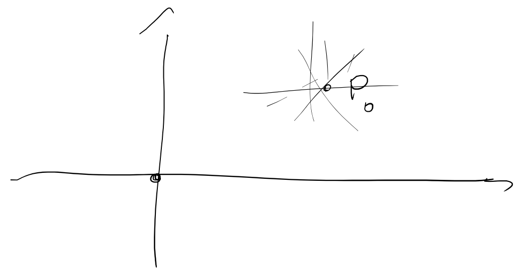
infatti $\lim\limits_{t \to 0} \frac{\sin t}{t} = 1 \Rightarrow \forall \varepsilon > 0 \ \exists r > 0$: se $0 < |t| < r$

si ha $\left| \frac{\sin t}{t} - 1 \right| < \varepsilon$

si può avere $\sqrt{x^2+y^2} < r$? Sì purché (x,y) stia nel cerchio di centro l'origine e raggio r

se $0 < \sqrt{x^2+y^2} < r$ si ha $\left| \dfrac{\sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} - 1 \right| < \varepsilon \Rightarrow$ TS.

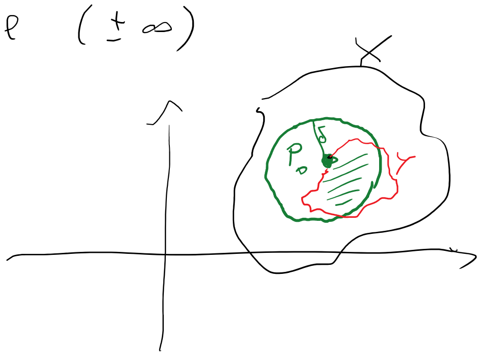




IP. $f : X \to \mathbb{R}$ $\quad (x_0, y_0) \in DX \quad \lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x,y) = \ell \quad (\pm\infty)$

$Y \subseteq \cancel{X} \quad (x_0, y_0) \in DY \quad g(x,y) = f(x,y) \quad \forall (x,y) \in Y$
(restrizione) $\quad g = f_{|Y}$

TS. $\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} g(x,y) = \ell \quad (\pm\infty)$



Il viceversa NON E' VERO $\Rightarrow$ basta trovare due restrizioni con due limiti diversi per concludere che f non è dotata di lim