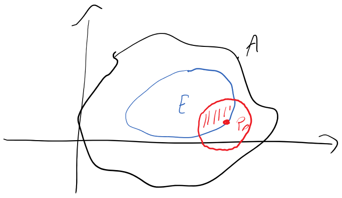
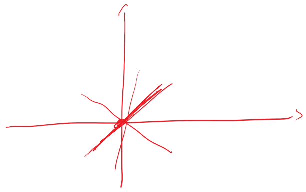
19 ott 2022

mercoledì 19 ottobre 2022 07:59





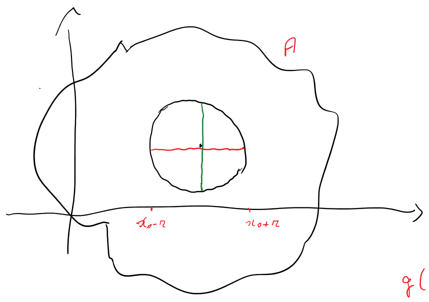
$$y = mx$$

$$E_m = \{ (x, mx) : x \in \mathbb{R} \}$$

$$f_{|E_m} = f(x, mx) = g_m(x) \qquad \lim_{x \to 0} g_m(x) = \ell$$

se ℓ dipende da m, f non è regolare

se ℓ NON " " f POTREBBE tendere ad ℓ



$$d(P, P_0) < r \Rightarrow P \in A$$

$$\Downarrow$$

se $x \in]x_0 - r, x_0 + r[\Rightarrow$

$\Rightarrow (x, y_0) \in A$

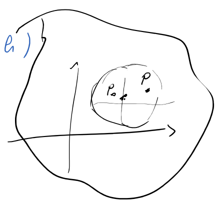
$$g(x) = f(x, y_0)$$

se $\exists \, g'(x_0) \Rightarrow \exists \, f_x(x_0, y_0) = g'(x_0)$

$(x_0, y) \qquad y_0 - r < y < y_0 + r \qquad f_y(x_0, y_0)$

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = f'(c) \Rightarrow \lim_{h \to 0} \frac{\Delta f - f'(c) h}{h} = 0$$

$$df = f'(c)\, h \qquad \Delta f - df = o(h)$$



$$df = (\nabla f, (h, k))$$

$$f_x(x_0, y_0) h + f_y(x_0, y_0) k$$

$P_0(x_0, y_0)$

$P(x_0 + h, y_0 + k)$

$$\Delta f = f(x_0 + h, y_0 + k) - f(x_0, y_0)$$

$\nabla f \qquad (h, k)$

$$\lim_{(h,k) \to (0,0)} \frac{\Delta f - df}{\sqrt{h^2 + k^2}} = 0 \ ?$$

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & (x, y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$f_x(0,0) = f_y(0,0) = 0$$

$$\frac{\Delta f - df}{\sqrt{h^2 + k^2}} = \frac{h k}{(h^2 + k^2)\sqrt{h^2 + k^2}}$$

$k = 0 \qquad 0$

$k = h \qquad \dfrac{h^2}{2h^2 \sqrt{2h^2}}$ div.

il limite non esiste

Esercizi

i) estr rel

$f(x,y) = 2x^2 \ldots 2x^2 y$

Esercizi

(1) $f(x,y) = 2xy^2 - 3x^2 y$

i) estr rel

ii) estr ass nel triangolo di vertici $A(0,1)$, $B(1,0)$, $C(0,0)$

$f_x(x,y) = 2y^2 - 6xy$

$f_y(x,y) = 4xy - 3x^2$

$$\begin{cases} 2y(y-3x) = 0 \\ x(4y-3x) = 0 \end{cases}$$

$$\begin{cases} y=0 \\ x=0 \end{cases} \quad \begin{cases} y=0 \\ 4y-3x=0 \end{cases} \quad \begin{cases} y-3x=0 \\ x=0 \end{cases} \quad \begin{cases} y-3x=0 \\ 4y-3x=0 \end{cases} \quad \Rightarrow C(0,0)$$

$f_{xx}(x,y) = -6y \qquad f_{xy}(x,y) = 4y - 6x \qquad H(0,0) = 0$

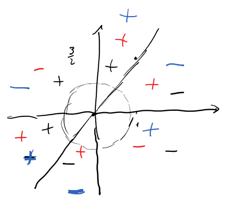
$f_{yy}(x,y) = 4x$

STUDIO LOCALE

$f(x,y) = 2xy^2 - 3x^2 y = xy(2y-3x)$

$2y - 3x = 0$

$y = \frac{3}{2}x$

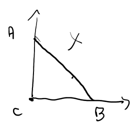


- segno del fattore $2y - 3x$
- " " xy
- segno di f

$\underline{f(0,0) = 0}$

$(0,0)$ non è di estr. rel.

ii)



$A(0,1)$

$B(1,0)$

$C(0,0)$

$X_1 = \{P \in int(X) : \nabla f(P) = 0\} = \emptyset$

$X_2 = \{P \in int(X) : \nexists \nabla f(P)\} = \emptyset$

$X_3 = \partial(X)$

AB $\quad y = 1 - x \qquad g(x) = f(x, 1-x) = 2x(1-x)^2 - 3x^2(1-x) = \qquad 0 \le x \le 1$

$= 2x + 2x^3 - 4x^2 - 3x^2 + 3x^3 = 5x^3 - 7x^2 + 2x$

$g'(x) = 15x^2 - 14x + 2 \qquad \frac{7 \pm \sqrt{19}}{15}$

$\frac{7+\sqrt{19}}{15} < 1$? $\qquad \sqrt{19} < 8$ sì

$0 < \frac{7-\sqrt{19}}{15} < 1$? $\qquad 7 - \sqrt{19} < 15$

$-\sqrt{19} < 8$ sì

$$\boxed{\begin{array}{l} P_1\left(\frac{7-\sqrt{19}}{15}, 1 - \frac{7-\sqrt{19}}{15}\right) \\ P_2\left(\frac{7+\sqrt{19}}{15}, 1 - \frac{7+\sqrt{19}}{15}\right) \end{array}}$$

BC $\quad y = 0 \qquad l(x) = f(x,0) = 0 \quad \forall x$

AC $\quad x = 0 \qquad l(y) = f(0,y) = 0 \quad \forall y$

$(a,0) \quad 0 \le a \le 1 \qquad f(a,0) = 0$

$(0,b) \quad 0 \le b \le 1 \qquad f(0,b) = 0$

$P_1 \quad P_2 \qquad f(P_1) = \dots$

$f(P_2) = \dots$

OSSERV. 1) Se non sono richieste gli estr. rel. non calcolate le derivate seconde.

2) se $f(x,y) = e^{g(x,y)} \qquad \log g(x,y)$

basta calcolare gli estremi di g

$2xy^2 - 3x^2 y \qquad$ se $M = \max g$

basta calcolare g ...

$f(x,y) = e^{2xy^2 - 3x^2 y}$

se $M = \max g$
dato che e^t è cresc.
$\Rightarrow e^M = \max f$

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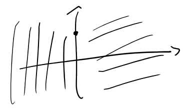
(2) $f(x,y) = |x|(xy^2 - y^3)$

ii) estr. ass. in $[-1,2] \times [-1,1]$

i) der. e diff.



i)

$$f(x,y) = \begin{cases} x^2y^2 - xy^3 & x \geq 0 \\ xy^3 - x^2y^2 & x < 0 \end{cases}$$

$x > 0$ $\quad f_x = 2xy^2 - y^3$
$\quad f_y = 2x^2y - 3xy^2$

$x < 0$ $\quad f_x = y^3 - 2xy^2$
$\quad f_y = 3xy^2 - 2x^2y$

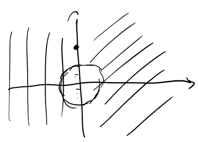
$P(0,b)$ $\exists f_x(P)$? $\quad f(x,b)$ $\quad f_x(x,b) = \begin{cases} 2xb^2 - b^3 & x > 0 \\ b^3 - 2xb^2 & x < 0 \end{cases}$

$\lim_{x \to 0^+} f_x(x,b) = -b^3 \qquad \lim_{x \to 0^-} f_x(x,b) = b^3$

$-b^3 = b^3 \iff b = 0$

Allora $f_x(0,0) = 0 \qquad \nexists f_x(0,b)$ se $b \neq 0$

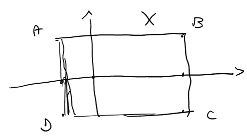
$P(0,b)$ $\exists f_y(P)$? $\qquad f(0,y) = 0 \ \forall y \Rightarrow \exists f_y(0,b) = 0 \ \forall b$



in tutti i punti in cui $x \neq 0$ f è diff.
in $(0,b)$ con $b \neq 0$ f non è diff perché
manca $f_x(0,b)$
in $(0,0)$ f è diff per il teor del diff totale
(una almeno delle der. $\exists$ in un intorno)

in ogni caso se vogliamo usare la def.

$$\frac{\Delta f - df}{\sqrt{h^2+k^2}} = \frac{|h|(\overset{\to 0}{hk^2 - k^3})}{\underbrace{\sqrt{h^2+k^2}}_{\leq 1}} \to 0$$



$A(-1,1) \quad B(2,1) \quad C(2,-1) \quad D(-1,-1)$

ii) $\quad X_1 = \{P \in \mathrm{int}(X) : \nabla f = 0\}$

$$\begin{cases} 2xy^2 - y^3 = 0 \\ 2x^2y - 3xy^2 = 0 \end{cases} \qquad \begin{cases} y^2(2x - y) = 0 \\ xy(2x - 3y) = 0 \end{cases}$$

$$\begin{cases} y = 0 \\ \forall x \end{cases} \quad \begin{cases} 2x - y = 0 \\ x = 0 \end{cases} \quad \begin{cases} 2x - y = 0 \\ y = 0 \end{cases} \quad \cancel{\begin{cases} y = 0 \\ x = 0 \end{cases}} \quad \cancel{\begin{cases} 2x - y = 0 \\ y = 0 \end{cases}} \quad \begin{cases} 2x - y = 0 \\ 2x - 3y = 0 \end{cases}$$

$(0,0)$ sol. di tutti questi sistemi

$X_1 = \{(0,0)\}$

$X_2 = \{P \in \mathrm{int}(X) : \nexists f_x(P),\ f_y(P) = 0$ o viceversa $\}$

$X_2 = \{(0,b) : -1 < b < 0,\ 0 < b < 1\}$

$X_3 = f(X)$

AB $\quad y = 1 \quad g(x) = f(x,1) = |x|(x-1) \qquad -1 \leq x \leq 2 \qquad g(x) = \begin{cases} x^2 - x & x \geq 0 \\ x - x^2 & x < 0 \end{cases}$

$g'(x) = \begin{cases} 2x - 1 & x > 0 \\ 1 - 2x & x < 0 \end{cases} \qquad = 0$ per $x = \frac{1}{2} \qquad \left(\frac{1}{2}, 1\right), (0,1) \in X_3$

$\nexists g'(0)$

BC $\quad x = 2 \quad h(y) = f(2,y) = 4y^2 - 2y^3 \qquad -1 \leq y \leq 1$

$h'(y) = 8y - 6y^2 = -2y(3y - 4) = 0$ per $y = 0,\ \cancel{y = \frac{4}{3}} \qquad (2,0) \in X_3$

CD $\quad y = -1 \quad \varphi(x) = |x|(x+1) = \begin{cases} x^2 + x & x \geq 0 \end{cases} \qquad -1 \leq x \leq 2$

$h'(y) = 8y - 6y = -2y(3y-4) = 0$ per $y = 0$ $\cancel{3}$ [illegible] 3

CD $y = -1$ $\quad \ell(x) = |x|(x+1) = \begin{cases} x^2 + x & x \geq 0 \\ -x^2 - x & x < 0 \end{cases}$ $\quad -1 \leq x \leq 2$

$\ell'(x) = \begin{cases} 2x+1 & x > 0 \\ -2x-1 & x < 0 \end{cases}$ $\quad \nexists\, \ell'(0)$ $\quad (0,-1),\ (-\frac{1}{2},-1) \in X_3$

AD $x = -1$ $\quad m(y) = y^2 - y^3$ $\quad -1 \leq y \leq 1$ $\quad (-1,0),\ (-1,\frac{2}{3}) \in X_3$

$m'(y) = 2y - 3y^2 = -y(3y-2) = 0$ per $y = 0,\ y = \frac{2}{3}$

dobbiamo calc.

$f(-1,1) = -2$ $\quad f(0,b) = 0$ $(-1 < b < 0,\ 0 < b < 1)$

$f(-1,-1) = 0$ $\quad f(\frac{1}{2},1) = -\frac{1}{4}$ $\quad f(0,-1) = 0$ $\quad f(-1,\frac{2}{3}) = -\frac{20}{27}$

$f(2,1) = 2$ $\quad f(0,1) = 0$ $\quad f(-\frac{1}{2},-1) = \frac{1}{4}$

$f(2,-1) = 6$ $\quad f(2,0) = 0$ $\quad f(-1,0) = 0$

$f(0,0) = 0$

$\max_X f = 6 = f(2,-1)$

$\min_X f = -2 = f(-1,1)$

$f(x,y) = |y|(xy^2 - x^2)$ $\quad$ der. diff

estr. ass. in $[-1,1] \times [-1,1]$