The use of advanced control methods in Quantitative Finance

Early research project report

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1 Motivation

Trading is a wide area of modern research with multiple approaches aimed at generating profits from financial markets. However, accurately measuring trading performance is a significant challenge. It is not sufficient to solely focus on the profitability represented by PnL. The risk associated with the trading strategy must also be carefully considered to ensure a comprehensive evaluation.

While a good PnL may indicate successful trades, it can be misleading if the achieved profits are driven by high levels of risk or simply due to chance. Therefore, it is essential to incorporate risk management into trading strategies and assess the combination of risk and return as the objective function. By doing so, traders can make informed decisions that balance profitability with risk exposure.

In this report, I explore the potential of control methods in managing position risk. These methods offer the advantage of enhancing the stability of trading positions and reducing the volatility of returns and PnL. By employing control techniques, traders can actively adjust their positions based on market conditions and risk thresholds, resulting in more controlled and predictable outcomes.

The integration of control methods into trading strategies provides a systematic approach to managing risk and can lead to improved trading performance. However, further research and analysis are necessary to explore advanced control algorithms, refine risk measurement models, enhance backtesting procedures, and conduct empirical studies using real-world trading data. These efforts will contribute to the development of more robust and effective approaches to managing risk in trading, ultimately leading to more successful outcomes for traders.

2 How to measure market risk of position

2.1 Value at Risk

Denote the the log-return of portfolio at time $t+1$ as

$$r_{t+1} = \log \frac{p_{t+1}}{p_t}$$

where p_t is the price at moment t. Assume it has some cumulative distribution F We are interested in managing the portfolio in such way that in the worst case our losses are not bigger than some value. This value is denoted as value at risk or VaR:

$$P(r_{t+1} \leq -VaR_\alpha) = \alpha \implies VaR_\alpha = -F^{-1}(\alpha)$$

Value at Risk (VaR) is a widely used risk management tool in finance that provides an estimate of the potential loss an investment or portfolio may incur over a specific time horizon, at a certain level of confidence. It quantifies the maximum loss in value that an investment or portfolio is expected to experience under normal market conditions.

The concept of VaR was developed in the late 1980s and early 1990s by financial institutions and academics as a response to the need for a more comprehensive and quantitative approach to risk management. Lately, it was considered as the most common way of managing the market risk. [1–4] Traditionally, risk management in finance had relied heavily on measures such as standard deviation and correlation, which did not adequately capture the tail risks and extreme events that could lead to significant losses.

The main goal of VaR is to provide a single number that represents the potential downside risk of an investment or portfolio, taking into account the statistical distribution of returns and the specified confidence level. It allows risk managers and investors to assess and compare the riskiness of different investments or portfolios and make informed decisions based on their risk appetite.

VaR is calculated by estimating the statistical distribution of historical returns or simulating future scenarios and determining the potential loss that would exceed a certain threshold at the desired confidence level. The most commonly used confidence levels are 95% and 99%, representing the level of certainty that the estimated VaR will not be exceeded. In the picture below there can be seen a graphical trivial visualization of VaR.

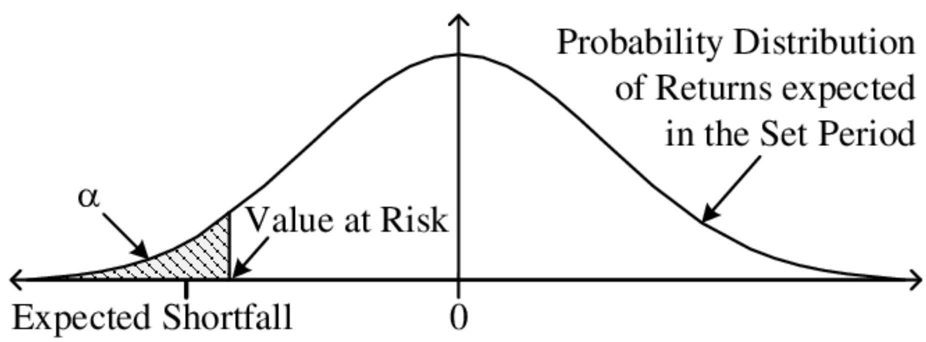
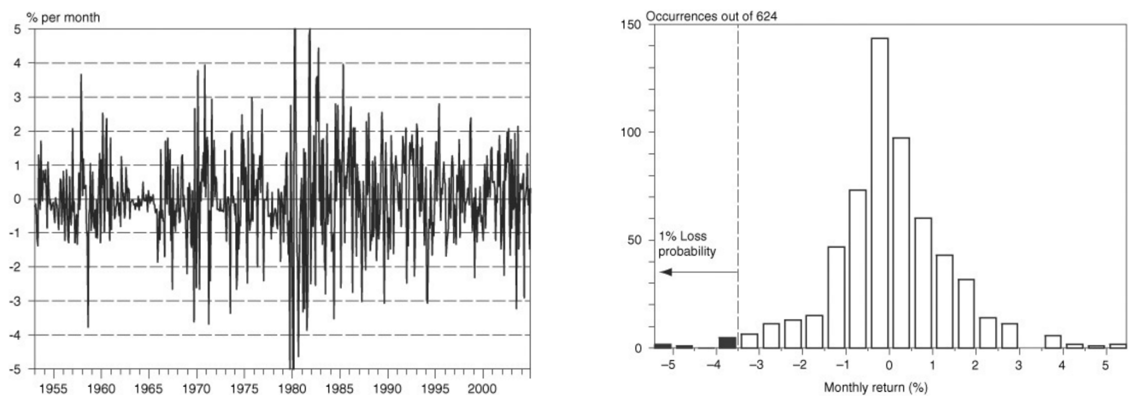


Figure 1: Visualization of VaR

2.2 Historical VaR

VaR can be measured in several ways, the most obvious one is historical: with some rolling window in the past look, what were the largest α losses. In the picture below there can be seen an example of calculating historical VaR of Medium-term bonds from 1955 to 2005



(a) Returns on medium-term bonds (b) Measuring value at risk

Figure 2: Straight forward historical VaR calculation

2.3 Real case: crypto VaR

There is a wonderful real-life example of VaR calculation on cryptocurrency market. Kaiko [1] offers an interactive report. Kaiko's new VaR estimator is one of the first risk management tools designed for cryptocurrencies, leveraging a proprietary and thoroughly backtested methodology that accounts for the idiosyncrasies of crypto market structure. In this article, we will review the background, methodology, and use cases for VaR, showing how cryptocurrency portfolio managers can better manage risk. In the figure below there can be seen PnL of Kaiko's portfolio and how the different levels (95% and 99%) was changing throught the time.

[1]https://www.kaiko.com/blogs/examples-and-case-studies/cryptocurrency-value-at-risk-var

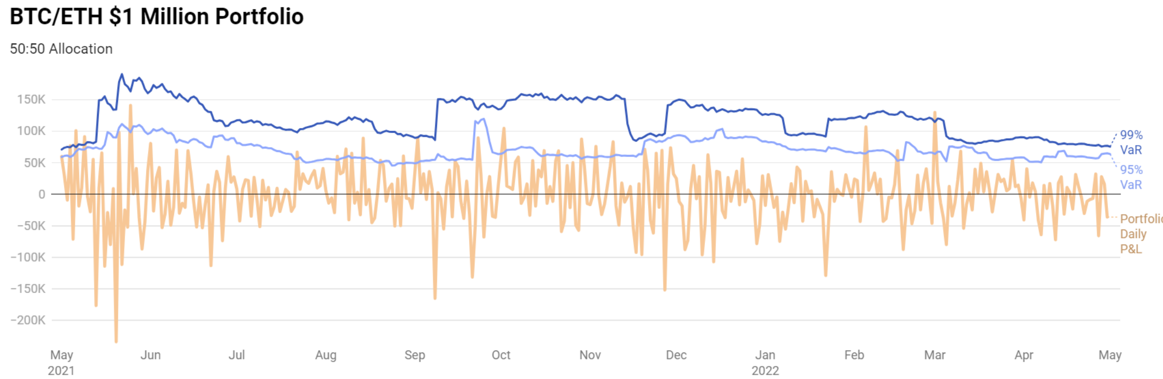


Figure 3: PnL, 95% and 99% Var of Kaiko's portfolio

In the picture below there can be found the historical VaR calculation by Kaiko

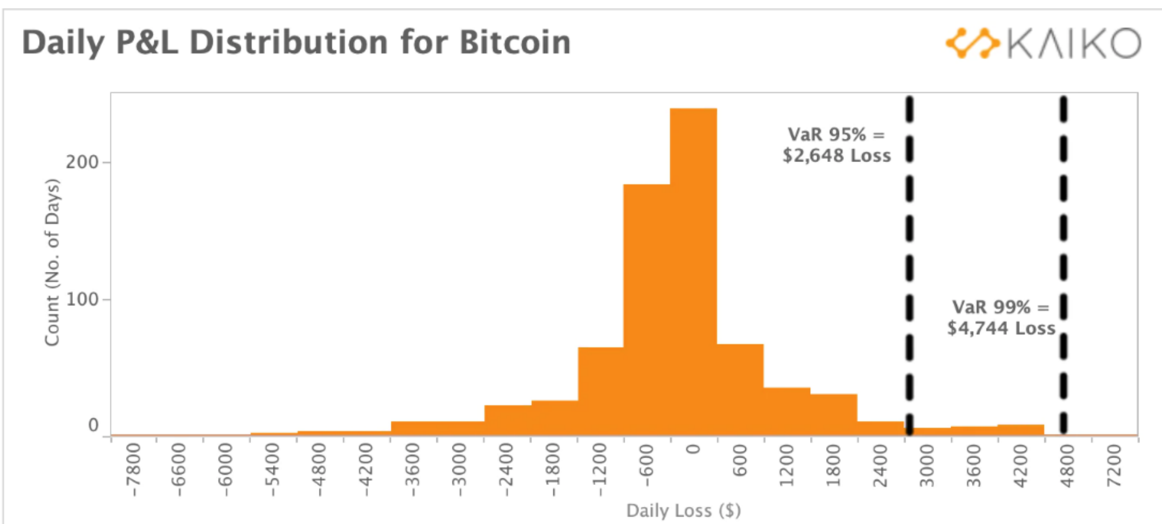


Figure 4: Historical VaR of Kaiko

2.4 Parametric VaR

Obviously Historical VaR is not our way, since in this case we cannot simulate the behavior of VaR, based on our actions, since did not know its nature, so some parametric methods can be used. The most simple parametric methods is RiskMetrics. The RiskMetrics [5, 6] model has played a significant role in advancing risk management practices and has been widely adopted by financial institutions for measuring and monitoring market risk. Its simplicity and transparency make it accessible to a broad range of users, although its assumptions and limitations should be carefully considered in practice.

The RiskMetrics model is a widely used method for measuring and managing financial market risk. It was developed by J.P. Morgan in the early 1990s and has since become an industry standard for calculating market risk metrics, particularly Value at Risk (VaR).

The RiskMetrics model is based on the assumption that financial market returns follow a multivariate normal distribution. It focuses on measuring the volatility and correlation of asset returns to estimate the potential losses in a portfolio. The key components of the RiskMetrics model include:

Volatility Estimation: The model uses historical data to estimate the volatility of asset returns. It employs a moving window approach, where a fixed period of historical returns is used to calculate the volatility. The exponentially weighted moving average (EWMA) method is commonly used to assign more weight to recent observations and capture time-varying volatility.

Correlation Estimation: The RiskMetrics model also estimates the correlation between asset returns. Correlations measure the degree to which the returns of different assets move together. Historical correlation matrices are constructed based on the same historical return data used for volatility estimation. The model assumes that correlations remain relatively stable over time.

Covariance Matrix Calculation: The volatility estimates and correlation matrices are combined to form a covariance matrix, which captures the joint risk of the portfolio. The covariance matrix represents the relationships between the individual assets in the portfolio and is a crucial input for calculating VaR.

VaR Calculation: Once the covariance matrix is obtained, the RiskMetrics model calculates VaR, which quantifies the potential losses that a portfolio may experience over a specified time horizon with a given confidence level. VaR is calculated by multiplying the portfolio's volatility by a quantile of the standard normal distribution, considering the portfolio's exposure and correlation structure.

The RiskMetrics model offers a practical and straightforward approach to estimate and manage market risk. It provides risk managers with a framework to measure the potential downside risk of portfolios and make informed risk management decisions. However, it's important to note that the RiskMetrics model has certain limitations. It assumes normality in returns and stable correlations, which may not hold during periods of extreme market conditions or financial crises. Additionally, the model's accuracy heavily relies on the quality and representativeness of the historical data used for estimation.

Let us formally consider the use of RiskMetrics model. The general model for our returns is

$$r_{t+1} = \mu_{t+1} + \sigma_{t+1}\nu_{t+1}$$

$$\nu_{t+1} \sim i.i.d, N(0,1)$$

RiskMetrics propose the following way of calculating VaR:

$$VaR_\alpha = -\sigma_{t+1}^2\Phi^{-1}(\alpha) = -(\lambda\sigma_t^2 + (1-\lambda)r_t^2)\Phi^{-1}(\alpha)$$

, where λ is a hyperparameter, specified by us. In this case, given the observation with current return and standard deviation of the position, next state VaR can be simulated.

3 How control can help

As a control method, MPC will be considered, since its predictive nature is well-suited for managing VaR as it enables the controller to anticipate potential future scenarios and make proactive adjustments to mitigate risks. By simulating future market behavior based on the current position and market observations, MPC can optimize position adjustments over a defined prediction horizon, taking into account the dynamic nature of the market and the desired risk targets.

Model Predictive Control (MPC) [7–9] is a control strategy used in engineering and process control to optimize the performance of a dynamic system. It involves predicting the future behavior of the system based on a mathematical model and using this prediction to compute an optimal control action. The mechanism of work in MPC can be summarized by the following steps:

Model formulation: A mathematical model of the dynamic system is developed, typically in the form of differential equations or transfer functions. This model describes the relationship between the system's inputs, outputs, and states.

Prediction: The MPC algorithm uses the mathematical model to predict the future behavior of the system over a specific prediction horizon. This horizon determines how far into the future the predictions are made. The predictions are based on the current state of the system and the previously applied control actions.

Objective function formulation: An objective function is defined to quantify the performance of the system. It represents the goals and requirements that the control algorithm aims to achieve.

Optimization: The predicted future behavior of the system and the defined objective function are used to solve an optimization problem. The optimization algorithm searches for the control actions that iteratively minimize the objective function while satisfying constraints on the system's inputs, outputs, and states. The solution provides the optimal control actions to be applied over the control horizon.

Control action implementation: The first control action from the optimized sequence is applied to the system. The system's outputs and states are measured, and the process is repeated by updating the state estimation and repeating the prediction and optimization steps at the next sampling instant.

In the picture below there can be seen a formal visualization of MPC work

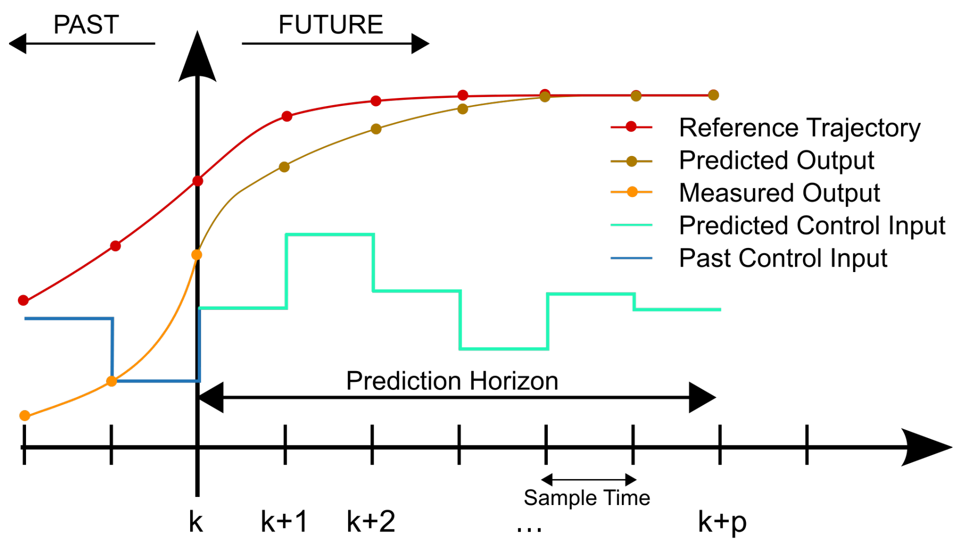


Figure 5: Key elements of MPC

As a baseline of control, simple strategy can be proposed: increase the short or long position by $x\%$ each time price crosses some level. What the control can do here is put the action on this x with action boundaries [0,1], so that controlling our position. As an observation, current return and standard deviation of current position can be given to controller (other state variables like current price, volume can be specified too). The euler prediction can be used here for linearized predicting of the next state: the increment will be step size multiplied by action multipled by $x\%$. As a running objective $(VaR_{0.05} - \omega)^2$, where ω is the level of risk we want to target.

As a controller MPC can easily be used here with some prediction horizon and discount factor. In this case our controller will iteratively simulate the future behaviour of the market, optimizing sequence of actions, what will be extremely beneficial in this setup.

As a practical example, during the ACM cource the MPC control on more simpler strategy was made. That project was based on the paper [10], which tried to implement MPC towards trading, just maximizing PnL. In that project only price and volume was considered, however, this allowed to get positive PnL, the expecatations of the strategy described in this report are much higher.

4 Backtesting

Backtesting of Value at Risk (VaR) [11,12] is a critical process in risk management that involves assessing the accuracy and effectiveness of VaR models. VaR is a widely used measure to quantify and manage market risk, providing an estimate of the potential losses that a portfolio or financial institution may face under adverse market conditions. Backtesting serves as a crucial tool for evaluating the VaR model's performance and ensuring its reliability.

The importance of backtesting VaR lies in its ability to validate the accuracy and credibility of risk models. By comparing the estimated VaR with the actual losses incurred over a specific period, backtesting enables risk managers to assess the model's predictive capabilities and identify any potential weaknesses. A robust backtesting framework ensures that VaR models are properly calibrated, helping institutions make informed risk management decisions and maintain regulatory compliance.

However, backtesting VaR can be challenging due to various reasons. One of the primary challenges arises from the assumptions and limitations of VaR models. VaR calculations are based on statistical methods and historical data, assuming that future market conditions will resemble the past. This assumption may not hold true during periods of significant market volatility or structural shifts, leading to inaccurate VaR estimates. Consequently, backtesting results may be affected by the model's inherent limitations and the presence of unforeseen events.

Another challenge in backtesting VaR is the choice of appropriate statistical tests. Selecting the right methodology to evaluate the VaR model's performance is crucial for obtaining meaningful results. Commonly used backtesting approaches include the Kupiec test, Christoffersen's conditional coverage test, and the conditional independence test. Each method has its own assumptions and statistical properties, and careful consideration is required to ensure the reliability of the chosen test..

To overcome these challenges, financial institutions employ different approaches in backtesting VaR. These approaches include unconditional and conditional backtesting. Unconditional backtesting assesses the overall performance of the VaR model, comparing the number of violations (actual losses exceeding VaR) against the expected number under a certain confidence level. Conditional backtesting, on the other hand, considers the magnitude and timing of the violations, evaluating the model's ability to capture extreme losses accurately.

So, let us formally describe backtesting and the corresponding testing procedures.

A big advantage of using Var as the goal function is a rather intuitive back testing procedure. We are interested in cases when our return goes below the target Var: Hits:

$$Hit_{t+1} := I(r_{t+1} < -Var^{\alpha}_{t+1})$$

And we should test the hypothesis:

$$H_0 : Hit_{t+1} \sim i.i.dBernoulli(\alpha)$$

vs

$$H_1 : Hit_{t+1} \sim Bernoulli(\pi), \pi \neq \alpha$$

what can be tested with likelihood test:

$$L(\Pi) = \prod_{t=1}^{T} (1-\pi)^{1-Hit} \Pi^{Hit}$$

$$LR_{uc} = -2 \log \frac{L(\alpha)}{L(\Pi)}$$

$$LR_{uc} \xrightarrow{H_o} \chi(1)$$

What is more, independence should additionally tested with the help of Markov chain methods

Markov chain transition probability matrix:

$$\Pi_1 = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix}$$

Then transition probabilities can be rewritten and likelihood can be calculated:

$$L(\Pi_1) = \prod_{t=2}^{T} (\pi_{00}^{(1-h_t)(1-h_{t+1})} \pi_{01}^{(1-h_t)h_{t+1}} \pi_{10}^{h_t(1-h_{t+1})} \pi_{11}^{h_t h_{t+1}})$$

The following null hypothesis is tested:

$$H_0 : \pi_{11} = \pi_{01} \text{ vs } H_1 : \pi_{11} \neq \pi_{01}$$

Then independence can be tested using the following test:

$$LR_{ind} = -2 \log \frac{L(\Pi)}{L(\Pi_1)}$$

$$LR_{ind} \xrightarrow{H_o} \chi(1)$$

And of course in the end the joint hypothesis can be tested using Christoffersen's conditional coverage test:

$$H_0 : \pi_{01} = \pi_{11} = \alpha$$

$$LR_{cc} = -2 \log \frac{L(\alpha)}{L(\Pi_1)}$$

$$LR_{cc} \xrightarrow{H_o} \chi(2)$$

Note that $LR_{cc} = LR_{uc} + LR_{ind}$ that means that everything is in harmony with each other

5 Conclusion

In conclusion, integrating control methods in trading risk management holds promise for achieving more stable returns and minimizing volatility. By leveraging market observations and predictive models, controllers can effectively optimize position adjustments to align with desired risk levels. Thorough backtesting and rigorous statistical analysis are vital to validate the effectiveness of these strategies in real-world trading environments. Overall, the use of control methods enhances risk mitigation efforts and improves trading performance by providing a systematic approach to managing positions. By actively adjusting positions based on market conditions and risk thresholds, we can strive for greater stability and reduced exposure to unexpected market fluctuations. Incorporating control methods into trading strategies represents a valuable avenue for maximizing returns while effectively managing risk, leading to more robust and reliable trading outcomes.

As a future work, practical implementation of this strategy should be carefully tested. As a first step, the strategy should be tested on historical data and the on real online data in any crypto testnet. Careful bactesting, described in this report will help to accurately evaluate the model performance - indeed how good it manages the risk of a given portfolio.

However, it should be pointed out that during the review of papers it was obtained that the use of control methods in modern finance is quite innovatory: both topics of risk management [1–6] and control methods [7–10] are well-developed by separate but not in union. So, this ambiguity carries certain risks, however, by the same reason I'm quite enthusiastic about it.

References

[1] Duffie, D., & Pan, J. (1997). An overview of value at risk. Journal of derivatives, 4(3), 7-49.

[2] Linsmeier, T. J., & Pearson, N. D. (2000). Value at risk. Financial Analysts Journal, 56(2), 47-67.

[3] Jorion, P. (2007). Value at risk: the new benchmark for managing financial risk. The McGraw-Hill Companies, Inc..

[4] Jorion, P. (1996). Risk2: Measuring the risk in value at risk. Financial analysts journal, 52(6), 47-56.

[5] Mina, J., & Xiao, J. Y. (2001). Return to RiskMetrics: the evolution of a standard. RiskMetrics Group, 1, 1-11.

[6] Pafka, S., & Kondor, I. (2001). Evaluating the RiskMetrics methodology in measuring volatility and Value-at-Risk in financial markets. Physica A: Statistical Mechanics and its Applications, 299(1-2), 305-310.

[7] Camacho, E. F., & Alba, C. B. (2013). Model predictive control. Springer science & business media.

[8] Rawlings, J. B. (2000). Tutorial overview of model predictive control. IEEE control systems magazine, 20(3), 38-52.

[9] Qin, S. J., & Badgwell, T. A. (1997, June). An overview of industrial model predictive control technology. In AIche symposium series (Vol. 93, No. 316, pp. 232-256). New York, NY: American Institute of Chemical Engineers, 1971-c2002..

[10] Clinet, S., Perreton, J. F., & Reydellet, S. (2021). Optimal trading: a model predictive control approach. arXiv preprint arXiv:2110.11008.

[11] Christoffersen, P., & Pelletier, D. (2004). Backtesting value-at-risk: A duration-based approach. Journal of Financial Econometrics, 2(1), 84-108.

[12] Candelon, B., Colletaz, G., Hurlin, C., & Tokpavi, S. (2011). Backtesting value-at-risk: a GMM duration-based test. Journal of Financial Econometrics, 9(2), 314-343.