tp2

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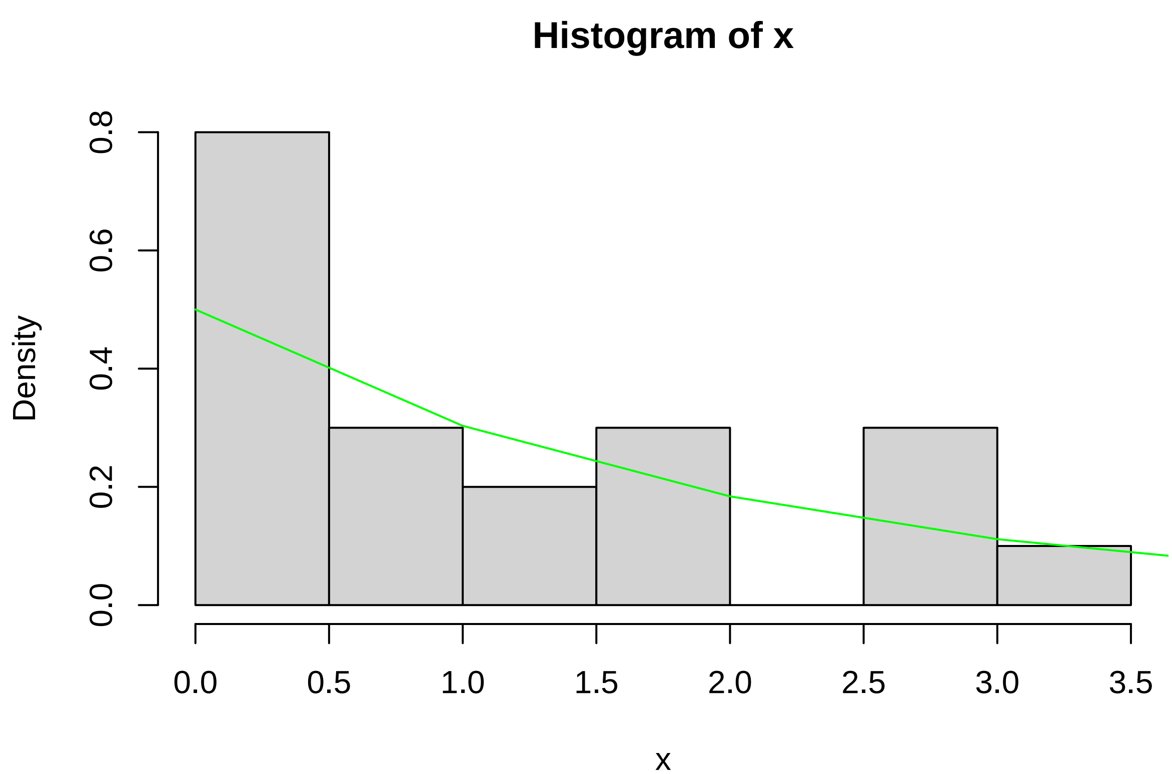
I. Variation sous-jacente et échantillonnage répété

1. Si X E(0.5), quelle est la probabilité qu'on observe une valeur supérieure à 3?

2.Simulez un échantillon de taille n = 20 d'un loi de E(0,5), créez un histogramme de votre échantillon et commentez la forme de votre histogramme. Superposer la vrai densité. Quelle est la probabilité empirique qu'on observe une valeur supérieure à 3 ?

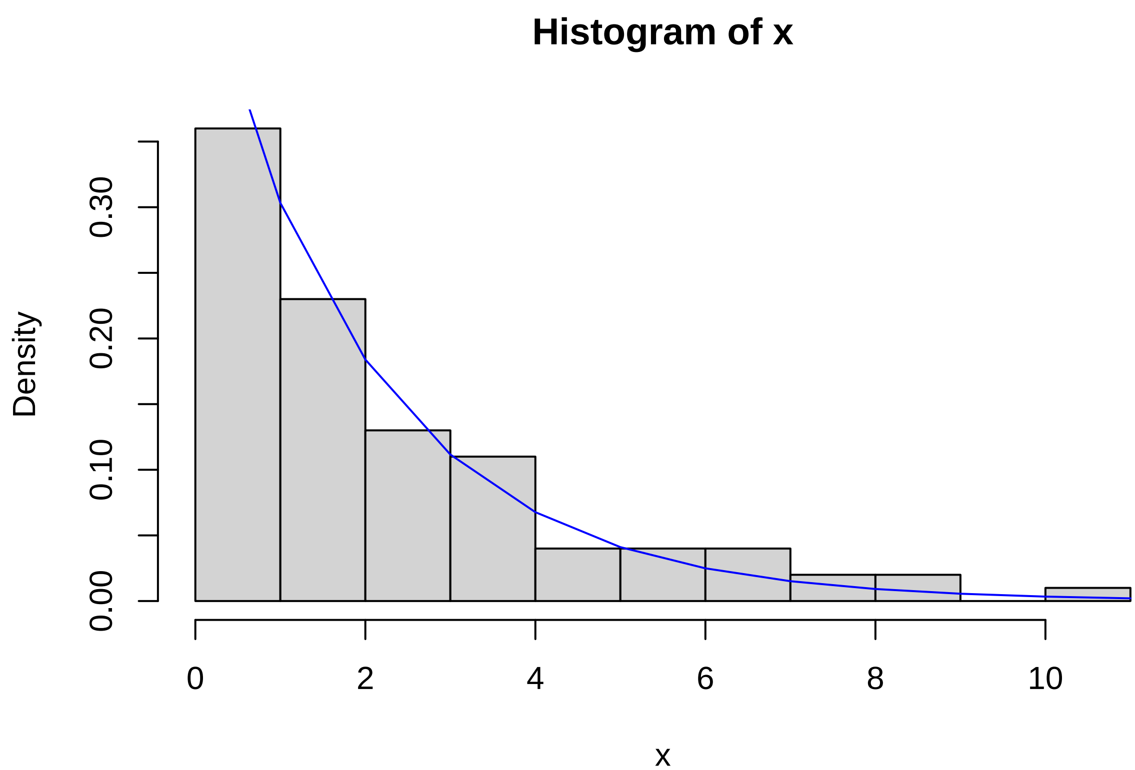
```
x<-rexp(20,0.5)
hist(x, freq=FALSE)
maxvalue <- ceiling(max(x))
lines(0:maxvalue,dexp(0:maxvalue, 0.5), col="green",)
```



Commentaire histogramme à insérer

3.Répétez cette opération 5 ou 6 fois et commentez les différences entre les histogrammes que vous obtenez à chaque fois. Utilisez la même limite sur les axes pour faciliter la comparaison. Notez également comment la probabilité empirique qu'on observe une valeur supérieure à 3 change.

4. Augmentez la taille de votre échantillon à 100 et répétez votre expérience. Que remarquez-vous?



II. Variabilité aléatoire du maximum de l'échantillon

1. Simuler un échantillon de taille n = 10 d'une loi U(−1, 1) et enregistrez le maximum de l'échantillon.

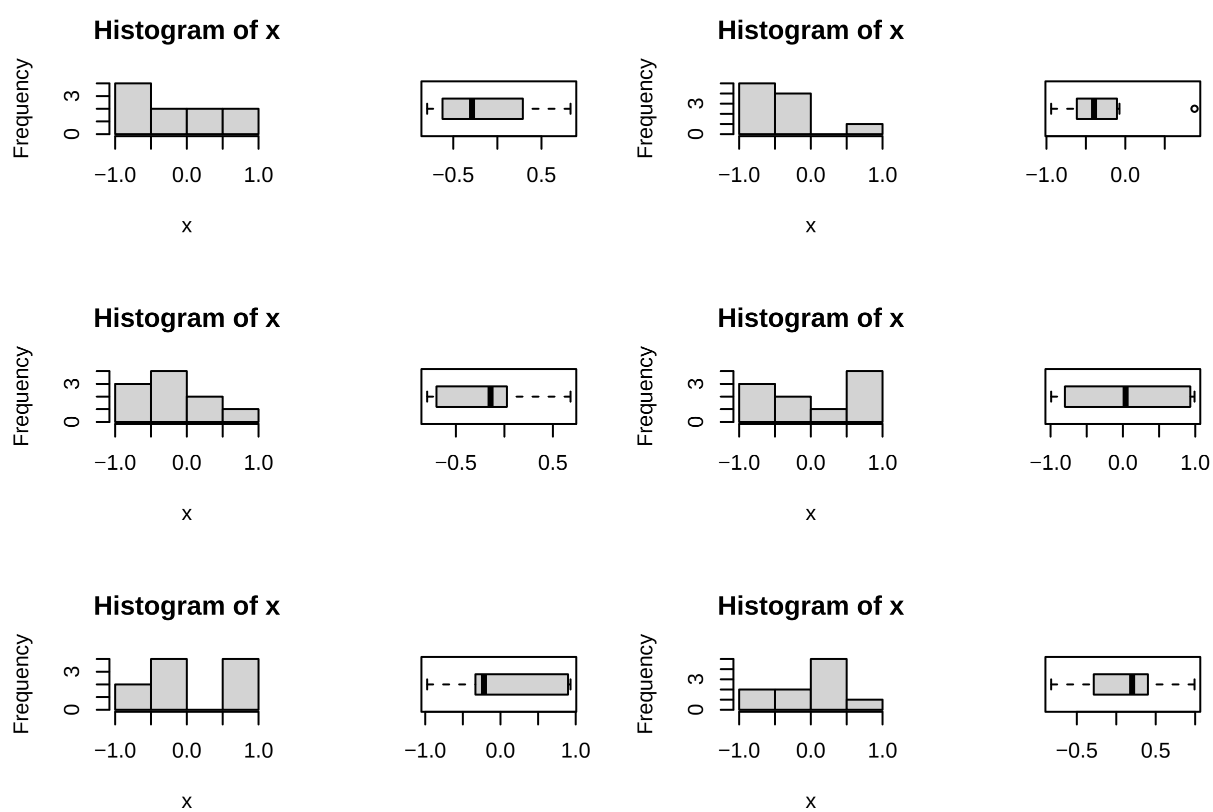
```
x <- runif(10, -1, 1)
max <- max(x)
```

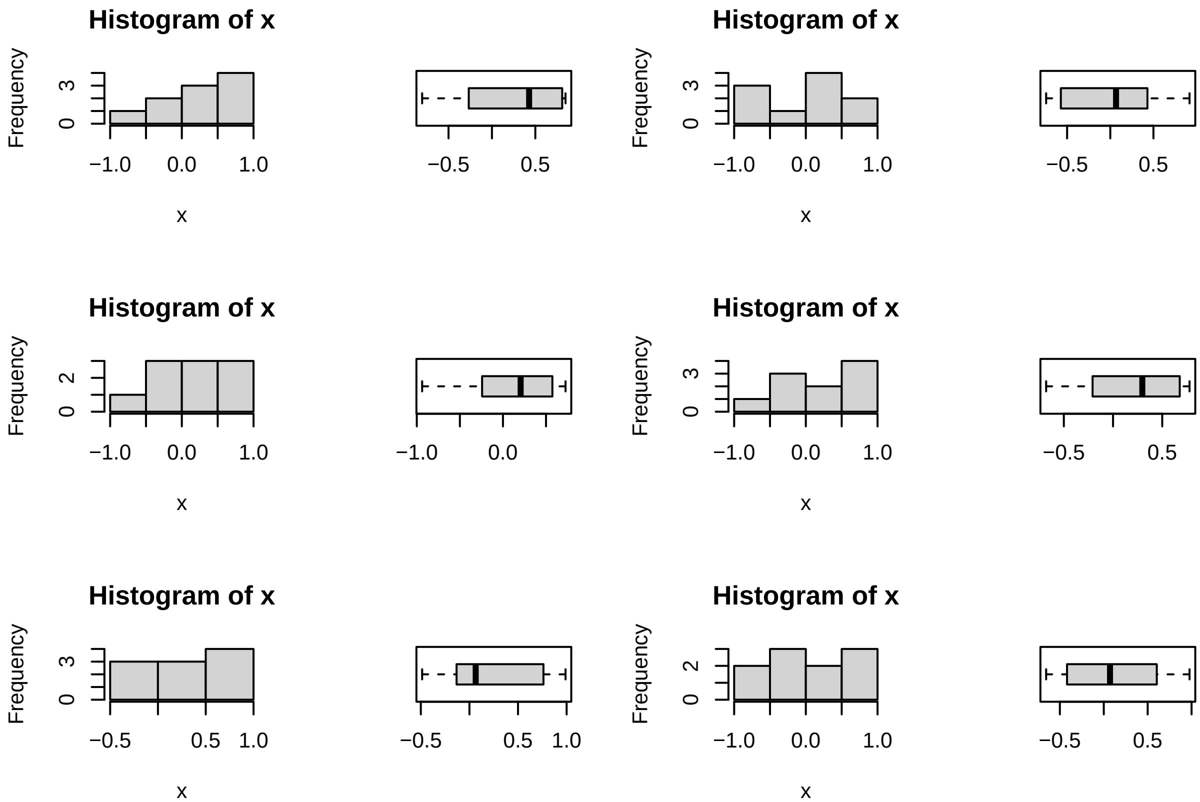
2. Répétez les deux étapes ci-dessus dix fois, en écrivant le maximum de l'échantillon à chaque fois. Commentez la variabilité des valeurs que vous obtenez pour les maxima de votre échantillon.

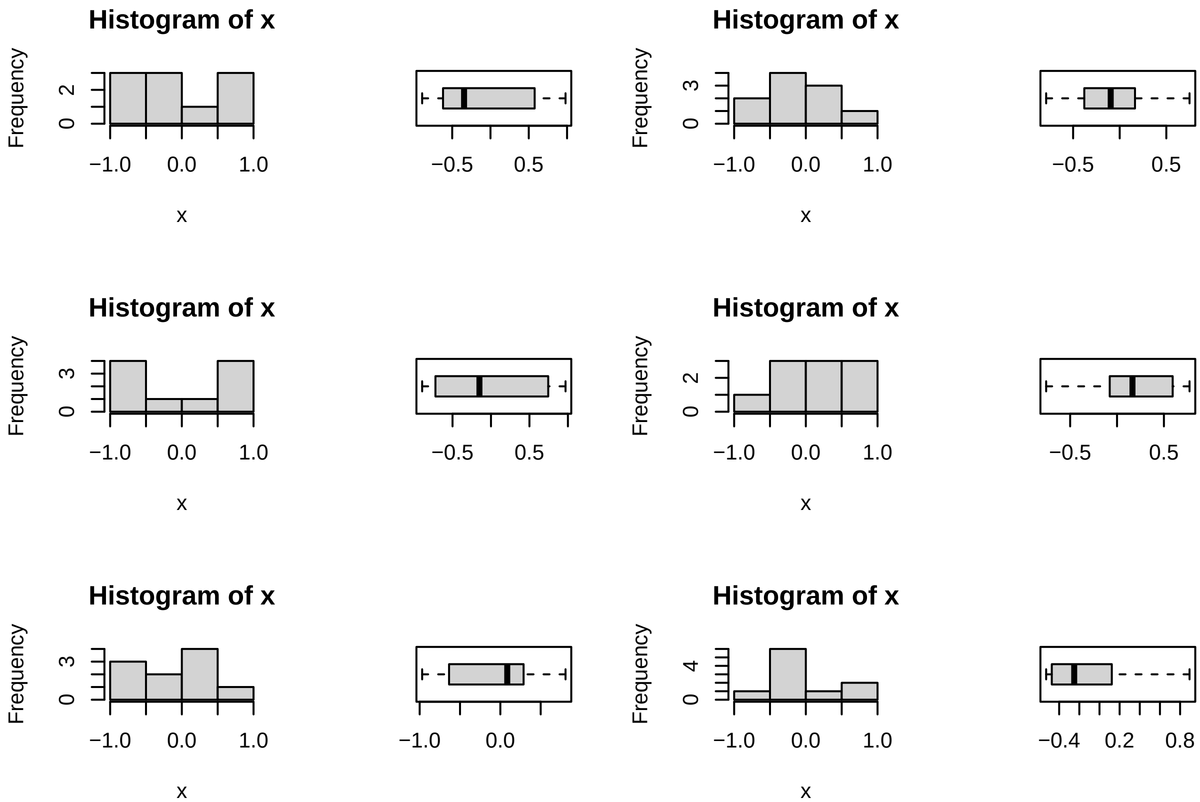
```r
for (i in 1:10) {
  x <- runif(10, -1, 1)
}
```

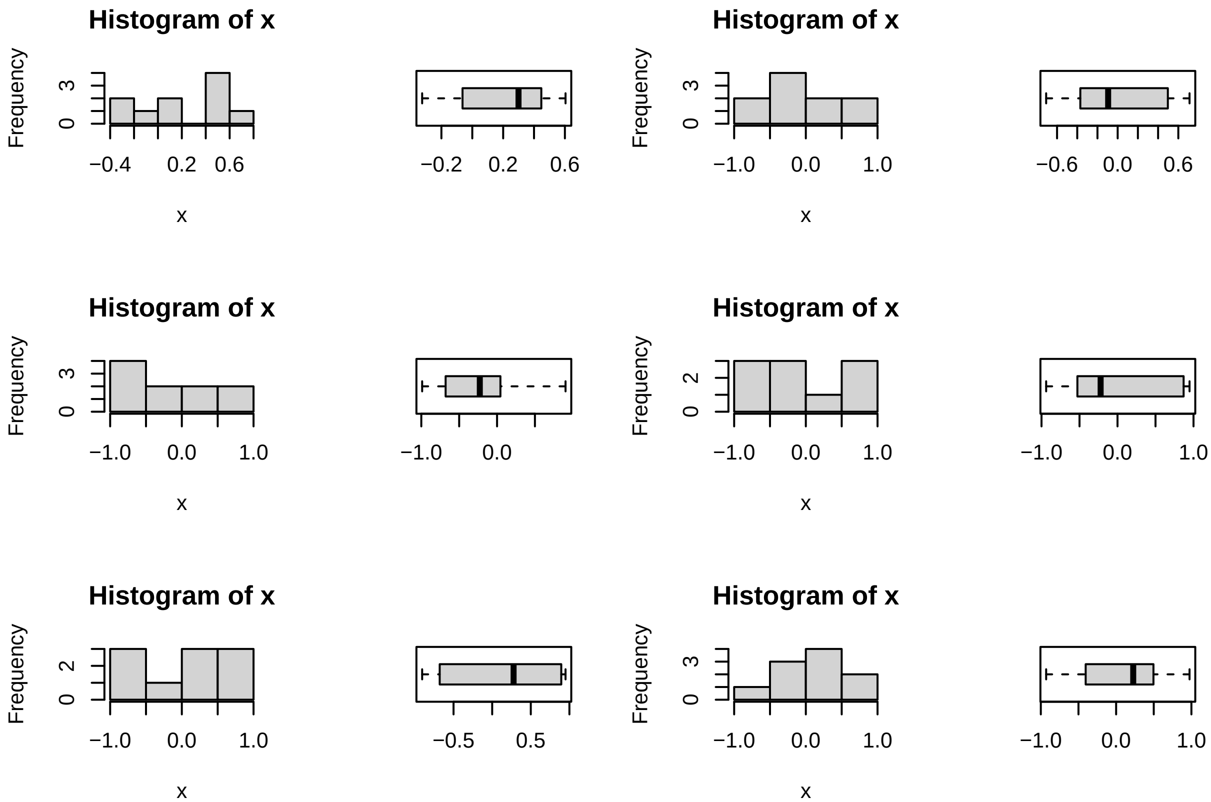
3. Répétez 100 fois et construisez un histogramme et une boîte à moustaches. Quelle est la loi dumaximum, M = max 1 i n X i où X i U(−1, 1) (TD1) ? Superposer la densité théoreique sur l'histogramme. Que remarquez-vous ?

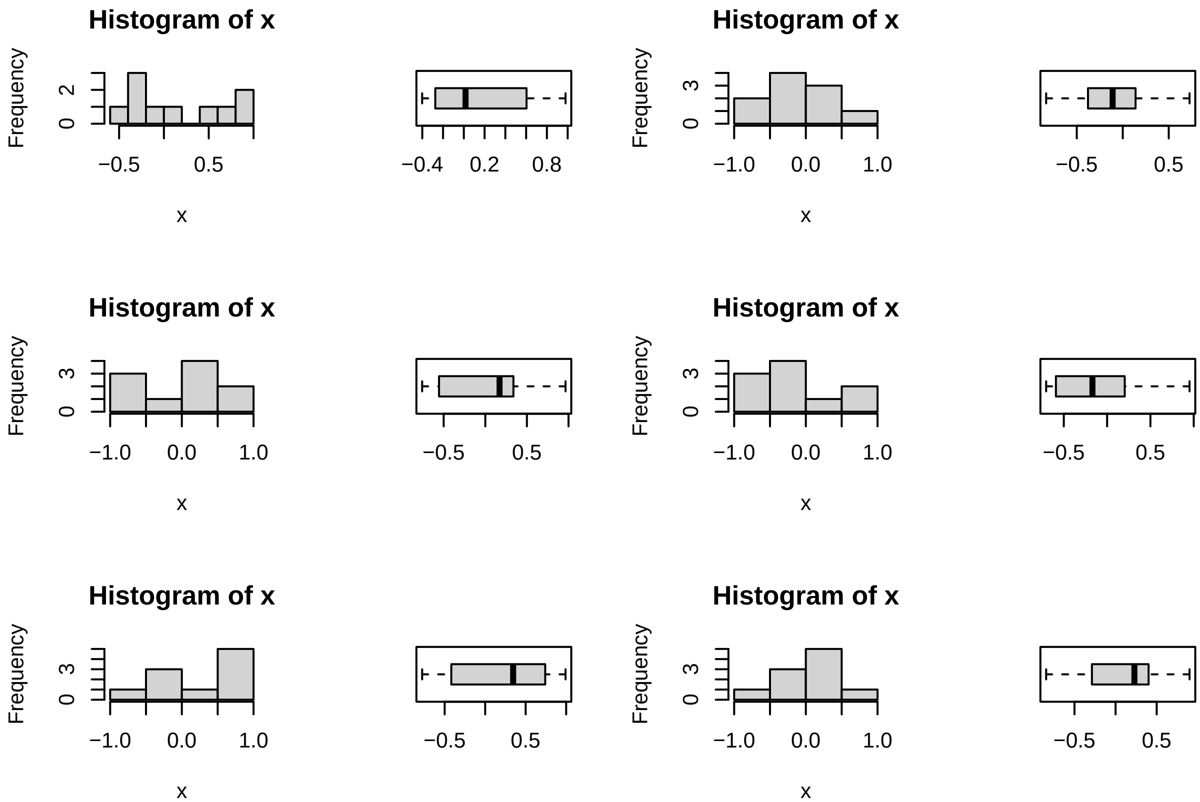
```r
par(mfrow=c(3,4))
for (i in 1:100) {
  x <- runif(10, -1, 1)
  hist(x)
  boxplot(x, horizontal = TRUE)
}
```



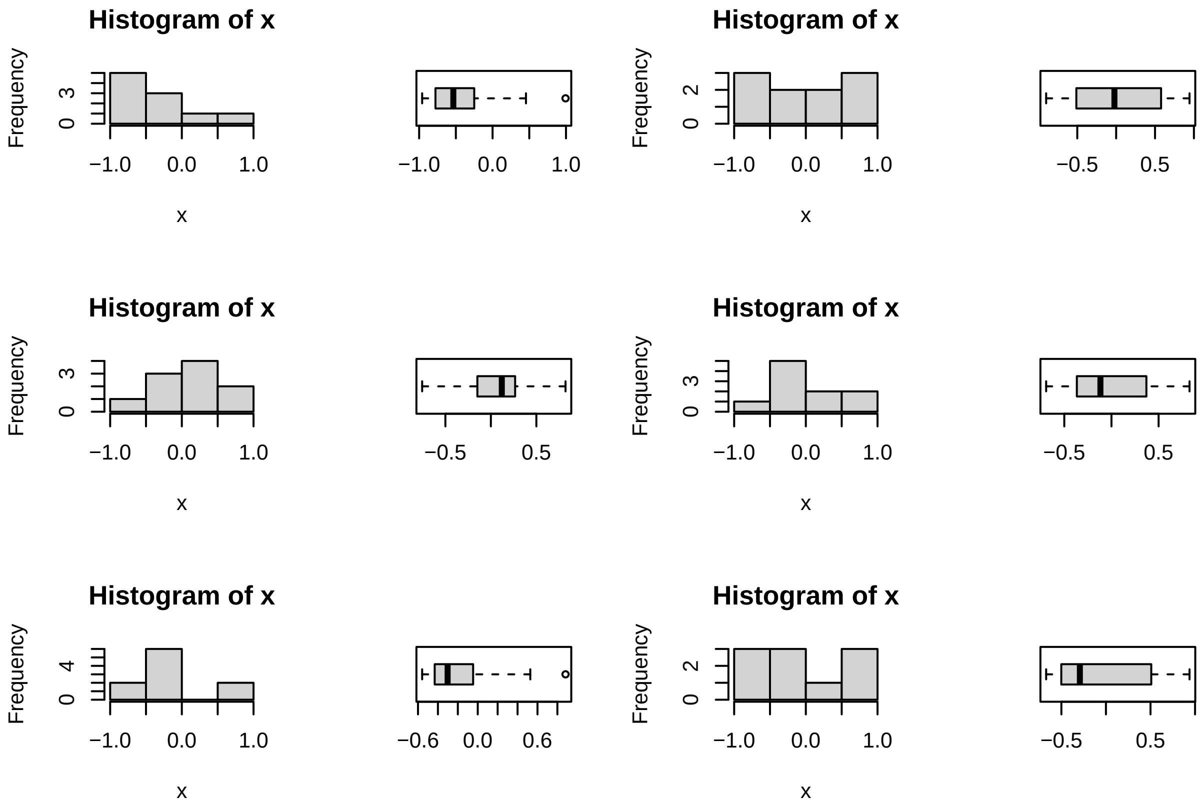

Histogram of x
Frequency
3
0
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
3
0
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
2
0
−1.0
0.0
1.0
x
−1.0
0.0
Histogram of x
Frequency
3
0
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
3
0
−0.5
0.5
1.0
x
−0.5
0.5
1.0
Histogram of x
Frequency
2
0
−1.0
0.0
1.0
x
−0.5
0.5

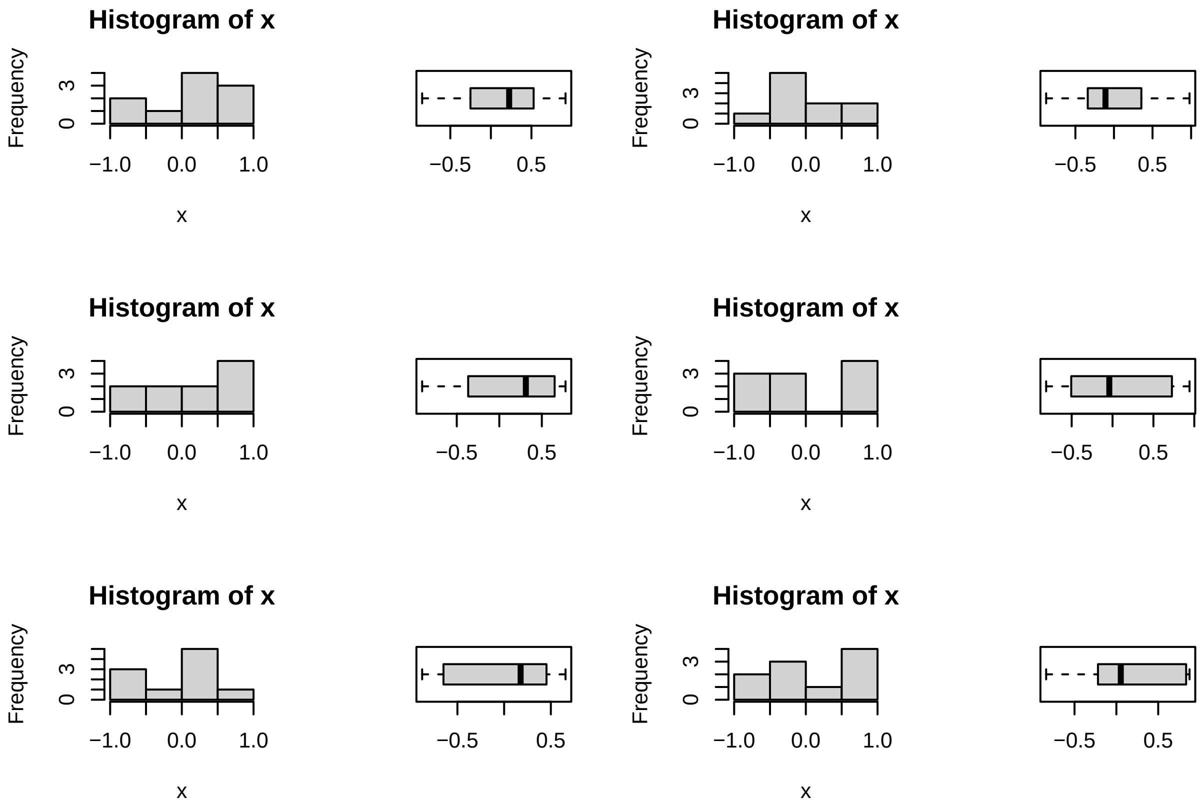

Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x

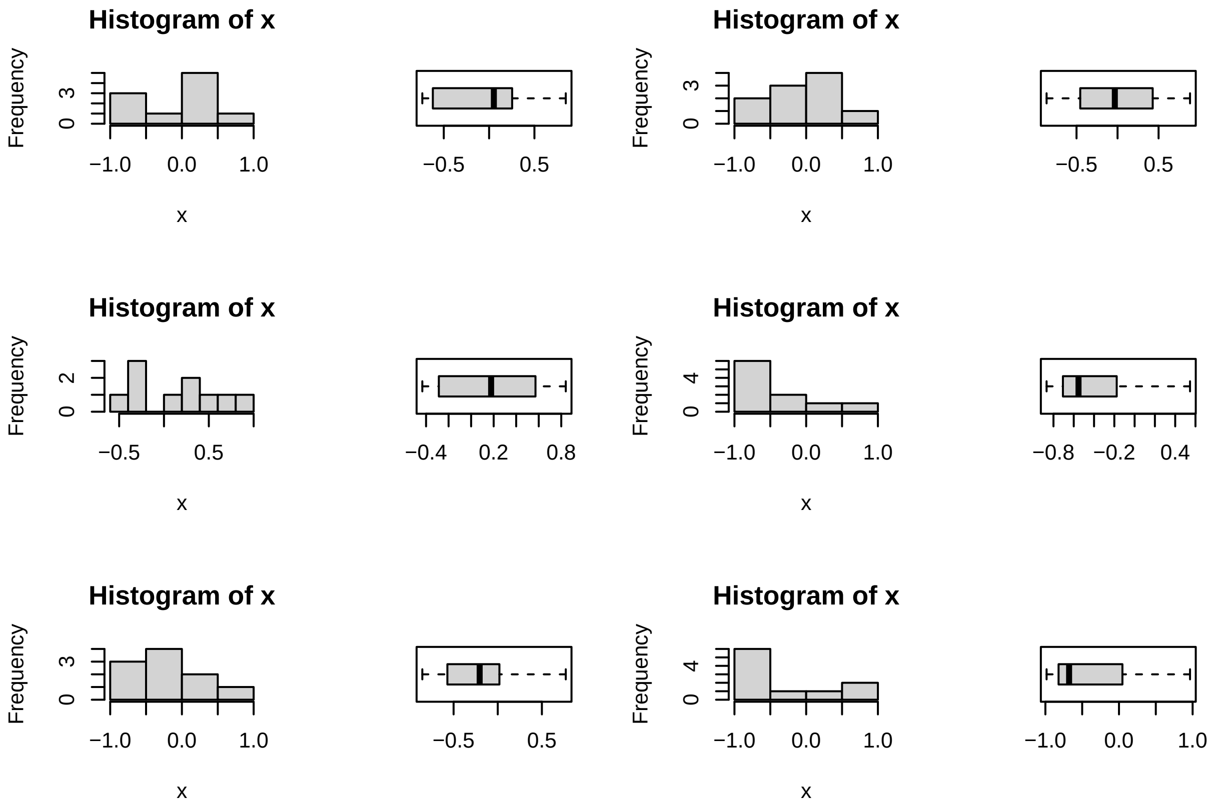

Histogram of x
Frequency
3
0
−0.4
0.2
0.6
x
−0.2
0.2
0.6
Histogram of x
Frequency
3
0
−1.0
0.0
1.0
x
−0.6
0.0
0.6
Histogram of x
Frequency
3
0
−1.0
0.0
1.0
x
−1.0
0.0
Histogram of x
Frequency
2
0
−1.0
0.0
1.0
x
−1.0
0.0
1.0
Histogram of x
Frequency
2
0
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
3
0
−1.0
0.0
1.0
x
−1.0
0.0
1.0

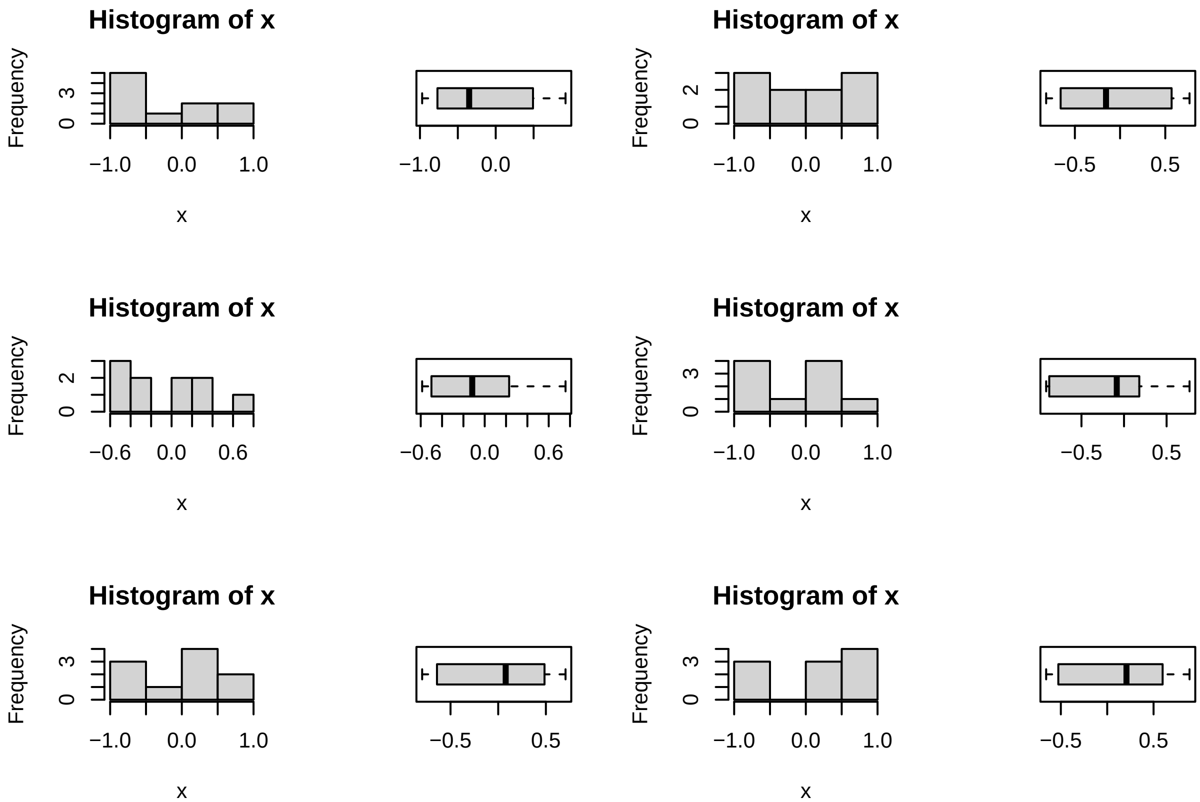


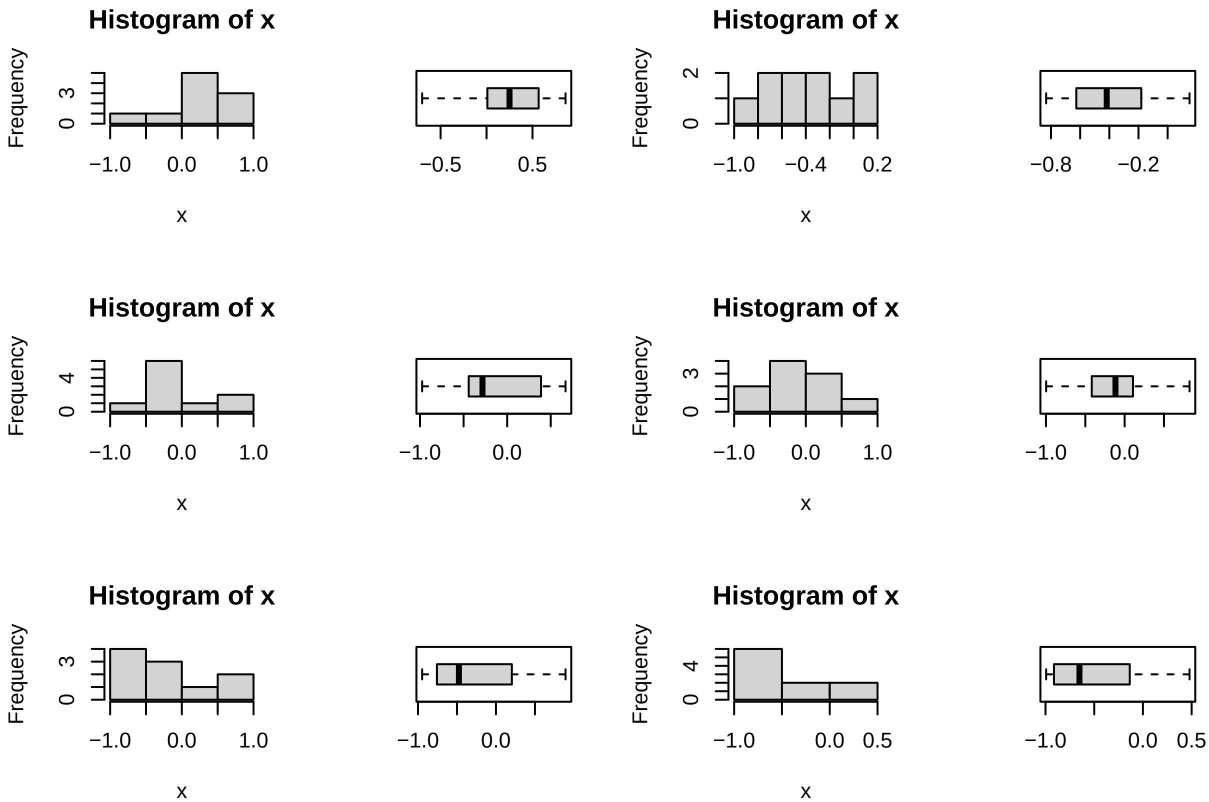
Histogram of x
Frequency
0
2
−0.5
0.5
x
−0.4
0.2
0.8
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5

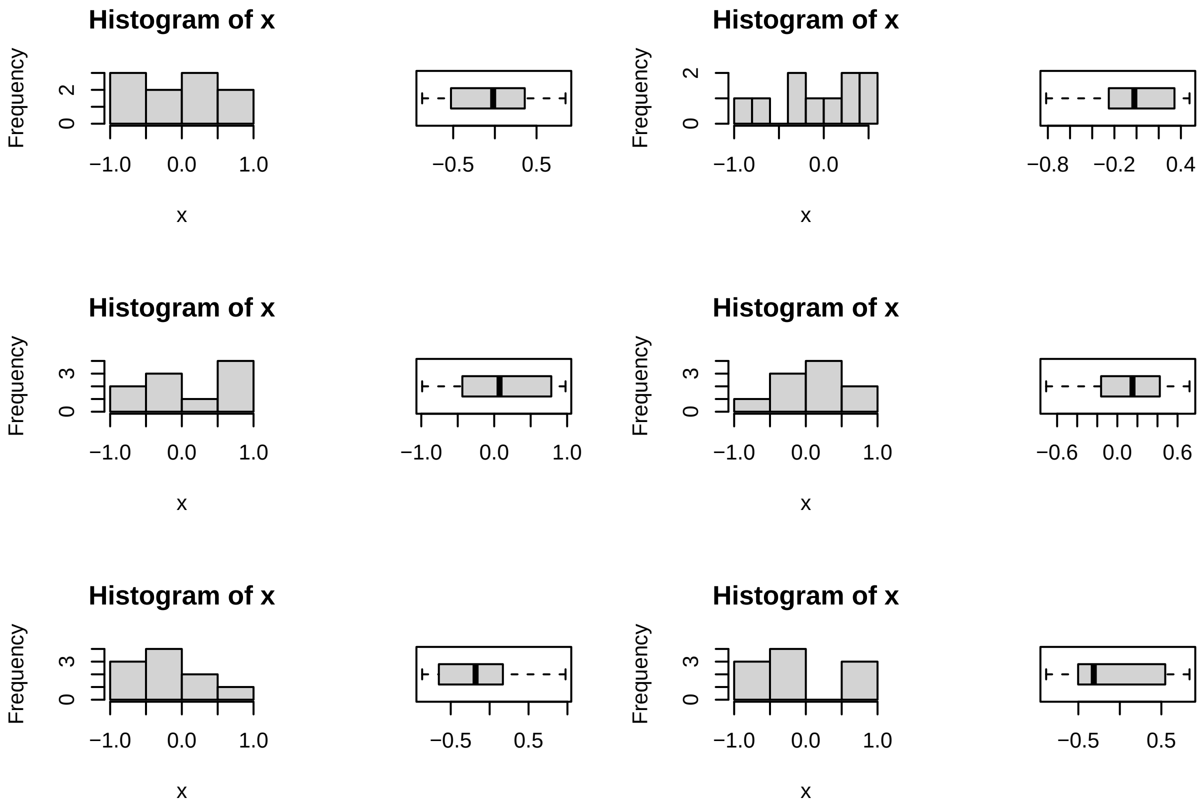

Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
Histogram of x
Frequency
0
2
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
4
−1.0
0.0
1.0
x
−0.6
0.0
0.6
Histogram of x
Frequency
0
2
−1.0
0.0
1.0
x
−0.5
0.5

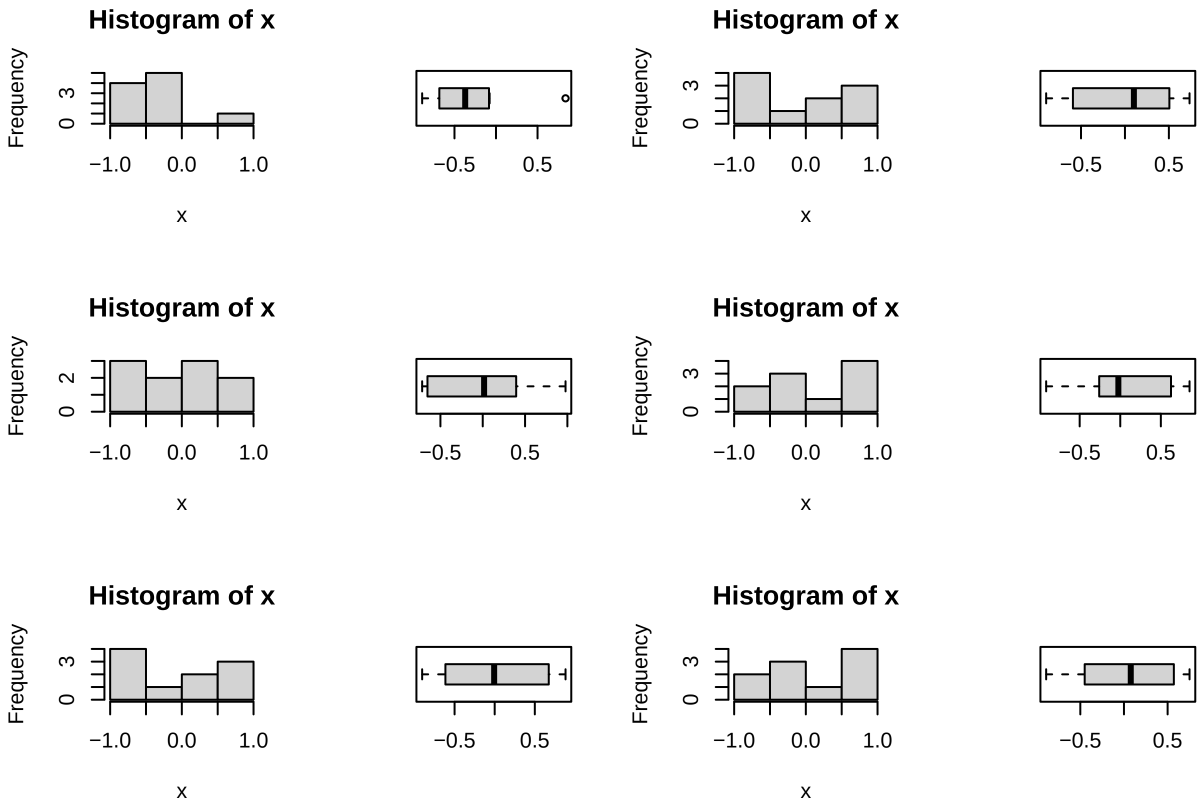

Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5

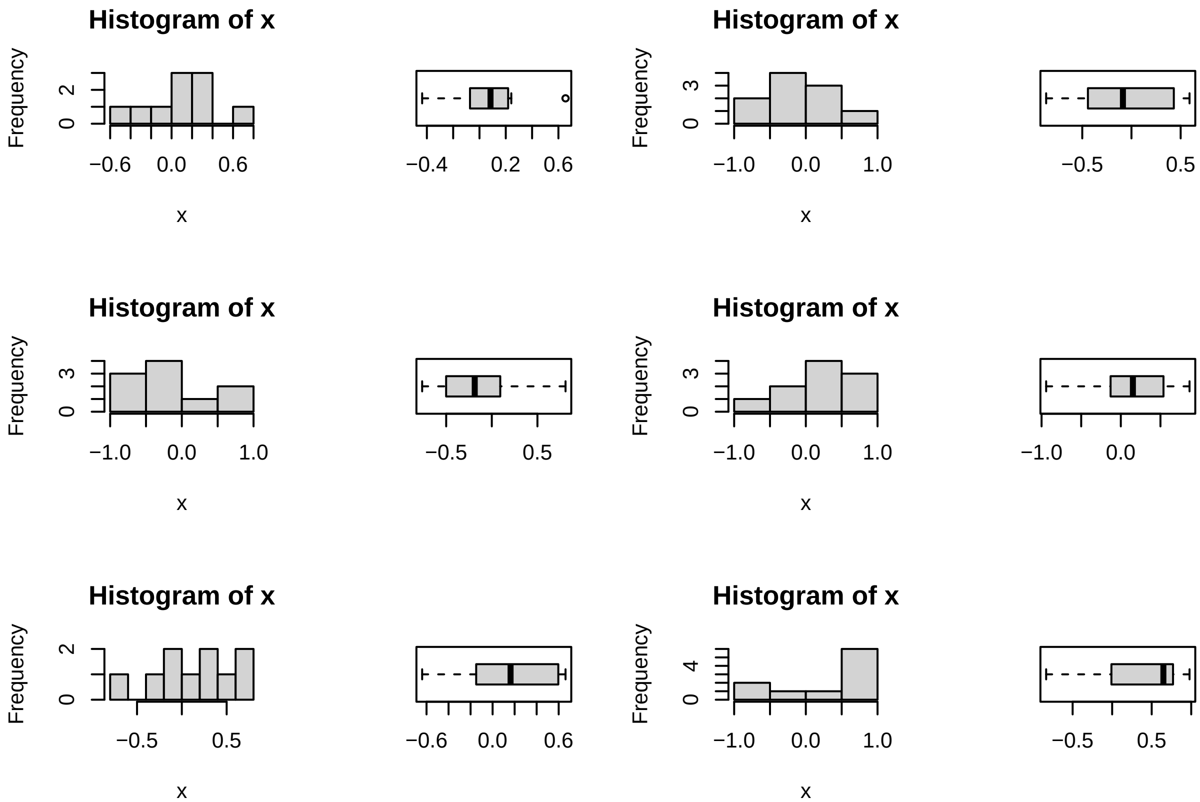

Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
2
−0.5
0.5
x
−0.4
0.2
0.8
Histogram of x
Frequency
0
4
−1.0
0.0
1.0
x
−0.8
−0.2
0.4
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
4
−1.0
0.0
1.0
x
−1.0
0.0
1.0

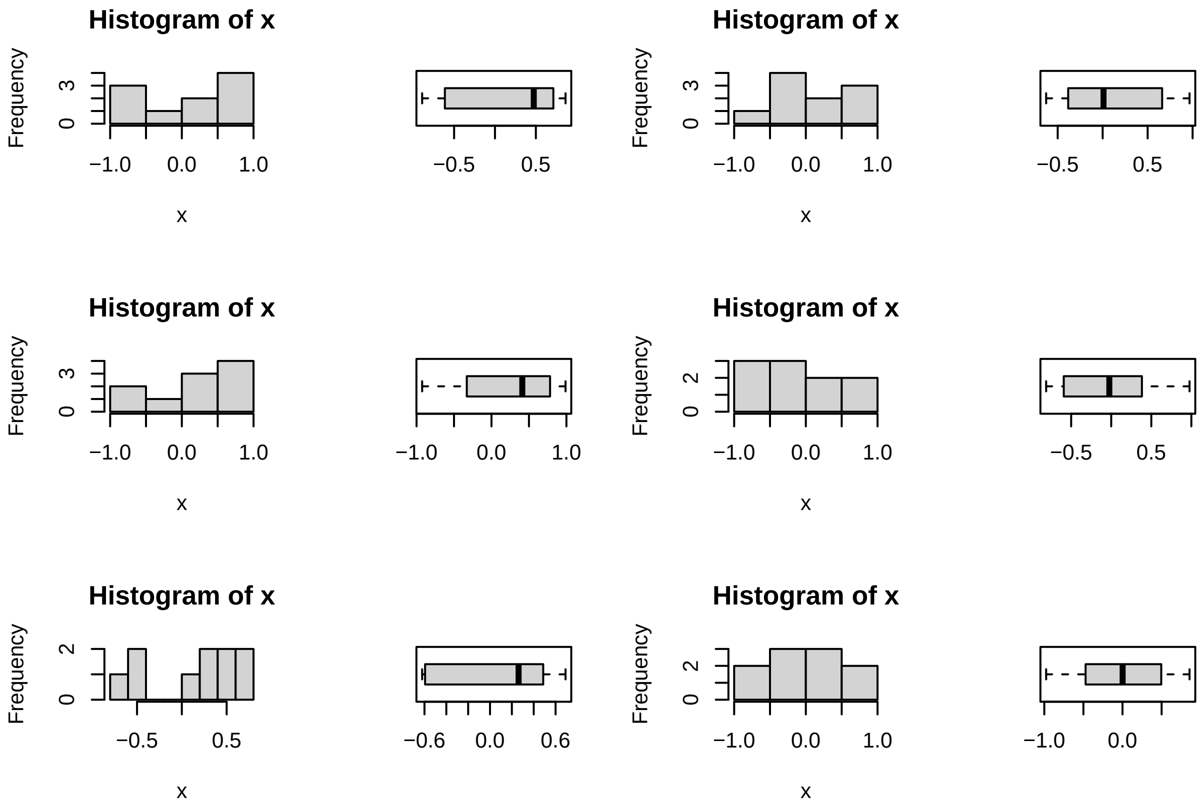

Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x

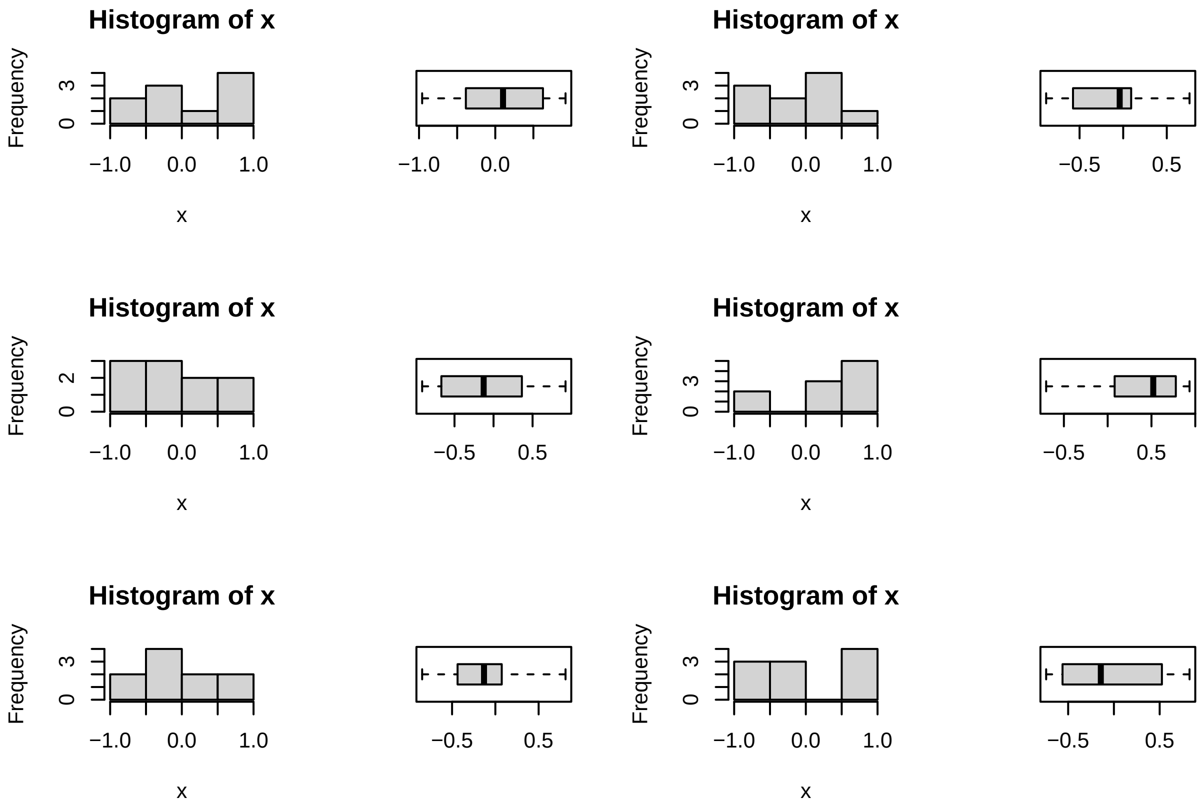

Histogram of x
Frequency
0 3
−1.0 0.0 1.0
x
−0.5 0.5
Histogram of x
Frequency
0 2
−1.0 −0.4 0.2
x
−0.8 −0.2
Histogram of x
Frequency
0 4
−1.0 0.0 1.0
x
−1.0 0.0
Histogram of x
Frequency
0 3
−1.0 0.0 1.0
x
−1.0 0.0
Histogram of x
Frequency
0 3
−1.0 0.0 1.0
x
−1.0 0.0
Histogram of x
Frequency
0 4
−1.0 0.0 0.5
x
−1.0 0.0 0.5

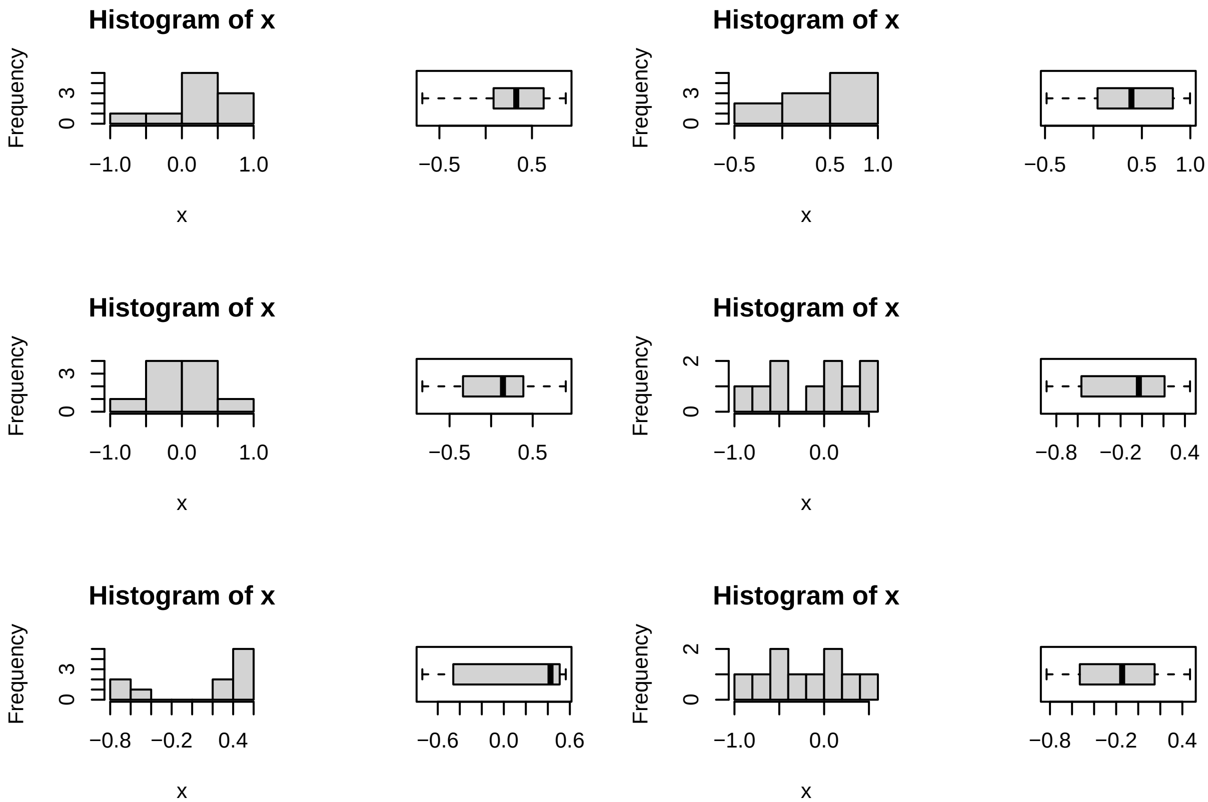

Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x
Histogram of x
Frequency
x

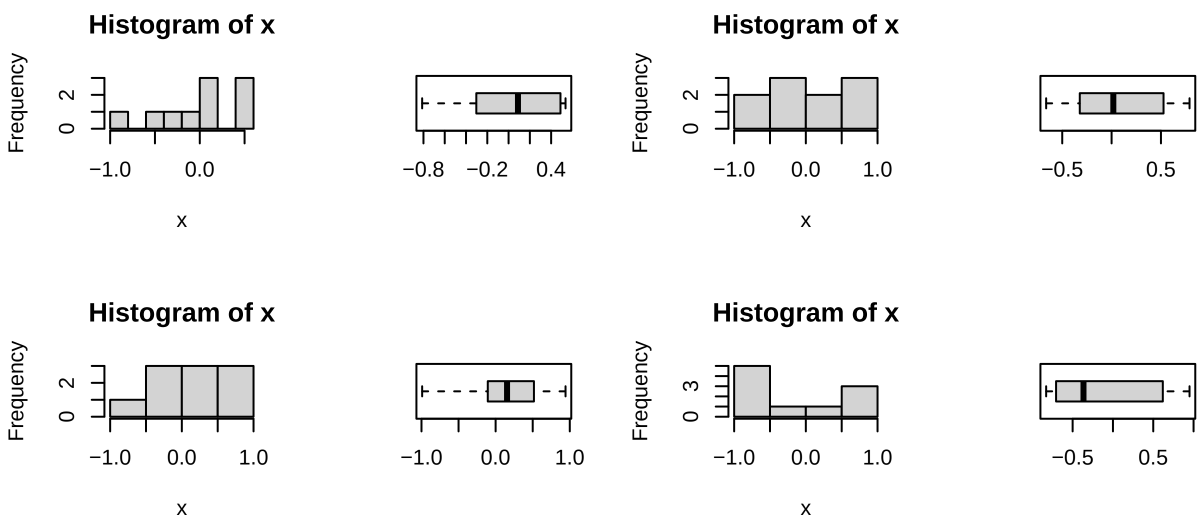

Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
2
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5


Histogram of x
Frequency
0
2
−0.6
0.0
0.6
x
−0.4
0.2
0.6
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−1.0
0.0
Histogram of x
Frequency
0
2
−0.5
0.5
x
−0.6
0.0
0.6
Histogram of x
Frequency
0
4
−1.0
0.0
1.0
x
−0.5
0.5


Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−1.0
0.0
1.0
Histogram of x
Frequency
0
2
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
2
−0.5
0.5
x
−0.6
0.0
0.6
Histogram of x
Frequency
0
2
−1.0
0.0
1.0
x
−1.0
0.0


Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
2


Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
3
−0.5
0.5
1.0
x
−0.5
0.5
1.0
Histogram of x
Frequency
0
3
−1.0
0.0
1.0
x
−0.5
0.5
Histogram of x
Frequency
0
2
−1.0
0.0
x
−0.8
−0.2
0.4
Histogram of x
Frequency
0
3
−0.8
−0.2
0.4
x
−0.6
0.0
0.6
Histogram of x
Frequency
0
2
−1.0
0.0
x
−0.8
−0.2
0.4



4. Augmentez la taille de votre échantillon à 50 et répétez votre expérience. Que remarquez-vous? Sont-ils proches de la symétrie ?

```r
x <- runif(50, -1, 1)
```

Monte Carlo Methods

Moyenne et phénomène de concentration.

1. Supposons que la variance 2 = V [(X)] < ∞. Donner une borne de cette quantité en utilisant l'inégalité de Bienaymé Chebychev.

On trouve grace à l'inégalité de Bienaymé Chebychev:

$$a^2 P(|\psi(X) - \theta| \geq a) \leq V[\psi(X)]$$

2. En supposant que a (Xi) b, donner une borne en utilisant l'inégalité de Hoeffding.