Learning in LTN ¶

This tutorial explains how to learn some language symbols (predicates, functions, constants) using the satisfaction of a knowledgebase as an objective. It expects basic familiarity of the first two turoials on LTN (grounding symbols and connectives).

In [1]:

```
import ltn
import tensorflow as tf
import numpy as np
import matplotlib.pyplot as plt
```

```
Init Plugin
Init Graph Optimizer
Init Kernel
```

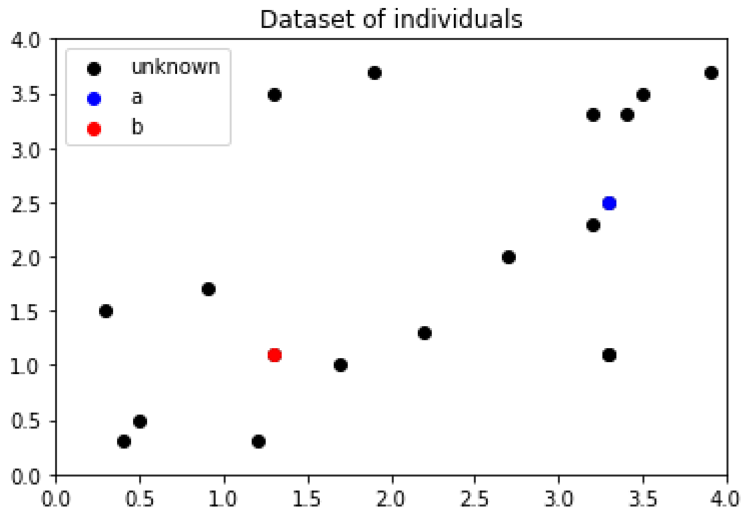
We use the following simple example to illustrate learning in LTN.

The domain is the square $[0,4] \times [0,4]$. We have one example of the class A and one example of the class B. The rest of the individuals are not labelled, but there are two assumptions:

- A and B are mutually exclusive,
- any two close points should share the same label.

In [2]:

```
points = np.array(
        [[0.4,0.3],[1.2,0.3],[2.2,1.3],[1.7,1.0],[0.5,0.5],[0.3, 1.5],[1.3, 1.1],[0.9,
        [3.4,3.3],[3.2,3.3],[3.2,2.3],[2.7,2.0],[3.5,3.5],[3.3, 2.5],[3.3, 1.1],[1.9,
point_a = [3.3,2.5]
point_b = [1.3,1.1]
fig, ax = plt.subplots()
ax.set_xlim(0,4)
ax.set_ylim(0,4)
ax.scatter(points[:,0],points[:,1],color="black",label="unknown")
ax.scatter(point_a[0],point_a[1],color="blue",label="a")
ax.scatter(point_b[0],point_b[1],color="red",label="b")
ax.set_title("Dataset of individuals")
plt.legend();
```



We define the membership predicate $C(x, l)$, where x is an individual and l is a onehot label to denote the two classes. C is approximated by a simple MLP. The last layer, that computes probabilities per class, uses a `softmax` activation, ensuring that the classes are mutually-exclusive.

We define the knowledgebase $\mathcal{K}$ composed of the following rules:

$$C(a, l_a) \tag{1}$$

$$C(b, l_b) \tag{2}$$

$$\forall x_1, x_2, l \left(\mathrm{Sim}(x_1, x_2) \rightarrow \left(C(x_1, l) \leftrightarrow C(x_2, l)\right)\right) \tag{3}$$

where a and b the two individuals already classified; x_1,x_2 are variables ranging over all individuals; l_a, l_b are the one-hot labels for A and B; l is a variable ranging over the labels. Sim is a predicate measuring similarity between two points. $\mathcal{G}(\mathrm{Sim}) : \vec{u}, \vec{v} \mapsto \exp(-\|\vec{u} - \vec{v}\|^2)$.

The objective is to learn the predicate C to maximize the satisfaction of $\mathcal{K}$. If θ denotes the set of trainable parameters, the task is :

$$\theta^* = \mathrm{argmax}_{\theta \in \Theta} \ \mathrm{SatAgg}_{\phi \in \mathcal{K}} \mathcal{G}_\theta(\phi)$$

where SatAgg is an operator that aggregates the truth values of the formulas in $\mathcal{K}$ (if there are more than one formula).

To evaluate the grounding of each formula, one has to define the grounding of the non-logical symbols and of the operators.

In [3]:

```
class ModelC(tf.keras.Model):
    def __init__(self):
        super(ModelC, self).__init__()
        self.dense1 = tf.keras.layers.Dense(5, activation=tf.nn.elu)
        self.dense2 = tf.keras.layers.Dense(5, activation=tf.nn.elu)
        self.dense3 = tf.keras.layers.Dense(2, activation=tf.nn.softmax)

    def call(self, inputs):
        """inputs[0]: point, inputs[1]: onehot label"""
        x, label = inputs[0], inputs[1]
        x = self.dense1(x)
        x = self.dense2(x)
        prob = self.dense3(x)
        return tf.math.reduce_sum(prob*label,axis=1)

C = ltn.Predicate(ModelC())
```

```
Metal device set to: Apple M1

systemMemory: 16.00 GB
maxCacheSize: 5.33 GB



2021-09-24 17:12:13.181397: I tensorflow/core/common_runtime/pluggable_dev
2021-09-24 17:12:13.181501: I tensorflow/core/common_runtime/pluggable_dev
```

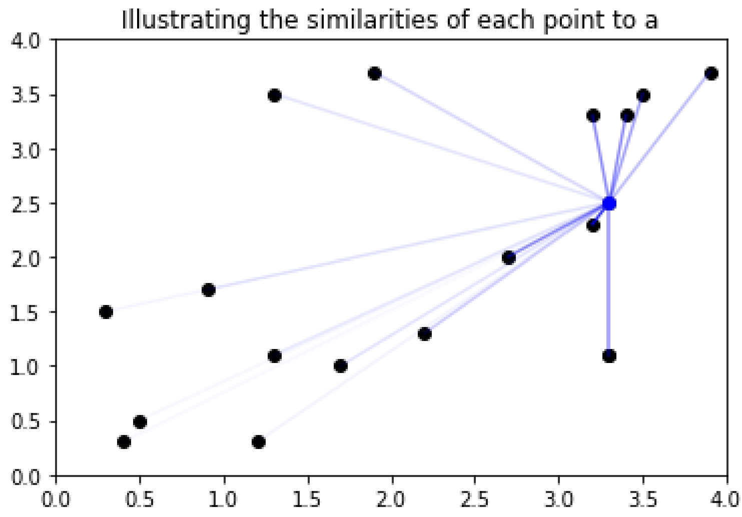
In [4]:

```
x1 = ltn.Variable("x1",points)
x2 = ltn.Variable("x2",points)
a = ltn.Constant([3.3,2.5], trainable=False)
b = ltn.Constant([1.3,1.1], trainable=False)
l_a = ltn.Constant([1,0], trainable=False)
l_b = ltn.Constant([0,1], trainable=False)
l = ltn.Variable("l",[[1,0],[0,1]])

Sim = ltn.Predicate.Lambda(
    lambda args: tf.exp(-1.*tf.sqrt(tf.reduce_sum(tf.square(args[0]-args[1]),axis=1)))
)
```

In [7]:

```
similarities_to_a = Sim([x1,a]).tensor
fig, ax = plt.subplots()
ax.set_xlim(0,4)
ax.set_ylim(0,4)
ax.scatter(points[:,0],points[:,1],color="black")
ax.scatter(a.tensor[0],a.tensor[1],color="blue")
ax.set_title("Illustrating the similarities of each point to a")
for i, sim_to_a in enumerate(similarities_to_a):
    plt.plot([points[i,0],a.tensor[0]],[points[i,1],a.tensor[1]], alpha=sim_to_a.numpy
```



Notice the operator for equivalence $p \leftrightarrow q$; in LTN, it is simply implemented as $(p \rightarrow q) \wedge (p \leftarrow q)$ using one operator for conjunction and one operator for implication.

In [8]:

```
Not = ltn.Wrapper_Connective(ltn.fuzzy_ops.Not_Std())
And = ltn.Wrapper_Connective(ltn.fuzzy_ops.And_Prod())
Or = ltn.Wrapper_Connective(ltn.fuzzy_ops.Or_ProbSum())
Implies = ltn.Wrapper_Connective(ltn.fuzzy_ops.Implies_Reichenbach())
Equiv = ltn.Wrapper_Connective(ltn.fuzzy_ops.Equiv(ltn.fuzzy_ops.And_Prod(),ltn.fuzzy_
Forall = ltn.Wrapper_Quantifier(ltn.fuzzy_ops.Aggreg_pMeanError(p=2),semantics="forall
Exists = ltn.Wrapper_Quantifier(ltn.fuzzy_ops.Aggreg_pMean(p=6),semantics="exists")
```

If there are several closed formulas in $\mathcal{K}$, their truth values need to be aggregated. We recommend to use the generalized mean inspired operator `pMeanError`, already used to implement $\forall$. The hyperparameter again allows flexibility in how strict the formula aggregation is ($p = 1$ corresponds to `mean`; $p \rightarrow +\inf$ corresponds to `min`).

The knowledgebase should be written inside of a function that is decorated with `tf.function`. This Tensorflow decorator compiles the function into a callable TensorFlow graph (static).

In [14]:

```
formula_aggregator = ltn.Wrapper_Formula_Aggregator(ltn.fuzzy_ops.Aggreg_pMeanError(p=

@tf.function
def axioms():
    axioms = [
        C([a,l_a]),
        C([b,l_b]),
        Forall(
            [x1,x2,l],
            Implies( Sim([x1,x2]),
                     Equiv(C([x1,l]),C([x2,l]))
                    )
        )
    ]
    kb = formula_aggregator(axioms)
    sat = kb.tensor
    return sat
```

It is important to always run (forward pass) the knowledgebase once before training, as Tensorflow initializes weights and compiles the graph during the first call.

In [15]:

```
axioms()
```

```
2021-09-24 17:15:46.561522: I tensorflow/compiler/mlir/mlir_graph_optimi
2021-09-24 17:15:46.566398: W tensorflow/core/platform/profile_utils/cpu
2021-09-24 17:15:46.567351: I tensorflow/core/grappler/optimizers/custom
```

Out[15]:

```
<tf.Tensor: shape=(), dtype=float32, numpy=0.4704498>
```

Eventually, one can write a custom training loop in Tensorflow.

In [16]:

```
trainable_variables = C.trainable_variables
optimizer = tf.keras.optimizers.Adam(learning_rate=0.001)

for epoch in range(2000):
    with tf.GradientTape() as tape:
        loss = 1. - axioms()
    grads = tape.gradient(loss, trainable_variables)
    optimizer.apply_gradients(zip(grads, trainable_variables))
    if epoch%200 == 0:
        print("Epoch %d: Sat Level %.3f"%(epoch, axioms()))
print("Training finished at Epoch %d with Sat Level %.3f"%(epoch, axioms()))
```

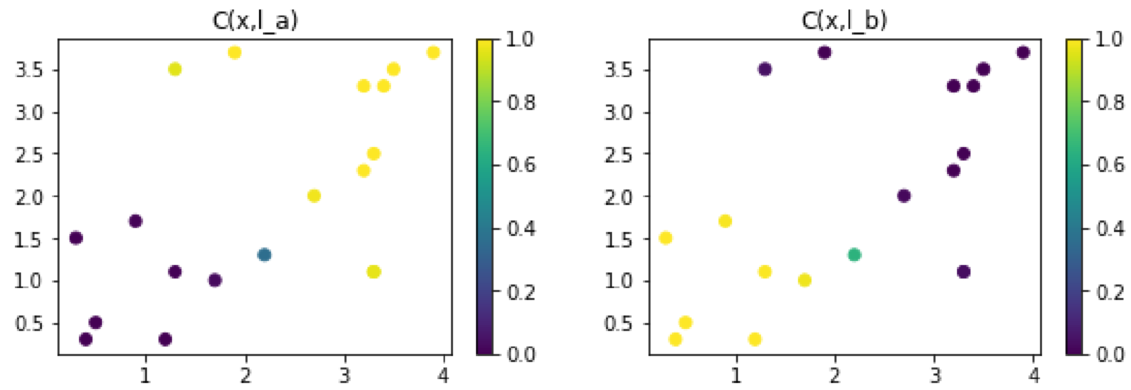
```
2021-09-24 17:16:07.536407: I tensorflow/core/grappler/optimizers/custom_g
2021-09-24 17:16:07.619984: I tensorflow/core/grappler/optimizers/custom_g
```

```
Epoch 0: Sat Level 0.471
Epoch 200: Sat Level 0.678
Epoch 400: Sat Level 0.931
Epoch 600: Sat Level 0.954
Epoch 800: Sat Level 0.958
Epoch 1000: Sat Level 0.959
Epoch 1200: Sat Level 0.959
Epoch 1400: Sat Level 0.959
Epoch 1600: Sat Level 0.959
Epoch 1800: Sat Level 0.959
Training finished at Epoch 1999 with Sat Level 0.959
```

After a few epochs, the system has learned to identify samples close to the point a (resp. b) as belonging to class A (resp. B) based on the rules of the knowledgebase.

In [18]:

```
fig = plt.figure(figsize=(10,3))
fig.add_subplot(1,2,1)
is_a = C([x1,l_a])
plt.scatter(x1.tensor[:,0],x1.tensor[:,1],c=is_a.tensor.numpy(),vmin=0,vmax=1)
plt.title("C(x,l_a)")
plt.colorbar()
fig.add_subplot(1,2,2)
is_b = C([x1,l_b])
plt.scatter(x1.tensor[:,0],x1.tensor[:,1],c=is_b.tensor.numpy(),vmin=0,vmax=1)
plt.title("C(x,l_b)")
plt.colorbar()
plt.show();
```



Special Cases

Variables grounded by batch

When working with batches of data, grounding the variables with different values at each step:

1. Pass the values in arguments to the knowledgebase function,
2. Create the ltn variables within the function.

```
@tf.function
def axioms(data_x, data_y):
    x = ltn.Variable("x", data_x)
    y = ltn.Variable("y", data_y)
    return Forall([x,y],P([x,y]))

...
for epoch in range(epochs):
    for batch_x, batch_y in dataset:
        with tf.GradientTape() as tape:
            loss_value = 1. - axioms(batch_x, batch_y)
        grads = tape.gradient(loss_value, trainable_variables)
        optimizer.apply_gradients(zip(grads, trainable_variables))
```

Variables denoting a sequence of trainable constants

When a variable denotes a sequence of trainable constants (embeddings):

1. Do not create the variable outside the scope of `tf.GradientTape()` ,
2. Create the variable within the training step function.

```
c1 = ltn.Constant([2.1,3], trainable=True)
c2 = ltn.Constant([4.5,0.8], trainable=True)
# Do not assign the variable here. Tensorflow would not keep track of the
# gradients between c1/c2 and x during training.
...
@tf.function
def axioms():
    # The assignation must be done within the tf.GradientTape,
    # inside of the training step function.
    x = ltn.Variable.from_constants("x", [c1,c2])
    return Forall(x,P(x))
...
for epoch in range(epochs):
    with tf.GradientTape() as tape:
        loss_value = 1. - axioms() # the training step function is called withi
    grads = tape.gradient(loss_value, trainable_variables)
    optimizer.apply_gradients(zip(grads, trainable_variables))
```