

```
# CAP 6635 Artificial Intelligence
# Local search (Hill Climbing and Simulated Annealing to solve N-Queens problem)
# X. Zhu, June. 15 2023
# Code adapted from: https://github.com/TranDatDT/n-queens-simulated-annealing/blob/master/main.py

# Modified by Matthew Acs for HW 2

# Set number of questions (N) as 8, 16, 32, 64, respectively.
# Repeat hill-climbing search (HC) and simulated annealing (SA) 10
# times for each N values, calculate average runtime and success rates
# (1 divided by number of repetitions before success).

# For simulated annealing method (SA), fix the number of Queen to 64,
# and vary the initial temperature from T=4000 to T=[400, 40, 4, 0.4],
# respectively.

# Code modified to run for N=8, 16, 32, and 64
# Code modified to loop through 10 iterations for each N value and take average runtime and success rate
# Code modified to test SA for T = 4000, 400, 40, 4, 0.4
```

▼ SA and HC with variable N

▼ N = 8

```
import random
import numpy as np
from math import exp
import time
from copy import deepcopy
import matplotlib.pyplot as plt

N_QUEENS = 8
TEMPERATURE = 40

def threat_calculate(n):
    '''Combination formular. It is choosing two queens in n queens'''
    if n < 2:
        return 0
    if n == 2:
        return 1
    return (n - 1) * n / 2

def create_board(n):
    '''Create a chess board with a queen on a row'''
    chess_board = {}
    temp = list(range(n))
    random.shuffle(temp) # shuffle to make sure it is random
    column = 0

    while len(temp) > 0:
        row = random.choice(temp)
        chess_board[column] = row
        temp.remove(row)
        column += 1
    del temp
    return chess_board

def cost(chess_board):
    '''Calculate how many pairs of threaten queen'''
    threat = 0
    m_chessboard = {}
    a_chessboard = {}

    for column in chess_board:
        temp_m = column - chess_board[column]
        temp_a = column + chess_board[column]
        if temp_m not in m_chessboard:
```

```

--f_m-----f_m-----
    m_chessboard[temp_m] = 1
else:
    m_chessboard[temp_m] += 1
if temp_a not in a_chessboard:
    a_chessboard[temp_a] = 1
else:
    a_chessboard[temp_a] += 1

for i in m_chessboard:
    threat += threat_calculate(m_chessboard[i])
del m_chessboard

for i in a_chessboard:
    threat += threat_calculate(a_chessboard[i])
del a_chessboard

return threat

def hill_climbing():
    '''Hill Climbing Search'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
            successor[index_1], successor[index_2] = successor[index_2], \
                successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0:
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True
            print("Successful Solution:")
            print_chess_board(answer)
            break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def simulated_annealing():
    '''Simulated Annealing'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch

```

```

    successor = deepcopy(answer)
    while True:
        index_1 = random.randrange(0, N_QUEENS - 1)
        index_2 = random.randrange(0, N_QUEENS - 1)
        if index_1 != index_2:
            break
    successor[index_1], successor[index_2] = successor[index_2], \
        successor[index_1] # swap two chosen queens

    delta = cost(successor) - cost_answer
    if delta < 0 or (random.uniform(0, 1) < exp(-delta / t)):
        answer = deepcopy(successor)
        cost_answer = cost(answer)
        Costs.append(cost_answer)
    if cost_answer == 0:
        solution_found = True
        print("Successful Solution:")
        print_chess_board(answer)
        break
if solution_found is False:
    print("Failed")
    return(False, Costs)
else:
    return(True, Costs)

def print_chess_board(board):
    '''Print the chess board'''
    showBoard = np.zeros([N_QUEENS, N_QUEENS], dtype = int)
    for column, row in board.items():
        showBoard[row][column] = 1
    #print("{} => {}".format(column, row))
    for i in range(N_QUEENS):
        print(showBoard[i])

def main(method='HC'):
    start = time.time()
    Success=False
    repetitions=0
    while not Success:
        if method=='SA':
            Success, Costs=simulated_annealing()
            repetitions=repetitions+1
        elif method=='HC':
            Success, Costs=hill_climbing()
            repetitions=repetitions+1
    print("It takes %d repetitions to succeed. Runtime in second %f:"% (repetitions,(time.time() - start)))
    return(repetitions, (time.time() - start), Success, Costs)

if __name__ == "__main__":
    print("Hill Climbing")
    print("")
    rate_HC = []
    runtime_HC = []
    for i in range(10):
        method='HC'
        Reps, run, Success, Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_HC.append(1/Reps)
        runtime_HC.append(run)
    plt.plot(Costs)
    plt.xlabel(method+' Board Updated Time')
    plt.ylabel('# of attacked Q pairs')
    plt.show()
    print()
    print("Average success rate: " + str(sum(rate_HC)/len(rate_HC)))
    print("Average runtime: " + str(sum(runtime_HC)/len(runtime_HC)))

if __name__ == "__main__":
    print("Simulated Annealing")
    print("")
    rate_SA = []
    runtime_SA = []
    for i in range(10):
        method='SA'
        Reps, run, Success, Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_SA.append(1/Reps)

```

```
runtime_SA.append(run)
plt.plot(Costs)
plt.xlabel(method+' Board Updated Time')
plt.ylabel('# of attacked Q pairs')
plt.show()
print()
print("Average success rate: " + str(sum(rate_SA)/len(rate_SA)))
print("Average runtime: " + str(sum(runtime_SA)/len(runtime_SA)))
```

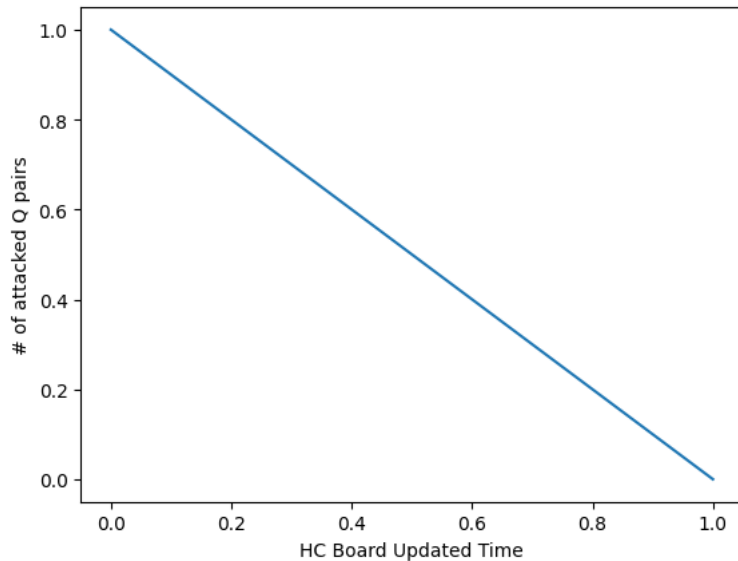
Hill Climbing

Failed

Successful Solution:

```
[0 0 0 1 0 0 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1]
[0 0 0 0 0 1 0 0]
[1 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
```

It takes 2 repetitions to succeed. Runtime in second 0.047978:

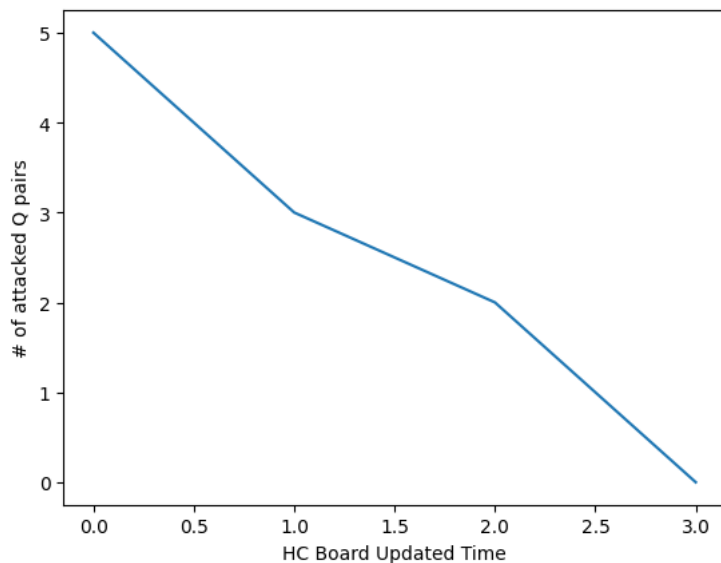


Failed

Successful Solution:

```
[0 0 0 0 0 1 0 0]
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 1 0]
[1 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1]
```

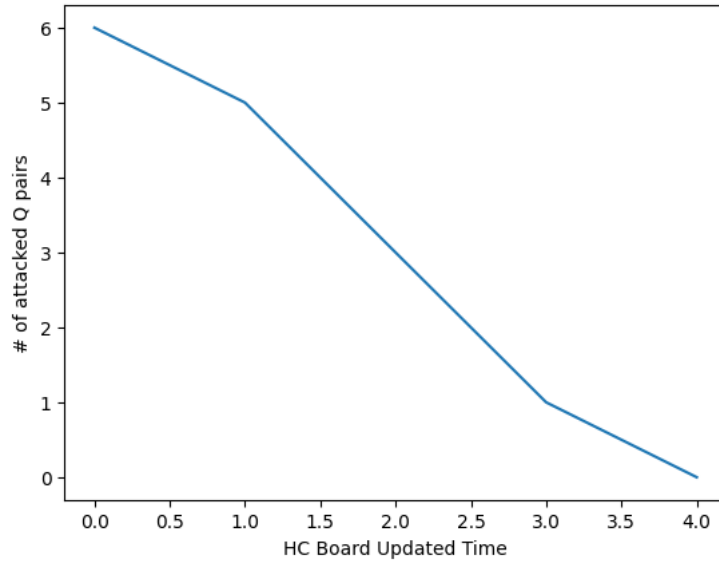
It takes 2 repetitions to succeed. Runtime in second 0.051243:



Successful Solution:

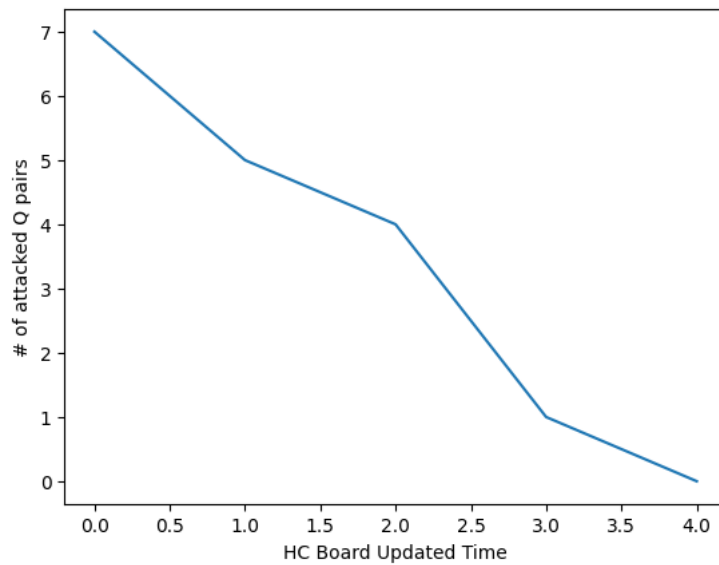
```
[1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1]
[0 1 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 1 0 0]
[0 0 0 1 0 0 0 0]
```

```
[ 0 0 0 0 0 0 0 0 0 0]
It takes 1 repetitions to succeed. Runtime in second 0.005311:
```



```
Failed
Failed
Successful Solution:
[0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1]
[0 0 0 0 1 0 0 0]
[1 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
```

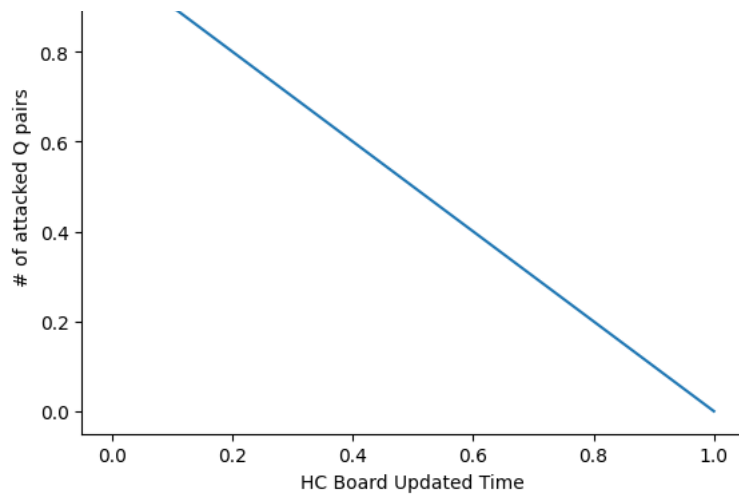
```
It takes 3 repetitions to succeed. Runtime in second 0.064399:
```



```
Failed
Failed
Failed
Failed
Failed
Failed
Successful Solution:
[0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0]
[1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1]
[0 1 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
```

```
It takes 7 repetitions to succeed. Runtime in second 0.261621:
```

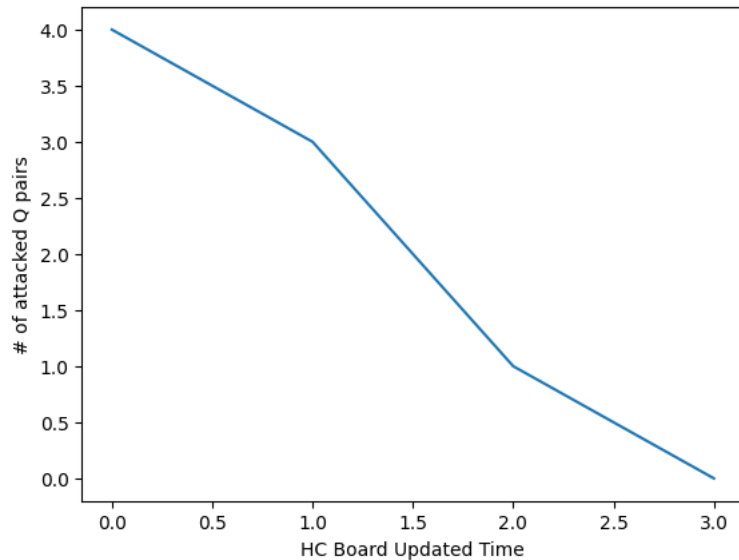




Successful Solution:

```
[0 0 1 0 0 0 0 0]
[0 0 0 0 0 1 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[0 0 0 0 1 0 0 0]
[1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1]
[0 0 0 1 0 0 0 0]
```

It takes 1 repetitions to succeed. Runtime in second 0.002003:



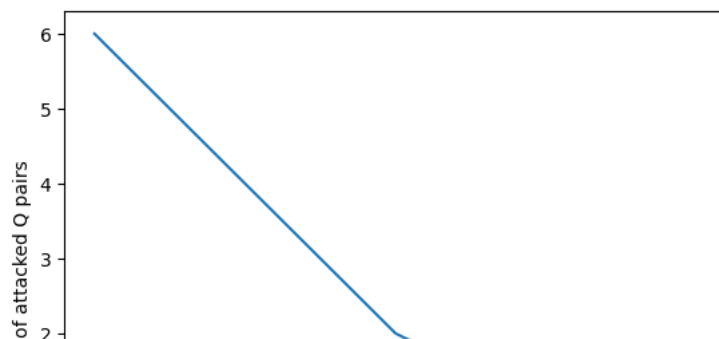
Failed

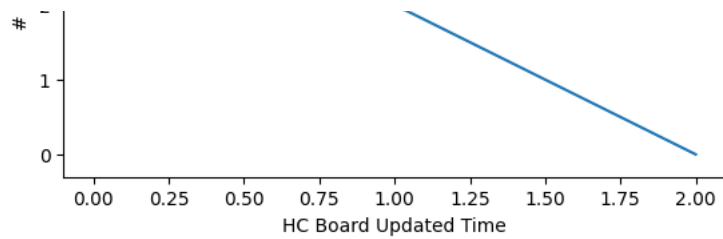
Failed

Successful Solution:

```
[0 0 0 0 1 0 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 1]
[0 0 1 0 0 0 0 0]
[1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
```

It takes 3 repetitions to succeed. Runtime in second 0.064966:



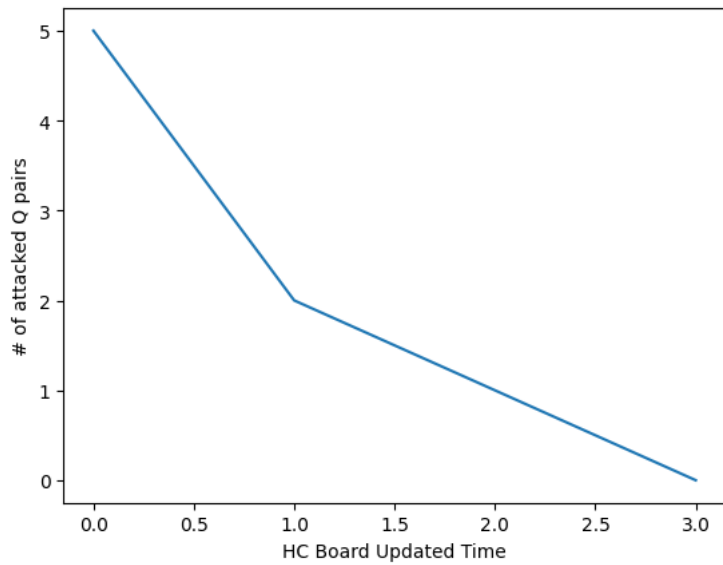


Failed
Failed
Failed
Failed
Failed
Failed
Failed

Successful Solution:

```
[0 0 0 1 0 0 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1]
[0 0 0 0 0 1 0 0]
[1 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
```

It takes 8 repetitions to succeed. Runtime in second 0.442849:

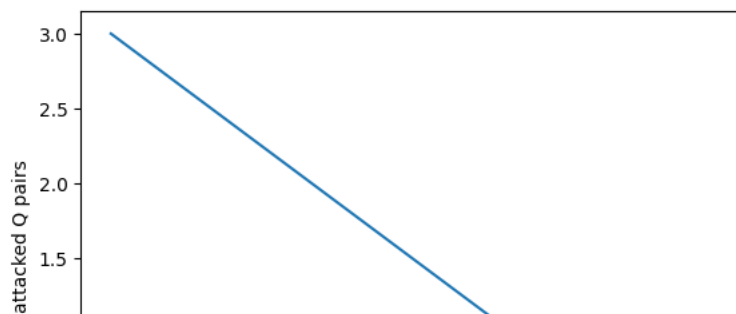


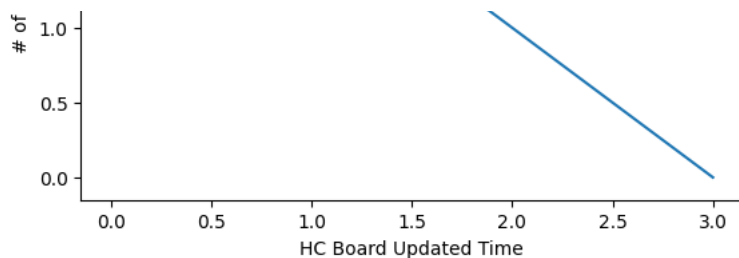
Failed
Failed
Failed
Failed
Failed
Failed
Failed

Successful Solution:

```
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 1]
[1 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 1 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0]
```

It takes 8 repetitions to succeed. Runtime in second 0.473745:





Failed

Failed

Successful Solution:

[0 1 0 0 0 0 0 0]

[0 0 0 0 0 0 1 0]

[0 0 0 0 1 0 0 0]

[0 0 0 0 0 0 0 1]

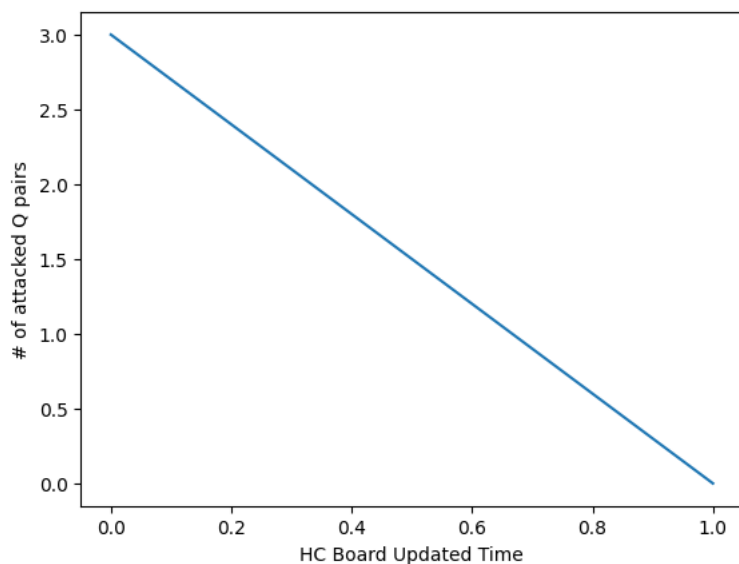
[1 0 0 0 0 0 0 0]

[0 0 0 1 0 0 0 0]

[0 0 0 0 0 1 0 0]

[0 0 1 0 0 0 0 0]

It takes 3 repetitions to succeed. Runtime in second 0.122559:



Average success rate: 0.43928571428571433

Average runtime: 0.1541755437850952

Simulated Annealing

Successful Solution:

[0 1 0 0 0 0 0 0]

[0 0 0 0 1 0 0 0]

[0 0 0 0 0 0 1 0]

[0 0 0 1 0 0 0 0]

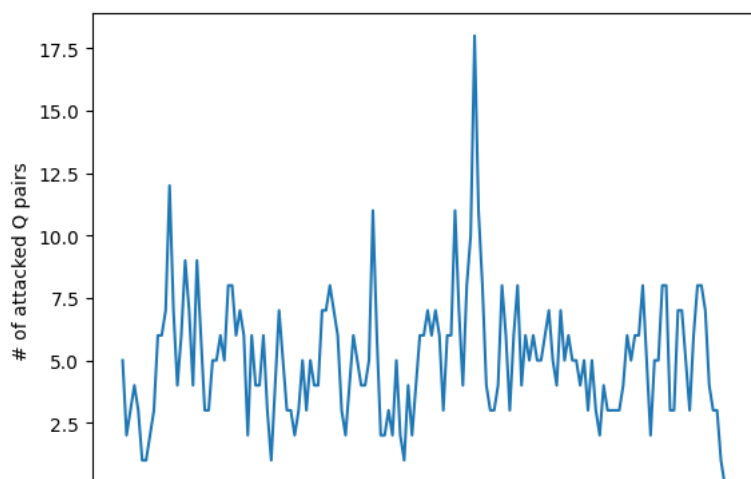
[1 0 0 0 0 0 0 0]

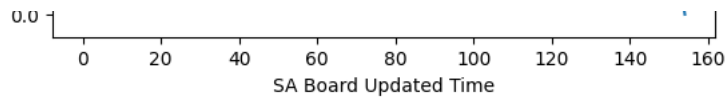
[0 0 0 0 0 0 0 1]

[0 0 0 0 0 1 0 0]

[0 0 1 0 0 0 0 0]

It takes 1 repetitions to succeed. Runtime in second 0.016500:



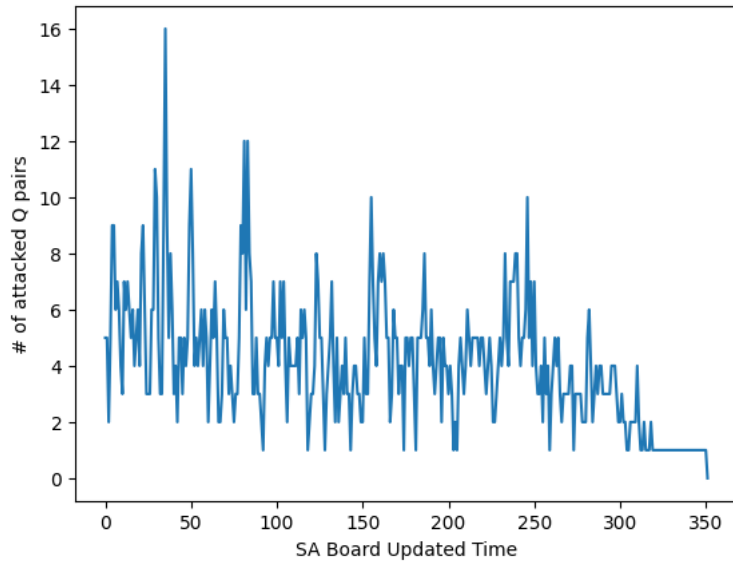


Failed

Successful Solution:

```
[0 0 0 0 0 0 0 1]
[0 1 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 0 1 0 0 0 0 0]
[1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 1 0 0]
```

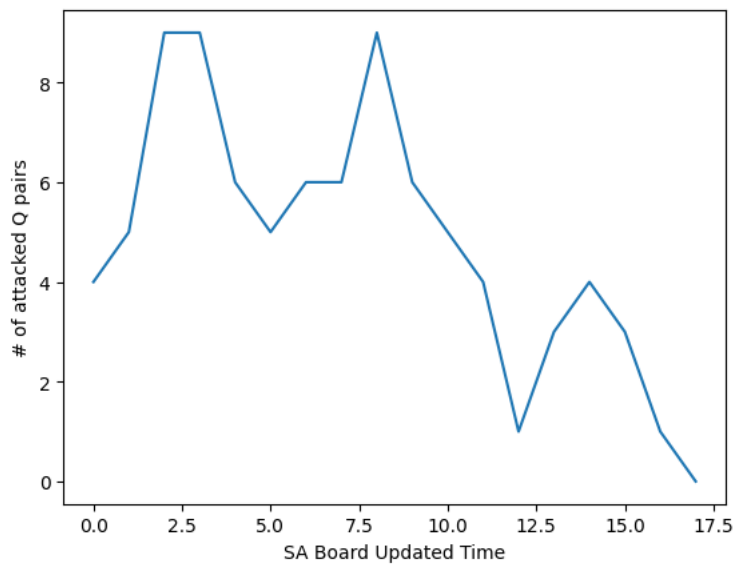
It takes 2 repetitions to succeed. Runtime in second 0.144954:



Successful Solution:

```
[0 0 0 0 0 1 0 0]
[0 0 0 1 0 0 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 1 0]
[1 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0]
```

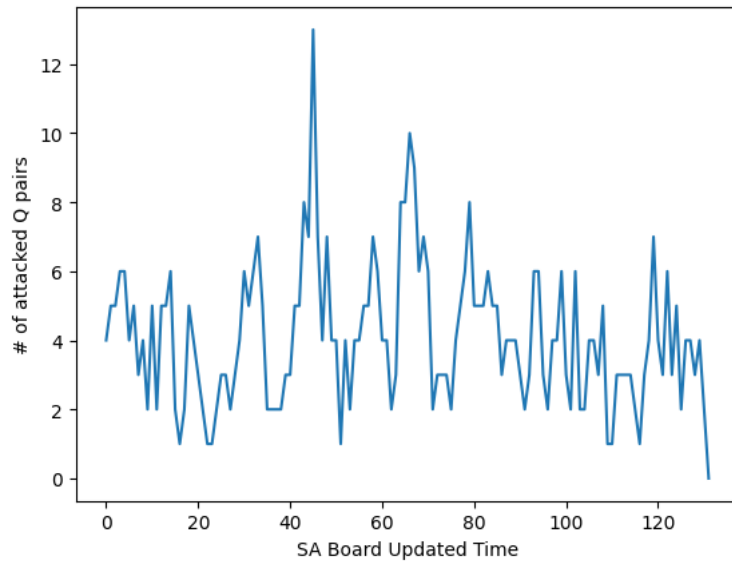
It takes 1 repetitions to succeed. Runtime in second 0.010169:



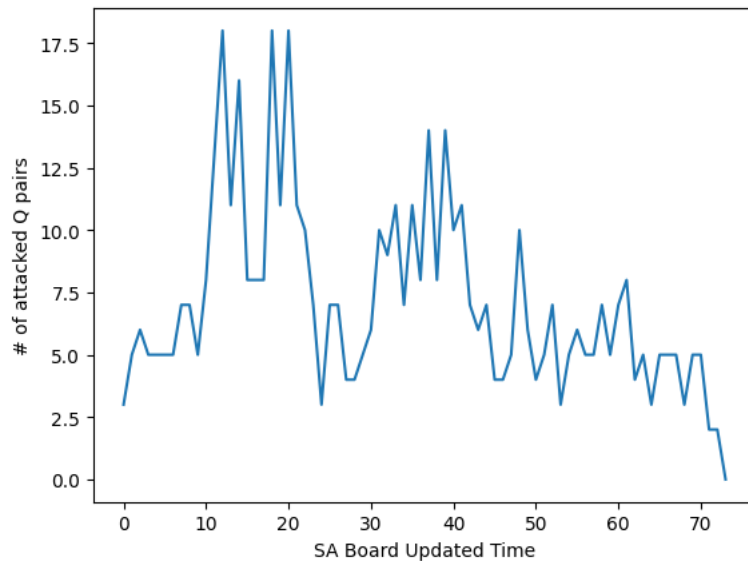
Successful Solution:

```
[0 0 0 0 0 1 0 0]
[1 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1]
[0 0 1 0 0 0 0 0]
```

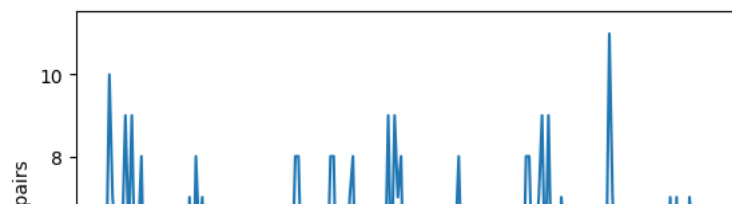
```
[0 0 0 0 0 0 1 0]
[0 0 0 1 0 0 0 0]
It takes 1 repetitions to succeed. Runtime in second 0.028919:
```

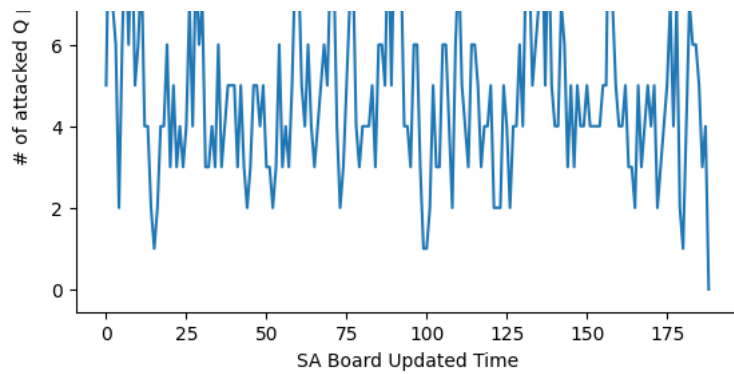


```
Failed
Successful Solution:
[0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 1 0]
[1 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1]
It takes 2 repetitions to succeed. Runtime in second 0.092557:
```



```
Successful Solution:
[0 0 0 0 0 1 0 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[1 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1]
[0 0 0 1 0 0 0 0]
It takes 1 repetitions to succeed. Runtime in second 0.036601:
```

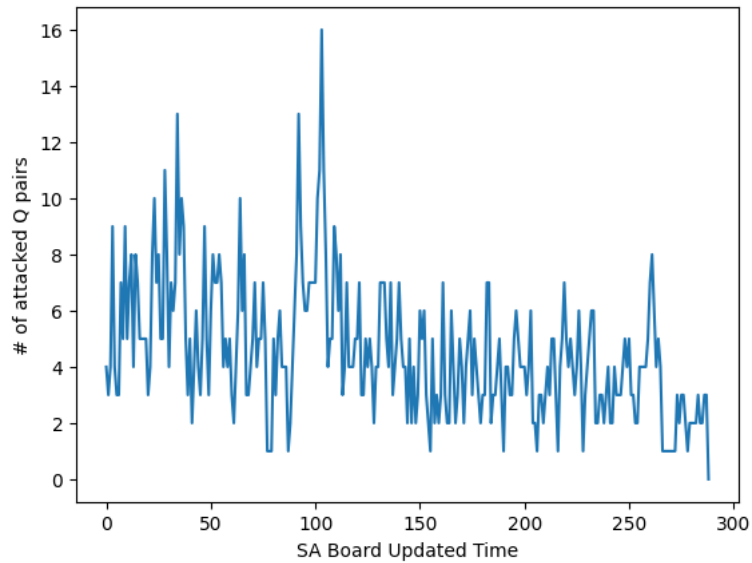




Successful Solution:

```
[0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 1 0]
[0 1 0 0 0 0 0 0]
[0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0]
[1 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 1]
```

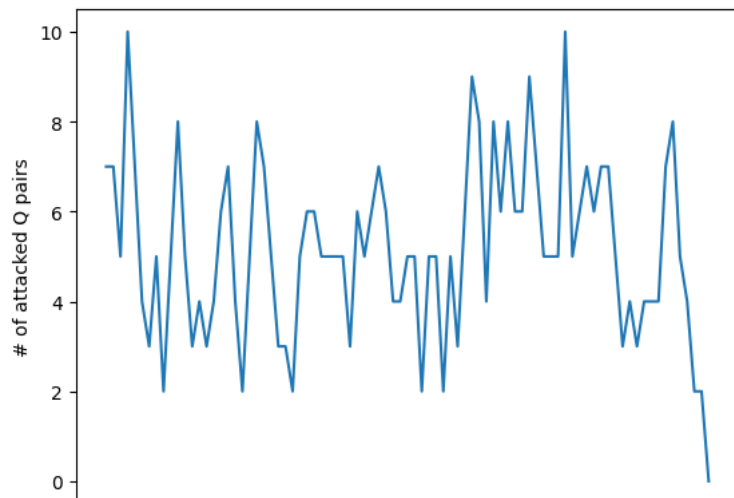
It takes 1 repetitions to succeed. Runtime in second 0.062470:

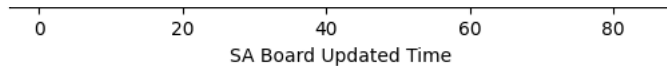


Successful Solution:

```
[0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[0 1 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 1]
[1 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0]
```

It takes 1 repetitions to succeed. Runtime in second 0.026093:





```

Successful Solution:
[0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 1 0]
[0 1 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 1]
.. ^ ^ ^ ^ ^ ^ ^

```

▼ N = 16

```

.. | |
import random
import numpy as np
from math import exp
import time
from copy import deepcopy
import matplotlib.pyplot as plt

N_QUEENS = 16
TEMPERATURE = 40

def threat_calculate(n):
    '''Combination formular. It is choosing two queens in n queens'''
    if n < 2:
        return 0
    if n == 2:
        return 1
    return (n - 1) * n / 2

def create_board(n):
    '''Create a chess board with a queen on a row'''
    chess_board = {}
    temp = list(range(n))
    random.shuffle(temp) # shuffle to make sure it is random
    column = 0

    while len(temp) > 0:
        row = random.choice(temp)
        chess_board[column] = row
        temp.remove(row)
        column += 1
    del temp
    return chess_board

def cost(chess_board):
    '''Calculate how many pairs of threaten queen'''
    threat = 0
    m_chessboard = {}
    a_chessboard = {}

    for column in chess_board:
        temp_m = column - chess_board[column]
        temp_a = column + chess_board[column]
        if temp_m not in m_chessboard:
            m_chessboard[temp_m] = 1
        else:
            m_chessboard[temp_m] += 1
        if temp_a not in a_chessboard:
            a_chessboard[temp_a] = 1
        else:
            a_chessboard[temp_a] += 1

    for i in m_chessboard:
        threat += threat_calculate(m_chessboard[i])
    del m_chessboard

    for i in a_chessboard:
        threat += threat_calculate(a_chessboard[i])
    del a_chessboard

```

```

    return threat

def hill_climbing():
    '''Hill Climbing Search'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0:
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True
            print("Successful Solution:")
            print_chess_board(answer)
            break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def simulated_annealing():
    '''Simulated Annealing'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0 or (random.uniform(0, 1) < exp(-delta / t)):
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True

```

```

        print("Successful Solution:")
        print_chess_board(answer)
        break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def print_chess_board(board):
    '''Print the chess board'''
    showBoard = np.zeros([N_QUEENS, N_QUEENS], dtype = int)
    for column, row in board.items():
        showBoard[row][column]=1
        #print("{} => {}".format(column, row))
    for i in range(N_QUEENS):
        print(showBoard[i])

def main(method='HC'):
    start = time.time()
    Success=False
    repetitions=0
    while not Success:
        if method=='SA':
            Success, Costs=simulated_annealing()
            repetitions=repetitions+1
        elif method=='HC':
            Success, Costs=hill_climbing()
            repetitions=repetitions+1
    print("It takes %d repetitions to succeed. Runtime in second %f:"% (repetitions, (time.time() - start)))
    return(repetitions, (time.time() - start), Success, Costs)

if __name__ == "__main__":
    print("Hill Climbing")
    print("")
    rate_HC = []
    runtime_HC = []
    for i in range(10):
        method='HC'
        Reps, run, Success, Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_HC.append(1/Reps)
        runtime_HC.append(run)
        plt.plot(Costs)
        plt.xlabel(method+' Board Updated Time')
        plt.ylabel('# of attacked Q pairs')
        plt.show()
        print()
    print("Average success rate: " + str(sum(rate_HC)/len(rate_HC)))
    print("Average runtime: " + str(sum(runtime_HC)/len(runtime_HC)))

if __name__ == "__main__":
    print("Simulated Annealing")
    print("")
    rate_SA = []
    runtime_SA = []
    for i in range(10):
        method='SA'
        Reps, run, Success, Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_SA.append(1/Reps)
        runtime_SA.append(run)
        plt.plot(Costs)
        plt.xlabel(method+' Board Updated Time')
        plt.ylabel('# of attacked Q pairs')
        plt.show()
        print()
    print("Average success rate: " + str(sum(rate_SA)/len(rate_SA)))
    print("Average runtime: " + str(sum(runtime_SA)/len(runtime_SA)))

```

Hill Climbing

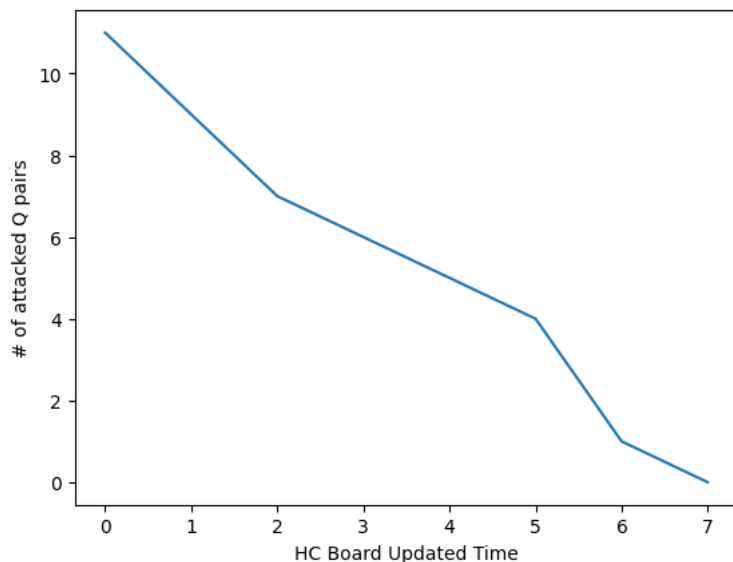
Failed

Failed

Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
```

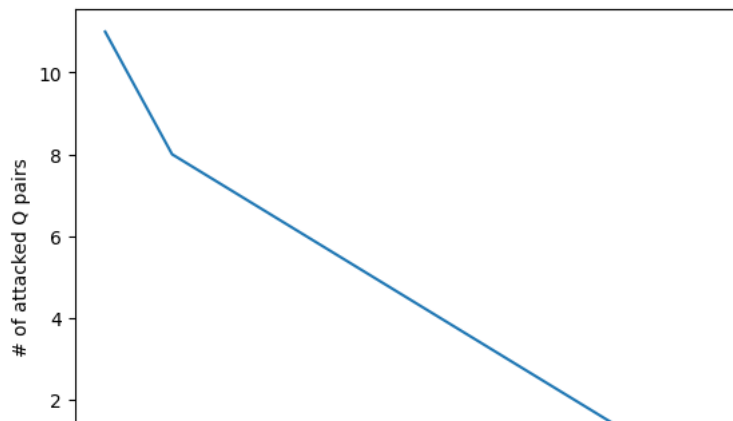
It takes 3 repetitions to succeed. Runtime in second 0.271998:

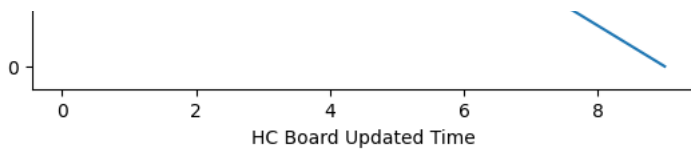


Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
```

It takes 1 repetitions to succeed. Runtime in second 0.022814:





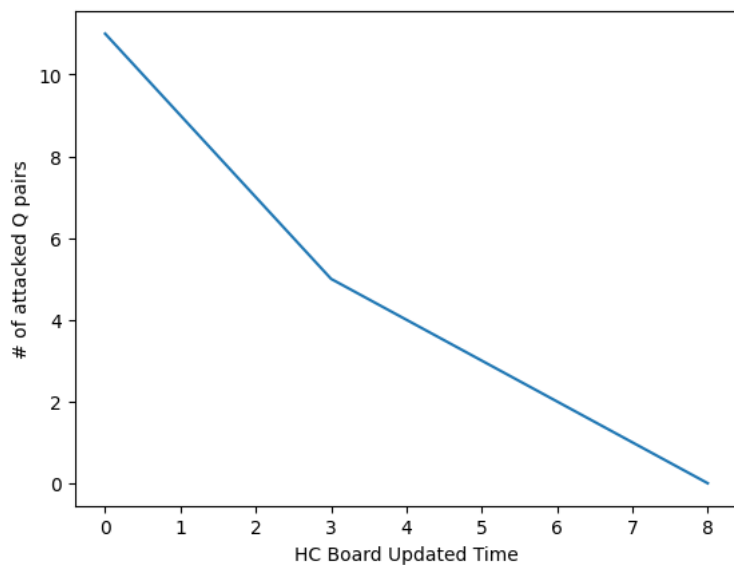
Failed

Failed

Successful Solution:

```
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
```

It takes 3 repetitions to succeed. Runtime in second 0.438193:



Failed

Failed

Failed

Failed

Failed

Failed

Failed

Failed

Failed

Failed

Failed

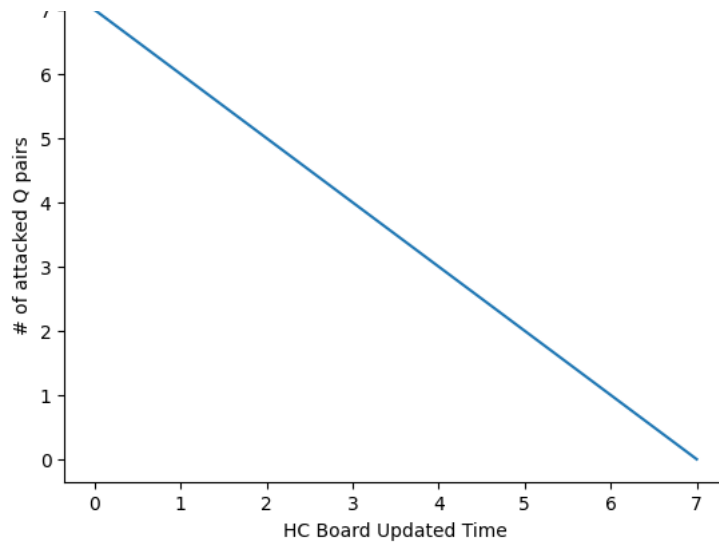
Failed

Failed

Successful Solution:

```
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
```

It takes 13 repetitions to succeed. Runtime in second 2.214340:



Failed

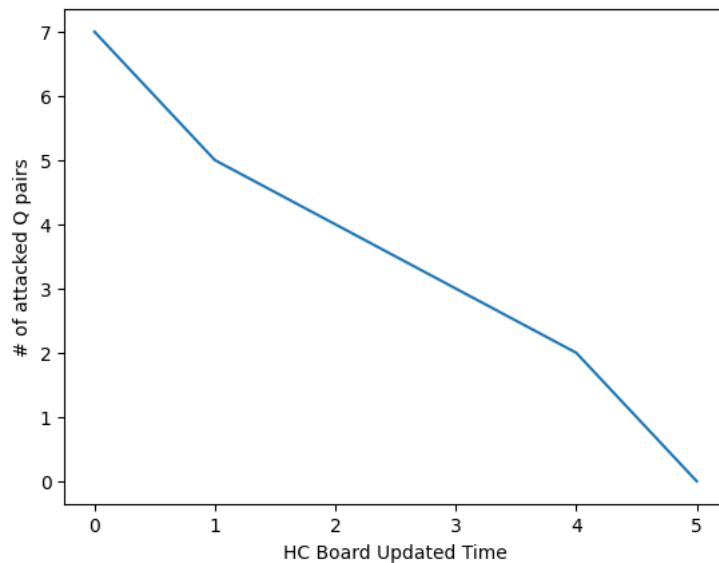
Failed

Failed

Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
```

It takes 4 repetitions to succeed. Runtime in second 0.487107:



Failed

Failed

Failed

Failed

Successful Solution:

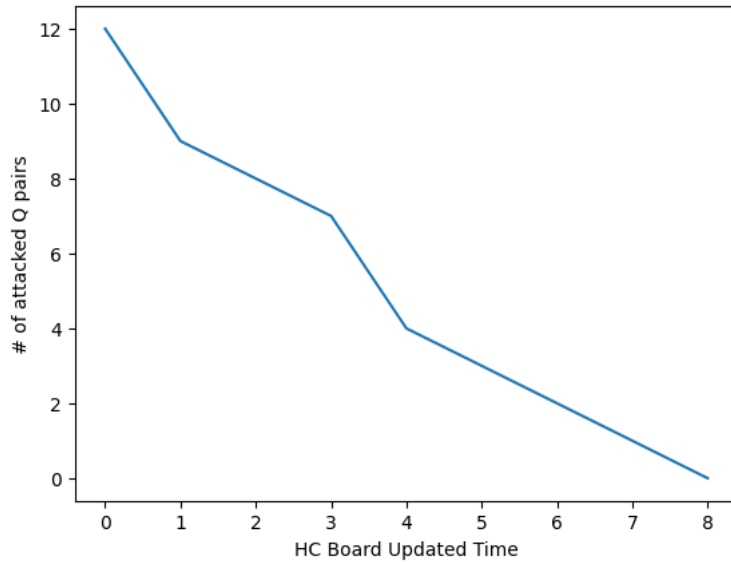
```
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

```

[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]

```

It takes 5 repetitions to succeed. Runtime in second 0.590032:



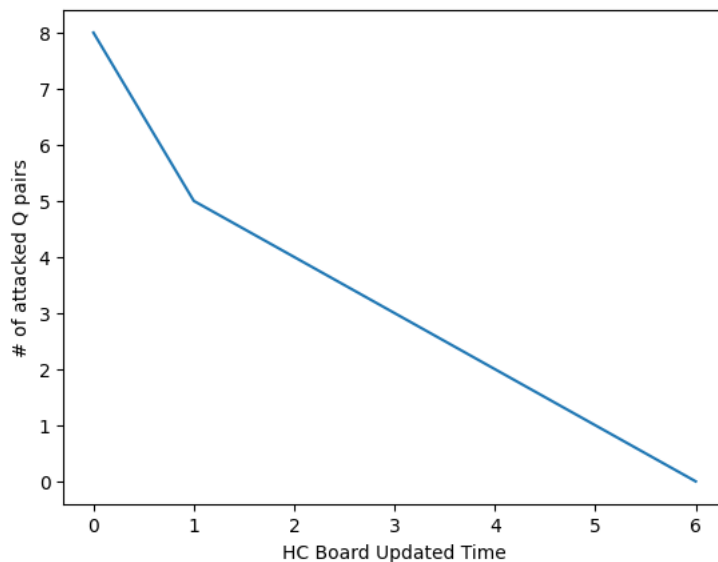
Successful Solution:

```

[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]

```

It takes 1 repetitions to succeed. Runtime in second 0.046898:



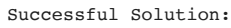
Successful Solution:

```

[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]

```

It takes 1 repetitions to succeed. Runtime in second 0.026606:



It takes 1 repetitions to succeed. Runtime in second 0.015300:



Successful Solution:

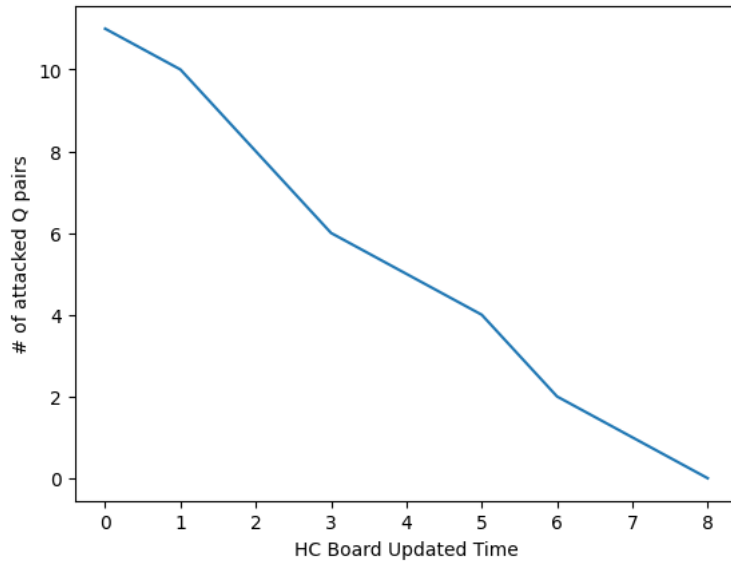
20/101

```

[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]

```

It takes 2 repetitions to succeed. Runtime in second 0.122010:



Average success rate: 0.5693589743589744

Average runtime: 0.42377538681030275

Simulated Annealing

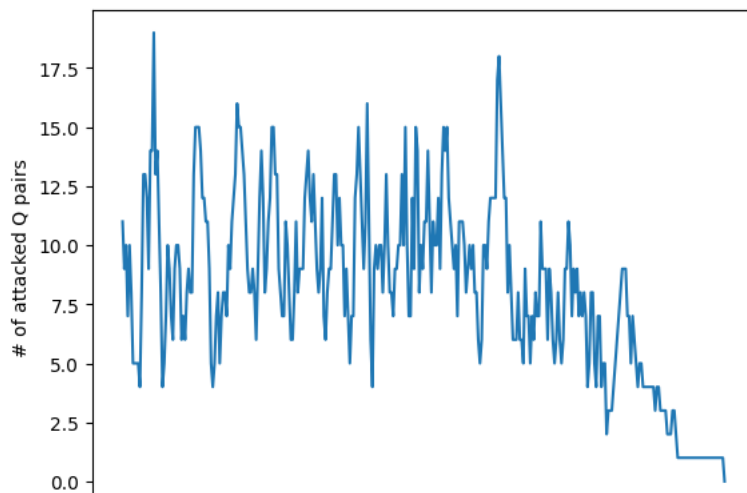
Successful Solution:

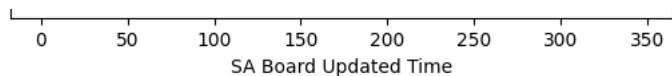
```

[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]

```

It takes 1 repetitions to succeed. Runtime in second 0.111677:

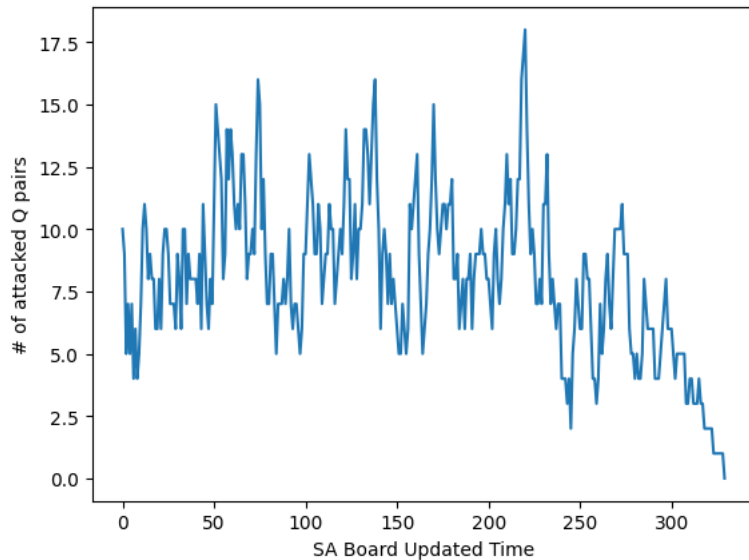




Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
```

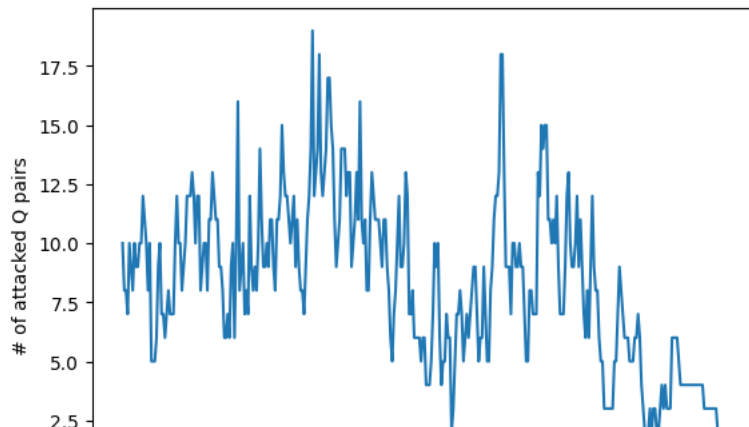
It takes 1 repetitions to succeed. Runtime in second 0.084882:

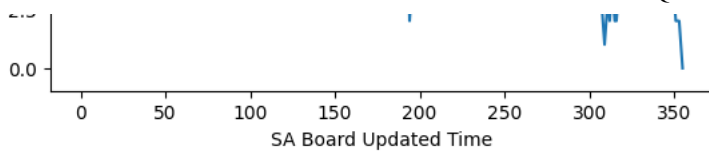


Successful Solution:

```
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

It takes 1 repetitions to succeed. Runtime in second 0.081211:



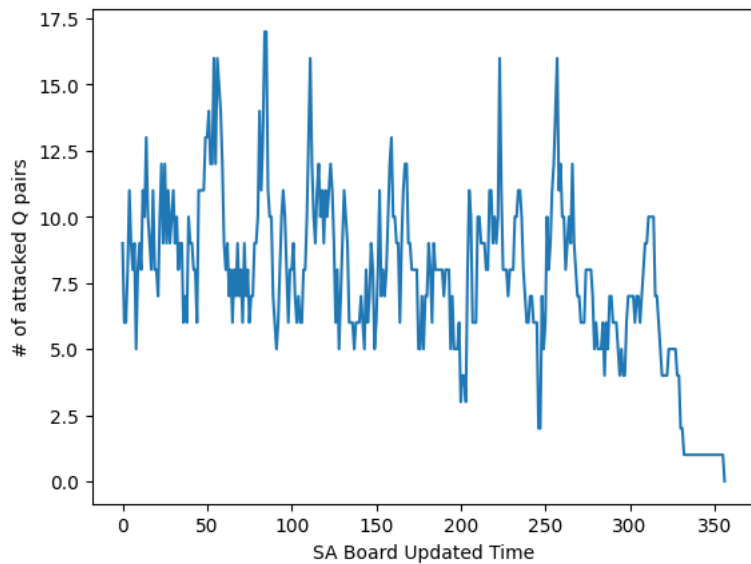


Failed

Successful Solution:

```
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
```

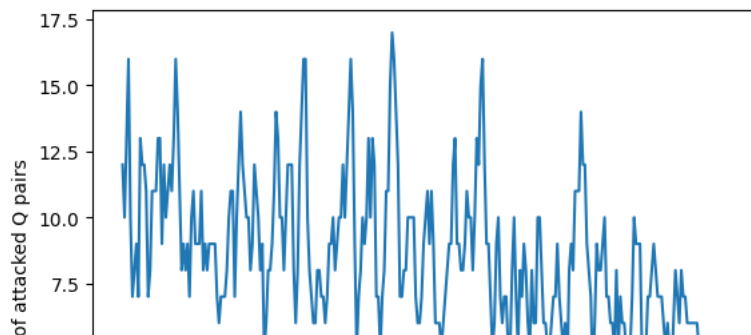
It takes 2 repetitions to succeed. Runtime in second 0.311804:

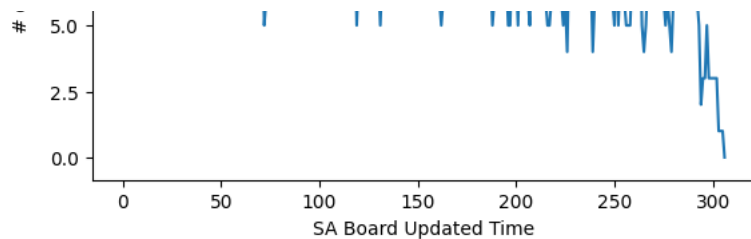


Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
```

It takes 1 repetitions to succeed. Runtime in second 0.126768:

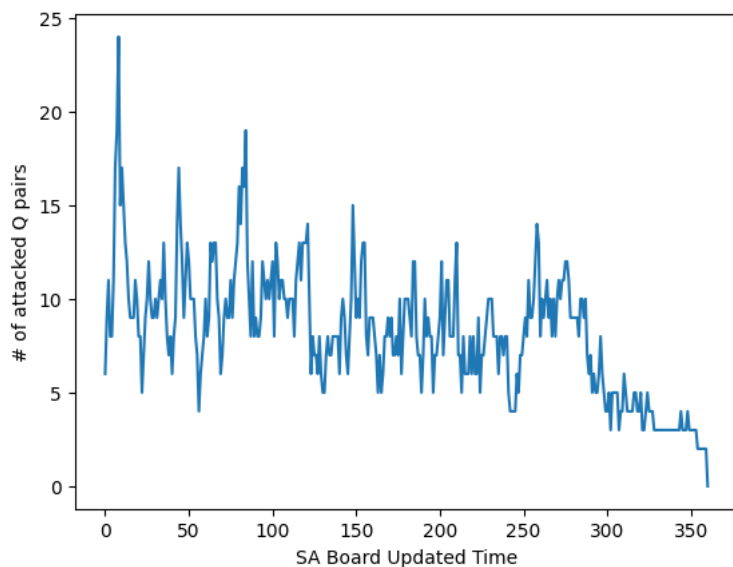




Successful Solution:

```
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

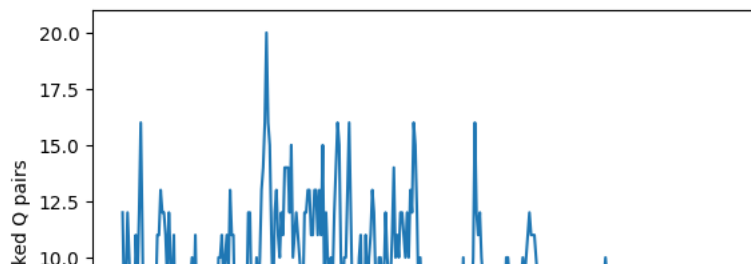
It takes 1 repetitions to succeed. Runtime in second 0.067981:

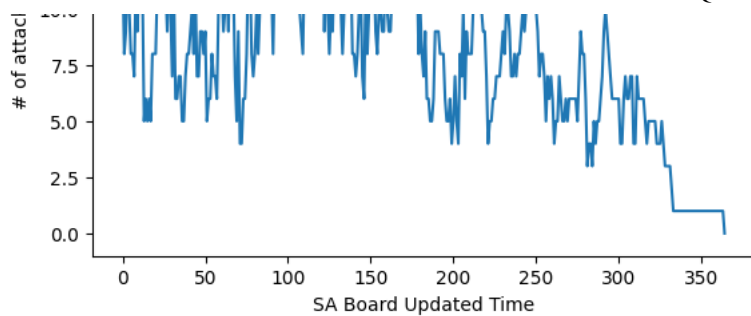


Successful Solution:

```
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
```

It takes 1 repetitions to succeed. Runtime in second 0.154989:

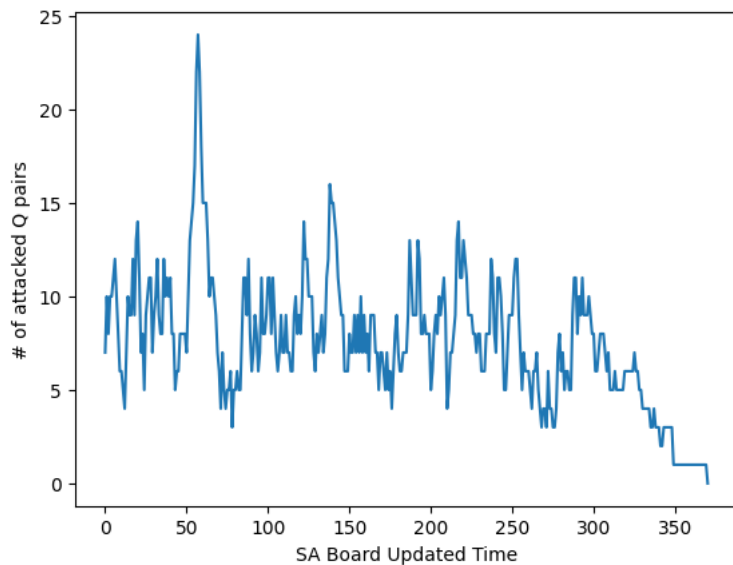




Successful Solution:

```
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
```

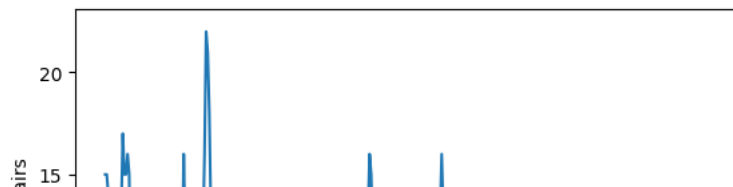
It takes 1 repetitions to succeed. Runtime in second 0.101357:

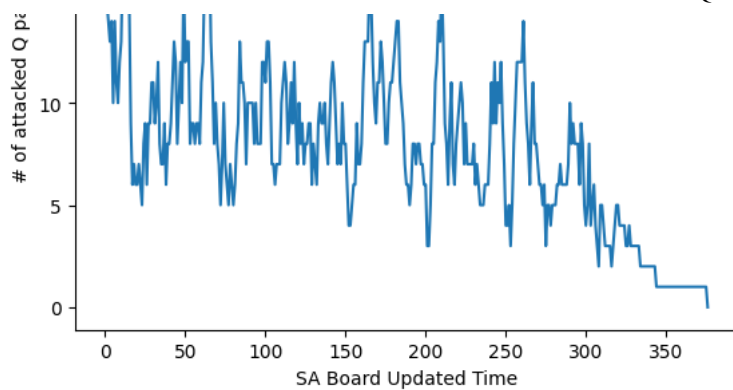


Successful Solution:

```
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

It takes 1 repetitions to succeed. Runtime in second 0.060917:

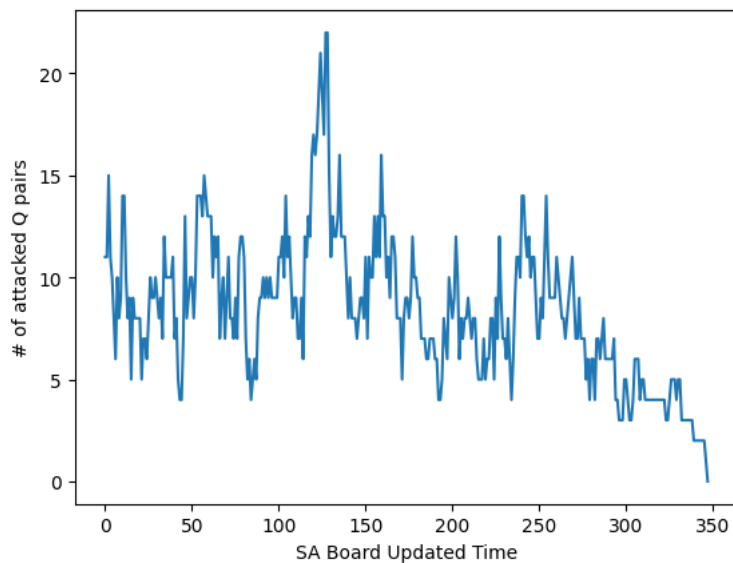




Successful Solution:

```
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

It takes 1 repetitions to succeed. Runtime in second 0.039786:



Average success rate: 0.95

Average runtime: 0.11488432884216308

▼ N = 32

```
import random
import numpy as np
from math import exp
import time
from copy import deepcopy
import matplotlib.pyplot as plt
```

```
N_QUEENS = 32
TEMPERATURE = 40
```

```

def threat_calculate(n):
    '''Combination formular. It is choosing two queens in n queens'''
    if n < 2:
        return 0
    if n == 2:
        return 1
    return (n - 1) * n / 2

def create_board(n):
    '''Create a chess board with a queen on a row'''
    chess_board = {}
    temp = list(range(n))
    random.shuffle(temp) # shuffle to make sure it is random
    column = 0

    while len(temp) > 0:
        row = random.choice(temp)
        chess_board[column] = row
        temp.remove(row)
        column += 1
    del temp
    return chess_board

def cost(chess_board):
    '''Calculate how many pairs of threaten queen'''
    threat = 0
    m_chessboard = {}
    a_chessboard = {}

    for column in chess_board:
        temp_m = column - chess_board[column]
        temp_a = column + chess_board[column]
        if temp_m not in m_chessboard:
            m_chessboard[temp_m] = 1
        else:
            m_chessboard[temp_m] += 1
        if temp_a not in a_chessboard:
            a_chessboard[temp_a] = 1
        else:
            a_chessboard[temp_a] += 1

    for i in m_chessboard:
        threat += threat_calculate(m_chessboard[i])
    del m_chessboard

    for i in a_chessboard:
        threat += threat_calculate(a_chessboard[i])
    del a_chessboard

    return threat

def hill_climbing():
    '''Hill Climbing Search'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break

```

```

        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

    delta = cost(successor) - cost_answer
    if delta < 0:
        answer = deepcopy(successor)
        cost_answer = cost(answer)
        Costs.append(cost_answer)
    if cost_answer == 0:
        solution_found = True
        print("Successful Solution:")
        print_chess_board(answer)
        break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def simulated_annealing():
    '''Simulated Annealing'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0 or (random.uniform(0, 1) < exp(-delta / t)):
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True
            print("Successful Solution:")
            print_chess_board(answer)
            break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def print_chess_board(board):
    '''Print the chess board'''
    showBoard = np.zeros([N_QUEENS, N_QUEENS], dtype = int)
    for column, row in board.items():
        showBoard[row][column]=1
        #print("{} => {}".format(column, row))
    for i in range(N_QUEENS):
        print(showBoard[i])

def main(method='HC'):
    start = time.time()
    Success=False
    repetitions=0
    while not Success:
        if method=='SA':

```

```

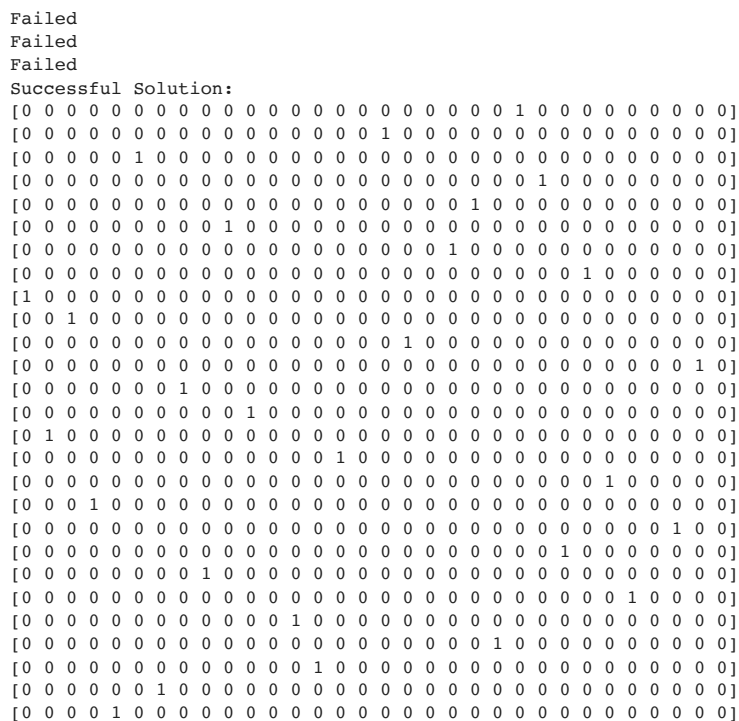
        Success, Costs=simulated_annealing()
        repetitions=repetitions+1
    elif method=='HC':
        Success, Costs=hill_climbing()
        repetitions=repetitions+1
    print("It takes %d repetitions to succeed. Runtime in second %f:"% (repetitions,(time.time() - start)))
    return(repetitions, (time.time() - start), Success, Costs)

if __name__ == "__main__":
    print("Hill Climbing")
    print("")
    rate_HC = []
    runtime_HC = []
    for i in range(10):
        method='HC'
        Reps, run, Success, Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_HC.append(1/Reps)
        runtime_HC.append(run)
        plt.plot(Costs)
        plt.xlabel(method+' Board Updated Time')
        plt.ylabel('# of attacked Q pairs')
        plt.show()
        print()
    print("Average success rate: " + str(sum(rate_HC)/len(rate_HC)))
    print("Average runtime: " + str(sum(runtime_HC)/len(runtime_HC)))

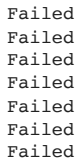
if __name__ == "__main__":
    print("Simulated Annealing")
    print("")
    rate_SA = []
    runtime_SA = []
    for i in range(10):
        method='SA'
        Reps, run, Success, Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_SA.append(1/Reps)
        runtime_SA.append(run)
        plt.plot(Costs)
        plt.xlabel(method+' Board Updated Time')
        plt.ylabel('# of attacked Q pairs')
        plt.show()
        print()
    print("Average success rate: " + str(sum(rate_SA)/len(rate_SA)))
    print("Average runtime: " + str(sum(runtime_SA)/len(runtime_SA)))

```

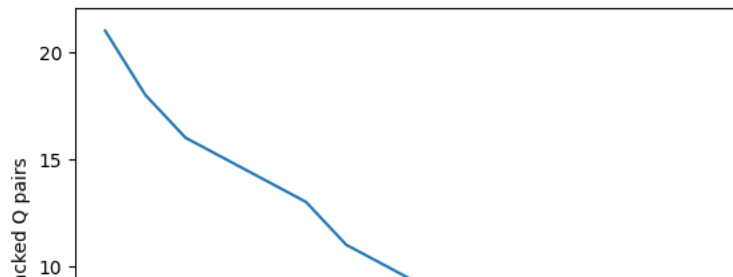

It takes 12 repetitions to succeed. Runtime in second 2.536137:

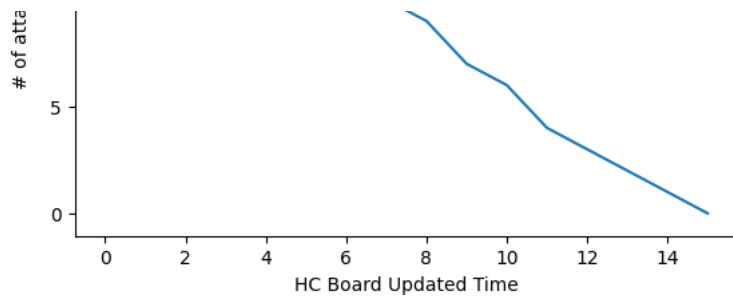


It takes 4 repetitions to succeed. Runtime in second 0.320097:

[illegible]

It takes 8 repetitions to succeed. Runtime in second 0.727934:



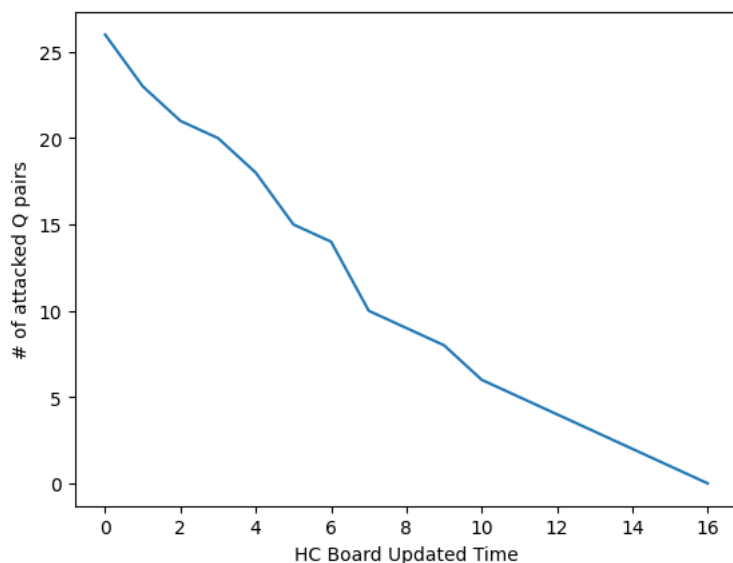


Failed
Failed
Failed
Failed
Failed
Failed
Failed
Failed

Successful Solution:

[illegible]

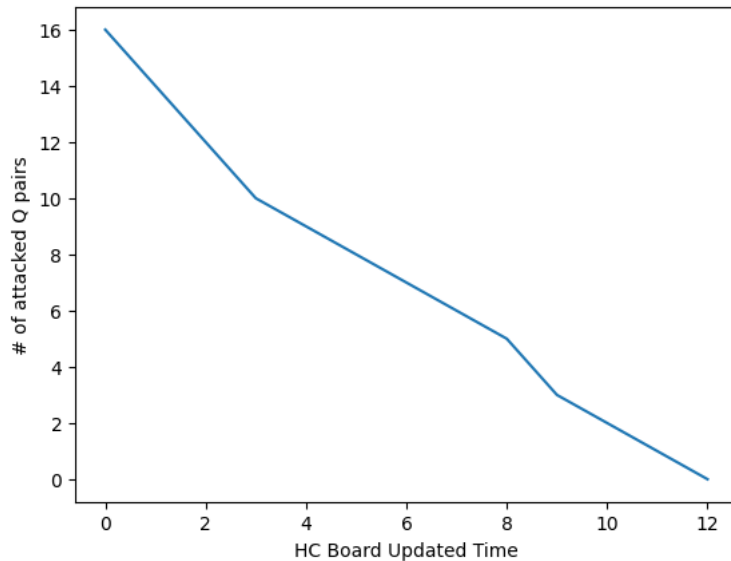
It takes 9 repetitions to succeed. Runtime in second 0.835045:



Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

It takes 1 repetitions to succeed. Runtime in second 0.053082:

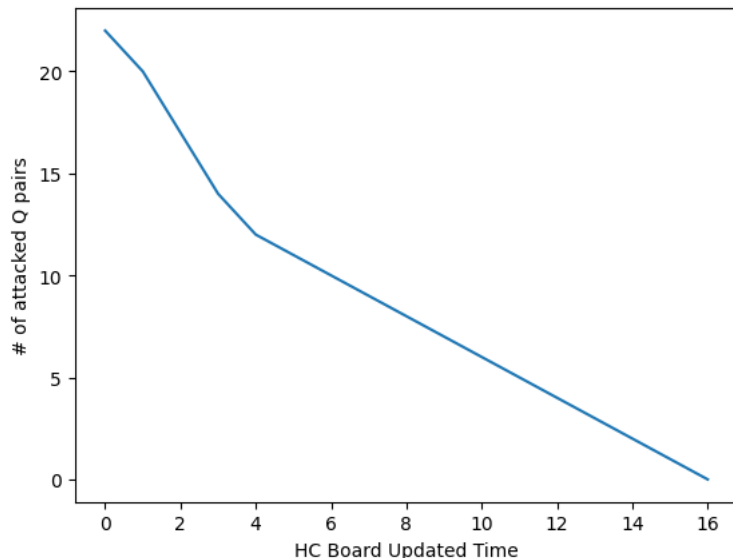


Successful Solution:

[illegible]

[illegible]

It takes 10 repetitions to succeed. Runtime in second 0.941591:



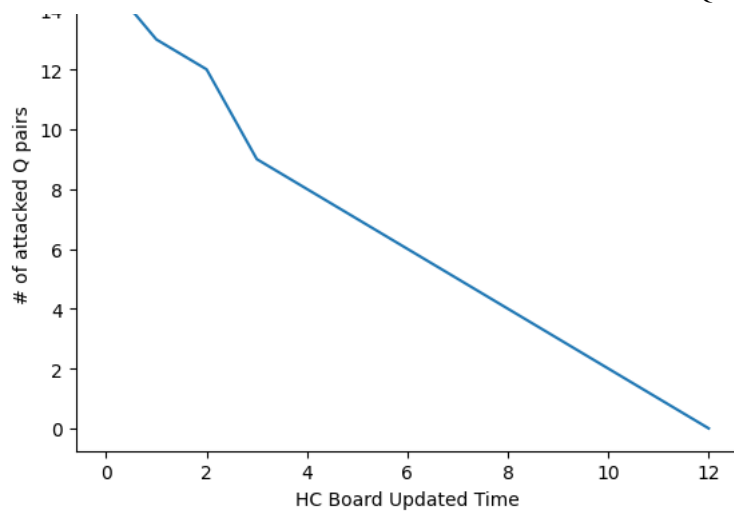
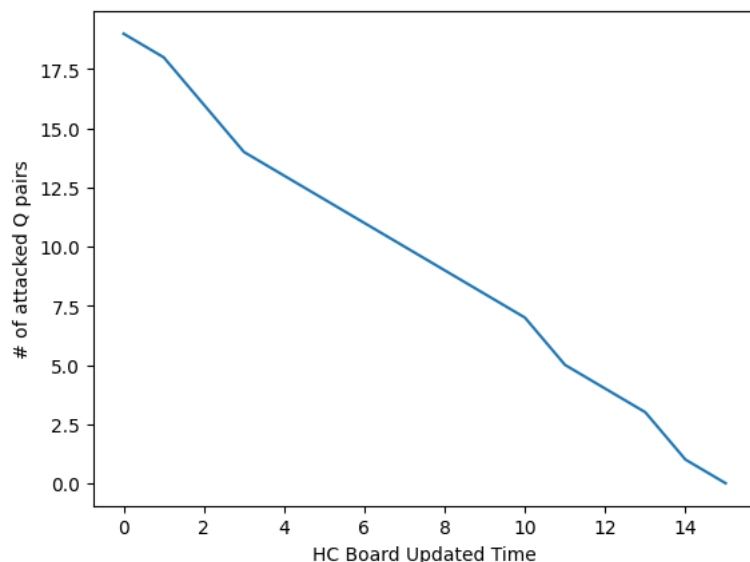
Failed
Failed
Failed
Failed
Failed
Failed
Failed

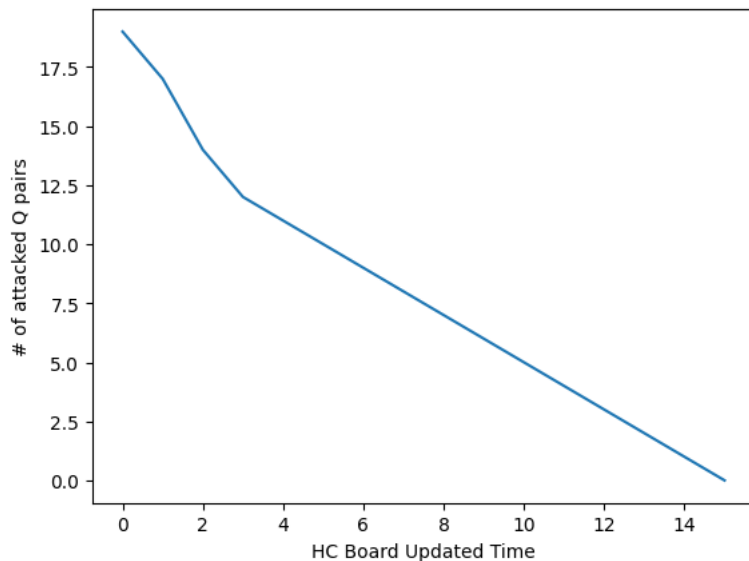
Successful Solution:

[illegible]

It takes 8 repetitions to succeed. Runtime in second 0.705604:



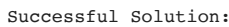
[illegible]

[illegible]

```
Average success rate: 0.24111111111111114
Average runtime: 0.798902940750122
Simulated Annealing
```

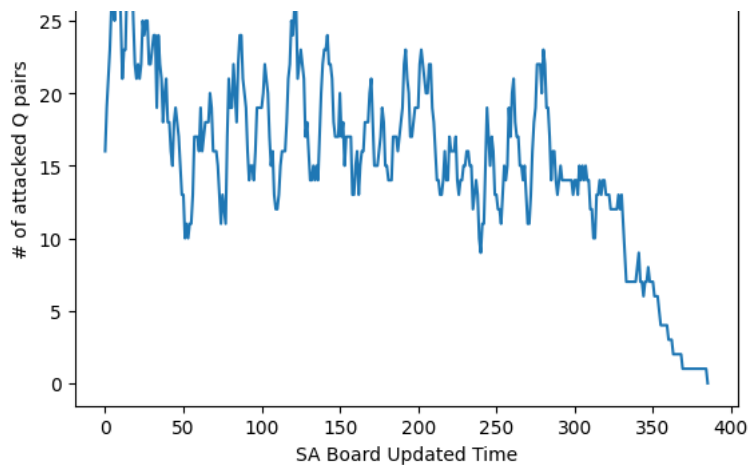
[illegible]

It takes 3 repetitions to succeed. Runtime in second 0.329597:



```
It takes 1 repetitions to succeed. Runtime in second 0.099040:
```

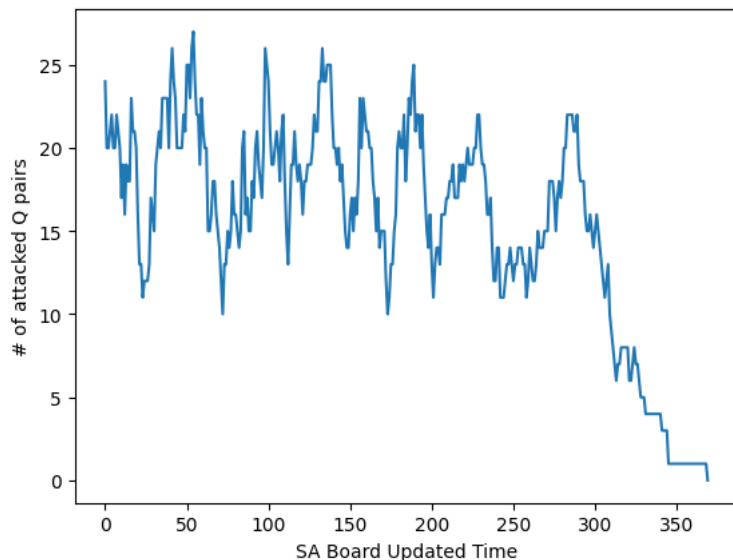




Successful Solution:

[illegible]

It takes 1 repetitions to succeed. Runtime in second 0.117313:

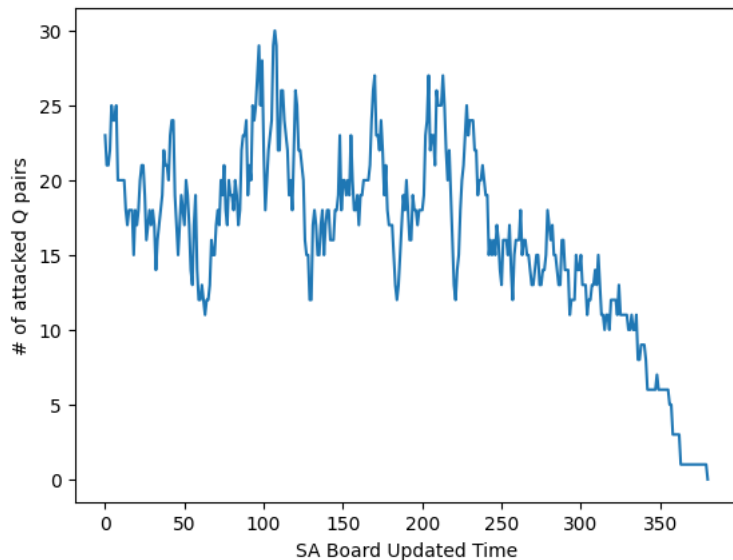


Failed

Failed

Successful Solution:

[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]

[illegible]

Failed

Successful Solution:

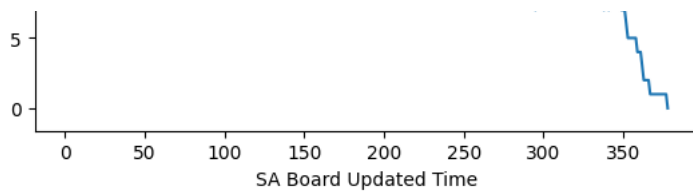
[illegible]

It takes 2 repetitions to succeed. Runtime in second 0.243398:



It takes 1 repetitions to succeed. Runtime in second 0.095723:

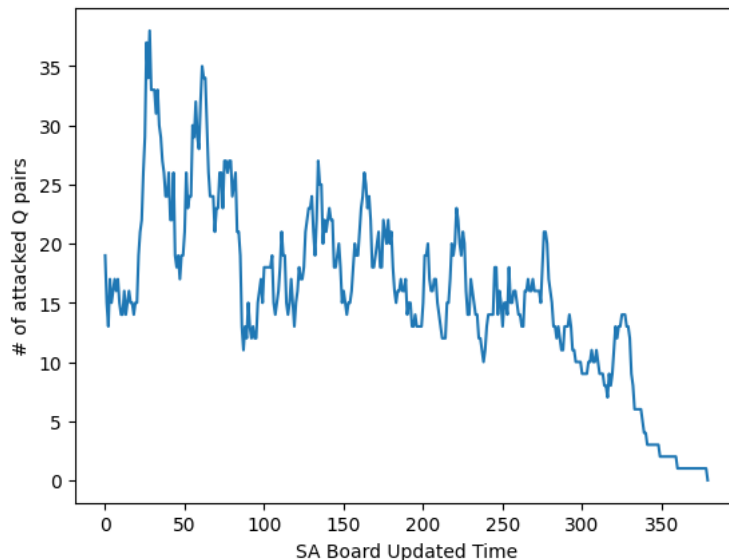




Successful Solution:

[illegible]

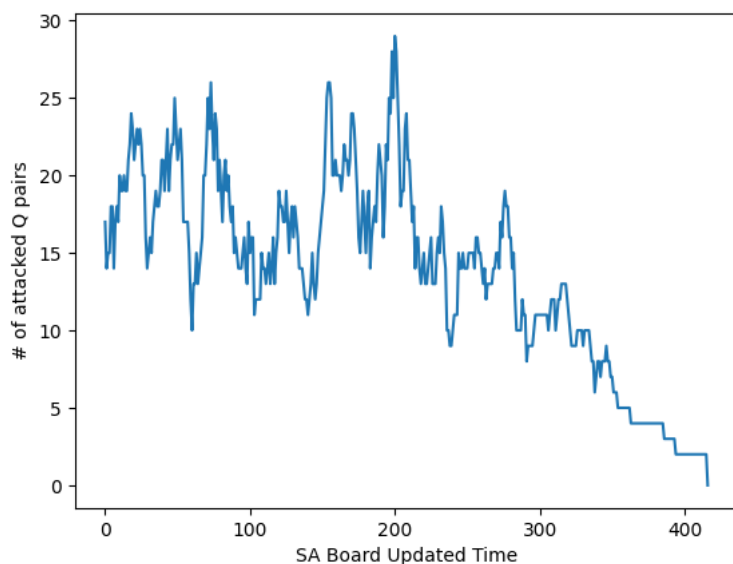
It takes 1 repetitions to succeed. Runtime in second 0.103137:



Successful Solution:

[illegible]

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0],
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0],
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0],
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
It takes 1 repetitions to succeed. Runtime in second 0.104951:
```



Failed

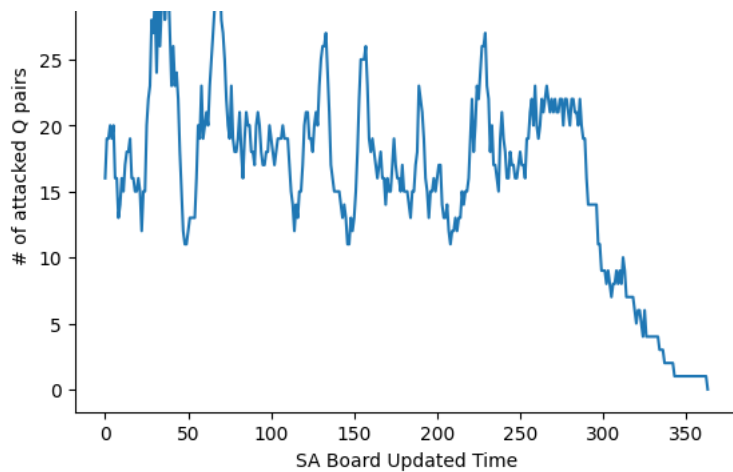
Failed

Successful Solution:

[illegible]

It takes 3 repetitions to succeed. Runtime in second 0.399633:



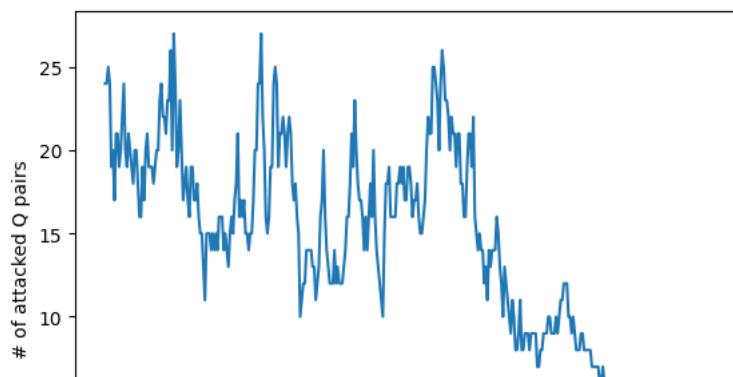


Failed

Successful Solution:

[illegible]

It takes 2 repetitions to succeed. Runtime in second 0.445862:



▼ N = 64

```
import random
import numpy as np
from math import exp
import time
from copy import deepcopy
import matplotlib.pyplot as plt
```

```

N_QUEENS = 64
TEMPERATURE = 40

def threat_calculate(n):
    '''Combination formular. It is choosing two queens in n queens'''
    if n < 2:
        return 0
    if n == 2:
        return 1
    return (n - 1) * n / 2

def create_board(n):
    '''Create a chess board with a queen on a row'''
    chess_board = {}
    temp = list(range(n))
    random.shuffle(temp) # shuffle to make sure it is random
    column = 0

    while len(temp) > 0:
        row = random.choice(temp)
        chess_board[column] = row
        temp.remove(row)
        column += 1
    del temp
    return chess_board

def cost(chess_board):
    '''Calculate how many pairs of threaten queen'''
    threat = 0
    m_chessboard = {}
    a_chessboard = {}

    for column in chess_board:
        temp_m = column - chess_board[column]
        temp_a = column + chess_board[column]
        if temp_m not in m_chessboard:
            m_chessboard[temp_m] = 1
        else:
            m_chessboard[temp_m] += 1
        if temp_a not in a_chessboard:
            a_chessboard[temp_a] = 1
        else:
            a_chessboard[temp_a] += 1

    for i in m_chessboard:
        threat += threat_calculate(m_chessboard[i])
    del m_chessboard

    for i in a_chessboard:
        threat += threat_calculate(a_chessboard[i])
    del a_chessboard

    return threat

def hill_climbing():
    '''Hill Climbing Search'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:

```

```

        index_1 = random.randrange(0, N_QUEENS - 1)
        index_2 = random.randrange(0, N_QUEENS - 1)
        if index_1 != index_2:
            break
    successor[index_1], successor[index_2] = successor[index_2], \
        successor[index_1] # swap two chosen queens

    delta = cost(successor) - cost_answer
    if delta < 0:
        answer = deepcopy(successor)
        cost_answer = cost(answer)
        Costs.append(cost_answer)
    if cost_answer == 0:
        solution_found = True
        print("Successful Solution:")
        print_chess_board(answer)
        break
if solution_found is False:
    print("Failed")
    return(False, Costs)
else:
    return(True, Costs)

def simulated_annealing():
    '''Simulated Annealing'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0 or (random.uniform(0, 1) < exp(-delta / t)):
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True
            print("Successful Solution:")
            print_chess_board(answer)
            break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def print_chess_board(board):
    '''Print the chess board'''
    showBoard = np.zeros([N_QUEENS, N_QUEENS], dtype = int)
    for column, row in board.items():
        showBoard[row][column]=1
        #print("{} => {}".format(column, row))
    for i in range(N_QUEENS):
        print(showBoard[i])

def main(method='HC'):
    start = time.time()

```

```

Success=False
repetitions=0
while not Success:
    if method=='SA':
        Success,Costs=simulated_annealing()
        repetitions=repetitions+1
    elif method=='HC':
        Success,Costs=hill_climbing()
        repetitions=repetitions+1
print("It takes %d repetitions to succeed. Runtime in second %f:"% (repetitions,(time.time() - start)))
return(repetitions, (time.time() - start), Success,Costs)

if __name__ == "__main__":
    print("Hill Climbing")
    print("")
    rate_HC = []
    runtime_HC = []
    for i in range(10):
        method='HC'
        Reps,run,Success,Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_HC.append(1/Reps)
        runtime_HC.append(run)
        plt.plot(Costs)
        plt.xlabel(method+' Board Updated Time')
        plt.ylabel('# of attacked Q pairs')
        plt.show()
        print()
    print("Average success rate: " + str(sum(rate_HC)/len(rate_HC)))
    print("Average runtime: " + str(sum(runtime_HC)/len(runtime_HC)))

if __name__ == "__main__":
    print("Simulated Annealing")
    print("")
    rate_SA = []
    runtime_SA = []
    for i in range(10):
        method='SA'
        Reps,run,Success,Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
        rate_SA.append(1/Reps)
        runtime_SA.append(run)
        plt.plot(Costs)
        plt.xlabel(method+' Board Updated Time')
        plt.ylabel('# of attacked Q pairs')
        plt.show()
        print()
    print("Average success rate: " + str(sum(rate_SA)/len(rate_SA)))
    print("Average runtime: " + str(sum(runtime_SA)/len(runtime_SA)))

```

Hill Climbing

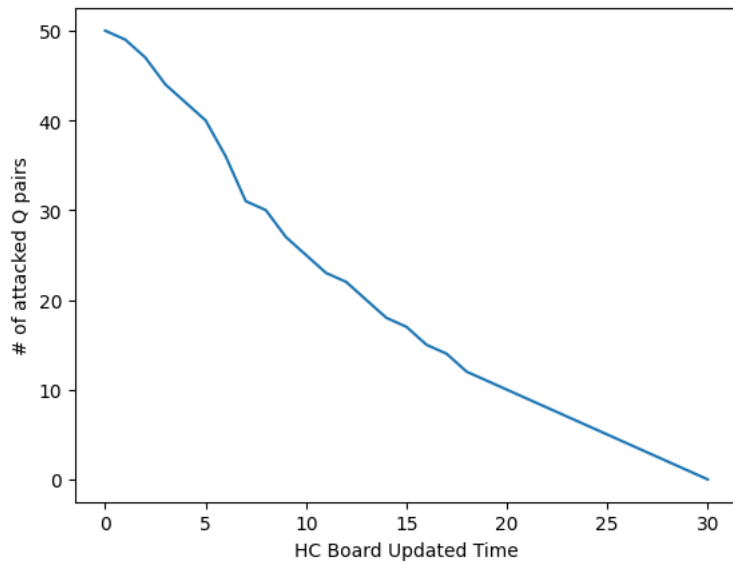
[illegible]

Successful Solution:

[illegible]

[illegible]

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
It takes 45 repetitions to succeed. Runtime in second 9.133700:
```

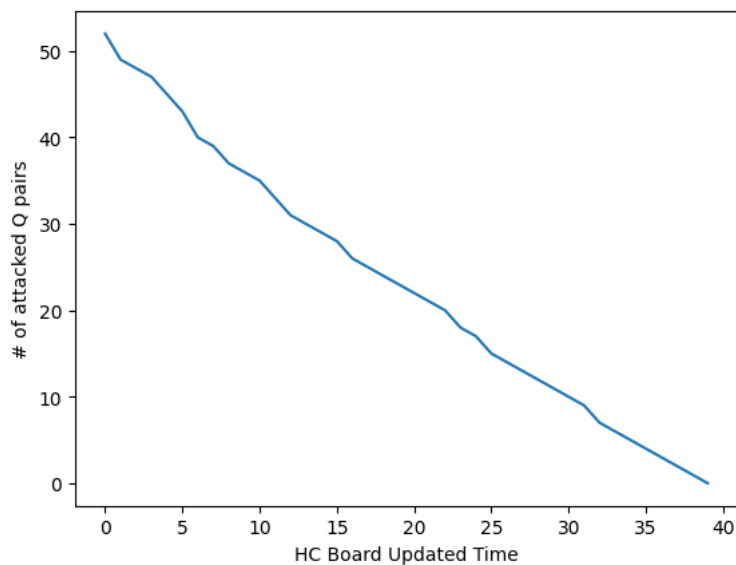
[illegible]

Successful Solution:

[illegible]

https://colab.research.google.com/drive/1oulTYbZyI77cHv0xTO2n_kQHC9rfkAj3?authuser=0#scrollTo=23NUfEyuuesO&printMode=true

It takes 42 repetitions to succeed. Runtime in second 9.055364:

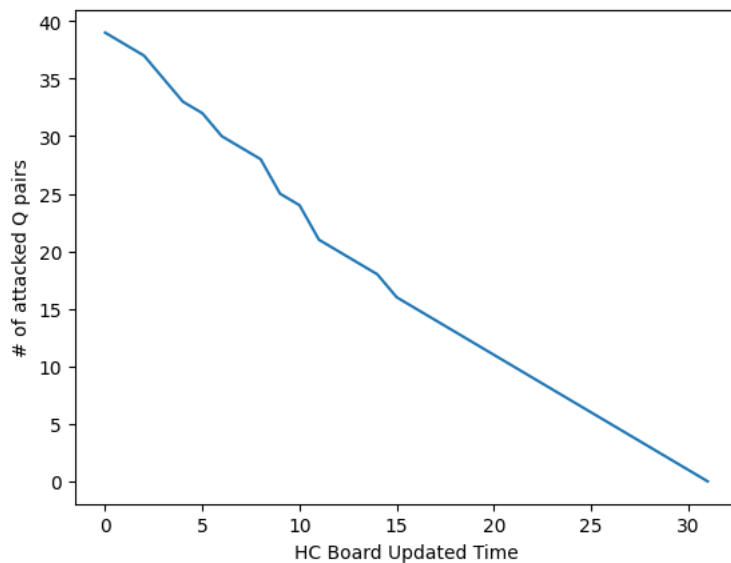


Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

100

It takes 17 repetitions to succeed. Runtime in second 3.061263:

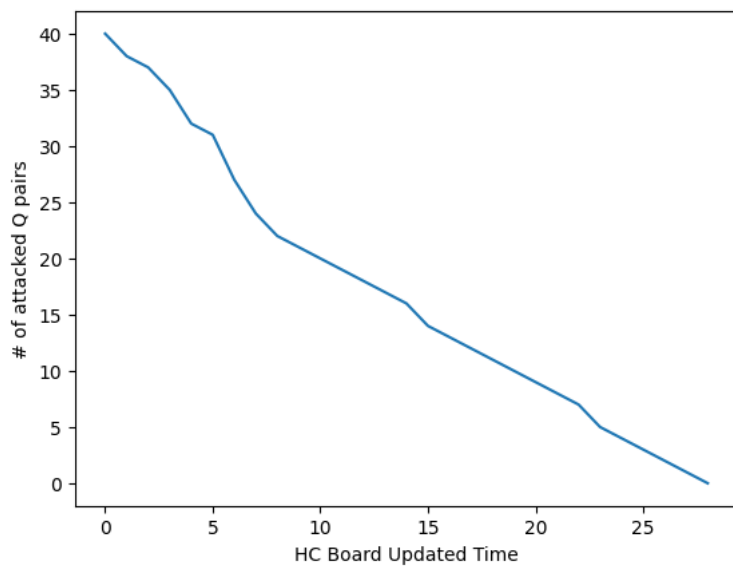


Successful Solution:

[illegible]

100

It takes 5 repetitions to succeed. Runtime in second 0.916729:



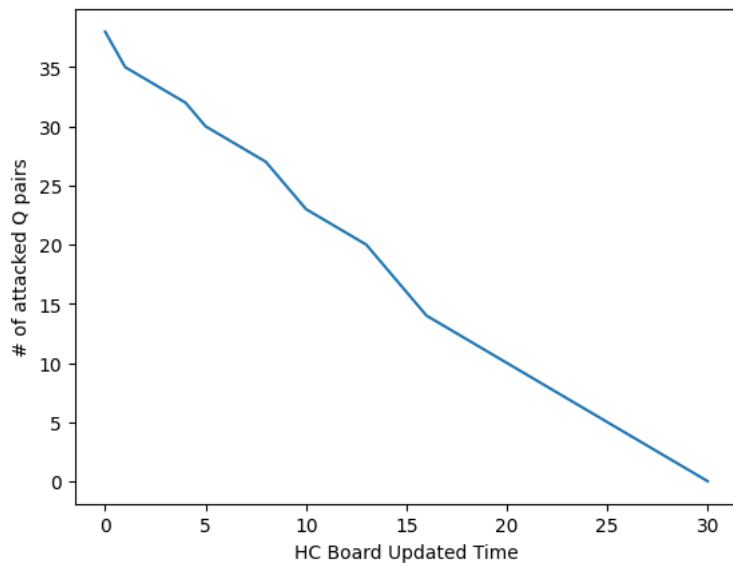
Successful Solution:

[illegible]

57/101

[illegible]

It takes 18 repetitions to succeed. Runtime in second 4.753929:



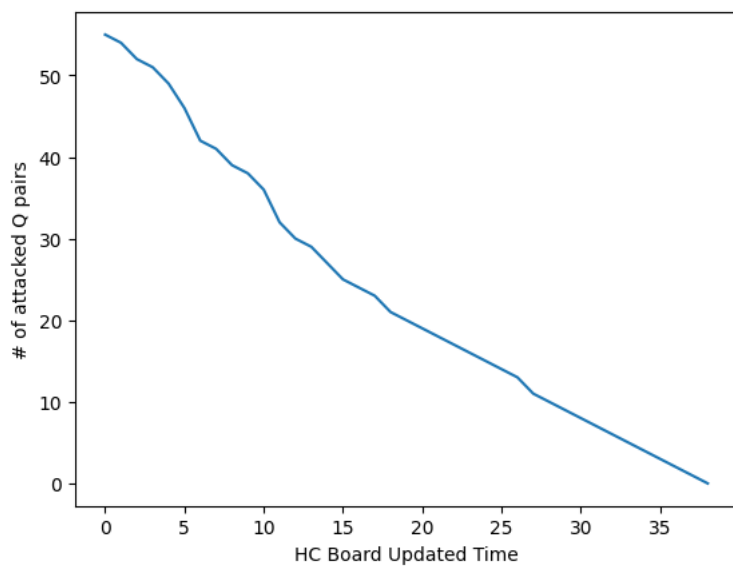
Failed
Failed
Failed
Failed
Failed
Failed
Failed
Failed
Failed
Failed

Successful Solution:

[illegible]

https://colab.research.google.com/drive/1oulTYbZyI77cHv0xTO2n_kQHC9rfkAj3?authuser=0#scrollTo=23NUfEyuuesO&printMode=true

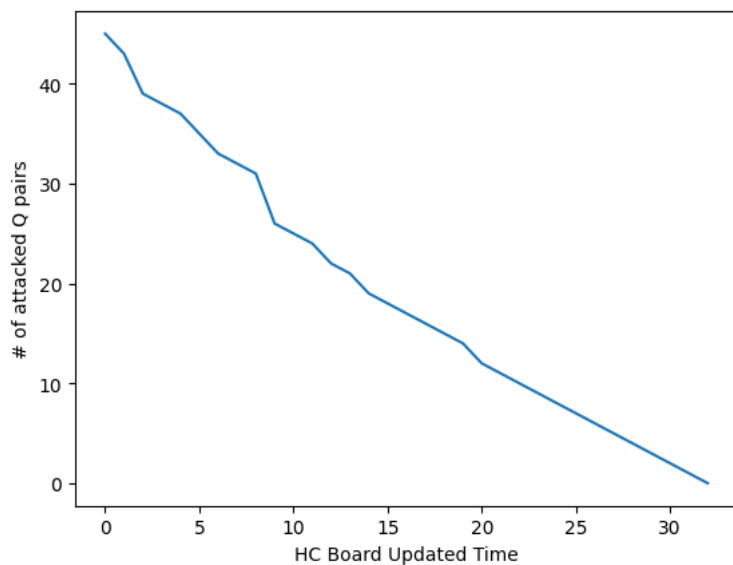
It takes 11 repetitions to succeed. Runtime in second 2.009667:



Successful Solution:

[illegible]

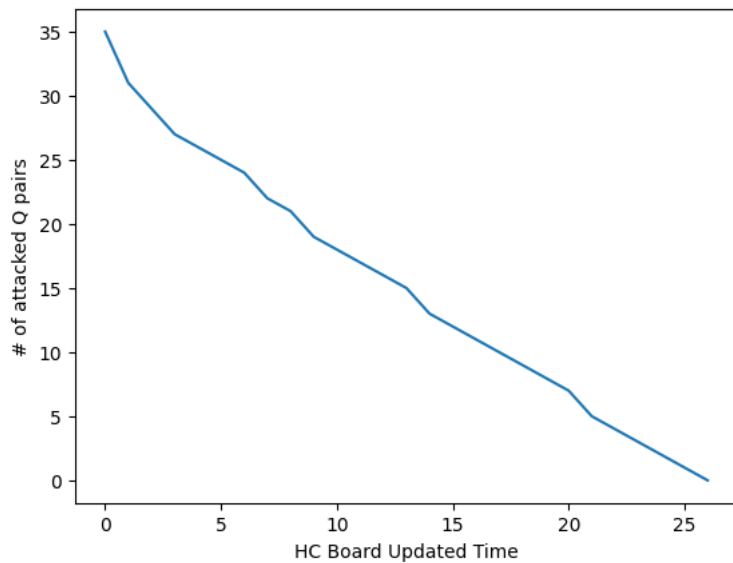
[illegible]

[illegible][illegible]

[illegible]

[illegible]

```
It takes 27 repetitions to succeed. Runtime in second 6.357987:
```

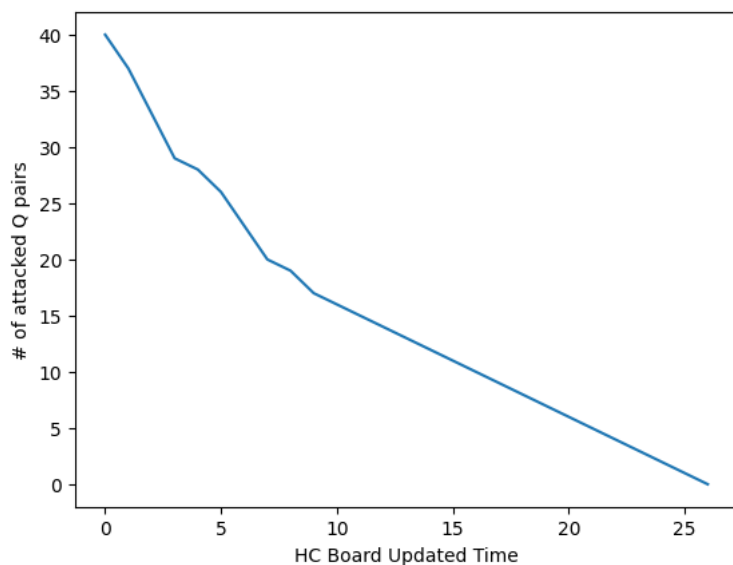


Failed
Failed
Failed
Failed
Failed
Failed
Failed
Failed
Failed

[illegible]

[illegible]

It takes 17 repetitions to succeed. Runtime in second 3.090264:



Failed

Failed

Failed

Failed

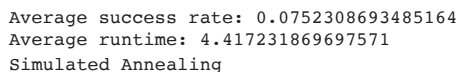
Failed

Successful Solution:

[illegible]

https://colab.research.google.com/drive/1oulTYbZyI77cHv0xTO2n_kQHC9rfkAj3?authuser=0#scrollTo=23NUfEyuuesO&printMode=true

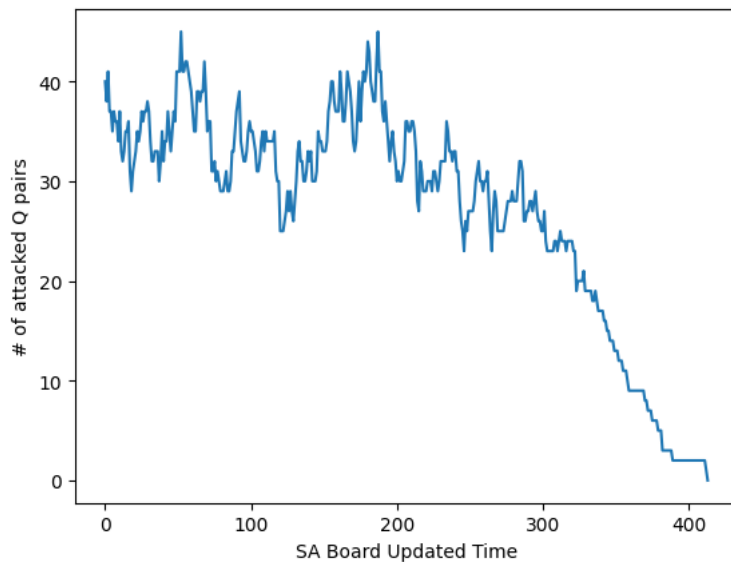
It takes 6 repetitions to succeed. Runtime in second 1.096360:

[illegible]

69/101

[illegible]

It takes 4 repetitions to succeed. Runtime in second 0.933234:

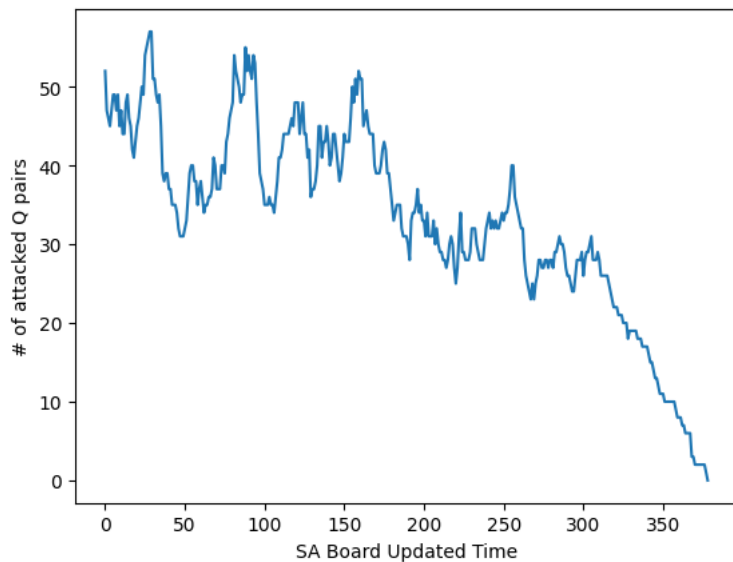
[illegible]

Successful Solution:

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

71/101

It takes 19 repetitions to succeed. Runtime in second 5.126971:



Failed

Failed

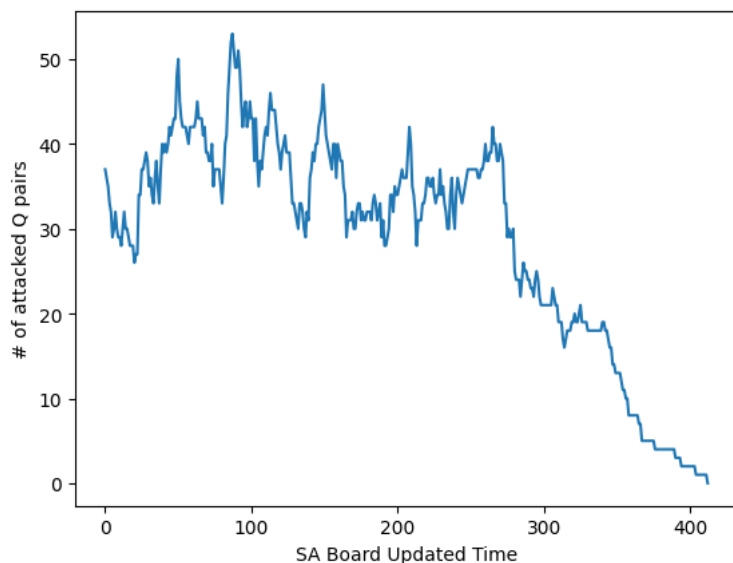
Failed

Successful Solution:

[illegible]

ab.research.google.com/drive/1oulTYbZvI77cHv0xTO2n_kOHC9rfkAj3?authuser=0#scrollTo=23

It takes 4 repetitions to succeed. Runtime in second 1.309794:

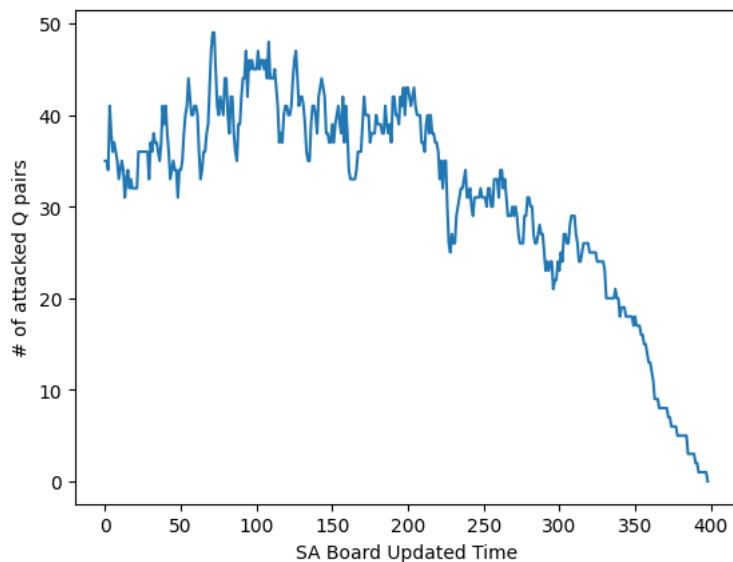


Successful Solution:

[illegible]

https://colab.research.google.com/drive/1oulTYbZyI77cHv0xTO2n_kQHC9rfkAj3?authuser=0#scrollTo=23NUfEyuuesO&printMode=true

It takes 2 repetitions to succeed. Runtime in second 0.450782:



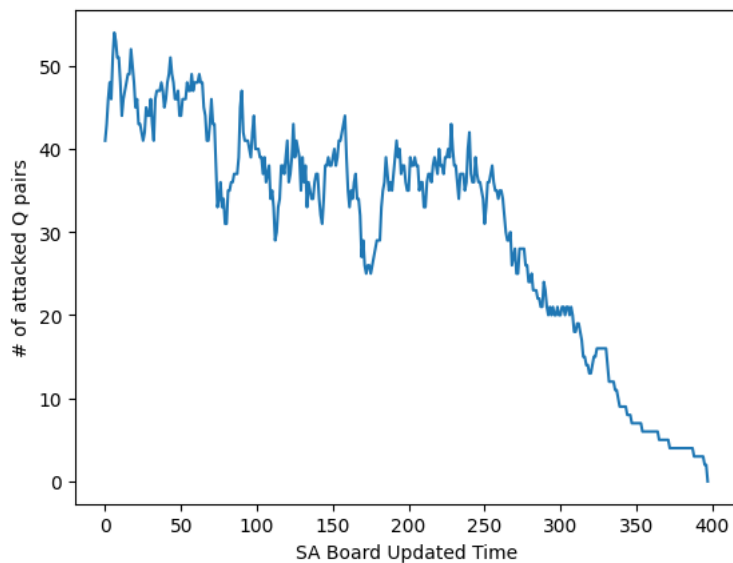
Successful Solution:

[illegible]

77/101

[illegible]

It takes 23 repetitions to succeed. Runtime in second 5.132225:

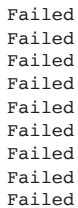
[illegible]

Successful Solution:

[illegible]

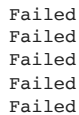
https://colab.research.google.com/drive/1oulTYbZyI77cHv0xTO2n_kQHC9rfkAj3?authuser=0#scrollTo=23NUfEyuuesO&printMode=true

It takes 14 repetitions to succeed. Runtime in second 3.185971:

[illegible]

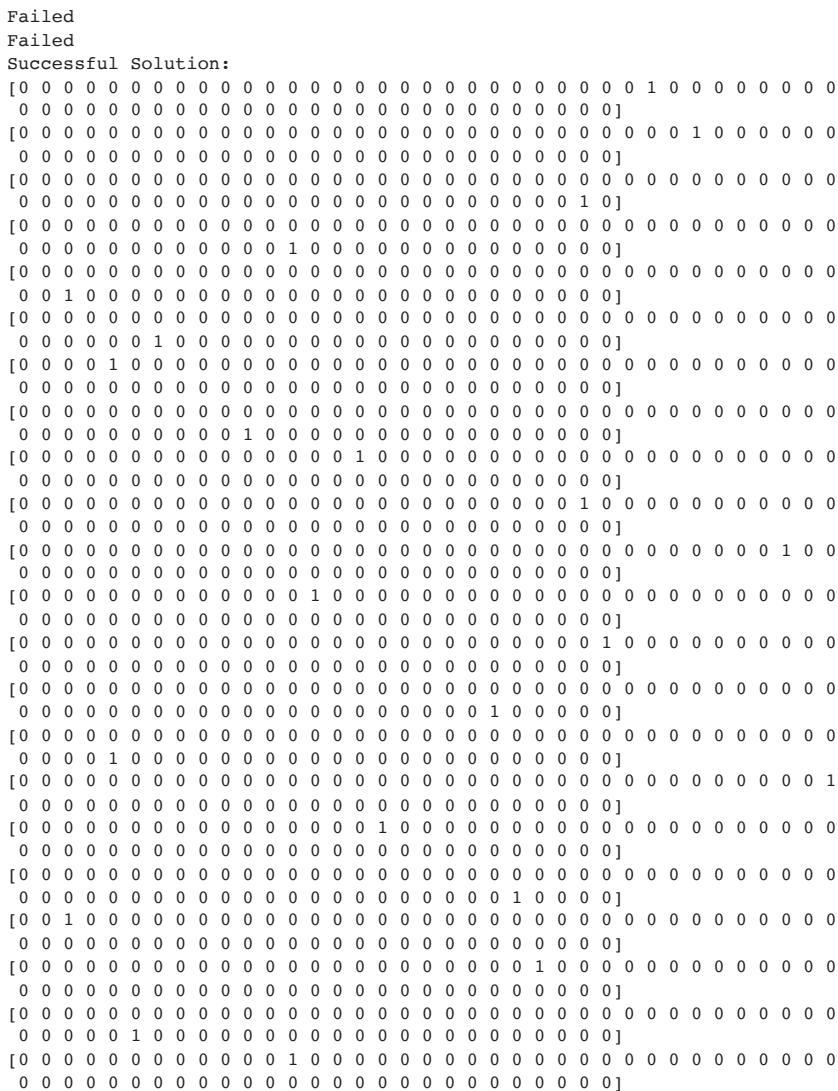
.....

It takes 10 repetitions to succeed. Runtime in second 3.659428:

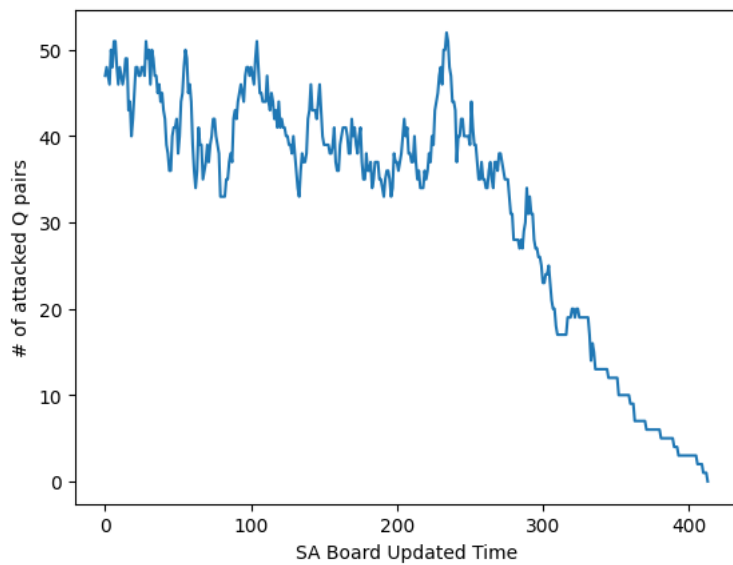
[illegible]

[illegible]

It takes 6 repetitions to succeed. Runtime in second 1.387645:



It takes 3 repetitions to succeed. Runtime in second 0.669878:

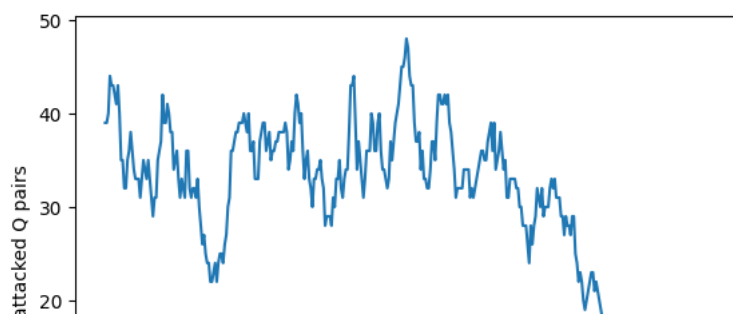


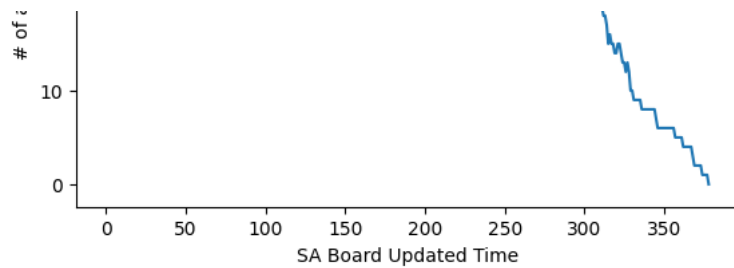
Successful Solution:

[illegible]

[illegible]

```
It takes 1 repetitions to succeed. Runtime in second 0.196121:
```





Average success rate: 0.27675384112455054
 Average runtime: 2.2052347898483275

▼ SA Temperature

▼ T = 4000

```
import random
import numpy as np
from math import exp
import time
from copy import deepcopy
import matplotlib.pyplot as plt
```



```

N_QUEENS = 64
TEMPERATURE = 4000

def threat_calculate(n):
    '''Combination formular. It is choosing two queens in n queens'''
    if n < 2:
        return 0
    if n == 2:
        return 1
    return (n - 1) * n / 2

def create_board(n):
    '''Create a chess board with a queen on a row'''
    chess_board = {}
    temp = list(range(n))
    random.shuffle(temp) # shuffle to make sure it is random
    column = 0

    while len(temp) > 0:
        row = random.choice(temp)
        chess_board[column] = row
        temp.remove(row)
        column += 1
    del temp
    return chess_board

def cost(chess_board):
    '''Calculate how many pairs of threaten queen'''
    threat = 0
    m_chessboard = {}
    a_chessboard = {}

    for column in chess_board:
        temp_m = column - chess_board[column]
        temp_a = column + chess_board[column]
        if temp_m not in m_chessboard:
            m_chessboard[temp_m] = 1
        else:
            m_chessboard[temp_m] += 1
        if temp_a not in a_chessboard:
            a_chessboard[temp_a] = 1
        else:
            a_chessboard[temp_a] += 1

    for i in m_chessboard:
        threat += threat_calculate(m_chessboard[i])
    del m_chessboard

    for i in a_chessboard:
        threat += threat_calculate(a_chessboard[i])
    del a_chessboard

    return threat

def hill_climbing():
    '''Hill Climbing Search'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:

```

```

        index_1 = random.randrange(0, N_QUEENS - 1)
        index_2 = random.randrange(0, N_QUEENS - 1)
        if index_1 != index_2:
            break
    successor[index_1], successor[index_2] = successor[index_2], \
        successor[index_1] # swap two chosen queens

    delta = cost(successor) - cost_answer
    if delta < 0:
        answer = deepcopy(successor)
        cost_answer = cost(answer)
        Costs.append(cost_answer)
    if cost_answer == 0:
        solution_found = True
        print("Successful Solution:")
        print_chess_board(answer)
        break
if solution_found is False:
    print("Failed")
    return(False, Costs)
else:
    return(True, Costs)

def simulated_annealing():
    '''Simulated Annealing'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0 or (random.uniform(0, 1) < exp(-delta / t)):
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True
            print("Successful Solution:")
            print_chess_board(answer)
            break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def print_chess_board(board):
    '''Print the chess board'''
    showBoard = np.zeros([N_QUEENS, N_QUEENS], dtype = int)
    for column, row in board.items():
        showBoard[row][column]=1
        #print("{} => {}".format(column, row))
    for i in range(N_QUEENS):
        print(showBoard[i])

def main(method='HC'):
    start = time.time()

```

```
Success=False
repetitions=0
while not Success:
    if method=='SA':
        Success,Costs=simulated_annealing()
        repetitions=repetitions+1
    elif method=='HC':
        Success,Costs=hill_climbing()
        repetitions=repetitions+1
print("It takes %d repetitions to succeed. Runtime in second %f:"% (repetitions,(time.time() - start)))
return(Success,Costs)

if __name__ == "__main__":
    print("Simulated Annealing")
    print("")
    method='SA'
    Success,Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
    plt.plot(Costs)
    plt.xlabel(method+' Board Updated Time')
    plt.ylabel('# of attacked Q pairs')
    plt.show()
    print()
```

Simulated Annealing

[illegible]

Successful Solution:

[illegible]

[illegible]

▼ $T = 400$

```
import random
import numpy as np
from math import exp
import time
from copy import deepcopy
import matplotlib.pyplot as plt

N_QUEENS = 64
TEMPERATURE = 400

def threat_calculate(n):
    '''Combination formular. It is choosing two queens in n queens'''
    if n < 2:
        return 0
    if n == 2:
        return 1
    return (n - 1) * n / 2

def create_board(n):
    '''Create a chess board with a queen on a row'''
    chess_board = {}
    temp = list(range(n))
    random.shuffle(temp) # shuffle to make sure it is random
    column = 0

    while len(temp) > 0:
        row = random.choice(temp)
        chess_board[column] = row
        temp.remove(row)
        column += 1

    del temp
    return chess_board
```

```

def cost(chess_board):
    '''Calculate how many pairs of threaten queen'''
    threat = 0
    m_chessboard = {}
    a_chessboard = {}

    for column in chess_board:
        temp_m = column - chess_board[column]
        temp_a = column + chess_board[column]
        if temp_m not in m_chessboard:
            m_chessboard[temp_m] = 1
        else:
            m_chessboard[temp_m] += 1
        if temp_a not in a_chessboard:
            a_chessboard[temp_a] = 1
        else:
            a_chessboard[temp_a] += 1

    for i in m_chessboard:
        threat += threat_calculate(m_chessboard[i])
    del m_chessboard

    for i in a_chessboard:
        threat += threat_calculate(a_chessboard[i])
    del a_chessboard

    return threat

def hill_climbing():
    '''Hill Climbing Search'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0:
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True
            print("Successful Solution:")
            print_chess_board(answer)
            break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def simulated_annealing():
    '''Simulated Annealing'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

```

```

# Record costs:
Costs=[]
Costs.append(cost_answer)

t = TEMPERATURE
sch = 0.99

while t > 0.0000001:
    t *= sch
    successor = deepcopy(answer)
    while True:
        index_1 = random.randrange(0, N_QUEENS - 1)
        index_2 = random.randrange(0, N_QUEENS - 1)
        if index_1 != index_2:
            break
    successor[index_1], successor[index_2] = successor[index_2], \
        successor[index_1] # swap two chosen queens

    delta = cost(successor) - cost_answer
    if delta < 0 or (random.uniform(0, 1) < exp(-delta / t)):
        answer = deepcopy(successor)
        cost_answer = cost(answer)
        Costs.append(cost_answer)
    if cost_answer == 0:
        solution_found = True
        print("Successful Solution:")
        print_chess_board(answer)
        break
if solution_found is False:
    print("Failed")
    return(False, Costs)
else:
    return(True, Costs)

def print_chess_board(board):
    '''Print the chess board'''
    showBoard = np.zeros([N_QUEENS, N_QUEENS], dtype = int)
    for column, row in board.items():
        showBoard[row][column]=1
    #print("{} => {}".format(column, row))
    for i in range(N_QUEENS):
        print(showBoard[i])

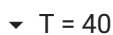
def main(method='HC'):
    start = time.time()
    Success=False
    repetitions=0
    while not Success:
        if method=='SA':
            Success, Costs=simulated_annealing()
            repetitions=repetitions+1
        elif method=='HC':
            Success, Costs=hill_climbing()
            repetitions=repetitions+1
    print("It takes %d repetitions to succeed. Runtime in second %f:"% (repetitions,(time.time() - start)))
    return(Success, Costs)

if __name__ == "__main__":
    print("Simulated Annealing")
    print("")
    method='SA'
    Success, Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
    plt.plot(Costs)
    plt.xlabel(method+' Board Updated Time')
    plt.ylabel('# of attacked Q pairs')
    plt.show()
    print()

```

[illegible]

It takes 10 repetitions to succeed. Runtime in second 2.879958:



```

import random
import numpy as np
from math import exp
import time
from copy import deepcopy
import matplotlib.pyplot as plt

N_QUEENS = 64
TEMPERATURE = 40

def threat_calculate(n):
    '''Combination formular. It is choosing two queens in n queens'''
    if n < 2:
        return 0
    if n == 2:
        return 1
    return (n - 1) * n / 2

def create_board(n):
    '''Create a chess board with a queen on a row'''
    chess_board = {}
    temp = list(range(n))
    random.shuffle(temp) # shuffle to make sure it is random
    column = 0

    while len(temp) > 0:
        row = random.choice(temp)
        chess_board[column] = row
        temp.remove(row)
        column += 1
    del temp
    return chess_board

def cost(chess_board):
    '''Calculate how many pairs of threaten queen'''
    threat = 0
    m_chessboard = {}
    a_chessboard = {}

    for column in chess_board:
        temp_m = column - chess_board[column]
        temp_a = column + chess_board[column]
        if temp_m not in m_chessboard:
            m_chessboard[temp_m] = 1
        else:
            m_chessboard[temp_m] += 1
        if temp_a not in a_chessboard:
            a_chessboard[temp_a] = 1
        else:
            a_chessboard[temp_a] += 1

    for i in m_chessboard:
        threat += threat_calculate(m_chessboard[i])
    del m_chessboard

    for i in a_chessboard:
        threat += threat_calculate(a_chessboard[i])
    del a_chessboard

    return threat

def hill_climbing():
    '''Hill Climbing Search'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE

```

```

sch = 0.99

while t > 0.0000001:
    t *= sch
    successor = deepcopy(answer)
    while True:
        index_1 = random.randrange(0, N_QUEENS - 1)
        index_2 = random.randrange(0, N_QUEENS - 1)
        if index_1 != index_2:
            break
    successor[index_1], successor[index_2] = successor[index_2], \
        successor[index_1] # swap two chosen queens

    delta = cost(successor) - cost_answer
    if delta < 0:
        answer = deepcopy(successor)
        cost_answer = cost(answer)
        Costs.append(cost_answer)
    if cost_answer == 0:
        solution_found = True
        print("Successful Solution:")
        print_chess_board(answer)
        break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def simulated_annealing():
    '''Simulated Annealing'''
    solution_found = False
    answer = create_board(N_QUEENS)

    # To avoid recounting when can not find a better state
    cost_answer = cost(answer)

    # Record costs:
    Costs=[]
    Costs.append(cost_answer)

    t = TEMPERATURE
    sch = 0.99

    while t > 0.0000001:
        t *= sch
        successor = deepcopy(answer)
        while True:
            index_1 = random.randrange(0, N_QUEENS - 1)
            index_2 = random.randrange(0, N_QUEENS - 1)
            if index_1 != index_2:
                break
        successor[index_1], successor[index_2] = successor[index_2], \
            successor[index_1] # swap two chosen queens

        delta = cost(successor) - cost_answer
        if delta < 0 or (random.uniform(0, 1) < exp(-delta / t)):
            answer = deepcopy(successor)
            cost_answer = cost(answer)
            Costs.append(cost_answer)
        if cost_answer == 0:
            solution_found = True
            print("Successful Solution:")
            print_chess_board(answer)
            break
    if solution_found is False:
        print("Failed")
        return(False, Costs)
    else:
        return(True, Costs)

def print_chess_board(board):
    '''Print the chess board'''
    showBoard = np.zeros([N_QUEENS, N_QUEENS], dtype = int)
    for column, row in board.items():
        showBoard[row][column]=1

```

```
#print("{} => {}".format(column, row))
for i in range(N_QUEENS):
    print(showBoard[i])

def main(method='HC'):
    start = time.time()
    Success=False
    repetitions=0
    while not Success:
        if method=='SA':
            Success,Costs=simulated_annealing()
            repetitions=repetitions+1
        elif method=='HC':
            Success,Costs=hill_climbing()
            repetitions=repetitions+1
    print("It takes %d repetitions to succeed. Runtime in second %f:"% (repetitions,(time.time() - start)))
    return(Success,Costs)

if __name__ == "__main__":
    print("Simulated Annealing")
    print("")
    method='SA'
    Success,Costs=main(method) # HC: hill_climbing or SA: Simulated annealing
    plt.plot(Costs)
    plt.xlabel(method+' Board Updated Time')
    plt.ylabel('# of attacked Q pairs')
    plt.show()
    print()
```

Failed
Failed
Failed
Failed
Failed
Failed

[illegible]