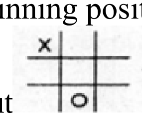
CAP 6635 Artificial Intelligence
Homework 3 | Matthew Acs

1. Question 1 [2 pts]: Figure 1 shows a portion of an imperfect tic-tac-toe game tree using evaluation function. Assume X is the Max player and O is the Min player. Assume a heuristic function is defined as the number of X's winning position subtract the number of O's winning position. For example, for board layout  , X has 6 winning positions, and O has 5 winning positions. Therefore, the heuristic value of this board layout is 6-5=1.

- Using defined heuristic function, list the heuristic values for all leaf nodes. [0.5 pt]
- Assume X is the root player, applying α-β pruning to the game tree in Figure 1, determine α value for Max node, and β value for Min nodes. [0.5]
- Determine nodes which are pruned by α-β pruning [0.5],
- Specify the best move for X. [0.5 pt]

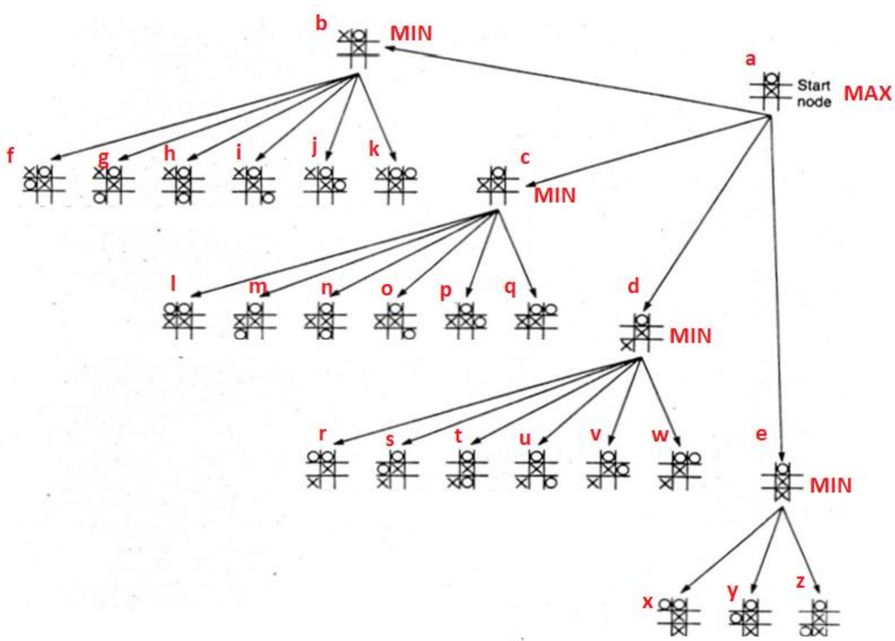


Figure 1

Heuristic Values for Leaf Nodes

Leaf Node	**Heuristic Value**
f	4 – 2 = **2**
g	3 – 2 = **1**
h	5 – 2 = **3**
i	3 – 2 = **1**
j	4 – 2 = **2**
k	4 – 2 = **2**
--	--
l	4 – 3 = **1**
m	3 – 3 = **0**
n	5 – 3 = **2**
o	3 – 3 = **0**
p	4 – 3 = **1**
q	4 – 3 = **1**
--	--
r	4 – 2 = **2**
s	4 – 2 = **2**
t	5 – 2 = **3**
u	3 – 2 = **1**
v	4 – 2 = **2**
w	4 – 2 = **2**
--	--
x	4 – 3 = **1**
y	4 – 3 = **1**
z	3 – 3 = **0**

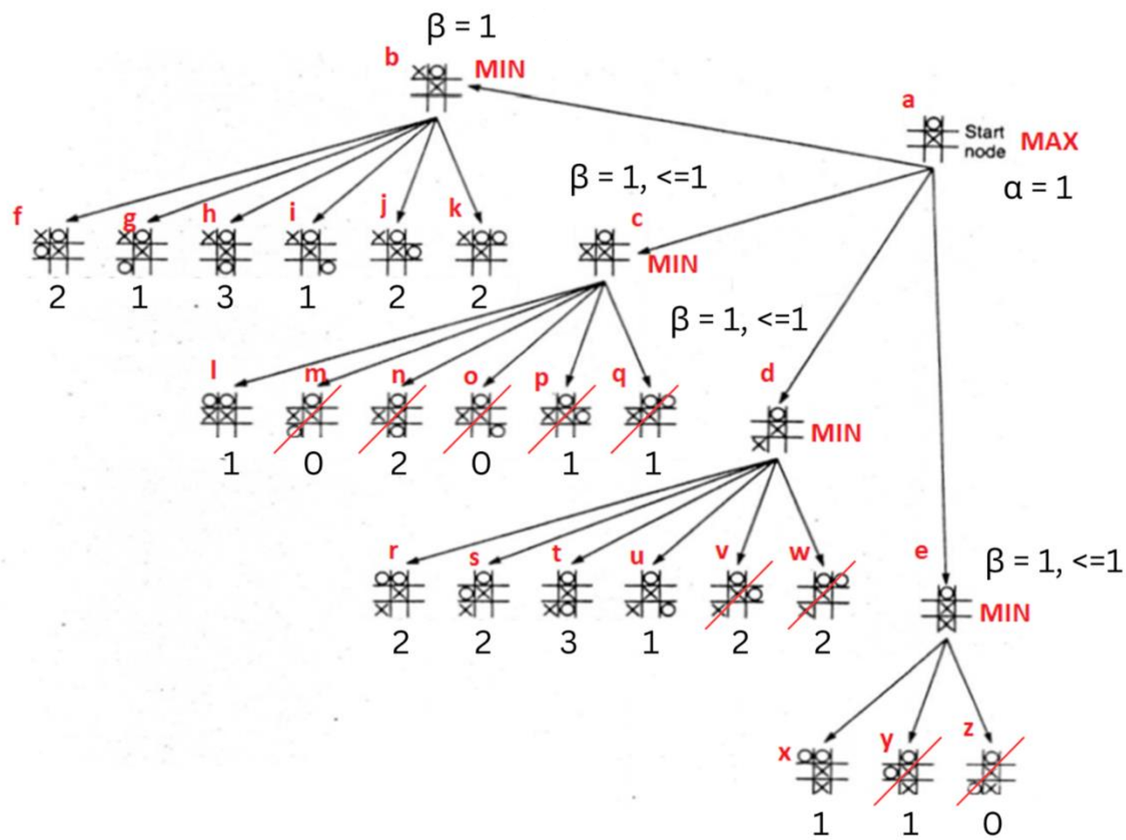
α-β Values

Node	**α-β Values**
a (MAX)	α = 1
b (MIN)	β = 1
c (MIN)	β = 1
d (MIN)	β = 1
e (MIN)	β = 1

Nodes Pruned
m, n, o, p, q, v, w, y, z

Best Move
The best move for x would be (b), which is to place an X in the upper left corner. This maximizes the score for X based on optimal play from O and the given heuristic.

Graphical Representation



Question 2 [2 pts]: Variable $W_{a,b}$ defines that a Wumpus is located as location [a,b], and $S_{x,y}$ denotes that a stench is sensed at location [x,y].

- Explain the semantic (meaning) of the following sentence? [1 pt]
- **S**how detailed steps to decompose sentence into CNF format [1 pt]

$$W_{1,3} \Leftrightarrow (S_{1,2} \vee S_{1,4} \vee S_{2,3})$$

Semantic meaning

The propositional logic sentence means that a Wumpus on location [1,3] implies that there is a stench on location [1,2], [1,4], or [2,3] and stench on location [1,2], [1,4], or [2,3] implies that there is a Wumpus on location [1,3].

Thus, the semantic meaning is -
A Wumpus in location [1,3] causes stench in one or more of the adjacent squares [1, 2], [1, 4], or [2, 3].

CNF format

1. Original Sentence
   $W_{1,3} \Leftrightarrow (S_{1,2} \vee S_{1,4} \vee S_{2,3})$

2. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(W_{1,3} \Rightarrow (S_{1,2} \vee S_{1,4} \vee S_{2,3})) \wedge ((S_{1,2} \vee S_{1,4} \vee S_{2,3}) \Rightarrow W_{1,3})$

3. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.
   $(\neg W_{1,3} \vee (S_{1,2} \vee S_{1,4} \vee S_{2,3})) \wedge (\neg (S_{1,2} \vee S_{1,4} \vee S_{2,3}) \vee W_{1,3})$

4. Move $\neg$ inwards using de Morgan's rules and double-negation:
   $(\neg W_{1,3} \vee (S_{1,2} \vee S_{1,4} \vee S_{2,3})) \wedge ((\neg S_{1,2} \wedge \neg S_{1,4} \wedge \neg S_{2,3}) \vee W_{1,3})$

5. Apply distributivity law ($\wedge$ over $\vee$) and flatten:
   $(\neg W_{1,3} \vee S_{1,2} \vee S_{1,4} \vee S_{2,3}) \wedge (\neg S_{1,2} \vee W_{1,3}) \wedge (\neg S_{1,4} \vee W_{1,3}) \wedge (\neg S_{2,3} \vee W_{1,3})$

6. CNF format (conjunction of disjunctions of literals clauses):
   $\mathbf{(\neg W_{1,3} \vee S_{1,2} \vee S_{1,4} \vee S_{2,3}) \wedge (\neg S_{1,2} \vee W_{1,3}) \wedge (\neg S_{1,4} \vee W_{1,3}) \wedge (\neg S_{2,3} \vee W_{1,3})}$

Question 3 [1 pt]: Given knowledge base KB which consists of following four rules:
R1: $P \Rightarrow Q$
R2: $Q \Rightarrow R$
R3: $R \Rightarrow T$
R4: $\neg T$
Prove that KB $\models \neg P$ (show detailed proof).

<u>Proof using Modus Tollens</u>

1. $R \Rightarrow T, \quad \neg T$ R4, R3, Modus Tollens

   $\therefore \neg R$

2. $Q \Rightarrow R, \quad \neg R$ 1, R2, Modus Tollens

   $\therefore \neg Q$

3. $P \Rightarrow Q, \quad \neg Q$ 2, R1, Modus Tollens

   $\therefore \neg P$

$\therefore$ KB $\models \neg$ P

Question 4 [4 pts] Figure 2 shows the Wumpus world game, where the agent starts from location [1,1], and does not sense breeze or stench. After that, the agent moves to location [1,2] and sense a Breeze (B). Then the agent moved back to [1,1], and further moved to location [2,1] and senses a Stench (S). Based on the above observations and the Wumpus world game rules, please use resolution algorithm to prove following entailment.

- $KB \models W_{3,1}$ (There is a Wumpus at location [3,1]) [1 pt]
- $KB \models P_{1,3}$ (There is a Pit at location [1,3]) [1 pt]
- $KB \models \neg P_{2,2}$ (There is no Pit at location [2,2]) [1 pt]
- $KB \models \neg W_{2,2}$ (There is a no Wumpus at location [2,2]) [1 pt]

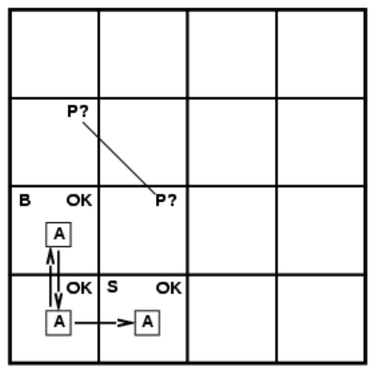


Figure 2

1. $KB = (S_{1,2} \Leftrightarrow (W_{1,3} \vee W_{2,2})) \wedge \neg S_{1,2}$

2. $KB \models \neg W_{1,3}$ 1

3. $\mathbf{KB} \models \mathbf{\neg W_{2,2}}$ 1

4. $KB = (S_{2,1} \Leftrightarrow (W_{3,1} \vee W_{2,2})) \wedge S_{2,1}$

5. $KB \models W_{3,1} \vee W_{2,2}$ 4

6. $W_{3,1} \vee W_{2,2}$, $\neg W_{2,2}$ 3, 5

$\therefore W_{3,1}$

$\therefore \mathbf{W_{3,1}, \neg W_{2,2}}$

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7. $KB = (B_{1,2} \Leftrightarrow (P_{1,3} \vee P_{2,2})) \wedge B_{1,2}$

8. $KB \models P_{1,3} \vee P_{2,2}$ 7

9. $KB = (B_{2,1} \Leftrightarrow (P_{3,1} \vee P_{2,2})) \wedge \neg B_{2,1}$

10. $KB \models \neg P_{3,1}$ 9

11. $\mathbf{KB} \models \mathbf{\neg P_{2,2}}$ 9

12. $P_{1,3} \vee P_{2,2}$, $\neg P_{2,2}$ 8, 11

$\therefore P_{1,3}$

$\therefore \mathbf{P_{1,3}, \neg P_{2,2}}$

Question 5 [2 pts] The following table lists defined predicates.

A(x)	x is an apple
B(x)	x is blue
T(x)	x is tasty
G(x)	x is green
R(x)	x is red
P(x)	x is people
F(x)	x is fruit
L(x,y)	x like y

Using first order logic to express following sentences

- Apples are fruit [0.25 pt]
- Some apples are red [0.25 pt]
- No apple is blue [0.25 pt]
- Green apples are tasty [0.25 pt]
- Some people do not like apple [0.25 pt]
- Not all apples are green [0.25 pt]
- Some people like fruit, except apples [0.25 pt]
- No fruit is liked by every people [0.25]

<u>Apples are fruit</u>
$\forall x\ (A(x) \rightarrow F(x))$

<u>Some apples are red</u>
$\exists x\ (A(x) \wedge R(x))$

<u>No apple is blue</u>
$\forall x\ (A(x) \rightarrow \neg B(x))$

<u>Green apples are tasty</u>
$\forall x\ (A(x) \wedge G(x) \rightarrow T(x))$

<u>Some people do not like apples</u>
$\exists x\ \forall y\ (P(x) \wedge A(y) \wedge \neg L(x,y))$

<u>Not all apples are green</u>
$\exists x\ (A(x) \wedge \neg G(x))$

<u>Some people like fruit, except apples</u>
$\exists x\ \forall y\ (P(x) \wedge F(y) \wedge L(x,y) \wedge \neg A(y))$

<u>No fruit is liked by every person</u>
$\neg\exists x\ \forall y\ (P(y) \wedge F(x) \wedge L(y,x))$

Question 6 [3 pts] Figure 2 shows the Wumpus world game, where the agent starts from location [1,1], and does not sense breeze or stench. After that, the agent moves to location [1,2] and sense a Breeze (B). Then the agent moves back to [1,1], and further moves to location [2,1] and senses a Stench (S).

$W_{a,b}$	Wumpus is located at located [a,b]
$S_{x,y}$	A stench is sensed at location [x,y]
$P_{a,b}$	A pit is located at located [a,b]
$B_{x,y}$	A breeze is sensed at location [x,y]
Adjacent([a,b], [c,d])	Cell located at [c,d] is adjacent to the cell located at [a,b]

Using first order logic (FOL) to express following sentences and/or derive respective conclusion.

- Write an FOL sentence to express that "all cells next to the cell of Wumpus will sense a stench" [0.5 pt].
- Write an FOL sentence to express that "all cells next to the cell of a pit will sense a breeze" [0.5 pt].
- Write an FOL sentence to express that "if there is no breeze sensed at a cell, there is no pit in adjacent cells" [0.5 pt].
- Write an FOL sentence to express that "if there is no stench sensed at a cell, there is no Wumpus in adjacent cells" [0.5 pt].
- Based on the above settings, derive a conclusion that there is no Wumpus at [1,3] [0.5 pt]
- Based on the above settings, derive a conclusion that there is no pit at [2,2] [0.5 pt]

<u>All cells next to the cell of Wumpus will sense a stench</u>
$\forall a,b,c,d\ (W_{a,b} \land \text{Adjacent}([a, b], [c, d]) \rightarrow S_{c,d})$

<u>All cells next to the cell of a pit will sense a breeze</u>
$\forall a,b,c,d\ (P_{a,b} \land \text{Adjacent}([a, b], [c, d]) \rightarrow B_{c,d})$

<u>If there is no breeze sensed at a cell, there is no pit in adjacent cells</u>
$\forall a,b,c,d\ (\neg B_{a,b} \land \text{Adjacent}([a, b], [c, d]) \rightarrow \neg P_{c,d})$

<u>If there is no stench sensed at a cell, there is no Wumpus in adjacent cells</u>
$\forall a,b,c,d\ (\neg S_{a,b} \land \text{Adjacent}([a, b], [c, d]) \rightarrow \neg W_{c,d})$

<u>No Wumpus at [1,3]</u>

$\forall a,b,c,d\ (W_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow S_{c,d})$
$\forall a,b,c,d\ (\neg S_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow \neg W_{c,d})$
$KB = \{S_{2,1};\quad \neg S_{1,2};\quad W_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow S_{c,d}\ \}$

$\neg S_{1,2} \wedge Adjacent([1, 2], [1, 3])$
$\neg S_{1,2} \rightarrow \neg (W_{1,3} \wedge Adjacent([1, 3], [1, 2]))$

$\neg S_{1,2} \rightarrow \neg (W_{1,3} \wedge Adjacent([1, 3], [1, 2])), \neg S_{1,2}$

$\therefore \neg (W_{1,3} \wedge Adjacent([1, 3], [1, 2]))$

$\neg W_{1,3} \vee \neg Adjacent([1, 3], [1, 2])), Adjacent([1, 3], [1, 2])$

$\therefore \neg W_{1,3}$

Since there is no stench sensed at cell [1, 2] and cell [1, 2] is adjacent to cell [1, 3], there is no Wumpus at [1, 3].

<u>No pit at [2,2]</u>

$\forall a,b,c,d\ (P_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow B_{c,d})$
$\forall a,b,c,d\ (\neg B_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow \neg P_{c,d})$
$KB = \{B_{1,2};\quad \neg B_{2,1};\quad P_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow B_{c,d}\ \}$

$\neg B_{2,1} \wedge Adjacent([2, 1], [2, 2])$
$\neg B_{2,1} \rightarrow \neg (P_{2,2} \wedge Adjacent([2, 2], [2, 1]))$

$\neg B_{2,1} \rightarrow \neg (P_{2,2} \wedge Adjacent([2, 2], [2, 1])), \neg B_{2,1}$

$\therefore \neg (P_{2,2} \wedge Adjacent([2, 2], [2, 1]))$

$\neg P_{2,2} \vee \neg Adjacent([2, 2], [2, 1])), Adjacent([2, 2], [2, 1])$

$\therefore \neg P_{2,2}$

Since there is no breeze sensed at cell [2, 1] and cell [2, 1] is adjacent to cell [2, 2], there is no pit at [2, 2].

Question 7 [2 pts] The following sentences describe interesting behaviors of a Hoffer club members.

- Tony, Shi-Kuo and Ellen belong to the Hoofers Club.
- Every member of the Hoofers Club is either a skier or a mountain climber or both.
- No mountain climber likes rain, and all skiers like snow.
- Ellen dislikes whatever Tony likes and likes whatever Tony dislikes.
- Tony likes rain and snow.

Query: Does Hoffer club have a member who is a mountain climber but not a skier?

(1) Define predicates and relations of the Hoffer using first order logic [0.5 pt]
(2) Using first order logic to express each sentence (including query) [0.5 pt]
(3) Converting each sentence to clause format [0.5 pt]
(4) Using Unification to answer the query [0.5]

Predicates **S(x): Skier, M(x): mountain climber** Relations **L(x,y): x likes y**

Every member of the Hoofers Club is either a skier or a mountain climber or both
$\forall$x (S(x) $\vee$ M(x)) 1. *Clause Format:* **S(x1) $\vee$ M(x1)**

No mountain climber likes rain, and all skiers like snow
$\neg$ $\exists$x (M(x) $\wedge$ L(x, Rain)) 2. *Clause Format:* **$\neg$ M(x2) $\vee$ $\neg$ L(x2, Rain)**
$\forall$x (S(x) $\rightarrow$ L(x, Snow)) 3. *Clause Format:* **$\neg$ S(x3) $\vee$ L(x3, Snow)**

Ellen dislikes whatever Tonty likes and likes whatever Tony dislikes
$\forall$x (L(Ellen, x) $\leftrightarrow$ $\neg$ L(Tony, x)) 4. *Clause Format:* **$\neg$ L(Ellen, x4) $\vee$ $\neg$ L(Tony, x4)**
5. *Clause Format:* **L(Ellen, x5) $\vee$ L(Tony, x5)**

Tony likes rain and snow
L(Tony, Rain) 6. *Clause Format:* **L(Tony, Rain)**
L(Tony, Snow) 7. *Clause Format:* **L(Tony, Snow)**

Does Hoofer Club have a member who is a mountain climber but not a skier?
$\exists$x (M(x) $\wedge$ $\neg$ S(x))?
Negation Query: **$\neg$ $\exists$x (M(x) $\wedge$ $\neg$ S(x))?** 8. *Clause Format:* **$\neg$ M(x6) v S(x6)?**

Clause 1	Clause 2	Resolvent	MGU
(8) $\neg$ M(x6) v S(x6)	(1) S(x1) $\vee$ M(x1)	(9) S(x1)	{x6/x1}
(9) S(x1)	(3) $\neg$ S(x3) $\vee$ L(x3, Snow)	(10) L(x1, Snow)	{x3/x1}
(10) L(x1, Snow)	(4) $\neg$ L(Ellen, x4) $\vee$ $\neg$ L(Tony, x4)	(11) $\neg$ L(Tony, Snow)	{x4/Snow, x1/Ellen}
(11) $\neg$ L(Tony, Snow)	(7) L(Tony, Snow)	False	{}

$\therefore$ $\exists$x (M(x) $\wedge$ $\neg$ S(x))
{x6/x1} $\rightarrow$ {x3, x1} $\rightarrow$ {x4/Snow, x1/Ellen} $\rightarrow$ M(Ellen) $\vee$ $\neg$ S(Ellen)
Ellen is a mountain climber but not a skier.

Question 8 [2 pts]: Table 1 shows probability values of different events. Using the table to calculate following values and show proof:

- The probability that a persona has no cavity [0.25 pt]
- The probability of no toothache [0.25 pt]
- The joint probability of cavity and no toothache [0.25 pt]
- Calculate conditional probability of no cavity, given the patient has toothache [0.25 pt]
- Calculate conditional probability of no cavity, given the patient does not have toothache [0.25 pt]
- Determine whether cavity and toothache are independent or not, why [0.25 pt]
- Given a patient has cavity, determine whether the tooth probe catch is conditionally independent of toothache or not, why [0.25 pt]
- Given a patient does not have cavity, determine whether the tooth probe catch is conditionally independent of toothache or not, why [0.25 pt]

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	**.108**	**.012**	**.072**	**.008**
¬ cavity	**.016**	**.064**	**.144**	**.576**

Table 1

<u>Probability that a person has no cavity</u>
0.016 + 0.064 + 0.144 + 0.576 = **0.80**

<u>Probability of no toothache</u>
0.072 + 0.008 + 0.144 + 0.576 = **0.80**

<u>The joint probability of cavity and no toothache</u>
0.072 + 0.008 = **0.08**

<u>Conditional probability of no cavity, given toothache P(no cavity | toothache)</u>
P(no cavity $\wedge$ toothache) / P(toothache) = (0.016 + 0.064)/(0.20) = **0.40**

<u>Conditional probability of no cavity, given no toothache P(no cavity | no toothache)</u>
P(no cavity $\wedge$ no toothache) / P(no toothache) = (0.144 + 0.576)/(0.80) = **0.90**

<u>Are cavity and toothache independent</u>
If cavity and toothache are independent, then P(cavity | toothache) = P(cavity)
(0.12) / (0.20) = (0.20)?
0.6 = 0.20? FALSE
Cavity and toothache are not independent because P(cavity | toothache) is not equal to P(cavity), and P(cavity | toothache) = P(cavity) if cavity and toothache are independent.

Given a patient has cavity is the tooth probe catch conditionally independent of toothache
P(catch | toothache, cavity) = P(catch | cavity)?
P(catch | toothache, cavity) = P(catch $\wedge$ toothache $\wedge$ cavity) / P(toothache $\wedge$ cavity) = 0.108 / 0.12 = 0.90
P(catch | cavity) = 0.18 / 0.20 = 0.90
Given a patient has a cavity, the tooth probe catch is conditionally independent of toothache because P(catch | toothache, cavity) = P(catch | cavity), which is a criterion of conditional independence.

Given a patient does not have cavity is the tooth probe catch conditionally independent of toothache
P(catch | toothache, no cavity) = P(catch | no cavity)?
P(catch | toothache, no cavity) = P(catch $\wedge$ toothache $\wedge$ no cavity) / P(toothache $\wedge$ no cavity) = 0.016 / 0.08 = 0.20
P(catch | no cavity) = 0.16 / 0.80 = 0.20
Given a patient has no cavity, the tooth probe catch is conditionally independent of toothache because P(catch | toothache, no cavity) = P(catch | no cavity), which is a criterion of conditional independence.

Question 9: [2 pts]: Figure 3 shows a Bayesian network, using first letter to denote each named variable, e.g, using T to denote tampering, and complete following questions. Assume x ⊥ y denotes that x are independent of y, x ⊥ y | z denotes that x and y are conditionally independent, given z. Complete Table 2, and use √ to mark correct answers. [2 pts]

	True	**False**
T ⊥ F		
A ⊥ S		
T ⊥ S		
$R \perp A$		
$R \perp A \mid L$		
$L \perp S$		
$L \perp S \mid F$		
$S \perp A \mid F$		
$R \perp S \mid L$		

Table 2

Figure 3

Question does not need to be answered according to lecture on 7/27/23