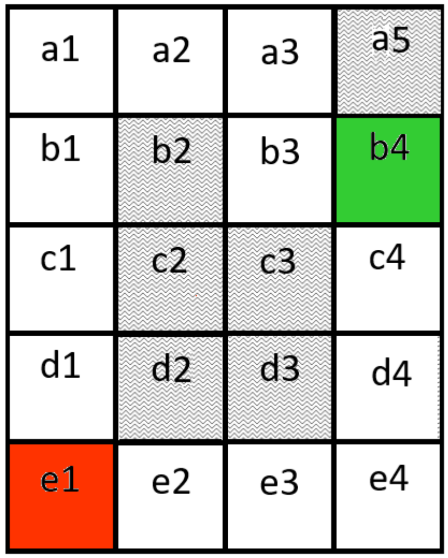

Figure 1

Question 1 [4 pts]: Figure 1 shows a robot navigation field, where the red square (e1) is the robot, and green square (b4) is the goal. The shade squares (such as b2, c2, etc.) are obstacles. The robot is not allowed to move in diagonal line. Nodes are coded using an alphabet letter followed by a digit (such as a1, b1, b2 etc.). When two sibling nodes are inserted into fringe (queue), use deque order to favor node with a lower alphabet and a lower digit. For example, if d1 and e2 are sibling nodes, d1 will be dequeued first (because "d" has a lower alphabetic order than "e"). If a1 and a2 are sibling nodes, a1 will be dequeued first (because "1" has a lower digit than "2"). Node expanded/visited does not need to be revisited.

- Use Best First Search to find path from e1 to b4 (Using Manhattan distance as the heuristic function)
  - Report nodes in the fringe (including their f(N) values) in the orders they are included in the fringe. **[1 pt]**
  - Report the order of the nodes being expanded. **[0.5 pt]**
  - Report the final path from e1 to b4. **[0.5 pt]**
- Use A* to find path from e1 to b4 (Using Manhattan distance as the heuristic function).
  - Report nodes in the fringe (including their f(N) values) in the orders they are included in the fringe. **[1 pt]**
  - Report the order of the nodes being expanded. **[0.5 pt]**
  - Report the final path from e1 to b4. **[0.5 pt]**

Best First Search

Fringe (Last → Next): f(N)=h(N)	Node visited/expanded
e1 (6)	e1
e2 (5), d1 (5)	d1
e2 (5), c1 (4)	c1
e2 (5), b1 (3)	b1
e2 (5), a1 (4)	a1
e2 (5), a2 (3)	a2
e2 (5), a3 (2)	a3
e2 (5), b3 (1)	b3
e2 (5), b4 (0)	**b4 FOUND**

Order of nodes expanded: e1, d1, c1, b1, a1, a2, a3, b3, b4
Final path: e1, d1, c1, b1, a1, a2, a3, b3, b4

A* Search

Fringe (Last → Next): f(N)=g(N)+h(N)	Node visited/expanded
e1 (6)	e1
e2 (6), d1 (6)	d1
e2 (6), c1 (6)	c1
e2 (6), b1 (6)	b1
a1 (8), e2 (6)	e2
a1 (8), e3 (6)	e3
a1 (8), e4 (6)	e4
a1 (8), d4 (6)	d4
a1 (8), c4 (6)	c4
a1 (8), b4 (6)	**b4 FOUND**

Order of node expansion: e1, d1, c1, b1, e2, e3, e4, d4, c4, b4
Final path: e1, e2, e3, e4, d4, c4, b4

Question 2 [4 pts]: Figure 2 shows the Wumpus world game, where the agent starts from location [4,4], and does not sense breeze or stench. After the agent enters [4,4], it moves to [4,3] and only senses a "Breeze". Then the agent travels back to [4,4], and then moves to [3,4]. At location [3,4], the agent only senses a "Stench".

$W_{a,b}$	Wumpus is located at located [a,b]
$S_{x,y}$	A stench is sensed at location [x,y]
$P_{a,b}$	A pit is located at located [a,b]
$B_{x,y}$	A breeze is sensed at location [x,y]
Adjacent([a,b], [c,d])	Cell located at [c,d] is adjacent to the cell located at [a,b]

Using first order logic (FOL) to express following sentences and/or derive respective conclusion.

- Write an FOL sentence to express that "all cells next to the cell of Wumpus will sense a stench" [0.5 pt].
- Write an FOL sentence to express that "if there is no breeze sensed at a cell, there is no pit in adjacent cells" [0.5 pt].
- Prove that there is no Wumpus at [4,3] (using FOL) [1 pt]
- Prove that there is no Wumpus at [3,3] (using FOL) [2 pts]

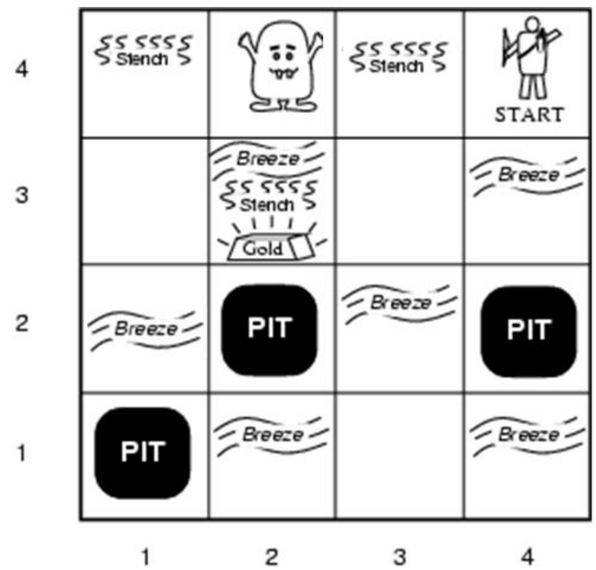

Figure 2

All cells next to the cell of Wumpus will sense a stench
$\forall a,b,c,d$: $W_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow S_{c,d}$

If there is no breeze sensed at a cell, there is no pit in adjacent cells
$\forall a,b,c,d$: $\neg B_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow \neg P_{c,d}$

Proof that there is no Wumpus at [4,3]
KB = {$\neg S_{4,4}$; Adjacent([4,3], [4,4]); $\forall a,b,c,d$: $W_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow S_{c,d}$;}

Using rule: $\forall a,b,c,d$: $W_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow S_{c,d}$

We have: $W_{4,3} \wedge Adjacent([4, 3], [4, 4]) \rightarrow S_{4,4}$

Using contraposition:

$\neg S_{4,4} \rightarrow \neg(W_{4,3} \wedge Adjacent([4, 3], [4, 4]))$

Using Modus Ponens:
$\neg S_{4,4} \rightarrow \neg(W_{4,3} \wedge Adjacent([4, 3], [4, 4]))$, $\neg S_{4,4}$

$\neg(W_{4,3} \wedge Adjacent([4, 3], [4, 4]))$

Using De Morgan's Theorem: $\neg W_{4,3} \vee \neg Adjacent([4, 3], [4, 4]))$

Using resolution:
$\neg W_{4,3} \vee \neg Adjacent([4, 3], [4, 4]))$, Adjacent([4,3], [4,4])

$\neg W_{4,3}$

$\therefore \neg W_{4,3}$

Proof that there is no Wumpus at [3,3]

KB = {$\neg S_{4,3}$; Adjacent([3,3], [4,3]); $\forall a,b,c,d$: $W_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow S_{c,d}$;}

Using rule: $\forall a,b,c,d$: $W_{a,b} \wedge Adjacent([a, b], [c, d]) \rightarrow S_{c,d}$

We have: $W_{3,3} \wedge Adjacent([3, 3], [4, 3]) \rightarrow S_{4,3}$

Using contraposition:

$\neg S_{4,3} \rightarrow \neg(W_{3,3} \wedge Adjacent([3, 3], [4, 3]))$

Using Modus Ponens:

$\neg S_{4,3} \rightarrow \neg(W_{3,3} \wedge Adjacent([3, 3], [4, 3])), \neg S_{4,3}$

$\neg(W_{3,3} \wedge Adjacent([3, 3], [4, 3]))$

Using De Morgan's Theorem: $\neg W_{3,3} \vee \neg Adjacent([3, 3], [4, 3]))$

Using resolution:
$\neg W_{3,3} \vee \neg Adjacent([3, 3], [4, 3])), Adjacent([3,3], [4,3])$

$\neg W_{3,3}$

$\therefore \neg W_{3,3}$

Question 3 [3 pts] Givin following sentences,

1. (1) Some cancers are caused by smoking.
2. (2) Some people like smoking.
3. (3) Not all people like smoking.
4. (4) Sam likes smoking.

Defined predicates/variables:

Cause(x,y)	x cause y
C(x)	x is cancer
P(x)	x is people
L(x,y)	x like y
Sam	Sam
S	Smoking

(1) Using defined predicates/variables and first order logic to express each sentence [2 pts]

(2) Converting each sentence to clause format [1 pt]

Sentence	*FOL*	*Clause Format*
Some cancers are caused by smoking	$\exists x$: **C(x) $\wedge$ Cause(S, x)**	**C(x) $\wedge$ Cause(S, x)**
Some people like smoking	$\exists x$: **P(x) $\wedge$ L(x, S)**	**P(x) $\wedge$ L(x, S)**
Not all people like smoking	$\neg \forall x$: **P(x) $\rightarrow$ L(x, S)**	**P(x) $\wedge \neg$ L(x, S)**
Sam likes smoking	**L(Sam, S)**	**L(Sam, S)**

Question 4 [4 pts]: Figure 3 shows a Bayesian network where all random variables have binary values. Assume x ⊥ y denotes that x are independent of y, and x ⊥ y | z denotes that x and y are conditionally independent, given z. Answer following questions and complete Table 1 (use √ to mark correct answers).

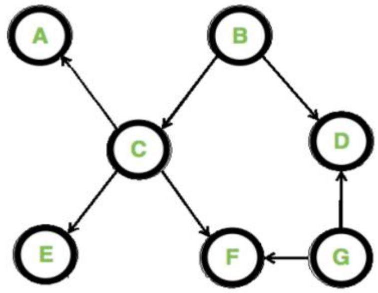

Figure 3

1. Write joint probability of the network showing in Figure 3 [0.5 pt]
2. How many probability values are needed to calculate the network's join probability, why [1 pt]
3. Complete Table 1 [2.5 pts]

Joint probability
P(A, B, C, D, E, F, G) =
P(B) * P(G) * P(A|C) * P(C|B) * P(E|C) * P(D | B, G) * P(F|C, G)

Number of probability values needed to calculate joint probability
1+ 1+ 2 + 2 + 2 + 4 + 4 = **16**

16 probability values. This is because the number of probability values needed to calculate the joint probability is the sum of 2^k where k is the number of parents for each node. In general, a Bayesian network with a maximum of k parents for a node needs $O(n * 2^k)$ probabilities. In contrast, unconstrained joint distributions require $O(2^n)$ probabilities.

Relationship	**True**	**False**
A⊥B		√
A⊥B | C	√	
A⊥D		√
C⊥D |B	√	
E⊥D | B	√	
E⊥F | C	√	
A⊥F | C	√	
C⊥G	√	
A⊥E| C	√	
C⊥D|B	√	

Question 5 [3 pts]: Table 2 shows joint probability distributions between three random variables A, B, and C, and their observed values (each has binary values 1 or 0).

Table 2

A	B	C	P(A, B, C)
0	0	0	0.2
0	0	1	0.05
0	1	0	0.2
0	1	1	0.05
1	0	0	0.1
1	0	1	0.15
1	1	0	0.1
1	1	1	0.15

(1) Calculate probability value P(A=1, B=1), using only variables shown in Table 2. **[1 pt]**
(2) Calculate probability value P(A=1 | B = 1), using only variables shown in Table 2. **[1 pt]**
(3) Are B and C independent or not? Explain your answer? **[1 pt]**

P(A=1, B=1)
0.1 + 0.15 = **0.25**

P(A=1 | B=1)
(0.1 + 0.15) / (0.2 + 0.05 + 0.1 + 0.15) = (0.25) / (0.50) = **0.5**

Are B and C independent?
P(B=1, C = 1) = (0.05 + 0.15) = **0.20**
P(B=1) = (0.2 + 0.05 + 0.1 + 0.15) = 0.50
P(C=1) = (0.05 + 0.05 + 0.15 + 0.15) = 0.40
0.50 * 0.40 = **0.20**
P(B=1, C=1) = P(B=1) * P(C=1)

B and C are independent. This is because P(B=1, C=1) = P(B=1) * P(C=1). Two random variables A and B are independent if P(A, B) = P(A) P(B), which is the case for B and C. Therefore, they are independent.

Question 6 [2 pts]: Figure 4 shows a Bayesian network where X, Y, Z are three random variables. Based on the network given in the Figure 4, answer following questions:

1. Write joint distribution P(X, Y, Z) [0.5 pt]
2. Is X and Z independent or not? Explain why [0.5 pt]
3. Prove that X and Z are conditional independent given Y [1 pt]

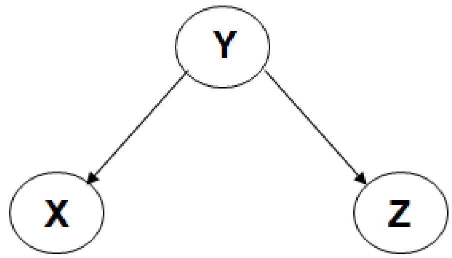

Figure 4

Joint Distribution

P(X, Y, Z) = P(X|Y) * P(Z|Y) * P(Y)

Is X and Z independent

No, X and Z are not independent. This is because in a 3-way common cause Bayesian network the two leaf nodes are not independent in general, but they are conditionally independent given Y. In such a network, X and Z are frequency associated with each other

Proof that X and Z are conditionally independent given Y

P(X|Z, Y) = P(X, Z, Y) / P(Z, Y) = (P(X|Y)P(Z|Y)P(Y)) / (P(Y)P(Z|Y)) = **P(X|Y)**

Since P(X|Z, Y) = P(X|Y), X is conditionally independent of Z given Y. This is denoted by the absence of an edge between X and Z in the Bayesian network.

Question 7 [5 pts]: A house installed an automatic water-saving sprinkler which monitors weather (r: rain or not) to turn the sprinkler on or off (s: on or off). This forms a Bayesian network shown in Figure 5 with three random variables:

r: 1/0 denoting raining (r=1) or not (r=0).

s: 1/0 denoting sprinkler is on (s=1) or off (s=0).

w: 1/0 denoting lawn is wet (w=1) or dry (w=0).

In the past, the house owner has observed following probability values:

$P(r=1)=0.1$	The probability of raining is 0.1
$P(s=1\|r=0)=0.8$	The probability of sprinkler is on given there is NO rain is 0.8
$P(s=1\|r=1)=0.3$	The probability of sprinkler is on given there is raining is 0.3
$P(w=1\|r=1,s=1)=0.9$	The probability of lawn being wet given rain and sprinkler is on
$P(w=1\|r=1,s=0)=0.8$	The probability of lawn being wet given rain and sprinkler is off
$P(w=1\|r=0,s=1)=0.6$	The probability of lawn being wet given No rain and sprinkler is on
$P(w=1\|r=0,s=0)=0.1$	The probability of lawn being wet given No rain and sprinkler is off

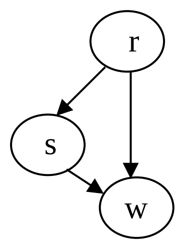

Figure 5: A Bayesian network with three random variables

Based on the above descriptions, answer following questions (show your solutions/calculations)

1. Write formula calculating joint probability value of the network P(r,s,w) [1 pt] (just write the formula, no need to show calculation)
2. Calculate joint probability of raining and the sprinkler is on: P(r=1, s=1) [1 pt]
3. Calculate joint probability of raining and the lawn is wet: P(r=1, w=1) [1 pt]
4. Calculate joint probability of No rain and the lawn is wet: P(r=0, w=1) [1 pt]
5. Calculate probability of raining, given the lawn is wet: P(r=1| w=1) [1 pt]

<u>Joint probability value of the network</u>

P(r, s, w) = P(r) * P(s|r) * P(w|r,s)

<u>Joint probability of raining and sprinkler on</u>

P(X, Y) = P(Y|X) * P(X)

P(r=1, s=1) = P(s=1|r=1) * P(r=1) = 0.3 * 0.1 = **0.03**

Joint probability of raining and the lawn is wet

$P(r=1, w=1, s) = P(r=1, w=1, s=1) + P(r=1, w=1, s=0)$
$P(r=1, w=1, s=1) = P(r=1) * P(s=1|r=1) * P(w=1|r=1, s=1) = 0.1 * 0.3 * 0.9 = 0.027$
$P(r=1, w=1, s=0) = P(r=1) * P(s=0|r=1) * P(w=1|r=1, s=0) = 0.1 * 0.7 * 0.8 = 0.056$
$0.027 + 0.056 = 0.083$

0.083

Joint probability of no rain and the lawn is wet

$P(r=0, w=1, s) = P(r=0, w=1, s=1) + P(r=0, w=1, s=0)$
$P(r=0, w=1, s=1) = P(r=0) * P(s=1|r=0) * P(w=1|r=0, s=1) = 0.9 * 0.8 * 0.6 = 0.432$
$P(r=0, w=1, s=0) = P(r=0) * P(s=0|r=0) * P(w=1|r=0, s=0) = 0.9 * 0.2 * 0.1 = 0.018$
$0.432 + 0.018 = 0.45$

0.45

Probability of rain, given the lawn is wet

$P(r=1| w=1) = P(r=1, w=1)/P(w=1) = 0.083 / P(w=1)$
$P(w=1) = P(w=1 | r=0, s=0) * P(r=0, s=0) + P(w=1 | r=0, s=1) * P(r=0, s=1) + P(w=1 | r=1, s=0) * P(r=1, s=0) + P(w=1 | r=1, s=1) * P(r=1, s=1)$

$P(r=0, s=1) = P(s=1|r=0) * P(r=0) = 0.8 * 0.9 = 0.72$

$P(w=1 | r=0, s=0) * P(r=0, s=0) = 0.1 * 0.18 = 0.018$
$P(w=1 | r=0, s=1) * P(r=0, s=1) = 0.6 * 0.72 = 0.432$
$P(w=1 | r=1, s=0) * P(r=1, s=0) = 0.8 * 0.07 = 0.056$
$P(w=1 | r=1, s=1) * P(r=1, s=1) = 0.9 * 0.03 = 0.027$

$0.018 + 0.432 + 0.056 + 0.027 = 0.533$

$0.083 / 0.533 = 0.1557$

0.1557