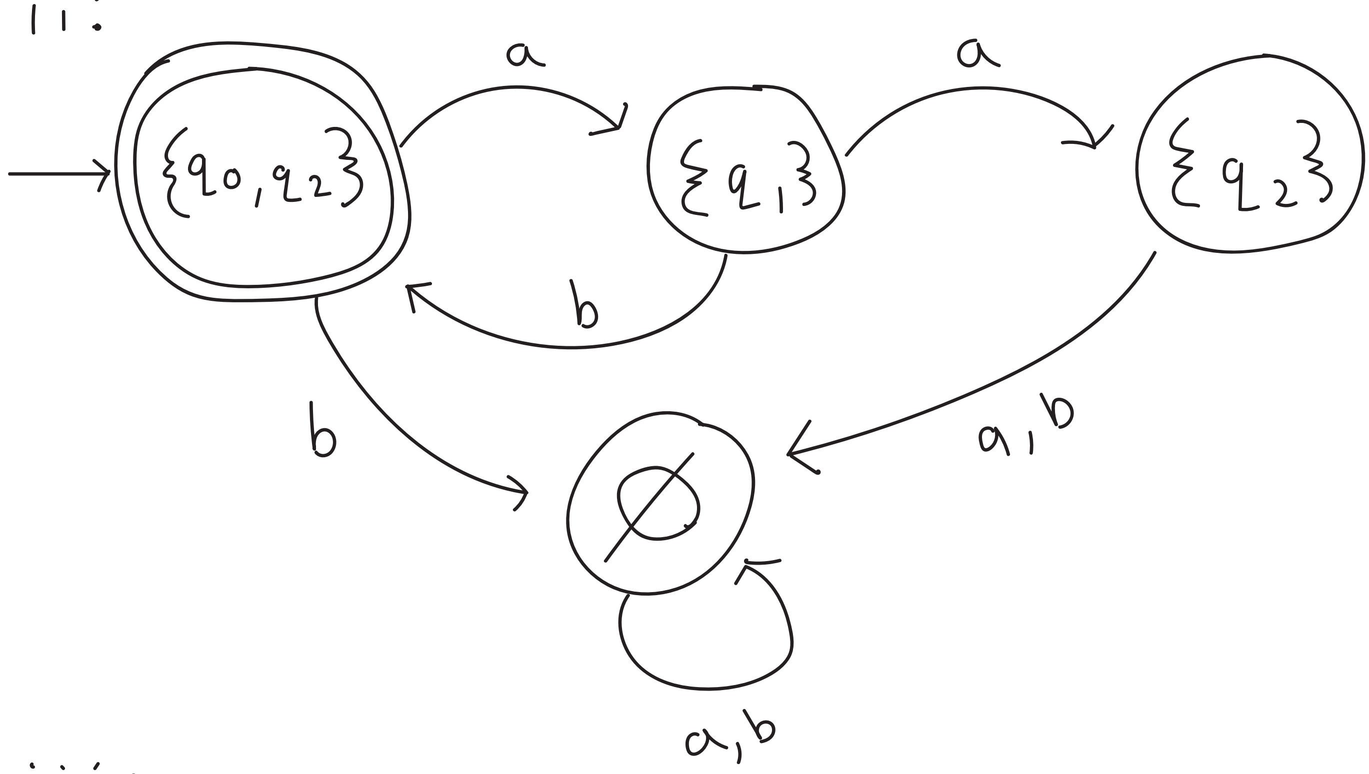
Hw 3 Matthew Acs 2/23/2021

Problem 1)

i: $L = \{(ab)^n : n \geq 0\}$

ii:



iii:

$$Q - F = \{\{q_1\}, \{q_2\}, \varnothing\} \quad F = \{\{q_0, q_2\}\}$$

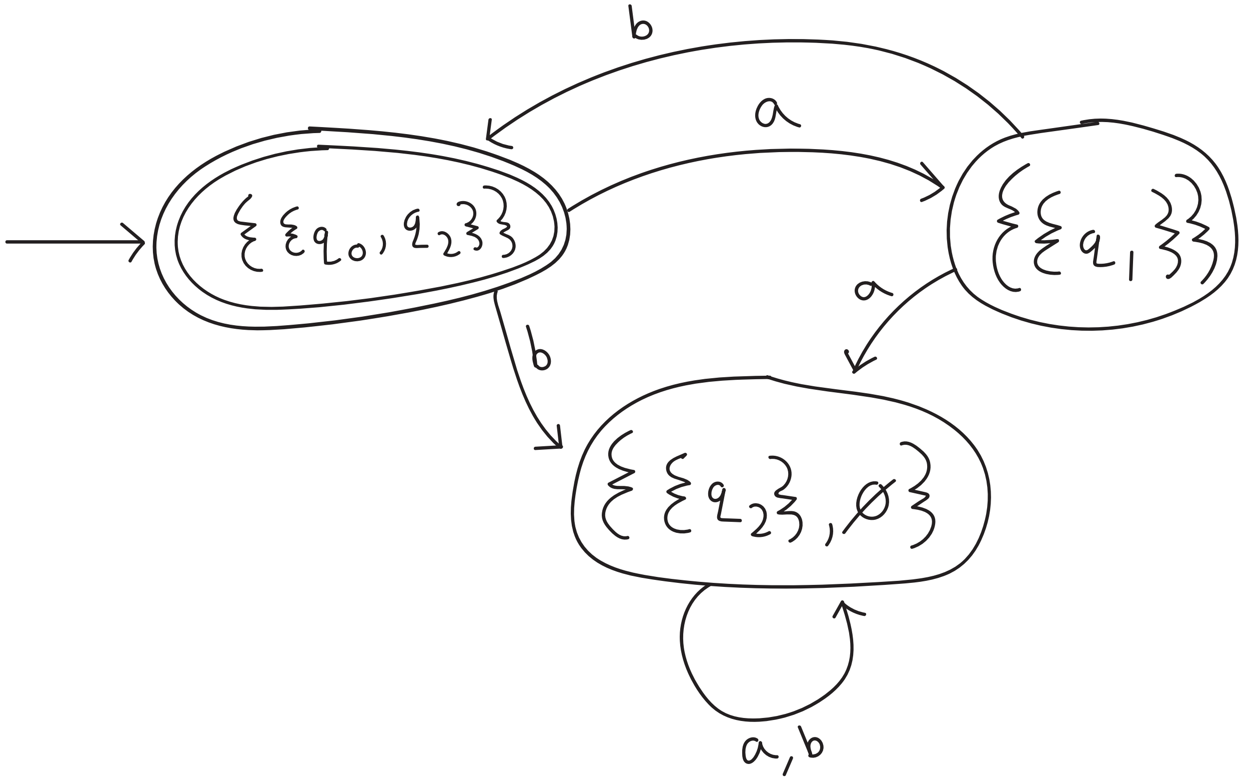
$$\left.\begin{array}{l} \delta(\{q_1\}, b) = \{q_0, q_2\} \in F \\ \delta(\{q_2\}, b) = \varnothing \notin F \end{array}\right] \Rightarrow \{q_1\}, \{q_2\} \text{ distinguishable}$$

$$\{\{q_1\}\}, \{\{q_2\}, \varnothing\}$$

iii (continued):

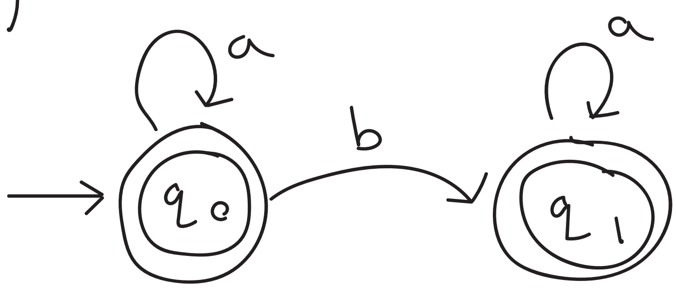
$\{q_2\}$ and $\emptyset$ are indistinguishable

Partition: $\{\{q_1\}\}, \{\{q_2\}, \emptyset\}, \{\{q_0, q_2\}\}$

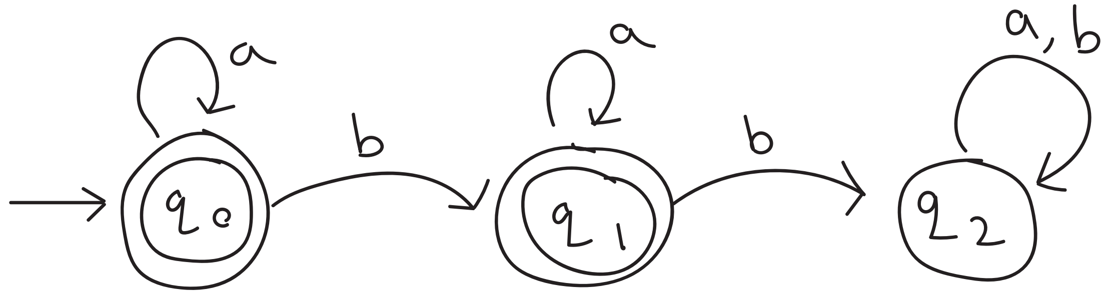


Problem 2)

NFA:



DFA:



Regular expression: $r_1 = a^*(b+\lambda)a^*$

Right linear grammar: $G(\{S, A\}, \{a, b\}, S, P)$

P: $S \to aS$

$S \to bA$

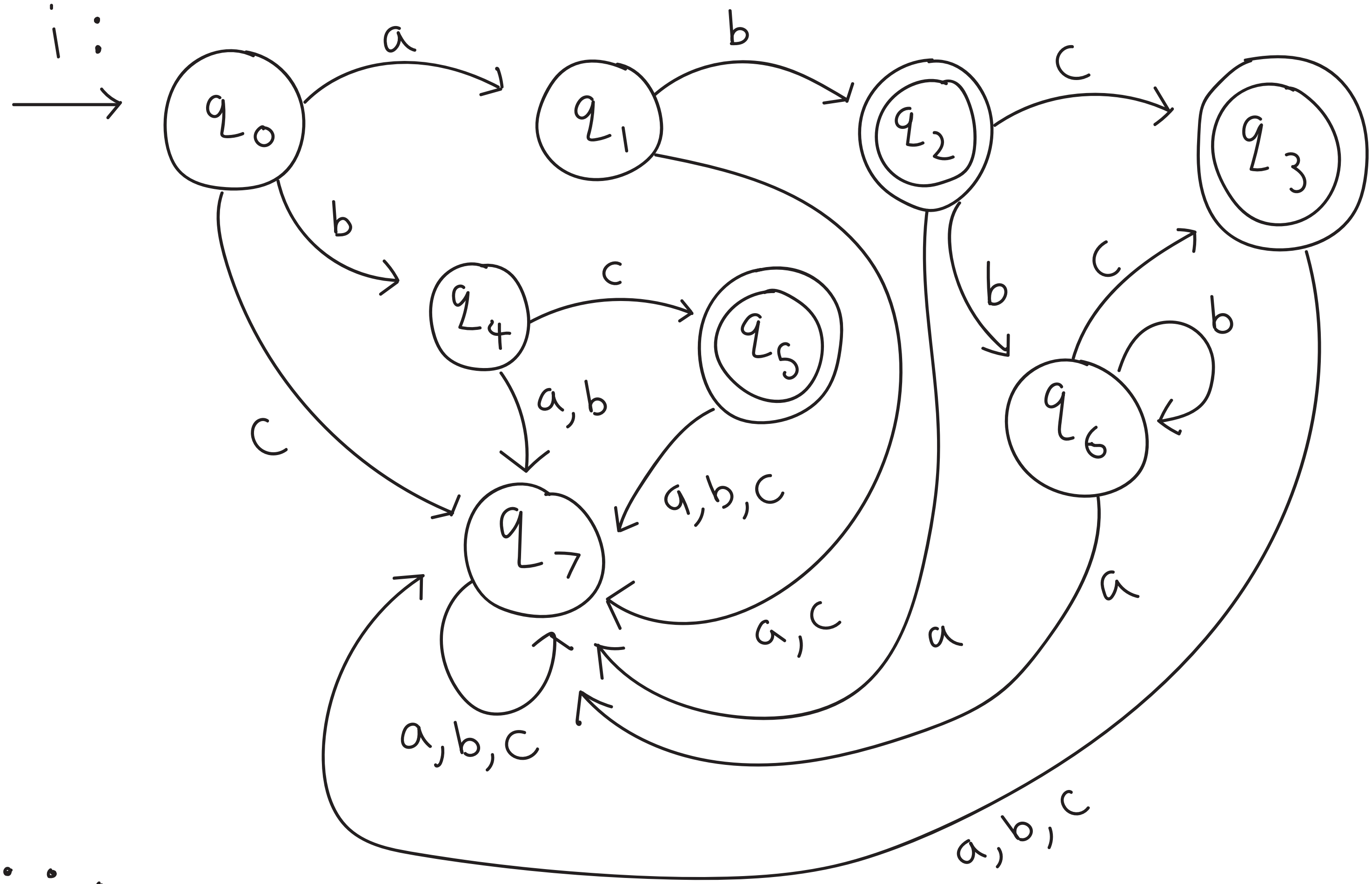
$S \to \lambda$

$A \to aA$

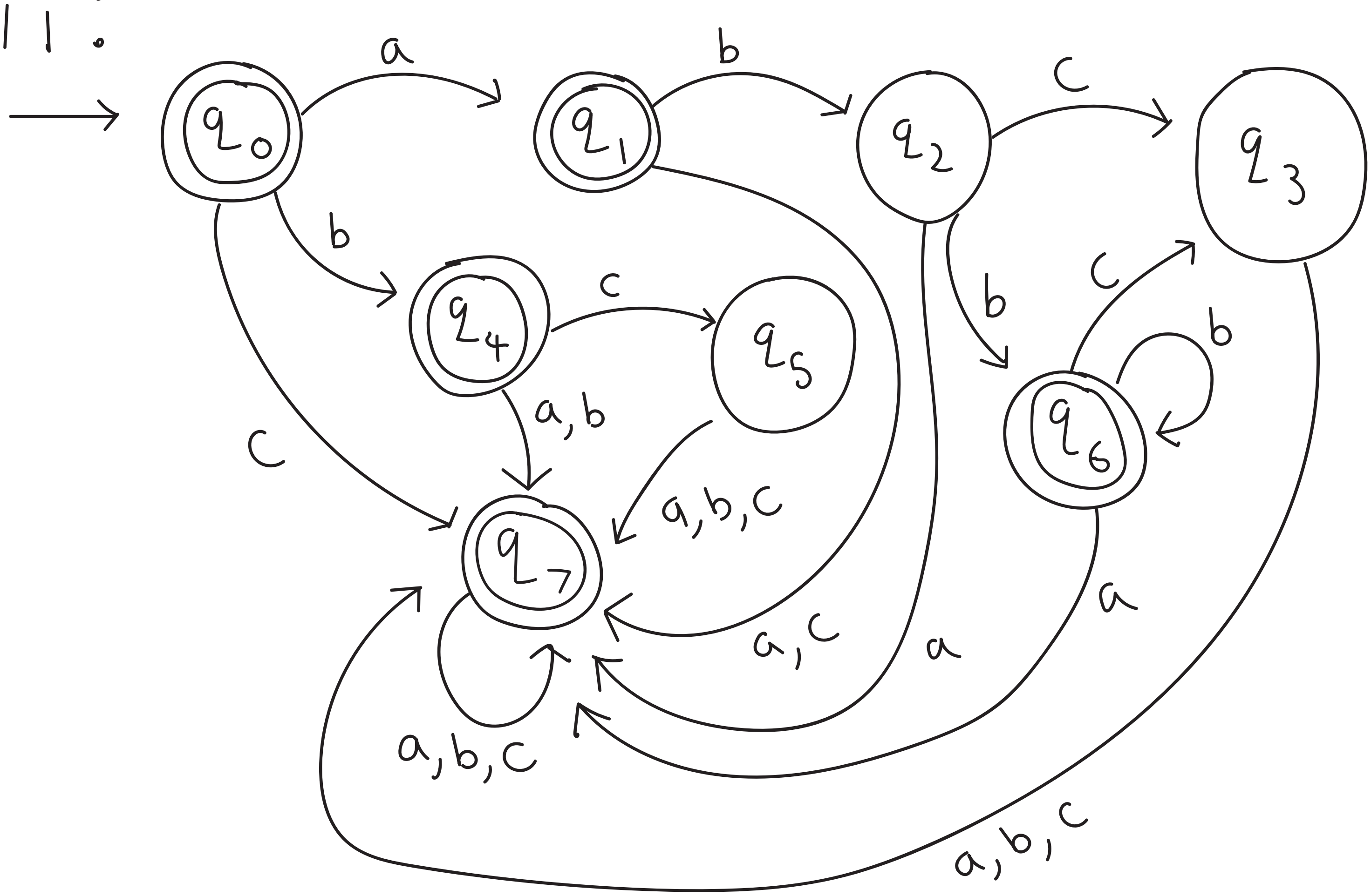
$A \to \lambda$

Problem 3)

i:

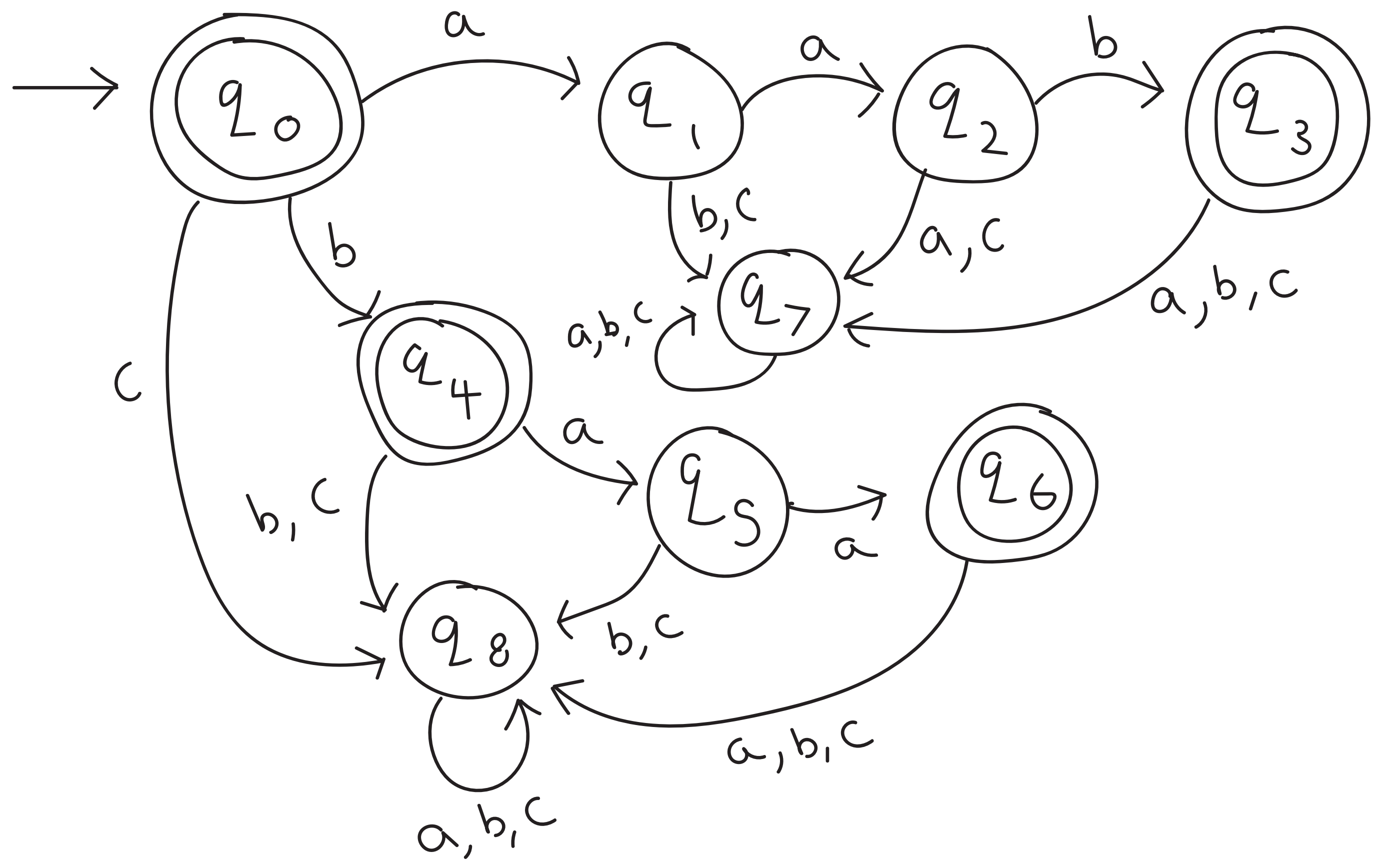


ii:

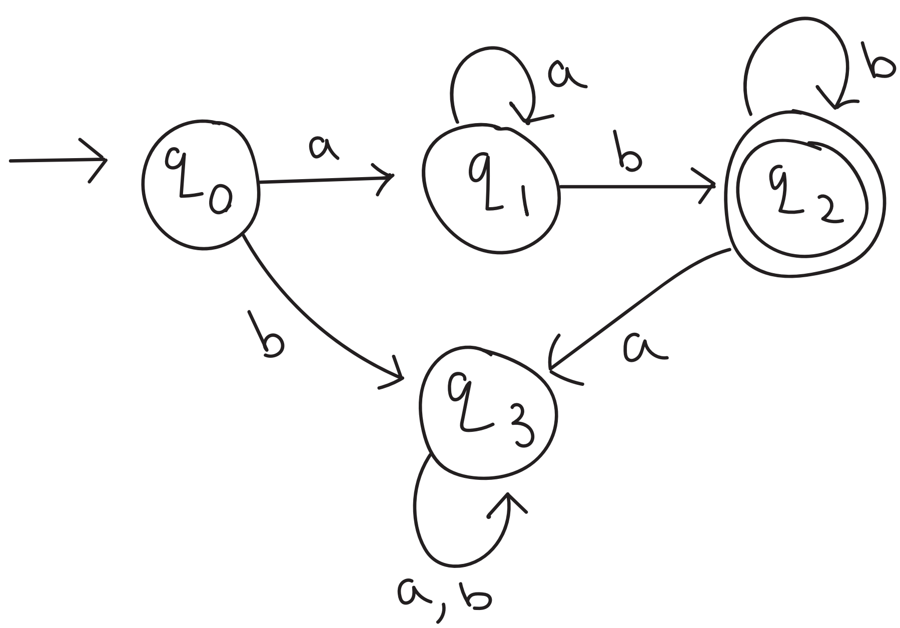


Problem 4)

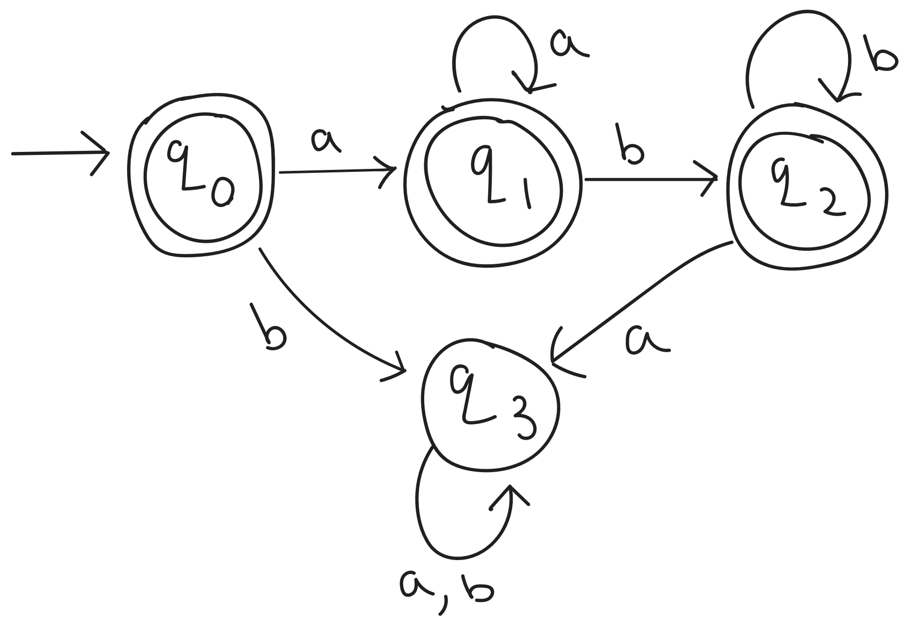
a: $L_1 = \{aab, baa, b, \lambda\}$



b: $L_2 = \{a^n b^m : n \geq 1, m \geq 1\}$

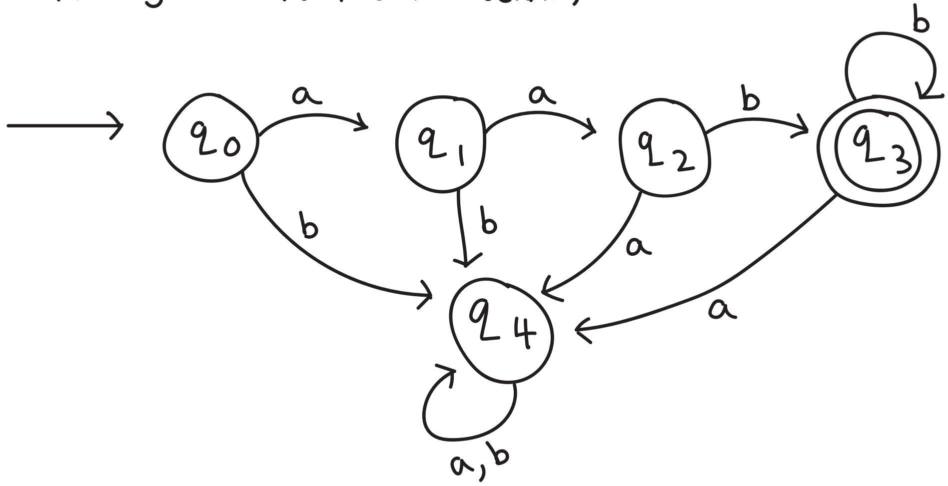


c :

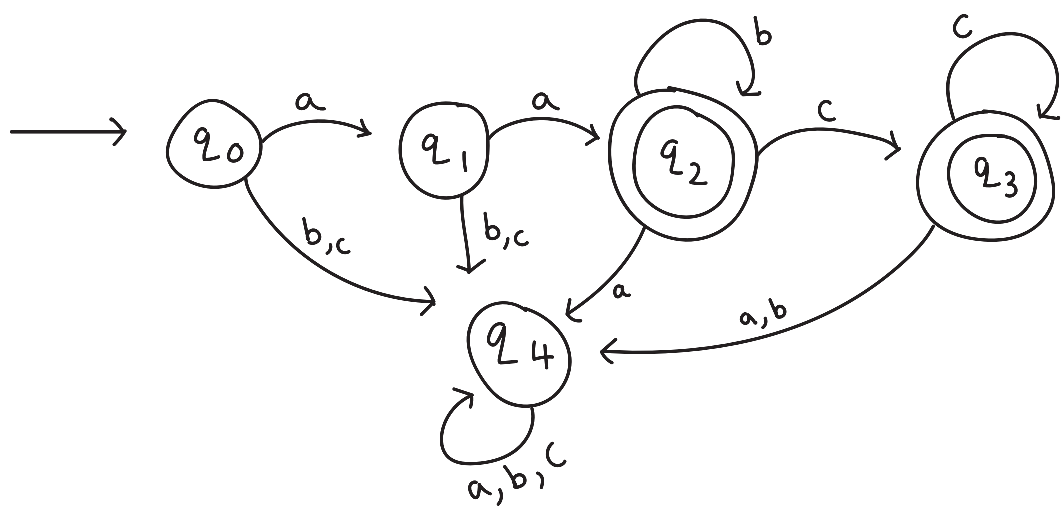


Problem 5)

a : The Language is regular because I can create a DFA that represents the language and any languge that can be represented by a DFA is regular. (DFA shown below)



b : The Language is regular because I can create a DFA that represents the language and any languge that can be represented by a DFA is regular. (DFA shown below)



C: The language is nonregular. This is because the language provides a contradiction if it assumed to be regular and the pumping lemma is applied. (Proof below)

Proof: $\underbrace{W = a^2 b^n c^n, \ W \in L_3}_{\text{"PICK"}}$ Assume that L_3 is infinite regular

$\underbrace{a^2 b^n | c^n}_{\text{"CUT"}} \longrightarrow \underbrace{a^2 b^{(n-k)} b^k | c^n}_{\text{"SPLIT"}}$ for $k \geq 1$, $k < n$

$\underbrace{a^2 b^{(n-k)} (b^k)^2 | c^n}_{\text{"PUMP WITH } i=2\text{"}} = a^2 b^{(n+k)} c^n$ $\quad \underbrace{a^2 b^{(n+k)} c^n \in L_3}_{\text{Contradiction because } n+k \neq n}$

$\therefore$ L_3 is not a regular language because it provided a contradiction on the pumping lemma based on the assumption that the language is regular, making the assumption false.

d : The language is nonregular. This is because the language provides a contradiction if it assumed to be regular and the pumping lemma is applied. (Proof below)

Proof: $\underbrace{W = a^2 b^n c^n, \; W \in L_4}_{\text{"PICK"}}$ Assume that L_4 is infinite regular

$\underbrace{a^2 b^n | c^n}_{\text{"CUT"}} \longrightarrow \underbrace{a^2 b^{(n-k)} b^k | c^n}_{\text{"SPLIT"}}$ for $k \geq 1, k < n$

$\underbrace{a^2 b^{(n-k)} (b^k)^2 | c^n}_{\text{"PUMP WITH } i=2\text{"}} = a^2 b^{(n+k)} c^n$ $\quad \underbrace{a^2 b^{(n+k)} c^n \in L_4}_{\text{Contradiction because } n+k \neq n}$

$\therefore$ L_4 is not a regular language because it provided a contradiction on the pumping lemma based on the assumption that the language is regular, making the assumption false.

e : The language is nonregular. This is because the language provides a contradiction if it assumed to be regular and the pumping lemma is applied. (Proof below)

Proof : $W = \underbrace{a^2(ab)^n(a^2c)^n}_{\text{"PICK"}}$, $W \in L_5$ Assume that L_5 is infinite regular

$$\underbrace{a^2(ab)^n|(a^2c)^n}_{\text{"CUT"}} \longrightarrow \underbrace{a^2(ab)^{(n-k)}(ab)^k|(a^2c)^n}_{\text{"SPLIT"}} \quad \text{for } k \geq 1,\ k < n$$

$$\underbrace{a^2(ab)^{(n-k)}((ab)^k)^2|(a^2c)^n}_{\text{"PUMP WITH } i=2\text{"}} = a^2(ab)^{n+k}(a^2c)^n \quad \underbrace{a^2(ab)^{n+k}(a^2c)^n \in L_5}_{\text{Contradiction because } n+k \neq n}$$

$\therefore$ L_5 is not a regular language because it provided a contradiction on the pumping lemma based on the assumption that the language is regular, making the assumption false.