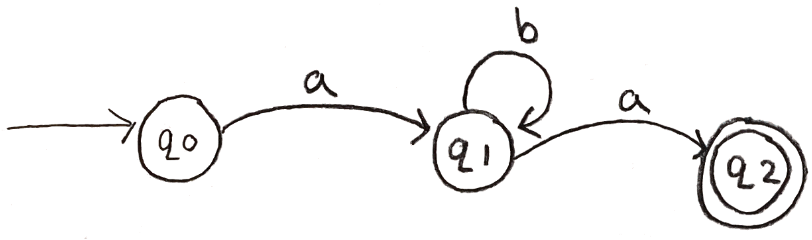
HW # 5 Matthew AcS

Problem 1:



The language is regular because there exists an NFA that represents the language and any language that can be represented by an NFA is regular.

Problem 2:

$G = (V, T, S, P)$

$V = \{S, A\}$ $T = \{a, b\}$

P:

$S \to aA$

$A \to aAb$

$A \to \lambda$

The language is linear because there exists a linear grammar that represents the language and any language that can be represented by a linear grammar is linear.

Problem 3:

Pumping lemma for linear languages

Pick) $w = a^{m/2} b^{m-2} a^{m/2} \in L$, $|w| \geq m$, assume L is linear

Decompose) $\underbrace{a^{m/4}}_{u} \underbrace{a^{m/4}}_{v} \underbrace{b^{m-2}}_{x} \underbrace{a^{m/4}}_{y} \underbrace{a^{m/4}}_{z}$

Pump i=2) $a^{m/4}(a^{m/4})^2 b^{m-2} (a^{m/4})^2 a^{m/4} = a^{m/4} a^{m/2} b^{m-2} a^{m/2} a^{m/4}$

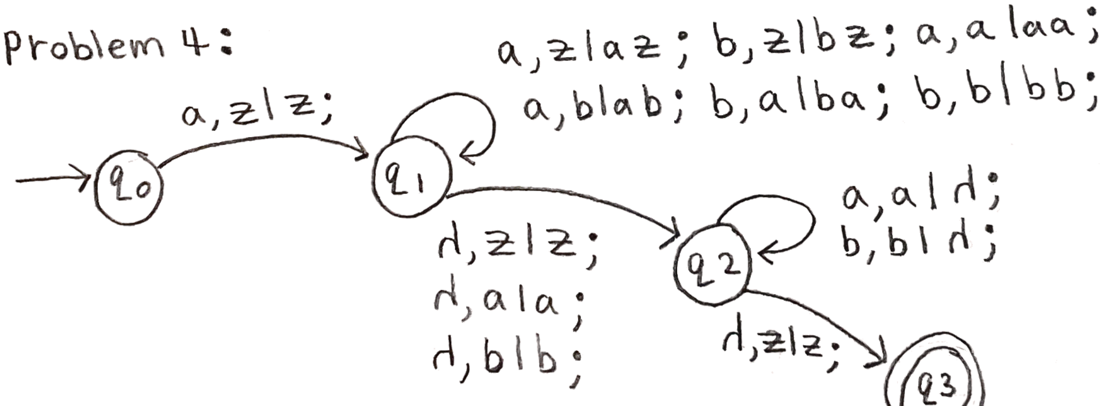
$= a^{3m/4} b^{m-2} a^{3m/4} = a^{6m/4} b^{m-2} = a^{3m/2} b^{m-2}$

$a^{\frac{3m}{2}} b^{m-2} \in L$

Contradiction because the $n_a(w) \neq n_b(w)+2$

∴ The language is not linear because it provides a contradiction under the pumping lemma for linear languages

Problem 4:



The language is context-free because there is an NPDA that represents the language and any language that can be represented by an NPDA is context-free.

Problem 5:

Pumping lemma for Context-free languages

Pick) $W = a a^m b^m a^m b^m \in L$, $|W| \geq m$, assume L is Context-free

Decompose) $\underset{u}{a a^m} | \underset{vxy}{b^m} | \underset{z}{a^m b^m} = \underset{u}{a a^m} | \underset{vxy}{b^{k_1} b^{m-k_1-k_2} b^{k_2}} | \underset{z}{a^m b^m}$

Pump $i=2$) $a a^m | (b^{k_1})^2 b^{m-k_1-k_2} (b^{k_2})^2 | a^m b^m$

$= a a^m | b^{2k_1} b^{m-k_1-k_2} b^{2k_2} | a^m b^m$

$= a a^m b^{m+k_1+k_2} a^m b^m$

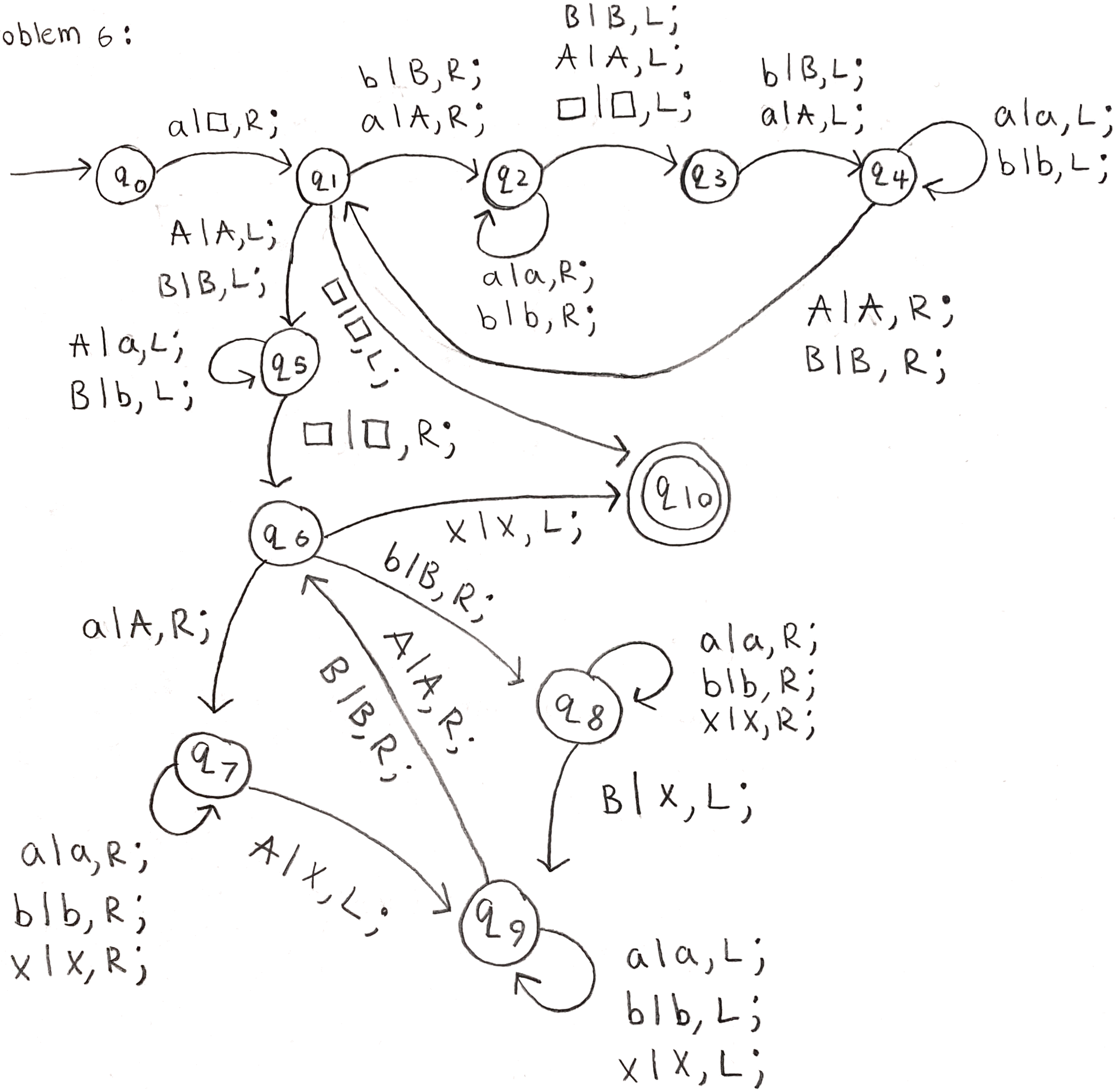
$a a^m b^{m+k_1+k_2} a^m b^m \in L$

↑

Contradiction

$\therefore$ The language is not Context-free because it provides a Contradiction under the pumping lemma for Context-free languages

Problem 6:



The Turing machine recognizes strings from language L if they are input onto the tape in the form:

|□|a|w|w|□|, where aww is the string.