COT 4420 - Formal Languages and Automata Theory
Midterm Exam - February 25, 2021
Instructor: Dimitris A. Pados

Student Name: Matthew Acs

Duration: 1 hour and 20 minutes

Proctoring: Students off-line; open text book and class notes only

Student Attestation and Signature: I followed all proctoring instructions. I neither gave nor received help during the exam. Matthew Acs

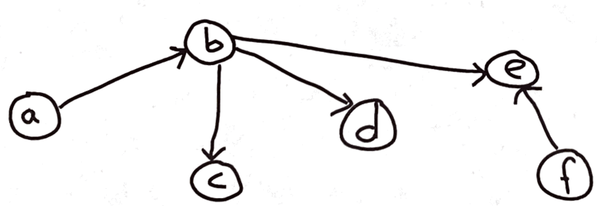
Problem 1 (10%) - Set Theory and Operations
Say that for this problem the universal set is $U = \{a, b, c, d, e\}$. If $A = \{a\}$ and $B = \{b\}$, calculate:

(i) (5%) $\overline{A \cap B} = \overline{A} \cup \overline{B} = \{b,c,d,e\} \cup \{a,c,d,e\}$
$= \{a,b,c,d,e\}$

(ii) (5%) $\overline{A} \cup \overline{B} =$

$\{b,c,d,e\} \cup \{a,c,d,e\} = \{a,b,c,d,e\}$

Problem 2 (10%) - Graphs
Consider the following directed graph:



(i) (5%) Identify a walk, a path and a simple path.

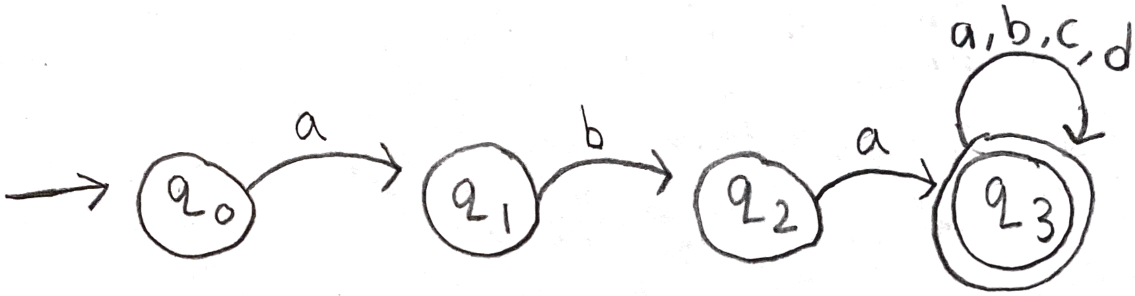
Walk: (a,b),(b,e)   Path: (a,b),(b,d)

Simple path: (a,b),(b,c)

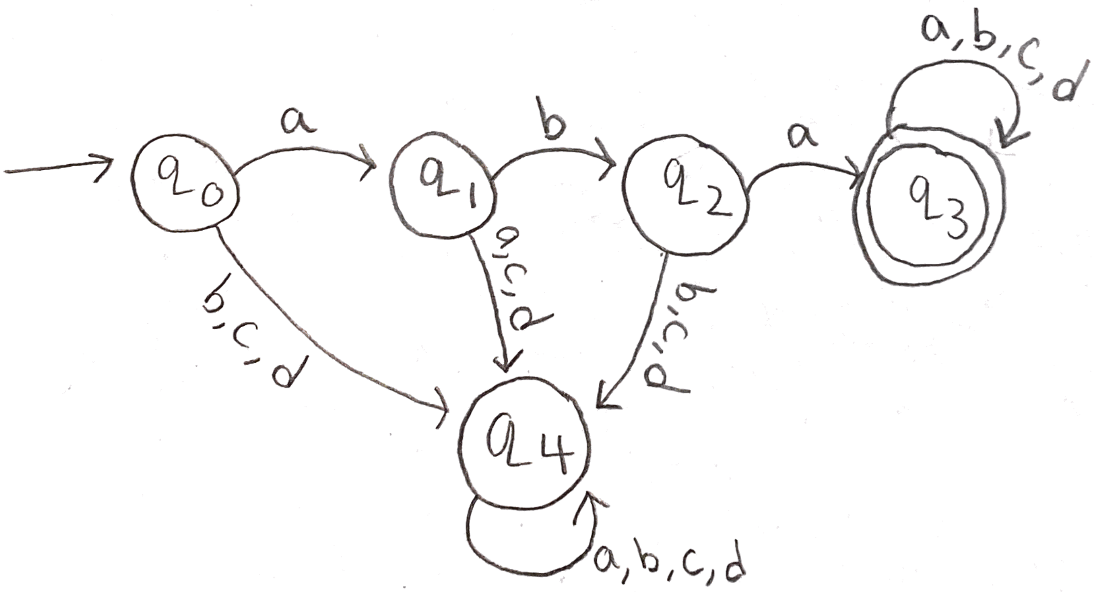
(ii) (5%) Is this graph a tree? Why?

No, because there is no node from which there is exactly one path to every other node. In other words, there is no root.

Problem 3 (10%) - Non-determinstic Finite Acceptors (NFAs)
Consider the alphabet $\Sigma = \{a, b, c, d\}$. Design an NFA that accepts all strings with prefix *aba*.



Problem 4 (10%) - Determinstic Finite Acceptors (DFAs)
Design a DFA for the language in Problem 3.

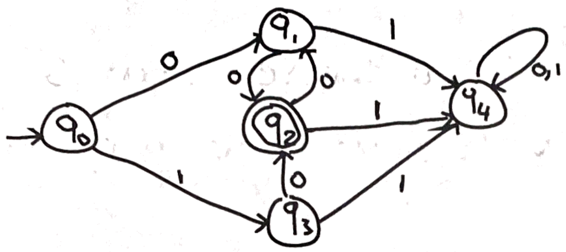


Problem 5 (10%) - Regular Expressions
Give a regular expression for the language in Problem 3.

$r_1 = aba(a+b+c+d)^*$.

Problem 6 (10%) - DFAs and Regular Expressions

(i) (5%) Simplify the DFA below.



$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$F = \{q_2\}$

$F = \{q_2\} \quad Q - F = \{q_0, q_1, q_3, q_4\}$

$$\left.\begin{array}{l}\delta(q_0, 0) = q_1 \notin F \\ \delta(q_1, 0) = q_2 \in F\end{array}\right\} \Rightarrow q_0, q_1 \text{ are distinguishable}$$

$\{q_2\}, \{q_0\}, \{q_1, q_3, q_4\}$

$$\left.\begin{array}{l}\delta(q_3, 0) = q_2 \in F \\ \delta(q_4, 0) = q_4 \notin F\end{array}\right\} \Rightarrow q_3, q_4 \text{ are distinguishable}$$

(continued on seperate paper)

(ii) (5%) Write a regular expression for the language of this given DFA.

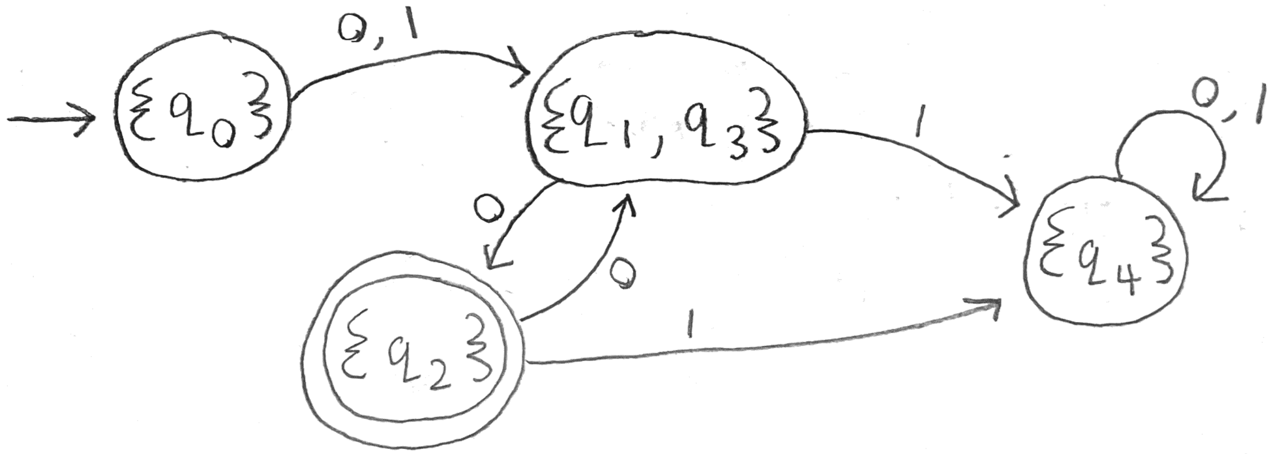
$r_1 = (0+1)(0)(00)^*$

Problem 6 part i continued)

$\{q_2\}, \{q_0\}, \{q_4\}, \{q_1, q_3\}$

q_1 and q_3 are indistinguishable because with 0 they both go to final states and with 1 they both go to non-final states. Thus the "output" partition is:

$\{q_2\}, \{q_0\}, \{q_4\}, \{q_1, q_3\}$



Problem 7 (15%) - Regular versus Non-regular Languages

(i) (5%) Is the language $L = \{a, abb, abbb, \lambda\}$ regular or non-regular? Why? A formal statement/proof is needed.

Language L is regular because a theorem states that all finite languages are regular and language L is indeed a finite language.

(ii) (5%) Is the language $L = \{(ab)^n(cd)^n : n \geq 0\}$ regular or non-regular? Why? A formal statement/proof is needed.

Language L is non-regular because if it is assumed to be regular and the pumping lemma is applied, a contradiction occurs.

(Continued on seperate paper)

(iii) (5%) Is the language $L = \{(ab)^n(cd)^n d^2 : n \geq 0\}$ regular or non-regular? Why? A formal statement/proof is needed.

Language L is non-regular because if it is assumed to be regular and the pumping lemma is applied, a contradiction occurs.

(Continued on seperate paper)

Problem 7 part ii continued)

$\underbrace{w = (ab)^n (cd)^n \in L}_{\text{"Pick"}}$, Assume that L is infinite regular

$\underbrace{(ab)^n \mid (cd)^n}_{\text{"Cut"}} \rightarrow \underbrace{(ab)^{n-k}(ab)^k \mid (cd)^n}_{\text{"Split"}}$ for $k \geq 1, k < n$

$\underbrace{(ab)^{n-k}((ab)^k)^2 \mid (cd)^n}_{\text{"Pump with } i=2\text{"}} = (ab)^{n+k}(cd)^n$

$\underbrace{(ab)^{n+k}(cd)^n \in L}$

Contradiction because $n+k \neq n$

Problem 7 part iii continued)

$\underbrace{w = (ab)^n (cd)^n d^2 \in L}_{\text{"Pick"}}$, Assume that L is infinite regular

$\underbrace{(ab)^n \mid (cd)^n d^2}_{\text{"Cut"}} \rightarrow \underbrace{(ab)^{n-k}(ab)^k \mid (cd)^n d^2}_{\text{"Split"}}$ for $k \geq 1, k < n$

$\underbrace{(ab)^{n-k}((ab)^k)^2 \mid (cd)^n d^2}_{\text{"Pump with } i=2\text{"}} = (ab)^{n+k}(cd)^n d^2$

$\underbrace{(ab)^{n+k}(cd)^n d^2 \in L}$

Contradiction because $n+k \neq n$

Problem 8 (10%) - Pumping Lemma
Assume that the alphabet is $\Sigma = \{a, b\}$. Use the Pumping Lemma to prove that the language over the alphabet Σ that contains all strings with equal number of a's and b's is non-regular.

$L = \{a^n b^n : n \geq 0\}$ $\underbrace{W = a^n b^n \in L}_{\text{"pick"}}$, Assume L to be infinite regular

$\underbrace{a^n | b^n}_{\text{"cut"}} \rightarrow \underbrace{a^{n-k} a^k | b^n}_{\text{"split"}}$ for $k \geq 1,\ k < n$

$\underbrace{a^{n-k} (a^k)^2 | b^n}_{\text{"pump with } i=2\text{"}} = a^{n+k} b^n$ $\quad \underbrace{a^{n+k} b^n \in L}_{\text{Contradiction because } n \neq n+k}$

$\therefore$ L is non regular

Problem 9 (10%) - Grammars
Consider the language $L = \{abb^n(ab)^n : n \geq 0\}$.
(i) (5%) Is this a regular or non-regular language? Why?

L is a non-regular language because if it is assumed to be regular and the pumping lemma is applied, a contradiction will occur. (continued on seperate page)

(ii) (5%) Give a grammar G with $L(G) = L$.

$G = (\{S, A\}, \{a, b\}, S, P)$

P:
$S \rightarrow abA$
$A \rightarrow \lambda$
$A \rightarrow bAab$

Problem 10 (15% - includes 10% bonus) - Grammars

Consider the grammar $G = \{\{S\}, \{a, b, c\}, S, P\}$ with production rules
$S \to abSc$
$S \to \lambda$.

(i) (5%) What type of grammar is this? Be as specific as possible. Generate two strings in $L(G)$.

The grammar G is a Contex free - linear grammar. It is not a regular grammar because it is not right or left linear.

$S \Rightarrow abSc \Rightarrow ababScc \Rightarrow abababSccc \Rightarrow abababccc$

$S \Rightarrow abSc \Rightarrow ababScc \Rightarrow ababcc$

(ii) (Bonus 5%) What language does G represent? That is, what is $L(G)$?

$L(G) = \{(ab)^n c^n : n \geq 0\}$

(iii) (Bonus 5%) What type of language is $L(G)$? Explain fully.

L(G) is a nonregular language. This is because if the language is assumed to be regular and the pummping lemma is applied, a contradiction will occur. Also, the language is a Context-free language because it was produced by a Context-free grammar. (Continued on Seperate page)

Problem 9 part i continued)

$\underbrace{W = ab\,b^n (ab)^n \in L}_{\text{"Pick"}}$, Assume L to be infinite regular

$\underbrace{ab\,b^n \mid (ab)^n}_{\text{"Cut"}} \rightarrow \underbrace{ab(b^{n-k})\,b^k \mid (ab)^n}_{\text{"Split"}}$ for $k \geq 1, k < n$

$\underbrace{ab(b^{n-k})(b^k)^2 \mid (ab)^n}_{\text{"Pump with } i=2\text{"}} = ab\,b^{n+k}(ab)^n$

$\underbrace{ab\,b^{n+k}(ab)^n \in L}$

Contradiction because $n+k \neq n$

Problem 10 part iii continued)

$\underbrace{W = (ab)^n c^n \in L}_{\text{"Pick"}}$, Assume L to be infinite regular

$\underbrace{(ab)^n \mid c^n}_{\text{"Cut"}} \rightarrow \underbrace{(ab)^{n-k}(ab)^k \mid c^n}_{\text{"Split"}}$ for $k \geq 1, k < n$

$\underbrace{(ab)^{n-k}((ab)^k)^2 \mid c^n}_{\text{"Pump with } i=2\text{"}} = (ab)^{n+k} c^n$

$\underbrace{(ab)^{n+k} c^n \in L}$

Contradiction because $n+k \neq n$