**COT 4420 - Formal Languages and Automata Theory**
Final Exam - April 21, 2021
Instructor: Dimitris A. Pados

Student Name: Matthew Acs

Duration: 1 hour and 45 minutes

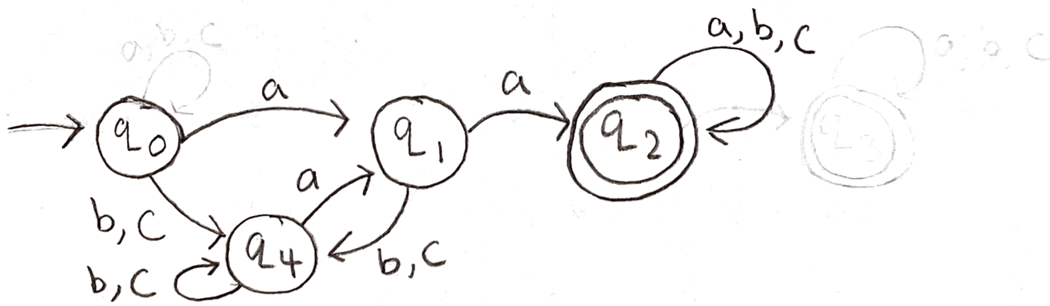
Proctoring: Students off-line; open text book and class notes only

Student Attestation and Signature: I followed all proctoring instructions. I neither gave nor received help during the exam. Matthew Acs

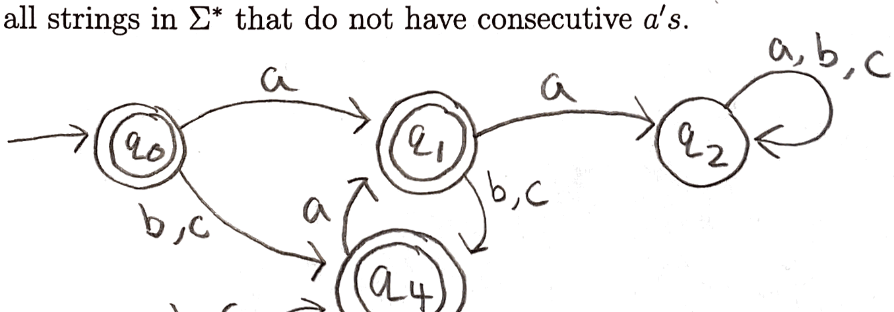
**Problem 1 (15%)**
You are given the alphabet $\Sigma = \{a, b, c\}$. Consider the language $L_1$ that contains all strings in $\Sigma^*$ that have at least one pair of consecutive $a's$.
(i) (4%) Give a regular expression for the language $L_1$.

$L_1 = (a+b+c)^*(aa)(a+b+c)^*$

(ii) (5%) Design a deterministic finite acceptor (dfa) for $L_1$.




(iii) (3%) Design a deterministic finite acceptor (dfa) for the language $L_2$ defined as the set of all strings in $\Sigma^*$ that do not have consecutive $a's$.




(iv) (3%) Where do $L_1$ and $L_2$ fall within the Chomsky Hierarchy of Languages and why? (See Appendix on the last page of the test.)

$L_1$ and $L_2$ fall within the regular language portion of the Champsky Hierarchy because they can be represented by DFAs and any language that can be represented by a DFA is regular.

**Problem 2 (15%)**

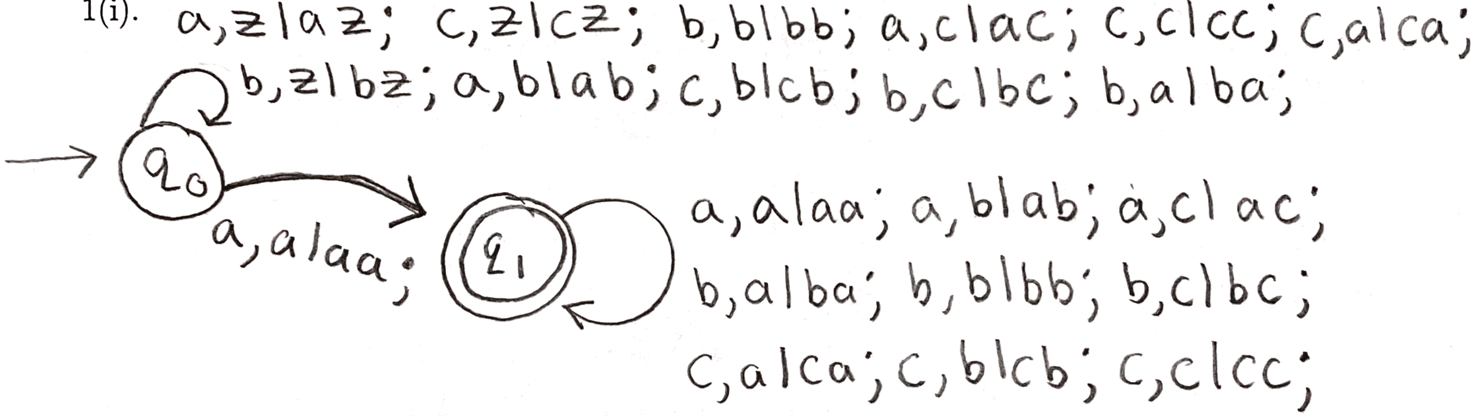
(i) (5%)Is it true that every regular language is deterministic context-free (dcf)? Yes or No (circle your answer.)

Yes, every regular language is DCF.

(ii) (5%) Give a formal argument/proof of your answer in Problem 2(i) above.

Every regular language is DCF because in the Chompsky hierarchy regular languages are a subclass of DCF languages. Also, a DPDA can be used to emulate a DFA, and DFAs represent all regular languages.

(iii) (5%) Design a deterministic pushdown automaton (dpda) for the language $L_1$ in Problem 1(i).




**Problem 3 (15%)**

(i) (5%) Show formally that the language $L_3 = \{a^{n+1}b^n : n \geq 0\}$ is not regular.

The language $L_3$ can be shown to be non-regular using the pumping lemma for regular languages. (use of pumping lemma shown on next page)

(ii) (3%) Write a grammar for this language (i.e., give the production rules.)

$G = \{V, T, S, P\}$ $V = \{S, A\}$ $T = \{a, b\}$ $L(G) = L_3$

$P$: $S \to aA$

$A \to aAb \mid \lambda$

Problem 3 part i Continued

The language is nonregular. This is because the language provides a contradiction if it is assumed to be regular and the pumping lemma is applied.

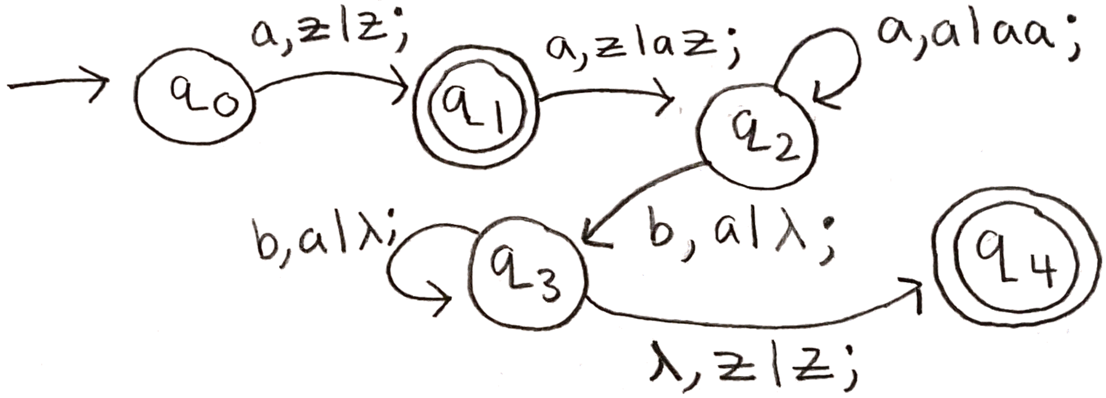
$\underbrace{w = a^{n+1} b^{n},\ w \in L_3}_{\text{"Pick"}}$ Assume that $L_3$ is regular

$\underbrace{a^{n+1} \mid b^{n}}_{\text{"Cut"}} \rightarrow \underbrace{a^{n-k+1} a^{k} \mid b^{n}}_{\text{"Split"}}$ for $k \geq 1,\ k < n$

$\underbrace{a^{n-k+1} (a^{k})^{2} \mid b^{n}}_{\text{"Pump with } i=2\text{"}} = a^{n+k+1} b^{n}$ $\underbrace{a^{n+k+1} b^{n} \in L_3,}_{\text{Contradiction because } n+k+1 \neq n+1}$

∴ $L_3$ is not a regular language because it provided a contradiction on the pumping lemma based on the assumption that the language is regular, making the assumption false.

(iii) (5%) Design a deterministic pushdown automaton (dpda) for the language $L_3$.




(iv) (2%) Place the language $L_3$ within the Chomsky taxonomy (see Appendix on the last page of the test) and justify formally your placement.

$L_3$ is in the Linear and DCF position because there exists a linear grammar and a DPDA that describes the language, however it is not regular (as shown via the pumping lemma).

**Problem 4 (15%)**

(i) (6%) Consider the grammmar $G$ with alphabet $\{0, 1\}$ and production rules $S \to 0SS \,|\, 0 \,|\, 1$. Convert the grammar to a Chomsky normal form.

$\hat{P}$: $S \to N_0 W_1$  $S \to 1$  $W_1 \to SS$  $N_0 \to 0$  $S \to 0$

$\hat{G} = \{\hat{V}, T, S, \hat{P}\}$  $L(G) = L(\hat{G})$

$\hat{V} = \{S, N_0, W_1\}$

(ii) (9%) Run below the Chomsky membership algorithm to see if the string 0011 is in the language $L(G)$.

$w = 0011$

$V_{11}$ $\{S, N_0\}$  $V_{22}$ $\{S, N_0\}$  $V_{33}$ $\{S\}$  $V_{44}$ $\{S\}$

$\therefore w \notin L(G)$

$V_{12}$ $\{W_1\}$  $V_{23}$ $\{W_1\}$  $V_{34}$ $\{W_1\}$

$S \notin V_{1n}$

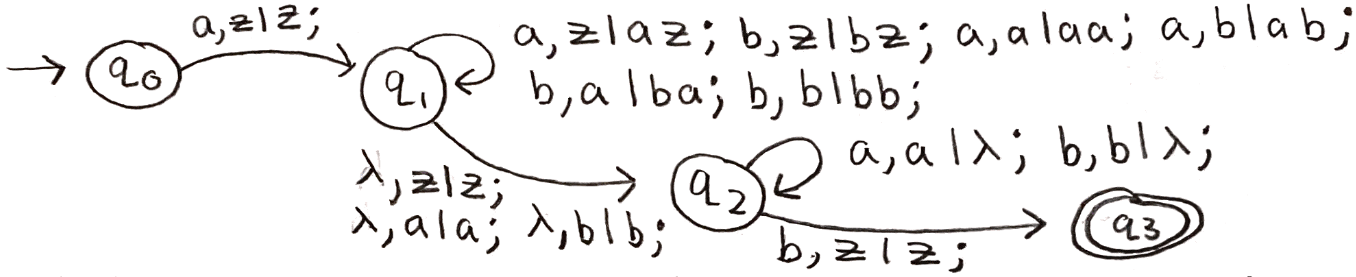
$V_{13}$ $\{S\}$  $V_{24}$ $\{S\}$



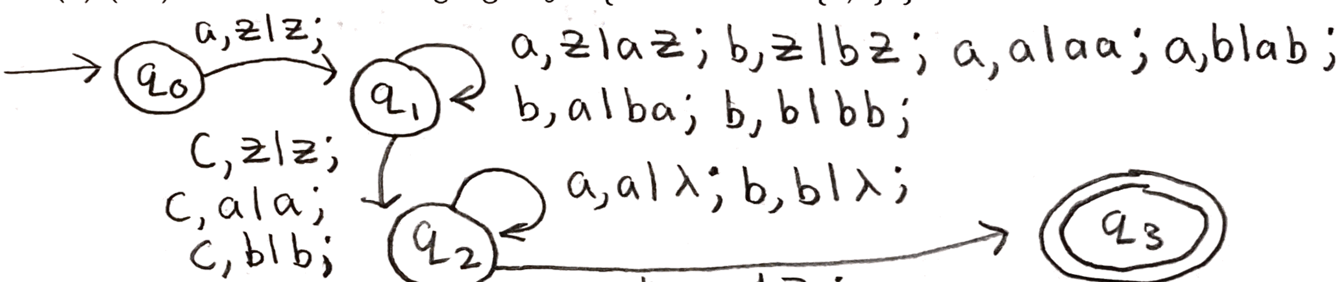
$V_{14}$ $\{W_1\}$

**Problem 5 (15%)**

(i) (8%) Show that the language $L_4 = \{aww^Rb : w \in \{a,b\}^*\}$ is context-free.




(ii) (7%) Show that the language $L_5 = \{awcw^Rb : w \in \{a,b\}^*\}$ is deterministic context-free.

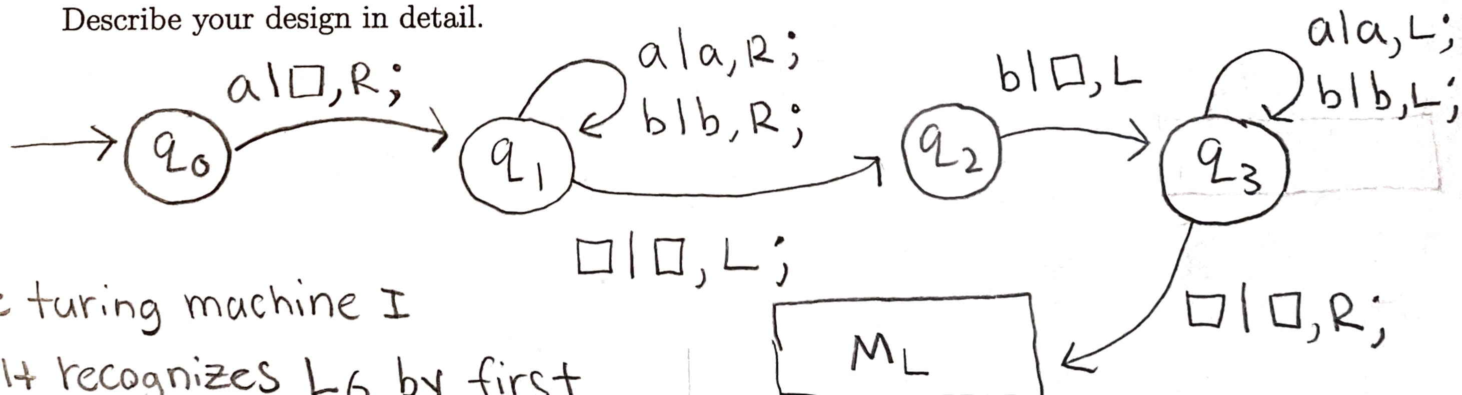



**Problem 6 (15%)**

(i) (8%) Show that the language $L_6 = \{awwb : w \in \{a,b\}^*\}$ is not context-free.

The language $L_6$ can be shown to be not CF using the pumping lemma for CF languages.
(use of pumping lemma shown on next page)

(ii) (7%) Assume that you have a Turing machine that decides membership in the language $L = \{ww : w \in \{a,b\}^*\}$. Think of this machine as a blackbox $M_L$ with initial state $q_0$ and final state $q_f$ and use $M_L$ to build a Turing machine that recognizes the language $L_6$ above. Describe your design in detail.




The turing machine I built recognizes $L_6$ by first removing an a from the front of the input and a b from the back of the input. Then it returns to the front of the input and feeds it to $M_L$. This works because the machine converts the input into a form that $M_L$ can decide membership for.

Problem 6 part i Continued

Pumping Lemma for CF languages

Pick) $W = a a^m b^m a^m b^m b \in L$, $|W| \geq m$, assume L is CF

Decompose) $\underset{u}{aa^m} | \underset{vxy}{b^m} | \underset{z}{a^m b^m b} = \underset{u}{aa^m} | \underset{v}{b^{k_1}} \underset{x}{b^{m-k_1-k_2}} \underset{y}{b^{k_2}} | \underset{z}{a^m b^m b}$

Pump i=2) $aa^m | (b^{k_1})^2 b^{m-k_1-k_2} (b^{k_2})^2 | a^m b^m b$

$= a a^m | b^{2k_1} b^{m-k_1-k_2} b^{2k_2} | a^m b^m b$

$= a a^m b^{m+k_1+k_2} a^m b^m b$

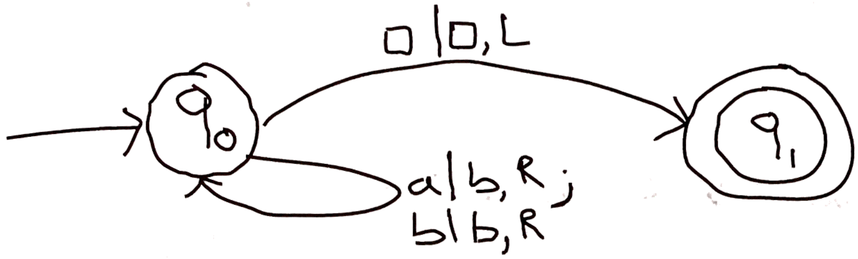
$a a^m b^{m+k_1+k_2} a^m b^m b \in L_6$

↗

Contradiction

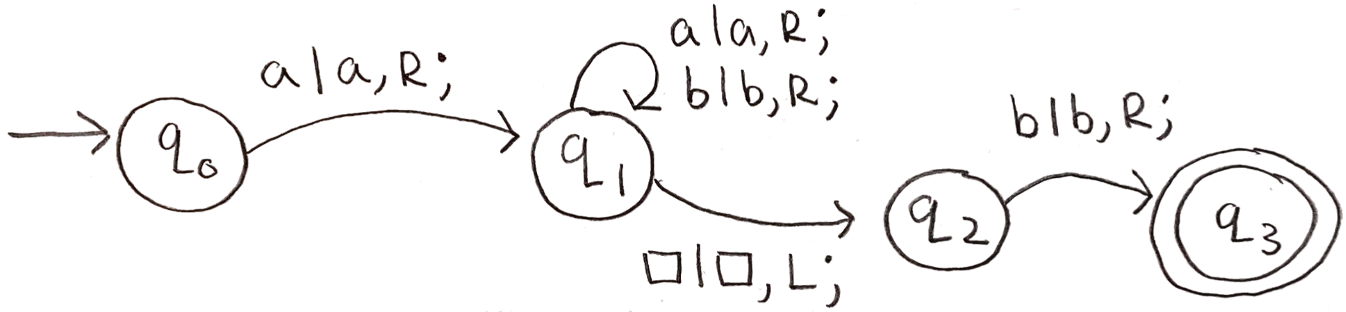
∴ The language is not CF because it provides a Contradiction under the pumping lemma fo CF languages.

**Problem 7 (15%)**
(i) (10%) What language does the following Turing machine decide?




The TM decides membership for $L = (a+b)^*$, while converting the input into a string of bs.

(ii) (5% Bonus) Design a Turing machine that decides memebrship in the regular language $a(a+b)^*b$.



**Appendix**

Taxonomy of formal languages (Chomsky Hierarchy) where R stands for regular, L for linear, DCF for deterministic context-free, CS for context-sensitive, REC for recurscive, and REC-E for recursive enumerable languages.