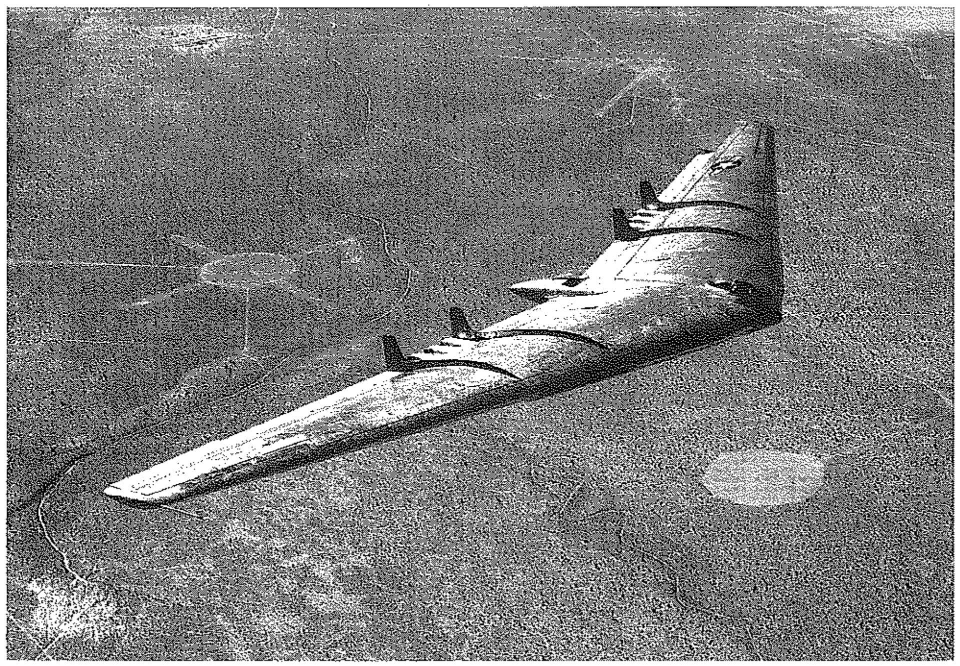
CHAPTER 11

Static Stability and Control



Photograph of a Northrop YB-49 Flying Wing. This aircraft was designed on a true flying wing principle, with no tail surfaces. It used "elevon" controls and was the first aircraft to incorporate artificial stability augmentation. (U.S. Air Force Museum Archives)

In the previous chapter, it was necessary to consider the pitching stability in order to estimate the force on the fuselage due to the horizontal tail lift, which counters the pitch-down moment. For this, rough estimates were made of the major weight components.

This chapter examines a more complete analysis of the static stability, which includes all three directions of motion consisting of

1. pitch;
2. roll;
3. yaw.

The application of these will lead to a method for sizing control surfaces. In addition, a more extensive set of empirical relations are presented to obtain improved estimates of the component weights. The locations of these components will provide a final estimate of the location of the center of gravity. In addition, the sum of these weights will provide a final check on the original structure weight that was presumed in the initial estimate of the weight at take-off.

11.1 REFINED WEIGHT ESTIMATE

The refined weight estimates are based on formulas that relate different characteristics of aircraft to their component weights. These formulas involve coefficients found by a minimum error fit to a large set of aircraft. In order to improve the result of the fits, the total set has been subdivided into smaller sets based on general mission requirements. These correspond to categories of aircraft consisting of combat/fighter, long-range/transport, and general aviation.

The formulas and coefficients are generally trade secrets of aircraft manufacturers because of their value in the conceptual design. However, useful estimates can be obtained based on the formulas published by Staton (1968, 1969) and Jackson (1971). The following are based on these.

The formulas are presented in a general format that involves coefficients whose values are given in subsequent tables. The different rows in the tables correspond to the different categories of aircraft. This approach is a convenient way for utilizing automated spreadsheets for evaluating the formulas for all of the types of aircraft.

11.1.1 Wing Weight

The general formula for the wing weight is

$$W_{\text{wing}} = C_1 C_2 C_3 W_{\text{dg}}^{C_4} n^{C_5} S_w^{C_6} A^{C_7} (t/c)^{C_8} (C_9 + \lambda)^{C_{10}} (\cos\Lambda)^{C_{11}} S_f^{C_{12}} q^{C_{13}} W_{\text{fw}}^{C_{14}} \quad (11.1)$$

with coefficients given in Table 11.1.
The parameters used in Eq. [11.1] correspond to

- A being the aspect ratio of the wing;
- K_{dw} being a coefficient, which is 0.768 for a delta wing and 1.0 otherwise;
- K_{vs} being a coefficient, which is 1.19 for a variable-sweep wing and 1.0 otherwise;
- n being the design load factor;
- q being the dynamic pressure at cruise conditions (lbs/f^2);
- S_w being the planform area of the main wing (f^2);
- S_f being the planform area of the flapped portion of the main wing (f^2);

TABLE 11.1: Coefficients for Eq. [11.1].

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
Fighter	0.0103	K_{dw}	K_{vs}	0.500	0.500	0.622	0.785	−0.4	1.0
Transport	0.0051	1	1	0.557	0.577	0.649	0.500	−0.4	1.0
Gen. Av.	0.0090	1	1	0.490	0.490	0.758	0.600	−0.3	0

	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
Fighter	0.050	−1.0	0.04	0	0
Transport	0.100	−1.0	0.10	0	0
Gen. Av.	0.004	−0.9	0	0.006	0.0035

t/c being the maximum thickness-to-chord ratio of the wing;
W_{dg} being the design gross weight (lbs);
W_{fw} being the weight of fuel stored in the wing (lbs);
Λ being the sweep angle of the maximum thickness line (°);
λ being the taper ratio.

11.1.2 Horizontal Tail Weight

The general formula for the weight of the horizontal tail is

$$W_{h-tail} = C_1\left(1+\frac{F_w}{b_{ht}}\right)^{C_2} W_{dg}^{C_3} n^{C_4} S_{ht}^{C_5} L_{ht}^{C_6} K_y^{C_7} (\cos\Lambda_{ht})^{C_8} A_{ht}^{C_9} (t/c)^{C_{10}} \lambda_{ht}^{C_{11}} q^{C_{12}}, \tag{11.2}$$

with coefficients given in Table 11.2.
The parameters used in Eq. [11.2] correspond to

A_{ht} being the aspect ratio of the horizontal stabilizer;
b_{ht} being the horizontal stabilizer span dimension (f);
F_w being the fuselage width at the location of the horizontal stabilizer (f);

TABLE 11.2: Coefficients for Eq. [11.2].

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
Fighter	0.5503	−2.00	0.260	0.260	0.806	0	0	0	0
Transport	0.0379	−0.25	0.639	0.100	0.750	−1	0.704	−1	0.116
Gen. Av.	0.0092	0	0.414	0.414	0.896	0	0	0.034	0.043

	C_{10}	C_{11}	C_{12}
Fighter	0	0	0
Transport	0	0	0
Gen. Av.	−0.120	−0.020	0.168

K_y being the pitching radius of gyration ($\simeq 0.3L_t$) (f);
L_{ht} being the distance between the 1/4-m.a.c locations of the main wing and horizontal tail (f);
n being the design load factor;
q being the dynamic pressure at cruise conditions (lbs/f^2);
S_{ht} being the planform area of the horizontal tail (f^2);
t/c being the maximum thickness-to-chord ratio of the horizontal tail;
W_{dg} being the design gross weight (lbs);
Λ_{ht} being the sweep angle of the maximum thickness line (°);
λ_{ht} being the taper ratio.

11.1.3 Vertical Tail Weight

The general formula for the weight of the vertical tail is

$$W_{v-tail} = C_1 K_{rht}\left(1+C_2\frac{H_{ht}}{H_{vt}}\right)^{C_3} W_{dg}^{C_4} n^{C_5} S_{vt}^{C_6} M^{C_7} L_{vt}^{C_8}\left(1+\frac{S_r}{S_{vt}}\right)^{C_9}$$
$$\cdot A_{vt}^{C_{10}} (C_{11}+\lambda_{vt})^{C_{12}} (\cos\Lambda_{vt})^{C_{13}} (t/c)^{C_{14}} K_z^{C_{15}} q^{C_{16}}, \tag{11.3}$$

with coefficients given in Table 11.3.
The parameters used in Eq. [11.3] correspond to

A_{vt} being the aspect ratio of the vertical stabilizer;
H_{ht} being the height of the horizontal stabilizer above the fuselage centerline (0 for a conventional tail) (f);
H_{vt} being the height of the vertical stabilizer above the fuselage centerline (same as vertical tail span) (f);
K_z being the yawing radius of gyration ($\simeq L_{vt}$) (f);
K_{rht} being a coefficient, which is 1.047 for a rolling ("flying") tail and 1.0 otherwise;
L_{vt} being the distance between the 1/4-m.a.c locations of the main wing and vertical tail (f);

TABLE 11.3: Coefficients for Eq. [11.3].

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
Fighter	0.4520	1	0.500	0.488	0.488	0.718	0.341	−1	0.348
Transport	0.0026	1	0.225	0.556	0.536	0.500	0	−0.5	0
Gen. Av.	0.0076	0.2	1	0.376	0.376	0.873	0	0	0

	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
Fighter	0.223	1	0.250	−0.323	0	0	0
Transport	0.350	0	0	−1	−0.50	0.875	0
Gen. Av.	0.357	0	0.039	−0.224	−0.49	0	0.122

M being cruise Mach number;
n being the design load factor;
q being the dynamic pressure at cruise conditions (lbs/f^2);
S_r being the planform area of the vertical tail rudder (f^2);
S_{vt} being the planform area of the vertical tail (f^2);
t/c being the maximum thickness-to-chord ratio of the vertical tail;
W_{dg} being the design gross weight (lbs);
Λ_{vt} being the sweep angle of the maximum thickness line (°);
λ_{vt} being the taper ratio.

11.1.4 Fuselage Weight

The general formula for the weight of the fuselage is

$$W_{\mathrm{fusc}} = C_1 C_2 C_3 W_{\mathrm{dg}}^{C_4} n^{C_5} L^{C_6} L_t^{C_7} D^{C_8} S_f^{C_9} W^{C_{10}} (1 + K_{\mathrm{ws}})^{C_{11}} q^{C_{12}} + C_{13}, \qquad (11.4)$$

with coefficients given in Table 11.4.
The parameters used in Eq. [11.4] correspond to

b_w being the main wing span (f);
D being the structural depth of the fuselage (f);
K_{ws} being defined as $0.75[(1+2\lambda)/(1+\lambda)][(b_w/L)\tan\Lambda]$;
K_{dwf} being equal to 0.774 for a delta wing, 1.0 otherwise;
K_{door} being equal to 1 for no cargo door, 1.06 for one side door, 1.12 for two doors or a single clam-shell door, and 1.25 for two doors and one clam-shell door;
K_{lg} being equal to 1.12 for fuselage mounted landing gear and 1.0 otherwise;
L being the length of the fuselage (f);
L_t being the fuselage tail length defined as the distance between the 1/4-m.a.c locations of the main wing and tail sections (f);
n being the design load factor;
q being the dynamic pressure at cruise conditions (lbs/f^2);

TABLE 11.4: Coefficients for Eq. [11.4].

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
Fighter	0.499	K_{dwf}	1	0.35	0.25	0.50	0	0.849	0
Transport	0.328	K_{door}	K_{lg}	0.50	0.50	0.35	0	−0.100	0.302
Gen. Av.	0.052	1	1	0.177	0.177	−0.072	−0.051	0.072	1.086

	C_{10}	C_{11}	C_{12}	C_{13}
Fighter	0.685	0	0	0
Transport	0	0.04	0	0
Gen. Av.	0	0	0.241	W_p

S_f being the fuselage wetted area (f^2);
S_{vt} being the planform area of the vertical tail (f^2);
W being the structural width of the fuselage (f);
W_p being a weight penalty due to having a pressurized fuselage, $W_p = 11.9 + (V_{\mathrm{pr}}\Delta P)^{0.271}$ (lbs), with V_{pr} being the volume of the pressurized section (f^3) and ΔP being the pressure difference ($\Delta P \simeq 8$ lb/f^2);
W_{dg} being the design gross weight (lbs);
Λ being the sweep angle of the maximum thickness line of the main wing (°);
λ being the taper ratio of the main wing.

11.1.5 Main Landing Gear Weight

The general formula for the weight of the main landing gear is

$$W_{\mathrm{main\ lg}} = C_1 C_2 C_3 W_l^{C_4} n^{C_5} L_m^{C_6} N_{\mathrm{mw}}^{C_7} N_{\mathrm{mss}}^{C_8} V_s^{C_9}, \qquad (11.5)$$

with coefficients given in Table 11.5.
The parameters used in Eq. [11.5] correspond to

K_{cb} being a coefficient equal to 2.25 for a cross-beam gear (similar to the F-111), or otherwise 1;
K_{mp} being a coefficient equal to 1.126 for a kneeling gear, or otherwise 1;
K_{tpg} being a coefficient equal to 0.826 for a tripod gear (similar to the A-7), or otherwise 1;
L_m being the length of the main landing gear (in.);
n being the design load factor;
N_{mw} being the number of main wheels;
N_{mss} being the number of main gear shock struts;
V_s being the stall velocity (f/s);
W_l being the landing design gross weight (lbs).

11.1.6 Nose Landing Gear Weight

The general formula for the weight of the nose landing gear is

$$W_{\mathrm{nose\ lg}} = C_1 C_2 W_l^{C_3} n^{C_4} L_n^{C_5} N_{\mathrm{nw}}^{C_6}, \qquad (11.6)$$

with coefficients given in Table 11.6.

TABLE 11.5: Coefficients for Eq. [11.5].

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
Fighter	1	K_{cb}	K_{tpg}	0.250	0.25	0.973	0	0	0
Transport	0.0106	1	K_{mp}	0.888	0.25	0.400	0.321	−0.5	0.1
Gen. Av.	0.0344	1	1	0.768	0.768	0.409	0	0	0

TABLE 11.6: Coefficients for Eq. [11.6].

	C_1	C_2	C_3	C_4	C_5	C_6
Fighter	1	1	0.290	0.290	0.500	0.525
Transport	0.032	K_{np}	0.646	0.200	0.500	0.450
Gen. Av.	1	0.0153	0.566	0.566	0.845	0

TABLE 11.7: Weight multipliers for different groups of aircraft.

Type	Engine W_{inst}/W_{uninst}	W_{remain}/W_{TO}
Fighter	1.3	0.17
Transport	1.3	0.17
Gen. Av.	1.4	0.14

The parameters used in Eq. [11.6] correspond to

K_{np} being a coefficient equal to 1.15 for a kneeling gear, or otherwise 1;
L_n being the length of the nose landing gear (in.);
n being the design load factor;
N_{nw} being the number of nose wheels;
W_l being the landing design gross weight (lbs).

The other significant weight is due to the engine and fuel. As in the previous chapter, the installed engine weight is estimated as the uninstalled weight times a factor. These factors are reproduced in Table 11.7.

The total weights of all the other components of the aircraft are simply estimated as a fixed fraction of the take-off weight. These fractions are also listed in Table 11.7 for the three different groups of aircraft. They range from 14 to 17 percent of the take-off weight. The components making up this weight are distributed all over the aircraft. In order to refine the estimate of the location of the center of gravity, the resultant load of the remaining weight can be assumed to act at the fuselage $x/L = 0.5$.

11.2 STATIC STABILITY

In the design, it is generally important that the aircraft be statically stable in flight. However, excessive stability can have adverse effects on maneuverability and performance. The static stability of the aircraft is presented by considering each of the three directions of motion separately. These are defined in Figure 11.1, along with the sign convention.

11.2.1 Longitudinal (Pitch) Stability

The longitudinal stability is the measure of the response of the aircraft due to a changing pitch angle condition. As discussed in the previous chapter, positive stability in this case