Aircraft Design 1 – Fall 2021

Assignment 4 Stability, Control, Weight & Balance

Student Names and USCIDs:

Instance 1
Hours spent on assignment: Approximately 3 Full Days

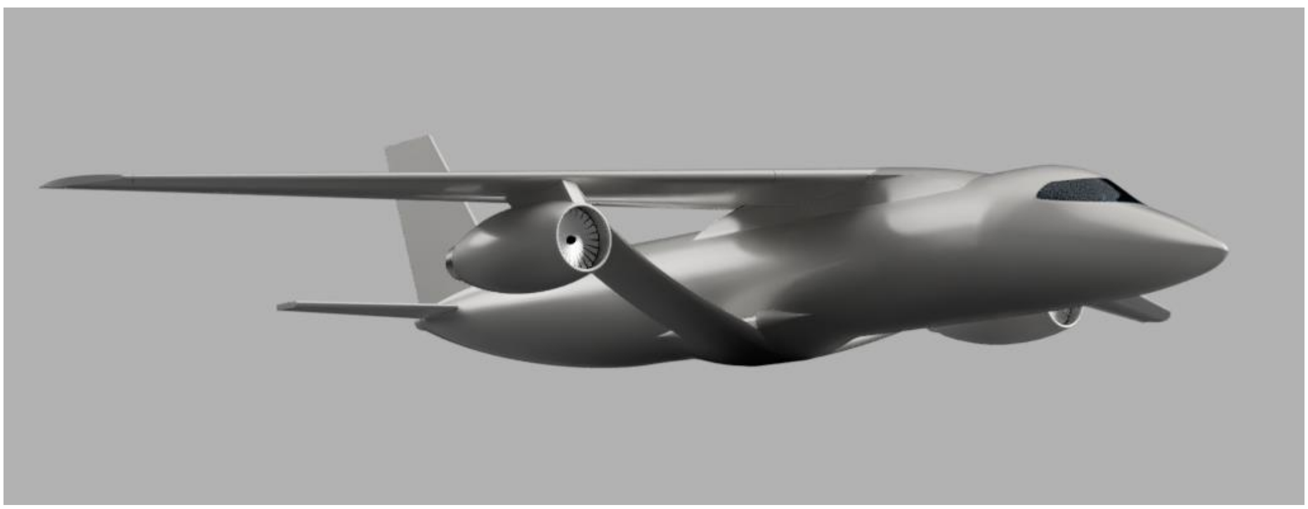

Figure 1.1: Aircraft Render

UNIVERSITY OF SOUTH CAROLINA

Aircraft type: Business Jet
Aircraft number: 105

Table 1: Requirements Table

Requirement type	Value	Unit
Payload	900	kg
Range	3200	km
Cruise speed	770	Km/h
Take-off distance	900	m
Landing distance	800	m

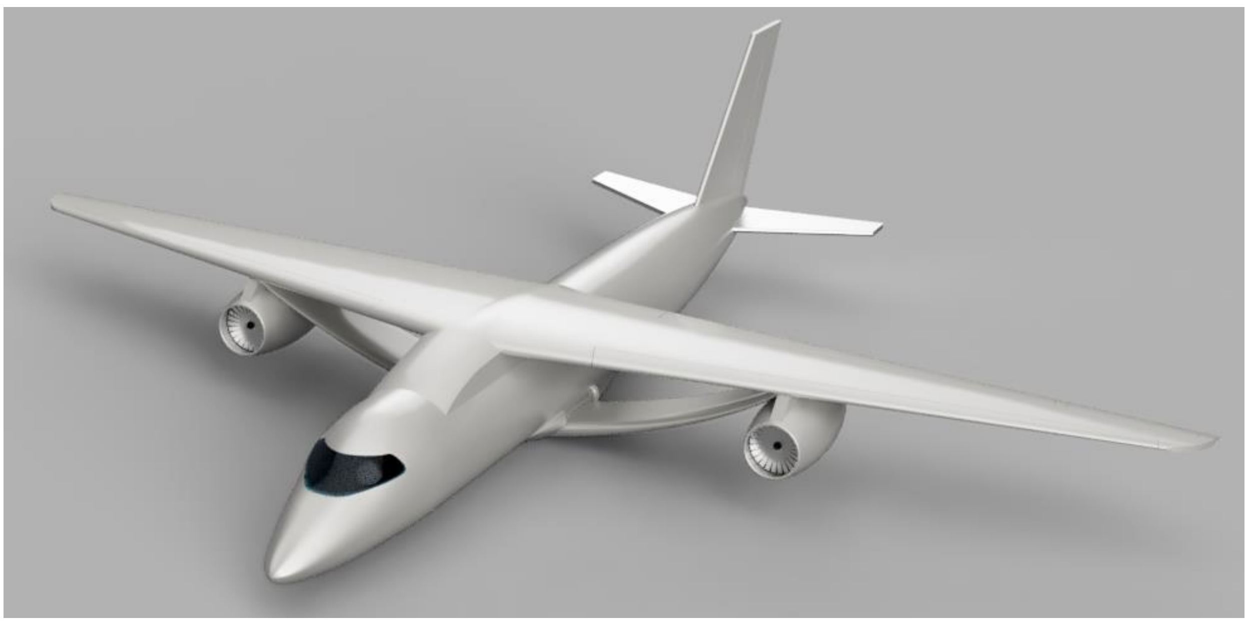

Figure 1.2: Aircraft Configuration

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1. Introduction

Throughout this report, the weight and balance of the proposed aircraft will be analysed. This will be done by initially creating the loading diagrams for the aircraft thought its flight envelopes. Next, preliminary sizing of the empennage will be conducted utilizing the V bar method. From there, class II weight estimations can begin and the center of gravity and its travel can be found. Lastly, stability of the aircraft based upon tail size will be considered and any empennage adjustments will be applied. Note that all units in this report will be presented in imperial terms due to the extensive reliance upon imperial units due to both FAA regulations and the derived empirical formulas for weight estimation.

2. The Load Factor

In this chapter the load factors that the aircraft will experience throughout various envelopes of flight will be explored. The aircraft must be designed with the knowledge that the structure will endure accelerations much greater than that of gravity due to manoeuvring and/or gusts. Furthermore, essentially all class II weight estimations utilize the expected maximum load factor as a variable for accurate estimations. To find the expected load factors experienced by the aircraft, a manoeuvre, or V-n, diagram can be constructed and combined with a gust loading diagram to discover the max loads.

2.1 Manoeuvre loading

The most evident form in which aircraft structures endure loading is though manoeuvring. For that reason, it is critical to understand the max possible load subjected to the aircraft at various flight speed. The value n is used to represent the acceleration in terms of gravitational acceleration. These loads are found by understanding the max possible load achievable by the wings at various speeds and upper required limits set by the FAA. As stated by CFR CS-25, the max design load for aircraft with a max takeoff weight, W_{TO}, of between 4100 and 50000 lbs is given by Equation 2.1 below.

$$n_{max} = 2.1 + \left(\frac{24000}{W_{TO} + 10000}\right) \qquad Eq. 2.1$$

From this, it is evident that the aircraft will need to be rated to a max upper load of positive 2.825. Similarly, lower bounds are set by the FAA. In this case, the aircraft is required to sustain a lower limit of -1. Now that the upper and lower limits have been determined, the remaining load factors based on the flight envelope can be evaluated. At zero velocity, no aerodynamic load is acting on the aircraft. As the aircraft begins to accelerate, the wings begin to generate lift, in turn exerting a load on the structure. This initial rise is directly correlated due to the stall characteristics of the clean wing of the aircraft and is given by Equation 3.2 below where q is the dynamic pressure, C_{Lmax} is the max coefficient of lift of the clean wing, and W/S is the wing loading as set previously.

$$n = \frac{qC_{Lmax}}{\frac{W}{S}} \qquad Eq. 2.2$$

The inverse of this can be applied to the lower portion of the diagram to achieve the negative loading characteristics due to stall conditions. The loading will continue to rise past 1, at which point the wing is producing enough lift to takeoff and continue to the upper set limit. The aircraft additionally has an upper speed limit, known as the max dive speed or Vd. The aircraft is not expected to be able to withstand loads much past this value so this will be the upper bounds on the velocity of the aircraft in the diagram. Lastly, a linear approximation is used to connect the 0 load at Vd to the max negative load at cruise speed, thus completing the V-n diagram. This diagram is displayed below in red in combination with the subsequent gust diagram in Figure 2.2.

2.2 Gust loading

While the V-n diagram takes into accounts of loading due to manoeuvring, the gust loading diagram accounts for the increase in load due to turbulence at various stages of flight. The loading experienced by an aircraft due to various gusts is defined by the FAA in CS 23.341 where the load is given by Equation 2.3.

$$n = 1 + \frac{K_g * U_{de} * V * C_{L\alpha}}{498 * \left(\frac{W}{S}\right)} \qquad Eq. 2.3$$

The wing loading of 44.98 lbs/ft^2 along with the lift curve slope of 5.835 and 5.646 rad^-1 for the clean and flapped wing respectively have already been determined. First, Kg, or the gust alleviation factor must be determined. Kg is defined by the FAA in Equation 2.4 below where μ is further defined by Equation 2.5.

$$K_g = \frac{0.88 * \mu}{5.3 + \mu} \qquad Eq. 2.4$$

$$\mu = \frac{2 * \left(\frac{W}{S}\right)}{\rho * MAC * C_{L\alpha} * g} \qquad Eq. 2.5$$

The three points of analysis are that of Vd, Vc, and Vb, where Vb is defined by a value not less than that which is defined by Equation 2.6 below. For the analysis, Vd and Vc have a clean wing configuration while Vb has full flaps deployed.

$$V_B \geq \sqrt{1 + \frac{K_g * U_{ref} * V * C_{L\alpha}}{498 * \left(\frac{W}{S}\right)}} \qquad Eq. 2.6$$

Uref and Ude above are both gust velocity values that are determined statistically for various envelopes of flight. Uref is set at 56 ft/sec while Ude can be found utilizing Figure 2.1 for the various flight combinations. The various data points for the 3 conditions can be found in Table 2.1 below.

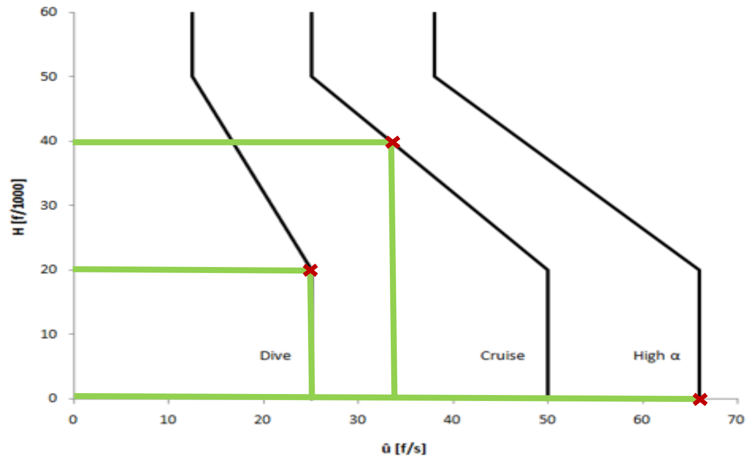


Figure 2.1: Gust Velocity for Various Configurations

Table 2.1: Gust Loading Analysis Values

	High a	Cruise	Dive	Unit
V	176	416	467	knots
Altitude	0	40000	20000	ft
U	66	33	25	ft/s
ρ	2.38E-03	5.87E-04	1.27E-03	slugs/ft^3
μ	27	106	39	
K	0.736	0.838	0.775	
n +	3.109	3.932	3.306	Gs
n -	-1.109	-1.932	-1.206	Gs

2.3 Combined Loading

Now that both the V-n diagram and the Gust loading diagram have been constructed, they can be combined to visualize the maximum loading scenarios the aircraft may experience. The combined diagrams are presented below in Figure 2.2.

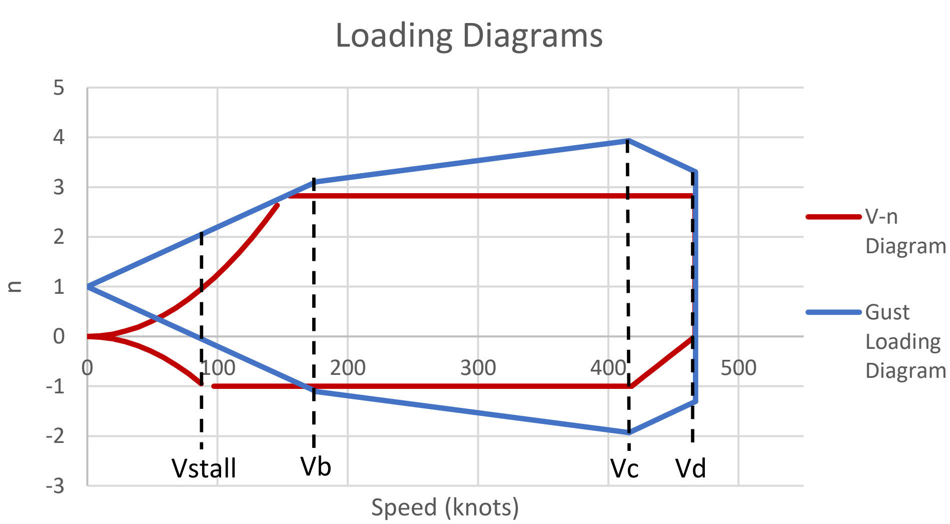


Figure 2.2: V-n and Gust Loading Diagram

From the figure the maximum loading the aircraft will experience can be determined and later used to estimate weight more accurately. To obtain the ultimate load factor, the maximum load of 3.932 and negative 1.932 at cruise speed should be multiplied by the FAA mandated safety factor of 1.5. This give the required ultimate load factor of 5.898 and negative 2.898 that the aircraft must be designed to withstand.

3. Tail Plane Geometry

Before weight estimations can begin, a preliminary estimation of the empennage geometry must be conducted. Initial estimations can be made on the tail sizing base on statistical volume estimation methods. The V bar method can be used to estimate tail sizing by comparing the volume coefficients of similar aircraft, as similar aircraft tend to have similar coefficients. This is however a brief estimation and does not account for unique features of aircraft that are in development.

3.1 Horizontal tail

The horizontal tail volume coefficient is given by Equation 3.1 below where Sh is the tail area and lh is the tail arm, or distance between the aerodynamic center of the wing and tail.

$$\overline{V_h} = \frac{S_h * l_h}{S * \bar{c}} \qquad Eq. 3.1$$

S and $\bar{c}$ are known, Sh and $\overline{V_h}$ can be estimated based on reference aircraft, and finally lh can be calculated. As these values have already been recorded for many aircraft, the important values are displayed below in Table 3.1.

Table 3.1: Horizontal Tail Volume Estimation References

Aircraft	$\overline{V_h}$	Sh (ft^2)
Cessna Citation II	0.64	73.1
Cessna Citation III	0.99	69.6
Learjet 55	0.76	57.8
Dassault Falcon 50	0.68	144

From these reference aircraft an initial $\overline{V_h}$ value of 0.7 and Sh value of 80 ft^2 can be selected. With the surface are and volume coefficient, the tail arm can now be calculated to be 34.15 ft. With the wing where it currently is with its aerodynamic center at approximately 40% of the fuselage there is enough space to accommodate the horizontal stabilizers. Further wing geometry can be selected based on typical values. Jet aircraft typically have a horizontal tail aspect ratio of 5 due to the fact that lower aspect ratios increase the stall angle which is beneficial in allowing the tail more time to correct a stall. A typical taper ratio of 0.4 is also selected. Finally, a thickness to chord ratio of 0.1 and sweep of 30 degrees is selected, this is to ensure the stabilizer does not experience Mach effects before that of the thicker less swept main wing. With these values selected the span, root chord, and tip chord can be calculated utilizing Equations 3.2, 3.3, and 3.4 respectively.

$$b = \sqrt{A * S} \qquad Eq. 3.2$$

$$C_r = \frac{2 * S}{b * (1 + \lambda)} \qquad Eq. 3.3$$

$$C_t = \lambda * C_r \qquad Eq. 3.4$$

The span of 20 ft is the only important value for weight estimations so root and tip chord values will be ignored until tail sizing is later confirmed upon further analysis.

3.2 Vertical tail

Similar to the horizontal tail, the vertical tail size can be estimated using a slightly different volume coefficient given by Equation 3.5 where the mean aerodynamic center is substituted with the wingspan.

$$\overline{V_v} = \frac{S_v * l_v}{S * b} \qquad Eq. 3.5$$

Reference values are similarly displayed below in Table 3.2.

Table 3.2 Vertical Tail Volume Estimation References

References	Vv bar	Sv (ft^2)
Cessna Citation II	0.062	53
Cessna Citation III	0.086	70.2
Learjet 55	0.086	52.4
Dassault Falcon 50	0.064	106

From there, a $\overline{V_v}$ of 0.066 and Sv of 85 ft^2 can be selected for analysis. The subsequent tail arm is 31.65 ft, which is acceptable for the length of the aircraft. The aspect ratio is set at 1.6 due to the fact that the aircraft has a low set tail but is based upon aircraft that nearly exclusively feature T-tail designs. For similar reasons, a taper ratio of 0.5 is selected. Finally, a standard sweep of 40 degrees is selected, along with a thickness to chord of 0.1. Using these values, the span can be found to be 10.95 ft, while the other values will be ignored for similar reasons to the horizontal tail.

4. OEW estimation using class II method

The main difference between class I and class II estimations is the availability of relative geometry. Class I solely utilizes statistics and requirements while class II uses empirical methods based upon the available characteristics of the aircraft. Many different resources have developed various methods for estimating the weight of various aircraft components. For the purposes of this report, Raymer's class II weight estimations will be utilized as many of his methods have been previously utilized in the design of this aircraft. Raymer's method breaks up the operational empty weight of Cargo/Transport aircraft into 20 categories that are as follows: wing, horizontal tail, vertical tail, fuselage, main gear, nose gear, nacelles, engine controls, engine starter, fuel systems, flight control systems, auxiliary power unit, instruments, hydraulics, electronics, avionics, furnishings, air conditioning, de-icing, and handling gear. Additionally, the engines and brace structure are to be added on top of the other analyses. Explanations for certain variables and values will be presented in this chapter however the full equation and variable reference can be found below in Appendix E. Note all equations presented below must be in imperial units.

4.1 Wing Weight

The weight of the aircraft's wing, which must be structurally sound to carry the entire weight of the aircraft in addition to supporting itself on the ground, is given by Equation 4.1 with the variable values presented in Table 4.1.

$$W_{wing} = 0.0051 * \left(W_{dg} * N_z\right)^{0.557} * S_w^{0.649} * AR^{0.5} * \left(\frac{t}{c}\right)^{-0.4} * (1+\lambda)^{0.1} * (\cos(\Lambda))^{-1} * S_{csw}^{0.1} \qquad Eq. 4.1$$

Table 4.1: Wing Weight Variable Values

Variable	Value	Units
W_{dg}	23108	lbs
N_z	5.9	Gs
S_w	497.66	ft^2
AR	10	
t/c root	0.12	
λ	0.265	
$\Lambda_{\frac{1}{4}}$	0.297	rad
S_{csw}	12.92	ft^2

The max design weight, Wdg, was determined based on class I statistical analysis from earlier along with much of the wing geometry. Subsequently, the wing weight comes out to be 2122.89 lbs.

The brace structure of the aircraft will also be analysed as a wing for the purposes of this report. The altered variables for the brace structure considered as a wing are presented in Table 4.2 below.

Table 4.2: Brace Weight Variable Values

Variable	Value	Units
S_{brace}	127.21	ft^2
AR brace	6.85	
t/c root	0.4	
λ	0	
$\Lambda_{\frac{1}{4}}$	0.2	rad
$S_{csw\ brace}$	0	ft^2

These values for the brace were estimated based on CAD of the required connection. The weight of the brace analysed as a wing then comes out to be 330.48 lbs.

4.2 Horizontal Tail Weight

The weight of the aircraft's tail, sized utilizing the V bar method, is given by Equation 4.2 with the variable likewise found in Table 4.3 below.

$$W_{Horizontal\,Tail} = 0.0379 * K_{uht} * \left(1 + \frac{F_w}{B_h}\right)^{-0.25} * W_{dg}^{0.639} * N_z^{0.1} * S_{ht}^{0.75} * L_t^{-1} * K_y^{0.704} * (\cos(\Lambda_{ht}))^{-1} * AR_h^{0.166} * \left(1 + \frac{S_e}{S_{ht}}\right)^{0.1} \qquad Eq. 4.2$$

Table 4.3: Horizontal Tail Weight Variables

Variable	Value	Unit
K_{uht}	1	
F_w	4.06	ft
B_h	19.36	ft
S_{ht}	75	ft^2
L_t	34.15	ft
K_y	10.93	ft
Λ_h	0.454	rad
AR h	5	
S_e	13.62	ft^2

The variable K_{uht} is a factor that applies to all moving tail constructions, as the chosen tail for this aircraft is simpler adjustable tail, the value is 1. F_w is the width of the fuselage at the tail intersection and is slightly smaller than that of the typical cross section at 4.06 ft. K_y represents the pitching radius of gyration which is estimated to be 1/3 the length of the tail arm. With all that, the weight of the horizontal tail comes out to be 158 lbs.

4.3 Vertical Tail Weight

Similar to the horizontal tail in that the V bar method was utilized to estimate the size, the vertical weight is estimated using slightly different values as shown in Equation 4.3 with the accompanying values in Table 4.4.

$$W_{Vertical\,Tail} = 0.0026 * \left(1 + \frac{H_t}{H_v}\right)^{0.225} * W_{dg}^{0.556} * N_z^{0.536} * S_{vt}^{0.5} * L_t^{-0.5} * K_z^{0.875} * (\cos(\Lambda_{vt}))^{-1} * A_v^{0.35} * \left(\frac{t}{c}\right)_{root}^{-0.5} \qquad Eq. 4.3$$

Table 4.4: Vertical Tail Weight Variables

Variable	Value	Unit
$\frac{H_t}{H_v}$	0	
L_t	31.65	ft
S_{vt}	85	ft^2
K_z	31.65	ft

Λ_V	0.628	rad
AR v	1.6	
t/c root	0.1	

For the vertical tail, $\frac{H_t}{H_v}$ is a factor applied only to T-tail aircraft, and as such is zero in this case. K_z, the other variable of note, is the yawing radius of gyration and is estimated to be the length of the tail arm. Together, the weight of the vertical tail comes out to be 70 lbs.

4.4 Fuselage Weight

The heaviest and largest part of the aircraft is that of the fuselage. Similar to the rest, the weight estimation for the fuselage is given below by Equation 4.4 along with the variable values in Table 4.5.

$$W_{fuselage} = 0.3280 * K_{door} * K_{lg} * \left(W_{dg} * N_z\right)^{0.5} * L^{0.25} * S_f^{0.302} * (1 + K_{ws})^{0.04} * \left(\frac{L}{D}\right)^{0.1} \qquad Eq. 4.4$$

Table 4.5: Fuselage Weight Variables

Variable	Value	Unit
k_{door}	1	
K_{lg}	1	
L	56.4332	ft
S_f	1000.667715	ft^2
K_{ws}	0.336527534	
D	0.166666667	ft

In this case, K_{door} is a factor that accounts for the increased weight of reinforced cargo door. Due to no such doors existing on this aircraft the value is 1. K_{lg} accounts for landing gear that may be attached to the fuselage, however, as the gear are attached to the brace structure, which is more akin to a wing, this value is also 1. K_{ws} accounts for the wing geometry while D represents the structural depth of the fuselage which was set at 2 in early on. All of this culminates in the fuselage weighing in at 4844 lbs.

4.5 Main Landing Gear Weight

Taking the majority of the brunt during landings and ground operations is that of the main landing gear. Weight estimations for the gear can be found utilizing Equation 3.5 with its accompanying variable found below in Table 4.6.

$$W_{main\ landing\ gear} = 0.0106 * K_{mp} * W_l^{0.888} * N_l^{0.25} * L_m^{0.4} * N_{mw}^{0.321} * N_{mss}^{-0.5} * V_{stall}^{0.1} \quad Eq. 4.5$$

Table 4.6: Main Landing Gear Weight Variables

Variable	Value	Unit
K_{mp}	1	

W_l	20797	lbs
N_l	3	Gs
L_m	3.281	ft
N_{mw}	4	
N_{mss}	2	
V stall	71.5392	knots

K_{mp} is a factor that is applied in the case that the gear can kneel. As there is no need for that on this aircraft, this value is 1. W_l is the max allowable landing weight and was set at 90% of the W_{TO}. N_l is the max landing load factor that the gear is expected to experience, and anything harder than 3 Gs of acceleration might as well be considered a crash landing. Lastly, N_{mw} and N_{mss} represent the number of main gear wheels and shock struts, with each of the two gears containing 2 wheels and 1 strut. Altogether, the weight of the main gear comes out to be 259 lbs.

4.6 Nose Gear Weight

Responsible for steering control and supporting approximately 1/8 the weight of the aircraft is that of the nose gear. The weight estimations for this gear are given by Equation 4.6 with the variable values found in the following Table 4.7.

$$W_{nose\ gear} = 0.032 * K_{np} * W_l^{0.646} * N_t^{0.2} * L_n^{0.5} * N_{nw}^{0.45} \qquad Eq. 4.6$$

Table 4.7: Nose Gear Weight Variables

Variable	Value	Unit
Ln	2.95	ft
N_{nw}	2	

Once again, the nose gear does not kneel and therefore has a K_{np} value of 1. Differences here include a reduced strut length to 2.95 ft and the presence of 2 wheels as a part of the nose gear. In all, the nose gear comes out to weigh 58 lbs.

4.7 Engine and Nacelle Group Weight

Here the weight of the nacelles and the engines themselves are analysis. From previous sizing, the dry weight of the engines was found to be 865 lbs each, for a total weight of 1730 lbs. The nacelle weight can be found using Equation 4.7 below with the accompanying variables found in Table 4.8.

$$W_{nacelle\ group} = 0{,}6724 * K_{ng} * N_{Lt}^{0.1} * N_w^{0.294} * N_z^{0.119} * W_{ec}^{0.611} * N_{en}^{0.984} * S_n^{0.224} \quad Eq. 4.7$$

Table 4.8: Nacelle Group Weight Variables

Variable	Value	Unit
K_{ng}	1.017	
N_{Lt}	9.1868	ft
N_w	3.83877	ft
K_p	1	
K_{tr}	1.18	

W_{ec}	1218.349	
N_{en}	2	
S_n	93.45063	ft^2

As for the K factors, $K_{ng}$is a factor accounting for pylon mounted engines, $K_p$is a factor for engines that might have propellers, and K_{tr} is a factor that accounts for jet engines with thrust reversers, which this aircraft will have. K_p and K_{tr} are accounted for in W_{ec} which is a factor that accounts for the weight of the engines. The nacelle dimensions were obtained from CAD of a presumed covering of the engines that were previously sized. Through the equation, it can be found that the nacelle group weight comes out to be 657 lbs.

4.8 Engine Control, Starter, and APU Weight

The engines of the aircraft require further subsequent systems to allow for seamless operation. For the purposes of this weight estimation, the starter, engine control and APU are analysed. The weights for the engine controls, starter, and APU are given by Equations 4.8, 4.9, and 4.10 respectively while the required variable values can be found together in Table 4.9.

$$W_{engine\ controls} = 5 * N_{en} + 0.8 * L_{ec} \qquad Eq. 4.8$$

$$W_{starter} = 49.19 * \left(\frac{N_{en} * W_{en}}{1000}\right)^{0.541} \qquad Eq. 4.9$$

$$W_{APU\ installed} = 2.2 * W_{APU\ uninstalled} \qquad Eq. 4.10$$

Table 4.9: Engine Control, Starter, and APU Weight Variables

Variable	Value	Unit
L_{ec}	36.09	ft
$W_{APU\ uninstalled}$	75	lbs

The only new variable here is the distance from the cockpit to the engines, L_{ec}, and the weight of the APU. The APU chosen for this application is that of a Honeywell Micro Power Unit. Overall, the engine control, starter, and APU systems weigh 39, 66, and 165 lbs respectively.

4.9 Fuel System Weight

While the fuel itself is responsible for a large percentage of the weight of the aircraft, the systems responsible for handling the fuel also make up a bit of the weight. The weight of the fuel can be found by Equation 4.11 presented below by utilizing the variable values given by Table 4.10.

$$W_{fuel\ system} = 2.405 * V_t^{0.606} * \left(1 + \frac{V_i}{V_t}\right)^{-1} * \left(1 + \frac{V_p}{V_t}\right) * N_t^{0.5} \qquad Eq. 4.11$$

Table 4.10: Fuel System Weight Variables

Variable	Value	Unit
V_t	1595.131	gal
V_i	1595.131	gal
V_p	0	gal

N_t	2	

The aircraft contains two integral fuel tanks located in either wing of the aircraft. The max possible volume of the wing tanks was calculated at nearly 1600 gallons, however, since less fuel will be required to meet the requirements, the tanks may not be fully loaded for every mission. Overall, the fuel system comes out to weigh 148 lbs.

4.10 Flight Control System Weight

The flight control systems are all the surfaces responsible for aerodynamic and mechanical control of the aircraft. These come in the form of primary control surfaces, secondary control surfaces, and more. The weight of all of these control systems can be given by Equation 4.12 with the corresponding variable in Table 4.11.

$$W_{flight\ controls} = 145.9 * N_f^{0,554} * \left(1 + \frac{N_m}{N_f}\right)^{-1} * S_{cs}^{0.2} * \left(I_y * 10^{-6}\right)^{0.07} \qquad Eq. 4.11$$

Table 4.11: Flight Control System Weight Variables

Variable	Value	Unit
N_f	5	
N_m	2	
S_{cs}	165.36	ft^2
I_y	338790	$lb * ft^2$

For the flight control systems, N_f represents the number of functions performed by the controls, which was set at 5 for the 3 primary control surfaces and 2 secondary ones. N_m is the number of mechanical functions performed which is 2 for the deflect and extend functions performed. The control surface area was estimated using CAD and I_y, the yawing moment of inertia, was estimated using Equation 4.12 provided by Raymer where $\overline{R_z}$ is an aircraft type constant of 0.44.

$$I_y = \left(\frac{b+L}{2}\right)^2 * \frac{W * \overline{R_z}^2}{4 * g} \qquad Eq. 4.12$$

In the end, the total weight of all the aircraft's control systems comes out to be 655 lbs.

4.11 Instrument and Avionics Weight

To properly fly the aircraft, instruments must be installed to monitor and display flight information and avionics must be installed to process and utilize the provided data. The weight of the instrument systems and avionics are given by Equations 4.13 and 4.14 respectively with the combined variables found in Table 4.12.

$$W_{instruments} = 4.509 * K_r * K_{tp} * N_c^{0.541} * N_{en} * \left(L_f + B_w\right)^{0.5} \qquad Eq. 4.13$$

$$W_{avionics} = 1.73 * W_{uav}^{0.983} \qquad Eq. 4.14$$

Table 4.12: Instrument and Avionics Weight Variables

Variable	Value	Unit
W_{uav}	1000	lbs

K_r	1	
K_{tp}	1	
N_c	2	

The new factors in this instance are that of K_r and K_{tp} which are multipliers in the case that the aircraft has a turboprop or reciprocating engine, neither of which this aircraft has. N_c is the number of pilots, of which there are 2. The weight of the uninstalled avionics, W_{uav}, was estimated to be at approximately 1000 lbs with the assumption that the aircraft will take advantage of modern technologies. Ultimately, the instrument weight and avionics weight come out to be 148 and 1538 lbs respectively.

4.12 Hydraulic System Weight

To operate many of the control systems, a hydraulic system must be installed. The weight of the system is given by Equation 4.15. All the utilize variables have already been found elsewhere.

$$W_{hydraulics} = 0.2673 * N_f * \left(L_f + B_w\right)^{0.937} \qquad Eq. 4.15$$

Here, the hydraulic systems come out to weigh 125 lbs.

4.13 Electrical System Weight

In addition to the hydraulic system, aircraft power and communication is done through extensive electrical systems. The weight of the aircraft's electrical system can be found through Equation 4.16 with the variables listed in Table 4.13 below.

$$W_{electrical} = 7.291 * R_{kva}^{0.782} * L_a^{0.346} * N_{gen}^{0.1} \qquad Eq. 4.16$$

Table 4.13: Electrical System Weight Variables

Variable	Value	Unit
R_{kva}	60	KW
L_a	25	ft
N_{gen}	2	

R_{kva} here is the power rating of the aircraft electrical system which was assumed to be around 60 KW. L_a is the routing distance for wires from the 2 generators located in the engines to the avionics and then on to the cockpit. For this reason, it will be beneficial to place the avionics between the cockpit and engines as to decrease this distance, thus saving weight. Overall, the electrical system comes out to be 585 lbs.

4.14 Furnishings Weight

To comfortably seat the passengers and accommodate any cargo, the aircraft must be furnished. This weight can be found by utilizing Equation 4.17 with its accompanying variable in Table 4.14.

$$W_{furnishings} = 0.0577 * N_c^{0.1} * W_c^{0.393} * S_f^{0.75} \qquad Eq. 4.17$$

Table 4.14: Furnishing Weight Variable

Variable	Value	Unit

W_c	1985	lbs

The variable W_c is simply the weight of all of the cargo, most of which are passengers, of the aircraft which was set at 1985 lbs. With this, the weight to furnish the aircraft is 214 lbs.

4.15 Air Conditioning System Weight

To keep the passengers comfortable and the avionics operating optimally, air-conditioning must be installed. Additionally, this serves to pressurize the aircraft to allow for high altitude operations at which the aircraft will be cruising. The weight of this system can be found with Equation 4.18 while the variable values can be found in Table 4.15.

$$W_{ac} = 62.36 * N_p^{0.25} * \left(\frac{V_{pr}}{1000}\right)^{0.604} * W_{uav}^{0.1} \qquad Eq. 4.18$$

Table 4.15: AC System Weight Variables

Variable	Value	Unit
N_p	10	
V_{pr}	916	ft^3

Being a small business jet aircraft, the max assumed number of people is 10, including 2 pilots and 8 passengers. V_{pr} on the other hand, is the pressurized volume which includes the cabin and cockpit and was found utilizing CAD. In all, the weight of the AC system comes in at 210 lbs.

4.16 Anti-Ice and Handling Gear Weight

Lastly is that of the weight estimation of the anti-ice and handling gear. Both are given simply in terms of the M_{TO} weight. The weight for the anti-ice systems is given by Equation 4.19 while the handling gear is given by Equation 4.20.

$$W_{anti-ice} = 0.002\, W_{dg} \qquad Eq. 4.19$$

$$W_{handling\ gear} = 3 * 10^{-4} * W_{dg} \qquad Eq. 4.20$$

In the end, these systems come out to be 46 and 7 lbs respectively ultimately concluding the individual component weight estimations.

4.17 Operational Empty Weight and 'Fudge Factors'

With the weight of all the individual components found, the total OEW of the aircraft can be discovered. The weight of all the individual components along with the summed up OEW are presented below in Table 4.16.

Table 4.16: Component Weights

Component	Weight (lbs)
Brace	330
Wing	2123
Horizontal Tail	158
Vertical Tail	70

Fuselage	4844
Main Gear	259
Nose Gear	58
Nacelles	657
Engine Control	39
Engine Starter	66
Fuel System	148
W flight control	655
APU	165
Instruments	148
Hydraulics	125
Electrical	585
Avionics	1538
Furnishings	214
AC	210
De-Ice	46
Handling Gear	7
Engines	1730
Total	14176

This total weight is less than a percent different than the predicted OEW of 14291 lbs obtained from the class I estimation method. However, as it is always imperative to reduce weight, some further factors can be taken into consideration. Raymer provides what are called 'fudge factors', or factors that alter the value of certain components based on general trends and the development of new technology. These factors are displayed below in Figure 4.1.

Category	Weight group	Fudge factor (multiplier)
Advanced composites	Wing	0.85
	Tails	0.83
	Fuselage/nacelle	0.90
	Landing gear	0.95
	Air induction system	0.85
Braced wing	Wing	0.82
Wood fuselage	Fuselage	1.60
Steel tube fuselage	Fuselage	1.80
Flying boat hull	Fuselage	1.25

Figure 4.1: Raymer Weight Estimation Fudge Factors [2]

Important factors for this aircraft include the Braced Wing factor and Advanced Composites factors. By employing advanced composites, the OEW of the aircraft can be significantly reduced. By applying these factors, the new OEW of the aircraft comes out to be 12855 lbs. This new value that is exactly 10% less than the original estimation, which doesn't account for composites use, will be the weight that is used going forward.

5. Center of Gravity Estimation

Of important factors that contribute to the stable flight of an aircraft is that of the location of its center of gravity, COG. The COG of a body can be found by summing the product of all the individual weights times their distance from a reference divided by the total weight as depicted by Equation 5.1 below.

$$COG = \frac{\sum W_{component} * X_{copmponent}}{\sum W_{component}} \qquad Eq. 5.1$$

One important consideration to take into account though is that not all weights are fixed on an aircraft, such as passengers that might be inclined to move around. Furthermore, there exist multiple ways in which an aircraft can be loaded. With that in mind, the COG for the empty aircraft will be presented for 3 different configurations.

5.1 The empty aircraft

With all the weights of the components and an estimated location of the component COGs with reference to the tip of the nose of the aircraft, the overall aircraft COG may be found. The component weights along with their relative X, Y, and Z coordinate in reference to the nose are displayed below in Table 5.1. Note, the number 1 component refers to that instance of the component on the left side while the number 2 refers to the tight side.

Table 5.1: COG Calculations

Component	Weight (lbs)	X (ft)	Y (ft)	Z (ft)
Brace 1	140	28.4	-8.6	0.15
Brace 2	140	28.4	8.6	0.15
Wing 1	740	23.7	-14.2	5
Wing 2	740	23.7	14.2	5
H Tail 1	66	53.85	-4.45	2.85
H Tail 2	66	53.85	4.45	2.85
V Tail	58	55.75	0	7.2
Engine 1	898	24.75	-15.5	1.7
Engine 2	898	24.75	15.5	1.7
Nacelle 1	296	23.25	-15.5	1.7
Nacelle 2	296	23.25	15.5	1.7
Fuselage	4359	22.57	0	1.7
Main Gear 1	130	29.75	-3	-1.6
Main Gear 2	130	29.75	3	-1.6
Nose Gear	58	6.5	0	-0.8
Instruments	148	6	0	2
APU	165	55.75	0	2.85
Misc.	3529	22.57	0	1.7
Center of Gravity		23.94	0	2.02

These values have been further visualized in Figures 5.1 and 5.2 below. To proceeded further with analysis, the location of the center of gravity in the X direction must be nondimensionalized with respect to the mean aerodynamic chord, MAC. The start of the MAC, X_{LeMAC}, is located 20.45 ft back from the nose. The difference between the COG and X_{LeMAC} is therefore 3.49 ft. Divided by the length of the MAC of 7.849 ft gives a COG location at 44.5% that of the MAC. This value will be important for the loading diagrams that will be generated.

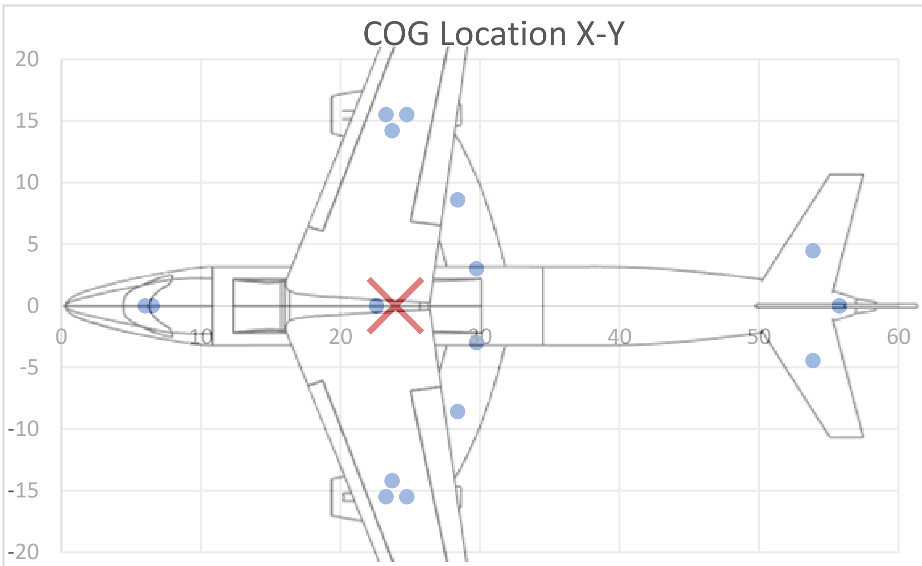


Figure 5.1: COG of Aircraft in X-Y Plane (distance in ft)

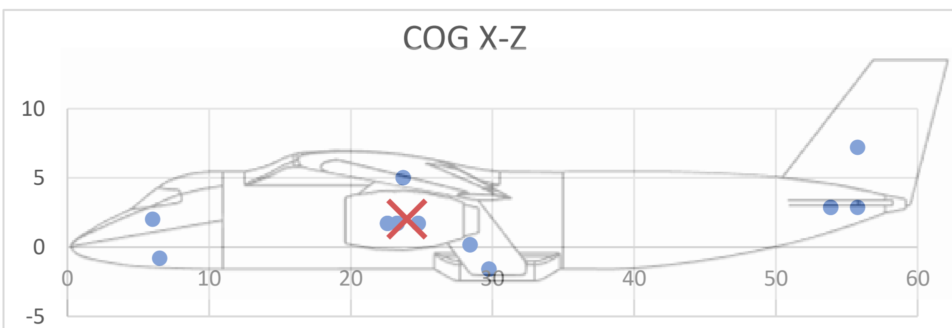


Figure 5.2: COG of Aircraft in X-Z Plane (distance in ft)

5.2 The loading diagram

Now that the initial location of the COG with respect to the MAC has been found, how the COG will shift as the aircraft is loaded can be determined. This is important to understand as it will demonstrate the most forward and aft ward COG that the aircraft will experience. This will ultimately lead to the required tail sizing for stable flight. To create the loading diagram the COG is initially set at the location found previously in section 5.1. From there, the contribution in the shift of the COG due to loading cargo is calculated. Next, the shift in COG due to loading passengers is found. Lastly, the shift in COG due to loading fuel is calculated. The locations of the seats, cargo hold, and fuel tanks, along with their respective max weight are show in Table 5.2.

The weight of a passenger was taken as the FAA guideline of 185 lbs. Remaining was the required cargo payload of 500 lbs. 9000 lbs of fuel was loaded, despite being less than the max possible load as it is still more than enough to accomplish the requirements while not pushing the aircraft far over the set takeoff weight.

Table 5.2: Useful Weight COG

Item	X (ft)	Weight (lbs)
Seat 1,2	17.75	370
Seat 3,4	21.75	370
Seat 5,6	25.75	370
Seat 7,8	29.75	370
Cargo	38	500
Fuel	23.7	9000

As there are few expected passengers for this business jet, these calculations are rather simple and only contain 1 loading configuration. Below, 3 wing locations will be evaluated as to determine the optimal wing location based on COG travel.

5.2.1 Wing Configuration 1

The first wing configuration is that of the neutrally located wing presented in section 5.1 with the COG at 44.5% of the MAC. The loading plot for the aircraft is presented below in Figure 5.3. From

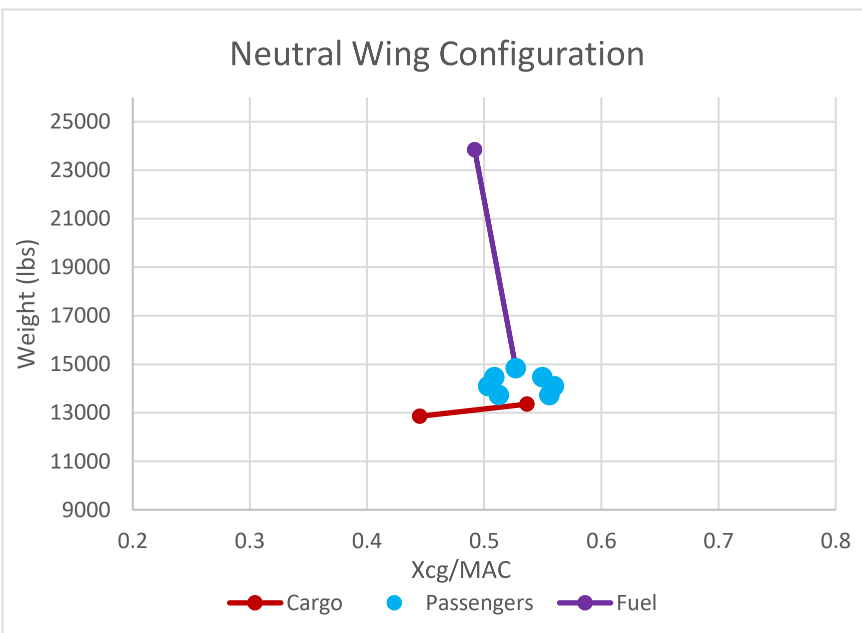


Figure 5.3: Neutral Wing COG Loading Diagram

here it can be seen that most of the weight shift is due to loading the far aftward located cargo. To account for possible shifts, or occupants moving around within the aircaft, a 2% safety margin will be applied to the maximum and minimum COG location. Ultiamtely this gives this configuration a minimum COG location of 43.6% of the MAC and a maximum location of 56.7% of the MAC for a range of travel of 13.1% of the MAC.

5.2.2 Wing Configuration 2

Configuration 2 sees the wings and all attached components moved 5 ft backward. This shift in components has the effect of changing the COG of the empty aircraft which now occurs 25.47 ft behind the nose of the aircraft. The new X_{LeMAC} is 25.45 ft. With that, the COG now occures at 0.2% the MAC. One more point to consider now is that the location of the fuel tanks also occurs 5 ft back, meaning the new location of the fuel is at X=28.7 ft. With this information, the second loading diagram can be constructed in a similar fashion to the first as shown in Figure 5.4.

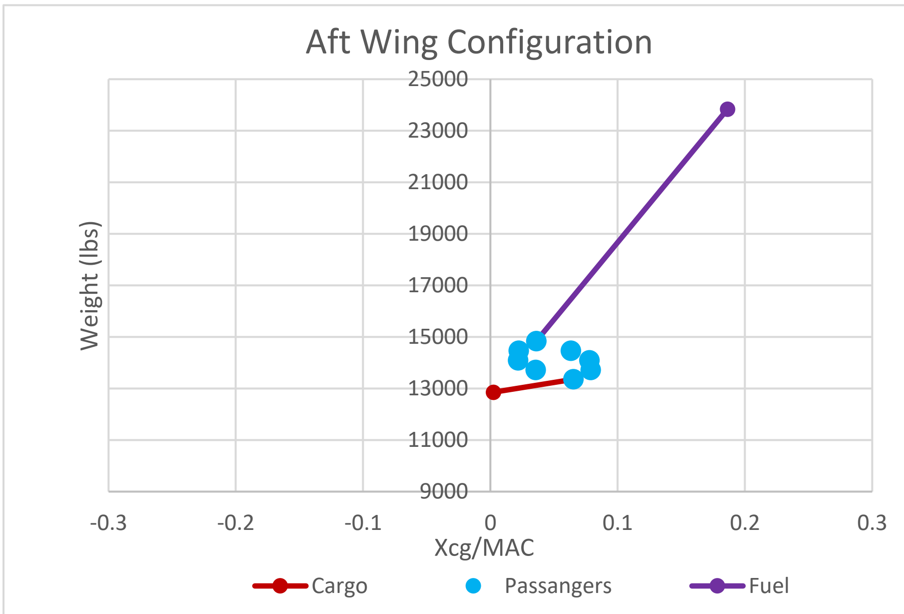


Figure 5.4: Aft Wing COG Loading Diagram

In this instance, the fuel load is largely responsible for the reward shift in COG. This is due to the location of the COG of the fuel tanks occurring at roughly 50% that of the MAC. By applying the same 2% safety factor, the minimum COG location of 0.2% of the MAC and maximum COG location of 19% the MAC can be found with a range of travel of 18.8% of the MAC.

5.2.2 Wing Configuration 3

Lastly is that of configuration 3, where the wings and associated components are moved forward 5 ft. X_{LeMAC} is now at 15.45 ft with the COG location at 86.9% of the MAC. The fuel location has also been shifted 5 ft forward to 18.75 ft. The final loading diagram can now be displayed below in Figure 5.5. The occurrence of the fuel tanks in front of the COG now causes a forward shift in COG upon loading. In the final configuration a minimum COG value of 77.1% of the MAC and a maximum value of 104% of the MAC can be observed. Together this gives the largest range of COG travel of 27.3% of the MAC. The min and max COG values for the 3 different configurations can then be plotted together for later use in Figure 5.6. The upper left points on the figure represent a wing that has been shifted forward while points on the lower right side of the figure represent a wing that has been shifted back.

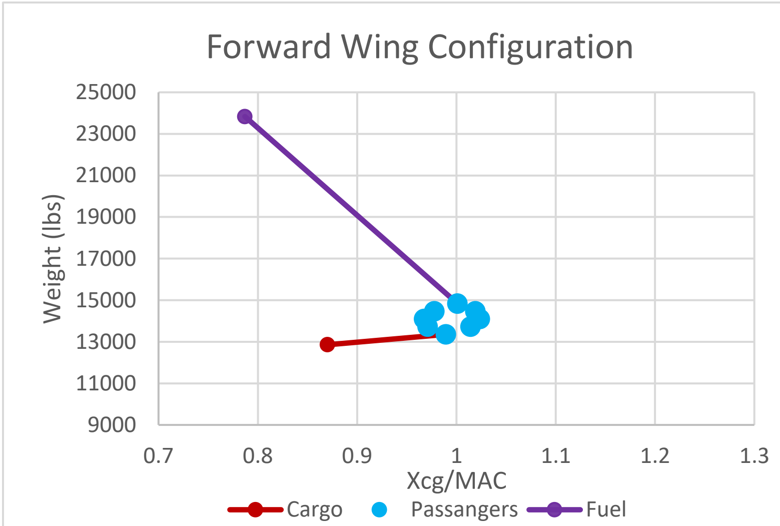


Figure 5.5: Forward Wing COG Loading Diagram

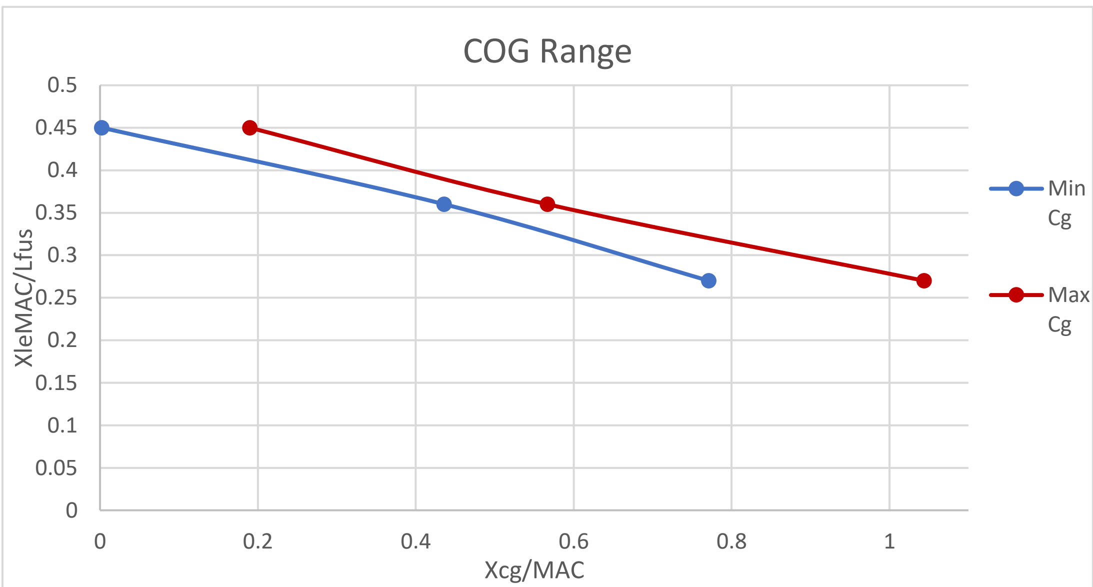


Figure 5.6: COG Range at Various Wing Locations

6. The scissor plot

Now that the center of gravity has been defined for the aircraft, stability and controllability of the aircraft can be assessed. This can be done by evaluating the required tail size and the subsequent effects it has on the aircraft.

6.1 The Stability Curve

There exists a point in between the wing of an aircraft and its tail such that when a gust or disturbance is applied to the aircraft, no rotation is applied, only linear translations. This point is known as the neutral point, NP, of the aircraft and is defined by Equation 6.1 below.

$$\bar{x}_{np} = \bar{x}_{ac} + \frac{C_{L\alpha_h}}{C_{L\alpha}} * \left(1 - \frac{d\varepsilon}{d\alpha}\right) * \frac{S_h * L_h}{S * \bar{c}} * \left(\frac{V_H}{V}\right)^2 \qquad Eq. 6.1$$

To have a stable aircraft when the stick is fixed, or the control surfaces do not fluctuate with turbulence, the COG should be in front of the NP. This difference between the distance of the COG and NP can represented as a safety margin where S.M. = Xnp-Xcg. For the purpose of designing a stable aircraft this value will be set at 0.05 to ensure that no fly by wire systems will be required to fly the aircraft safely. Eq 6.1 can be rearranged to obtain a nondimensionalized graph of tail size vs the location of the COG as a percentage of the MAC. This equation is depicted in Equation 6.2.

$$\frac{S_h}{S} = \frac{\bar{x}_{cg} - \bar{x}_{ac} + S.M.}{\frac{C_{L\alpha_h}}{C_{L\alpha}} * \left(1 - \frac{d\varepsilon}{d\alpha}\right) * \frac{l_h}{\bar{c}} * \left(\frac{V_h}{V}\right)^2} \qquad Eq. 6.2$$

Before this equation may be graphed, a few values must be found. First, and most simple is that of $\left(\frac{V_h}{V}\right)^2$, or the ratio of the speed of the free stream air over the tail vs the speed of the air over the wing. This value is typically quoted at being 0.85 for fuselage mounted stabilizers. Next, is that of the lift curve slope for the horizontal tail at cruise speed. To estimate this, the DATCOM method, shown by Equation 6.3, will be used with the cruise speed Prandtl Glauert correction factor.

$$C_{L\alpha_h} = \frac{2 * \pi * AR_h}{2 + \sqrt{4 + \left(\frac{AR_h * \beta}{\eta}\right)^2 * \left(1 + \frac{\tan^2 \Lambda_{\frac{1}{2}c_h}}{\beta^2}\right)}} \qquad Eq. 6.3$$

Here $\beta = 0.695$, AR was set at 5, an airfoil efficiency factor of 0.95 was used, and the half chord sweep was found to be 22 degrees from CAD. In all, the lift curve slope for the horizontal stabilizer comes out to be 4.703 rad^{-1} or 0.0821 $\deg^{-1}$. Next, the lift curve slope of the aircraft must be determined. This value is given by Equation 6.4.

$$C_{L\alpha} = C_{L\alpha_{A-h}} + C_{L\alpha_h} * \frac{S_h}{S} * \left(1 - \frac{d\varepsilon}{d\alpha}\right) * \left(\frac{V_H}{V}\right)^2 \qquad Eq. 6.4$$

To find this value, the lift curve slope of the aircraft without a tail must be found. This value can be determined through Equation 6.5

$$C_{L\alpha_{A-h}} = C_{L\alpha_w} * \left(1 + 2.15 * \frac{b_f}{b}\right) * \frac{S_{net}}{S} + \frac{\pi}{2} * \frac{b_f^2}{S} \qquad Eq. 6.5$$

$C_{L\alpha_w}$ is the value of the lift curve slope of the wing at high speeds and is 5.835 rad^{-1} as determined in earlier analysis. The variable bf is the intersection width of the fuselage and wing and is equal to 6.33 ft. Lastly, b and S are the wingspan and area respectively at 70.54 ft and 453.66 ft^2. Altogether, a lift curve slope for the tail less aircraft is found to be 7.997 rad^{-1}. Working back up, $\frac{d\varepsilon}{d\alpha}$, or the downwash effect on the tail, must be found. To utilize the empirical formula for $\frac{d\varepsilon}{d\alpha}$ presented by Equation 6.6, values for r and m_{tv} must be found. r is the distance between the wing MAC and the tail MAC divided by the half span of the wing. This value comes out to be approximately 0.968 for this aircraft. m_{tv} is likewise the vertical distance between the zero lift angle of the wing and tail divided by the half span. This comes out to be about-0.019.

$$\frac{d\varepsilon}{d\alpha} = \frac{K_{\varepsilon\Lambda}}{K_{\varepsilon\,\Lambda=0}} * \frac{C_{L\alpha_w}}{\pi * AR}\left(\frac{r}{r^2+m_{tv}^2} * \frac{0.4879}{\sqrt{r^2+0.6319+m_{tv}^2}} + \left[1+\left(\frac{r^2}{r^2+0.7915+5.0734*m_{tv}^2}\right)^{0.3113}\right] * \left(1-\sqrt{\frac{m_{tv}^2}{1+m_{tv}^2}}\right)\right) \qquad Eq. 6.6$$

The values $K_{\varepsilon\Lambda}$ and $K_{\varepsilon\,\Lambda=0}$ are factors that account for the sweep of the wing and are given by Equations 6.7 and 6.8 respectively below.

$$K_{\varepsilon\Lambda} = \frac{0.1124+0.1265\Lambda+0.1766\Lambda^2}{r^2} + \frac{0.1204}{r} + 2 \qquad Eq. 6.7$$

$$K_{\varepsilon\,\Lambda=0} = \frac{0.1124}{r^2} + \frac{0.1204}{r} + 2 \qquad Eq. 6.8$$

The effect of the downwash comes out to be 0.42. With that, the lift curve slope for the entire aircraft can be found to be 8.344 rad^{-1}. Continuing on, the location of the aerodynamic center of the entire aircraft minus the tail must be determined. This can be done by breaking up the aerodynamic center into the part depicted by Equation 6.9.

$$\bar{x}_{ac} = (\bar{x}_{ac})_{wf} + (\bar{x}_{ac})_{nacelle} + (\bar{x}_{ac})_{jet} \qquad Eq. 6.9$$

The location of the center of the aerodynamic center of the wing and fuselage group can be further broken down as shown by Equation 6.10. Also note, from here on out, the location of the aerodynamic center will be nondimensionalized with respect to the MAC.

$$\left(\frac{X_{ac}}{\bar{c}}\right)_{wf} = \left(\frac{X_{ac}}{\bar{c}}\right)_{w} - \frac{1.8*b_f*h_f*l_{fn}}{C_{L\alpha_{wf}}*S*\bar{c}} + \frac{0.273}{1+\lambda} * \frac{b_f*c_g*(b-b_f)}{\bar{c}^2*(b+2.15*b_f)} * \tan(\Lambda) \qquad Eq. 6.10$$

The aerodynamic center of the wing can be estimated using the carpet plots displayed in Figure 6.1. Note that because $\beta * AR = 7$ and only values for 6 and 8 are displayed, both will be utilized to linearly interpolate between the two. These two values for the main wing are shown in green below.

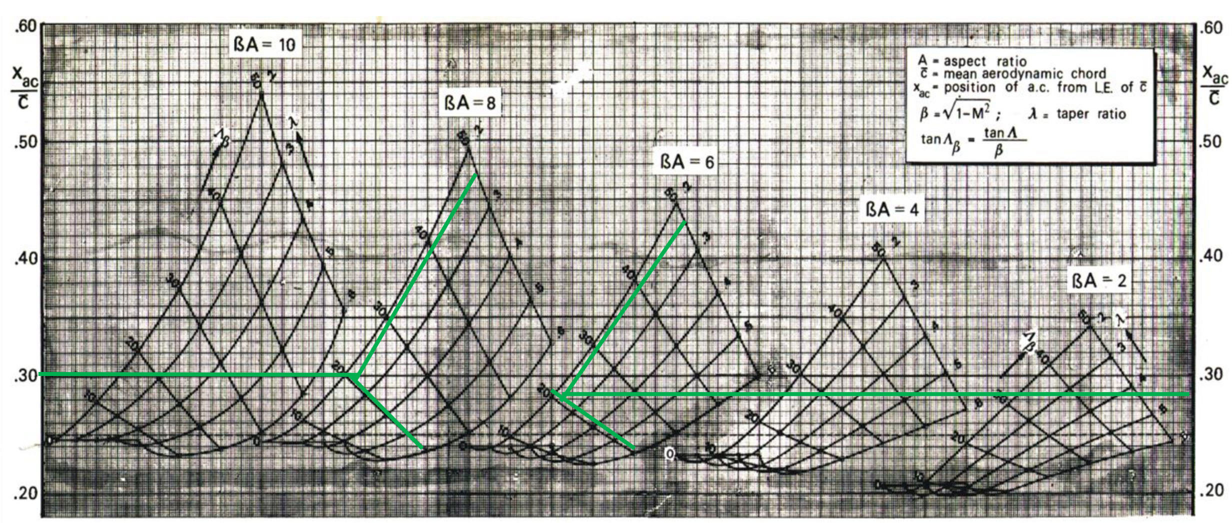


Figure 6.1: Carpet Plot for Finding Aerodynamic Surface of Wing

From the figure it can bee seen that Xac/c can be approximated to be 0.285. As for the other new variables, hf is the height of the fuselage at 6.33 ft, Lfn is the length to the start of the wing at 16 ft, and cg is the geometric chord S/b at 7.06 ft. Ultimately, $\left(\frac{X_{ac}}{\bar{c}}\right)_{wf}$ can be found to be equal to 0.285. The nacelle contribution to the aerodynamic center can be given by Equation 6.11.

$$\left(\frac{X_{ac}}{\bar{c}}\right)_{nacelle} = \sum k_n * \frac{b_n * l_n}{S * \bar{c} * C_{L\alpha_{wf}}} \qquad Eq. 6.11$$

Kn is a factor equal to -4 that accounts for nacelles mounted in front of the aircraft wings. ln is similarly, the distance from the MAC to the front of the nacelle which is 3 ft. bn is the width of the nacelle at 3.5 ft. This value is then multiplied by two to account for the two nacelles and comes out to be -0.003. Lastly, the contribution of the thrust is analysed. The contribution from the jet stream can be found with Equation 6.12.

$$\left(\frac{X_{ac}}{\bar{c}}\right)_T = -\sum \frac{T}{W} * \frac{z_T}{\bar{c}} \qquad Eq. 6.12$$

The thrust at cruise was determined to be 1461 lbs with a COG vertical offset, zt, of -0.3 ft. This results in a $\left(\frac{X_{ac}}{\bar{c}}\right)_T$ of 0.002. Ultimately, the total aerodynamic center, $\frac{\bar{x}_{ac}}{\bar{c}}$ is found to be 0.285. Now all the required values have been determined to construct the stability curve utilizing the neutral point and separately with the safety margin. This graph is depicted in Figure 6.5 at the end of section 6.3 as to allow for the combination with the controllability curve.

6.2 The Controllability Curve

While tail sizing is important for static stability, it is also extremely important in controlling the aircraft. Similar to how stability was defined around tail sizing and the NP, controllability can be defined in terms of nondimensionalized tail size verses the COG location. This is shown by Equation 6.13.

$$\frac{S_h}{S} = \frac{\frac{C_{m_{ac}}}{C_{L_{A-h}}} + \bar{x}_{cg} - \bar{x}_{ac}}{\frac{C_{L\alpha_h}}{C_{L\alpha}} * \frac{l_h}{\bar{c}} * \left(\frac{V_h}{V}\right)^2} \qquad Eq. 6.13$$

One main difference here is that the worst-case scenario that the aircraft will be design around is now during landing/takeoff conditions. With that in mind, a few variables will change. $C_{L_{A-h}}$ is now considered to be 5.436, or the lift curve slope of the wing in landing configuration. Furthermore, the aerodynamic center is recalculated using the methods previously discussed to be 0.322. $C_{L_h}$is set to -0.8 since an adjustable tail is to be implemented on the aircraft. The last variable that needs to be found is that of the moment coefficient, $C_{m_{ac}}$. Similar to the aerodynamic center location, the moment coefficient can be broken up into components as describe by Equation 6.14.

$$C_{m_{ac}} = C_{m_{ac_w}} + \Delta_f C_{m_{ac}} + \Delta_{fus} C_{m_{ac}} + \Delta_{nac} C_{m_{ac}} \qquad Eq. 6.14$$

The overall wing moment coefficient is assumed to be -0.1. with the change in moment due to the flaps assumed to be -0.05. The change in moment due to the fuselage is given below by Equation 6.15.

$$\Delta_{fus}C_{m_{ac}} = -1.8 * \left(1 - 2.5 * \frac{b_f}{l_f}\right) * \frac{\pi * b_f * h_f * l_f}{4 * S * \bar{c}} * \frac{C_{L0}}{C_{L\alpha_{wf}}} \qquad Eq. 6.15$$

The only new value here is that of C_{L0}, or the coefficient of lift of the flapped wing at zero angle of attack. As previously found, this value is 1.155. In the end, it is found that the contribution on the moment coefficient due to the fuselage is essentially 0. Lastly, the contribution from the flaps can be considered. The contribution can be calculated from Equation 6.16.

$$\Delta_{\mathrm{f}}C_{m_{ac}} = \mu_2\left(-\mu_1 * \Delta C_{lmax} * \frac{c'}{c} - \left[C_L + \Delta C_{lmax}\left\{1 - \frac{S_{wf}}{S}\right\}\right] * \frac{1}{8} * \frac{c'}{c} * \left[\frac{c'}{c} - 1\right]\right) + 0.7 * \frac{AR}{1 + \frac{2}{AR}} * \mu_3 * \Delta C_{lmax} * tan\Lambda_{\frac{1}{4}} - C_L * \left(0.25 - \frac{x_{ac}}{\bar{c}}\right) \qquad Eq. 6.16$$

c'/c was determined to be 1.1875 when selecting single slotted fowler flaps. The ΔC_{lmax} was similarly found to be 1.544. CL, or the coefficient of lift required for landing was set early on to be 2.4. $\frac{S_{wf}}{S}$ was also found earlier to be 0.6 for the trailing edge devices that are primarily responsible for the change in moment. The remaining Mu factors 1, 2, and 3 can be determined through Figures 6.2, 6.3, and 6.4 respectively. From the figures it can be seen that Mu 1 = 0.2125, Mu 2 = 0.95, and Mu 3 = 0.05. All of this together can be used to find the moment coefficient contribution from the flaps to be -0.188. With all the new values solved for, the controllability curve can be plotted alongside the stability curve.

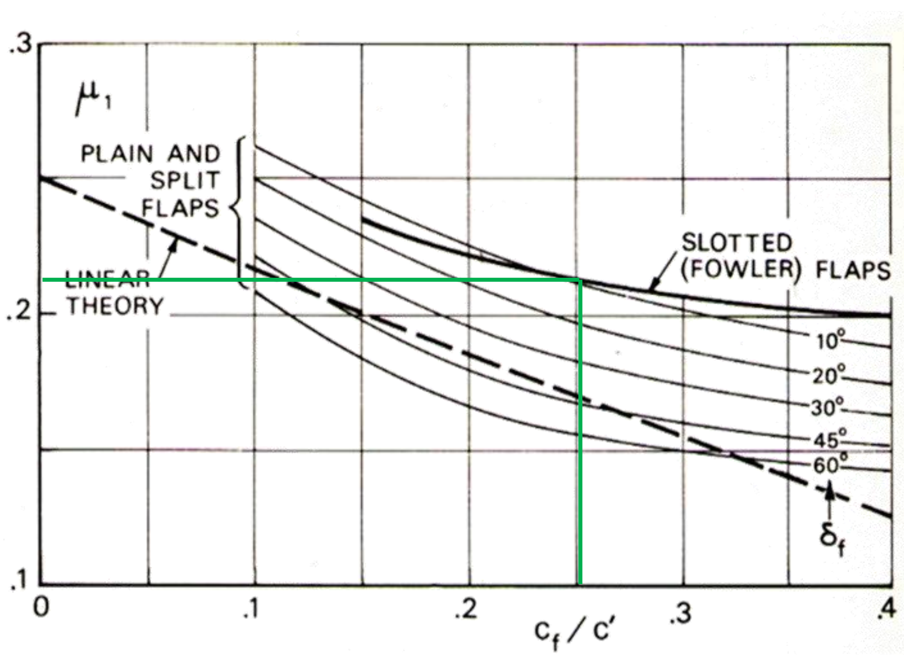


Figure 6.2: Flap Length, over Extended Chord vs Mu 1

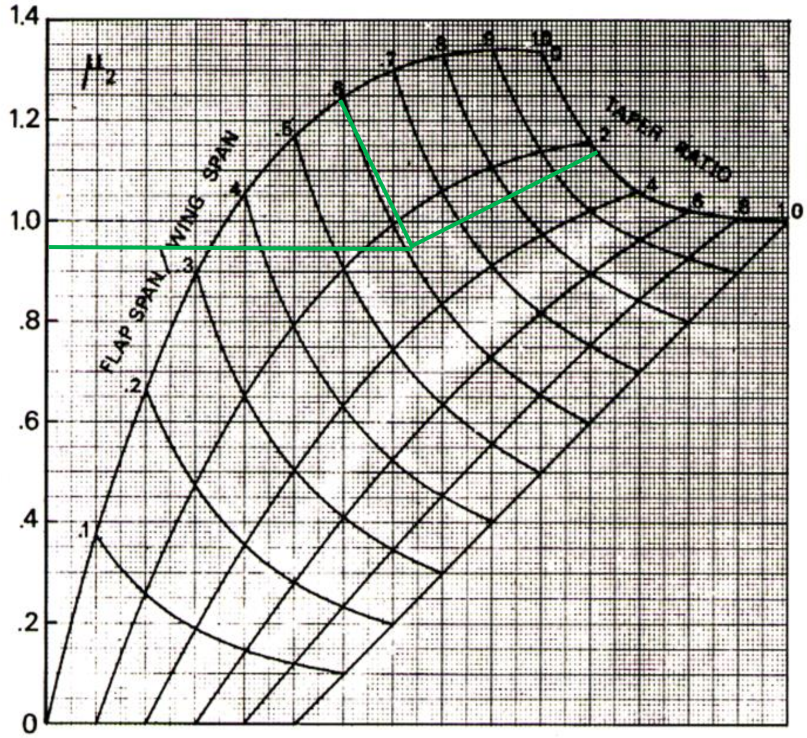


Figure 6.3: Mu 2 vs Flap Span over Wingspan and Taper Ratio

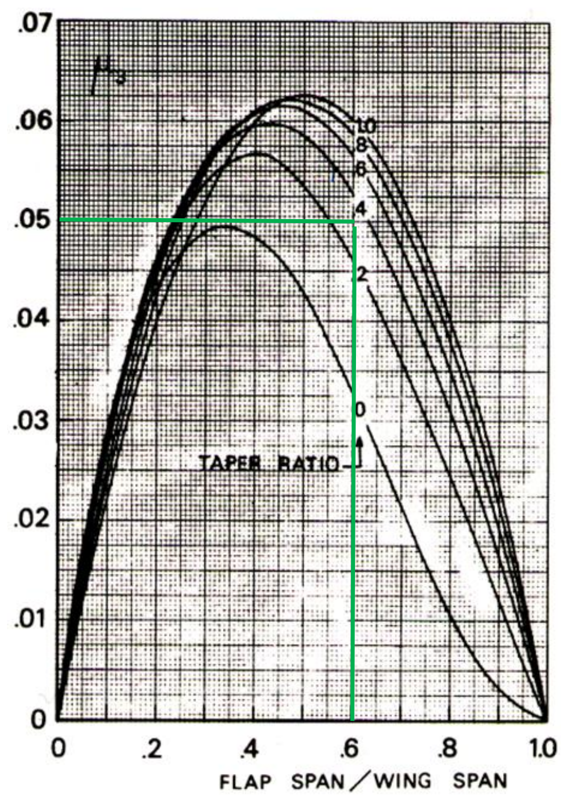


Figure 6.4: Mu 3 vs Flap Span over Wingspan and Taper Ratio

6.3 The Scissor Plot

With both the Eq. 6.2 and 6.13 fully satisfied, the stability and controllability curves can be plotted. The two curves are displayed below in Figure 6.5.

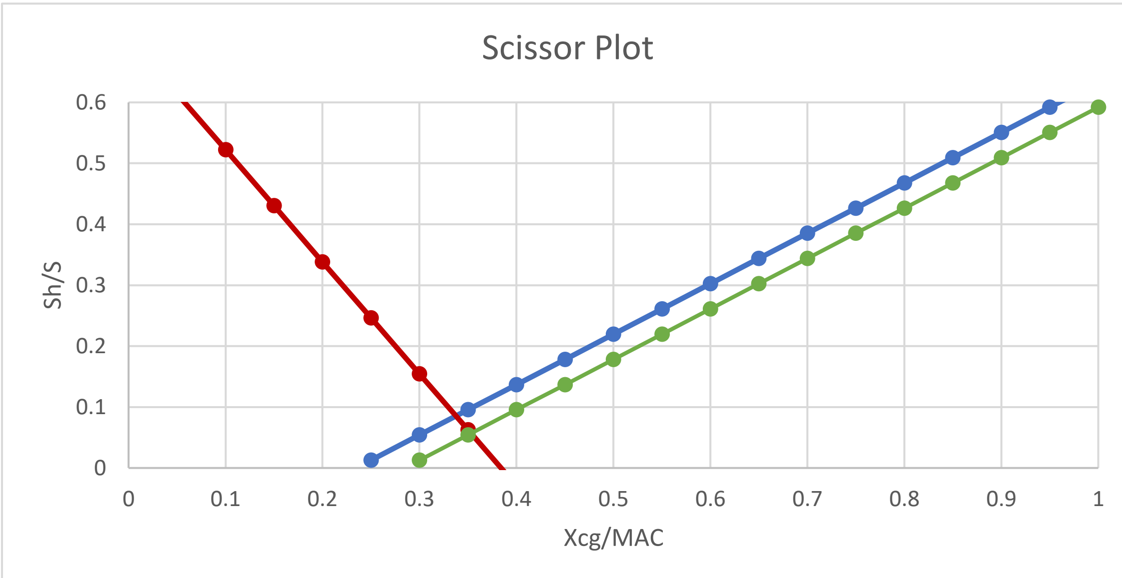


Figure 6.5: Stability Curve Overlayed Upon Controllability Curve

The negative slope line represents the controllability curve. Anything to the right of this slope represents a tail and COG combination that is controllable. Anything to the left is uncontrollable. The positive sloped lines represent the stability curves with the left line representing that of the applied safety margin. To the left of these curves represent an aircraft that is stable, while anything to the right of these lines represents an aircraft that is unstable. Note, that there are some special considerations that may add on top of these curves that will not be taken into account for the purpose of this design. With the Scissor plot fully realised, the above figure can now be overlayed upon the range of COG graph depicted by Figure 5.6. By adjusting the scaling, the two figures can be lined up to find a point at which the linear COG travel is fully acceptable by a given tail size. This overlay and intersection zone is depicted below in Figure 6.6 From the intersections on the graph it can be concluded that the minimum tail size required is 0.15 time the wing surface area or 75 ft^2. Furthermore, the optimal location for the wing is at $\frac{X_{LeMAC}}{L_{fus}} = 0.395$, or 22.12 ft back from the nose of the aircraft.

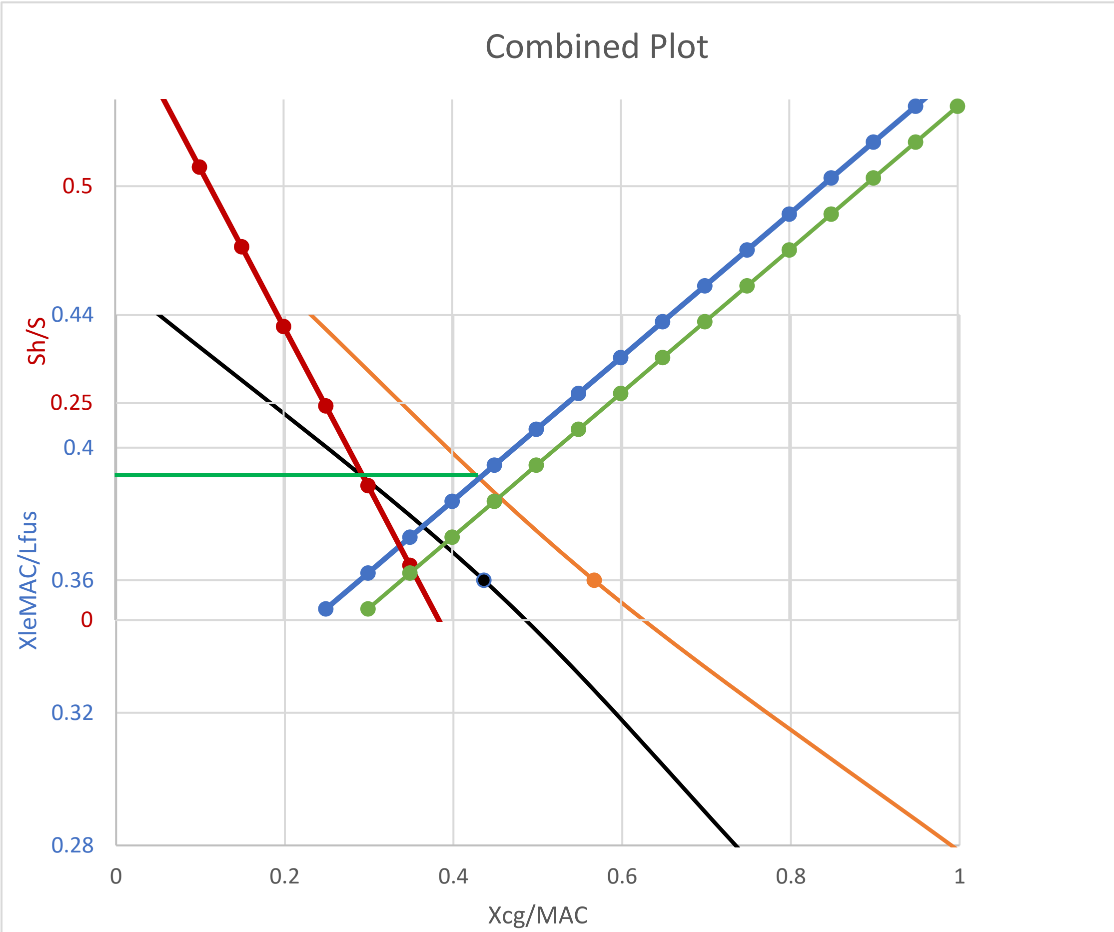


Figure 6.6 Scissor Plot Overlayed on top of COG Travel Plot

7. Empennage Revaluation

Now that slight changes to the aircraft have been taken into consideration, the empennage can be revaluated to see if it meets requirements. The horizontal tail is considered to be acceptable as the assumed size from the V bar method was only slightly larger than that of the found minimum required size for stability and controllability. The vertical tail on the other hand will require slight further investigation. The vertical tail is integral in keeping the plane under control during an engine out condition and in preventing unwanted swaying motions during flight. The vertical tail sizing can be aided using Figure 7.1 below which is a historical trend based upon other jet aircraft with wing mounted engines. First the X value must be determined. Ye is horizontal offset of the engines from the fuselage centerline and is 15.42 ft. lv is the distnace between COG and tail MAC and is 28.97 ft. ΔT_e is the differential thrust that could be experienced and is set at 2922 lbs to ensure the aircraft has the capability to climb safely in the case of a sing engine faliure. The CLmax at takeoff is 1.8 and the difference in MTOW and payload weight is 21870 lbs. Ultimately, this gives an X value of 0.128. This in turn corrilates to a Y value of 0.124 for a 2 engine aircraft as seen in Figure 7.1. Now these variable can be unpacked to find the required tail size in terms of the wing surface area. Here, k_∂ is a factor based

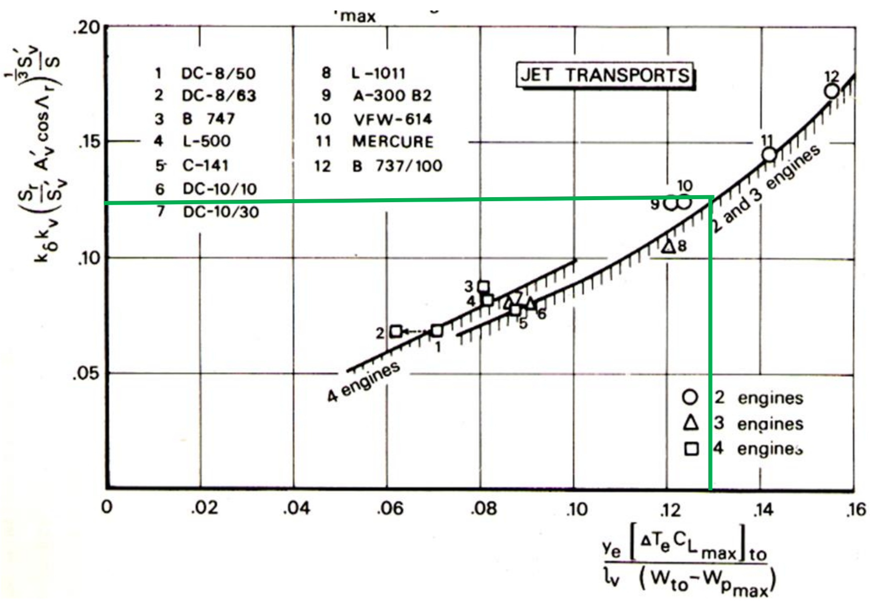


Figure 7.1: Trend for Tail Sizing of Jet Aircraft with Wing Mounted Engines

on rudder defleciton angles and can be found through Figure 7.2. With a max allowed deflection of 25 degrees, the k_∂ value is found to be 0.95.

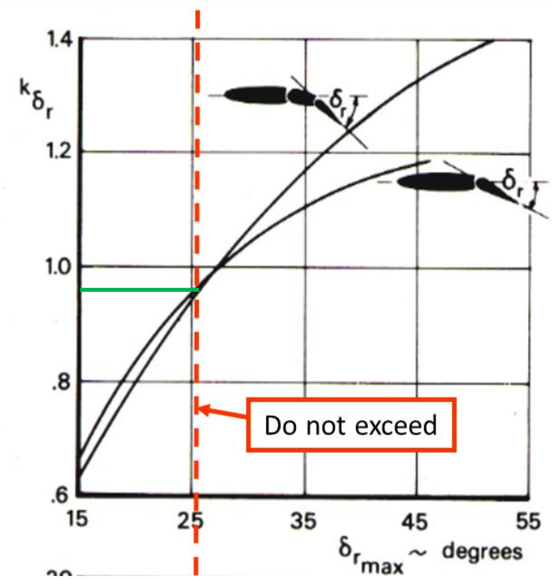


Figure 7.2: k_∂ vs flap deflection

Kv is a factor based on tail type and is 1 for the selected conventional tail. The rudder area vs the vertical tail area is estimated to be about 26% as to ensure a reasonably sized tail. The aspect ratio of the rudder is defined as the square of the height divided by the area and is assumed to be 1.6. Lastly the sweep of the rudder is assumed to be 20 degrees. With all of that. The minimum required rudder area can be determined to be roughly 90 ft^2. As 85 ft^2 is less that 6% different than the required size, no changes will be made to the vertical tail. Supplimentary fins may be added however to increase the rudder surface area. Ultimately this concludes the weight and balance anylasis of the aircarft.

References

[1] W. De Backer, "Wing Sizing University of South Carolina, 2021.

AESP415_001_Fall2021_09-WeightAndBalance_Files

AESP415_001_Fall2021_09-WeightAndBalance_V3

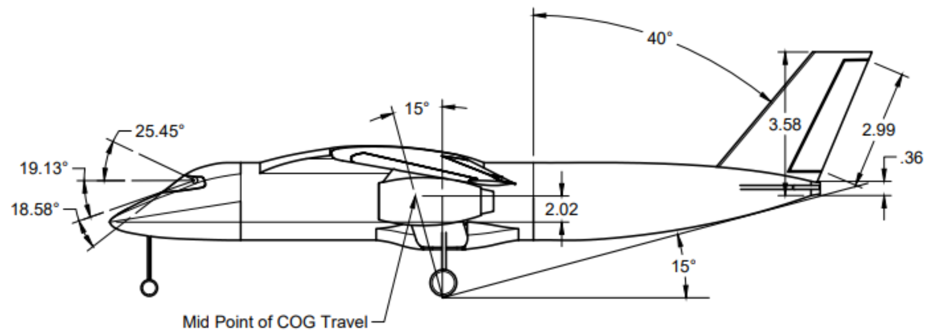
AESP415_001_Fall2021_10-TailPlaneDesign_V1

[2] D. Raymer, *Aircraft design*. Reston: American Institute of Aeronautics and Astronautics, 2021.

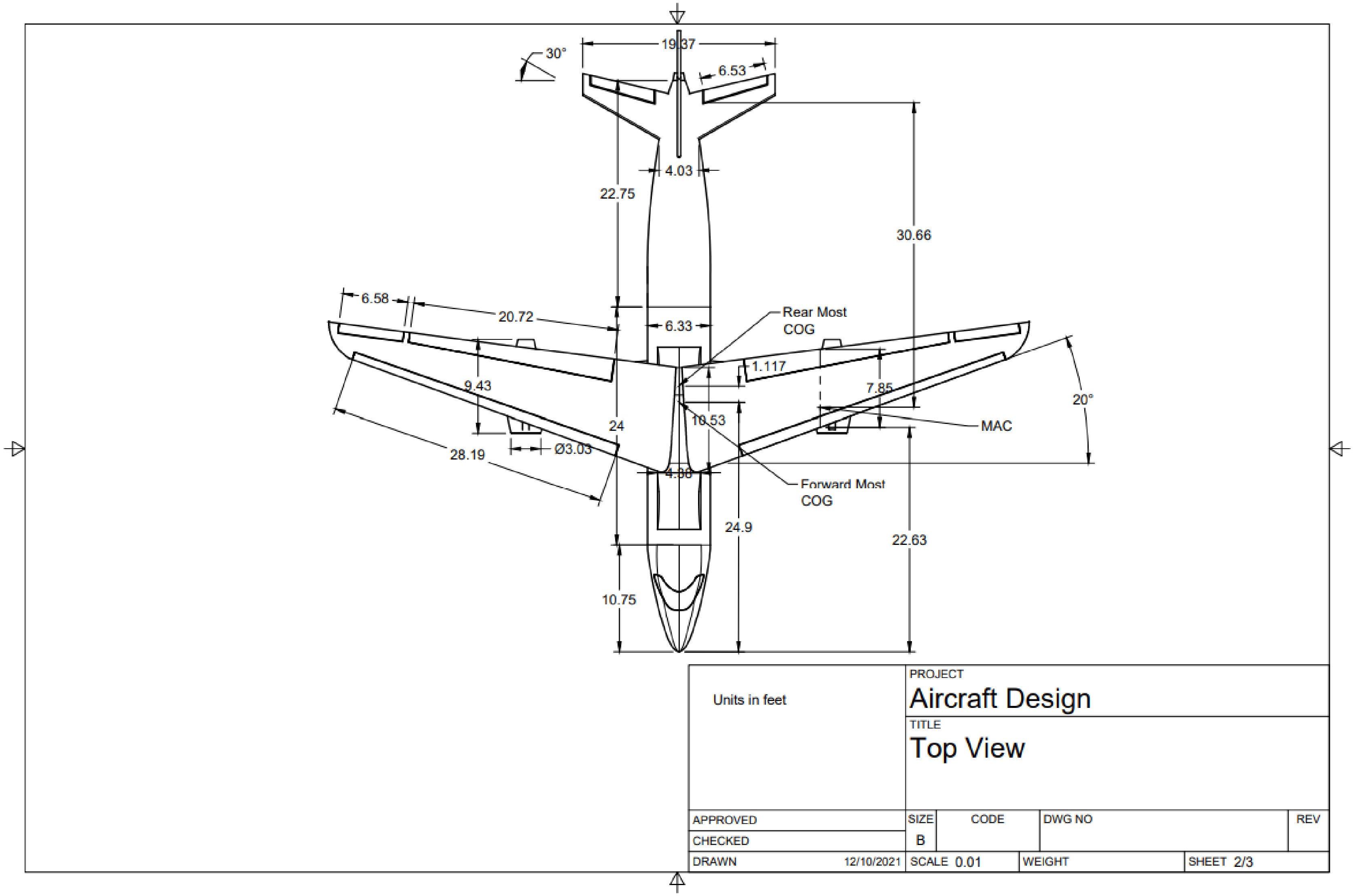
Appendix A: Updated Drawings

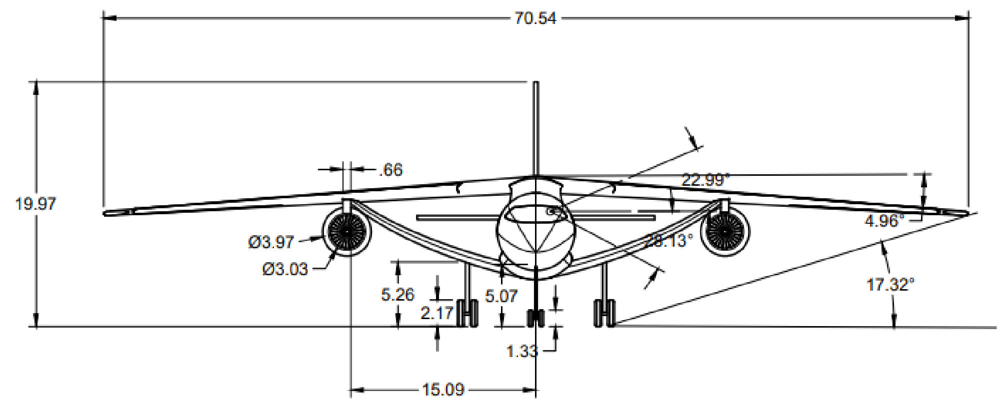
Below the technical Drawings for the aircraft are presented in the following order:

1. Top View
2. Side View
3. Front View
4. Fuselage Cross Section

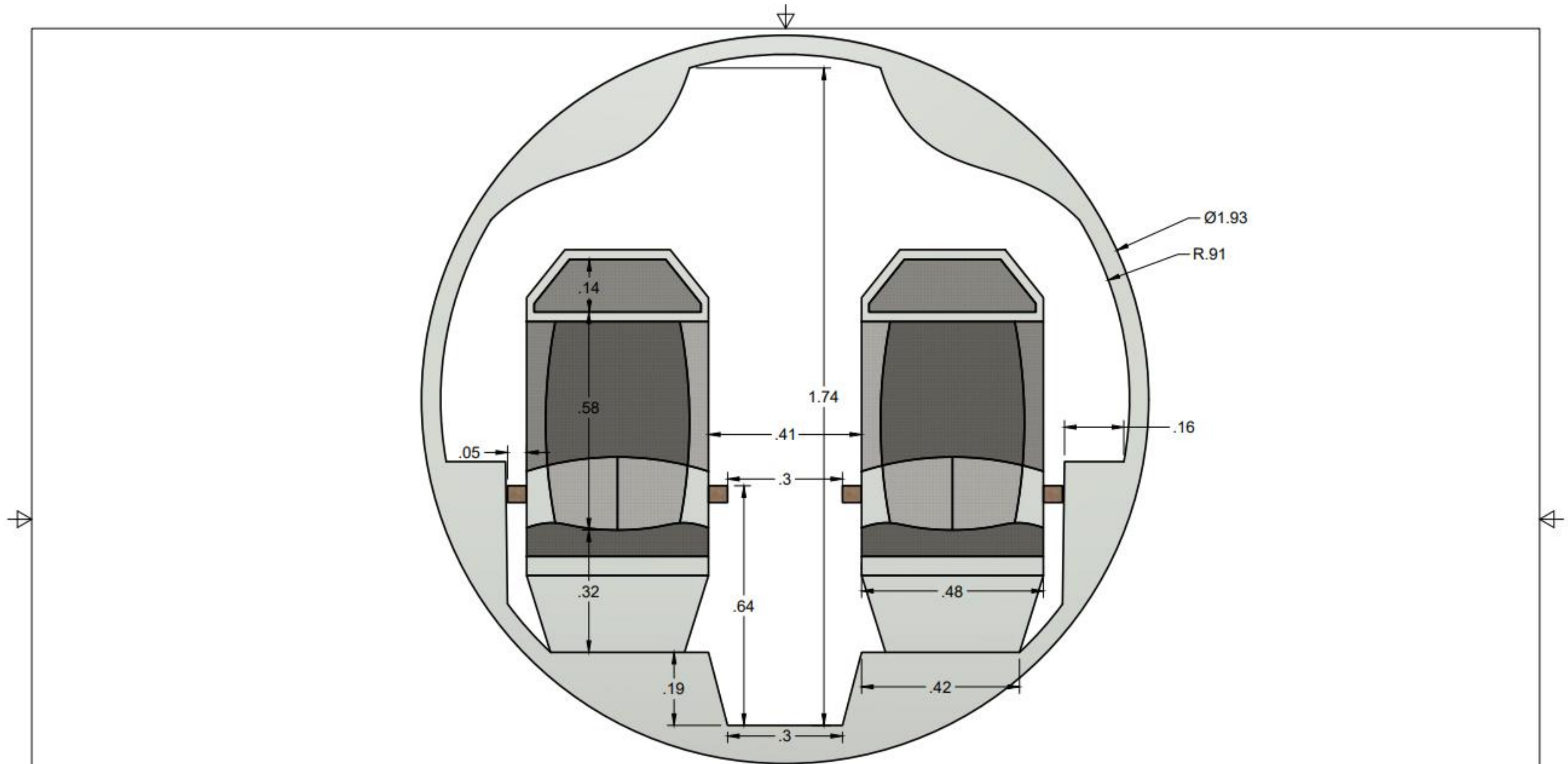


Units in meters	PROJECT Aircraft Design			
	TITLE Side View			
APPROVED	SIZE	CODE	DWG NO	REV
CHECKED	B			
DRAWN 12/10/2021	SCALE 0.01	WEIGHT	SHEET 3/3	


30°
19.37
6.53
4.03
22.75
30.66
6.58
20.72
6.33
Rear Most
COG
1.117
7.85
20°
9.43
MAC
24
10.53
28.19
Ø3.03
4.38
Forward Most
COG
24.9
22.63
10.75
Units in feet
PROJECT
Aircraft Design
TITLE
Top View
APPROVED
CHECKED
DRAWN 12/10/2021
SIZE
B
CODE
DWG NO
REV
SCALE 0.01
WEIGHT
SHEET 2/3



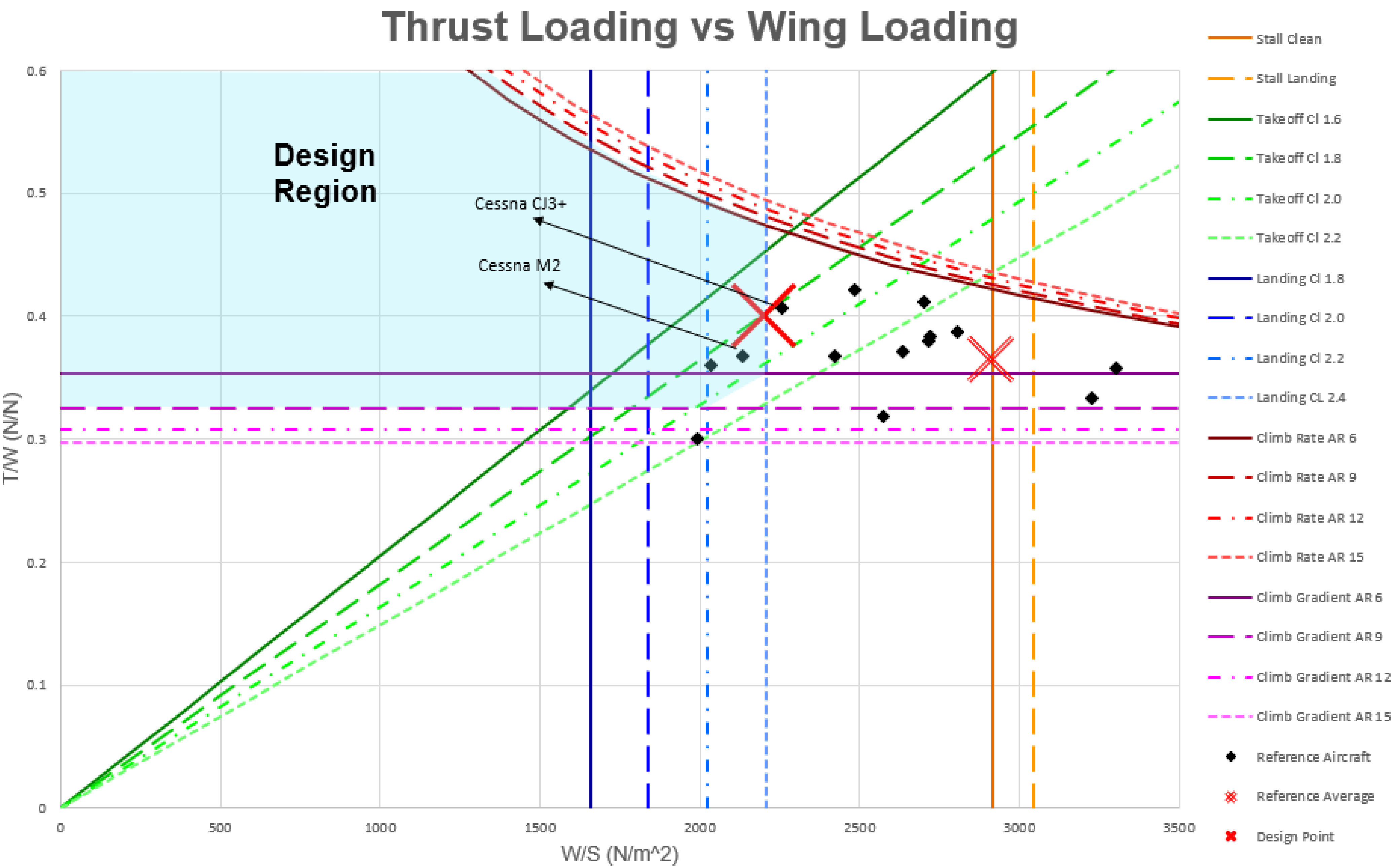
Units in feet	PROJECT Aircraft Design				
	TITLE Front View				
APPROVED	SIZE	CODE	DWG NO		REV
CHECKED	B				
DRAWN 12/10/2021	SCALE 0.01		WEIGHT	SHEET 1/3	



Units in Meters	PROJECT Aircraft Design					
	TITLE Fuselage Cross Section					
APPROVED	SIZE	CODE	DWG NO			REV
CHECKED	B					
DRAWN 11/9/2021	SCALE 1:10		WEIGHT		SHEET 1/1	

Appendix B: T/W – W/S graph

Below the Thrust Loading vs Wing Loading Graph is presented.


Thrust Loading vs Wing Loading
Design Region
Cessna CJ3+
Cessna M2
T/W (N/N)
W/S (N/m^2)
0
0.1
0.2
0.3
0.4
0.5
0.6
0
500
1000
1500
2000
2500
3000
3500
Stall Clean
Stall Landing
Takeoff Cl 1.6
Takeoff Cl 1.8
Takeoff Cl 2.0
Takeoff Cl 2.2
Landing Cl 1.8
Landing Cl 2.0
Landing Cl 2.2
Landing CL 2.4
Climb Rate AR 6
Climb Rate AR 9
Climb Rate AR 12
Climb Rate AR 15
Climb Gradient AR 6
Climb Gradient AR 9
Climb Gradient AR 12
Climb Gradient AR 15
Reference Aircraft
Reference Average
Design Point

Appendix C: Table of Reference Aircraft

Aircraft Name	A/C Type	Payload [lb]	range [nm]	cruise speed [kn]	Height [ft]	Length [ft]	Wingspan [ft]	MTOW [lb]	Take off Length (sea level) [ft]	Landing field length at max landing weight [ft]
105	Business Jet	1984.5	1728	415.77	16.25	54.5	40	TBD	2953	2624.8
CJ3+	Business Jet	1971.27	2040	416	15.16	51.15	53.35	13871.65	3179	2769.16
Cessna Citation CJ4	Business Jet	2220.44	2165	451	15.39	53.35	50.82	17113.00	3409	2939.77
Nextant 400XT	Business Jet	2469	2108	460	13.91	48.42	43.50	16300	3821	4045
Learjet 75	Business Jet	2902	2080	465	14.42	56.42	50.92	21500	4440	2296
Learjet 40	Business Jet	2305	1552	457	14.14	55.55	47.77	21000	4250	2660
Cessna Citation XLS+	Business Jet	2240.28	2100	441.14	17.06	52.50	56.33	21002.62	3560	3179.28
Phenom 300	Business Jet	2216	2077	430	16.41	52.12	53.12	17968	3138	3700
Eclipse 500/550	Business Jet	1800	1125.26	374.73	11.15	33.46	38.05	6000	2434.50	2788.85
Pilatus PC-24	Business Jet	2500.47	2000	440	17.38	55.12	55.77	18301.5	2929.93	2375.44
Pilatus PC-12	Business Jet	2235.87	1802.91	290	13.97	47.24	53.41	10451.7	2486.99	2168.74

Hawker 400	Business Jet	2000	1519	443	13.75	48.13	43.41	16300	3950	2960
Premier 1A	Business Jet	4000	1350	451	15.33	46	44.5	12500	3792	3170
HA-420	Business Jet	1400	1223	422	14.9	42.6	39.8	9964.39	3934	3047
Spectrum S-40 Freedom	Business Jet	2401.24	2200	440	11.81	53.8084	45.6059	9550	2998.83	2700.26
Stratos 714/716x	Business Jet	2160.9	1777	415	11.41	35.79571	40.48754	8423.1	2089.99	1509.26
Syberjet SJ30	Business Jet	1500	2129	436	14.20	46.78706	42.3249	13951.03	3937.2	2552.61
Gulfstream G150	Business Jet	2400	3130	459	19.08	56.75	55.583	26100	5012	2431
Preator 500	Business Jet	1680.21	3250	466	21.08	64.583	70.5	37567	4222.64	2091
Phenom 100EV	Business Jet	1000	1178	406	14.27	42.06242	40.3563	10852	3190	2430
Learjet 60	Business Jet	2200	2408.74	453	14.5	58.583	43.66	23500	5920	3120
Fairchild Dornier 328JET	Business Jet	2952	1800	400	23.12	69.65563	68.83538	34530.3	4485.12	4284.98
Citation Latitude	Business Jet	2544	2699.78	446	20.93	62.24057	72.34605	30806.05	3579.57	2480
Citation M2	Business Jet	1510.42	1550.21	403.88	13.91	42.58738	47.2464	10700.86	3208.81	2590
				AVERAGE:	15.53	51.08	50.33	17750.14		

Appendix D: Current Aircraft parameters Table

Symbol	Parameters	Value	Unit
	Flight Parameters		
h_{cruise}	Cruise altitude	12192	m
V_{cruise}	Cruise speed	770	km/h
CL_{cruise}	Cruise lift coefficient	1.8	
CL_{max}	Max lift coefficient	2.6	
s_{TO}	Take-off distance	900	m
S_L	Landing distance	800	m
R	Range	3200	km
	Weight Parameters		
W_{MTOW}	Maximum Take-off Weight	101.7	kN
W/S	Wing Loading	2200	N/m^2
T/W	Thrust to weight ratio	0.4	
W_F	Fuel Weight	32.79	kN
W_{BO}	Basic operating weight	62.9	kN
	Power Plant Parameters		
T	Take-off Thrust	20.4	kN
D	Fan Diameter	906	mm
L	Length	1933	mm
W dry	Dry Weight	392	kg
	SFC Cruise	0.07239	kg/N*h

Appendix E: Raymer Weight Estimation Variables and Equations

Cargo/Transport Weights

$$W_{\text{wing}} = 0.0051\,(W_{dg}N_z)^{0.557} S_w^{0.649} A^{0.5} (t/c)_{\text{root}}^{-0.4} (1+\lambda)^{0.1} \times (\cos\Lambda)^{-1.0} S_{csw}^{0.1} \quad (15.25)$$

$$W_{\substack{\text{horizontal}\\ \text{tail}}} = 0.0379\,K_{uht}(1+F_w/B_h)^{-0.25} W_{dg}^{0.639} N_z^{0.10} S_{ht}^{0.75} L_t^{-1.0} \times K_y^{0.704} (\cos\Lambda_{ht})^{-1.0} A_h^{0.166} (1+S_e/S_{ht})^{0.1} \quad (15.26)$$

$$W_{\substack{\text{vertical}\\ \text{tail}}} = 0.0026\,(1+H_t/H_v)^{0.225} W_{dg}^{0.556} N_z^{0.536} L_t^{-0.5} S_{vt}^{0.5} K_z^{0.875} \times (\cos\Lambda_{vt})^{-1} A_v^{0.35} (t/c)_{\text{root}}^{-0.5} \quad (15.27)$$

$$W_{\text{fuselage}} = 0.3280\,K_{\text{door}} K_{Lg} (W_{dg}N_z)^{0.5} L^{0.25} S_f^{0.302} (1+K_{ws})^{0.04} (L/D)^{0.10} \quad (15.28)$$

$$W_{\substack{\text{main landing}\\ \text{gear}}} = 0.0106\,K_{mp} W_l^{0.888} N_l^{0.25} L_m^{0.4} N_{mw}^{0.321} N_{mss}^{-0.5} V_{\text{stall}}^{0.1} \quad (15.29)$$

$$W_{\substack{\text{nose landing}\\ \text{gear}}} = 0.032\,K_{np} W_l^{0.646} N_l^{0.2} L_n^{0.5} N_{nw}^{0.45} \quad (15.30)$$

$$W_{\substack{\text{nacelle}\\ \text{group}}} = 0.6724\,K_{ng} N_{Lt}^{0.10} N_w^{0.294} N_z^{0.119} W_{ec}^{0.611} N_{en}^{0.984} \underset{\text{(includes air induction)}}{S_n^{0.224}} \quad (15.31)$$

$$W_{\substack{\text{engine}\\ \text{controls}}} = 5.0\,N_{en} + 0.80\,L_{ec} \quad (15.32)$$

$$W_{\substack{\text{starter}\\ \text{(pneumatic)}}} = 49.19\left(\frac{N_{en}W_{en}}{1000}\right)^{0.541} \quad (15.33)$$

$$W_{\substack{\text{fuel}\\ \text{system}}} = 2.405\,V_t^{0.606} (1+V_i/V_t)^{-1.0} (1+V_p/V_t) N_t^{0.5} \quad (15.34)$$

$$W_{\substack{\text{flight}\\ \text{controls}}} = 145.9\,N_f^{0.554} (1+N_m/N_f)^{-1.0} S_{cs}^{0.20} (I_y \times 10^{-6})^{0.07} \quad (15.35)$$

$$W_{\substack{\text{APU}\\ \text{installed}}} = 2.2\,W_{\substack{\text{APU}\\ \text{uninstalled}}} \quad (15.36)$$

$$W_{\text{instruments}} = 4.509\,K_r K_{tp} N_c^{0.541} N_{en} (L_f + B_w)^{0.5} \quad (15.37)$$

$$W_{\text{hydraulics}} = 0.2673\,N_f (L_f + B_w)^{0.937} \quad (15.38)$$

$$W_{\text{electrical}} = 7.291\,R_{kva}^{0.782} L_a^{0.346} N_{gen}^{0.10} \quad (15.39)$$

$$W_{\text{avionics}} = 1.73\,W_{uav}^{0.983} \quad (15.40)$$

$$W_{\text{furnishings}} = 0.0577\,N_c^{0.1} W_c^{0.393} S_f^{0.75} \quad (15.41)$$

$$W_{\substack{\text{air}\\ \text{conditioning}}} = 62.36\,N_p^{0.25} (V_{pr}/1000)^{0.604} W_{uav}^{0.10} \quad (15.42)$$

$$W_{\text{anti-ice}} = 0.002\,W_{dg} \quad (15.43)$$

$$W_{\substack{\text{handling}\\ \text{gear}}} = 3.0 \times 10^{-4}\,W_{dg} \quad (15.44)$$

$$W_{\substack{\text{military cargo}\\ \text{handling system}}} = 2.4 \times (\text{cargo floor area, ft}^2) \quad (15.45)$$

K_{cb} = 2.25 for cross-beam (F-111) gear; = 1.0 otherwise
K_d = duct constant (see Fig. 15.2)
K_{door} = 1.0 if no cargo door; = 1.06 if one side cargo door; = 1.12 if two side cargo doors; = 1.12 if aft clamshell door; = 1.25 if two side cargo doors and aft clamshell door
K_{dw} = 0.768 for delta wing; = 1.0 otherwise
K_{dwf} = 0.774 for delta wing aircraft; = 1.0 otherwise
K_{Lg} = 1.12 if fuselage-mounted main landing gear; = 1.0 otherwise
K_{mc} = 1.45 if mission completion required after failure; = 1.0 otherwise
K_{mp} = 1.126 for kneeling gear; = 1.0 otherwise
K_{ng} = 1.017 for pylon-mounted nacelle; = 1.0 otherwise
K_{np} = 1.15 for kneeling gear; = 1.0 otherwise
K_p = 1.4 for engine with propeller or 1.0 otherwise
K_r = 1.133 if reciprocating engine; = 1.0 otherwise
K_{rht} = 1.047 for rolling tail; = 1.0 otherwise
K_{tp} = 0.793 if turboprop; = 1.0 otherwise
K_{tpg} = 0.826 for tripod (A-7) gear; = 1.0 otherwise
K_{tr} = 1.18 for jet with thrust reverser or 1.0 otherwise
K_{uht} = 1.143 for unit (all-moving) horizontal tail; = 1.0 otherwise
K_{vg} = 1.62 for variable geometry; = 1.0 otherwise
K_{vs} = 1.19 for variable sweep wing; = 1.0 otherwise
K_{vsh} = 1.425 if variable sweep wing; = 1.0 otherwise
K_{ws} = $0.75[1 + 2\lambda)/(1 + \lambda)]$ $(B_w \tan\Lambda/L)$
K_y = aircraft pitching radius of gyration, ft ($\cong 0.3L_t$)
K_z = aircraft yawing radius of gyration, ft ($\cong L_t$)
L = fuselage structural length, ft (excludes radome, tail cap)
L_a = electrical routing distance, generators to avionics to cockpit, ft
L_d = duct length, ft
L_{ec} = length from engine front to cockpit—total if multiengine, ft
L_f = total fuselage length
L_m = length of main landing gear, in.
L_n = nose gear length, in.
L_s = single duct length (see Fig. 15.2)
L_{sh} = length of engine shroud, ft
L_t = tail length; wing quarter-MAC to tail quarter-MAC, ft
L_{tp} = length of tailpipe, ft
M = Mach number
N_c = number of crew
N_{ci} = 1.0 if single pilot; = 1.2 if pilot plus backseater; = 2.0 pilot and copassenger
N_{en} = number of engines
N_f = number of functions performed by controls (typically 4–7)
N_{gen} = number of generators (typically = N_{en})
N_l = ultimate landing load factor; = $N_{gear} \times 1.5$
N_{Lt} = nacelle length, ft
N_m = number of mechanical functions (typically 0–2)
N_{mss} = number of main gear shock struts
N_{mw} = number of main wheels
N_{nw} = number of nose wheels

Weights Equations Terminology

A = aspect ratio
B_h = horizontal tail span, ft
B_w = wing span, ft
D = fuselage structural depth, ft
D_e = engine diameter, ft
F_w = fuselage width at horizontal tail intersection, ft
H_t = horizontal tail height above fuselage, ft
H_t/H_v = 0.0 for conventional tail; 1.0 for "T" tail
H_v = vertical tail height above fuselage, ft
I_y = yawing moment of inertia, lb-ft^2 (see Chap. 16)

N_p = number of personnel onboard (crew and passengers)
N_s = number of flight control systems
N_t = number of fuel tanks
N_u = number of hydraulic utility functions (typically 5–15)
N_w = nacelle width, ft
N_z = ultimate load factor; = 1.5 × limit load factor
q = dynamic pressure at cruise, lb/ft^2
R_{kva} = system electrical rating, kv · A (typically 40–60 for transports, 110–160 for fighters & bombers)
S_{cs} = total area of control surfaces, ft^2
S_{csw} = control surface area (wing-mounted), ft^2
S_e = elevator area, ft
S_f = fuselage wetted area, ft^2
S_{fw} = firewall surface area, ft^2
S_{ht} = horizontal tail area
S_n = nacelle wetted area, ft^2
S_r = rudder area, ft^2
S_{vt} = vertical tail area, ft^2
S_w = trapezoidal wing area, ft^2
SFC = engine specific fuel consumption—maximum thrust
T = total engine thrust, lb
T_e = thrust per engine, lb
V_i = integral tanks volume, gal
V_p = self-sealing "protected" tanks volume, gal
V_{pr} = volume of pressurized section, ft^3
V_t = total fuel volume, gal
W = fuselage structural width, ft
W_c = maximum cargo weight, lb
W_{dg} = design gross weight, lb
W_{ec} = weight of engine and contents, lb (per nacelle), $\cong 2.331\, W_{engine}^{0.901}\, K_p K_{tr}$
W_{en} = engine weight, each, lb
W_{fw} = weight of fuel in wing, lb
W_l = landing design gross weight, lb
W_{press} = weight penalty due to pressurization, $= 11.9 + (V_{pr}P_{delta})^{0.271}$, where P_{delta} = cabin pressure differential, psi (typically 8 psi)
W_{uav} = uninstalled avionics weight, lb (typically = 800–1400 lb)
Λ = wing sweep at 25% MAC