Linear Algebra - Worksheet

Read this article before beginning the exercises: Linear Algebra Explained in 4 Pages

This assignment consists of 3 parts:

- Matrix Dimensions

- Vector Operations

- Matrix Operations

After completing the exercises by hand, use Python to check your work.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix} \quad v = \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix} \quad w = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 5 \end{bmatrix}$$

1. Matrix Dimensions
Write the dimensions of each matrix.

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1.1) A 2×3

1.2) B 2×2

1.3) C 3×2

1.4) D 2×3

1.5) u 1×4

1.6) w 4×1

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2. Vector Operations
Perform the following operations. Assume $\alpha = 6$.

2.1) $\vec{u} + \vec{v} = \begin{bmatrix} 9, & 7, & -4, & 9 \end{bmatrix}$

2.2) $\vec{u} - \vec{v} = \begin{bmatrix} 3, & -3, & -2, & 1 \end{bmatrix}$

2.3) $\alpha \vec{u} = \begin{bmatrix} 36, & 12, & -18, & 30 \end{bmatrix}$

2.4) $\vec{u} \cdot \vec{v} = 18 + 10 + 3 + 20 = 51$

2.5) $\|\vec{u}\| = \sqrt{36 + 4 + 9 + 25} = \sqrt{74}$

3. Matrix Operations

Evaluate each of the following expressions, if it is defined; else fill in with "not defined." Do your work by hand on scratch paper.

3.1) $A + C =$ Not defined

3.2) $A - C^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -4 & -7 & -3 \\ 3 & 6 & 4 \end{bmatrix}$

3.3) $C^T + 3D = \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 9 & -6 & -3 \\ 3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 14 & 3 & 3 \\ 2 & 7 & 9 \end{bmatrix}$

3.4) $BA = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 1-2 & 2-7 & 3-4 \\ 2 & 7 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -5 & -1 \\ 2 & 7 & 4 \end{bmatrix}$

3.5) $BA^T =$ Not defined

Optional

3.6) $BC =$ Not defined

3.7) $CB = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -6 \\ 9 & -8 \\ 6 & -6 \end{bmatrix}$

3.8) $B^4 = B^2 B^2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = B^2 \quad \therefore \quad B^4 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

3.9) $AA^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 7 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 28 \\ 28 & 69 \end{bmatrix}$

3.10) $D^T D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & -4 & 0 \\ -4 & 0 & 8 \\ 0 & 8 & 10 \end{bmatrix}$