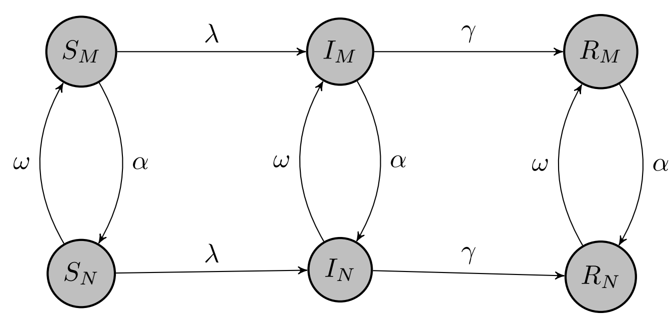
SIR and mask

Basic idea

- Simple SIR model disease progression with strata for mask-wearing status, resulting in compartments for S-masked, S-no mask, I-masked, I-no mask, R-masked, R-no mask.
- Supposing wearing a mask reduces both susceptibility and infectiousness (depending on who is wearing it)
- Assume transitions between masked-not masked based on social influence and fear of disease (noting FD type “infection” structure for fear of disease, when could actually be more about absolute number/density dependence – interesting question to explore how structure affects dynamics)
- Work in proportions such that S + I + R = 1
- no demography

State diagram



Parameter description

- λ is the force of infection, assuming a form of FD
- $\frac{1}{\gamma}$ is average rate of recovery
- α is transition rate from wearing a mask to not wearing a mask
- ω is transition rate from not wearing a mask to wearing a mask

Further parameters that will pop up

- β is the transmission rate
- c is the reduction in chance of catching disease if S is wearing a mask # this description is inconsistent with code comment definition and `1-c` model structure (I think)
- p is the reduction in chance of spreading disease if I is wearing a mask # this description is inconsistent with code comment definition and `1-p` model structure (I think)
- α_1 is the rate of transition to no mask based on social influence of no mask
- α_2 is the rate of transition to no mask based on fear of disease
- ω_1 is the rate of transition to mask based on social influence of mask
- ω_2 is the rate of transition to no mask based on fear of disease

Basic system of ODEs

$$
\begin{aligned}
\dot{S}_M &= -\lambda(1-c)S_M - \alpha S_M + \omega S_N \\
\dot{S}_N &= -\lambda S_N + \alpha S_M - \omega S_N \\
\dot{I}_M &= \lambda(1-c)S_M - \alpha I_M + \omega I_N - \gamma I_M \\
\dot{I}_N &= \lambda S_N + \alpha I_M - \omega I_N - \gamma I_N \\
\dot{R}_M &= \gamma I_M - \alpha R_M + \omega R_N \\
\dot{R}_N &= \gamma I_N + \alpha R_M - \omega R_N \, ,
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda(t) &= \beta\left(I_N + (1-p)I_M\right) \\
\omega(t) &= \omega_1(S_M + I_M + R_M) + \omega_2(I_M + I_N) \\
\alpha(t) &= \alpha_1(S_N + I_N + R_N) + \alpha_2(1 - (I_M + I_N)) \, .
\end{aligned}
$$

Next generation matrix

- The disease states are I_M and I_N
- Cannot ignore S_M and S_N!

The flow in is

$$
\mathscr{F} = \begin{bmatrix} \beta(1-c)\left(I_N + (1-p)I_M\right) S_M \\ \beta\left(I_N + (1-p)I_M\right) S_N \end{bmatrix}
$$

and flow out is

$$
\mathscr{V} = \begin{bmatrix} -\omega I_N + (\alpha+\gamma)I_M \\ (\omega+\gamma)I_N - \alpha I_M \end{bmatrix} \, .
$$

The differential of these with respect to the disease states (remembering that α and ω depend on I_N and I_M) are:

$$
F = \begin{bmatrix} \beta(1-c)(1-p)S_M & \beta(1-c)S_M \\ \beta(1-p)S_N & \beta S_N \end{bmatrix} \, ,
$$

$$
V = \begin{bmatrix} \gamma + \alpha - \alpha_2 I_M - (\omega_1+\omega_2)I_N & (\alpha_1-\alpha_2)I_M - \omega - \omega_2 I_N \\ (\omega_1+\omega_2)I_N - \alpha + \alpha_2 I_M & \gamma - (\alpha_1-\alpha_2)I_M + \omega + \omega_2 I_N \end{bmatrix} \, .
$$

Then $R(t) = \sigma(t)$ where $\sigma(t)$ is the largest (absolute) eigenvalue of $(FV^{-1})(t)$. Defining

- $x = \alpha - \alpha_2 I_M - (\omega_1+\omega_2)I_N$, (rate of change in I_M due to mask transitions)
- $y = \omega - (\alpha_1-\alpha_2)I_M + \omega_2 I_N$, (rate of change in I_N due to mask transitions)
- $a = (1-p)(\gamma+y) + x$, (total rate of change in I_N accounting for reduced infections due to mask $(1-p)$?)
- $b = (1-p)y + \gamma + x$, and (total rate of change in I_M accounting for reduced infections due to mask $(1-p)$?)
- $\Gamma = \frac{\beta}{\gamma(\gamma+x+y)}$ (R_0 with no mask effects ($p=c=0$))

- Can look at $I = I_N + I_M$ and $M = I_M + S_M + R_M$ and maybe work out some equilibriums? First (wrong) solution thinks that

$$S_N = \frac{\gamma(1 + \frac{\omega}{\alpha})}{\beta(1 + (1-p)\frac{\omega}{\alpha})(1 + (1-c)\frac{\omega}{\alpha})}$$

Problem is that ω and α depend on the disease states

we can succinctly write

$$\sigma(t) = \Gamma\left((1-c)S_M a + S_N b\right) .$$