EXAMPLE OF A RENEWAL APPROACH TO R_0

Suppose that a time τ after infection, an individual makes infectious contacts at a rate $\beta\lambda(\tau)$ where we have normalised λ s.t. $\int_0^\infty \lambda(\tau)\,d\tau = 1$.

One such λ is the Weibull, $\lambda(\tau) = \frac{k}{\ell}\left(\frac{\tau}{\ell}\right)^{k-1} e^{-(\tau/\ell)^k}$, which has $\Lambda(t) = \int_0^t \lambda(\tau)\,d\tau = 1 - \exp\left(-(x/\ell)^k\right)$.

Suppose each individual has a probability p of testing at time T and successfully reducing their infectivity to zero.

Then $R_0 = \int_0^\infty \beta\lambda(\tau)\,d\tau = \beta$

$$R_T = \beta\left((1-p)\int_0^\infty \lambda(\tau)\,d\tau + p\int_0^T \lambda(\tau)\,d\tau \right)$$

$$= R_0\left(1-p + p\left(1-\exp\left(-\left(\tfrac{T}{\ell}\right)^k\right)\right)\right)$$

(for the Weibull)

This can be relatively straightforwardly extended to multiple times & probabilities, if we let p_i = Prob tests at time T_i & not earlier times etc:

$$R_{\underline{T}} = R_0\left(1 - \sum_i p_i + \sum_i p_i \Lambda(T_i)\right)$$

or with a reduction to ε of infectivity after a successful test,

$$R_{\underline{T}} = R_0\left[1 - \sum_i p_i + \sum_i \left\{ p_i\left(\Lambda(T_i) + \varepsilon(1-\Lambda(T_i))\right)\right\}\right].$$

Basically, this approach shifts calculation from Eigensystem $\to$ Integral.

Now suppose the trajectories are switched according to a stochastic process, i.e. if in the a-th state there is overall infection rate $\propto \beta_a$.

$$R_0 = \mathbb{E}\left[\int_0^\infty \beta_X \lambda(t)\, dt\right]$$

Letting $\pi_a = \Pr(X=a)$ ~~[illegible]~~ be the stationary distribution of this stochastic process, we have that

$$R_0 = \underline{\beta} \cdot \underline{\pi} \int_0^\infty \lambda(\tau)\, d\tau \; .$$

Now, how this interacts with testing on different days is not trivial if we let people move between behavioral classes during infection ($a \in \{B, N\}$?)