Chapter 5 Project: Apply Nelder-Mead to the Rheology Problem

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Nelder-Mead Algorithm

Given $f : \mathbb{R}^n \mapsto \mathbb{R}$ and the vertices of an initial simplex $\mathrm{Y}^0 = \{\mathrm{y}^0, \mathrm{y}^1, \ldots, \mathrm{y}^n\}$

0. Initialize:

$\delta^e, \delta^{oc}, \delta^{ic}, \gamma$	parameters
$k \leftarrow 0$	iteration counter

1. Order and create centroid:

   reorder Y^k so $f(y^0) \leq f(y^1) \leq \ldots \leq f(y^n)$

   set $x^c = \frac{1}{n} \sum_{i=0}^{n-1} y^i$, the centroid of all except the worst point

2. Reflect:

   test reflection point $x^r = x^c + (x^c - y^n)$

   if $f(y^0) \leq f(x^r) < f(y^{n-1})$, then accept x^r and goto 1

3. Expand:

   if $f(x^r) < f(y^0)$, then test expansion point $x^e = x^c + \delta^e(x^c - y^n)$

4a). Outside Contraction:

   if $f(y^{n-1}) \leq f(x^r) < f(y^n)$, then test outside contraction $x^{oc} = x^c + \delta^{oc}(x^c - y^n)$

4b). Inside Contraction:

   if $f(x^r) \geq f(y^n)$, then test inside contraction point $x^{ic} = x^c + \delta^{ic}(x^c - y^n)$

5. Shrink:

   if all tests fail, then shrink

   $\mathrm{Y}^{k+1} = \{\mathrm{y}^0, \mathrm{y}^0 + \gamma(\mathrm{y}^1 - \mathrm{y}^0), \mathrm{y}^0 + \gamma(\mathrm{y}^2 - \mathrm{y}^0), \ldots, \mathrm{y}^0 + \gamma(\mathrm{y}^n - \mathrm{y}^0)\}$

Algorithm from lecture slides

0. Initialize

1. Order, create centroid, and compute x^r

2. Reflect

3. Expand

4.a) Outside Contraction

4.b) Inside Contraction

5. Shrink