Trust Region Policy Optimization (TRPO)

Original Paper by Schulman et al. [2017]

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Robot Learning Seminar Presentation

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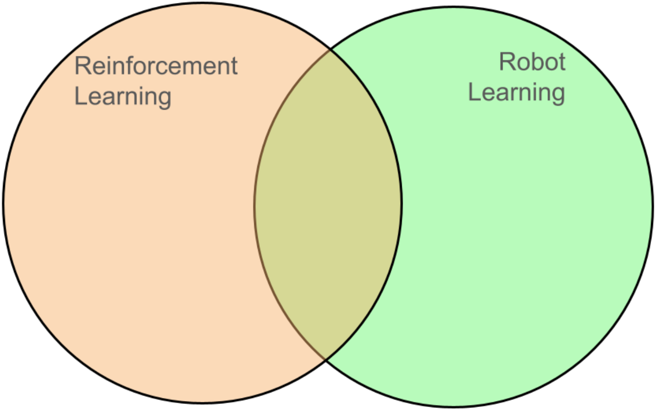
Motivation



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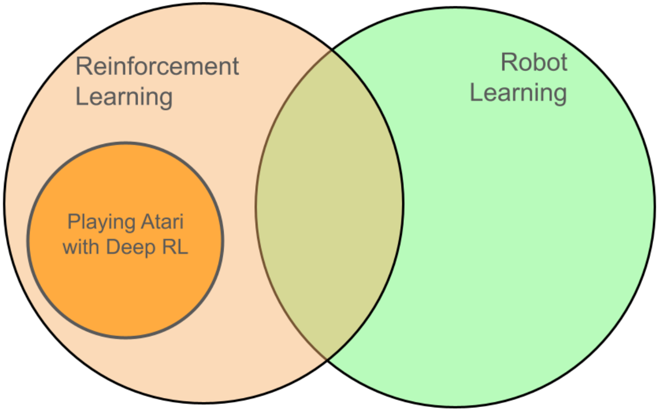
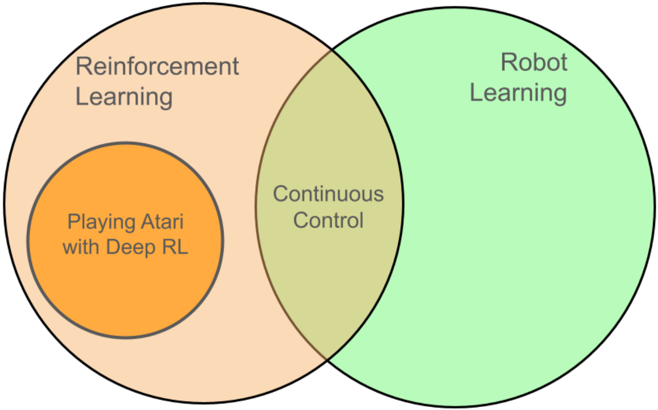

Reinforcement Learning
Playing Atari with Deep RL
Robot Learning

Motivation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Existing Solutions

- Reinforce
- Basic Actor-Critic Algorithms
- Natural Policy Gradients
- Derivative Free Methods: cross-entropy method, covariance matrix adaptation.

Once again, ... RL

"RL is computational framework for learning from interaction" Sutton and Barto [2018]. The agent interacts with an environment with the goal of maximizing expected return. Let us denote expected return for a particular policy by $\eta(\pi)$.

Advantage Function

Recall the advantage function $A_\pi(s, a)$ defined as,

$$A_\pi(s, a) = Q_\pi(s, a) - V_\pi(s)$$

This function tells us how "good" taking action is compared to what we would have done otherwise.

Policy Improvement via Advantage

Since policies are just collections of actions, we can use advantage function to evaluate them. Let π and π' be two different policies. Equation 1 gives a way to write the performance of π' using the performance of π and the advantage function.

$$\eta(\pi') = \eta(\pi) + \sum_s \mu_{\pi'}(s) \sum_a \pi'(a|s) A_\pi(s, a). \tag{1}$$

Proof in appendix

RL is Solved!

At first glance we have a solution!

Algorithm The Perfect RL Algorithm

Require: π and $\eta(\pi)$

1: $\max_{\pi'} \left(\eta(\pi) + \sum_s \mu_{\pi'}(s) \sum_a \pi'(a|s) A_\pi(s, a)\right)$

This doesn't work because we have a dependence on the policy distribution which is not something we have access to when considering π'.

Dealing with $\mu_{\pi'}$

- Let's use μ_π instead of $\mu_{\pi'}$
- Define $L_\pi(\pi') = \eta(\pi) + \sum_s \mu_\pi(s) \sum_a \pi'(a|s) A_\pi(s, a)$
- Assume π is a parameterized by weights θ.
- Gives us a local first order approximation, $\nabla_\theta L_{\pi_{\theta_0}}(\theta_\theta)|_{\theta=\theta_0} = \nabla_\theta \eta_{\pi_{\theta_0}}(\theta_\theta)|_{\theta=\theta_0}$
- If we take **small** steps in θ then we can use our 'Perfect RL algorithm'.

What is a small step?

A Major Contribution of the Paper is the following bound,

Theorem

Let $D_{KL}^{max}(\pi, \pi') := max_s D_{KL}\left(\pi(*|S)||\pi'(*|s)\right)$. *We then have that,*
$\eta(\pi') \geq L_\pi(\pi') - CD_{KL}^{max}(\pi, \pi')$ *where* $C = \frac{4\epsilon\gamma}{(1-\gamma)^2}$

This means we can bound the improvement between any-two policies based on their KL divergence. This means if we optimize within a certain KL distance we can *guarantee improvement*.

A More Perfect RL Algorithm

Algorithm 1 Policy iteration algorithm guaranteeing non-decreasing expected return η

Initialize π_0.
for $i = 0, 1, 2, \dots$ until convergence **do**
  Compute all advantage values $A_{\pi_i}(s, a)$.
  Solve the constrained optimization problem

$$\pi_{i+1} = \arg\max_{\pi} \left[L_{\pi_i}(\pi) - C D_{\mathrm{KL}}^{\max}(\pi_i, \pi) \right]$$

where $C = 4\epsilon\gamma/(1-\gamma)^2$

and $L_{\pi_i}(\pi) = \eta(\pi_i) + \sum_s \rho_{\pi_i}(s) \sum_a \pi(a|s) A_{\pi_i}(s, a)$

end for

The constraint is not computable due to $\max_s f(s)$.

Make RL in Practice

- Computable constraint
- Cheap cost function
- Cheap constrained optimization solver

Computable Constraint

We want to make our constrained optimization solvable.

- Get rid of max KL constraint over the whole state space.
- Define 'Average' KL,
  $\bar{D}_{KL} := \mathbb{E}_{s \sim \mu_{\pi_\theta}} \left[D_{KL} \left(\pi_\theta(*|S) || \pi_{\theta_{old}}(*|s) \right) \right]$
- Estimate this Expectation using roll-outs under the policy.

This gives us,

$$\max_\theta \; L_{\theta_{old}}(\theta)$$
$$\text{subject to} \; \bar{D}_{KL}(\theta_{old}, \theta) \leq \delta$$

Cheap Cost Function

We want to make $L_{\theta_{old}}(\theta)$ fast to compute.

$$L_{\theta_{old}}(\theta) = \sum_s \mu_{\pi_{\theta_{old}}} \sum_a \pi_\theta(a|s) A_{\theta_{old}}(s,a) \quad \text{(Definition of } L\text{)}$$

We replace the sums by an expectation, and A with estimator $\hat{A}$

$$L_{\theta_{old}}(\theta) = \mathbb{E}_{a\sim\pi_{\theta_{old}}, s\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a|s)}{\pi_{\theta_{old}}(a|s)} \cdot \hat{A}_{\theta_{old}}(s,a)\right]$$

Cheap constrained optimization solver

$$\max_\theta L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}\left(\pi_{\theta_{old}}, \pi_\theta\right) \leq \delta$$

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$$f(x) = \overbrace{f(0) + f'(0)x}^{\text{Actual Taylor expansion}} + \underbrace{\cancel{\frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3}}_{\text{Lies invented by mathematicians to feel superior to physicists}} \ldots$$

Cheap constrained optimization solver

$$\max_\theta L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}(\pi_{\theta_{old}}, \pi_\theta) \leq \delta$$

First Order of Taylor Expansion to $L_{\theta_{old}}(\theta)$

Cheap constrained optimization solver

$$\max_\theta L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}(\pi_{\theta_{old}}, \pi_\theta) \leq \delta$$

First Order of Taylor Expansion to $L_{\theta_{old}}(\theta)$

$$L_{\theta_{old}}(\theta) \approx L_{\theta_{old}}(\theta_{old}) + g^T(\theta - \theta_{old})$$

Cheap constrained optimization solver

$$\max_\theta \ L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}\left(\pi_{\theta_{old}}, \pi_\theta\right) \leq \delta$$

First Order of Taylor Expansion to $L_{\theta_{old}}(\theta)$

$$L_{\theta_{old}}(\theta) \approx L_{\theta_{old}}(\theta_{old}) + g^T(\theta - \theta_{old})$$
$$L_{\theta_{old}}(\theta) \approx 0 + g^T(\theta - \theta_{old})$$

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Let $\Delta\theta = (\theta - \theta_{old})$

Cheap constrained optimization solver

$$\max_\theta \ L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}\left(\pi_{\theta_{old}}, \pi_\theta\right) \le \delta$$

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Let $\Delta\theta = (\theta - \theta_{old})$

$$L_{\theta_{old}}(\theta) \approx g^T \Delta\theta$$

Cheap constrained optimization solvers

$$\max_\theta \; L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}\left(\pi_{\theta_{old}}, \pi_\theta\right) \leq \delta$$

Second Order of Taylor Expansion of Contraint

Cheap constrained optimization solvers

$$\max_\theta L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}(\pi_{\theta_{old}}, \pi_\theta) \leq \delta$$

Second Order of Taylor Expansion of Contraint

$$\bar{D}_{KL}(\pi_{\theta_{old}}, \pi_\theta) \approx \bar{D}_{KL}(\pi_{\theta_{old}}, \pi_{\theta_{old}}) + \nabla_\theta \hat{D}_{KL}(pi_{\theta_{old}}||\pi_\theta)|_{\theta_{old}} \cdot$$

$$only < 5- > \bar{D}_{KL}(\pi_{\theta_{old}}, \pi_\theta) \approx \frac{1}{2}\Delta\theta^T H \Delta\theta$$

Cheap constrained optimization solvers

$$\max_\theta L_{\theta_{old}}(\theta) \text{ subject to } \bar{D}_{KL}\left(\pi_{\theta_{old}}, \pi_\theta\right) \leq \delta$$

Second Order of Taylor Expansion of Contraint

Cheap constrained optimization solvers

We now have a new constrained optimization problem,

$$\text{argmax}_{\theta} \; g^T \Delta\theta$$
$$\text{subject to } \frac{1}{2}\Delta\theta^T H \Delta\theta \leq \delta$$

Now since it's a nice quadratic/convex contrainst we can solve,

$$\theta = \theta_{old} + \sqrt{\frac{2\delta}{g^t H^{-1} g}} H^{-1} g.$$

skip derivation

Steps to Solve

$$\text{argmax}_\theta \; g^T \Delta\theta \quad \text{subject to } \frac{1}{2}\Delta\theta^T H \Delta\theta \leq \delta$$

Use Lagrangian to solve the constrained optimization

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Use Lagrangian to solve the constrained optimization

$$L = g^T \Delta\theta + \lambda \left(\frac{1}{2}(\theta - \theta_{old})^T H(\theta - \theta_{old}) - \delta \right)$$

Steps to Solve

$$\text{argmax}_\theta \; g^T \Delta\theta \quad \text{subject to } \frac{1}{2}\Delta\theta^T H \Delta\theta \leq \delta$$

Use Lagrangian to solve the constrained optimization

$$L = g^T \Delta\theta + \lambda \left(\frac{1}{2}(\theta - \theta_{old})^T H(\theta - \theta_{old}) - \delta \right)$$

$$\nabla_\theta L = 0 \implies g^T + \frac{\lambda}{2} H \Delta\theta = 0$$

Steps to Solve

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$$\Delta\theta = -\frac{2}{\lambda} H^{-1} g$$

Steps to Solve

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This looks like SGD, set $\alpha = -\frac{2}{\lambda}$

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Steps to Solve

We have $\Delta\theta = \alpha H^{-1} g$, and we solve for α by setting contraint $= \delta$

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Now we simply re-arrange

$$\alpha^2 g^T H g = 2\delta$$

Steps to Solve

We have $\Delta\theta = \alpha H^{-1} g$, and we solve for α by setting contraint $= \delta$

$$\frac{1}{2}\Delta\theta^T H \Delta\theta = \delta$$
$$\frac{1}{2}\alpha H^{-1} g^T H \alpha H^{-1} g = \delta$$

Now we simply re-arrange

$$\alpha^2 g^T H g = 2\delta$$
$$\alpha = \sqrt{\frac{2\delta}{g^t H g}}$$

Steps to Solve

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Subbing this into $\Delta\theta = \theta - \theta_{old} = \alpha H^{-1} g$ we get

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$H^{-1}g$

An Introduction to
the Conjugate Gradient Method
Without the Agonizing Pain
Edition $1\frac{1}{4}$

Jonathan Richard Shewchuk
August 4, 1994

Line Search

Our approximations have gone back to bite us. We need to check that our ubdpates don't exceed our *actual* constraint using a line search.

Trust Region Policy Optimization

Algorithm 3 Trust Region Policy Optimization

Input: initial policy parameters θ_0

for $k = 0, 1, 2, ...$ **do**

Collect set of trajectories $\mathcal{D}_k$ on policy $\pi_k = \pi(\theta_k)$

Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm

Form sample estimates for

- policy gradient $\hat{g}_k$ (using advantage estimates)
- and KL-divergence Hessian-vector product function $f(v) = \hat{H}_k v$

Use CG with n_{cg} iterations to obtain $x_k \approx \hat{H}_k^{-1} \hat{g}_k$

Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$

Perform backtracking line search with exponential decay to obtain final update

$$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$

end for

Experimental Results in TRPO Paper

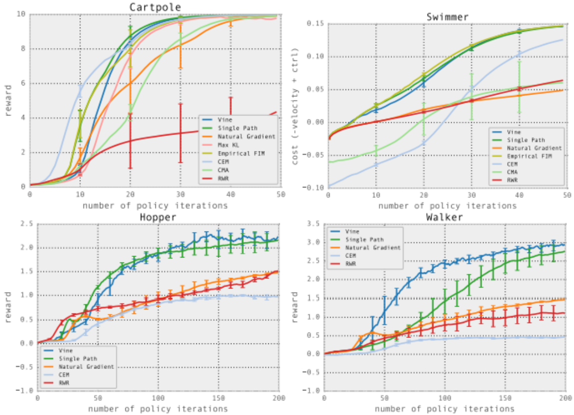


Figure 4. Learning curves for locomotion tasks, averaged across five runs of each algorithm with random initializations. Note that for the hopper and walker, a score of -1 is achievable without any forward velocity, indicating a policy that simply learned balanced standing, but not walking.

External TRPO Robotics Applications

Mahmood et al. [2018] wrote a paper bencmarking policy gradient algorithms for robotics, including TRPO.

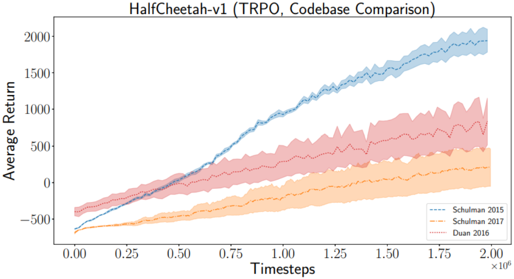
- "TRPO achieving near-best final learning performance in all tasks."
- "Among these algorithms, the final performance of TRPO was never substantially worse compared to the best in each task."
- "TRPO's performance was the least sensitive to hyper-parameter variations with the smallest interquartile range on both tasks."

Next Steps

- The Chosen Path
  - Clipping
  - Hyperparameter tuning
- Other Approaches:
  - More precise approximations $\rightarrow$ no more line search
  - Assumptions about policy structure $\rightarrow$ tighter bound
  - Assumptions about function approximator $\rightarrow$ better notation of small step

Critiques of the Paper

- They leave out critical implementation details [Henderson et al., 2019].
- They compare 'Vine' and 'Path', and Vine is a different problem formulation



Thank you for listening

Robots following TRPO Policies

References

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