Hash Tables

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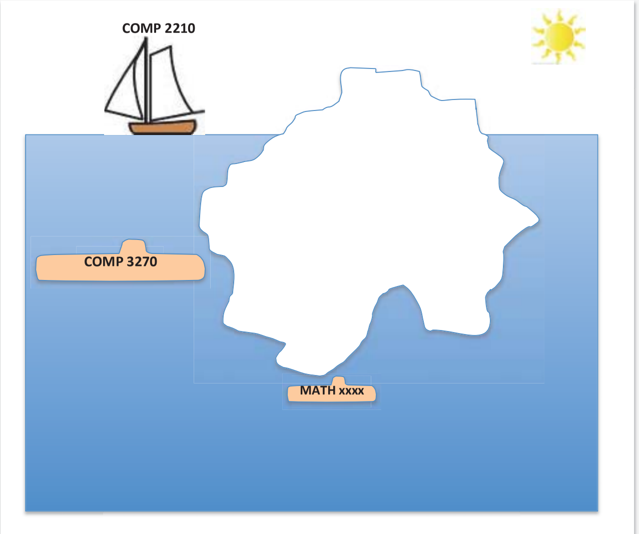
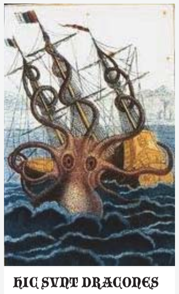
Introduction to Hash Tables

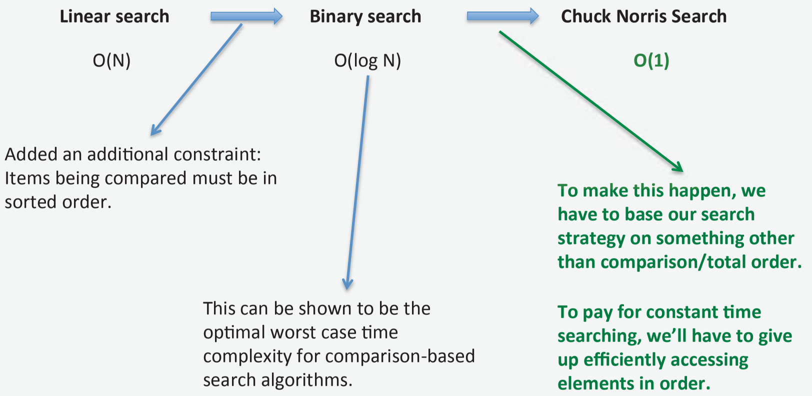
Very simple on the surface, significantly complex beneath.





Motivation: Constant time searching

So far, the basic operation at the heart of our search strategies has been the *comparison* of one item to another, based on a total ordering of the elements.



Linear search

O(N)

Binary search

O(log N)

Chuck Norris Search

O(1)

Added an additional constraint: Items being compared must be in sorted order.

This can be shown to be the optimal worst case time complexity for comparison-based search algorithms.

To make this happen, we have to base our search strategy on something other than comparison/total order.

To pay for constant time searching, we'll have to give up efficiently accessing elements in order.

Context: The Map ADT

A **map** (or *associative array,* or *dictionary*) is an abstract data type composed of key-value pairs, where each possible key occurs at most once.

Example key-value pairs: (SS#, person info) (ISBN, book info) (phone#, address)

(word, definition) (filename, location on disk)

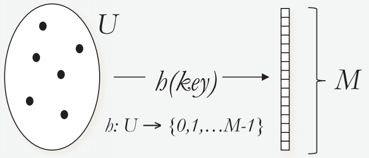
Different data structures could be used to implement a map, but we can get a dramatic speed-up by considering a new type of data structure, one that's not based on comparison of elements with respect to total order.

Map method	Ordered List	Balanced Search Tree	Hash table
add(key, value)	O(N)	O(log N)	O(1)
remove(key)	O(N)	O(log N)	O(1)
search(key)	O(log N)	O(log N)	O(1)

This is average, not worst-case. But we will take steps to avoid the worst case, just as we did with quicksort.

Hash tables

A **hash table** is a data structure that implements the map behavior by using a mathematical function (the *hash function*) to associate keys with the location in a table where that key and its associated value are stored.

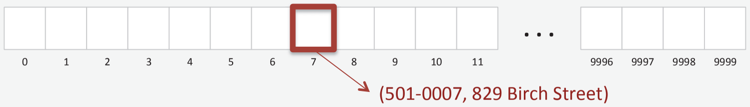


Example: (phone#, address)   *Assume phone numbers in the 501 prefix of Auburn.*

(501-1234, 102 Oak Street)   (501-7834, 245 Elm Street)   . . .   (501-0007, 829 Birch Street)

hash function: Take the last four digits of the phone number and use it for the array index.

h(501-1234) = 1234   h(501-0563) = 563   **h(501-0007) = 7**



```
add(key, value) {
   index = h(key);
   table[index] = (key, value);
}
```

```
search(key) {
   index = h(key);
   return table[index];
}
```

```
remove(key) {
   index = h(key);
   table[index] = null;
}
```

Hash tables

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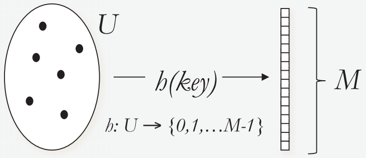

```
add(key, value) {
   index = h(key);
   table[index] = (key, value);
}
```

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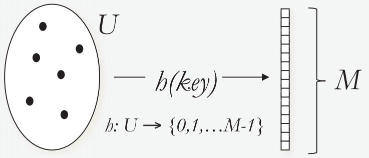
search(key) {
   index = h(key);
   return table[index];
}
```

```
remove(key) {
   index = h(key);
   table[index] = null;
}
```

This is simple and all three operations are O(1) in the worst case, but **ONLY** if the hash function **is one-to-one** (if h(x) = h(y) then x = y).

Hash tables

A **hash table** is a data structure that implements the map behavior by using a mathematical function (the *hash function*) to associate keys with the location in a table where that key and its associated value are stored.



Example: (sessionid, session info)   *Assume sessionid is a string of at most 80 ascii char.*

(5CpPi1g3FV, session info)   (KBWocaE7mj, session info)   . . .   (DsbAIdOWGj, session info)

hash function: ???   *We'll have to come back to this one …*

Some observations:

Since U is the set of all strings of at most 80 ascii char, |U| = 128^{80}.

Thus, |U| > M and by the pigeonhole principle we know that some elements of the table will be assigned more than one sessionid.

We say that a **collision** occurs when $k_1 \neq k_2$ and $h(k_1) = h(k_2)$.

Collisions degrade hash table performance.

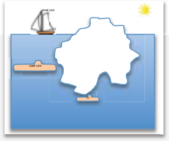
Hash tables

Most of the work in designing and implementing a hash table implementation goes into avoiding, mitigating, and resolving collisions and their effects.

We'll discuss these ideas and organize the rest of the note set around the following elements of hash table implementations.

- Implementing hash functions
- Collision resolution strategies
- The add, remove, and search methods in context
- The uniform hashing assumption
- Performance analysis
- Generating and using hash codes

Hash functions



A good hash function must:

1. Be **deterministic**. (Always give the same index for the same key.)
2. Be **fast**. (Both asymptotically and literally.)
3. Provide a **uniform distribution** of keys over indexes. (Each table index is equally likely for each key.)

We'll think of the hash function working in two stages:

1. Convert the key into an int value called the hash code.
2. Map the hash code onto a legal index value.

For 2210:

1. Use the hashCode method inherited from Object to compute hash codes.
2. Use modulus division (remainder operator % in Java) to map the hash code onto a legal index value.

```
public int h(Object key) {
   int hashCode = key.hashCode();
   int index = hashCode % table.length;
   return index;
}
```

The approach to hashing that we're using is called **modular hashing**.

In general, this is an example of the **division method**, with the alternative being **multiplicative hashing**.

Hashing example

Example: table length M = 11, h(hashcode) = hashcode % M, hashcodes = {35, 22, 18, 94, 56}

22	56	35				94	18			
0	1	2	3	4	5	6	7	8	9	10

h(35) = 35 % 11 = 2   h(22) = 22 % 11 = 0   h(18) = 18 % 11 = 7

h(94) = 94 % 11 = 6   h(56) = 56 % 11 = 1

What would happen if an object with hashcode 12 were inserted?

h(12) = 12 % 11 = 1   **collision**

Collisions

Example: table length M = 11, h(hashcode) = hashcode % M, hashcodes = {35, 22, 18, 94, 56}

22	56	35				94	18			
0	1	2	3	4	5	6	7	8	9	10

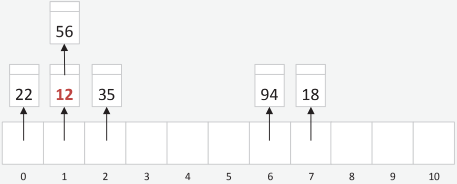
h(12) = 12 % 11 = 1   **collision**

How to handle this?

Put the colliding element in an unused location in the table.

22	56	35	12			94	18			
0	1	2	3	4	5	6	7	8	9	10

Let the table elements store linked lists of colliding elements.

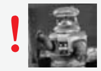


Collision resolution

Collision = distinct keys hash to the same index

The most intuitive time that a collision could occur is during insertion. That is, a new element being inserted hashes to an index that is already occupied by another (non-duplicate) element. For insertion, resolving the collision means answering the question: Where does the new element go?

Two broad categories of collision resolution:



These terms or similar can have opposite meanings!

Open addressing Find an alternate to the home index *inside* the hash table (that is, find another index). Note that in this scheme each index contains a reference to an element, and at any point in time an index contains at most one element reference.

Closed addressing Store all elements *outside* the hash table. That is, each index contains a reference to an auxiliary data structure that stores all the elements that hash to a given index.

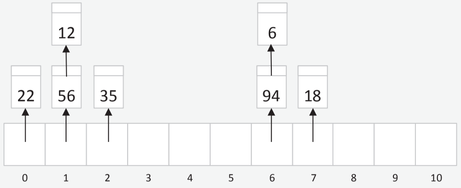
Note that this same situation happens during a search or a deletion. For example, say you're searching for a specific element but its home index contains something else. Does this mean that the table doesn't contain the element you're searching for? Not necessarily. For search (and deletion), resolving the collision means answering the question: Where could the target element be since something else occupies its home index?

Closed addressing – chaining

Store all elements outside the hash table in linked lists.

Each index in the table points to a linked list that stores all elements that hash to that index.

Example: M = 11, h(hashcode) = hashcode % M, hashcodes = {35, 22, 18, 94, 56, 12, 6}



h(35) = 35 % 11 = 2 h(22) = 22 % 11 = 0 h(18) = 18 % 11 = 7

h(94) = 94 % 11 = 6 h(56) = 56 % 11 = 1

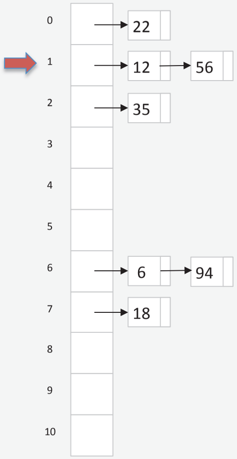
h(12) = 12 % 11 = 1 *collision* h(6) = 6 % 11 = 6 *collision*

Closed addressing – chaining

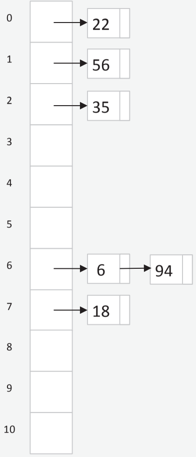
Removing elements Trivial – just delete the element from the list.

Example: Delete the element with hashcode 12.

12 % 11 = 1



Search the list in bucket 1 and delete the element with hashcode 12 if present.



Closed addressing – chaining

The performance of closed addressing with chaining is intuitively straightforward.

By always inserting onto the front of a collision chain linked list, the **add** method will be **O(1)** in the worst case.

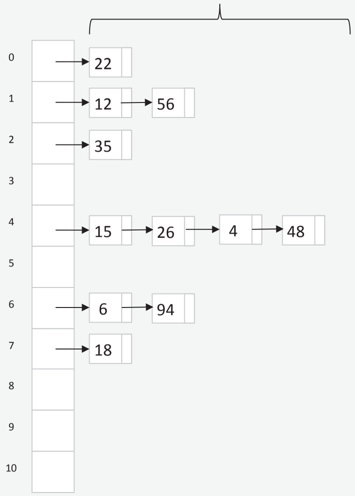
The time required for both the **remove** method and the **search** method is **proportional to the length of the collision chains**.

How do we quantify these lengths?

O(1) ?
O(N) ?
something else?

[more later …]

Collisions cause these linked lists to grow, thus slowing down search and remove.



Open addressing

Find an open (unused) index in the table.

The process of searching for this open index is called **probing**.

Probing repeatedly applies the following formula to resolve the collision:

```
index = (home + i*C) % M
```

where **home** is the index at which the collision occurred, **i** is the number of probe attempts made so far, **C** is a constant multiplier, and **M** is the number of indexes in the table (table.length).

Different choices for the constant multiplier C results in three distinct probing strategies:

Linear Probing (C = 1) `index = (home + i) % M`

Quadratic Probing (C = i) index = (home + i^2) % M

Double Hashing (C = h_2) index = (home + i*h_2) % M
h_2(hashcode) = 1 + (hashcode % (M-1))

Open addressing – linear probing

Example: M = 11, h(hashcode) = hashcode % M, hashcodes = {35, 22, 18, 94, 56, 12, 6}

22	56	35	12			94	18	6		
0	1	2	3	4	5	6	7	8	9	10

h(35) = 35 % 11 = 2   h(22) = 22 % 11 = 0   h(18) = 18 % 11 = 7

h(94) = 94 % 11 = 6   h(56) = 56 % 11 = 1

h(12) = 12 % 11 = 1 *collision*

Probe sequence:

index = (home + i) % M

index = (1 + 1) % 11 = 2

index = (1 + 2) % 11 = 3

h(6) = 6 % 11 = 6 *collision*

Probe sequence:

index = (home + i) % M

index = (6 + 1) % 11 = 7

index = (6 + 2) % 11 = 8

Open addressing – quadratic probing

Example: M = 11, h(hashcode) = hashcode % M, hashcodes = {35, 22, 18, 94, 56, 12, 6}

22	56	35			12	94	18			6
0	1	2	3	4	5	6	7	8	9	10

h(35) = 35 % 11 = 2   h(22) = 22 % 11 = 0   h(18) = 18 % 11 = 7

h(94) = 94 % 11 = 6   h(56) = 56 % 11 = 1

h(12) = 12 % 11 = 1 *collision*

Probe sequence:

index = (home + i^2) % M

index = (1 + 1^2) % 11 = 2

index = (1 + 2^2) % 11 = 5

h(6) = 6 % 11 = 6 *collision*

Probe sequence:

index = (home + i^2) % M

index = (6 + 1^2) % 11 = 7

index = (6 + 2^2) % 11 = 10

Open addressing – double hashing

Example: M = 11, h(hashcode) = hashcode % M, hashcodes = {35, 22, 18, 94, 56, 12, 6}

22	56	35		12		94	18		6	
0	1	2	3	4	5	6	7	8	9	10

h(35) = 35 % 11 = 2   h(22) = 22 % 11 = 0   h(18) = 18 % 11 = 7

h(94) = 94 % 11 = 6   h(56) = 56 % 11 = 1

h(12) = 12 % 11 = 1 *collision*

Probe sequence:
index = (home + ih_2) % M*
h_2 = 1 + (hashcode % (M-1))

h_2 = 1 + 12 % 10 = 3

index = (1 + 1*3) % 11 = 4

h(6) = 6 % 11 = 6 *collision*

Probe sequence:
index = (home + ih_2) % M*
h_2 = 1 + (hashcode % (M-1))

h_2 = 1 + 6 % 10 = 7

index = (6 + 1*7) % 11 = 2

index = (6 + 2*7) % 11 = 9

Open addressing – self-check exercise

M = 13, h(hashcode) = hashcode % M, hashcodes = {26, 51, 92, 37, 19, 60, 95, 78, 79, 90}

Linear probing index = (home + i) % M

26	92	78	79	95	90	19		60			37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

Quadratic probing index = (home + i^2) % M

26	92	79	90	95		19		60	78		37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

Double Hashing index = (home + i*h_2) % M   h2 = 1 + (hashcode % (M-1))

26	92	90		95	79	19	78	60			37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

Open addressing – search

1. Hash the key that is being searched for to find its home index.
2. If the home index is empty, return not found.
3. If the home index contains the key being searched for, return found.
4. If the home index contains anything else, follow the probe sequence.

Example: M = 11, linear probing

22	56	35	12			94	18	6		
0	1	2	3	4	5	6	7	8	9	10

Target = 21	h(21) = 21 % 11 = 10	**not found**	1 look
Target = 16	h(16) = 16 % 11 = 5	**not found**	1 look
Target = 35	h(35) = 35 % 11 = 2	**found**	1 look
Target = 33	h(33) = 33 % 11 = 0	**not found**	5 looks

Two more adds …

22	56	35	12	77	88	94	18	6		
0	1	2	3	4	5	6	7	8	9	10

Target = 33	h(33) = 33 % 11 = 0	**not found**	10 looks

Open addressing – load factor

A the table becomes more full collisions become more likely.

Load factor (λ) = the ratio of the number of elements in the table to the capacity of the table.

λ = N / M   empty table: λ = 0   full table: λ = 1

Most hash tables maintain the load factor between 0.5 and 0.8. (java.util.HashMap 0.75)

Example:

M = 11, linear probing, λ = 7 ÷ 11 = 0.64, λ max threshold = 0.70

22	56	35	12			94	18	6		
0	1	2	3	4	5	6	7	8	9	10

add(33)   resulting λ would be = 8 ÷ 11 = 0.73

Rehash, then insert:

M = 23, linear probing, λ = 8 ÷ 23 = 0.35, λ threshold = 0.70

		94				6				56	33	35	12					18				22
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

Removing elements

Open addressing – Linear Probing   We can't simply delete an element. Why?

Example: Delete the element with hashcode 79.

26	92	78	79	95	90	41		60			37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

79 % 13 = 1   Search the probe sequence for 79.   *Now what?*

Option 1: Immediate deletion

Make index 3 empty, then move the elements in the remaining probe sequence closer to their home bucket (if possible) by using the new empty bucket.

26	92	78		95	90	41		60			37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

95 % 13 = 4   90 % 13 = 12   41 % 13 = 2

26	92	78	90	95	41			60			37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

Removing elements

Open addressing – Linear Probing   We can't simply delete an element. Why?

Example: Delete the element with hashcode 79.

26	92	78	79	95	90	41		60			37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

79 % 13 = 1   Search the probe sequence for 79.   *Now what?*

Option 2: Lazy deletion

Don't make index 3 empty. Instead, mark it as "unoccupied but not empty" or replace 79 with a "sentinel key". These marked indexes are emptied either periodically or when the table is rehashed.

26	92	78	⊘	95	90	41		60			37	51
0	1	2	3	4	5	6	7	8	9	10	11	12

Search terminates on empties, add terminates on either empty or unoccupied.

Removing elements

Open addressing – Double Hashing

Lazy deletion is our only practical option. Why?

Example: Delete the element with hashcode 79.

26	92		51	95		19	78	60	79		37	90
0	1	2	3	4	5	6	7	8	9	10	11	12

79 % 13 = 1   Search the probe sequence for 79.   *Now what?*

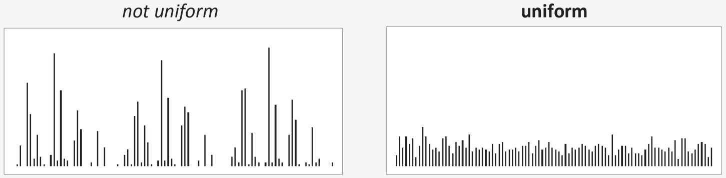
$h_2 = 1 + (79 \% 12) = 8$ probe 1 = 9

26	92		51	95		19	78	60	⦸		37	90
0	1	2	3	4	5	6	7	8	9	10	11	12

Uniform hashing

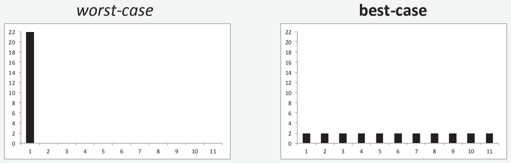
Uniform hashing assumption: For a hash table with M indexes, each key is equally likely to hash to an integer between 0 and M-1 inclusive.

Example: Same data, different hash functions



Keys = first 1000 unique words in Charles Dickens' *A Tale of Two Cities.*   **M** = 96

Example: **N** = 22 **M** = 11

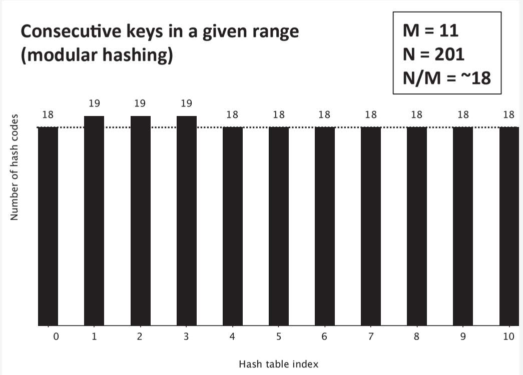


Uniform hashing will ensure that each index in the table has, *to within a small constant factor*, **N/M** elements.

Uniform hashing

Working definition For a hash table with M indexes and N keys, each index has to within a small constant factor N/M keys hashed to it.

Example:



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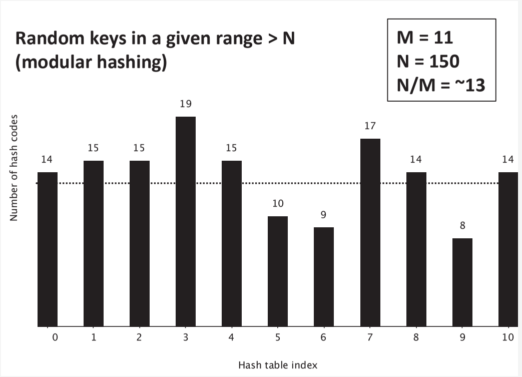

Uniform hashing

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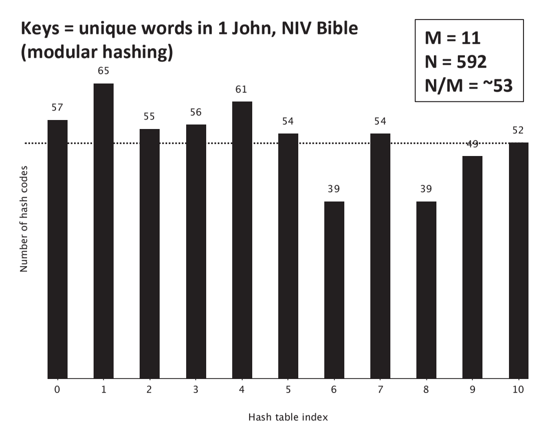


Uniform hashing

```
public int h(Object key) {
   int hashCode = key.hashCode();
   int index = (hashCode & 0x7fffffff) % table.length;
   return index;
}
```

In practice, with modular hashing, the divisor (table size) is typically a prime number not too close to a power of 2 or 10.

Sometimes the divisor (table size) is a power of 2, but there’s extra care and feeding required.

java.util.HashMap uses the power of 2 approach:

In the put method:

```
int hash = hash(key.hashCode());
```

The hash function:

```
static int hash(int h) {
   h ^= (h >>> 20) ^ (h >>> 12);
   return h ^ (h >>> 7) ^ (h >>> 4);
}
```

!

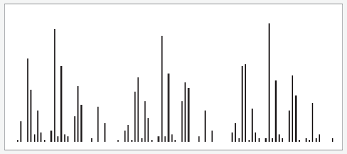
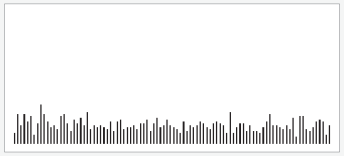
Kraken!

Uniform hashing

Being able to attain O(1) time complexity depends on uniform hashing.

Uniform hashing ensures that the probability of a collision between two distinct keys in a hash table of size M is 1/M.

Uniform hashing ensures that the size of each “collision chain” is approximately N/M.

Performance analysis

!

Kraken!

Perfect hashing is obvious – O(1) for add, remove, and search.

Typical hashing, not so much.

Don Knuth’s study of hash table performance was truly a watershed moment in algorithm analysis and in our ability to understand complex performance.

Knuth’s student Leonidas Guibas developed a deep mathematical analysis of random hashing and double hashing, which extended Knuth’s work to those cases.

We will use the following terms and make the following assumptions to characterize the average number of probes we can expect in tables using different collision resolution strategies.

N = number of elements in the table
M = number of table indexes (table size)
λ = N/M
ln = the natural logarithm
e = ~2.71828 (the base of ln)
Uniform hashing assumption is true.

Performance analysis

Closed addressing – Separate chaining

Because of uniform hashing, we know that each list has to within a small constant factor λ elements.

If we keep the lists unordered, then insert is O(1) and search is O(N/M).

By managing λ appropriately, we can ensure that no search will require more than some constant amount of comparisons.

For example, in a separate-chaining table with 1000 elements and and 200 indexes (λ = 5), it is highly unlikely that any search will take much more than 5 comparisons.

Specifically, the probability that some list in the table has as many as 10 elements is ~0.00098.

For the interested reader

In a separate-chaining hash table with N elements and M indexes, the probability that a given list will have *k* items on it is

$$\binom{N}{k}\left(\frac{1}{M}\right)^k\left(1-\frac{1}{M}\right)^{N-k}$$

We can express this in terms of λ = N/M as

$$\binom{N}{k}\left(\frac{\lambda}{N}\right)^k\left(1-\frac{\lambda}{N}\right)^{N-k}$$

Using the Poisson approximation, we know this is less than

$$\frac{\lambda^k e^{-\lambda}}{k!}$$

Thus, the probability that any list in the table has more than *t* λ elements is less than

$$\left(\frac{\lambda e}{t}\right)^t e^{-\lambda}$$

Performance analysis – average number of probes

Open addressing – Linear Probing

$$\sim\frac{1}{2}\left(1+\frac{1}{1-\lambda}\right)$$

search hit

$$\sim\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$$

search miss / insert

λ	1/2	2/3	3/4	9/10
hit	1.5	2.0	3.0	5.5
miss/insert	2.5	5.0	8.5	55.5

Open addressing – Double Hashing

$$\frac{1}{\lambda}\ln\left(\frac{1}{1-\lambda}\right)$$

search hit

$$\frac{1}{1-\lambda}$$

search miss / insert

λ	1/2	2/3	3/4	9/10
hit	1.4	1.6	1.8	2.6
miss/insert	1.5	2.0	3.0	5.5

Double hashing provides better performance in the same space.

To guarantee an average cost of no more than a constant *t* on any search:

$\lambda < 1 - \frac{1}{\sqrt{t}}$ for linear probing

$\lambda < 1 - \frac{1}{t}$ for double hashing

How empty must we keep a hash table to guarantee no more than 5 probes for any search?

~45% empty for linear probing (λ = 0.55)
~20% empty for double hashing (λ = 0.80)

Double hashing provides the same performance in less space.

hashCode to legal index

```
public int h(Object key) {
   int hashCode = key.hashCode();
   int index = hashCode % table.length;
   return index;
}
```

hashCode() returns an int value.

[-2,147,483,648 … 2,147,483,647]

!

hashCode can be negative and thus illegal for an index.

```
add(key, value) {
   index = hash(key);
   hashTable[index] = (key, value);
}
```

```
public int h(Object key) {
   int hashCode = key.hashCode();
   int index = Math.abs(hashCode) % table.length;
   return index;
}
```

Math.abs() can return a negative value!

```
public static int abs(int a) {
        return (a < 0) ? -a : a;
}
```

`Math.abs(Integer.MIN_VALUE) = -2,147,483,648`

But this is a 1-in-4billion value. Do we really have to worry about it??

`"polygenelubricants".hashCode() = -2,147,483,648`

!

We can't rely on Math.abs() ensuring only positive values.

Find another way to ensure positive values, even in this special case.

bit masking ints

Java's int data type is a 32-bit signed two's complement integer.

The left-most bit is the sign bit (1 = negative, 0 = positive).

Negation in two's complement notation is accomplished by inverting all the bits in the number being negated, then adding one.

```
       Decimal     Binary
Integer.MAX_VALUE  0111 1111 1111 1111 1111 1111 1111 1111    2,147,483,647

               15  0000 0000 0000 0000 0000 0000 0000 1111

                1  0000 0000 0000 0000 0000 0000 0000 0001

                0  0000 0000 0000 0000 0000 0000 0000 0000

               -1  1111 1111 1111 1111 1111 1111 1111 1111

              -15  1111 1111 1111 1111 1111 1111 1111 0001

   -2,147,483,647  1000 0000 0000 0000 0000 0000 0000 0001

Integer.MIN_VALUE  1000 0000 0000 0000 0000 0000 0000 0000   -2,147,483,648
```

hashCode to legal index

To ensure positive hashCode values, we will mask the sign bit using the bitwise AND operator (&) and treat the int like a 31-bit unsigned integer.

```
  1010
& 0111
------
  0010
```

Mask the left-most bit, leave everything else unchanged.

```
hashCode & Integer.MAX_VALUE
```

```
public int h(Object key) {
   int hashCode = key.hashCode();
   int index = (hashCode & Integer.MAX_VALUE) % table.length;
   return index;
}
```

Correct! Will always be positive.

You often see Integer.MAX_VALUE expressed as a hexadecimal literal:

```
public int h(Object key) {
   int hashCode = key.hashCode();
   int index = (hashCode & 0x7fffffff) % table.length;
   return index;
}
```

Generating hash codes

We described hashing a key onto an index as working in two steps:

1. Convert the key into an int value called the hash code.
2. Map the hash code onto a legal index value.

And we implemented these steps as:

Generating good hash codes is an important part of ensuring uniform hashing.

```
public int h(Object key) {
   int hashCode = key.hashCode();
   int index = (hashCode & 0x7fffffff) % table.length;
   return index;
}
```

If the set of keys to be inserted is known in advance, and if the size of this set is reasonable, then we can write hashCode() to yield *perfect hashing*, leading to worst case O(1) performance.

Typically, perfect hashing is not an option so ensuring hashCode() yields *uniform hashing* is crucial.

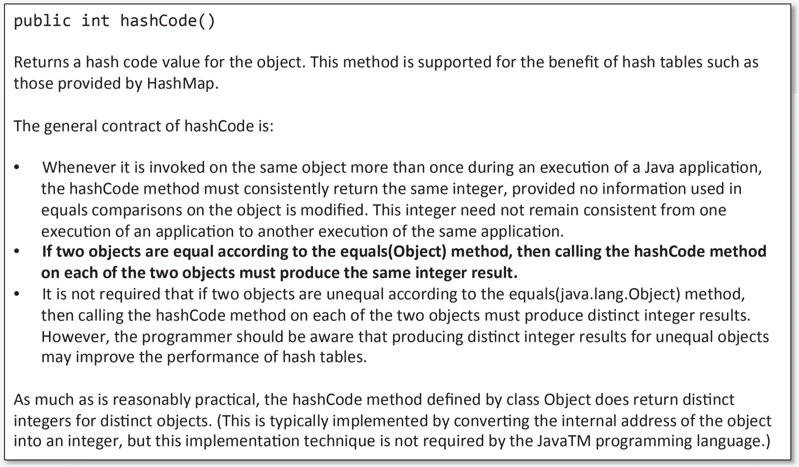
If the set of keys contained random integers, then “return key” is a good hashCode() implementation.

For a general purpose hash table, we can’t assume anything about the keys. So we write for the typical case of keys being of some unknown type and the keys’ values are constructed in a non-random way.

Overriding hashCode()

The hashCode() contract from Object:

ORACLE http://docs.oracle.com/javase/7/docs/api/java/lang/Object.html



```
public int hashCode()
```

Returns a hash code value for the object. This method is supported for the benefit of hash tables such as those provided by HashMap.

The general contract of hashCode is:

- Whenever it is invoked on the same object more than once during an execution of a Java application, the hashCode method must consistently return the same integer, provided no information used in equals comparisons on the object is modified. This integer need not remain consistent from one execution of an application to another execution of the same application.
- **If two objects are equal according to the equals(Object) method, then calling the hashCode method on each of the two objects must produce the same integer result.**
- It is not required that if two objects are unequal according to the equals(java.lang.Object) method, then calling the hashCode method on each of the two objects must produce distinct integer results. However, the programmer should be aware that producing distinct integer results for unequal objects may improve the performance of hash tables.

As much as is reasonably practical, the hashCode method defined by class Object does return distinct integers for distinct objects. (This is typically implemented by converting the internal address of the object into an integer, but this implementation technique is not required by the JavaTM programming language.)

Generating hash codes

A few Java wrappers …

```
public final class Integer ... {
   private final int value;

   public int hashCode() {
      return value;
   }
}
```

```
> Integer i = new Integer(32);
> i.hashCode()
  32
```

```
public final class Boolean ... {
   private final boolean value;

   public int hashCode() {
      return value ? 1231 : 1237;
   }
}
```

```
> Boolean t = new Boolean(true)
> Boolean f = new Boolean(false)
> t.hashCode()
  1231
> f.hashCode()
  1237
```

```
public final class Double ... {
   private final double value;

   public int hashCode() {
      long bits = doubleToLongBits(value);
      return (int)(bits ^ (bits >>> 32));
   }
}
```

```
> Double d = new Double(3.14159)
> d.hashCode()
  -1340954729
```

A double in Java is a double-precision 64-bit IEEE 754 floating point. So, the hashCode method:

1. Convert to IEEE 754 floating-point “double format” bit layout.
2. xor the most significant 32 bits with the least significant 32 bits
3. Return the xor’d 32 bits as an int.

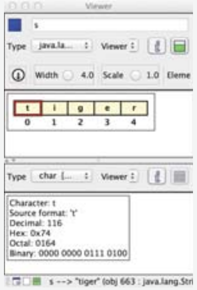
Generating hash codes – Strings

Strings are a common choice for keys and they offer a good pattern for us to think about hashing other composite types.

Strings are composed of individual char values, and a char in Java is represented as a 16-bit Unicode character. Java interprets a char as a numeric value from 0 to 65, 535 (2^{16} - 1).

```
public final class String ... {
   private final char[] s;
     . . .
}
> String s = new String(“tiger”)
```

To compute a String’s hash code, we could sum up the Unicode decimal values of its constituent characters.



```
Character: t
Source format: 't'
Decimal: 116
Hex: 0x74
Octal: 0164
Binary: 0000 0000 0111 0100
```

s --> "tiger" (obj 663 : java.lang.Stri

t		i		g		e		r		
116	+	105	+	103	+	101	+	114	=	539

```
public int hashCode(String s) {
   int hash = 0;
   for (int i = 0; i < s.length(); i++) {
      hash = hash + s.charAt(i);
   }
   return hash;
}
```

Generating hash codes – Strings

```
public int hashCode(String s) {
   int hash = 0;
   for (int i = 0; i < s.length(); i++) {
      hash = hash + s.charAt(i);
   }
   return hash;
}
```

hashCode(“top”) = 339
hashCode(“pot”) = 339

Make the order of the characters matter with a “place value” – choose a radix > 1 to compute the hash code.

Example: Radix = 11

t		o		p		
$11^2 \bullet 116$	+	$11^1 \bullet 111$	+	$11^0 \bullet 112$	=	15369
p		o		t		
$11^2 \bullet 112$	+	$11^1 \bullet 111$	+	$11^0 \bullet 116$	=	14889

```
public int hashCode(String s) {
   int hash = 0;
   for (int i = 0; i < s.length(); i++) {
      hash = 11*hash + s.charAt(i);
   }
   return hash;
}
```

hashCode(“top”) = 15369
hashCode(“pot”) = 14889

String in Java ...

```
public final class String ... {
   private final char[] s;

   public int hashCode() {
      int hash = 0;
      for (int i = 0; i < length(); i++) {
         hash = (31*hash) + s[i];
      }
      return hash;
   }
}
> String s = new String("Hello")
> s.hashCode()
  99162322
```

Generating hash codes – user-defined types

A decent recipe: Use the **Horner’s Rule** pattern.

```
public class Book implements Comparable {
    private String author = new String("no title");
    private String title = new String("none");
    private int pages = 0;



   public int hashCode() {
       int hash = 17;
       hash = 31*hash + author.hashCode();
       hash = 31*hash + title.hashCode();
       hash = 31*hash + pages;
       return hash;
   }
```

17 and 31 are arbitrary primes.

The Map ADT

A **map** (or *associative array,* or *dictionary*) is an abstract data type composed of key-value pairs, where each possible key occurs at most once.

Example key-value pairs: (SS#, person info) (ISBN, book info) (phone#, address)

(word, definition) (filename, location on disk)

Different data structures could be used to implement a map, but we can get a dramatic speed-up by considering a new type of data structure, one that’s not based on comparison of elements.

Map method	Ordered List	Balanced Search Tree	Hash table
add(key, value)	O(N)	O(log N)	**O(1)**
remove(key)	O(N)	O(log N)	**O(1)**
search(key)	O(log N)	O(log N)	**O(1)**
	worst case	worst case	average case (but highly probable)