Notation for Stochastic Dynamic Programming (Markov Decision Processes, Approximate Dynamic Programming, Reinforcement Learning)

	Bertsekas	Sutton and Barto	Puterman	Powell
Stages	k	t	t	t
First Stage	N	1	1	1
Final Stage	0	T	N	T
State Space			S	$\mathcal{S}$
State	$i, i_k$	s	s	$s, S_t$
Action Space	U(i)		$A = \cup_{s \in S} A_s$	$\mathcal{A}$
Action		a	a	a
Policy	$\mu_k(i), \pi$	$\pi(s,a), \pi$	$\pi, d_t^{MD}(s)$	$\pi$
Transitions	$p_{ij}(\mu_k(i))$	$\mathcal{P}^a_{ss'}$	$p_t(\cdot \mid s, a)$	$  \mathbb{P}(s'   S_t, a_t)  $
Cost	g(i,u,j)	$\mathcal{R}^a_{ss'}$	$r_t(s,a)$	$C_t(S_t, a_t)$
Terminal Cost	$G(i_N)$	$r_T$	$r_N(s)$	$V_T(S_T)$
Discount	$\alpha$	$\gamma$	$\lambda$	$\gamma$
Q-Value (Policy)	$J_k^{\pi}(i)$	$\mathcal{Q}^{\pi}(s,a)$		
Q-Value (Optimal)				$Q(S^n,a)$
Value (Policy)	$J_k^{\pi}(i)$	$V^{\pi}(s)$	$u_t^{\pi}$	$V_t^{\pi}(S_t)$
Value (Optimal)	$J_k^*(i)$	$V^*(s)$	$u_t^*$	$V_t(S_t)$
Bellman Operator	T		$\mathscr{L}, L$	$\mathcal{M}$

## **Optimal Value Function**

• Bertsekas [2007]

$$J_k^* = \min_{u \in U(i)} \sum_{j=1}^n p_{ij}(u) \left( g(i, u, j) + \alpha J_{k-1}^*(j) \right)$$

• Sutton and Barto [1998]

$$V^*(s) = \max_{a} \mathcal{P}_{ss'}^{a} \left[ \mathcal{R}_{ss'}^{a} + \gamma V^*(s') \right]$$

• Puterman [1994]

$$u_t^*(s_t) = \max_{a \in A_{s_t}} \left\{ r_t(s_t, a) + \sum_{j \in S} p_t(j \mid s_t, a) u_{t+1}^*(j) \right\}$$

• Powell [2011]

$$V_t(S_t) = \max_{a_t} \left\{ C_t(S_t, a_t) + \gamma \sum_{s' \in \mathcal{S}} \mathbb{P}(s' \mid S_t, a_t) V_{t+1}(s') \right\}$$

## References

- D.P. Bertsekas. *Dynamic Programming and Optimal Control*. Number v. 2 in Athena Scientific Optimization and Computation Series. Athena Scientific, 2007. ISBN 9781886529304. URL http://books.google.com/books?id=eL01YAAACAAJ.
- W.B. Powell. Approximate Dynamic Programming: Solving the Curses of Dimensionality. Wiley Series in Probability and Statistics. John Wiley & Sons, 2011. ISBN 9781118029152. URL http://books.google.com/books?id=VBuZhne7pmwC.
- M.L. Puterman. Markov decision processes: discrete stochastic dynamic programming. Wiley series in probability and statistics. Wiley-Interscience, 1994. ISBN 9780471727828. URL http://books.google.com/books?id=Y-gmAQAAIAAJ.
- R.S. Sutton and A.G. Barto. Reinforcement Learning: An Introduction. Adaptive Computation and Machine Learning. Mit Press, 1998. ISBN 9780262193986. URL http://books.google.com/books?id=CAFR61BF4xYC.