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32) Expressar o vetor $u = (-1, 4, -4, 6) \in \mathbb{R}^4$ como combinação linear dos vetores $v_1 = (3, -3, 1, 0)$, $v_2 = (0, 1, -1, 2)$ e $v_3 = (1, -1, 0, 0)$

$$\vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 + a_4\vec{v}_4$$

$$(-1, 4, -4, 6) = a_1(3, -3, 1, 0) + a_2(0, 1, -1, 2) + a_3(1, -1, 0, 0) + a_4(0, 0, 0, 0)$$

$$= (3a_1, -3a_1, a_1, 0) + (0, a_2, -a_2, 2a_2) + (a_3, -a_3, 0, 0) + (0, 0, 0, 0)$$

$$= (3a_1 + 0 + a_3 + 0, -3a_1 + a_2 - a_3 + 0, a_1 - a_2 + 0 + 0, 0 + 2a_2 + 0 + 0)$$

$$(-1, 4, -4, 6) = (3a_1 + a_3, -3a_3 + a_2 - a_3, a_1 - a_2, 2a_2)$$

$$\begin{cases} 3a_1 + a_3 = -1 \\ -3a_1 + a_2 - a_3 = 4 \\ a_1 - a_2 = -4 \\ 2a = 6 \end{cases} \Rightarrow \begin{cases} 3a_1 + a_3 = -1 \\ -3a_1 + a_2 - a_3 = 4 \\ a_1 - a_2 = -4 \\ 2a_2 = 6 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 3 & 0 & 1 & -1 \\ -3 & 1 & -1 & 4 \\ 1 & -1 & 0 & -4 \\ 0 & 2 & 0 & 6 \end{array}\right)$$

$$\begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \qquad \begin{array}{ccc} a_1 & a_3 & a_2 \end{array} \qquad \begin{array}{ccc} a_3 & a_1 & a_2 \end{array}$$

$$\left(\begin{array}{ccc|c} -3 & 1 & -1 & 4 \\ 3 & 0 & 1 & -1 \\ 1 & -1 & 0 & -4 \\ 0 & 2 & 0 & 6 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} -3 & 1 & -1 & 4 \\ 0 & 1 & 0 & 3 \\ 1 & -1 & 0 & -4 \\ 0 & 2 & 0 & 6 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} -3 & -1 & 1 & 4 \\ 0 & 0 & 1 & 3 \\ 1 & 0 & -1 & -4 \\ 0 & 0 & 2 & 6 \end{array}\right)$$

$$nL_2 = L_2 + L_1$$

$$\Rightarrow \left(\begin{array}{ccc|c} -1 & -3 & 1 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 2 & 6 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} -1 & -3 & 1 & 4 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 6 \end{array}\right) \Rightarrow \begin{cases} -a_3 - 3a_1 + a_2 = 4 \\ a_1 - a_2 = -4 \\ a_2 = 3 \\ 2a_2 = 6 \end{cases}$$

4 eq e 3 var **SPD** $a_2 = 3$ $\quad a_1 - 3 = -4 \therefore a_1 = -4 + 3$

$-a_3 - 3(-1) + 3 = 4$ $\qquad a_1 = -1$

$-a_3 + 3 + 3 = 4 \therefore -a_3 = 4 - 6 \therefore$ $\qquad a_3 = 2$

R: $u = -1v_1 + 3v_2 + 2v_3$

35) Determinar os subespaços do $\mathbb{R}^3$ gerados pelos seguintes conjuntos

c) $A = \{(1,0,1), (0,1,1), (-1,1,0)\}$

$$(x,y,z) = a_1(1,0,1) + a_2(0,1,1) + a_3(-1,1,0)$$
$$= (a_1, 0, a_1) + (0, a_2, a_2) + (-a_3, a_3, 0)$$
$$= (a_1 + 0 - a_3,\ 0 + a_2 + a_3,\ a_1 + a_2 + 0)$$
$$(x,y,z) = (a_1 - a_3,\ a_2 + a_3,\ a_1 + a_2)$$

$$\begin{cases} a_1 - a_3 = x \\ a_2 + a_3 = y \\ a_1 + a_2 = z \end{cases} \Leftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & x \\ 0 & 1 & 1 & y \\ 1 & 1 & 0 & z \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 1 & -1 & x+z \\ 0 & 1 & 1 & y \\ 1 & 1 & 0 & z \end{array}\right)$$

$nL_1 = L_1 + L_3$

$nL_3 = 2L_3 - L_1$

$$\begin{array}{cccc} 2 & 2 & 0 & 2z \\ -2 & -1 & +1 & -x-z \\ \hline 0 & 1 & 1 & -x+z \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 2 & 1 & -1 & x+z \\ 0 & 1 & 1 & y \\ 0 & 1 & 1 & z-x \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 1 & -1 & x+z \\ 0 & 1 & 1 & y \\ 0 & 1 & 1 & z-x \end{array}\right)$$

$nL_3 = L_3 - L_2$

$$\begin{array}{cccc} 0 & 1 & 1 & z-x \\ 0 & -1 & -1 & -y \\ \hline & 0 & 0 & z-x-y \end{array}$$

Após escalonar o sistema verificamos que ele somente se torna possível se

$z - x - y = 0$ ou $x + y - z = 0$

R: $S = \{(x,y,z) \in \mathbb{R}^3 \ / \ x + y - z = 0\}$

35)

d) $A = \{(-1,1,0); (0,1,-2); (-2,3,1)\}$

$$(x,y,z) = a_1(-1,1,0) + a_2(0,1,-2) + a_3(-2,3,1)$$
$$= (-a_1, a_1, 0) + (0, a_2, -2a_2) + (-2a_3, 3a_3, a_3)$$
$$= (-a_1 + 0 - 2a_3,\ a_1 + a_2 + 3a_3,\ 0 - 2a_2 + a_3)$$
$$(x,y,z) = (-a_1 - 2a_3,\ a_1 + a_2 + 3a_3,\ -2a_2 + a_3)$$

$$\begin{cases} -a_1 - 2a_3 = x \\ a_1 + a_2 + 3a_3 = y \\ -2a_2 + a_3 = z \end{cases} \Rightarrow \left(\begin{array}{ccc|c} -1 & 0 & -2 & x \\ 1 & 1 & 3 & y \\ & -2 & 1 & z \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} -1 & -2 & 1 & x+z \\ 1 & 1 & 3 & y \\ & -2 & 1 & z \end{array}\right)$$

$mL_1 = L_1 + L_3$
$mL_2 = L_2 + L_1$
$mL_3 = L_3 - 2L_2$

$$\Rightarrow \left(\begin{array}{ccc|c} -1 & -2 & 1 & x+z \\ 0 & -1 & 4 & x+y+z \\ & -2 & 1 & z \end{array}\right)$$

$$\begin{array}{cccc} 0 & -2 & 1 & z \\ 0 & +2 & -8 & -2x-2y-2z \\ \hline 0 & 0 & -7 & -2x-2y-z \end{array} +$$

$$\Rightarrow \left(\begin{array}{ccc|c} -1 & -2 & 1 & x+z \\ 0 & -1 & 4 & x+y+z \\ 0 & 0 & -7 & -2x-2y-z \end{array}\right)$$

Como após escalonar, não existe restrição para a resolução do sistema, pois $-2x-2y-z$ pode assumir qualquer valor

R: $S = \{(x,y,z) \in \mathbb{R}\}$ ou $S = \mathbb{R}^3$