Com S 477/577 Fall 2022

Assignment 1 (50 pts)

Due online before class

Thursday, September 1

1.(10 pts) Consider a point $\boldsymbol{p} = (2, 6)$.

(a) (4 pts) Suppose that the point undergoes first a translation by $\boldsymbol{b} = (1, -2)$ and then a rotation about the origin through $\frac{\pi}{3}$. Give the coordinates of the resulting point $\boldsymbol{p}'$.

(b) (4 pts) In a second scenario, $\boldsymbol{p}$ at its original position $(2, 6)$ first rotates about the origin through $\frac{\pi}{3}$, and then translates by $\boldsymbol{b}$. Give the coordinates of the resulting point $\boldsymbol{p}''$.

(c) (2 pts) Are rotation and translation commutative?

2.(10 pts) Consider three points $\boldsymbol{p}_1 = (2, 3)$, $\boldsymbol{p}_2 = (5, 1)$, and $\boldsymbol{q} = (1, 1)$. Find the point on the line ℓ through $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ that is the closest to $\boldsymbol{q}$. You are required to use homogeneous coordinates.

3.(8 pts) Consider a planar displacement T that starts with a rotation about the origin through $\pi/3$ and follows with a translation $\binom{1}{0}$. Locate the point $\boldsymbol{s}$ that is not changed under T. (This point is called the *pole* of T.)

4.(10 pts) Two coordinate frames x-y and u-v share the same origin in the plane. Let $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ be the unit vectors along the u- and v-axes, respectively. They are represented in x- and y-coordinates.

(a) (5 pts) Give the matrix R describing the rotation from the xy-frame to the uv-frame, that is, the matrix mapping a point $\boldsymbol{p}$ in the uv-coordinates $\binom{p_u}{p_v}$ to its xy-coordinates $R\binom{p_u}{p_v}$.

(b) (5 pts) Let $\hat{\boldsymbol{\imath}} = \binom{0}{1}$ and $\hat{\boldsymbol{\jmath}} = \binom{-1}{0}$. Suppose that a point $\boldsymbol{p}$ has coordinates $\binom{p_x}{p_y} = \binom{3}{4}$ in the xy-frame. What are its coordinates $\binom{p_u}{p_v}$ in the uv-frame?

5.(12 pts) In this problem you are required to use homogeneous coordinates.

(a) (6 pts) Determine the line passing through $(2, -4)$ and $(11, 13)$.

(b) (6 pts) Give the point of intersection of the lines $-2x + 3y - 8 = 0$ and $5x + 2y + 6 = 0$.