Matthew Bentz
Stat 3600-140

1.1-1 $P(pt) \cap P(ch) = \frac{28}{100}$

$(P(pt') \cap P(ch') = \frac{8}{100}$; $P(pt \cup ch)' = \frac{8}{100}$ $pt \cup ch = \frac{92}{100}$

$P(pt)$ ~~[illegible]~~ $= P(ch) + \frac{16}{100}$

$P(ch) + \frac{16}{100} \cap P(ch) = \frac{28}{100}$

$P(ch) = \frac{12}{100}$

$Pt \cup Ch = 92\%$

$Pt + Ch - (Pt \cap Ch) = 92\%$

$Pt + Ch - (28\%) = 92\%$

$Pt + Ch = 120\%$

$Ch + Ch = 104\%$

$Ch = 52\%$ $+16 = Pt$

$\boxed{P(Pt) = 68\%}$

1.1-3 $S = 52$

A.) $\frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{52}$

B.) $(\frac{4}{52} + \frac{4}{52} + \frac{4}{52}) \div 2 = \frac{6}{52}$

C.) $\frac{13}{52}$

D.) $\frac{39}{52}$

↳ a.) $P(A) = \frac{12}{52}$

b.) $P(A \cap B) = \frac{2}{52}$ (red Jacks)

c.) $P(A \cup B) = \frac{16}{52}$ $(\frac{18}{52} - \frac{2}{52})$

d.) $P(C \cup D) = 1$

e.) $P(C \cap D) = 0$

1.1-10 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
$- P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$A \cup B \cup C = A \cup (B \cup C)$

$= A \cup (B + C - B \cap C)$

$= A + (B + C - (B \cap C)) - (A \cap (B + C - B \cap C))$

$= A + B + C - (B \cap C) - (A \cap B) - (A \cap C) + (A \cap B \cap C)$

1.2-4 a.) 1 flavor × 3 toppings = $4 \times \binom{6}{3} = \boxed{80}$

b.) 1 flavor × 0–6 toppings = $4 \times 2^6 = \boxed{256}$

c.) 3 flavors w/ replacement = $\binom{6+3-1}{3} = \boxed{20}$

1.2-11 a.) $\frac{9!}{(9-9)!} = 9! = \boxed{362,880}$

b.) $\frac{9!}{3!6!} = \boxed{84}$

c.) $2^9 = \boxed{512}$

1.2-17 a.) $\frac{\binom{13}{1}\binom{3}{3}\binom{48}{1}}{\binom{52}{5}} = \boxed{0.00024}$

b.) $\frac{\binom{4}{2}\binom{4}{3}\binom{13}{2}}{\binom{52}{5}} = \boxed{0.0014}$

c.) $\frac{\binom{13}{1}\binom{3}{2}\binom{49}{2}}{\binom{52}{5}} = \boxed{0.02113}$

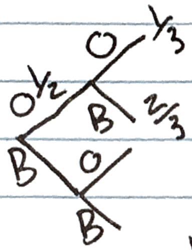
d.) $\frac{\binom{13}{2}\binom{3}{1}\binom{3}{1}\binom{48}{1}}{\binom{52}{5}} = \boxed{0.04754}$

e.) $\frac{\binom{13}{1}\binom{3}{1}\binom{50}{3}}{\binom{52}{5}} = \boxed{0.42257}$

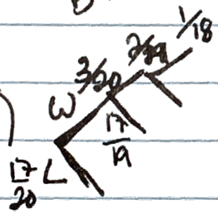
1.3-4 a.) $\frac{1}{4} \times \frac{13}{51} = \frac{12}{204} = \boxed{\frac{3}{51}}$

b.) $\frac{13}{52} \times \frac{13}{51} = \boxed{\frac{169}{2652}} =$

c.) $\frac{1}{4} \times \frac{4}{51} = \boxed{\frac{4}{204}}$

1.3-7 

$\frac{1}{2} \times \frac{1}{3} = \boxed{\frac{1}{6}}$ $\frac{1}{5}$?

1.3-8 a.) 

$P(W|W|W) = \frac{3}{20} \times \frac{2}{19} \times \frac{1}{18} = \boxed{.0877\%}$

b.) P(Winning on 2^{nd} draw) $= \boxed{0}$ must choose 3!

c.) $P(W) = P(W_2) \dots P(W_{10})$

$$P(W) = \sum_{K=1}^{9} \frac{\binom{3}{2}\binom{17}{2K-2}}{\binom{20}{2K}} \cdot \frac{1}{20-2K} = \frac{35}{76} = \boxed{.4605}$$

d.) $P(W$ on $1^{st}) = .4605$ $P(W$ on $2^{nd}) = \boxed{.5395}$

Second, higher chance of winning

1.3-16 $P(R_B) = P(R_A \cap R_B) + P(W_A \cap R_B)$

$P(R_A \cap R_B) = P(R_A) \times P(R_B|R_A) = \frac{3}{5} \times \frac{5}{8} = \frac{3}{8}$

$P(W_A \cap R_B) = P(W_A) \times P(R_B|W_A) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$

$P(R_B) = \frac{3}{8} + \frac{1}{5} = \frac{15+8}{40} = \frac{23}{40} = \boxed{.575}$

1.4-1 a.) $P(A \cap B) = .7 \times .2 = \boxed{.14}$

b.) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.7 + .2 - .14 = .9 - .14 = \boxed{.76}$

c.) $P(A' \cup B') = P(A \cap B)' = 1 - .14 = \boxed{.86}$

1.4-7 a.) $P(\text{Only one successful}) = P(A_1) \times P(A_2)' \times P(A_3)' +$

$P(A_1)' \times P(A_2) \times P(A_3)' +$

$P(A_1)' \times P(A_2)' \times P(A_3)$

$= (.5 \times .3 \times .4) + (.5 \times .7 \times .4) + (.5 \times .3 \times .6)$

$= \boxed{.29}$

b.) $P(2 \text{ success and } 1 \text{ fail}) = P(A_1) \times P(A_2) \times P(A_3)' +$

$P(A_1) \times P(A_2)' \times P(A_3) +$

$P(A_1)' \times P(A_2) \times P(A_3)$

$= (.5 \times .7 \times .4) + (.5 \times .3 \times .6) + (.5 \times .7 \times .6)$

$= \boxed{.44}$

1.4-17 a.) $P(\text{matches} \geq 1) = 1 - P(\text{matches} = 0)$

$P(\text{matches} = 0) = \frac{11}{12} \cdot \frac{11}{12} \ldots = \frac{11}{12}^{12} = .352$

$1 - .352 = \boxed{.648}$

b.) $P(\text{match with at least 1 other student}) = 1 - P(\text{matches} = 0 \text{ for } 11 \text{ students})$

$P(0 \text{ matches } \text{'} \, 11 \text{ students}) = \frac{11}{12}^{11} = .384$

$1 - .384 = \boxed{.616}$

(b) Proof:

$$P(A^c \cap B) = P(A^c)P(B) \quad \text{-Multiplication rule}$$
$$= P(A^c|B)P(B)$$
$$= [1-P(A|B)]P(B) \quad \text{-Conditional Prob}$$
$$= [1-P(A)]P(B) \quad \text{-Since } A \,\&\, B \text{ are independent}$$
$$= P(A^c)P(B) \quad \square$$

(c) Proof:

$$P(A^c \cap B^c) = P(A^c)P(B^c)$$
$$= [P(A^c|B^c)P(B^c|A^c)] \quad \text{-Multiplication rule}$$
$$= [1-P(A|B^c)][1-P(B|A^c)] \quad \text{-Conditional prob}$$
$$= [1-P(A)][1-P(B)] \quad \text{-independence rule}$$
$$= P(A^c)P(B^c) \quad \square$$

1.5-1 $P(B_1) = 1/2 \quad P(B_2) = 1/4 \quad P(B_3) = 1/8 \quad P(B_4) = 1/8$

a.) $P(W) = \sum_{i=1}^{x} P(B_i) \times P(W|B_i)$

$= (1/2 \cdot 1) + (1/4 \cdot 0) + (1/8 \cdot 1/2) + (1/8 \cdot 3/4)$

$= 1/2 + 1/16 + 3/32 = \boxed{21/32}$

b.) $P(B_1|W) = \dfrac{P(B_1) \times P(W|B_1)}{21/32} = \dfrac{1/2}{21/32} = \dfrac{32}{42} = \boxed{\dfrac{16}{21}}$

1.5-6 Standard = .6 $\quad P(\text{Dying}|S) = .01$

Preferred = .3 $\quad P(D|P) = .008$

Ultrapref = .1 $\quad P(D|U) = .007$

$\sum_{i=1}^{m} P(\text{Plan } i) \times P(\text{Dying}|\text{Plan } i) = (.6 \times .01) + (.3 \times .008) + (.1 \times .007)$

$= .0091$

$P(S|D) = .006/.0091 = \boxed{.659}$

$P(P|D) = .3 \times .008/.0091 = \boxed{.264}$

$P(U|D) = .0007/.0091 = \boxed{.0769}$

1.5-11 $P(H) = .15 \quad P(\text{Death}|H) = 8x$

$P(L) = .3 \quad P(\text{Death}|L) = 5x$

$P(N) = .55 \quad P(\text{Death}|N) = x$

$P(N|\text{Death}) = \dfrac{P(N) \times P(\text{Death}|N)}{\sum_{i=1}^{m} P(\text{Smoker}_i) P(\text{Death}|\text{smoker}_i)}$

$= \dfrac{.55x}{(.15 \times 8x) + (.3 \times 5x) + (.55x)} = \dfrac{.55x}{2.2x} = \boxed{.25}$

2.1-3 a.) $f(x) = x/c \quad x = 1,2,3,4$

$f(x) = \frac{1}{c} + \frac{2}{c} + \frac{3}{c} + \frac{4}{c} = \frac{10}{c} = 1 \quad \boxed{c = 10}$

b.) $f(x) = cx \quad x = 1,2,3,\ldots 10.$

$c(1) + c(2) + c(3) \ldots + c(10) = c(1+2+3+\ldots 10)$
$= 55c = 1 \quad \boxed{c = 1/55}$

c.) $f(x) = c\left(\frac{1}{4}\right)^x \quad x = 1,2,3\ldots\ldots$

$c(1/4) + c(1/16) + c(1/64)\ldots\ldots = c\sum_{x=1}^{\infty}(1/4)^x = c \cdot 1/3$
$c/3 = 1 \quad \boxed{c = 3}$

d.) $f(x) = c(x+1)^2 \quad x = 0,1,2,3$

$c + 4c + 9c + 16c = 30c = 1 \quad \boxed{c = 1/30}$

e.) $f(x) = x/c \quad x = 1,2,3\ldots n$
$1/c \sum_{x=1}^{n} x = 1/c \cdot \frac{n(n+1)}{2} = 1 \quad \boxed{c = \frac{n(n+1)}{2}}$

2.1-10 $X \sim$ Hypergeometric $(10, 3, 47)$
a.)
$f(1) = \frac{\binom{3}{1}\binom{47}{9}}{\binom{50}{10}} = \boxed{\frac{38}{98}}$

b.) $f(x=0) + f(x=1) \quad \frac{247}{490} + \frac{38}{98} = \boxed{.902}$

$f(0) = \frac{\binom{3}{0}\binom{47}{10}}{\binom{50}{10}} = \frac{247}{490}$

2.2-1 a.) $f(x)=\frac{x}{c}$ $c=10$ $E[x]=\sum x\cdot f(x)$ $x=1,2,3,4$

$\frac{1}{10}\sum x^2 = \frac{1}{10}(1+4+9+16)=\frac{30}{10}=\boxed{3}$

b.) $f(x)=cx$ $E[x]=\sum x c x$ $x=1-10$ $c=\frac{1}{55}$

$\frac{1}{55}\sum x^2=\frac{1}{55}(1+4+9+16+25+36+49+64+81+100)$

$=\frac{385}{55}=\boxed{7}$

c.) $f(x)=c\frac{1}{4}^x$ $x=1-\infty$ $c=3$ $E[x]=3\sum x\frac{1}{4}^x$

$3\sum_{x=1}^{\infty}\frac{x}{4}^x=3\cdot\frac{4}{9}=\boxed{\frac{4}{3}}$

d.) $f(x)=c(x+1)^2$ $x=0,1,2,3$ $E[x]=\sum x\, c(x+1)^2$ $c=\frac{1}{30}$

$\frac{1}{30}\sum_{x=0}^{3}x(x+1)^2=\frac{1}{30}(0+4+18+48)=\frac{70}{30}=\boxed{\frac{7}{3}}$

e.) $f(x)=\frac{x}{c}$ $x=1-n$ $c=\frac{n(n+1)}{2}$ $E[x]=x\frac{x}{c}=\frac{x^2}{c}$

$\frac{2}{n(n+1)}\sum_{x=1}^{n}x^2=\frac{2}{\cancel{n(n+1)}}\cdot\frac{\cancel{(n)(n+1)}(2n+1)}{6}=\boxed{\frac{2n+1}{3}}$

2.2-4 $f(x)=\begin{cases} .9 & x=0 \\ \frac{c}{x} & x=1..6\end{cases}$

$.9+\frac{c}{1}+\frac{c}{2}+\frac{c}{3}+\frac{c}{4}+\frac{c}{5}+\frac{c}{6}=1$

$c+\frac{c}{2}+\frac{c}{3}+\frac{c}{4}+\frac{c}{5}+\frac{c}{6}=.1$

$60c+30c+20c+15c+12c+10c=6$

$147c=6$

$c=\frac{6}{147}=\boxed{\frac{2}{49}}$ ~~$E[x]=\sum$~~

↓

pmf

$f(x) = f(0) = 9/10$

$f(1) = 2/49$

$f(2) = 2/49 \cdot 1/2$

$f(3) = 2/49 \cdot 1/3$

$f(4) = 2/49 \cdot 1/4$

$f(5) = 2/49 \cdot 1/5$

$f(6) = 2/49 \cdot 1/6$

$u(x) = \begin{cases} 0 & x = 0,1 \\ x-1 & x = 2-6 \end{cases}$

$f(x) = \frac{c}{x}$

$E[x] = \sum_0^6 u(x) f(x) + (2 \cdot 2/147) + (3 \cdot 1/98) + (4 \cdot 2/245) + (5 \cdot 1/147)$

$= 0 + 0 + 1/49$

$= \boxed{.145}$

2.2-8 $h(c) = E[|X-c|] = \sum_{x \in (1 \ldots 50)} |X-c| P(X=x)$

$1/7(|1-c| + |2-c| + |3-c| + |5-c| + |15-c| + |25-c| + |50-c|)$

$= 12 + \underbrace{|5-c|}_{7}$ c is minimized at 5

2.2-9 a.) $E[x] = \sum x P(X=x) = 1 \cdot \frac{18}{38} + -1 \cdot \frac{10}{19} = \boxed{-.0526}$

b.) $E[x] = \sum x P(X=x) = 1 \cdot \frac{18}{37} + -1 \cdot \frac{19}{37} = \boxed{-.027} = \boxed{-\frac{1}{37}}$

2.2-12 a.) $\frac{1000 \text{ students}}{20 \text{ classes}} = \boxed{50}$ avg

b.) $P(X = \text{student in 25 person class}) = 16 \cdot 25/1000 = \boxed{2/5}$

$P(X = 100) = \boxed{3/10}$

$P(X = 300) = \boxed{3/10}$

2.3-8 $f(x) = \frac{2x-1}{16}$ $x = 1,2,3,4$

mean $= \cancel{\frac{1}{16} + \frac{2}{16} + \frac{5}{16} + \frac{9}{16} = 1}$

$1+3+5+9 \div 16 = \frac{50}{16} = \boxed{3.125}$

variance $= E[x^2] - E[x]^2 = E[x^2] - 3.125^2$

$E[x^2] = \sum x^2 f(x) = 1 \cdot \frac{1}{16} + 4 \cdot \frac{3}{16} + 9 \cdot \frac{5}{16} + 16 \cdot \frac{9}{16}$
$= 10.625$

$10.625 - 3.125^2 = \boxed{.859}$

$\sigma = \sqrt{.859} = \boxed{.927}$

2.3-11 mean $= M'(0) = \frac{2}{5} + \frac{2}{5} + \frac{6}{5} = \frac{10}{5} = \boxed{2}$

variance $= M''(0) = \frac{2}{5} + \frac{4}{5} + \frac{18}{5} = \cancel{\boxed{\frac{22}{5}}}$ $\frac{24}{5}$

$E[x^2] - E[x]^2 = \frac{24}{5} - 2^2 = \boxed{\frac{4}{5}}$

pmf $= M(t) =$ $f(x) = \begin{cases} \frac{2}{5} & x=1 \\ \frac{1}{5} & x=2 \\ \frac{2}{5} & x=3 \end{cases}$

2.3-12 a.) $X \sim$ geometric $p = \frac{1}{365}$

$f(x) = (1-p)^{x-1}(p) = \left(\frac{364}{365}\right)^{x-1}\left(\frac{1}{365}\right)$

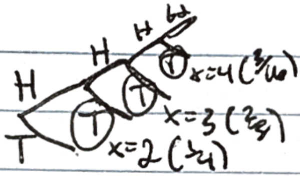
b.) $\mu = \sum_{x=1}^{\infty} x \left(\frac{364}{365}\right)^{x-1}\left(\frac{1}{365}\right) = \boxed{365}$

$E[x^2] = \sum_{x=1}^{\infty} x^2 P(x) = \frac{1}{365} \sum_{x=1}^{\infty} x^2 \left(\frac{364}{365}\right)^{x-1}$
$= 266{,}085$

$\sigma^2 = 266{,}805 - 365^2 = \boxed{132{,}860}$
$\sigma = \sqrt{\sigma^2} = \boxed{364.6}$

$P(X>400) = 1 - \sum_{x=1}^{399} (1-p)^{x-1}(p) = .33$
$P(X<300) = \sum_{x=1}^{299} (1-p)^{x-1}(p) = .56$

2.3-17



$f(x) = \frac{x-1}{2^x}$

b.) $M(t) = E[e^{tx}] = \sum_{x=2}^{\infty} e^{tx}\left(\frac{x-1}{2^x}\right)$

$= x-1\left(\frac{e^t}{2}\right)^x$
$= \sum x\left(\frac{e^t}{2}\right)^x - \sum\left(\frac{e^t}{2}\right)^x$
$= \frac{e^{2t}}{(e^t-2)^2}$

c.) (i) $\mu = M'(0) = \frac{4e^0}{(2-e^0)^3} = 4$

(ii) $M''(0) = \frac{4e^0 + 16e^0}{(2-e^0)^4} = 20$

d.) (i) $P(X \le 3) = P(2) + P(3) = \frac{1}{4} + \frac{2}{8} = \boxed{\frac{1}{2}}$
(ii) $P(X \ge 5) = 1 - P(2) - P(3) - P(4)$
$1 - \frac{1}{4} + \frac{2}{8} + \frac{3}{16} = \boxed{\frac{5}{16}}$
(iii) $P(X=3) = \boxed{\frac{1}{4}}$

2.4-6 a.) Binomial distribution $b(15, .75)$

b.) $P(x \geq 10) = P(10) + P(11) \ldots P(15)$

$$\binom{n}{x} p^x (1-p)^{n-x}$$

$$\binom{15}{10} .75^{10} (1-.75)^5 \ldots \binom{15}{15} (.75^{15})(1-.75)^0 = \boxed{.8516}$$

c.) $P(x \leq 10) = 1 - P(x > 10) = \boxed{.3135}$

d.) $P(x=10) = \binom{15}{10} (.75)^{10} (.25)^5 = \boxed{.1651}$

e.) $\mu = n \cdot p = 15 \cdot .75 = \boxed{11.25}$

$$\sigma^2 = 11.25(1-p) = 11.25 \times .25 = \boxed{2.8125}$$

$$\sigma = \sqrt{2.8125} = \boxed{1.68}$$

2.4-14 a.) $P(x \geq 1) = 1 - P(x=0)$

$$P(x=0) = \binom{n}{0} .1^0 (.9)^n = 0.9^n$$

$$1 - .9^n = 0.5$$

$$+.9^n = +.5$$

$$n \ln .9 = \ln .5$$

$$n = \frac{\ln .5}{\ln .9} = \boxed{7} \text{ tickets}$$

b.) $1 - .9^n = .95$

$$.9^n = .05$$

$$n = \frac{\ln .05}{\ln .9} = \boxed{29} \text{ tickets}$$

2.5-5 a.) $\mu = R'(0) = \frac{M'(0)}{M(0)} = \frac{\mu}{M(0)} = \frac{\mu}{E[e^{0x}]} = \frac{\mu}{E[1]} = \frac{\mu}{1} = \mu$

b.) $\sigma^2 = R''(0) = \frac{M(0)M''(0) - M'(0)^2}{M(0)^2}$

$= \frac{1\, M''(0) - M'(0)^2}{1}$

$= E[x^2] - \mu^2 = \sigma^2$

2.5-6 a.) Bernoulli $M(t) = 1 - p + pe^t$ $M(0) = 1$

$M'(t) = pe^t$ $M'(0) = p$

$M''(t) = pe^t$ $M''(0) = p$

$\mu = \frac{M'(0)}{M(0)} = \boxed{p}$

$\sigma^2 = \frac{M(0)M''(0) - [M'(0)]^2}{[M(0)]^2} = \frac{p - p^2}{1} = \boxed{p(1-p)}$

b.) Binomial distribution $M(t) = (1-p+pe^t)^n$

$M'(t) = n(1-p+pe^t)^{n-1}pe^t$ $M'(0) = np$

$M''(0) = n^2p^2 - np^2 + np$

$\mu = \frac{M'(0)}{M(0)} = \boxed{np}$

$\sigma^2 = (n^2p^2 - np^2 + np) - (np)^2 = \boxed{np(1-p)}$

c.) Geometric $M(t) = \frac{pe^t}{1-(1-p)e^t}$

$M(0) = 1$

$M'(0) = 1/p$

$M''(0) = \frac{2-p}{p^2}$

$\mu = \frac{1/p}{1} = \boxed{1/p}$

$\sigma^2 = \frac{(2-p)/p^2 - (1/p)^2}{1} = \boxed{\frac{1-p}{p^2}}$