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pdf $\Rightarrow f(x) = xe^{-x} \quad 0 < x < \infty$

mgf $\Rightarrow M(t) = E[e^{tx}] = \int_0^\infty e^{tx} x e^{-x} dx$

$\Rightarrow \lim_{b\to\infty} \int_0^b x e^{-(1-t)x} dx$

$u = x \qquad dv = e^{x(t-1)}$
$du = 1 \qquad v = \frac{1}{t-1} e^{x(t-1)}$

$\Rightarrow M(t) = \lim_{b\to\infty} \int_0^b u\,dv$

$= \lim_{b\to\infty} \left(\int_0^b (uv)' dx - \int_0^b v\,du\,dx \right) = \frac{1}{(1-t)^2}$

3.1.1 pdf $\Rightarrow f(x) = \frac{1}{b-a} \quad a \le x \le b$

$\mu = \int_a^b x \cdot f(x) dx = \int_a^b x \cdot \frac{1}{b-a} dx = \frac{a+b}{2}$

$\sigma^2 = E[x^2] - (E[x])^2 = \int_a^b x^2 \frac{1}{b-a} dx - \left(\frac{a+b}{2}\right)^2 = \frac{(b-a)^2}{12}$

$M(t) = \int_a^b e^{tx} \frac{1}{b-a} dx = \frac{e^{tb} - e^{ta}}{t(b-a)}$

3.1.4 a.) $E[x] = \frac{a+b}{2} \qquad x \sim U(4,5) \qquad \mu = 9/2$

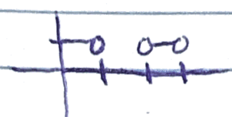
b.) $Var(x) = \frac{(b-a)^2}{12} = 1/12$

c.) $P(4.2 < X \le 4.7) = \int_{4.2}^{4.7} \frac{1}{5-4} dx = 4.7 - 4.2 = 0.5$

3.1-13 $\text{cdf} = (1+e^{-x})^{-1} \quad -\infty < x < \infty$

$\text{pdf} = -(1+e^{-x})^{-2}(-e^{-x}) = \dfrac{e^{-x}}{(1+e^{-x})^2}$

3.1.14 $f(x) = \begin{cases} \frac{1}{2} & 0 < x < 1 \\ 0 & 1 \le x \le 2 \\ \frac{1}{2} & 2 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$

a.) 

b.) $F(x) = \cancel{\begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{2} & 0 < x < 1 \\ \frac{1}{2} & 1 \le x \le 2 \\ 1 & 2 < x < 3 \\ 1 & 3 \le x \le \infty \end{cases}} \quad \begin{cases} 0 & x < 0 \\ \int_0^x \frac{t}{2}\,dt = \frac{x}{2} & 0 \le x \le 1 \\ \frac{1}{2} & 1 \le x \le 2 \\ \frac{x}{2} - \frac{1}{2} & 2 \le x \le 3 \\ 1 & x \ge 3 \end{cases}$

c.) $F(\pi_{.25}) = \frac{\pi_{.25}}{2} = \frac{1}{4} \quad \pi_{.25} = \frac{1}{2}$

d.) $m = \pi_{.5} = \quad 1 \le x \le 2 \quad x \in [1,2]$, so it is not unique

e.) $F(\pi_{.75}) = \frac{\pi_{.75}}{2} - \frac{1}{2} \quad \pi_{.75} = (\frac{3}{4} + \frac{1}{2})2 = \frac{5}{2}$

3.1-20 a.) $\int_0^1 x\,dx + \int_1^\infty \frac{c}{x^3}\,dx = 1$

$\left(\frac{x^2}{2}\right)\Big|_0^1 + c\left(-\frac{x^{-2}}{2}\right)\Big|_1^\infty = 1$

$\frac{1}{2} + \frac{c}{2} = 1 \quad ; \quad c = 1$

b.) $\mu = \int_0^1 x(x)\,dx + \int_1^\infty x\left(\frac{c}{x^3}\right)dx = \frac{4}{3}$

c.) $E[X^2] = \infty \quad \sigma^2$ does not exist

d.) $P(\frac{1}{2} \le X \le 2) = 0.75$

3.2-2 a.) $f(x) = \frac{2}{3}e^{-\frac{2x}{3}}$

b.) $P(X>2) = \int_2^\infty \frac{2}{3}e^{-\frac{2x}{3}}dx = \frac{1}{e^{4/3}}$

$u = \frac{2}{3}x \quad du = \frac{2}{3}du$

3.2-5 a.) $F(x) = \begin{cases} 0 & x < \delta \\ \int_\delta^\infty \frac{1}{\theta}\exp\{-\frac{t-\delta}{\mu}\}dt & \delta \leq x < \infty \end{cases}$

b.) $\mu = \int_\delta^\infty x \frac{1}{\theta}\exp\{-\frac{x-\delta}{\theta}\} = \delta + \theta$

$\sigma^2 = E[x^2] - (E[x])^2 = \theta^2$

3.2.10 mgf gamma $= \frac{1}{(1-\theta t)^\alpha}$

$E[x] = M'(0) = \alpha\theta$

$Var(x) = \sigma^2 = E[x^2] - (E[x])^2 = \alpha\theta^2$

3.2-23 $M = -1000\ln(.92)$

3.3-1 a.) .2784
b.) .7209
c.) .9616
d.) .0019
e.) .9500
f.) .6826
g.) .9544
h.) .9974

3.3-3 a.) 1.96
b.) 1.96
c.) 1.645
d.) 1.645

3.3-8 $f''(x) = 0$ when $x = \mu \pm \sigma$

3.3-10 $Y = aX + b \sim N(a\mu + b, a^2\sigma^2)$

3.3-14 cdf: $\frac{1}{\sqrt{2\pi}} \int_{\infty}^{\ln(x)} \exp\{-(z-10)^2/2\}\, dz$

pdf: $\frac{1}{x\sqrt{2\pi}} \exp\{-(\ln x - 10)^2/2\}$

$P(10{,}000 < X < 20{,}000) = .2454$

4.1-1 a.) $c = 1/32$
b.) $c = 1/24$
c.) $c = 1/18$
d.) $c = 6$

4.1.3 a.) $f_X(x) = \frac{2x+5}{16}$
b.) $f_Y(y) = \frac{3+2y}{32}$

c.) $P(X > Y) = 3/32$
d.) $P(Y = 2X) = 9/32$
e.) $P(X+Y=3) = 3/16$
f.) $P(X \le 3-Y) = 1/4$
g.) $f_X(1)f_Y(1) \neq f(1,1)$ so X & Y are not independent
h.) $E[X] = 25/16$
$E[Y] = 45/16$
$V[X] = 63/256$
$V[Y] = 295/256$

4.1.5 c.) $x = 1,2,3,4$; $f_X(x) = .25$
d.) $x = 2-8$ $f_Y(y) = .0625, .125, .1875, .25, .1875, .125, .0625$ resp.
e.) not independent

4.2-1 $\mu_x = \frac{25}{16}$

$\mu_y = \frac{45}{16}$

$\sigma_x^2 = 63/256$

$\sigma_y^2 = 295/256$

$\sigma_{xy} = -5/256$

$\rho = -5/256 \div \sqrt{\frac{63}{256}\left(\frac{295}{256}\right)}$

4.2-5 $b = \rho \frac{\sigma_y}{\sigma_x}$ $a = \mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x$

4.2-7 a.) no

b.) $\sigma_{xy} = \rho = 0$

4.2-9 a.) $c = 1/154$

c.) $f_x(0) = 6/77$, $f_x(1) = 21/77$, $f_x(2) = \frac{30}{77}$, $f_x(3) = \frac{20}{77}$

d.) No