Homework 2

Matthew Bentz

2/16/2022

Part 1

1. Let the random variable X denote the number of semiconductor wafers that need to be analyzed before a large particle of contamination is detected. Assume that the probability that a wafer has a large particle on it is .1 and that the wafers are independent. Determine the probability distribution of X. HINT: Let p denote a wafer with a large particle present and a denote a wafer with the large particle absent. The sample space for the random experiment is: $\mathcal{S} = \{p, ap, aap, aaap, \ldots\}$. Calculate the probability of getting the first few values and see if you can find the emerging pattern to help create the probability mass function.

X ~ Geometric

$$\begin{aligned} P(1) &= (.1) \\ P(2) &= (.9)(.1) \\ P(3) &= (.9)^2(.1) \\ P(4) &= (.9)^3(.1) \\ pmf = P(X) &= (.9)^{x-1}(.1) \end{aligned}$$

2. Let X be a continuous random variable. Say if the following sentence is true or false, and give a justification to your answer. Remember that probability is represented as area under a curve for a continuous random variable.

"$P(X \leq x) = P(X < x)$ because $P(X = x) = 0$ for all possible values of X."

This statement is true. Since probability is represented as an area under a curve for continuous random variables, that means the probability can be calculated as the integral between two points. The probability of X=x means the integral from x to x, which is zero.

3. Four identical electronic components are wired to a controller that can switch from a failed component to one of the remaining spares. Let X denote the number of components that have failed after a specified period of operation. Is X a binomial random variable? State your assumptions and justify why or why not.

X is a binomial random variable since it refers to a success/failure of the 4 components that make up the sample space (finite amount). The events are also independent, and the probability of success in each component remains constant.

Part 2

4. Solve for a value of c such that the following is a probability density function:

$$f(x) = \begin{cases} c(1 - \frac{1}{x^2}) & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Here we integrate (Because it is a density function) from 1 to 2 (Sample Space) and set the probability equal to 1. The integration multiplied by c will be equal to 1, so we can find c from there.

$P(1 \leq x \leq 2) = c(1 - \frac{1}{x^2}) = 1$

```
integrand <- function(x) {(1 - {1}/{x^2})}
integrate(integrand, lower=1, upper=2)
```

```
## 0.5 with absolute error < 5.6e-15
```

$$0.5c = 1$$
$$c = 2$$

5. Suppose that X is a continuous random variable with pdf $f(x) = e^{-x}$ for all $x > 0$ and 0 otherwise. Determine a value of x such that $P(X \leq x) = .1$.

To find x, we need to take the integral from 0 to a of the function f(x) and set this equal to 0.1. We can calculate a to find x where $P(X \leq x) = 0.1$.

```
integrand <- function(x) {exp(1)^{-x}}
#Solve for the upper bound. x becomes -ln(0.9) which gives us 0.1 when integrated to.
integrate(integrand, lower=0, upper=-log(0.9, base=exp(1)))
```

```
## 0.1 with absolute error < 1.1e-15
```

6. The number of messages sent per hour over a computer network has the distribution described below. Determine the mean and standard deviation of the number of messages sent per hour.

$$P(X = x) = f(x) = \begin{cases} .08 & x = 10 \\ .15 & x = 11 \\ .30 & x = 12 \\ .20 & x = 13 \\ .27 & x = 14 \\ 0 & \text{otherwise} \end{cases}$$

```r
#mean = E[X] = sum(x[i] * f(x), i=10, 14)$

mu <- (10)*(.08) + (11)*(.15) + (12)*(.30) + (13)*(.20) + (14)*(.27)
#mu = 12.43

#variance = E[(x - mu)^2] = sum((x[i] - mu)^2 * f(x))
variance <- ((10 - 12.43)^2 * (.08)) + ((11 - 12.43)^2 * (.15)) +
  ((12 - 12.43)^2 * (.30)) + ((13 - 12.43)^2 * (.20)) + ((14 - 12.43)^2 * (.27))
#variance = 1.5651

#standard deviation = sqrt(variance)
sd <- sqrt(variance)
#sd = 1.25104
```

7. Suppose $f(x) = .125x$ for all $0 < x < 4$. Determine the mean and variance of X.

Mean = integral from 0 to 4 of x*f(x)dx = integral 0,4 .125x^2dx Variance = integral from 0 to 4 of (x-mu)^2*f(x)dx = (x-(64/24))^2*.125xdx

```r
integrand2 <- function(x) {.125*x^2}
integrate(integrand2, lower=0, upper=4)
```

```
## 2.666667 with absolute error < 3e-14
```

```r
#mean = 2.666667 or 64/24

integrand3 <- function(x) {(x-(64/24))^2 * .125*x}
integrate(integrand3, lower=0, upper=4)
```

```
## 0.8888889 with absolute error < 9.9e-15
```

```r
#variance = .8888889
```

8. The diameter of a shaft in an optical storage drive is normally distributed with a mean of .2508 inches and a standard deviation of .0005 inches. The specifications on the shaft are $.2500 \pm .0015$ inches. What proportion of shafts conform to the specifications?

To get the mean = 0 and variance = 1, we standardize using the Z variable (x-mu / sd). We can then calculate the probability x + or - .0015 from .2500.

```r
# Left of the curve, probability to be subtracted
leftBound <- pnorm((.2485-.2508)/.0005)

# Right of the curve
rightBound <- pnorm((.2515-.2508)/.0005)

proportion <- rightBound - leftBound
#proportion = 0.91924122831152

# Altering the mean/sd we can find these values as well.
pnorm(.2515, mean = .2508, sd = .0005) - pnorm(.2485, mean = .2508, sd = .0005)
```

```
## [1] 0.9192412
```

9. In a state lottery, a single digit is drawn independently from each of four containers. Each container has 10 balls numbered 0-9. You choose a 4 digit number and bet $1, if you win you will receive $4999, so a total of $5000 (this includes your original bet). If you lose than you lose your original bet. What is your expected value for playing the lottery?

E[X] = sum(x * f(x))

$(-1 * (9999/10000)) + (5000 * (1/10000)) = -.4999$

Expected value for playing the lottery is ≈ -$0.49.

10. Let Z be a standard normal distribution. Using R, solve for: $P(Z < 1.32)$, $P(Z > 1.45)$, $(-2.76 < Z < 1.34)$, and $P(Z < z) = .68$.

```r
#P(Z < 1.32)
pnorm(1.32)
```

```
## [1] 0.9065825
```

```r
#P(Z > 1.45)
pnorm(1.45, lower.tail = F)
```

```
## [1] 0.07352926
```

```r
#P(-2.76 < Z < 1.34)
pnorm(1.34) - pnorm(-2.76)
```

```
## [1] 0.9069873
```

```r
#P(Z < z) = .68
qnorm(0.68)
```

```
## [1] 0.4676988
```