

1. **Claim:**

$$(\text{append } (\text{append } xs \text{ } ys) \text{ } zs) = (\text{append } xs \text{ } (\text{append } ys \text{ } zs))$$

Proof: We will prove this by induction on the set xs

Base Case: $xs = '()$

$$\begin{aligned} & (\text{append } (\text{append } xs \text{ } ys) \text{ } zs) \\ &= \{\text{Append-nil Law}\} \\ & (\text{append } ys \text{ } zs) \\ &= \{\text{Append-nil Law, right to left}\} \\ & (\text{append } '() \text{ } (\text{append } ys \text{ } zs)) \\ &= \{\text{Base case assumption}\} \\ & (\text{append } xs \text{ } (\text{append } ys \text{ } zs)) \end{aligned}$$

Inductive Hypothesis: Assume the statement is true for some xs' :

$$(\text{append } (\text{append } xs' \text{ } ys) \text{ } zs) = (\text{append } xs' \text{ } (\text{append } ys \text{ } zs))$$

Now we prove that it is true for $xs = (\text{cons } x \text{ } xs')$:

$$\begin{aligned} & (\text{append } (\text{append } xs \text{ } ys) \text{ } zs) \\ &= \{\text{Definition of } xs\} \\ & (\text{append } (\text{append } (\text{cons } x \text{ } xs') \text{ } ys) \text{ } zs) \\ &= \{\text{append-cons law}\} \\ & (\text{append } (\text{cons } x \text{ } (\text{append } xs' \text{ } ys)) \text{ } zs) \\ &= \{\text{append-cons law}\} \\ & (\text{cons } x \text{ } (\text{append } (\text{append } xs' \text{ } ys) \text{ } zs)) \\ &= \{\text{Inductive Hypothesis}\} \\ & (\text{cons } x \text{ } (\text{append } xs' \text{ } (\text{append } ys \text{ } zs))) \\ &= \{\text{append-cons law, right to left}\} \\ & (\text{append } (\text{cons } x \text{ } xs') \text{ } (\text{append } ys \text{ } zs)) \\ &= \{\text{Definition of } xs\} \\ & (\text{append } xs \text{ } (\text{append } ys \text{ } zs)) \end{aligned}$$

Thus, we have proven the statement is true for $xs = (\text{cons } x \text{ } xs')$, so the claim is always true. QED