Opsem-Theory

1)

a) The variable x exists either within the scope of some function, or it exits as a global variable

b) Given what variables and functions we have defined at present, expression e evaluates to single value v.

2a) There exists ξ such that $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle$, if $x \in dom\ \xi$

b) $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho' \rangle$ given e is a valid expression

c) $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$ where ξ' may have different mappings, but $dom\ \xi = dom\ \xi'$

3) a) For simplicity, we can see the only differeence in the 2 inference rules is the premises:

$\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi', \phi, \rho' \rangle \quad v_1 = 0$ VS $\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi', \phi, \rho' \rangle$

- In the new (2nd) premise, we're saying that e_1 evaluates to 0 under ξ, ϕ, ρ, where ξ, ϕ, ρ can potentially change. In the original, in the same conditions, we say e_1 evaluates to v_1, but in the next line say $v_1 = 0$, so it is equivalent to saying e_1 evaluates to 0.

b) $$\frac{x \in dom\ \rho \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle set(x,e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle} \text{ formal assign'}$$ is not equivalent to the true formal assign,

due to the difference in the conclusion. In this new formal assign, the formal environment ρ at the conclusion is ρ', where ρ' is defined in the premise as the formal environment after e is evaluated. ρ' does not account for x being set to the value v, as the prime only denotes any changes when e is evaluated, not when x is set.

B

4a) AWK

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, \rho \rangle} \text{ (AWK UNBOUND VAR)}$$

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle Set(x,e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, \rho' \rangle}$$

(AWK UNBOUND ASSIGN)

4b) Icon

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho\{x \mapsto 0\} \rangle} \text{ (ICON UNBOUND VAR)}$$

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle set(x,e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho'\{x \mapsto v\} \rangle} \text{ (ICON UNBOUND ASSIGN)}$$

2a) There exists ξ such that $\langle e, \xi, \phi, \rho\rangle \Downarrow \langle v, \xi, \phi, \rho\rangle$, if $x \in \mathrm{dom}\ \xi$

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b)
$$\frac{x \in \mathrm{dom}\ \rho \quad \langle e, \xi, \phi, \rho\rangle \Downarrow \langle v, \xi', \phi, \rho'\rangle}{\langle \mathrm{set}(x,e), \xi, \phi, \rho\rangle \Downarrow \langle v, \xi', \phi, \rho'\rangle}\ \text{formal assign'}$$
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B

4a) Awk

$$\frac{x \notin \mathrm{dom}\ \rho \quad x \notin \mathrm{dom}\ \xi}{\langle \mathrm{VAR}(x), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, \rho\rangle}\ (\text{AWK UNBOUND VAR})$$

$$\frac{x \notin \mathrm{dom}\ \rho \quad x \notin \mathrm{dom}\ \xi \quad \langle e, \xi, \phi, \rho\rangle \Downarrow \langle v, \xi', \phi, \rho'\rangle}{\langle \mathrm{Set}(x,e), \xi, \phi, \rho\rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, \rho'\rangle}\ (\text{AWK UNBOUND ASSIGN})$$

4b) Icon

$$\frac{x \notin \mathrm{dom}\ \rho \quad x \notin \mathrm{dom}\ \xi}{\langle \mathrm{VAR}(x), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\{x \mapsto 0\}\rangle}\ (\text{I UNBOUND VAR})$$

$$\frac{x \notin \mathrm{dom}\ \rho \quad x \notin \mathrm{dom}\ \xi \quad \langle e, \xi, \phi, \rho\rangle \Downarrow \langle v, \xi', \phi, \rho'\rangle}{\langle \mathrm{set}(x,e), \xi, \phi, \rho\rangle \Downarrow \langle v, \xi', \phi, \rho'\{x \mapsto v\}\rangle}\ (\text{ICON UNBOUND ASSIGN})$$

3) a) For simplicity, we can see the only difference in the 2 inference rules is the premises:

$\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi', \phi, \rho' \rangle \quad v_1 = 0 \qquad vs \qquad \langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi', \phi, \rho' \rangle$

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$$\frac{x \notin dom\,\rho \quad x \notin dom\,\xi}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, \rho \rangle} \text{(AWK UNBOUND VAR)}$$

$$\frac{x \notin dom\,\rho \quad x \notin dom\,\xi \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle set(x,e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, \rho' \rangle}$$
(AWK UNBOUND ASSIGN)

4b) Icon

$$\frac{x \notin dom\,\rho \quad x \notin dom\,\xi}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho\{x \mapsto 0\} \rangle} \text{(ICON UNBOUND VAR)}$$

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$$\frac{x \notin \operatorname{dom} \rho \quad x \notin \operatorname{dom} \xi}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, \rho \rangle} \text{ (AWK UNBOUND VAR)}$$

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4b) <u>Icon</u>

$$\frac{x \in \operatorname{dom} \rho \quad x \notin \operatorname{dom} \xi}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho\{x \mapsto 0\} \rangle} \text{ (ICON UNBOUND VAR)}$$

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4c) I prefer the changes in the ICON Language, of (4b). In a scenario in which you are reffering to an unbound variable, it is likely not as central to the program, and is possibly used as shorthand in a loop or something local. In these cases there would be no apparent need to have these vars assigned globally in my opinion

5)

$$\dfrac{\dfrac{x \in dom\,\rho \quad \langle LIT(3), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\rangle}{\langle (set\ x\ 3), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\}\rangle} \qquad \dfrac{x \in dom\,\rho\{x \mapsto 3\} \quad \rho\{x \mapsto 3\}(x) = 3}{\langle VAR(x), \xi, \phi, \rho\{x \mapsto 3\}\rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\}\rangle}}{\langle begin\ (set\ x\ 3)\ x, \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\rangle}$$

6) Case 1: $x = 0$

6.1:

$$\dfrac{\dfrac{x \in \rho \quad \rho(x) = 0}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}\text{(formal var)} \qquad \dfrac{}{\langle LIT(0), \xi, \phi, \rho\rangle \Downarrow \langle v_1, \xi, \phi, \rho\rangle}\text{(LITERAL)}}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho\rangle \Downarrow \langle v_1, \xi, \phi, \rho\rangle}\text{(If False)}$$

from case

6.2:

$$\dfrac{x \in dom\,\rho \quad \rho(x) = v_2}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle v_2, \xi, \phi, \rho\rangle}\text{(formal Var)}$$

Here we can see $v_1 = v_2 = 0$, because in 6.1, $v_1 = 0$ by the (LITERAL) rule, which is the exp we evaluate because in this case we start w/ (If False)

Case 2: $x \neq 0$: this is clearly the compliment of case 1

6.3

$$\dfrac{\text{(formal var)}\dfrac{x \in dom\,\rho}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle} \qquad \rho(x) \neq 0 \quad \dfrac{x \in dom\,\rho}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\text{formal var}}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\text{(IF TRUE)}$$

Comparing to $\rho(x)$,

There is a

$x=0$ and x

these proofs to

equally valid

6) Case 1: $x = 0$

6.1: $$\dfrac{\dfrac{x \in \rho \quad \rho(x) = 0}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho \rangle}\ \text{(Formal Var)} \qquad \dfrac{}{\langle LIT(0), \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi, \phi, \rho \rangle}\ \text{(LITERAL)}}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi, \phi, \rho \rangle}\ \text{(If False)}$$

from case

6.2: $$\dfrac{x \in dom\,\rho \quad \rho(x) = v_2}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle v_2, \xi, \phi, \rho \rangle}\ \text{(Formal Var)}$$

Here we can see $v_1 = v_2 = 0$, because in 6.1, $v_1 = 0$ by the (LITERAL) rule, which is the exp we evaluate because in this case we start w/ (If False)

Case 2: $x \neq 0$: this is clearly the compliment of case 1

Case: $$\dfrac{\dfrac{x \in dom\,\rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ \text{(Formal Var)} \qquad \rho(x) \neq 0 \quad \dfrac{x \in dom\,\rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ \text{formal var}}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ \text{(IF TRUE)}$$

Comparing this 6.2, we see that in both cases the expressions both evaluate to $\rho(x)$. thus, $v_1 = v_2$.

There is an additional 2 cases where x is global. In the previous derivations of cases $x = 0$ and $x \neq 0$, there was nothing specific about ρ was required, and we could extend these proofs to consider $x \notin dom\ \rho$ and $x \in dom\ \xi$, and we would have nearly identical & equally valid proofs.