

Claim: $((\circ ((\text{curry map}) f) ((\text{curry map}) g)) == ((\text{curry map}) (\circ f g)))$

We will prove these functions are equal by proving they always equate to the same thing for any input. We will do this by inducting over general input

$xs = (\text{cons } x \text{ } ys)$

First, we will use laws to simplify both sides of the equation to make the rest of the proof more readable, without making any assumptions about xs :

LHS:

$$\begin{aligned} & ((\circ ((\text{curry map}) f) ((\text{curry map}) g)) xs) \\ &= \{ \text{apply-compose law} \} \\ & (((\text{curry map}) f) (((\text{curry map}) g) xs)) \\ &= \{ \text{apply-curry law} \} \\ & (((\text{curry map}) f) (\text{map } g \text{ } xs)) \\ &= \{ \text{apply-curry law} \} \\ & (\text{map } f \text{ } (\text{map } g \text{ } xs)) \end{aligned}$$

RHS:

$$\begin{aligned} & (((\text{curry map}) (\circ f g)) xs) \\ &= \{ \text{apply-curry law} \} \\ & (\text{map } (\circ f g) \text{ } xs) \end{aligned}$$

Now, we can use induction to prove that these expressions are equivalent:

Base Case: $xs = '()$

$$\begin{aligned} & (\text{map } f \text{ } (\text{map } g \text{ } xs)) \\ &= \{ \text{Definition of } xs \} \\ & (\text{map } f \text{ } (\text{map } g \text{ } '())) \\ &= \{ \text{map-nil law} \} \\ & (\text{map } f \text{ } '()) \\ &= \{ \text{map-nil law} \} \\ & '() \\ &= \{ \text{map-nil law, right to left} \} \\ & (\text{map } f' \text{ } '()) \\ &= \{ \text{define function } f' \text{ as } (\circ f g) \} \\ & (\text{map } (\circ f g) \text{ } '()) \end{aligned}$$

Now that we have proven it for a base case, we can assume it to be true for a general xs' and prove it to be true for $xs = (\text{cons } x \text{ } xs')$:

Proof:

Inductive Hypothesis: Assume $(\text{map } f (\text{map } g \text{ xs}')) == (\text{map } (o \ f \ g) \text{ xs}')$

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(map f (map g xs))
= { Definition of xs }
(map f (map g (cons x xs'))))
= { map-cons law }
(map f (cons (g x) (map g xs'))))
= { map-cons law }
(cons (f (g x)) (map f (map g xs'))))
= { apply-compose law right to left }
(cons ((o f g) x) (map f (map g xs'))))
= { Inductive Hypothesis }
(cons ((o f g) x) (map (o f g) xs'))
= { map-cons law right to left }
(map (o f g) (cons x xs'))
= { Definition of xs }
(map (o f g) xs)
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Thus, we have proven that the claim holds for any general xs, So the claim is always true. QED