Opsem-Theory

1)

a) The variable x exists either within the scope of some function, or it exists as a global variable

b) Given what variables and functions we have defined at present, expression e evaluates to single value v.

2a) There exists ξ such that $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle$, if $x \in dom\ \xi$

b) $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho' \rangle$ given e is a valid expression

c) $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$ where ξ' may have different mappings, but $dom\ \xi = dom\ \xi'$

3) a) For simplicity, we can see the only differecence in the 2 inference rules is the premises:

$\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi', \phi, \rho' \rangle \quad v_1 = 0$ vs $\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi', \phi, \rho' \rangle$

- In the new (2nd) premise, we're saying that e_1 evaluates to 0 under ξ, ϕ, ρ, where ξ, ϕ, ρ can potentially change. In the original, in the same conditions, we say e_1 evaluates to v_1, but in the next line say $v_1 = 0$, so it is equivalent to saying e_1 evaluates to 0.

b) $$\frac{x \in dom\ \rho \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle set(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}\ \text{formal assign'}$$ is not equivalent to the true formal assign,

due to the difference in the conclusion. In this new formal assign, the formal environment ρ at the conclusion is ρ', where ρ' is defined in the premise as the formal environment after e is evaluated. ρ' does not account for x being set to the value v, as the prime only denotes any changes when e is evaluated, not when x is set.

B

4a) Awk

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, \rho \rangle}\ (\text{AWK UNBOUND VAR})$$

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle Set(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, \rho' \rangle}\ (\text{AWK UNBOUND ASSIGN})$$

4b) Icon

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho\{x \mapsto 0\} \rangle}\ (\text{ICON UNBOUND VAR})$$

$$\frac{x \notin dom\ \rho \quad x \notin dom\ \xi \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle set(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho'\{x \mapsto v\} \rangle}\ (\text{ICON UNBOUND ASSIGN})$$

4c) I prefer the changes in the ICON Language, of (4b). In a scenario in which you are reffering to an unbound variable, it is likely not as central to the program, and is possibly used as shorthand in a loop or something local. In these cases there would be no apparent need to have these vars assigned globally in my opinion

Recursi

Scheme

2 categ

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$$5)\quad \frac{\dfrac{x \in dom\,\rho \quad \dfrac{}{\langle LIT(3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}}{\langle (set\ x\ 3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle} \qquad \dfrac{x \in dom\,\rho\{x \mapsto 3\} \quad \rho\{x \mapsto 3\}(x) = 3}{\langle VAR(x), \xi, \phi, \rho\{x \mapsto 3\} \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle}}{\langle begin\ (set\ x\ 3)\ x, \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}$$

6) Case 1: $x = 0$

6.1:

$$\frac{\dfrac{x \in \rho \quad \rho(x) = 0}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho \rangle}\ (\text{formal var}) \qquad \dfrac{}{\langle LIT(0), \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi, \phi, \rho \rangle}\ (\text{LITERAL})}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi, \phi, \rho \rangle}\ (\text{If False})$$

from case

6.2:

$$\frac{x \in dom\,\rho \quad \rho(x) = v_2}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle v_2, \xi, \phi, \rho \rangle}\ (\text{formal Var})$$

Here we can see $v_1 = v_2 = 0$, because in 6.1, $v_1 = 0$ by the (LITERAL) rule, which is the exp we evaluate because in this case we start w/ (If False)

Case 2: $x \neq 0$: this is clearly the compliment of case 1

6.3:

$$\frac{\dfrac{x \in dom\,\rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ (\text{formal var}) \qquad \rho(x) \neq 0 \qquad \dfrac{x \in dom\,\rho}{\langle VAR(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ (\text{formal var})}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ (\text{IF TRUE})$$

Comparing this 6.2, we see that in both cases the expressions both evaluate to $\rho(x)$, thus, $v_1 = v_2$.

There is an additional 2 cases where x is global. In the previous derivations of cases $x = 0$ and $x \neq 0$, there was nothing specific about ρ was required, and we could extend these proofs to consider $x \notin \mathrm{dom}\, \rho$ and $x \in \mathrm{dom}\, \xi$, and we would have nearly identical & equally valid proofs.