Procedural Plant Generation and Simulated Plant Growth

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Abstract

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Chapter 1

Introduction

Procedurally generating 3D models of plant-life is a challenging task, largely due to the complex branching structures and variation between different types of plant species. Up until recently, all assets within 3D graphics applications either had to be sculpted using 3D modeling software or scanned using photogrammetry, laser triangulation or some form of contact-based 3D scanning. These methods are still used today but tend to be very time consuming and extremely costly. With the increase in computational power over the last few decades more emphasis has been placed on the use of procedural generation. Which can be used to create complex structures such as terrain, architecture, sound and 3D models with far greater speed than previous techniques, and often much better realism than would be possible with artists. Plant-life stands as a challenge due to the thousands of species, each with their unique structure and features. It is difficult to define a system that can represent them all in a way that is simple, understandable and accurate. The Lindenmayer System (L-system) stands as a solution to this problem, it was originally developed by Aristed Lindenmayer as a method of representing the development of multicellular organisms [Lindenmayer, 1968]. This has since gained popularity in the area of procedural generation and has been adapted to represent different types of structures. L-systems have been adapted to represent plant-life, such as trees, flowers, algae, and grasses. Whilst still applying to non-organic structures such as music, artificial neural networks, and tiling patterns [Prusinkiewicz and Hanan, 1989].

1.1 Motivations

The L-system, in its most basic form, is a formal grammar that contains a set of symbols or letters that belong to an *alphabet*. A starting string or *axiom* is created using the alphabet, as well as a set of production rules. The production rules are applied to each symbol within the axiom string, and each rule dictates whether or not the symbol can be rewritten and what they will be rewritten with. In essence, an L-system uses the set of production rules to generate a resulting string of symbols which follow those production rules. What the resulting strings' is ultimately going to represent depends on how it is interpreted. In this case, the string of symbols can be interpreted to generate a model of a plant. This thesis develops upon the L-system concepts described by Przemyslaw Prusinkiewicz and Aristid Lindenmayer to procedurally generate structures of plant-life in real-time. The L-system grammar allows the construction of a plant to be described in a human-readable, formal grammar. The grammar

can be used to specify variation in shape, size, and branching structure within a particular species. Furthermore, this thesis will also investigate the use of a parameterised L-systems to provide physical properties using string rewriting. Which in turn will enable the animation and physical behavior of the plant that it generates, thus making it possible to simulate external forces such as gravity and wind.

This chapter will provide an overview for how to improve the procedural generation of plant-life in 3D applications and the motivations doing so. It will then introduce the concepts of procedural generation, rewriting systems, and formal grammars. This chapter will briefly describe how to apply procedural generation to the development of plant-life, and will provide sufficient background as to the use of formal grammars as a means of describing complex L-system languages. Finally, there will be an outline as to the structure of this thesis.

1.2 Introduction to Procedural Generation

Procedural generation is used in many different areas and applications in computer graphics, particularly when generating naturally occurring structures such as plants or terrain. An effective procedural generator is capable of taking input in the form of a relatively simple description of what it should be generating; its job is then to computationally create the structure in a way that is accurate to the description given. Currently, there are three main methods for procedurally generating models of plant-life; these are genetic algorithms [Haubenwallner et al., 2017], space colonisation algorithms[Juuso, 2017], and L-systems. The genetic algorithm and space colonisation algorithms are similar in that they require the overall shape of the plant to be described by simple 3D shapes; the algorithm then creates a branching structure that matches these shapes. The limitation of these methods is that the 3D description is not very specific, and although it can get good results for trees, it may not be able to generate different types of plant-life, such as flowers. The L-system, on the other hand, relies on a method of string rewriting, whereby the rewriting is based on a set of production rules to generate a string of symbols that obey those rules. A separate system can later interpret this string to create the model. The L-system procedural generation, therefore, has two different systems within it, one of string rewriting and one of interpretation of the generated string. This makes it quite easy for the same L-system to generate very different results based upon the interpretation.

Plant-life can have very complex and seemingly random structures; however, with closer observation, trees of a similar species have distinct traits and features. For instance, a palm tree has long straight trunks with large compound leaves exclusively near the top, branching in all different directions. Comparatively, a pine tree has a long straight trunk with many branches coming off in different directions perpendicular to the ground, from its base to the top of the trunk. These are two very different species of trees; the palm belongs to the Arecaceae family, whereby the pine belongs to the Pinaceae family. They look different; however, they share very similar properties, such as their long straight trunks. The challenge behind the procedural generation of plant-life is providing a human-readable grammar that

describes in sufficient detail, how to generate a 3D model. Whilst allowing for randomness and variety within the generation process, such that variations of a particular species can be created without repetition. The grammar for procedural generation should also be relatively straightforward and intuitive, and must accurately represent what it is going to generate. Furthermore, the description must not be limited to only known species of trees, as some graphics applications may require something other-worldly.

1.3 Introduction to Rewriting Systems

Rewriting systems are the fundamental concept behind L-systems. In their most basic form, rewrite systems are a set of symbols or states, and a set of relations or production rules that dictate how to transform from one state to the other [Prusinkiewicz and Lindenmayer, 2012]. These production rules can be used to generate complex structures by successively replacing parts of a simple initial object with more complex parts. Rewrite systems can be non-deterministic, meaning that there could be a transition that depends on a condition being met or on neighbouring states. The rewriting concept means that any next state can rely upon some conditions necessary for transformation. If the condition evaluates true, the state is rewritten; otherwise, it remains the same and is checked in the next rewriting stage. A graphical representation of an object defined in rewriting rules can be seen below in figure 1.1 below, called the snowflake curve proposed by Von Koch [Koch et al., 1906].

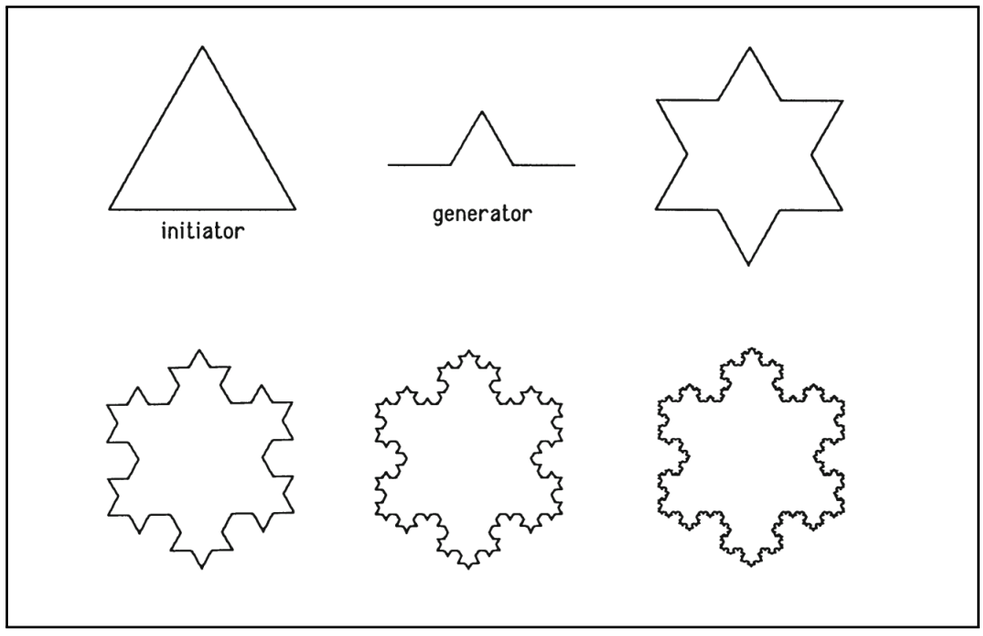


Figure 1.1: Construction of the snowflake curve[Prusinkiewicz and Hanan, 2013].

The snowflake curve starts with two parts, the initiator and the generator. The initiator is the initial set of edges forming a certain shape, whereas the generator is a set of edges that can be used to replace each edge of the initiator to form a new shape. That new shape then becomes the initiator for the next generation, where the generator again replaces each edge. The result is a complex shape similar to that of a snowflake. The initiator, generator concept, is a graphical representation of how rewriting systems operate; rather than the initiator and generator being a set of edges, a set of symbols or strings instead represents them.

1.4 Introduction to Formal Grammars

In the context of computer science, a grammar is defined as a set of rules governing which strings are valid or allowable in a language or text. They consist of syntax, morphology, and semantics. Formal languages have been defined in the form of grammars to suit particular problem domains. It is natural for humans to communicate a problem or solution in the form of language; it is intuitive to use a language to describe the desired outcome when dealing with the procedural generation of plant-life. In the past, formal grammars have been used extensively in computer science in the form of programming languages in which humans can provide a computer with a set of instructions to carry out to gain an expected result. The challenge is to procedural generation of plant-life by creating a grammar in the form of a rewriting system. A rewriting system such as the L-system operates in a way that is consistent with a context-free class of Chomsky grammar [Chomsky, 1956], similar to that of the programming language ALGOL-60 introduced by Backus and Naur in 1960[Backus et al., 1960]. In figure 1.2 below, two types of L-system grammars overlap the classes of Chomsky grammars, the OL-system, and the 1L-system. The details of these two systems will be discussed in detail chapter 2, but in summary, 0L-systems are grammars that can represent a context-sensitive Chomsky grammar but generally tend to be context-free, the main difference between the 0L-system and the 1L-system is that 1L-systems can be recursively enumerable. Furthermore, a 1L-system can represent any 0L-system, and 1L-system languages tend to be more complex and verbose when compared to 0L-systems. These two different types of L-systems each have trade-offs, 1L-systems are more powerful and complex, and 0L-systems are less powerful but make for a simpler language.

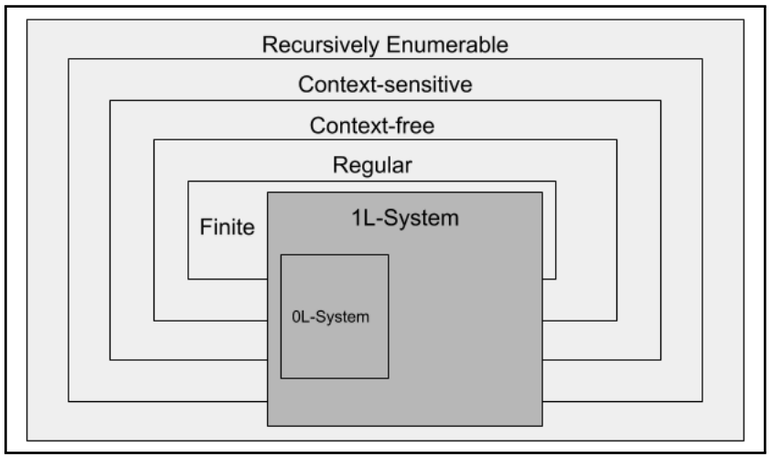


Figure 1.2: Diagram of the Chomsky hierarchy grammars with relation to the 0L and 1L systems generated by L-systems.

1.5 Structure of Thesis

This thesis begins by delving into the underlying concepts of L-systems. It makes sense to firstly start by defining the simplest type of L-system named the DOL-system. Then to provide details about how to interpret DOL-systems to produce graphical representations. The L-system chapter provides a formal definition for more complex types of L-systems, such as parametric L-systems. In conjunction, the L-system chapter talks about major features

and improvements that aid the procedural generation of plant life. These include branching, conditionals, randomness, and stochastic rules.

The next major part of the thesis focuses on the L-system rewriter implementation. The string rewriter section includes the definition of the grammar and syntax for a parametric L-system. It also describes the process of string rewriting, and computationally understanding the L-system grammar using lexical analysis, parsing. The rewriter implementation describes a method of implementing the rewriting system and its connection to the string interpretation process.

The next chapter covers specific mathematics concepts necessary for working with 3D graphics. The chapter includes vectors, matrix transformations, and quaternions. The mathematics chapter is there to provide a brief overview of the mathematics concepts often used when rendering or animating 3D graphics.

Chapter 5 discusses the three main stages of L-system string interpretation with regards to the procedural generation of 3D plant-life. These three stages are the turtle graphics interpreter, model generator, and renderer. The turtle graphics interpreter goes into detail about the skeletal and joint structure of plants. The model generator talks about how to generate the vertex data for the 3D model of the plant using a skeletal structure, which can create a realistic-looking plant. Finally, the renderer covers the specifics of rendering models on the screen in the OpenGL framework.

The physics chapter which focuses on the physics behind the simulation of 3D generated plants. This chapter includes details of Hook's Law and the equations of motion within a 3D application.

Chapter 2

Lindenmayer Systems

An L-system at its core is a formal grammar. The term grammar refers to the structure or definition of a language. Grammars consist of syntax and semantics and allow the formalisation of a language. L-systems can be seen as a grammar for a language that can be used to describe the properties and structure of plant-life. The L-system grammar specifies an *alphabet* of characters which are concatenated together into collections of symbols, called strings. The L-system describes a starting string called an *axiom* and a set of production rules. The production rules decide whether or not another symbol or string should replace a symbol within the L-system string. This process of replacing symbols in a string depending on the production rules is called a rewriting step. The *axiom* is used in the first rewriting step. Each symbol within the axiom is matched to the production rules. If a match is found, the axioms symbol is replaced by the string described by that production rule. This process is carried out for each symbol in the *axiom* until the end of the string is reached. The resulting string created by the rewriting process then becomes the next string for rewriting, and the next rewritten step will begin. This process of rewriting using production rules is the mechanism for generating a structure of symbols that obey the production rules, similar to that of a context-free grammar. The symbols can represent plant-life because each symbol represents a particular state or feature of the plant-life. The resulting strings' symbols generated by the L-system can then be read by a different system called the interpreter. The interpreter understands the meaning of each symbol, and will use each symbol as an instruction to generate the plants structure in 3D space.

This chapter will go into detail about the L-system concept, the rewriting process and a simple interpreter. It will then discuss several different types of L-systems, and their features and limitations. This chapter focuses on the mechanics behind the rewriting system and different techniques that can be used to represent plant-life better. It will also provide sufficient background by briefly touching on how the resulting strings generated by the L-system can be interpreted. The interpretation of an L-system is a separate system to the L-system; however, it is essential to note that the L-system has no concept of what it is trying to represent, it is merely a string rewriting system. It is left up to the interpreter to carry out the L-systems' interpretation. The interpreter is responsible for interpreting the resulting string to create a suitable representation for that problem domain. For instance, the symbols for an L-system trying to represent a tree may be interpreted very different to the symbols trying to represent music; however, the L-systems may be identical. Although the interpreter is not necessarily

part of the L-system, it is important to understand the reliance of the L-system on the string interpreter. The string interpreter will be explored in great detail in chapter 5.

The diagram below details the relationship between the L-system grammar and how it conforms to a number of different classes of grammars. In the L-system grammar the symbol "N" indicates the number of rewriting steps follow. "W" states that what follows is the axiom. Finally, "p1" and "p2" each indicate a production rule follows. It shows how the L-system is sent to the rewriter as input. There are three stages of rewritting, starting with the axiom. Each stage develops an increasingly complex string of symbols. The resulting string of symbols is then interpreted. In this example the symbol "A" draws a line and the symbol "B" draws a circle, resulting in an image that can be drawn on the screen.

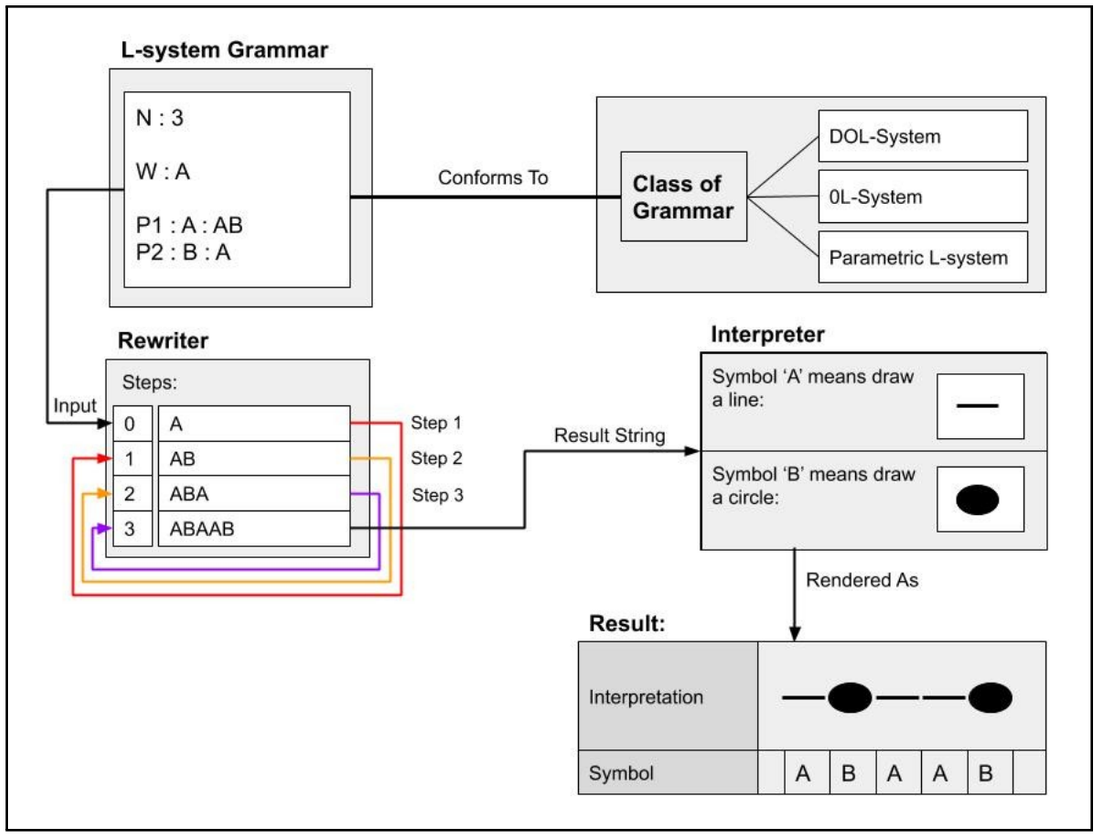


Figure 2.1: Diagram showing the relationships between the L-system grammar, language, rewriter, and interpreter.

A well-known biologist, Aristid Lindenmayer, started work on the Lindenmayer System or L-system in 1968, he sought to create a new method of simulating the growth in multicellular organisms such as algae and bacteria [Lindenmayer, 1968]. He later defined a formal grammar for simulating multicellular growth, which he called the 0L-system [Lindenmayer, 1971]. In the last twenty years, the concept has been adapted to be used to describe larger organisms, such as plants and trees, as well as other nonorganic structures like music [Worth and Stepney, 2005]. There have also been studies to use an L-system for creating and controlling the growth of a connectionist model to represent human perception and cognition [Vaario et al., 1991]. Similarly, Kókai et al. (1999) have created a method of using a parametric L-system to describe a human retina. This method can be combined with evolutionary operators and applied to patients with diabetes who are being monitored [Kókai et al., 1999].

2.1 Simple DOL-system

According to Prusinkiewicz and Hanan, the most simple type of L-system is known as the D0L-system. The term 'D0L system' abbreviates 'Deterministic Lindenmayer system with zero-sided interactions.' It is deterministic because each symbol has an associated production rule, and there is no randomness in determining the production rule. A zero-sided interaction refers to the multicellular representation of an L-system, where each symbol refers to a type of cell, each cell does not account for the state of its directly neighbouring cells, making it zero-sided. There are three major parts to a D0L system. Firstly there is a finite set of symbols known as the *alphabet*, a starting string or *axiom* and the state transition rules *production rules*. The alphabet is a set of characters that represent a state in a system. The axiom is the starting point of the system, which contains one or more characters from the alphabet. The transition rules dictate whether a state should remain the same, or transition into a different state, or even disappear completely. [Prusinkiewicz and Hanan, 2013].

The DOL-system serves as a context-free grammar, to represent the development of multicellular organisms. The DOL-system shown in 2.3 below is an example formulated by Prusinkiwicz and Lindenmayer to simulate Anabaena Catenula, which is a type of filamentous cyanobacteria which exists in plankton. According to Prusinkiewicz and Lindenmayer "Under a microscope, the filaments appear as a sequence of cylinders of various lengths, with a-type cells longer than b-type cells. The subscript l and r indicate cell polarity, specifying the positions in which daughter cells of type a and b are produced." [Prusinkiewicz and Lindenmayer, 2012].

$$\begin{aligned} \omega &: a_r \\ p_1 &: a_r \rightarrow a_l b_r \\ p_2 &: a_l \rightarrow b_l a_r \\ p_3 &: b_r \rightarrow a_r \\ p_4 &: b_l \rightarrow a_l \end{aligned} \tag{2.1}$$

With the definition above, the ":" symbol separates the axiom and production names from their values, furthermore, the $\rightarrow$ can be verbalised as "is replaced by" or "rewritten with". The DOL-system states that $w \;:\; a_r$, where the symbol w signifies that what follows is the axiom, therefore, the starting point is the cell a_r. The production rules then follow and are $p1, p2, p3$ and $p4$. In production rule 1 ($p1$) the cell a_r will be rewritten with cells $a_l b_r$. Production rule $p2$ states that a_l will be rewritten with cells $b_l a_r$. Production rule $p3$ states b_r will rewritten with cell a_r and finally production rule 4 ($p4$), states that b_l will be rewritten with cell a_l. To simulate Anabaena Catenula there are four rewriting rules required, due to the four types of state transitions. The resultant strings for five generations of the rewriting process can be seen in 2.2 below:

$$
\begin{aligned}
G_0 &: a_r \\
G_1 &: a_l b_r \\
G_2 &: b_l a_r a_r \\
G_3 &: a_l a_l b_r a_l b_r \\
G_4 &: b_l a_r b_l a_r a_r b_l a_r a_r \\
G_5 &: a_l a_l b_r a_l a_l b_r a_l b_r a_l a_l b_r a_l b_r
\end{aligned}
\tag{2.2}
$$

During the rewriting process, generation zero (G_0) is the axiom. In subsequent generations, the resultant string of the previous generation is taken, and each symbol in the string is compared to the production rules. If they match the production rule, the symbol is rewritten with the successor symbol or string, which is specified by the production rule. For instance, the previous generation for G_1 is G_0, and the resultant string is for G_0 is a_r, the first symbol in this resultant string is compared with the production rules. In this case a_r matches rule $p1$ with the rule being $p1 \ : \ a_r \ \rightarrow \ a_l b_r$ and therefore, a_r is rewritten with $a_l b_r$. The resultant string of G_0 only has one symbol, so it can be concluded that the string of G_1 is $a_l b_r$, this string is stored for the next rewriting step and is later rewritten to produce generation two and so on, until the desired number of generations is reached.

The D0L-system is very simple and minimalist in design, which comes with some limitations. The D0L-system production rules merely state that if the symbol matches the production rule, then that symbol is rewritten. Often this is not the case; there may be some other conditions that may need to be checked before it can be concluded that a rewrite should take place. Furthermore, the symbols within a D0L-system does not supply much information. For instance, how does the D0L-system indicate how many times a given string has been rewritten? The D0L-system is also deterministic, meaning that there is no randomness in the rewriting process, and therefore, it always yields the same result with no variation. This can be seen as a limitation as variation within the system may be seen as a good thing, such as variation within the branching structure of plants.

2.2 Interpreting the DOL-system String

Section 2.1 outlined a simple type of L-system known as the DOL-system. This type of L-system specifies an alphabet, an axiom, and a set of production rules. This concept allows the representation of a problem as a set of states. The production rules can express valid state transitions, eventually produces a resulting string of symbols that obey the L-systems production rules.

This functionality is compelling; however, the L-system's symbols are only useful if they represent some meaning. Furthermore, the L-system does not supply this meaning; each symbol's meaning is interpreted after the rewriting process by the interpreter. Due to this, there are two separate systems involved in taking an L-systems input, such as the alphabet, axiom, and production rules, turning it into something that can model plant-life. These two systems are the L-system rewriter and the string interpreter. The L-system rewriter is responsible for using L-system to rewrite a string from its axiom by a certain number of

generations, eventually providing a resulting string of symbols. The string interpreter takes the resulting string from the L-system rewriter and interprets it in a way that can represent the model we are trying to render.

A paper by Przemyslaw Prusinkiewicz outlines a method for interpreting the L-system in a way that can model fractal structures, plants, and trees. The method interprets the resultant string of the L-system. Where each symbol represents an instruction that is carried out one after the other to control a 'turtle' [Prusinkiewicz, 1986]. When talking about a turtle, Prusinkiewicz is referring to turtle graphics. Turtle graphics is a type of vector graphics that can be carried out with instructions. It is named a turtle after one of the main features of the Logo programming language. The simple set of turtle instructions listed below can be displayed as figure 2.3. The turtle starts at the base or root of the tree and interprets a set of rotation and translation movements. When all executed one after the other, they trace the points which make up the plants' structure. When these points are then joined together, the result is a fractal structure such as a plant or tree.

Instruction Symbol	Instruction Interpretation
F	Move forward by a specified distance whilst drawing a line
f	Move forward by a specified distance without drawing a line
+	Yaw to the right specified angle.
-	Yaw to the left by a specified angle.
/	Pitch up by specified angle.
\	Pitch down by a specified angle.
^	Roll to the right specified angle.
&	Roll to the left by a specified angle.

Table 2.1: Table of turtle graphics instructions symbols and their meaning to the interpreter

In the OL-system, several symbols represent a particular meaning to the L-system interpreter. Whenever the interpreter comes across one of these symbols in the resultant string, it is interpreted as a particular turtle instruction, which can be seen in table 2.2.

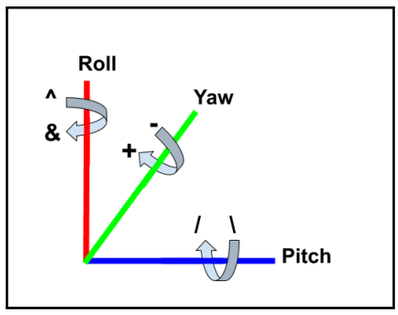


Figure 2.2: Diagram of the 3D rotations of the turtle.

The turtle instructions are presented in such a way that allows movement in three dimensions. The rotations are represented as yaw, pitch, and roll. Where yaw is a rotation around the Z-axis, the pitch is rotation around the X-axis, and roll is rotation around the Y-axis. There are two symbols for each rotation, which represent positive and negative rotations, respectively. Rotations are expected to be applied before a translation; that way, the rotations change the orientation of the turtle, and then the forward instructions move the turtle in the Y direction using the current orientation. The orientation is maintained from one translation to the next, and subsequent rotations are concatenated together to create a global orientation. In this way, when the turtle moves forward again, it moves in the direction of this global orientation.

Figure 2.2 shows the yaw, pitch and roll rotations as well as their axis and the instruction symbols for the L-system.

The turtle instructions in the table 2.1, can be used as the alphabet for the rewriting system defined in the L-system grammar below:

$$\begin{aligned} &\text{Generations: } 1 \\ &\text{Angle: } 90^\circ \\ &\omega \quad : F \\ &p_1 \ : F \ \rightarrow \ F + F - F - F + F \end{aligned} \tag{2.3}$$

This L-system makes use of the alphabet "F, +, -". The meaning of these symbols is not relevant to the rewriting system. The main piece of information that is relevant to the interpreter is the angle to rotate by when it comes across the symbols + and -. This value is specified in the definition of the L-system with the Angle: 90° statement. The resulting string would be "F+F-F-F+F"; this string is passed to the interpreter system, which uses turtle graphics to execute the list of instructions. These instructions can be articulated in table 2.2 below.

Instruction Number	Instruction Symbol	Instruction Interpretation
I1	F	Move forward by 1
I2	+	Yaw right by 90 degrees
I3	F	Move forward by 1
I4	-	Yaw left by 90 degrees
I5	F	Move forward by 1
I6	-	Yaw left by 90 degrees
I7	F	Move forward by 1
I8	+	Yaw right by 90 degrees
I9	F	Move forward by 1

Table 2.2: Table showing each instruction symbols and their interpretation for the L-system 2.3

These instructions are carried out one after the other, moving the turtle around the screen in three dimensions. Tracing the structure which the 0L-system has generated, these instructions generate the traced line shown in figure 2.3 below.

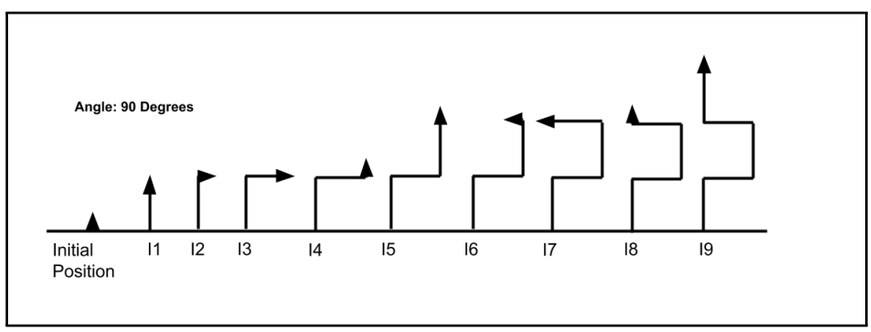


Figure 2.3: Diagram showing a turtle interpreting simple L-system string.

As we can see from the turtle interpretation above, the turtle moves around as if it is an entity within a 3D world following a set of instructions that tell it where to move. This is the basic concept of turtle graphics and how it is implemented in the interpreter system. What also becomes apparent is that there are several assumptions which the interpreter makes to

produce the final image in I9. It is assumed that that the + and - symbols mean a change in yaw of 90 degrees, and the second assumption is that the F symbol means to move forward by a distance of 1 unit measurement. The angle and distance values are assumed because the resultant string does not explicitly define the angle or the distance; it leaves that up to the interpretation of the string.

In a simple DOL-system like the one above, there is no explicit way of providing this additional information to the interpreter. This means that it must be hardcoded into the interpretation or assumed by some other means. This highlights one of the primary considerations when creating an L-system. There is a difference in complexities between the L-system rewriter and the interpreter. It is possible to create a very complex rewriting system with extensive rule systems, which can supply a large amount of information to the interpreter. The interpreter, on the other hand, can be rudimentary and follow the instructions exactly. Conversely, we could have a system where the L-system rewriter is quite simple, but the interpreter is very complicated. The interpreter must be capable of representing the L-system, despite the lack of information in the resultant string. Alternatively, it should be able to obtain this information by other means.

It may be tempting to leave the complexity to the interpreter to make the L-system rewriter and its rules more simple. However, the drawback of this is that the information needed for modeling branch diameters, branching angles, and the type of objects that need to be rendered have to be supplied to the interpreter in some way. If not through the resulting string of information, how is this information meant to be provided to the interpreter? An answer may be to build a system within the interpreter that is capable of assuming the general look of a plant, for instance, branches that decrement in diameter and branching angles, which are consistent. This could result in a very inflexible system that may work for a portion of plant-life but might struggle to represent certain classes of plant-life. Therefore, the benefit of using a system with most of its complexity within the rewriting system is the L-system is responsible for some of the details of the interpretation, such as angles, branch diameters, and other details. In the next few sections, different types of L-systems are described, explaining their benefits and limitations, as well as developing a system integrating these separate systems into a single L-system grammar.

Several well-known fractal geometry patterns have been explored. They are particularly interesting because of how they seemingly imitate nature [Mandelbrot, 1982]. An example of this is the is with edge-rewriting patterns like the Koch curve and the Sierpiński gasket. The Koch curve can be represented using the L-system defined in 2.4 below. This is an adaption of the Koch snowflake, which can be generated by the 0L-system. It is important to note that as the number of rewrite generations increases, the complexity of the patterns becomes increasingly intricate.

Koch Curves:
Generations: 2,3,4
Angle: 90°
Distance: 1 cm
ω : F
$p1$: F $\rightarrow$ F+F-F-F+F

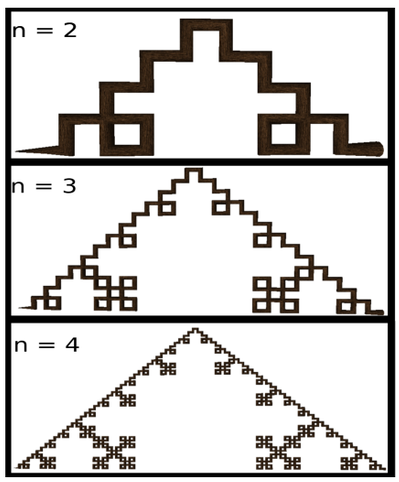


Figure 2.4: Koch Curve.

The Sierpińkski gasket is another example of an edge-rewriting pattern which can show the power of a rewriting system like the L-system. This example is interesting as with each generation, the even-numbered generations face left, and the odd-numbered generations face right.

Sierpiński Gasket:
Generations: 2,3,4,5
Angle: 60°
Distance: 1 cm
ω : F
$p1$: F $\rightarrow$ X-F-X
$p2$: X $\rightarrow$ F+X+F

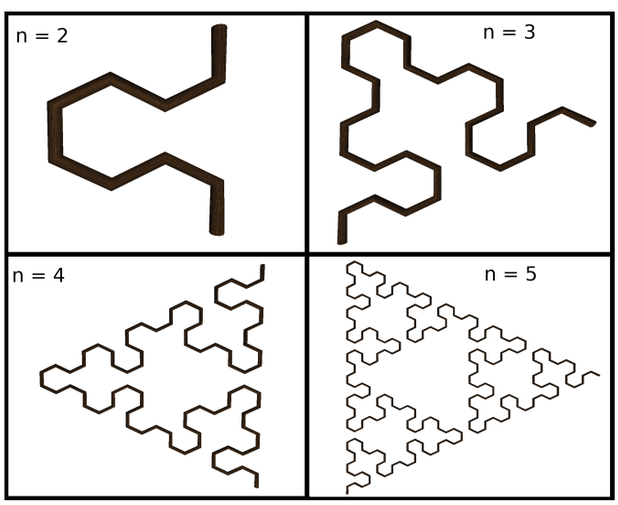


Figure 2.5: Sierpiński Triangles.

2.3 Branching

The simplistic D0L-system defined in previous sections can trace a 3D pattern. The D0L-systems interpretation provides a way of tracing a path or structure in 3D space. These types of L-systems are useful; however, to trace the branching structure of plants, there needs to

be a way of branching off in one or more directions. A simple solution may be for the turtle object to trace its steps back to a particular branching point and then branch off in a different direction. Branching like this may get the desired result but is slow and inefficient.

Lindenmayer proposed a better solution to the branching problem. He introduced two symbols that have special meanings within the alphabet of the DOL-system, which make branching much easier [Lindenmayer, 1968]. These are the square bracket symbols "[", "]". The open square bracket "[" symbol instructs the turtle object to save its current state (position and orientation) and be able to go back to that saved state later. The close square bracket "]" instructs the turtle to load the saved state and continue from that position and orientation. The save and load states allow the turtle to jump back to a previously saved position, facing in the same direction as it was before. The orientation can later be changed, allowing the turtle to branch off in a different direction. This method was originally used by Lindenmayer to imitate the branching that occurs within algae but was later adapted by Smith to represent larger plant-life as well [Smith, 1984].

The main advantage of using the save and load position functionality within the alphabet is that the rewriting system itself handles branching. The production rules often contain the next generations branching structure by using the save and load symbols, and thus the branching structure becomes more intricate from one generation to the next.

Each save state symbol must have a corresponding load state symbol within the string. This is not a requirement by the L-system language, but a requirement during interpretation because the load and save state symbols have no special meaning to the rewriter. It is treated the same as any other symbol in the alphabet. This being said, during interpretation, for the turtle object to jump back to a saved state, those save and load states should correspond. For instance, the resultant string "F[+F-F]-F" has both a load, and a save state, meaning there is a single branch off the main branch. An example of this can be seen in figure 2.6 below. Additionally, using nested save and load states in the string, for instance, "F[+F[+F]-F]-F", there can be two branches off the main branch twice as seen in figure 2.7.

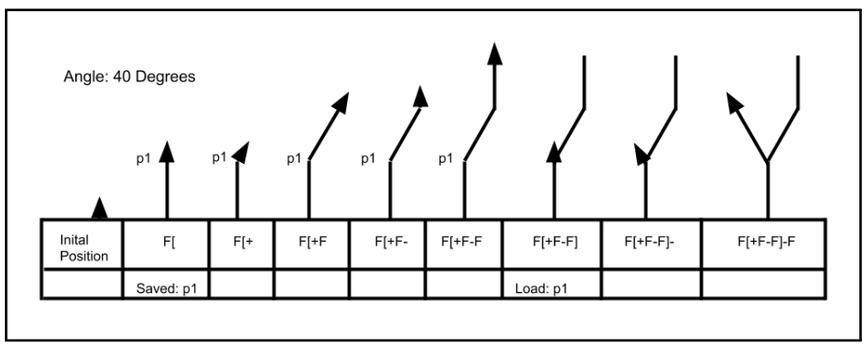


Figure 2.6: Diagram showing a turtle interpreting an L-system using the branching symbols.

Save and load operations are handled using the Last In First Out (LIFO) principle. LIFO states that when using the save symbol, it saves the current position and orientation at $p1$. The next load state restores $p1$'s position and orientation. Unless there is another save that takes place before the load state, in which case the most recent save has to be loaded before $p1$ can be loaded. In this way, the position saves are stacked, and the most recent save is always loaded first. An example of this can be seen in figure 2.7 below:

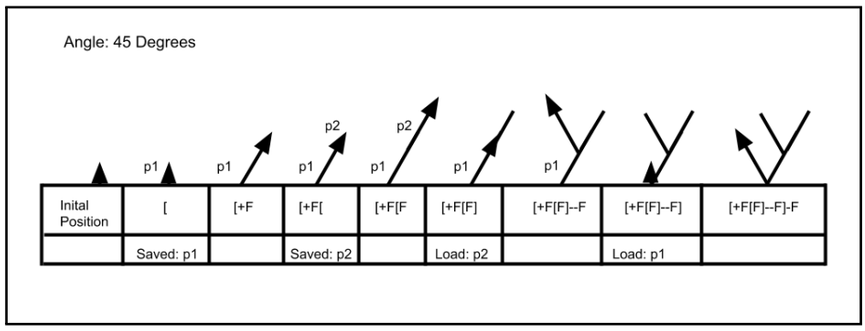


Figure 2.7: Diagram showing a turtle interpreting an L-system with nested branching.

The save and load state symbols can be used within a simple L-systems to create a more complex plant-like fractal pattern. In the following examples, there are two L-systems. One can generate a fractal pattern similar to that of a bush, and the other a fractal representing a tree. In figure 2.8, the F symbol can be rendered as a branch segment. The L-system only consists of a single rewriting rule; thus, each generation results in exponentially more branches. Each generation results in eight times more branches than the previous generation.

Fractal Bush:
Alphabet: F, +, -, [,]
Axiom: F
Angle: 25°
Rules:
F → FF+[+F-F-F]-[-F+F+F]

Figure 2.8: Fifth generation of the fractal bush L-system.

In figure 2.9 below, there are two different rewriting rules. One for the symbol F and the other for symbol X. Symbol X is the axiom; however, it is not a rendered symbol meaning the interpreter ignores it. Unlike the symbol F, which is rendered as a branch. Instead, symbol X stands as a placeholder for the next rewriting step, where it is rewritten with "F-[[X]+X]+F[+FX]-X". The symbol F is replaced by FF, this means that existing branches get longer each generation, but new branching structures are created at the end "leaves" or ends of the branches due to the production rule for symbol X.

Fractal tree:
Alphabet: X, F, +, -, [,]
Axiom: X
Angle: 25°
Rules:
X → F-[[X]+X]+F[+FX]-X
F → FF

Figure 2.9: Fifth generation of the fractal tree L-system

2.4 Parametric OL-systems

Simplistic L-systems, like the algae representation in section 2.1, give enough information to create the fundamental structure of plant life. Many details necessary for rendering the plant are not included with a simple OL-system. Things like the width, length, and branching angles of each section. These details have to be assumed or are defined somewhere as a constant value. The interpreter is left to find the details of the branching structure. The question becomes, is there a type of L-system that is capable of providing these details? The answer lies with parametric 0L-systems.

This section will outline the definition and significant concepts of the parametric L-system formulated by Prusinkiewicz and Hanan in 1990 [Prusinkiewicz and Hanan, 1990], and developed upon in 2012 by Prusinkiewicz and Lindenmayer [Prusinkiewicz and Lindenmayer, 2012]. This section talks about the changes and improvements to the parametric L-system. As well as explains why these changes are necessary for this thesis.

2.4.1 Formal Definition of a Parametric 0L-system

Prusinkiewicz and Hanan define the parametric 0L-systems as a system of parametric words, where a string of letters make up a module name A, each module can have several parameters associated with it. The module names belong to an alphabet V; therefore, $A \in V$, and the parameters belong to a set of real numbers $\Re$. If $(a_1,\ a_2,\ ...,\ a_n) \in R$ are parameters of A, the module can be stated as $A(a_1,\ a_2,\ ...,\ a_n)$. Each module is an element of the set of modules $M = V \times \Re^*$. $\Re^*$ represents the set of all finite sequences of parameters, including the case where there are no parameters. We can then infer that $M^* = (V \times \Re^*)^*$ where M^* is the set of all finite modules.

Each parameter of a given module corresponds to a formal definition of that parameter defined within the L-system productions. Let the formal definition of a parameter be Σ. $E(\Sigma)$ can be said to be an arithmetic expression of a given parameter.
Similar to the arithmetic expressions in the programming languages C/C++, we can make use of the arithmetic operators $+,\ -,\ *,\quad \wedge$. Furthermore, we can have a relational expression $C(\Sigma)$, with a set of relational operators. In the literature by Prusinkiewicz and Hanan the set

of relational operators is said to be $<$, $>$, $=$, I have extended this to include the relational operators $>$, $<$, $>=$, $<=$, $==$, $!=$. Where $==$ is the 'equal to' operator, $!=$ is the 'not equal' operator, the symbols $>=$ and $<=$ are 'greater than or equal to' and 'less than or equal to' respectively. The parentheses () specify precedence within an expression. A set of arithmetic expressions can be said to be $\hat{E}(\Sigma)$, these arithmetic expressions can be evaluated and result in the real number parameter $\Re$, and the relational expressions can be evaluated to either true or false.

The parametric 0L-system can be shown as follows as per Prusinkiewicz and Hanan's definition:

$$G = (V, \Sigma, \omega, P) \tag{2.4}$$

G is an ordered quadruplet that describes the parametric OL-system. V is the alphabet of characters for the system. Σ is the set of formal parameters for the system. $\omega \in (V \times \Re^*)^+$ is a non-empty parametric word called the axiom. Finally, P is a finite set of production rules which can be fully defined as:

$$P \subset (V \times \Sigma^*) \times C(\Sigma) \times (V \times \hat{E}(\Sigma))^* \tag{2.5}$$

Where $(V \times \Sigma^*)$ is the predecessor module, $C(\Sigma)$ is the condition and $(V \times E(\Sigma))^*$ is the set of successor modules. For the sake of readability we can write out a production rule as *predecessor* : *condition* $\rightarrow$ *successor*. I will be explaining the use of conditions in production rules in more detail in section 2.4.4. A module is said to match a production rule predecessor if they meet the three criteria below.

- The name of the axiom module matches the name of the production predecessor.
- The number of parameters for the axiom module is the same as the number of parameters for the production predecessor.
- The condition of the production evaluates to true. If there is no condition, then the result is true by default.

In the case where the module does not match any of the production rule predecessors, the module is left unchanged, effectively rewriting itself.

2.4.2 Defining Constants and Objects

There are some other features covered by Prusinkiewicz and Lindenmayer that are not specific to the parametric L-systems definition itself but serve as quality of life. In the literature, they refer to the #define, which is said: "To assign values to numerical constants used in the L-system." The #include statement specifies what type of shape to draw by referring to a library of predefined shapes [Prusinkiewicz and Lindenmayer, 2012]. For instance, if we have

a value for an angle that we would like to use within the production rules, we can use the #define statement as follows:

$$
\begin{aligned}
& n = 4 \\
& \text{\#define angle 90} \\
& \omega \quad : F(5) \\
& p_1 \ : F(x) \qquad : * \rightarrow \ F(w) + (angle)F(w) + (angle)F(w) + (angle)F(w)
\end{aligned}
\tag{2.6}
$$

Here you can see that the #define acts like a declaration, where a variable is going to be defined, which is used later. Essentially we are replacing any occurrences of the variable *angle* with the value of 90 degrees. The define statement is written as #define *variable_name value*.

With regards to the #include statement, In the literature, the #include may be used by stating "#include H". This tells the turtle interpreter that the symbol "H" is a shape in a library of predefined shapes which should be rendered instead of the default shape. This functionality has been slightly modified, instead of the #include statement, the #object is used and serves a similar purpose, however, instead importing the symbol "H", denoting to the heterocyst object from a library of predefined shapes, The statement "#object H HETEROCYST" specifies that we are associating the symbol or module "H" with the object HETEROCYST. The HETEROCYST object is still stored in a predefined library; however, the advantage is that the object can be associated with multiple different symbols, it also does not limit us to a predefined name for an object. Below is an example using the #object statement:

$$
\begin{aligned}
& n = 1 \\
& \text{\#object F BRANCH} \\
& \text{\#object S SPHERE} \\
& \omega \quad : F(1) \\
& p_1 \ : F(x) \qquad : * \rightarrow \ F(w)F(w)F(w)F(w)S(w)
\end{aligned}
\tag{2.7}
$$

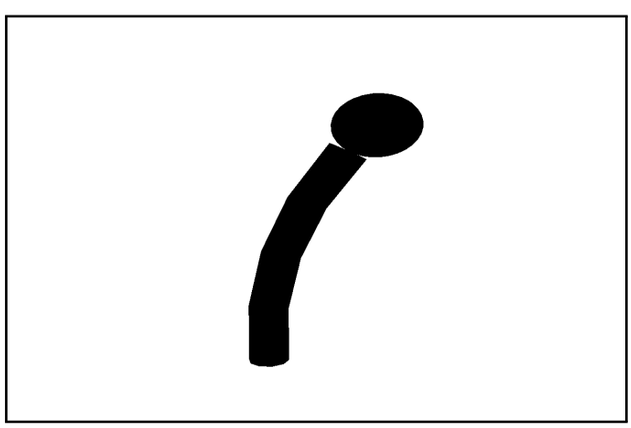


Figure 2.10: Diagram of an L-system Using Multiple Objects.

In the simple example in figure 2.7 above, you can see that the first three F modules render a branch segment with a length of 1.0; however, for the final S module renders a sphere of diameter 1.0. The geometric shape that is eventually rendered does not affect the L-system in any way, and the #object feature bears no meaning to the rewriting system, it merely

stands as an instruction to the interpreter which instructs that each time the symbols F or S are interpreted, a specific object should be rendered, such as BRANCH and SPHERE respectively. The position of the next object or branch can then be determined by moving forward by the diameter of the object and rendering the next object from that point. The details of the interpreter are discussed in more detail chapter 5.

2.4.3 Modules With Special Meanings

In the above section, I defined the details of a parametric 0L-system. In the paper by Prusinkiewicz and Lindenmayer, there are two operators which have not been discussed yet. These operators are the ! and the ‘. Prusinkiewicz and Lindenmayer state that “The symbols ! and ‘ are used to decrement the diameter of segments and increment the current index to the color table respectively” [Prusinkiewicz and Lindenmayer, 2012]. We have decided to modify this to work slightly differently, the ! and ‘ still performs the same operation; however, the ! and ‘ symbols are treated as a module that holds particular meaning to the interpreter, rather than a single operator. Furthermore, they share the same properties with modules; they can contain multiple parameters, and depending on the number of parameters, they can be treated differently. The module ! with no parameters could mean decrement the diameter of the segment by a default amount, whereas !(10) means set the diameter of the segment to 10. The length can also be manipulated similarly. The module with the name F has a default meaning to create a segment in the current direction by a default amount. If we provide the module F(10) we are specifying to create a segment of length 10.

Using the L-system below, we can create figure 2.8, the concepts discussed above have been used by decrementing the segment diameter during the rewriting process as well as by incrementing the branch length.

$$\begin{aligned} & n = 8 \\ \omega \quad & : A(5) \\ p_1 \; & : A(w) \qquad : * \rightarrow F(1)!(w)[+A(w * 0.707)][-A(w * 0.707)] \\ p_2 \; & : F(s) \qquad : * \rightarrow F(s * 1.456) \end{aligned} \tag{2.8}$$

The above l-system gives the resulting representation shown below in figure 3.8.

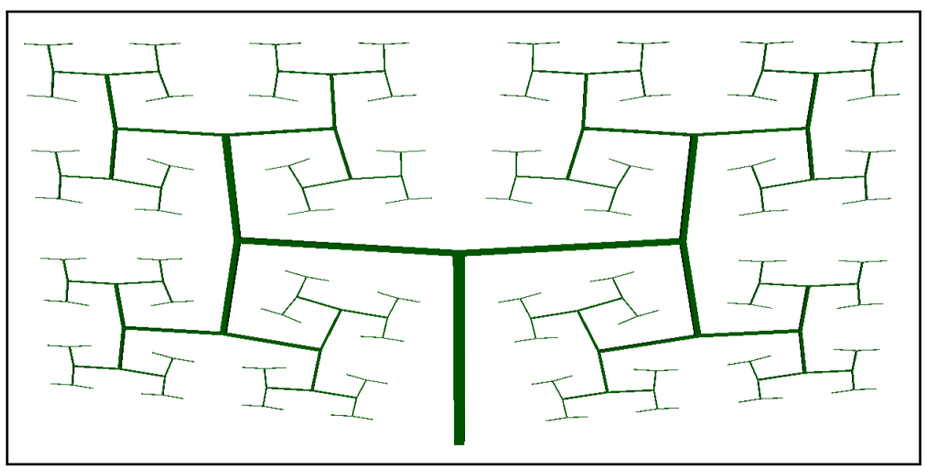


Figure 2.11: 3D Parametric L-system.

This gives a much more realistic looking tree structure as the branch segments become shorter

but also become thinner in diameter as they get closer to the end of the branch as a whole.

2.4.4 Representing L-system Conditions

A condition allows multiple production rules that are the same in terms of their module name and number of parameters. Furthermore, they require a particular condition to be met for the module to match that rule.

This section will detail the use of the condition statement, which lies between the predecessor and the successor in a production rule. It can be seen as a mathematical expression on either side of a relational operator. During the rule selection process, the expressions are evaluated, and the results are compared using the condition operator. If the result of the condition evaluates as true, then that rule is selected for rewriting; otherwise, it will check the next rule.

Below is an example of a parametric 0L-system using condition statements:

$$
\begin{aligned}
& n = 5 \\
& \omega \quad : A(0)B(0,4) \\
& p_1 \ : A(x) \qquad : \ x \ > \ 2 \ \rightarrow \ C \\
& p_2 \ : A(x) \qquad : \ x \ < \ 2 \ \rightarrow \ A(x \ + \ 1) \\
& p_3 \ : B(x,\ y) \ : \ x \ > \ y \ \rightarrow \ D \\
& p_4 \ : B(x,\ y) \ : \ x \ < \ y \ \rightarrow \ B(x \ + \ 1,\ y)
\end{aligned}
\tag{2.9}
$$

The L-system above in 2.9 is rewritten five times using the axiom specified by the symbol ω, as well as the four production rules p_1, p_2, p_3, p_4. Each generation of the rewriting process can be seen below in 2.10.

$$
\begin{aligned}
& g_0 : \ A(0)B(0,\ 4) \\
& g_1 : \ A(1)B(1,\ 4) \\
& g_2 : \ A(2)B(2,\ 4) \\
& g_3 : \ C \ B(3,\ 4) \\
& g_4 : \ C \ B(4,\ 4) \\
& g_5 : \ C \ D
\end{aligned}
\tag{2.10}
$$

The practical use of the condition statement might be to simulate different stages of growth. The condition statement is best illustrated using the L-system below:

$$
\begin{aligned}
&n = 2,\ 4,\ 6\\
&\#\text{object F BRANCH}\\
&\#\text{object L LEAF}\\
&\#\text{object S SPHERE}\\
&\#\text{define r 45}\\
&\#\text{define len 0.5}\\
&\#\text{define lean 5.0}\\
&\#\text{define flowerW 1.0}\\
&\omega \ :\ !(0.1)I(5)\\
&p_1\ : I(x)\ :\ x\ >\ 0\ \rightarrow\ F(len) - (lean)[R(0,100)]F(len)[R(0,100)]I(x-1)\\
&p_2\ : R(x)\ :\ x\ >\ 50\ \rightarrow\ -(r)/(20)!(2.0)L(2)!(0.1)\\
&p_3\ : R(x)\ :\ x\ <\ 50\ \rightarrow\ -(r)\backslash(170)!(2.0)L(2)!(0.1)\\
&p_4\ : I(x)\ :\ x\ <=\ 0\ \rightarrow\ F(len)!(flowerW)S(0.3)
\end{aligned}
\tag{2.11}
$$

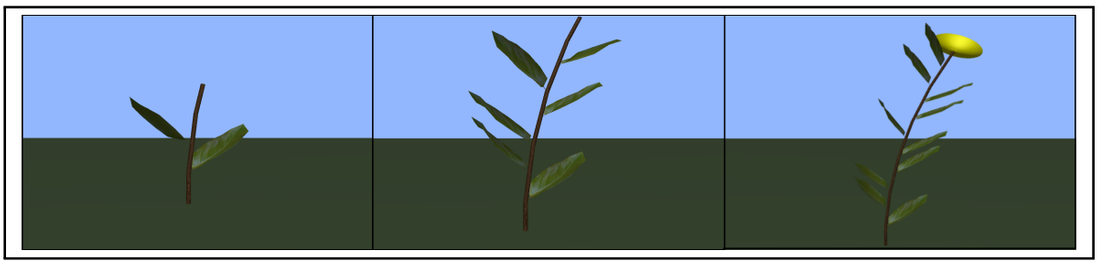


Figure 2.12: Condition statements used to simulate the growth of a flower. 2nd generation on the left, 4th generation in the center and 6th generation on the right

2.5 Randomness within L-systems

Randomness is an essential part of nature. If there is no randomness in plant life, it will end up with very symmetric and unrealistic. Randomness is also responsible for creating variation in the same L-system. An L-system essentially describes the structure and species of a plant. It describes how large the trunk of the tree is, how many leaves are on the end of a branch, or even if it has flowers or not. However, if there is no capability to have randomness in the generation of the L-system, then it will always end up with the same structure. Below is a simple example of how randomness can be used to create variation.

$$
\begin{aligned}
&n = 2\\
&\#\text{define r 25}\\
&\omega \ :\ !(0.2)F(1.0)\\
&p_1\ : F(x)\ :\ *\ \rightarrow\ F(x)[+(r)F(x)][-(r)F(x)] + (\{-20,20\})F(x) - (\{-20,20\})F(x)
\end{aligned}
\tag{2.12}
$$

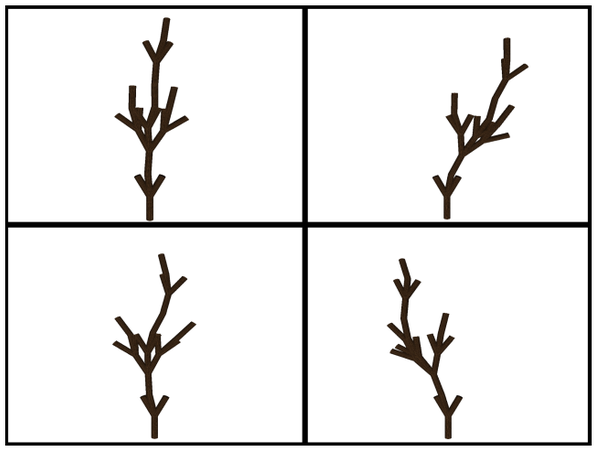


Figure 2.13: Different Variations of the Same L-system with Randomness Introduced in The Angles.

In figure 2.13, there are four variations of the same L-system using randomness. We can specify that we would like to create a random number by using the expression {-20.0, 20.0}. The curly braces signify that a random number range contains a number ranging from the minimum value, being the first floating-point value and the maximum value, being the second floating-point value, separated by a comma. If both values are the same for instance +({10.0, 10.0}) this is equivalent to +(10.0).

2.6 Stochastic Rules within L-systems

Similar to the previous section, stochastic L-systems fulfill a similar goal. On their own, 0L-systems are incapable of creating any variation. They follow a strict set of production rules that give the same result. Introducing randomness to an 0L-system for the width, length, and other parameters can result in a plant that looks slightly different but does not change to the overall structure of the plant. To create a different structure for a plant, we must introduce stochastic probability within the selection of production rules, thus effecting the rewriting of the plant's structure.

Eichhorst and Savitch introduced a new type of 0L-system called the S0L-system, this added two features to the existing 0L-system, firstly the S0L-system is not limited to defining a single axiom (starting point), a finite number of starting points can be defined, and a probability distribution is used to select the starting point at the start of the rewriting process. Secondly, the S0L-system allows the definition of a finite number of production rules which have a probability distribution to decide which rule should be chosen for rewriting [Eichhorst and Savitch, 1980]. Similarly, an article by Yokomori proposes a stochastic 0L-system which also proposes a measure of the entropy of a string generated by a 0L-system [Yokomori, 1980].

Later, Prusinkiewicz and Lindenmayer built upon this by creating a definition of a stochastic L-system, that makes use of the stochastic nature of the production rules from the SOL-system. This paper will be using the definition of the stochastic 0L-system defined by Prusinkiewicz and Lindenmayer and developing them into the existing parametric 0L-system. This paper will not allow multiple starting points as defined by Eichhorst and Savitch in the SOL-system, as it does not seem necessary and could overcomplicate the 0L-system. However,

this functionality could be added in the future if it is seen to be necessary.
Similarly to the 0L-system, the stochastic 0L-system is an ordered quadruplet, represented as $G_\pi = (V, \omega, P, \pi)$, where V is the alphabet of the 0L-system, ω is the axiom, P is the finite set of productions, and π represents a probability distribution for a set of production probabilities this can be shown as $\pi : P \rightarrow (0, 1)$ the production probabilities must be between 0 and 1 and the sum of all production probabilities must add up to 1.

The following L-system definition created by Prusinkiewicz and Lindenmayer states three production rules with each rule having a probability of 0.33 out of one. For a finite set of production rules to be stochastic, the production rules must share the same module name and the same number of parameters. There must be two or more production rules, and the total probability distribution must add up to 1.0 [Prusinkiewicz and Lindenmayer, 2012].

$$
\begin{aligned}
& n = 5 \\
& \#\text{define r } 25 \\
& \omega \ : \ F(1) \\
& p_1 \ : F(x) \ : \ \sim 0.33 \ \rightarrow \ F(x)[+(r)F(x)]F(x)[-(r)F(x)]F(x) \\
& p_2 \ : F(x) \ : \ \sim 0.33 \ \rightarrow \ F(x)[+(r)F(x)]F(x) \\
& p_3 \ : F(x) \ : \ \sim 0.34 \ \rightarrow \ F(x)[-(r)F(x)]F(x)
\end{aligned}
\tag{2.13}
$$

As seen above, the module F(x) is the predecessor for all three of the production rules, each rule has a probability which is defined using the $\sim$ symbol followed by a probability from 0 to 1. In the above example, each probability is approximately one third, and they are approximate to total an exact probability of 1.0. During the rewriting process, when module F with one parameter is found, a production rule is randomly selected using the probability distribution described within the production rules. The predecessor from the selected rule will then rewrite that module.

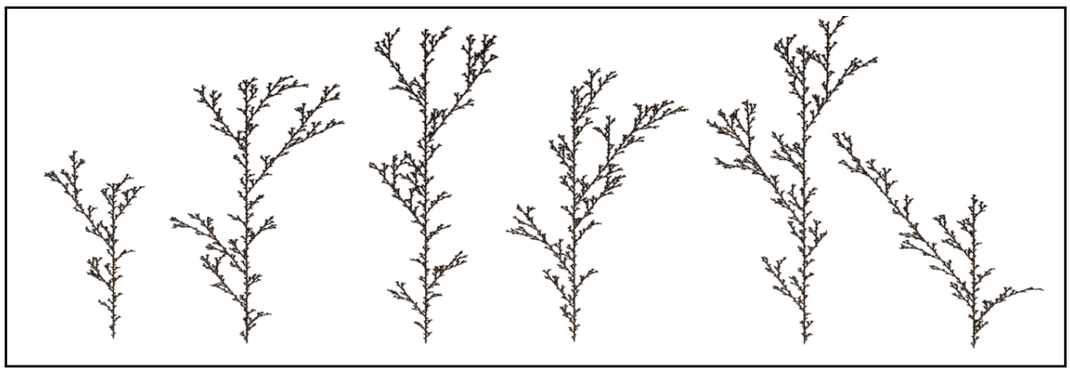


Figure 2.14: Representation of an L-system with a probability stochastic with a 0.33 probabability for each rule.

The stochastic L-system definition in 2.13, produces the following fractal structures seen in figure 2.14 below. The stochastic L-system will get a slightly different resultant string each time it is run, depending on which rules were selected for rewriting. The difference in resulting strings gives a different number of translation instructions, resulting in the plant having branches of different lengths. $p1$ has two extra F instructions, this results in some branches being much longer than others, and possibly producing plants of different sizes.

2.7 Computing L-systems

This thesis focuses on the different levels of complexity between the L-system rewriting and the L-system interpretation. It is essential to distinguish these two systems by their components, and how these components interact. The two systems will be called the L-system rewriter and the L-system interpreter. As discussed at the begining of this chapter, the L-system rewriter takes L-system language as input in the form of a text file. The rewriter has three significant parts the tokenizer, parser, and the rewriter. The tokenizer breaks the language into individual words, then checks the syntax of the language according to the grammar. The parser then uses these words to check the validity of the semantic structure of the language as well as build relevant data structures for the rewriter. Finally, the rewriter uses these data structures to rewrite the axiom string several times according to the production rules. The result of the string rewriter is a module string, as well as other bits of information that will be used by the interpreter. The interpreter must now assume any information that has not provided by the string rewriter.

The string interpreter also has three significant parts; however, the functions of these parts are very dependant on what the L-system is trying to represent. For the procedural generation of plant life, there is the turtle graphics interpreter, model generator, and the OpenGL renderer. The turtle graphics renderer takes each module from the module string and interprets its meaning as a set of instructions carried out by a turtle object. The model generator takes the information generated by the turtle graphics interpreter and generates the 3D branching model as well as leaves and other objects. Lastly, the OpenGL renderer takes the models generated and renders them on the screen for the user. This process can be visualised in figure 2.7 below.

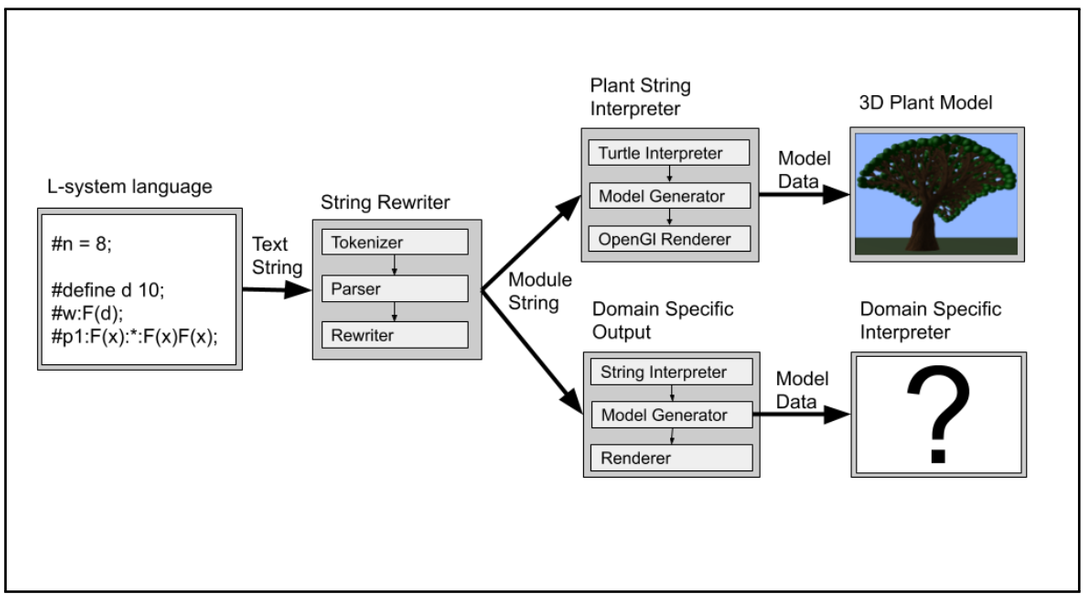


Figure 2.15: Diagram of the procedural generation process.

2.8 Summary

L-systems represent a set of state transitions based upon the production rules provided. These rules dictate how a string will be rewritten, which in turn determines the overall

structure of the plant it is trying to represent. The symbols in D0L-systems or modules in parametric 0L-systems represent particular instructions to be carried out by turtle graphics within the interpreter. The modules within an L-system do not change the behaviour of rewriting but instead matter to the interpreter. Additionally, the complexity of the L-system rewriter decides the complexity of the interpreter. If an L-system provides a large amount of information to the interpreter, fewer assumptions need to be made during the interpretation and, therefore, providing the ability to describe the plant-life it is representing accurately.

By using the parametric 0L-system, we can build in several features, otherwise used in other L-systems, such as branching, conditional production rules, randomness in parameters, stochasticity. These features allow the parametric 0L-system to represent plant-life with varying structures, branch lengths, branch widths, and production rule conditions, which gives further control over stages of growth.

Chapter 3

L-system Rewriter Implementation

There are two major parts necessary to procedurally generate plant-life using an L-system. These are the rewriter and the interpreter. The purpose of the L-system rewriter is to take an L-system file as input, and generate the resulting string that fits the L-system grammar. It does this by syntactically and semantically analysing the L-system input, and generating the structures and information necessary to carry out the rewriting process. The rewriting process uses the structures and information, such as the string of modules and the production rules, to step through each string and rewrite the symbols. This chapter focuses on each part of the string rewriters' implementation and will introduce a technique of processing the L-systems' input, similar to how computer languages are compiled. This chapter will also formally define the L-system grammar in Backus-Naur Form, and provide the pseudocode for the L-system rewriter.

For a simple D0L-system, like the one seen in section 2.3. Each symbol within the alphabet is made up of a single character, the productions rules then match against those characters. As the D0L-system is deterministic, there is no randomness when determining the matching rule. The simplicity of the L-system makes it quite easy to create a rewriting system for the D0L-system. All the rewriter must do is store the starting string and production rule predecessors and successors. It then iterates over a string of symbols and replace them with the successor. The implementation of a more sophisticated L-system, like the parametric 0L-system, is much more complex. A parametric L-system can have multiple modules that make up a string, where each module may have multiple parameters, and each parameter could be a mathematical expression. The added complexity makes developing a rewriting system considerably more difficult. The rewriter must better understand what the syntax of the L-system is specifying, based on the context of each symbol within the L-system.

Due to the complexity of the L-system grammar, it is difficult for a computer to tell the syntactic and semantic properties of each part of the L-system input, which makes it difficult to carry out the rewriting process. Using a system similar to a "compiler", an L-system "program" can be broken down into a three-stage process, as seen in figure 3.3 below. The first stage is *lexical analysis*, then a process called *parsing* and finally the string rewriting stage. The lexical analyser is responsible for splitting the input into syntactic words, and then assigning each word into its syntactic category. Any word within the L-system that does match a syntactic category will result in a lexical error. If there are no lexical errors the words and their syntactic categories are sent to the parser. The parser matches the

syntactical categories of each sentence in the language against a grammatical model. If any of the sentences within the language do not match the grammatical model, an appropriate error message can be displayed, similar to that of the lexical error. The error states where the syntax error occurred and what was grammatically incorrect. The parser also creates a syntax tree along with any data structures necessary for the rewriting process. These structures can then be used to carry out string rewriting or provide information to the interpreter.

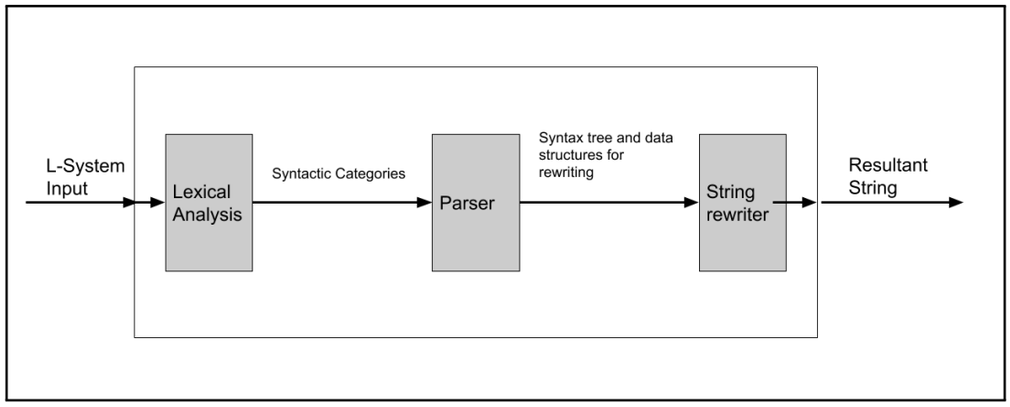


Figure 3.1: Diagram of the Parts of The Rewiting System.

3.1 Environment and Tools

The implementation of the string rewriter, and the string interpreter, is written in the C and C++ programming languages [Stroustrup, 2000]. The C and C++ languages are two of the most common programming languages that have stood the test of time with the first version of C being released in 1974. These languages are frequently used within computer graphics, with some of the most popular game engines supporting either C or C++. Such as CryEngine, Unreal Engine, Source Engine, and more. The main reason for this is the high performance and low-level memory management that C and C++ provide, and the graphics programming frameworks such as OpenGL, Vulkan, and DirectX all having direct support for either C or C++. The C and C++ languages also have a large number of useful libraries that provide extra functionality.

The implementation of the rewriter and the interpreter will use the modern Open Graphics Library (OpenGL). The OpenGL framework is one of the industry standards for creating 3D graphics applications. It is a cross-platform API for interacting with the GPU in a low-level way. The high-performance nature of OpenGL is essential, as displaying and simulating the L-system can be very graphically intensive [Sellers et al., 2013] [Movania et al., 2017]. OpenGL was initially intended to be an API for the C and C++ programming languages. Therefore, both the programming language and graphics API have a strong emphasis on performance, which is necessary when procedurally generating and simulating plant-life.

For more specialised mathematics capabilities, the OpenGL Mathematics Library (GLM) library holds many mathematics classes and functions for conveniently dealing with structures such as vectors, matrices, and quaternions. This thesis will cover these mathematical concepts in chapter ; however, it is convenient to have these implemented and tested within a C++ library. Another important library is Graphics Library Framework (GLFW) which is a multi-platform API for creating an managing user interface windows, events, and user-input

[GLFW development team, 2019]. To keep track of changes and manage versions. Git is a free and open-source version control software. It can keep track of changes that have been made to the files within a project folder as well as keep previous versions of the project throughout the development process. In conjunction with Git, Github is an online web application that stores git repositories. Git acts as a backup as well as containing all previous versions of the project [Torvalds,].

3.2 The L-system as an Interpreted Grammar

Traditionally an interpreter in computing is a program that takes program code as input. It is then analyzed and interpreted as it is encountered in the execution process. All of the previously encountered information is kept for later interpretations. The information about the program can be extracted by inspecting the program, such as the set of declared variables in a block or a function [Wilhelm and Seidl, 2010]. In essence, the L-system rewriter contains a type of interpreter. This should not be confused with the interpreter that processes the resultant string using turtle graphics. Due to this confusion of terms, the system containing the lexical analyser, L-system parser, and the string rewriter will be referred to as the L-system rewriter, instead of the interpreter in the computational sense.

A similarity can be drawn between traditionally interpreted languages and the L-system rewriter. The L-system rewriter defines a set of constant variables, a starting point, and then some production rules. This information can then be used to rewrite the starting string several times. Later on, it may be decided that, instead of five generations of rewriting, the rewriter should instead generate ten. Some information about the L-system is still valid, the production rules, axiom, and constants have not changed, and therefore this information can be used to interpret to the tenth generation. This concept can be used to go from the current state of the L-system rewriter and rewrite another five times. Instead of throwing all the information away and starting from scratch. Furthermore, if we would like to retrieve the resultant string, this can be requested from the L-system rewriter.

The lexical analyser and parser are a necessary part to carry out rewriting. Without the lexical analyser or parser, it would not be straightforward to find the syntactic roles of each part of the L-system. Take the example of the module: F(2*3, x * (2 + y)). Here there is a single module with two parameters, one parameter has the expression (2 * 3), and the other has the expression (x * (2+y)). These complex structures within a grammar require knowledge about the grammatical model it represents. The lexical analyser firstly makes sure that all the syntax within the L-system is correct and assigns each word or symbol to a syntactic category, the parser then splits the L-system into its components and is describes each parts syntactic roll. The lexical analyser provides the understanding that x and y are variables within a module and do not represent something else. It also provides knowledge about how to find the values of x and y.

The difficulty of creating an L-system with more complexity in the grammar is that it becomes more challenging to write a valid L-system to represent a particular structure. For example, imagine trying to write a C program where the compiler does specify why the program is incorrect. The advantage of using a rewriter similar to a compiler is that it makes

it simpler to debug any syntactic errors, as well as make the string rewriting much faster. This means that writing an L-system becomes similar to rewriting a recursive program, where any syntactic mistakes will result in a meaningful error describing what was incorrect.

3.3 The Syntax of a Parametric L-system

This section will specify the valid syntax for the parametric L-system rewriter. The syntax is similar to the definition of the parametric L-system definition given by Prusinkiewicz and Lindenmayer in section 2.4.1. There are some additions and modifications to the syntax definition provided by Prusinkiewicz and Lindenmayer to construct an L-system that includes branching, constant variable definitions, object specifications, parametric L-system concepts, randomness, and stochastic L-systems [Prusinkiewicz and Lindenmayer, 2012].

This L-system has five major parts. Each part is categorised as a statement. Valid statements are the *defines* , the *includes*, a single generation statement, a single axiom statement, and one or more production rules [Prusinkiewicz and Hanan, 2013]. All of these statements collectively form an L-system. Each statement starts with a '#' character and ends with a ';' symbol. These are used to indicate the start and end of a statement, even if multiple statements are written on the same line.

The order that statements should be listed is as follows:

$$\begin{aligned}
&\text{\#generations statement;}\\
&\text{\#define statements;}\\
&\qquad\ldots\\
&\text{\#include statements;}\\
&\qquad\ldots\\
&\text{\#axiom statement;}\\
&\text{\#production statements;}\\
&\qquad\ldots
\end{aligned} \tag{3.1}$$

The order for the statements does not always matter; for instance, the generation statement can be defined anywhere within the L-system. However, some parts are required to be in a particular order, such as the define and include statements, which must appear above the axiom and production rule statements as they define values used within the axiom and production rules. It is best practice to specify the L-system in the above order as to avoid any conflictions or errors.

All numbers within the L-system are represented as floating-point numbers. Using a single data-type keeps all numbers consistent. Other data types could be added in the future; however, there are added complexities in doing so, such as the conversion from one type to another, or having to specify which data type a variable represents. The floating-point data type provides all the necessary functionality needed for the L-system; therefore, it seems unnecessary to add more data types.

3.4 The L-system Lexical Analyser

In computer science, specifically the study of programming language compilers, the program responsible for carrying out lexical analysis is the lexer. Depending on the literature the lexer can also be known as the tokenizer or scanner. D. Cooper and L. Torczon write that "The scanner, or lexical analyser, reads a stream of characters and produces a stream of words. It aggregates characters to form words and applies a set of rules to determine whether each word is legal in the source language. If the word is valid, the scanner assigns it a syntactic category or part of speech" [Cooper and Torczon, 2011]. This is no different for the parametric 0L-system rewriter. For the rewriter to have enough information to carry out rewriting, it must first understand what each word or token within the L-system means, this requires assigning a syntactic category to each token, and whether or not the token is valid or not within the L-system grammar.

The scanner itself is quite complex, its main goal is to match the characters or strings within the language, to either a word or a regular expression defined in the grammar. When the match is made the token is given a syntactic category. The mechanism by which it achieves this is known as *finite automata* [Wilhelm et al., 2013]. It is possible to write custom lexer, however, it can be quite complicated and time-consuming to design and implement, and once a custom lexer has been created it is also difficult to change functionality at a later stage. There is a well known program known as the Fast Lexical Analyzer Generator (Flex). Flex takes in a file which contains the lexical rules of the language, this being the strings as well as the regular expression as well as its associated syntactic category. When Flex is executed it will create a lexer in the form of a C program. To create a lexer with Flex, the lexical rules must be defined. Below are the characters, strings and regular expressions and their associated syntactic categories, as well as a description as to its use in the parametric 0L-system.

Syntactic Word	Syntactic Category	Description
,	T_COMMA	Separation between module parameters
:	T_COLON	Separation between production rule parts
;	T_SEMI_COLON	End of a statement
#	T_HASH	Beginning of a statement
(	T_PARENL	Start of a modules parameters or specifies presidence in an expression
)	T_PARENR	End of a modules parameters or specifies presidence in an expression
{	T_BRACKETL	Start of a random range
}	T_BRACKETR	End of a random range
∼	T_TILDE	Stochastic operator
==	T_EQUAL_TO	Relational operator stating equal to
!=	T_NOT_EQUAL_TO	Relational operator for not equal to
<	T_LESS_THAN	Relational operator for less than
>	T_GREATER_THAN	Relational operator for greater than
<=	T_LESS_EQUAL	Relational operator for greater or equal
>=	T_GREATER_EQUAL	Relational operator for greater or equal
[	T_SQUARE_BRACEL	Module name (branching save state)
]	T_SQUARE_BRACER	Module name (branching load state)
+	T_PLUS	Arithmetic operator for addition, or Module name (Yaw right)
-	T_MINUS	Arithmetic operator for subtraction, or Module name (Yaw left)
/	T_FORWARD_SLASH	Arithmetic operator for division, or Module name (Pitch up)
\	T_BACK_SLASH	Module name (Pitch down)
*	T_STAR	Arithmetic operator for multiplication, or Condition in a production rule which is true
∧	T_HAT	Arithmetic operator for and exponent, or Module name (Roll right)
&	T_AMPERSAND	Module name (Roll left)
!	T_EXCLAMATION	Module name (Set size of branch)
$	T_DOLLAR	Module name
=	T_ASSIGN	Assignment operator used to set generations
#n	T_GENERATIONS	Declaration of the number of generations
#w	T_AXIOM	Declaration of the axiom
#define	T_DEFINE	Declaration of the define
#object	T_OBJECT	Declaration of the object
[0-9]+.[0-9]+|[0-9]+	T_FLOAT	Regular expression for a floating point number
[a-zA-Z_][a-zA-Z0-9_]*	T_VAR_NAME	Regular expression for a module or variable name

Table 3.1: Table of Valid Lexer Words

From the table above, several syntactic categories contain more than one meaning; for instance, the open and close parentheses have two meanings. They are used to either specify a modules' parameters or to specify precedence within an expression. It is not up to the scanner to determine what each parenthesis means, or that it has a meaning at all, the lexer only recognises that it falls into the syntactic categories, T_PARENL and T_PARENR. Deriving the meaning of a given token or syntactic category is decided by the parser. The parser is more aware of the context of each syntactic word. Similarly, the symbols [,],+,-,/,\, ∧, &, !, $, and T_VAR_NAME are valid module names. These symbols need to be specifically defined as their syntactic category, as they not only represent a module name but can also represent a different meaning depending on their context. For instance, the +, -, / are valid module names, but they also are mathematical symbols used within arithmetic expressions. The scanner must separate these symbols and keep them in their syntactic category for the parser to be able to understand the same symbol in multiple contexts.

It is also important to note that there are two unique types of tokens. These are the T_FLOAT and T_VAR_NAME. The regular expression for T_FLOAT will match any floating-point value, and the regular expression for T_VAR_NAME will match with any valid variable name. These unique tokens are valid syntactic categories but also contain an associated value. For instance, T_FLOAT has a floating-point value associated with it, and T_VAR_NAME has a string value associated with it. These values must be kept and provided to the parser for use later on.

3.5 The L-system Parser

The parsers' job is to find out if the input stream of words from the lexer is a valid sentence according to the grammar. If the syntactical categories from the lexer match the grammatical model, then the syntax is seen to be correct. If the syntax of the language is correct, the parser will generate a syntax tree and build the relevant data structures for use later on in the compilation process [Cooper and Torczon, 2011]. For the L-system rewriter, the syntax tree and data structures are not used for compilation but rather for the string rewriting process.

In order to describe a grammar, a suitable notation is necessary to express its syntactic structure and grammatical model. According to Cooper, the Backus-Naur Form(BNF) has traditionally been used by computer scientists to represent context-free grammars such as programming languages. Its origins are from the late 1950s and early 1960s. The BNF notation represents the context-free grammar by defining a set of non-terminal symbols that derive from a set of terminal or non-terminal symbols. Terminal symbols are elementary symbols of the language defined by the formal grammar. A terminal symbol will eventually appear in the resulting formal language. On the other hand, a non-terminal symbol exists only as a placeholder for patterns of terminal symbols but does not appear within the formal language itself. The syntactic convention for a BNF is for non-terminal symbols to be surrounded by angled brackets. For instance, <expression> and terminal symbols, such as the symbol for addition "+" to be underlined, but nowadays, it is not often underlined. The symbol ϵ represents an empty string, the ::= means "derives" and the | means "also derives" but is often articulated as an "or" [Cooper and Torczon, 2011]. The very first derivation must be a non-terminal symbol called the goal symbol. The goal symbol is a set of all valid derived strings. This means that the goal symbol is not a word within the language, but rather a syntactic variable in the form of a non-terminal symbol. The BNF notation below can be used to represent a simple grammar for arithmetic expressions, where the terminal "number" is any valid integer, and the goal symbol is <expression>. Below is the BNF notation for the syntax of an arithmetic expression that can represent addition and subtraction.

⟨expression⟩ ::= `number`
| `(`⟨expression⟩`)`
| ⟨expression⟩`+`⟨expression⟩
| ⟨expression⟩`-`⟨expression⟩

The BNF above states that the goal symbol, <expression> derives from one of four states. Either a terminal number, or an expression contained within two parentheses, or two expressions either side of an addition or subtraction terminal symbol. This type of notation is recursive and allows the formal language to write expressions that exist within other expressions. For example the expression "5 + 10 - (20 + 2)" can be broken down into using the BNF production rule forming a syntax tree as seen in figure 3.2 below. In this case, the whole expression fits the grammatical model of the language. Thus it can be parsed, forming the syntax tree. Computationally, when parsed, this expression will create a data structure, which will be discussed in more detail in section 3.5.4.

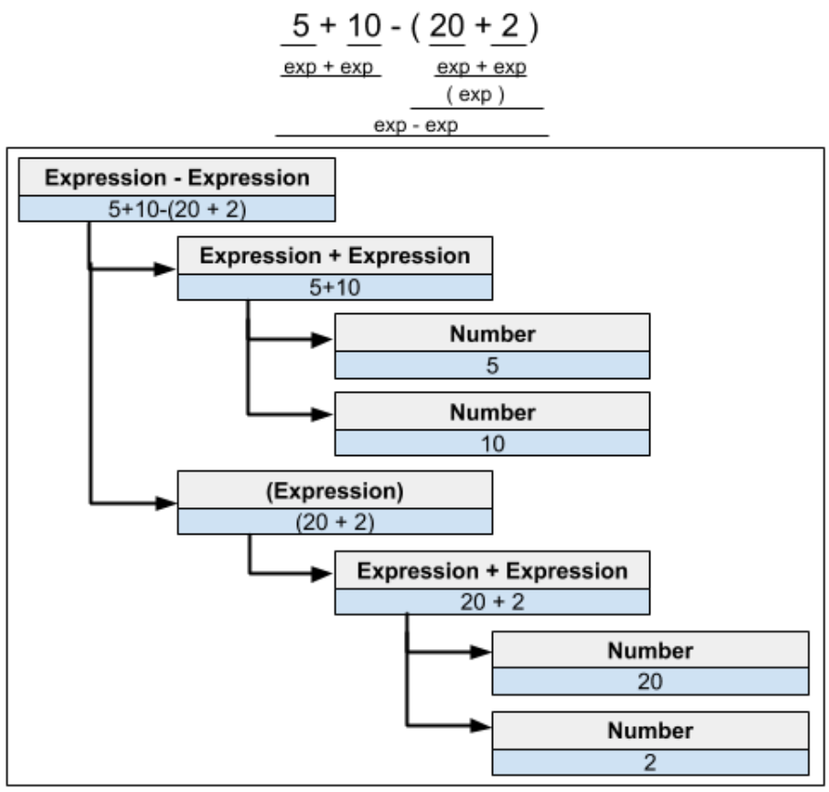


Figure 3.2: Diagram syntax tree for an expression.

Similar to the scanner, the parser program can be quite complex. It needs to find the associated terminal and non-terminal symbols and comply with the grammatical model. Furthermore, if there is a change in the grammar or there is a need to add features at a later date, it is frequently difficult to change the parser. Many studies have been conducted on creating a parsers; however this is beyond the scope of this thesis. Therefore, a program called a parser generator can be used to create the parser program. It uses a specification of the grammar similar to that of the BNF to generate a C program capable of parsing a given language. A popular implementation of a parser generator is called Bison.

3.5.1 Backus-Naur Form of the L-system Grammar

A BNF below is used to describe any possible valid L-system. The Bison program takes a definition similar to this one and creates the parser program. The parser takes in an L-system as input and will process and output the appropriate data structures and information necessary to carry out rewriting.

⟨lSystem⟩ ::= ϵ | ⟨statements⟩ `EOF`

⟨statements⟩ ::= ϵ | ⟨statement⟩⟨statements⟩

⟨statement⟩ ::= `EOL` | ⟨generation⟩ | ⟨definition⟩ | ⟨object⟩ | ⟨axiom⟩ | ⟨production⟩

⟨generation⟩ ::= `#define =` ⟨float⟩`;`

⟨float⟩ ::= `[0-9]+.[0-9]+|[0-9]+`

⟨variable⟩ ::= `[a-zA-Z_][a-zA-Z0-9_]*`

⟨number⟩ ::= ⟨float⟩ | `-` ⟨float⟩

⟨range⟩ ::= {⟨number⟩`,`⟨number⟩}

⟨definition⟩ ::= `#define` ⟨variable⟩ ⟨number⟩`;`

⟨object⟩ ::= `#object` ⟨variable⟩ ⟨variable⟩`;`

⟨module⟩ ::= ⟨variable⟩ | `+` | `-` | `/` | \ | ∧ | `&` | `$` | `[` | `]` | `!`
| `+(`⟨param⟩`,` ⟨paramList⟩`)`
| `-(`⟨param⟩`,` ⟨paramList⟩`)`
| `/(`⟨param⟩`,` ⟨paramList⟩`)`
| \`(`⟨param⟩`,` ⟨paramList⟩`)`
| ∧`(`⟨param⟩`,` ⟨paramList⟩`)`
| `&(`⟨param⟩`,` ⟨paramList⟩`)`
| `$(`⟨param⟩`,` ⟨paramList⟩`)`
| `[(`⟨param⟩`,` ⟨paramList⟩`)`
| `](`⟨param⟩`,` ⟨paramList⟩`)`
| `!(`⟨param⟩`,` ⟨paramList⟩`)`

⟨axiom⟩ ::= `#w :` ⟨axiomStatementList⟩`;`

⟨axiomStatementList⟩ ::= ϵ | ⟨axiomStatement⟩⟨axiomStatementList⟩

⟨axiomStatement⟩ ::= ⟨module⟩

⟨paramList⟩ ::= ϵ | ⟨param⟩⟨paramList⟩

⟨param⟩ ::= ⟨expression⟩

⟨expression⟩ ::= ⟨variable⟩ | ⟨number⟩ | ⟨range⟩
| ⟨expression⟩`+`⟨expression⟩
| ⟨expression⟩`-`⟨expression⟩
| ⟨expression⟩`*`⟨expression⟩
| ⟨expression⟩`/`⟨expression⟩
| ⟨expression⟩∧⟨expression⟩
| `(`⟨expression⟩`)`

⟨production⟩ ::= `#`⟨variable⟩ `:` ⟨predecessor⟩ `:` ⟨condition⟩ `:` ⟨successor⟩`;`

⟨predecessor⟩ ::= ⟨predecessorStatementList⟩

⟨predecessorStatementList⟩ ::= ϵ | ⟨predecessorStatement⟩⟨predecessorStatementList⟩

⟨predecessorStatement⟩ ::= ⟨module⟩

⟨condition⟩ ::= `*`
| ∼⟨float⟩
| ⟨leftExpression⟩⟨operator⟩⟨rightExpression⟩

⟨leftExpression⟩ ::= ⟨expression⟩

⟨rightExpression⟩ ::= ⟨expression⟩

⟨operator⟩ ::= `==` | `!=` | <= | >= | > | <

⟨successor⟩ ::= ⟨successorStatementList⟩

⟨successorStatementList⟩ ::= ϵ | ⟨successorStatement⟩⟨successorStatementList⟩

⟨successorStatement⟩ ::= ⟨module⟩

As seen above in the BNF notation for a L-system, the goal state is <lSystem>. The <lSystem> can be made up of <statements> beginning with the symbol "#" and ending with the symbol ";", or the End of File (EOF) character signifying the end of the L-system. Each non-terminal <statements> is made up of a <statement> followed by more <statements>, or an empty string (ϵ). The <statement> itself can either be an End of Line (EOL) character or a <generation>, <definition>, <object>, <axiom> or <production> statement. The non-terminal symbols <float> and <variable> specify a regular expression. Each statement then has a number of terminal and non-terminal derivatives that allow the production of all valid L-systems that follow this grammar.

In the previous chapter, the scanner defined the syntactic categories. These syntactic categories are all the valid terminal symbols within the L-system grammar. In essence, the parser takes these syntactic categories and finds if they fit the above BNF, and if so, it extracts the information from the L-system and generates the relevant data structures and syntax tree.

3.5.2 Dealing with Constant Values and Objects

Defining constants and objects is essential as it allows the specification of named variables and module names that have a particular meaning. To define a constant or an object is syntactically similar. The keyword define or include is used, then a variable name followed by a value. The value for a constant is a floating-point number, and the value for an *include* is a name of an object within the predefined object library. Seen below is an example of defining a constant and an object:

```
#define num 10;
#define pi 3.1415;

#include F BRANCH;
#include S SPHERE;
```
(3.2)

The definition variables can be stored as a table, called a constants table, which keeps track of all of the constant variable names as well as their values defined by the L-system, as seen in the table below:

Variable Name	Value
num	10.0
pi	3.1415

Table 3.2: Table of turtle instruction symbols and their meaning to the interpreter

The object table structure is very similar to the constants table. The object table holds the module name, and name of the object in the predefined object library. The object table is not used during rewriting, but it is necessary to provide information during the interpretation of the resulting string.

Module Name	Object Name
F	BRANCH
S	SPHERE

Table 3.3: Table of turtle instruction symbols and their meaning to the interpreter

3.5.3 Implementing Modules and Strings

For the rewriter it is important to understand that there are three major parts of a module, there is a module name, which is a string of characters or a symbol, secondly, there is a list of parameters signified by open and close parenthesis, there can be zero or more parameters listed. If there are no parameters for a module you can specify it without parenthesis, however, there should then be a space between the module without parenthesis and the next module. Thirdly, each parameter can either be made up of a number, variable, random number range or a mathematical expression containing numbers, variables and parentheses signifying precedence.

It is important to note that there are two types of modules. One being a module definition and the other a module call. Although these are two different types of modules, they can refer to the same thing. This is because the module definition stands as a template for a module within a production rule. These templates do not have to hold actual values but rather the variable names or random ranges which will be substituted during the rewriting process. Module calls, on the other hand, would appear either in the axiom or in the resultant string, the parameters of a module call will always hold actual numerical values. Below is an example outlining the difference between the module definition and module calls.

$$\begin{aligned} &\#\text{w} : \text{A}(10,\ 20); \\ &\#\text{p1} : \text{A}(\text{x},\ \text{y}) : * : \text{A}(\text{x+y},\ \text{y}); \end{aligned} \tag{3.3}$$

In the example 3.3 above, module A(10, 20) within the axiom, is a module call, as it contains two numerical values of 10 and 20. In the production rule p1, the predecessor is the module A(x, y), this is a module definition, it states that module A's first parameter has a local variable x, and its second parameter has the local variable y. The calling modules values 10 and 20 will substitute x and y respectively, anywhere within the successor statement. The production rule p1's successor has a single module A(x+y, y), this is also a module definition, however, the variables will be substituted with the calling modules values such that it is A(10+20, 20). This can then be evaluated to A(30, 20). After the successor module has been substituted and evaluated, the successors' modules must have a numerical value. They then become module calls within the resultant string ready for the next round of rewriting.

A string in the context of a parametric L-system is a vector of modules, the modules are linked one after the other creating a type of string, but instead of characters or symbols, we have a string of modules.

3.5.4 Implementing Arithmetic Expressions Trees

As stated within the BNF of the L-system grammar an expression is either a variable name, a number or a random range, it is also possible that an expression is part of another expression. Take the example: $5 \times 4 + n$, here there are three expressions 5, 4 and n however, 5×4 is also an expression, as well as $4 + n$. An expression can also be described as any of the aforementioned expressions between a set of parentheses such as $(4 + n)$. The result of the expression is calculated from left to right unless the parenthesis is used which prioritises the encapsulated expression to be calculated first. We can represent this expression as an expression tree in

the diagram below:

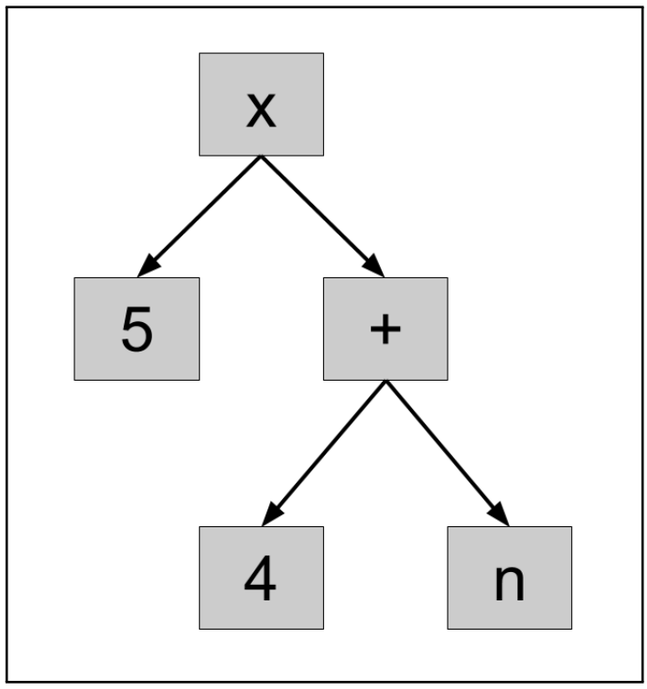


Figure 3.3: Diagram of an expression tree.

The parser provides a syntax tree, which makes it easy to generate the above expression tree, this can have four types of nodes, a variable, number, random range or an operator. The end nodes of the expression tree must be either a number, variable or random range; moreover, a connecting node within the tree must be an operator. We can then traverse the generated tree, and replace the variables with their associated value, and for random ranges, we can generate the random value and assign it to the node. A second traversal during the rewriting process can then computes the result of the expression.

3.5.5 Implementing Random Ranges

L-systems can be quite limited in the amount of variation that can be achieved from rewriting alone. In reality, the variation between the two plants depends on an enormous number of factors. Regardless of the cause, the factors ultimately change the variation mainly within the branching structure as well as a slight variation in the features of the branches themselves. These features include but are not limited to branching angles, branch width, branch length, and branch weight. To introduce variation in the branching structure the rules that are chosen need to be random now and then, which is discussed in section 3.5.6. This section introduces a method of providing variation in the features of branch segments, will be called random ranges.

A random range provides a method of declaring a variable that represents a number that should be randomly generated between two bounding numbers. The bounding numbers are the minimum and a maximum respectively. The main method used for generating a pseudo-random number using a uniform distribution within a range which can be seen below.

Several other types of pseudo-random number generators can be used to generate numbers according to certain distributions; such as normal, binomial, Poisson among others. To generate plant-life a uniform distribution should be sufficient.

```
1: procedure RANDOM RANGE(min, max)
2:     n ← (rand() % (max - min + 1)) + min
3:     return n
4: end procedure
```

A random range can be declared in the define statement, axiom parameter or a production rule successor parameter. If it is declared within a define statement, it will generate a random number when that constant variable is added to the constants table. A random range declared within the axiom will generate a random number before the string rewriting process begins, this ensures that the number. And finally, if a random range is defined within the successor of a production rule, the number should be generated during the rewriting process when the current module within the string is successfully matched to the predecessor at the same time as the expressions within the successors are being evaluated. The values are generated during the rewriting process rather than prior is so that each time a module is matched to the rule, the successor will generate a different value.

3.5.6 Implementing Stochastic Rules

Each rule belonging to a stochastic group of rules provides a probability value of how likely it is that the particular rule is selected during the rewriting process. For production rules to be part of the same stochastic group they are required to meet four criteria:

- The stochastic operator $\sim$ must be used with a probability between 0.0 and 1.0.
- The predecessor module name must match the other predecessor module names within that stochastic group.
- The number of parameters within the predecessor must match the number of parameters of other production rules within that stochastic group.
- The total probability of all of the production rules within the stochastic group must not exceed 1.0 or be less than 0.0.

Each time a rule is added to a stochastic group an entry a stochastic rule table is created to keep track of which rules are associated with which stochastic group as well as the probability of each rule. Using the stochastic rules below, we can generate a stochastic rule table as seen in table 3.4.

$$\begin{aligned} p_1 &: F(x) \;:\; \sim 0.33 \;:\; F(x)[+(r)F(x)]F(x)[-(r)F(x)]F(x) \\ p_2 &: F(x) \;:\; \sim 0.33 \;:\; F(x)[+(r)F(x)]F(x) \\ p_3 &: F(x) \;:\; \sim 0.34 \;:\; F(x)[-(r)F(x)]F(x) \end{aligned} \tag{3.4}$$

Stochastic Group	Rule Name	Probability
F1	p1	0.33
	p2	0.33
	p3	0.34

Table 3.4: Table of the stochastic rules probabilities within a stochastic group.

The stochastic name can be generated by using the module name of the predecessor of the production rule as well as the number of parameters within the predecessor module. In the

example above we can use the predecessor name F which has a single parameter, making the stochastic name F1. This serves as a unique identifier for the stochastic group. Once all of the production rules have been processed and added to the stochastic rule table, each groups' probabilities should be added together and the total should equal 1.0, certain tolerances should put in place to account for floating-point error.

During the rewriting process, the module that is to be rewritten will be matched to a particular stochastic group. A uniformly distributed random number is then generated between 0.0 and 1.0. A range for each rule will then be generated, for instance, p1 will be between 0.0 and 0.33, p2 will be between 0.33 and 0.66 and finally, p3 will be between 0.66 and 1.0. The production rule with the range that the random number falls between is then selected and used for rewriting.

3.6 The String Rewriter

Once the L-system has been processed by both the lexical analyser and the parser, the data structures and information is set up for the string rewriter. The string rewriter is the final stage which uses this data by starting with a current string of modules which is originally set to the axiom string. The string rewriter will then iterate over each module within the current string carrying matching it to the production rules and rewriting the module with the successor if the production rule matches. Once all of the modules have been rewritten, the current string is replaced by the result string for that iteration. This process is carried out for the number of generations specified within the L-system and will eventually provide the resultant string of modules.

When the string rewriter is run it should data for the number of generations, axiom string, production rules, constants table, and local variable table. Below is the pseudocode for the rewriting procedure as well as several useful functions for finding the matching production rule, replacing variables, evaluating expressions and adding variables to the local table.

```
struct node{
  enum Type {VARIABLE, OPERATOR, NUMBER, RANGE} type;
  union{
        string *variable;
        string *operator;
        float number;
        float range[3];
  };
  node *left;
  node *right;
};
```

```
struct condition{
    enum Type {EQUAL_TO, NOT_EQUAL_TO, LESS_THAN, GREATER_THAN,
        LESS_EQUAL, GREATER_EQUAL, STOCHASTIC, NO_CONDITION} type;
    node * leftExp;
    node * rightExp;

    float stochasticValue;
};
```

```
struct module{
  string name;
  int numParam;
  enum Type {CALL, DEFINITION} type;
  string object;
  vector<struct node*> params;
};
```

```
struct production{
    string name;
    module *predecessor;
    condition *condition;
    vector<module*> successor;
};
```

```
1: procedure REWRITER(N, A)

Ensure: N > 0                                   ▷ The number of generations to rewrite
Ensure: A ≠ empty                               ▷ A non empty Axiom, a list of modules

2:     n ← 0
3:     current ← A                                              ▷ Current string of modules
4:     while n < N do                                           ▷ For each generation
5:         next ← empty list
6:         for each mod in current do                  ▷ call is the calling module in current
7:             P ← FindProductionMatch(mod)            ▷ P is the matching production rule
8:             if P ≠ NULL then
9:                 pred ← P.predecessor                ▷ def is the defining module in predecessor
10:                for each succ in P.successor do
11:                    index ← 0
12:                    while index < number of predecessor parameters do
13:                        AddLocalVar(pred.param[index], mod.param[index])
14:                        index ← index + 1
15:                    end while
16:                    copy ← succ                                  ▷ Create a deep copy
17:                    for each parameter in copy do        ▷ parameter is an expression tree
18:                        ReplaceVariables(parameter)
19:                        EvaluateExpression(parameter)
20:                    end for
21:                    next ← next + copy
22:                end for
23:            else
24:                next ← next + mod
25:            end if
26:        end for
27:        n ← n + 1
28:        current ← next
29:    end while
30:    return current
31: end procedure
```

```
1: function FindProductionMatch(Module)
2:     for each P in productionTable do                          ▷ P is a production
3:         predecessor ← P.predecessor                  ▷ predecessor is a single module
4:         if predecessor.name ≠ Module.name then
5:             continue
6:         end if
7:         if predecessor.numParam ≠ Module.numParam then
8:             continue
9:         end if
10:        if P has no condition then
11:            return P.name                                          ▷ match found
12:        else if P has a stochastic condition then
13:            rand ← random float between 0.0 and 1.0
14:            total ← 0.0
15:            S ← list of pairs              ▷ pair(production name, probability value)
16:            for each s in S do          ▷ Loop through each tuple in the stochasic list
17:                if first item then
18:                    if rand ≥ 0.0 AND rand < s.value then
19:                        return s.name
20:                    end if
21:                else if last item then
22:                    if rand ≥ total AND rand ≤ 1.0 then
23:                        return s.name
24:                    end if
25:                else
26:                    if rand ≥ total AND rand < total + s.value then
27:                        return s.name
28:                    end if
29:                end if
30:                total ← total + s.value
31:            end for
32:        else                                                  ▷ Regular condition
33:            left ← P.condition.left                ▷ Deep copy left expression tree
34:            right ← P.condition.right              ▷ Deep copy right expression tree
35:            replaceVariables(left)
36:            replaceVariables(right)
37:            evaluateExpression(left)
38:            evaluateExpression(right)
39:            if left P.condition.op right then    ▷ Apply operator (==, ≠, <, >, ≤, ≥)
40:                return P.name
41:            end if
42:        end if
43:    end for
44: end function
```

```
function EvaluateExpression(TreeNode)  ▷ Recursively evaluate the expression tree
    left ← 0.0
    right ← 0.0
    if TreeNode.left == NULL OR TreeNode.right == NULL then
        return TreeNode.value
    end if
    left ← ReplaceVariables(TreeNode.left)
    right ← ReplaceVariables(TreeNode.right)
    if TreeNode.type is an operator then
        return left TreeNode.operator right    ▷ Apply arithmetic operator (+, -, *, /, ∧)
    end if
end function

function ReplaceVariables(TreeNode) ▷ Recursively replace expression tree variables
    if TreeNode == NULL then
        return
    end if
    if TreeNode.type is a variable then
        if TreeNode.value is in constants table then
            TreeNode.value ← numeric value in constants table
        end if
        if TreeNode.value is in local table then
            TreeNode.value ← numeric value in local table
        end if
    end if
    ReplaceVariables(TreeNode.left)
    ReplaceVariables(TreeNode.right)
end function

function AddLocalVar(TreeNodeCall, TreeNodeDef)
    if TreeNodeCall child nodes == NULL OR TreeNodeDef child nodes == NULL then
        if TreeNodeCall.type == Number AND TreeNodeDef.type == Variable then
            Add variable name and value to local table
        else if TreeNodeCall.type == Range AND TreeNodeDef.type == Variable then
            Add variable and generated random range value to local table
        end if
    end if
end function
```

3.7 Summary

The L-system rewriter is one of two major systems within the process or procedurally generating plant-life. The rewriter defined in this thesis acts as a type of compiler similar to that of a computer language. In a sense, the L-system itself becomes a language, that the L-system rewriter can understand and generate meaningful data structures with. The L-system rewriter follows the grammatical structure of the language very closely. Due to the lexer and the parser, it can give informative messages if there is a mistake either grammatically or syntactically. If all of these requirements are met, the rewriter can use this data to very quickly generate the resultant string of modules. The person writing that L-system, can know indefinitely that the result provided by the rewriting system is correct according to the grammar of the L-system.

The L-system languages can be used for many different applications and are not limited to that of procedural plant generation. The interpretation of the resultant string is really what creates the physical result such as the plant model. This means that this L-system rewriter need not change if the L-system is used for a different purpose, only the interpretation will need to change. This is the main reason behind using a compiler-like process to govern the string rewriting. It allows the L-system enough complexity to provide information to the interpreter but not too much that interpretation becomes reliant on the string rewriter.

Chapter 4

Mathematics For 3D Graphics

In any 3D application, mathematical models are used to represent the positions, rotations and scale of objects within a given scene. It is important for the purpose of this thesis to briefly touch on some of the core concepts, particularly with regards to representing the position and rotation of 3D objects. All objects within a 3D application are represented by a set of vertices or points, which can be represented with X, Y and Z coordinates. Three vertices can make up one triangle also called a face, multiple faces will then make up a whole 3D object. The use of mathematical methods in 3D graphics is to be able to manipulate all vertices of an object in a consistant way, thus rotating, translating or scaling the object within the scene. This section will provide sufficient background on some of most important concepts of 3D Mathematics, such as vectors, matrices and quaternions, which are widely used in the turtle graphics interpreter as well as the model generator.

4.1 Vectors

Vectors have many meanings in different contexts, in 3D computer graphics, often vectors are refering to the Euclidean vector. The Euclidean vector is a quantity in n-dimensional space that has both magnitude (the length from A to B) and direction (the direction to get from A to B). Vectors can be represented as a line segment pointing in a direction, with a certain length. A 3D vector can be written as a triple of scalar values eg: (x, y, z)

The most common operations on vectors are multiplication by a scalar, addition, subtraction, normalisation and the dot and cross product. The multiplication by a scalar value can be simply seen as scaling the magnitude of the vector itself, this can be done uniformly or non-uniformly as seen in the equation below:

$$a \otimes s = (a_x s_x, a_y s_y, a_z s_z) \tag{4.1}$$

Where $\otimes$ is the component-wise product of a vector a and the scaling vector s. Similar to the scalar product of a vector the addition and subtraction of two vectors is the component-wise sum or difference.

$$\begin{aligned} a \oplus b &= [(a_x + b_x), (a_y + b_y), (a_z + b_z)] \\ a \ominus b &= [(a_x - b_x), (a_y - b_y), (a_z - b_z)] \end{aligned} \tag{4.2}$$

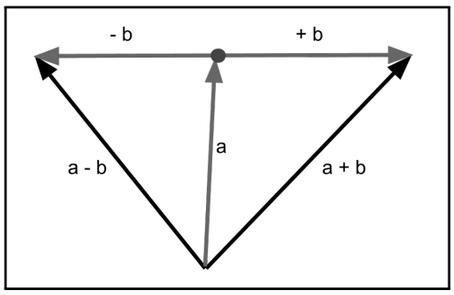


Figure 4.1: Table of common dot product tests between two vectors.

A useful type of vector is known as a unit vector. This is a type of vector which has a magnitude of 1. Unit vectors are used extensively in computer graphics particularly with ragards to shaders. Take the vector v its magnitude α can be calculated by taking the square root of the product its components squared, as seen below

$$\alpha = \quad | \mathbf{v} | \quad = \sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2 + \mathbf{v}_z^2} \tag{4.3}$$

The unit vector can then be calculated by taking the product of v and the reciprocal of its magnitude shown in the following equation.

$$\upsilon = \frac{\mathbf{v}}{\alpha} = \frac{1}{\alpha}\mathbf{v} \tag{4.4}$$

There are many different ways to multiply vectors, however, in 3D graphics there two main multiplications. These being the dot and cross product. The dot product yields a scalar by adding the products of the vector product components. The cross product on the other hand is the product of two vectors which gives a vector which is perpendicular. The dot product can be calculated using the formula below:

$$a \cdot b = a_x b_x + a_y b_y + a_z b_z = d \tag{4.5}$$

Some of the main uses for dot products within 3D graphics is to find whether two vectors are collinear, perpendicular, in the same direction or opposite directions. One possible use for this is to find the dot product of two branches directions in order to find out if they growing in the same direct or in opposite directions. In the table 4.1 below, there are each of the dot product test diagrams as well as the test equation where $ab =\mid a \mid\mid b \mid= a \cdot b$.

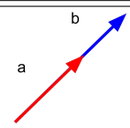
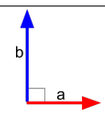
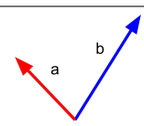
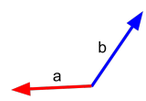
Test	Equation	Example
Collinear	$(a \cdot b) = ab$	
Opposite Collinear	$(a \cdot b) = -ab$	
Perpendicular	$(a \cdot b) = 0$	
Same Direction	$(a \cdot b) > 0$	
Opposite Direction	$(a \cdot b) < 0$	

Table 4.1: Table of turtle instruction symbols and their meaning to the interpreter

The cross product also known as the outer product takes two vectors and finds the perpendicular vector of the two vectors, this is only possible in 3D space and can be expressed in the following formula using the left-hand rule:

$$a \times b = [(a_y b_z - a_z b_y), (a_z b_x - a_x b_z), (a_x b_y - a_y b_x)] \tag{4.6}$$

The result of a cross product can be seen in figure 4.1 below. Where vectors a and b give the perpendicular vector $a \times b$. The cross product is very useful within physics calculations when it is necessary to find the rotational motion.

Some of the properties of the cross product are as follows:

- is non-commutative, meaning order matters($a \times b \neq b \times a$).
- is anti-commutative ($a \times b = -(a \times b)$).
- is distributive with addition ($a \times (b + c) = (a \times b) + (a \times c)$).

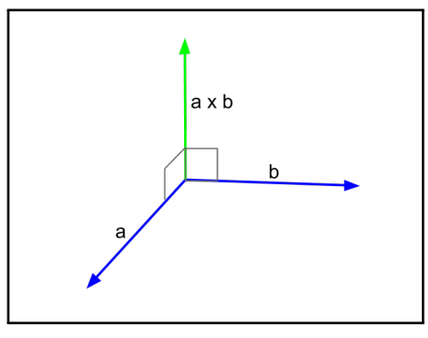


Figure 4.2: Diagram of the cross product of two vectors a and b.

4.2 Matrices

A model in 3D space will exist as a set of vertices which each have a position. Moving the model requires moving all of the the vertices of that model without distorting it in any way, this is called a model transform. There are four main types of transforms; translation, rotation, scale and shear. Matrices are a single matematical construct which is capable of carrying out all four of these transformations. This sections will only cover the first three as the shear transformation is likely not going to be useful for this thesis.

A matrix is an 2D array of numbers, arranged into rows and columns, which can come in many different sizes. In 3D graphics, matrices used for transformations are the 3 × 3 and 4 × 4 matrix as seen below. A 3 × 3 matrix can be used for linear transorms such as scaling and rotation, furthermore, a linear transform which contains translation is known as an affine transform and can be represented by a 4 × 4 matrix known as an Atomic Transform Matrix. An atomic Transfom matrix is the concatination of four 4 × 4 matrices, one for translations, rotations, scale and shear transforms. Resulting in a 4 × 4 matrix as shown below.

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \tag{4.7}$$

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \tag{4.8}$$

The affine matrix can be shown in the expression below where RS is a 3 × 3 matrix containing the rotation and scale where the 4^th elements are 0. The T elements represent the translation with the 4th element being 1.

$$\mathbf{M} = \begin{bmatrix} RS_{11} & RS_{12} & RS_{13} & 0 \\ RS_{21} & RS_{22} & RS_{23} & 0 \\ RS_{31} & RS_{32} & RS_{33} & 0 \\ T_1 & T_2 & T_3 & 1 \end{bmatrix} \tag{4.9}$$

The product of two linear transform matrices will be another linear transform matrix that carries out both of those tranformations. This is true for the multiplication of two affine transform matrices as well, and is why matrix multiplication is so powerful in 3D graphics. Take the two matrices A and B which give the product P. In order to multiply A and B together, the dot product of the row and the column is calculated as seen below. It is also imporant to know that matrix multiplication is non-commutative ($AB \neq BA$).

$$\mathbf{AB} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} = \begin{bmatrix} (A_{row1} \cdot B_{col1}) & (A_{row1} \cdot B_{col2}) & (A_{row1} \cdot B_{col3}) \\ (A_{row2} \cdot B_{col1}) & (A_{row2} \cdot B_{col2}) & (A_{row2} \cdot B_{col3}) \\ (A_{row3} \cdot B_{col1}) & (A_{row3} \cdot B_{col2}) & (A_{row3} \cdot B_{col3}) \end{bmatrix} \tag{4.10}$$

To translate a vertex in 3D space without causing any distortion. The vertex can be added the the matrix below as follows. These translations can be carried out on all vertices in order to translate a whole object model.

$$V + T = \begin{bmatrix} V_x \\ V_y \\ V_z \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix} = \begin{bmatrix} (V_x + T_x) \\ (V_y + T_y) \\ (V_z + T_z) \\ 1 \end{bmatrix} \tag{4.11}$$

In order to rotate a vertex in 3D space the vertex position and the rotation angle can be applied to the as a matrix depending on the axis about which it is rotating. These rotation matrices can be applied to the vertex itself in order to gain the new position of the vertex.

$$R_x(\theta) = \begin{bmatrix} (v_x) \\ (v_y) \\ (v_z) \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.12}$$

$$R_y(\theta) = \begin{bmatrix} (v_x) \\ (v_y) \\ (v_z) \\ 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.13}$$

$$R_z(\theta) = \begin{bmatrix} (v_x) \\ (v_y) \\ (v_z) \\ 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.14}$$

$$ST = \begin{bmatrix} S_x \\ S_y \\ S_z \\ 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (S_x R_x) \\ (S_y R_y) \\ (S_z R_z) \\ 1 \end{bmatrix} \tag{4.15}$$

4.3 Quaternions

In computer graphics there are a number of ways to represent 3D rotations. One method is to use matrix affine transforms, which is spoken about in the previous section. Matrices are a common way of represening rotation, however, it has a number of limitations. Matrices are represented by nine floating point values and can be computationally expensive to store and process, particularly when doing a vector to matrix multiplication. There are also situations where it is neccessary to smoothly interpolate from one rotation to another or to find the rotation somewhere between two different rotations. It is possible to make these calculations using matrices but it can become very complicated and even more computationally expensive. Quaterions are the answer to these challenges.

Quaternions look similar to a 4D vector. They contain four axes $q = [q_x, q_y, q_z, q_w]$, these are represented with a real axis (q_w) and three imaginary axes (q_x, q_y, q_z). A quaternion can

be represented in the complex form below:

$$q = (iq_x + jq_y + kq_z + qw) \tag{4.16}$$

For the purpose of this thesis it is not important to understand the derivation of quaterions in mathematics. However it is important to understand that any quaternion which obays the rule in 4.17 below is known as a unit quaternion.

$$q_x^2 + q_y^2 + q_z^2 + q_w^2 = 1 \tag{4.17}$$

Unit quaternions can be used for rotations, it is possible to convert a quaternion to a unit quaternion by taking the angle and the axis of a rotation and applying to the quaternion as seen in 4.18.

$$\begin{aligned} &q = [q_x, q_y, q_z, q_w] \\ &\text{where} \\ &q_x = a_x sin\frac{\theta}{2} \\ &q_y = a_y sin\frac{\theta}{2} \\ &q_z = a_z sin\frac{\theta}{2} \\ &q_w = cos\frac{\theta}{2} \end{aligned} \tag{4.18}$$

The scalar part (q_w) is the cosine of the half angle, and the vector part ($q_x q_y q_z$) is the axis of that rotation, scaled by sine of the half angle of rotation. The unit quaternion can be used for rotations in a number of ways. The most useful of which is to rotate vectors, interpolate between two rotations and concatonate rotations together similar to how matrix transformations can be multiplied.

The first operation for quaternions is that of addition. The addition of two quaternions is quite simple, it involves taking each component of each quaternion and adding them together. This is similar to that of matrices addition and can be expressed as follows:

$$p + q = [(p_w + q_w), (p_x + q_x), (p_y + q_y), (p_z + q_z)] \tag{4.19}$$

Multiplication of quaternions is also incredibly powerful and can be used to concatonate rotations. There are a number of different types of quaternion multiplication, however, the one most commonly used for quaternion rotaton is called the grassmann product. This can be described in the following formula below. Where p and q are quaternions and the subscript w indicates the scalar part and subscript x, y, z indicate the vector components of each quaternion.

$$
\begin{aligned}
&R = r_w + r_x + r_y + r_z \\
&\text{where} \\
&r_w = p_w q_w - (p_x q_x + p_y q_y + p_z q_z) \\
&r_x = p_w q_x + p_x q_w + p_y q_z - p_z q_y \\
&r_y = p_w q_y + p_y q_w - p_x q_z + p_z q_x \\
&r_z = p_w q_z + p_z q_w + p_x q_y - p_y q_x
\end{aligned}
\tag{4.20}
$$

To rotate a vector by a unit quaternion the vector will need to be converted into its quaternion form. This requires taking the unit vector v and using it as the vector part of the quaternion with a scalar part being equal to zero. This can be written as $Q_v = [v, 0] = [v_x, v_y, v_z, 0]$. The grassmann product can be used to calculate the rotation, by taking the product of the rotation quaternion q and the vector form quaternion v and the inverse of the rotation quaternion q^-1.

$$
V_q = qvq^{-1} \tag{4.21}
$$

For unit quaternions the conjugate and the inverse are identical. Quite simply the inverse of a unit quaternion can be calculated by negating the vector components of the quaternion whilst leaving the scalar component the same. This can be expressed as follows:

$$
q^{-1} = [-q_v, q_s] \tag{4.22}
$$

Quaternion rotations can be concatonated together similar to how matrix transforms can be multiplied together in affine transforms. The grassman product is noncommutative and therefore order matters. Using the grassmann product the rotations can easily be multiplied together to give the result of all of those rotations as if they were to happen one after the other. This can be expressed as follows:

$$
\begin{aligned}
Q_{net} &= Q_3 Q_2 Q_1 \\
v' &= Q_3 Q_2 Q_1 \; v \; Q_1^{-1} Q_2^{-1} Q_3^{-1}
\end{aligned}
\tag{4.23}
$$

The order by which the quaternions Q_1, Q_2 and Q_3 are applied is Q_3 then Q_2 and then Q_1. To apply this to a vector the product of the three quaternions is multiplied to the vector and then multiplied to the product of the inverse of each quaternion.

Another incredibly useful mathematical function is called rotational linear interpolation also known as LERP. The LERP function takes two quaternions, Q_1 and Q_2 and linearly interperpolates between those two rotations by a given percentage β. The LERP function can be defined as follows.

$$
\begin{aligned}
Q_{\text{LERP}} = \text{LERP}(Q_1, Q_2, \beta) &= \frac{(1-\beta)Q_1 + \beta Q_2}{\mid (1-\beta)Q_1 + \beta Q_2 \mid} \\
&= \text{normalize} \begin{pmatrix} \begin{bmatrix} (1-\beta)Q_{1x} + \beta Q_{2x} \\ (1-\beta)Q_{1y} + \beta Q_{2y} \\ (1-\beta)Q_{1z} + \beta Q_{2z} \\ (1-\beta)Q_{1w} + \beta Q_{2w} \end{bmatrix} \end{pmatrix}
\end{aligned}
\tag{4.24}
$$

Using the linear interpolation function will result in a rotation between Q_1 and Q_2 at a given percentage β that is between 0 and 1. Where 0 is the rotation of Q_1 and 1 is rotation of Q_2.

4.4 summary

This chapter covers the three major mathematical concepts used for representing 3D objects location, rotation and scale within a 3D graphics application. This includes moving objects around a scene for the purposes of animation or simulation. It is important to understand these concepts when implementing the string interpreter in order to be able to manipulate the branches or other objects. These concepts are also useful in the implementation of the physics simulations. The OpenGL Mathematics Library (GLM) library provides a large number of useful classes and functions for working with vertices, matrices and quaternions.

Chapter 5

L-system String Interpreter Implementation

The string interpreter is one of the major components of plant generation and it is the final step in the process of procedural generation. The output of this stage of processing is dependant on what the L-system is representing, in this case it is responsible for interpreting the resulting string of modules provided by the L-system rewriter, and uses this to generate the 3D models, structures and data of the resulting plant, which is then rendered and simulated on the screen using the OpenGL framework. The generation of plant-life has three main stages, the first part consists of a turtle graphics interpreter which takes the string of modules as a set of instructions, it starts from the root of the tree and generates a skeleton made up of joints, this is similar to the techniques used in skeletal rigging in animation [Gregory, 2014]. The joints within the tree skeleton each represents a branch segment which has some information about the properties of that segment. These segements can be used to generate the vertex, index and other data that make up the 3D models of the plant. These models can finally be passed to the final part of string interpreter which is the renderer, the renderer is reponsible for taking all the vertices, indices, textures and shaders and organising it in an optimal way that enables rendering the plant on the screen, as well as the physical simulation of the tree skeleton. The stages of string interpretation can be seen in figure 5.1 below.

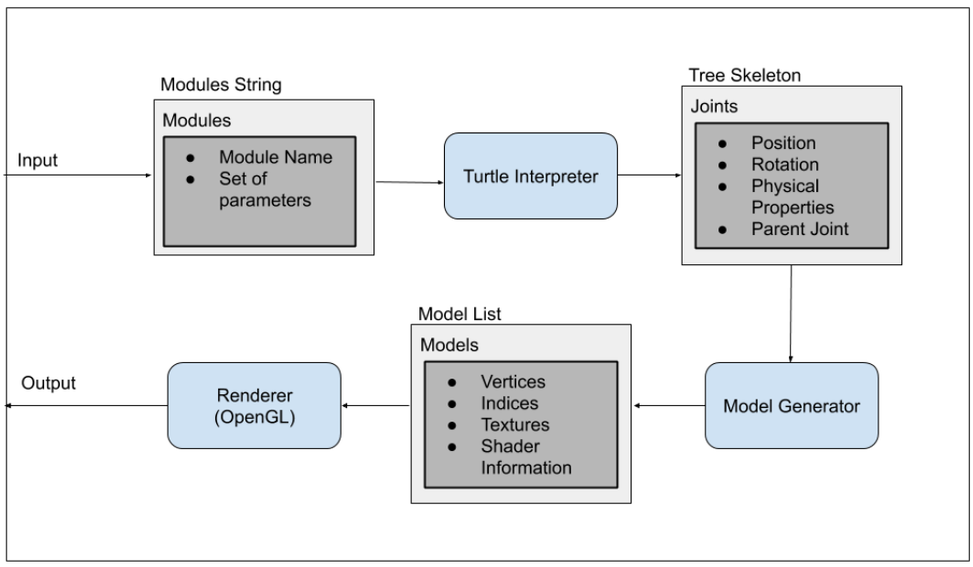


Figure 5.1: Diagram of the stages of L-system interpretation and rendering

This chapter will cover each stage of the string interpreter implementation in detail, as well as well as talk about how the interpreter is able to simulate and animation the plants movements under forces such as gravity and wind in real time.

5.1 Turtle Graphics Interpreter

The main purpose of the turtle graphics interpreter is to take the string of modules from the L-system rewriter, and interpret it as a list of turtle graphics instructions. As briefly covered in chapter 2, each module name within the L-systems resultant string represents a particular meaning to the turtle graphics interpreter. The meaning of the module names are predefined in the string interpreter and are dependant on what the L-system is trying to represent. The L-system defined for this thesis is a parametric L-system, which allows each module to also provide a number of optional parameters. These may also carry a particular meaning for the interpreter. For instance the forward instruction or module name "F" can have three parameters. The value of the first parameter is the distance to move forward, this can also be seen as the length of the branch segment. The second and third parameter is the spring constant of the branch and the mass of the branch repectively. These give some information to the physics simulation in order to animate the plant. Below is a table describing the L-system module names as well as the parameter meanings for the turtle graphics interpreter.

Instruction Name	Parameter 1	Parameter 2	Parameter 3
F	Distance	Spring Constant	Branch Mass
f	Distance	Spring Constant	Branch Mass
+	Angle of rotation		
-	Angle of rotation		
/	Angle of rotation		
\	Angle of rotation		
∧	Angle of rotation		
&	Angle of rotation		
!	Branch width		

Table 5.1: Table of turtle instruction symbols and their meaning to the interpreter

Each modules instruction is carried out one by one to generate the plants skeletal structure,

which is made up of joints. The joints hold information about the properties of each particular segement or object of the plant. The joints properties are the position, orientation, scale, parent joint as well as its physical characteristics. It is important to note that all of the scales and rotations must happen before the forward movement. As the rotations change the orientation of the brach and then the movement generates the joint itself. A joint is defined by the figure below:

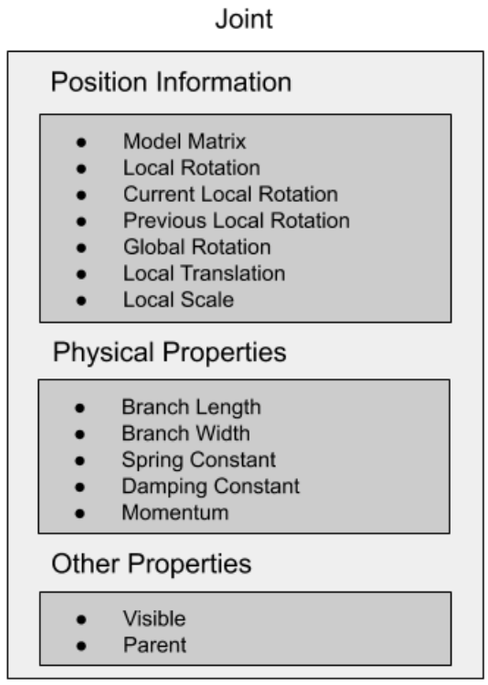


Figure 5.2: Diagram for the properties of a joint

Figure 5.2 shows that there is in fact a large amount of information stored for the position and orientation of each joint. This is because the rotation of the joint is stored in both a local and global space. Local space refers to the rotation of the joint relative to its parent rotation, this is useful as it allows the manipulation of subsequent child joints, whilst leaving other joints local rotation unchanged. Global space, also known as world space, is the rotation of each joint relative to the world itself this is useful for understanding the current rotation of the joint relative to the world for instance calculating the torque or force calculations due to gravity. It is important to store both the current and previous rotations as they are used to calculate the rate of change for physics calculations.

The physical properties for each joint are the parts are what will affect model generation as well as physics simulations. These properties include the length, width, spring constant, damping constant as well as the current momentum of the branch.

Take the following string of modules "F(1)[/(90)F(1)\ (90)F(1)]-(90)F(1)+(90)F(1)", the alphabet is made up of seven unique modules F, /, \, [,], + and -. According to the as discussed in previous chapters the "F" symbol represents a move forward, and "+", "-", "/", "\" symbolize different rotations, and the "[" and "]" represent save and load state respecively. The aforementioned symbols each have a single parameter except the load and save state. It is the turtle graphics interpreters job to understand what these parameters are and how to interpret them. In this case all of the "F" modules have the parameter value of 1, and all of the rotation modules have the parameter of 90. These are interpreted as the distance to move

forward and the change in angle from the previous joint in degrees. This interpretation can be represented with the joint structure shown in figure 5.3 below:

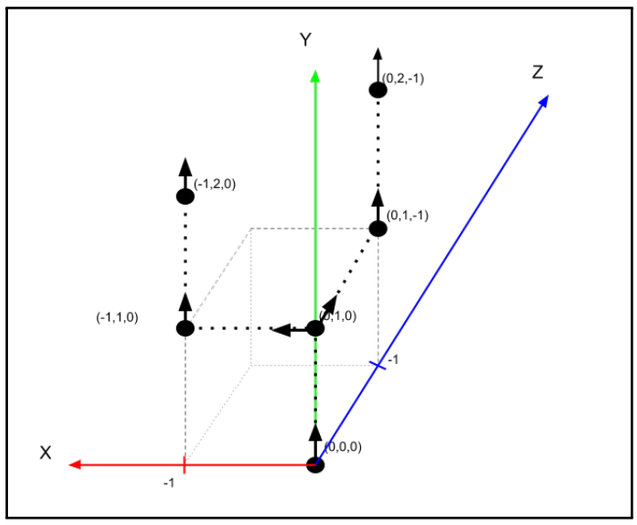


Figure 5.3: Diagram of a simple plant skeleton with joint position and orientation.

5.2 Model Generator

Modeling the branches of a plant is one of the most important parts for the overall look and feel of that plant that is being generated. The L-system described in the previous sections is able to describe the details about the plants structure, for instance the position, width, length, weight and other important information. The job of the model generator is to take this information and intelligently generate the models vertices, normals, texture coordinates and other information that can then be provided to the OpenGL renderer and finally to the GPU to be rendered on the screen.

The simplest way to generate a model for a branching structure of a plant would be to take a number of cylinders, and to rotate and stack them according to each joints position in 3D space. The up side to this approach is that every branch within the plant shares the same object model, depending on the position, rotation and scale of the branch the relavent matrix transforms can be applied. In this way we are able to represent the overall branching structure of the plant. However, there is a problem which is pointed out by Baele and Warzée "The branches junction causes a continuity problem: to simply stack up cylinders generates a gap" [Baele and Warzee, 2005]. This can be shown in the figure below:

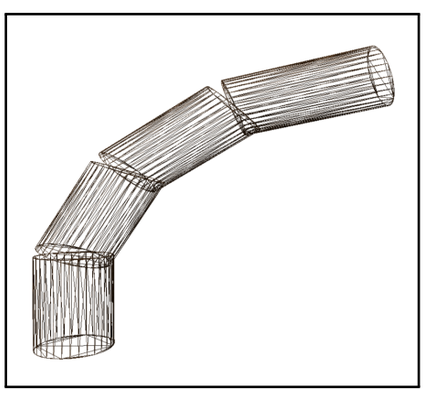


Figure 5.4: Example of the continuity problem faced with stacked branching with a 25° bend per joint.

This simple method of stacking cylinders gives a reasonable looking tree structure and it is usually good enough when the angles of branches are not more than 25° and the size of the branches do not change. However for a much more convincing tree structure there will need to be a better solution. The logical next step would be to actively link the branch segments together. This requires a number of things to take place, first of all the vertices from the previous branch top must be linked with the new top of the branch. These are the circles of vertices at either end of each branch segment. These circles will have to rotate depending on the bending direction of the branch. This means that the final model will not be made up of a large number of the same model but rather a single model with many linked branches. An example of this can be seen below:

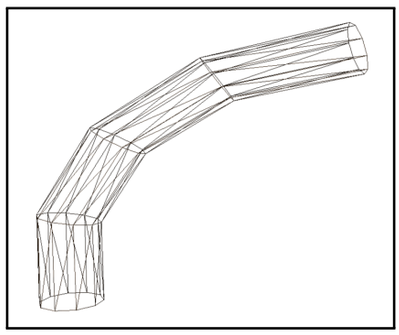


Figure 5.5: Example of linked branching with a 25° bend per joint.

This method of branch generation gives a very similar result at first glance to that of stacking cylinders. Although it does have a number of advantages, firstly it completely avoids the branch gap problem that happens with angle changes as well as branch size changes. It also means that the resolution is dynamic, meaning the number of vertices that make up a cylinder can be dynamically changed. This means that a very high resolution tree can be rendered which may look very smooth but will take a lot more computational resources, or a very low resolution tree can be rendered with more jagged edges but will require a lot less computational resources. This can be seen in figure 5.2 below, where similar a looking branch can be acheived using less than half the number of vertices, with joined branches instead of stacked branches.

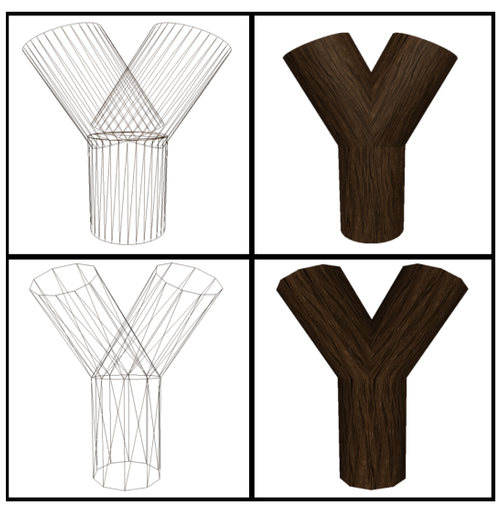


Figure 5.6: Stacked Vs Linked.

5.3 Renderer

The renderer is the final stage in the procedural generation pipeline. It takes all of the 3D models generated by the model generator, such as leaves, branches and flowers and renders them on the screen. For this thesis, the Open Graphics Library (OpenGL) application programming interface is use used to efficiently render the models on the screen using the Graphics Processing Unit (GPU).

The GPU is a specially designed piece of harware for processing computer graphics and image processing, it has hundereds or even thousands of individual compute cores which can be used in parallel. Due to the highly parallel nature of the GPU, the OpenGL framework helps to abstract the hardware and create an interface to interact with the GPU in a simpler way. There are a number of other types of graphics API such as Vulkan, Metal or DirectX. These APIs all provide a way of interacting with the hardware behind the scenes, However, they each have a different approach. Therefore, this section will not be be going into great detail about about the specifics of OpenGL but rather the general concepts required for rendering the plant model on the screen. The main parts of the rendering stage has to do with how model data is stored into buffer objects, secondly how textures are stored and then mapped to a certain object and thirdly how lighting can be calculated for a procedurally generated object.

5.3.1 Models and Buffer Objects

The model generator produces all of the information neccessary for the renderer to produce the result on the screen. In general the model data will consist of vertex data, texture co-ordinates and vertex normals. The vertex data is simply position of a point within a model, three vertices make up a face and the faces are what are ultimately rendered on the screen. The texture co-ordinates are the locations on a texture image which maps directly to the model vertices. Finally the vertex normals simply known as normals are the average normal vector. A normal vector being the vector that is purpendicular to the surface at a given point, and can be used for Phong shading or other types of lighting techniques.

One of the most important parts of the rendering process is buffering the model data onto the GPU. The Vertex Buffer Object (VBO) is a data structure within the OpenGL library which can be used to store this data on the GPU. Generally, the data is stored as a single buffer or array with the first 3 values being a vertex position, the second two being a texture co-ordinate and the last three being a vertex normal.

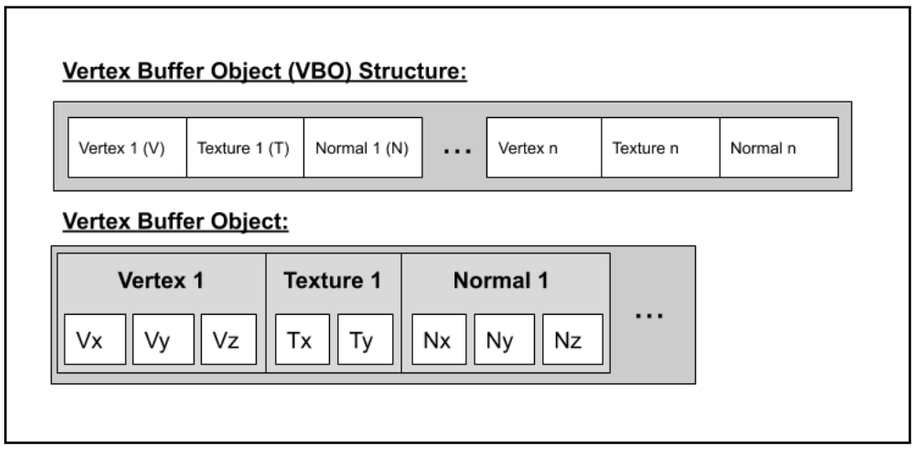


Figure 5.7: Diagram showing the structure of a vertex buffer object.

Buffer objects can be created not only for the plant as a whole but potentially for different parts of the plant. For instance, the leaves on the tree are all quite similar. There is no need to have thousands of copies of a leafs vertices and textures. This would be highly wasteful and unnecessary. Simply, there could be one copy of the vertex data and texture data and instanced renderering can be used to render many copies of this object in at different places in one scene.

5.4 Shaders

5.5 Summary

Chapter 6

Physics Simulation

The motion of plants is an important factor when looking to create realistic looking plant-life. It has been a topic of discussion and research for many years now, particularly with regards to grass, bushes and trees within video games. The movement is usually very subtle, but if it is missing, a scene can start looking very unnatural, making the user feel uncomfortable. This chapter will discuss a method of simulating the physical motion of plant-life, layed out by Barron et al [Barron et al., 2001]. This method will be built into the parametric L-system itself in such a way that the L-system can provide the physical parameters for the simulation. This will allow a physics simulation to be run on any plant generated by the L-system.

The main technique discussed by Barron et al for simulating the motion of a system like a tree or plant, is taken from that of a particle system, first described by Reeves [Reeves, 1983]. Particle systems can be applied to simulate phenomena like clouds, smoke, water and fire. The main advantage of particle systems is that the motion for each particle can be updated simaltaniously. This technique can be applied to the L-system representation of plant-life. Where branches are split into segments that make up a skeleton of segments or joints. Each joint can represent a "particle" within the system, which has a dependency on all of its parent branches.

Using the particle system concept, the motion of the plant can simulated by having each joint within the plant skeleton to be seen as a particular segment of a branch with some basic physical properties. These properties include but are not limited to the width, length, direction vector, spring consistant and dampening constant. The direction vector is the global direction of the branch in 3D space pointing in the direction that the branch itself is pointing. The spring constant and the dampening constant are used for Hook's Law. The spring force of the branch tries to prevent it from bending. Whereas gravity, wind and other forces cause torque, which generally acts against this spring force, causing the branch to bend.

6.1 Branch Physical Properties

The mass of each branch segment can be simply calculated by taking the volume of each branch and multiplying it by the density of the wood or material. To do this the volume of each branch needs to be calculated. This can be done by multiplying π by the radius r squared and the length l as sees below.

$$v = \pi r^2 l \tag{6.1}$$

This is not always the case, particularly if the branch segment is decreasing in size, however it gives a good inidication as to volume. This can now be used to calculate the mass, this requires knowing the density of the material that the plant is made of. For instance the density of pine wood is between 400 - 420 kg/m^3. Some woods being less dense at about 200 kg/m^3, and other hard wood being up to about 1000kg/m^3.

$$m = v \times d \tag{6.2}$$

The mass can be used to calculate the branch segments moment of inertia, this being the branches resistance to angular momentum. As the object is 3D the shape of the object needs to be taken into account. Each branch can be simply seen as a long thin cylinder, which can be expressed in the following equation.

$$I = \frac{1}{3} m l^2 \tag{6.3}$$

Where I is the inertia of the branch, m is the mass of and l is the lenght. Similarly an inertia tensor can be used for the sake of convenience and to better describe the objects rotational inertia which is used within vector and matrix calculations. The intertia will be used when calculating the velocity of each segment in section 6.3. Below is an inertia tensor for a shape that is similar to that of a branch segment.

$$I = \begin{bmatrix} \frac{1}{12} m(3r^2 + l^2) & 0 & 0 \\ 0 & \frac{1}{12} m(3r^2 + l^2) & 0 \\ 0 & 0 & \frac{1}{2} m r^2 \end{bmatrix} \tag{6.4}$$

The forward vector of the branch, this being the vector in the direction that the branch is pointing towards, can be used to calcuate the direction the torque is acting on the branch V, by taking the cross product of the forward vector v and the force vector w. This can be visualised using the right hand rule, where the index finger is the forward vector and the middle finger is the force vector. The direction of the thumb then points in the direction of the torque. The angular velocity is produced as spin in the direction around the torque vector.

$$V = v \otimes w \tag{6.5}$$

The dispacement can be calculated by keeping track of the starting local rotation of the branch p as well as the current rotation of the branch q in the form of two quaternions. We can then calculate a quaternion d for the differece of the two quaternions, by taking p and multiplying it by the inverse of q.

$$d = p \times q^{-1} \tag{6.6}$$

6.2 Hook's Law

Hook's law is a law of physics that states that the resultant force from compressing or extending a spring is equal to the product of the spring constant and the displacement of the spring. Each branch in a plant structure can be seen as a type of semi-rigid spring where external forces like gravity or wind bend the spring. Hook's law is used to then calculate the reaction force due to the displacement of the spring.

$$f = -k_s d + k_d v \tag{6.7}$$

Where f is the force exerted by the spring, k_s is the spring constant and x is the total displacement of the spring. The dampening force can be calculated as $k_d v$ part where k_d is the dampening constant and v is the velocity at the end of the spring.

6.3 Equations of Motion

The forces can then be multiplied together to get the net force f_net acting on the spring, this can be used to calculate the momentum and furthermore the velocity of the the branch. T_{delta} is that change in time between physics calculations.

$$M = M_0 + f_{net} * T_{delta} \tag{6.8}$$

The velocity v can be calculated by taking the inverse of the inertia tensor I and multiplying that by the momentum vector M.

$$v = I^{-1} * MQ_v = [0, v] \tag{6.9}$$

The velocity vector can be converted to its quaternion form Q_v in order to make the last step simpler. The scalar part of quaternion can be set to 0 and the vector part can be set to v. This allows the next rotation quaternion R to be calculated.

$$R = R_0 + (\frac{1}{2} * Q_v * R_0 * T_{delta}) \tag{6.10}$$

Where R is the next local rotation quaternion, R_0 is the previous local rotation quaternion, Q_v is the velocity quaternion and finally T_{delta} is the change in time since the previous physics update. This new rotation quaternion can then replace the current local rotation of the branch in turn simulating the motion of the branch.

6.4 Updating Branches

The particles in this system are the joints within the trees' skeleton. All of these joints have to be updated in each update step. This can happen as frequently as needed. A consideration is that if the branches aren't updated frequently enough the animations will not look smooth. Effectively each update step needs to take the forces acting on each branch, its current position and rotation and then calculate the next position and rotation of that

branch. This information is then used to generate the model of the tree once again. This is passed to the renderer which will render the result.

6.5 Results

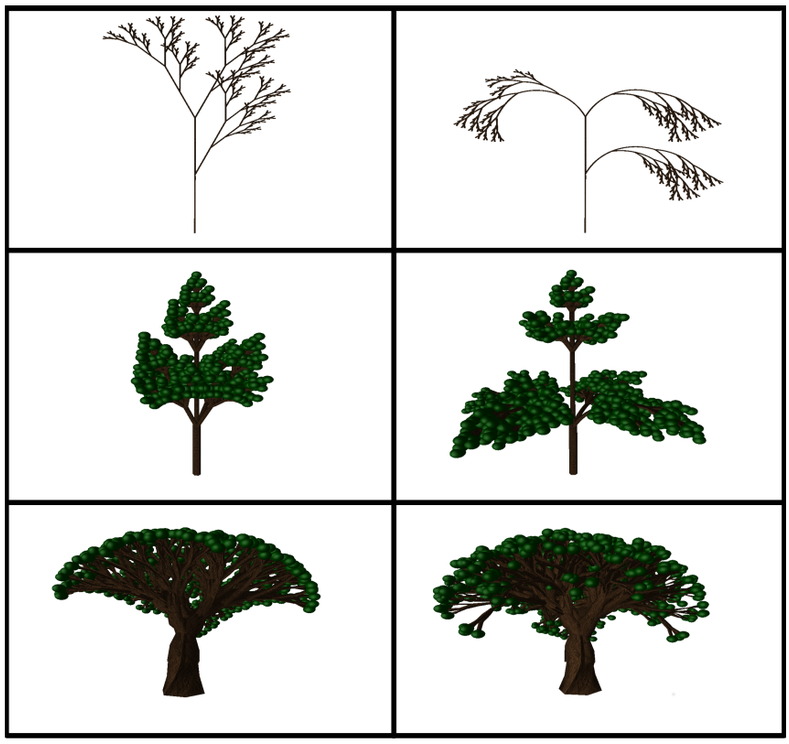


Figure 6.1: Examples simulating gravity on different 3D models

6.6 Summary

This chapter outlined a method of simulating and animating a procedurally generated plant-life by representing each branch as a particle in a larger particle system. The information about the location, rotation, dimensions and properties of each branch is provided by the tree skeleton created during the interpretation stage which is ultimately provided by the L-system. Using these properties each branch can be simulated as a particle within the entire tree system. Due to the embarrassingly parrallel nature of the system each branch update can be computed in parallel, either on the CPU or GPU.

Chapter 7

Discussion

The relationship between the string rewriting system and the the string interpreter system cannot be independant of one another. As complexity is added one system the other need not be as complex. This can be described with a simple example of determining the branch width of each segment. On the one hand the branch width could be determined within the L-system rewriter by decrementing the branch width within the production rules. On the other hand you could leave the process of determining the branch width to the interpreter. This type of relationship may require the interpreter to understand where the branch lies within the tree, as well as information as to the base width and rate at which the branches decrease in size. There are arguements that can be made for both sides of this discussion. It can be difficult to determine where the line of complexity should lie between the rewriter and the interpreter. Depending on what the L-system is representing, there may be a need for emphasis on one or the other side. The implementation contained within this thesis puts emphasis on providing a large amount of information in an interpreter independant way. This means that the information is provided through the parameters of modules and through declarations like the #object declaration. If more specific instructions are required they can be provided through the use of the #object defined module, which will point to a meaning defined within the interpreter. This allows an L-system to provide specific information to the interpreter without the grammar of the L-system dictating how it should be interpreted. Whilst leaving the rewriting system free of any interpretation.

The advantage of the approach within this thesis is that the L-system grammar does not need to change regardless of how it is interpretated. Additionally, the rewriting process is also kept independant of the interpretation. The interpretation of a particular module is definined by the interpreter itself, but can also be modified by the L-system. For instance, the module name “F” can be interpreted as a turtle graphics instruction to move forward. However, the statement “#object F BRANCH” can be used to modify the meaning of this within the interpreter. Such that the meaning now suggests a move forward whilst also indicating that a BRANCH object should be rendered at that position. This makes it clear not only to the interpreter but also to the person writing the L-system.

Chapter 8

Conclusions

This thesis aimed to explore the relationship between the two major systems within the procedural generation of plant-life using L-systems. These systems being the L-system rewiter and the interpreter.

Appendix A

Appendix

A.1 Appendix 1

A.2 Bibliography

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