Files:

The **Metric2Ricci.mlx** code contains the function that does the computations using the MATLAB® Symbolic Math Toolbox™.

The **ExamplesOfMetrics.mlx** code describes and sets up some examples of metric tensors and spacetime metrics.

Purpose:

The primary purpose of this code is to assist academics in solving Einstein's field equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

or

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

This can be a cumbersome task if done by hand. This code allows the user to input a spacetime metric or metric tensor of their own design and output the Christoffel symbols, Ricci tensor and Ricci scalar. They can then, for example, test if the metric is a valid solution to Einstein's field equations.

Details:

Einstein's Theory of General Relativity describes how matter and energy curve/deform spacetime, giving rise to what we observe as gravitational forces. To get the magnitude of a vector *x* in euclidean space, you take the dot product $x_\mu x^\mu$. If the space becomes curved, we need to describe that curvature with a metric *g*, and the dot product comes $g_{\mu\nu} x^\mu x^\nu$.

If we want to see if a given metric is a solution to the Einstein field equations, we compute the Einstein tensor. The Einstein tensor $G_{\mu\nu}$ describes the curvature of spacetime in how it relates to the presence of mass and energy described in the stress-energy tensor $T_{\mu\nu}$.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The Einstein tensor itself is composed of the Ricci curvature tensor $R_{\mu\nu}$ and Ricci scalar *R* and spacetime metric $g_{\mu\nu}$.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

The Ricci curvature tensor and Ricci scalar are both composed of Christoffel symbols $\Gamma^\beta_{\mu\nu}$, which are derived from the spacetime metric:

$$\Gamma^\beta_{\mu\nu} = \frac{1}{2} g^{\beta\alpha} (g_{\alpha\mu,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha})$$

The comma signifies a partial derivative: $\frac{\partial g_{\alpha\beta}}{\partial x^{\mu}} = g_{\alpha\beta,\mu}$

The Christoffel symbols play an explicit role in the geodesic equation:

$$\frac{d^2 x^{\beta}}{ds^2} + \Gamma^{\beta}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}$$

The Ricci curvature tensor and scalar can then be described in terms of the Christoffel symbols and spacetime metric:

$$R_{\alpha\beta} = \partial_{\rho}\Gamma^{\rho}_{\beta\alpha} - \partial_{\beta}\Gamma^{\rho}_{\rho\alpha} + \Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{\beta\alpha} - \Gamma^{\rho}_{\beta\lambda}\Gamma^{\lambda}_{\rho\alpha}$$

$$R = g^{\alpha\beta}R_{\alpha\beta}$$

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