Problem 1.1.28.

$$\begin{aligned} x - y &= 5 \\ 3y + z &= 11 \\ 4z &= 8 \end{aligned}$$

Proof. Start with equation 3. Divide both sides by 4:

$$4z = 8$$

$$\boxed{z = 2}$$

Substitute this into the second equation:

$$3y + z = 11$$

$$3y + 2 = 11$$

$$3y = 9$$

$$\boxed{y = 3}$$

Substitute this into the first equation:

$$x - y = 5$$

$$x - 3 = 5$$

$$\boxed{x = 2}$$

$$\boxed{(x, y, z) = (2, 3, 2)}$$

□

Problem 1.1.38.

$$\begin{aligned} 3x + 2y &= 2 \\ 6x + 4y &= 14 \end{aligned}$$

Proof. By inspection, the first equation appears to be very similar to the second.
Multiply the first equation by 2:

$$6x + 4y = 4$$

However, the second equation says that $6x + 4y = 14$. This set of equations is $\boxed{\text{inconsistent}}$. □

Problem 1.1.48.

$$\begin{aligned} x + y + z &= 2 \\ -x + 3y + 2z &= 8 \\ 4x + y &= 4 \end{aligned}$$

Proof. Add 4 times the first equation and the negative of the third equation.

$$3y + 4z = 4$$

Next, add the first and second equations:

$$4y + 3z = 10$$

This set of equations is easier to solve.

$$\begin{aligned} 3y + 4z &= 4 \\ 4y + 3z &= 10 \end{aligned}$$

Add 4 times the first equation and -3 times the second.

$$\begin{aligned} 12y + 16z &= 16 \\ -12y - 9z &= -30 \end{aligned}$$

Adding these, $7z = -14$ and $\boxed{z = -2}$. Substitute this back into equation 2 (the unomodified one):

$$3y + (-8) = 4$$

$$\boxed{y = 4}$$

Substitute both of these into the original first equation:

$$x + 2 + -4 = 2$$

$$\boxed{x = 0}$$

Therefore, the final solution set is $\boxed{(x, y, z) = (0, 4, -2)}$

□

Problem 1.2.14.

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Proof. This matrix represents the following set of simultaneous equations:

$$\begin{array}{lcr} x + 2y + z & = & 0 \\ z & = & -1 \\ 0 & = & 0 \end{array}$$

Substituting the value for z in the second equation into the first equation, we get a linear equation in terms of x and y.

$$\boxed{x + 2y = 1}$$

What this solution represents is a line in the xy-plane where every point on this line is an acceptable solution to the original matrix.

□

Problem 1.2.31.

$$\begin{array}{lcr} x - 3z & = & -2 \\ 3x + y - 2z & = & 5 \\ 2x + 2y + z & = & 4 \end{array}$$

Proof. Start by writing this as an augmented matrix: $\begin{bmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{bmatrix}$

First, $R_3 = R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 3 & 2 & -2 & 2 \end{bmatrix}$$

Next, $R_3 = R_3 - R_2$:

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 0 & 1 & 0 & -3 \end{bmatrix}$$

Next, $R_2 = R_2 - R_1$:

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 2 & 1 & 1 & 7 \\ 0 & 1 & 0 & -3 \end{bmatrix}$$

Swap R_2 and R_3:

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 0 & -3 \\ 2 & 1 & 1 & 7 \end{bmatrix}$$

Next, $R_3 = R_3 - 2R_1$:

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 1 & 7 & 11 \end{bmatrix}$$

Next, $R_3 = R_3 - R_2$:

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 7 & 14 \end{bmatrix}$$

Multiply the last row by $\frac{1}{7}$:

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Next, $R_1 = R_1 + 3R_3$:

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

This means the solution to the original set of equations is $\boxed{(x_1, x_2, x_3) = (4, -3, 2)}$ □