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Homework 2
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MA244-03

Problem 12. Find the inverse of this matrix, if it exists.

$$\begin{vmatrix} -1 & 1 \\ 3 & -3 \end{vmatrix}$$

Proof. Find the determinant of the matrix. If the determinant is 0, then there is no inverse.

$$det\begin{vmatrix} -1 & 1 \\ 3 & -3 \end{vmatrix} = (-1 \cdot -3) - (1 \cdot 3) = 3 - 3 = 0$$

$$\boxed{\text{This matrix has no inverse.}}$$

□

Problem 14. Find the inverse of this matrix.

$$\begin{vmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & 7 \end{vmatrix}$$

Proof. Begin by finding the determinant. Call this matrix A.

$$det(A) = 1 \cdot \begin{vmatrix} 7 & 9 \\ -4 & -7 \end{vmatrix} + 2 \cdot \begin{vmatrix} 3 & 9 \\ -1 & -7 \end{vmatrix} - 2 \cdot \begin{vmatrix} 3 & 7 \\ -1 & -4 \end{vmatrix}$$

$$det(A) = 1(7 \cdot -7 - 9 \cdot -4) + 2(3 \cdot -7 - 9 \cdot -1) - 2(3 \cdot -4 - 7 \cdot -1) = 1.5$$

As the determinant is not 0, there is an inverse to this matrix.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right]$$

$R_2 = R_2 - 3R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right]$$

$R_3 = R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

$R_2 = R_2 - 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

$R_1 = R_1 - 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 9 & 10 & 6 \\ 0 & 1 & 0 & -4 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

$$R_1 = R_1 - 2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 19 & 6 & 4 \\ 0 & 1 & 0 & -4 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

The right side of the augmented matrix is the inverse of the original matrix.

$$\boxed{\begin{vmatrix} 19 & 6 & 4 \\ -4 & -5 & -3 \\ -5 & 2 & 1 \end{vmatrix}}$$

□

Problem 34. Find the inverse of this matrix.

$$\begin{vmatrix} -12 & 3 \\ 5 & -2 \end{vmatrix}$$

Proof. Using the equation on page 66:

$$A^{-1} = \frac{1}{ad - bc} \cdot \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$A^{-1} = \frac{1}{(-12 \cdot -2) - (3 \cdot 5)} \begin{vmatrix} -2 & -3 \\ -5 & -12 \end{vmatrix}$$

$$\frac{1}{9} \cdot \begin{vmatrix} -2 & -3 \\ -5 & -12 \end{vmatrix}$$

$$\begin{vmatrix} \frac{-2}{9} & \frac{1}{3} \\ \frac{-5}{9} & \frac{-4}{3} \end{vmatrix}$$

□