Matt Fletcher MA385 Homework 1

1.

a) List all items in the sample space Ω:

{1, 0}, {1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {2, 0}, {2, 1}, {2, 2}, {2, 3}, {2, 4}, {2, 5}, {2, 6}, {3, 0}, {3, 1}, {3, 2}, {3, 3}, {3, 4}, {3, 5}, {3, 6}, {4, 0}, {4, 1}, {4, 2}, {4, 3}, {4, 4}, {4, 5}, {4, 6}, {5, 0}, {5, 1}, {5, 2}, {5, 3}, {5, 4}, {5, 5}, {5, 6}, {6, 0}, {6, 1}, {6, 2}, {6, 3}, {6, 4}, {6, 5}, {6, 6},

b) List outcomes where:

A) At least 1 four is rolled: {1, 4}, {2, 4}, {3, 4}, {4, 0}, {4, 1}, {4, 2}, {4, 3}, {4, 4}, {4, 5}, {4, 6}, {5, 4}, {6, 4}

B) Both Dice are even: {2, 0}, {2, 4}, {4, 0}, {4, 4}, {6, 0}, {6, 4}

C) $A \cap B$: {2, 4}, {4, 0}, {4, 4}, {6, 4}

D) $A \cup B$: {1, 4}, {2, 0}, {2, 4}, {3, 4}, {4, 0}, {4, 1}, {4, 2}, {4, 3}, {4, 4}, {4, 5}, {4, 6}, {5, 4}, {6, 0}, {6, 4}

E) {1, 0}, {1, 1}, {1, 2}, {1, 3}, {1, 5}, {1, 6}, {2, 1}, {2, 2}, {2, 3}, {2, 5}, {2, 6}, {3, 0}, {3, 1}, {3, 2}, {3, 3}, {3, 5}, {3, 6}, {5, 0}, {5, 1}, {5, 2}, {5, 3}, {5, 5}, {5, 6}, {6, 1}, {6, 2}, {6, 3}, {6, 5}, {6, 6}

2.

The probability that a pet comes from a house with 3 pets:

Total number of possible pets to pick: 50.

Number of pets that could come from a house with 3 pets: Because 7 houses each have 3 pets, there are 21 pets that could come from a house with 3 pets.

The probability, then, is $\boxed{\dfrac{21}{50}}$.

3.

a) $P(A)$: there are 3 desired outcomes (drawing a 2,4, or 6) and 8 possible outcomes. Therefore, the probability $P(A) = \frac{3}{8}$.

b) $P(A)'$: The complement of A is the opposite. There are 5 balls in the desired outcome; therefore, the probability $P(A)' = \boxed{\dfrac{5}{8}}$.

c) $P(A \cup B)$: The union of $\{3,4,5,6,7\}$ and $\{2,4,6\}$ is $\{2,3,4,5,6,7\}$. There are 6 outcomes in the desired set, and 8 in the total possibilities, making the probability $P(A \cup B) = \boxed{\dfrac{3}{4}}$.

d) $P(A \cap B)$: The intersect of $\{3,4,5,6,7\}$ and $\{2,4,6\}$ is $\{4,6\}$. There are 2 outcomes in the desired set, and 8 in the total possibilities, making the probability $P(A \cap B) = \boxed{\dfrac{1}{4}}$.

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4. The probability of a blackjack from a random deck:

There are 4 Aces in the deck. If one card is an Ace, the other card must be in $\{10, J, Q, K\}$. There are 16 cards in that set. Therefore, there are $4 * 16 = 64$ cases of a blackjack.

There are ${}^{52}C_2 = \frac{52!}{2!(52-2)!} = 52 \times 51/2 = 1326$ possible 2 card hands.

$\frac{64}{1326} = \boxed{\dfrac{32}{663}}$.

5.

There is exactly 1 case where both of the 2 contractors are selected.

There are $7 \times 6 = 42$ possible ways to select 2 council members.

The probability both contractors are selected is $\frac{2}{42} = \boxed{\dfrac{1}{21}}$.

6.

Simplify. See attached page for graphics.

a) $(E \cup F) \cap (E \cup F^C)$

I'll first work on the insides of the parentheses. $E \cup F$ is the union of E and F, so everything that is inside both E and F. $E \cup F^C$ is E unioned with everything not in F. So, this is everything that is in E that is also not in F.

The intersection of these two sets is everything contained in both sets. Because one set is everything in both E and F, and the other set is everything in E and NOT in F, this simplifies to the null set:

$\boxed{\{\varnothing\}}$

b) $(E \cap F) \cap (E \cap G)^C$

Again, I will evaluate the inside of the parentheses first.

$E \cap F$ is everything that is contained in both E and F. This is everything inside of both the E circle and the F circle.

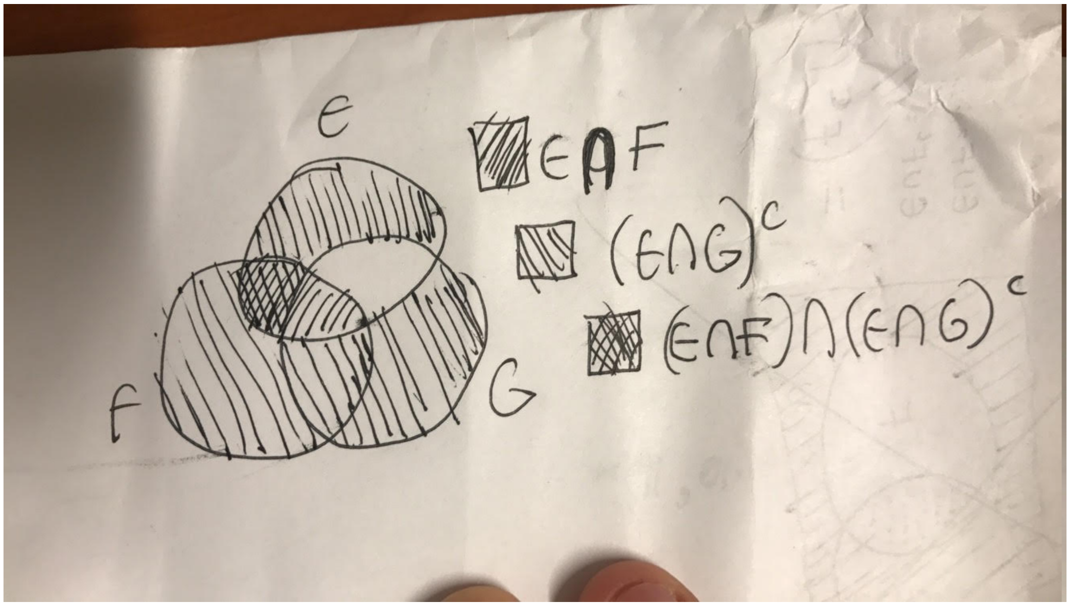

Figure 1: Picture for Problem 6, part B.

$E \cap G$ is everything that is contained in both E and G. The compliment of this is everything that is not in E and also not in G. In a Venn diagram, this is everything outside the circles representing E and G but inside the universal set U.

The intersect of these 2 sets is everything contained in both sets. This means that the simplification is $\boxed{\{E \cap F \cap G^C\}}$

See picture.

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7. $P(A \cup B) = 0.76$, and $P(A \cup B') = 0.87$.

For the first set, a simplification and placing in algebraic form gives:

$A + B = 0.76$

$A + B' = 0.87$

$B + B' = 1.00$

Placing in matrix form...

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

This is equal to

$$\begin{bmatrix} 0.76 \\ 0.87 \\ 1.00 \end{bmatrix}$$

Solving this system of matrices results in finding that $P(A) = 0.315$.

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8. There are $\boxed{12!}$ ways to line up the 12 children. If there are 6 boys and 6 girls, and the boys will all be in the back row, then there are 6! ways to arrange the boys, and 6! ways to arrange the girls. The order of the groups of children (not the order inside the groups) is already set, so there are $6! \times 6! = \boxed{518400}$ ways to arrange.