Matt Fletcher MA385 Homework 3

---

1. The probability he gets a hit at any at bat is 0.342. We need to find the probability that his 9th at bat is his 5th hit. This is a negative binomial distribution.

The probability it takes k trials to get r successes, with the probability of success on any one trial being p, is:

$$P(x=k) = \binom{k-1}{r-1}(p)^r(1-p)^{k-r}$$

$$P(x=9) = \binom{9-1}{5-1}(0.342)^5(1-0.342)^{9-5}$$

$$P(x=9) = \boxed{0.0613948}$$

2.

a) Find EV and Var of the sum of 2 dice:

2 dice rolled together can have any value from 2 to 12. In the following table, roll is the value of a roll, and roll count is the number of ways that roll can be achieved.

Roll	2	3	4	5	6	7	8	9	10	11	12
Roll Count	1	2	3	4	5	6	5	4	3	2	1

The expected value is $\Sigma \text{rollvalue} \cdot \frac{\text{rollcount}}{36}$

This results in an EV of $\boxed{7}$

The variance is represented by $E[x^2] - E[x]^2 = 54.833 - 49 = \boxed{5.8333}$

b) Find EV and Var of the min roll of 2 dice.

There are 11 ways that the minimum value is a 1:
Die 1 has a 1, Die 2 has 2,3,4,5, or 6. (5 ways)
Die 2 has a 1, Die 1 has 2,3,4,5, or 6. (5 ways)
Both die show a 1. (1 way)
There are 9 ways that the minimum value is a 2:
Die 1 has a 2, Die 2 has 3,4,5, or 6. (4 ways)
Die 2 has a 2, Die 1 has 3,4,5, or 6. (4 ways)
Both die show a 2. (1 way)
It is apparent there is a pattern: 11, 9, 7, 5, 3, 1. The sum of these is 36, which checks out.
Roll_count = 13-2x, where x is the minimum value.

Min Value	1	2	3	4	5	6
Roll Count	11	9	7	5	3	1

As in part A, we find the sum of the values multiplied by their probabilities.
$(1\times 11 + 2\times 9 + 3\times 7 + 4\times 5 + 5\times 3 + 6\times 1)\times \frac{1}{36} = \frac{91}{36} = \boxed{2.5277}$
To find variance, find the Expected value of the square of the results:
$E[x^2] = (1^2\times 11 + 2^2\times 9 + 3^2\times 7 + 4^2\times 5 + 5^2\times 3 + 6^2\times 1)\times \frac{1}{36} = 8.3611$
$E[x]^2 = 6.3896.$
$Var = 8.36111 - 6.3896 = \boxed{1.9714}$

---

3. a) The probability that all 12 customers order the regular chicken sandwich is $(0.75)^{12} = \boxed{0.031676}$

b) The probability that no more than 3 customers order the spicy chicken sandwich can be found more easily by finding 1 minus the probability that 0, 1, 2, or 3 customers order the spicy chicken sandwich.

This is a cumulative binomial random variable.

According to the *binomcdf* function in my TI84, with function arguments $trials = 12$, $p = 0.25$, and $x = 3$, the result is $\boxed{0.64877}$.

---

4. To find the probability that at least 1 double is rolled, it is easier to find the compliment of the probability that no doubles are rolled. Let $n = 24$, for the number of trials. Let $p = \frac{6}{36} = \frac{1}{6}$, for the probability that neither die lands on the same value as the other. Then, $\lambda = np = 24 \times \frac{5}{6} = 4$.

Now, we can use the Poisson equation:

$$P(x=k)=\frac{\lambda^k}{k!}e^{-\lambda} \tag{1}$$

Let $k=0$.

$$P(x=0)=\frac{4^0}{0!}e^{-4}$$

This results in 0.01831, and the compliment is $1-0.01831=98.16\%$

The exact value is given by the following logic: The probability at least one double is rolled is the compliment of the probability that no doubles are rolled. The probability of no doubles being rolled is $\frac{5}{6}$. This must happen 24 times in a row. Therefore, the probability of no doubles being rolled is $\frac{5}{6}^{24}=0.012579$, and the probability of at least one double being rolled is $1-0.012579=\boxed{98.742\%}$.

---

5. The sum of the probabilities in the bottom row must equal 1. Therefore, I know that $f(5)$ and $f(6)$ sum to 0.1.

Calculating μ for the known values, with a and b standing for $f(5)$ and $f(6)$ respectively:

$0\times0.1+1\times0.15+2\times0.2+3\times0.25+4\times0.2+5\times a+6\times b=2.64$ (this value is given)

$0\times0.1+1\times0.15+2\times0.2+3\times0.25+4\times0.2=2.1$

Therefore, $5a+6b=2.64-2.1=0.54$

Also, from above, $a+b=0.1$.

This is just simultaneous equations:

$f(5)=\boxed{0.06}$

$f(6)=\boxed{0.04}$.