COMP 5660 Fall 2023 Assignment 1d

Matthew Freestone
maf0083@auburn.edu

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1 Green: Without Crowding Deliverables

1.1 MOEA Parameters

See Table 1 for Green Non-Crowding MOEA Parameters.

Parameter Name	Value
μ	600
λ	500
Mutation Rate	0.60
Objectives	Length and Width
Use Crowding	False
Parent Selection Method	K-Tournament With Replacement ($k = 5$)
Survival Selection	K-Tournament Without Replacement ($k = 9$)
Recombination Method	n-point Crossover ($n = 3$)
Mutation Expected Loci Changed	3
Mutation Creep Normal Distribution σ	5

Table 1: Green Without Crowding MOEA Parameter Values

1.2 Results

See Figure 1 for an Evals vs Population Mean {Length, Width} score and Population Max {Length, Width} score. As the shapes get tigher, both objectives improve, leading to the maximum value for each objective being fairly close to the average value. The hypervolume for the Paretro front also increased across the runs.

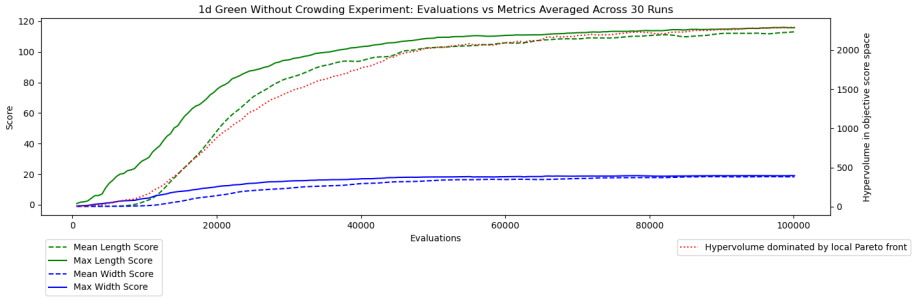

Figure 1: Green Without Crowding Evals vs Objectives

See Figure 2 for a plot of the final Pareto front from the run with the highest hypervolume. When crowding is not taken into consideration, the MOEA found a single solution that does best for both length and width, leading to a single individual in the Pareto Front.

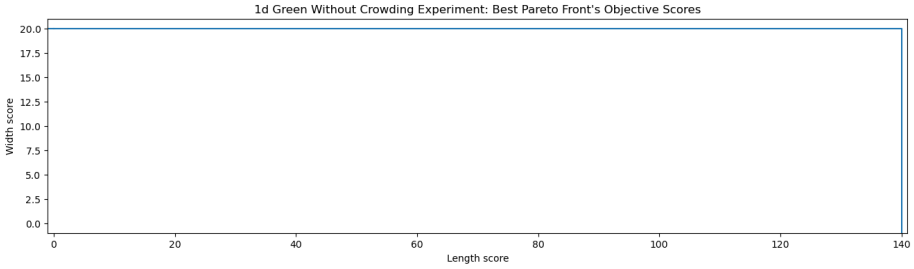

Figure 2: Best Pareto Front from Green Without Crowding

1.3 Solutions

The best run from Green Without Crowding yielded a total of 600 (identical in objective performance) solutions on the Pareto front. Here, we show the individuals with the best performance on each objective.

See Figure 3 for the minimum length solution generated by the Green Without Crowding parameters. This solution had length score 140 and width score 20.

Figure 3: Best Length from Green Without Crowding

See Figure 4 for the minimum width solution generated by the Green Without Crowding parameters. This solution had length score 120 and width score 20.

Figure 4: Best Width from Green Without Crowding

For the experiment without crowding, there was only one unique solution on the front, which is shown in both visualizations. Therefore, no solution exists that balances the two objectives, as the same solution maximized both.

1.4 Statistical Analysis

The comparison of this method to the Green With Crowding method can be found in the section for Green With Crowding.

2 Green With Crowding Deliverables

2.1 MOEA Parameters

See Table 2 for Green Crowding MOEA Parameters.

Parameter Name	Value
μ	600
λ	500
Mutation Rate	0.60
Objectives	Length and Width
Use Crowding	True
Parent Selection Method	K-Tournament With Replacement ($k = 5$)
Survival Selection	K-Tournament Without Replacement ($k = 9$)
Recombination Method	n-point Crossover ($n = 3$)
Mutation Expected Loci Changed	3
Mutation Creep Normal Distribution σ	5

Table 2: Green With Crowding MOEA Parameter Values

2.2 Results

See Figure 5 for an Evals vs Population Mean {Length, Width} score and Population Max {Length, Width} score. As the shapes get tighter, both objectives improve, leading to the maximum value for each objective being fairly close to the average value. The hypervolume for the Pareto front also increased across the runs.

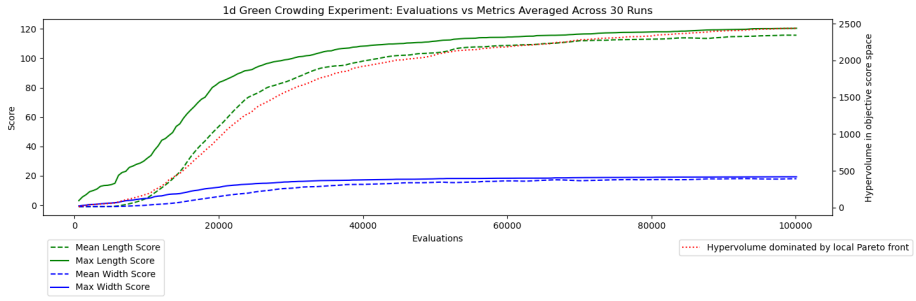

Figure 5: Green With Crowding Evals vs Objectives

See Figure 6 for a plot of the final Pareto front from the run with the highest hypervolume. When crowding is taken into consideration, the MOEA now finds multiple solutions that sit on the Pareto front.

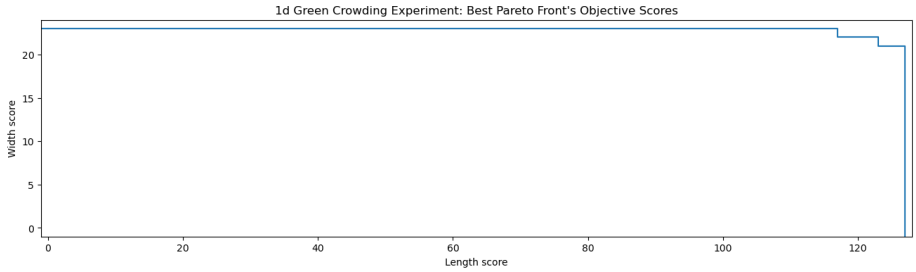

Figure 6: Best Pareto Front from Green With Crowding

2.3 Solutions

The best run from Green With Crowding yielded a total of 24 solutions on the Pareto front. Here, we show the individuals with the best performance on each objective, as well as one individual on the front that has decent performance on both.

See Figure 7 for the minimum length solution generated by the Green With Crowding parameters. This solution had length score 127 and width score 21.

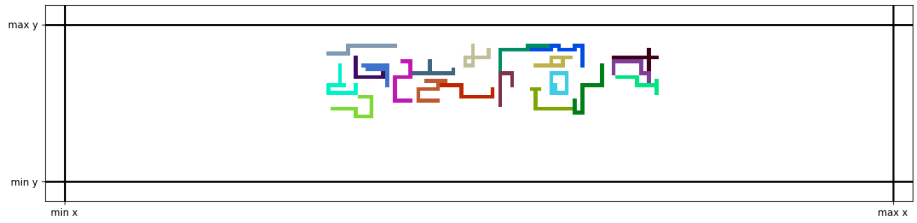

Figure 7: Best Length from Green With Crowding

See Figure 8 for the minimum width solution generated by the Green Without Crowding parameters. This solution had length score 117 and width score 23.

Figure 8: Best Width from Green With Crowding

See Figure 9 for a solution that balanced both objectives. This solution had length score 123 and width score 22.

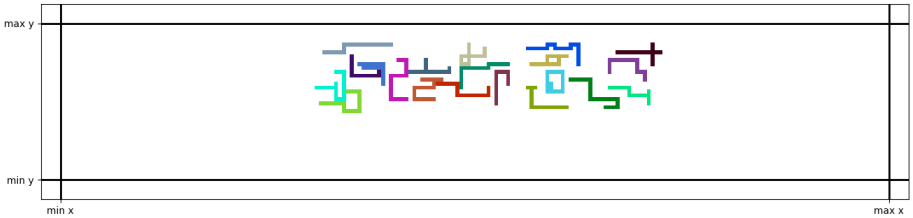

Figure 9: Balanced Solution from Green With Crowding

2.4 Statistical Analysis

In this section, the data from the experiment without Crowding is compared to the data from this experiment, with Crowding. The statistics for each dataset are shown in Table 3.

1d Green without Crowding mean:	2289.5666666666666
1d Green without Crowding stdv:	279.2300600522472
1d Green Crowding mean :	2437.3
1d Green Crowding data stdv:	362.7800844246724

Table 3: Green With vs Without Crowding Statistical Data

After performing a Welch's T-test, a p value was found of 0.08274536417598023. With $\alpha = 0.05/3$, the null hypothesis of the methods yielding the same results cannot be rejected, so it cannot be concluded that one method yields significantly better results. Based on the plots from each experiment, it seems likely that allowing more evaluations per run might lead to a larger difference between the experiments, but that is left for future work.

3 Yellow: Without Crowding Deliverables

3.1 MOEA Parameters

See Table 4 for Yellow Non-Crowding MOEA Parameters.

Parameter Name	Value
μ	600
λ	500
Mutation Rate	0.60
Objectives	Length, Width and Common Edges Count
Use Crowding	False
Parent Selection Method	K-Tournament With Replacement ($k = 5$)
Survival Selection	K-Tournament Without Replacement ($k = 9$)
Recombination Method	n-point Crossover ($n = 3$)
Mutation Expected Loci Changed	3
Mutation Creep Normal Distribution σ	5

Table 4: Yellow Without Crowding MOEA Parameter Values

3.2 Results

See Figure 10 for an Evals vs Population Mean {Length, Width, Edges} score and Population Max {Length, Width, Edges} score.

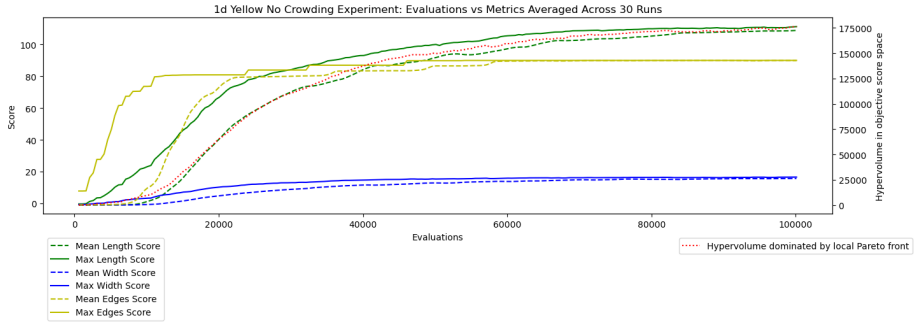

Figure 10: Yellow Without Crowding Evals vs Objectives

Due to the 3-dimensional nature of the Yellow MOEA, plotting a Pareto front is difficult. When crowding is not taken into consideration, the MOEA found a single solution that does best for both width and common edge count, but it located a different one for the best length.

3.3 Solutions

The best run from Yellow Without Crowding yielded a total of 600 (likely many being duplicate) solutions on the Pareto front. Here, we show the individuals with the best performance on each objective.

See Figure 11 for the minimum length solution generated by the Yellow Without Crowding parameters. This solution had length score 109, width score 18, and common edges score 90 .

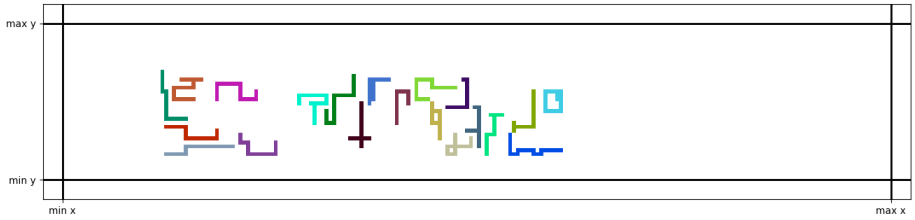

Figure 11: Best Length from Yellow Without Crowding

See Figure 12 for the minimum width solution generated by the Yellow Without Crowding parameters. This solution had length score 101, width score 20, and common edges score 90.

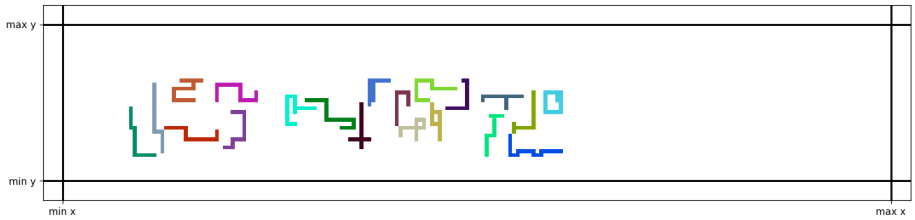

Figure 12: Best Width from Yellow Without Crowding

See Figure 13 for the minimum common edge solution generated by the Yellow Without Crowding parameters. This solution had length score 101, width score 20, and common edges score 90.

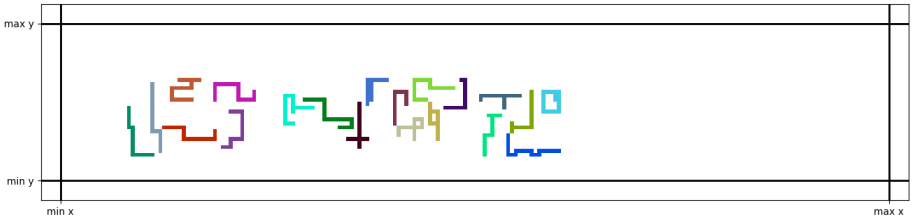

Figure 13: Best Common Edges from Yellow Without Crowding

See Figure 14 for a solution that balanced all 3 objectives. This solution had length score 107, width score 19, and common edges score 87.

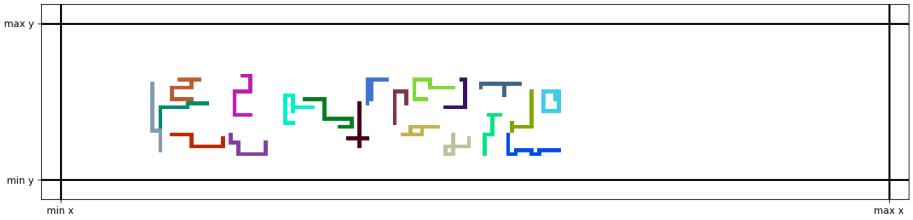

Figure 14: Balanced Solution from Yellow Without Crowding

3.4 Statistical Analysis

The comparison of this method to the Yellow With Crowding method can be found in the section for Yellow With Crowding.

4 Yellow: With Crowding Deliverables

4.1 MOEA Parameters

See Table 5 for Yellow Crowding MOEA Parameters.

Parameter Name	Value
μ	600
λ	500
Mutation Rate	0.60
Objectives	Length, Width and Common Edges Count
Use Crowding	True
Parent Selection Method	K-Tournament With Replacement ($k = 5$)
Survival Selection	K-Tournament Without Replacement ($k = 9$)
Recombination Method	n-point Crossover ($n = 3$)
Mutation Expected Loci Changed	3
Mutation Creep Normal Distribution σ	5

Table 5: Yellow With Crowding MOEA Parameter Values

4.2 Results

See Figure 15 for an Evals vs Population Mean {Length, Width, Edges} score and Population Max {Length, Width, Edges} score.

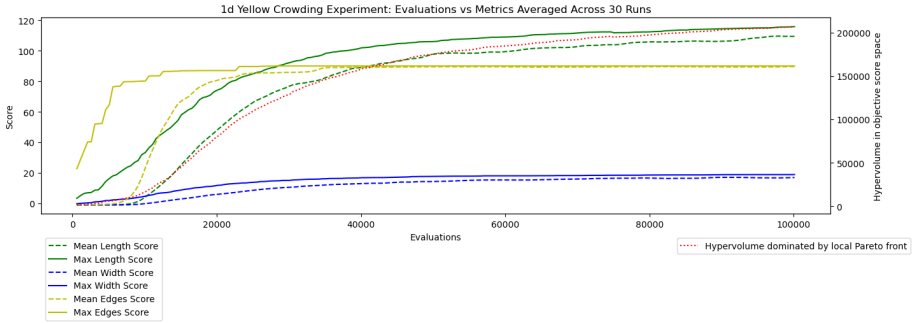

Figure 15: Yellow With Crowding Evals vs Objectives

Due to the 3-dimensional nature of the Yellow MOEA, plotting a Pareto front is difficult.

4.3 Solutions

The best run from Yellow With Crowding yielded a total of 84 solutions on the Pareto front. Here, we show the individuals with the best performance on each objective.

See Figure 16 for the minimum length solution generated by the Yellow With Crowding parameters. This solution had length score 122, width score 13, and common edges score 90 .

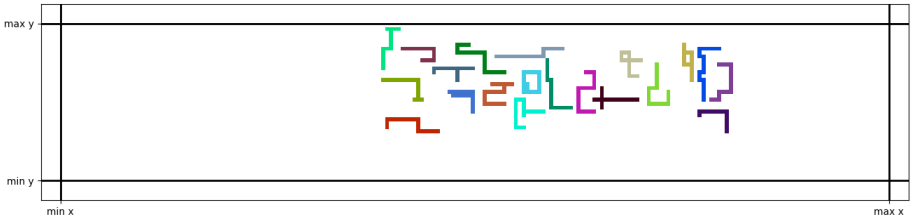

Figure 16: Best Length from Yellow With Crowding

See Figure 17 for the minimum width solution generated by the Yellow With Crowding parameters. This solution had length score 100, width score 22, and common edges score 90.

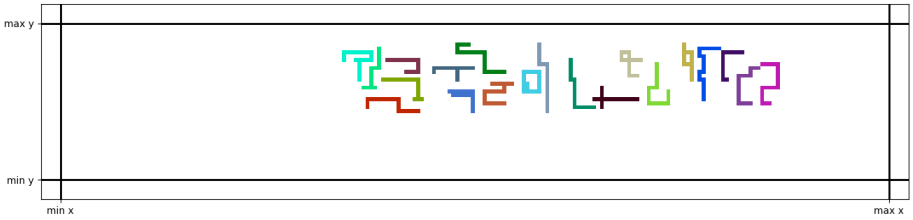

Figure 17: Best Width from Yellow With Crowding

See Figure 18 for the minimum common edge solution generated by the Yellow With Crowding parameters. This solution had length score 122, width score 22, and common edges score 90.

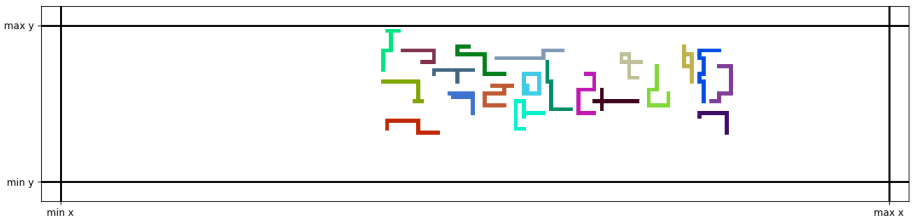

Figure 18: Best Common Edges from Yellow With Crowding

See Figure 19 for a solution that balanced all 3 objectives. This solution had length score 120, width score 17, and common edges score 86.

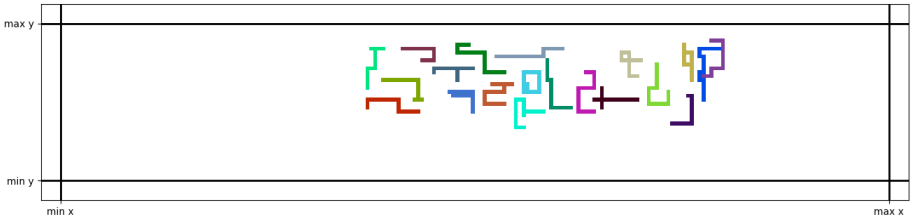

Figure 19: Balanced Solution from Yellow With Crowding

4.4 Statistical Analysis

In this section, the data from the experiment without Crowding is compared to the data from this experiment, with Crowding. The statistics for each dataset are shown in Table 6.

1d Yellow without Crowding mean:	176252.6
1d Yellow without Crowding stdv:	15860.994384620579
1d Yellow Crowding mean :	206970.13333333333
1d Yellow Crowding data stdv:	25423.143768721002

Table 6: Yellow With vs Without Crowding Statistical Data

After performing a Welch's T-test, a p value was found of $9.333916831970476 \cdot 10^{-7}$.
With $\alpha = 0.05/3$, the null hypothesis of the methods yielding the same results can be firmly rejected, as $\alpha >> p$. This indicates that using Crowding to encourage more distribution along the Pareto front lead to a significantly larger hypervolume.

5 Red: Random Search Deliverables

5.1 Parameters

See Table 7 for Red Random Search Parameters. In this context, μ is the number of individuals initially put in the population, the in each "generation," λ randomly generated individuals are added to the population, and only the Pareto front of the population survives.

Parameter Name	Value
μ	500
λ	500
Objectives	Length and Width
Survival Selection	Truncation, only Pareto Front survives

Table 7: Red Random Search Parameter Values

5.2 Results

See Figure 20 for a plot of the final Pareto front from the run with the highest hypervolume.

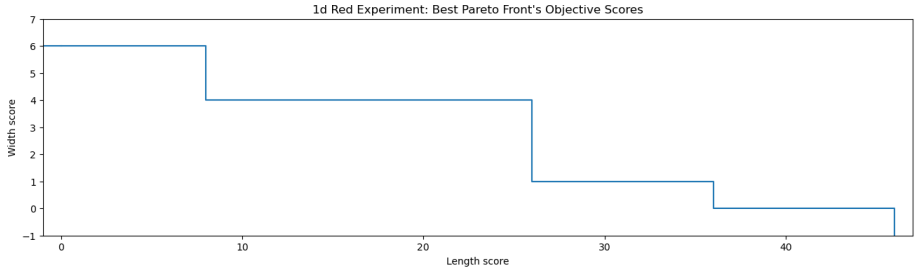

Figure 20: Best Pareto Front from Red Random Search

5.3 Solutions

The best run from Red yielded a total of 5 solutions on the Pareto front. Here, we show the individuals with the best performance on each objective, as well as one individual on the front that has decent performance on both.

See Figure 21 for the minimum length solution generated by the Green With Crowding parameters. This solution had length score 46 and width score 0.

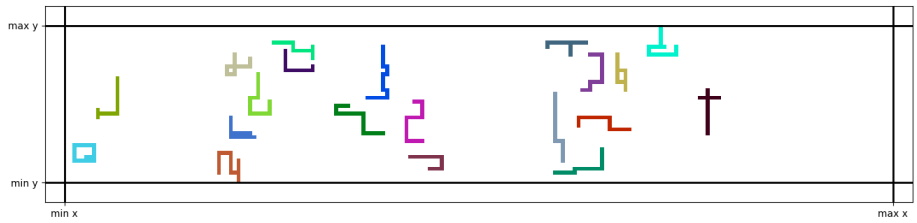

Figure 21: Best Length from Red

See Figure 22 for the minimum width solution generated by the Red parameters. This solution had length score 8 and width score 6.

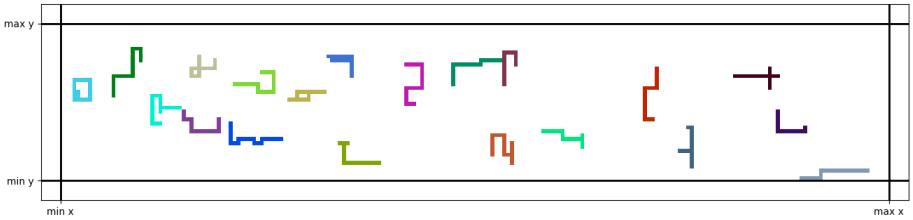

Figure 22: Best Width from Red

See Figure 23 for a solution that balanced both objectives. This solution had length score 26 and width score 4.

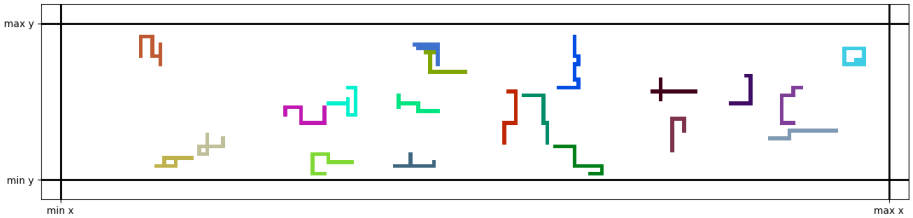

Figure 23: Balanced Solution from Red

5.4 Statistical Analysis

In this section, the data from the best experiment, Green with Crowding, is compared to the results from Red Random Search. The statistics for each dataset are shown in Table 8.

1d Red mean:	218.96666666666667
1d Red stdv:	59.57984114836467
1d Green Crowding mean :	2437.3
1d Green Crowding data stdv:	362.7800844246724

Table 8: Green With Crowding vs Red Statistical Data

After performing a Welch's T-test, a p value was found of $1.7535371803407102 \cdot 10^{-25}$.
With $\alpha = 0.05/3$, the null hypothesis of the methods yielding the same results can be firmly rejected, as $\alpha >> p$. This indicates that the Random Search method from Red performs significantly worse than a more intelligent MOEA method.