Settling Velocity Analysis for ECIV 767 Homework 1

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September 5, 2024

1 Introduction

The objective of this homework is to develop an empirical relationship for the drag coefficient c_D as a function of the particle Reynolds number Re_{vp}, and subsequently derive a relation for the settling velocity ratio R_f as a function of the particle Reynolds number Re_p. This relationship was then compared with the Dietrich (1982) formula. A special case for clay particles was analyzed, and the limitations of the empirical model for such fine particles were explored.

2 Derivation of Empirical Relationship for c_D

To derive an empirical relationship for c_D as a function of Re_{vp}, I first digitized the provided plot and extracted the necessary data points. Using Python, I fit a fourth-degree polynomial to the data in log-log space. This polynomial provides a functional relationship between $\log_{10}(c_D)$ and $\log_{10}(Re_{vp})$. The fitted equation is as follows:

$$\log_{10}(c_D) = 2.081 - 1.372\log_{10}(Re_{vp}) + 0.1545(\log_{10}(Re_{vp}))^2 + 0.02525(\log_{10}(Re_{vp}))^3 - 0.004918(\log_{10}(Re_{vp}))^4 \tag{1}$$

The Python code for this process is detailed in the accompanying Jupyter notebook file *HW1.ipynb*. The plot of Re_{vp} vs. c_D with the polynomial fit is shown below:

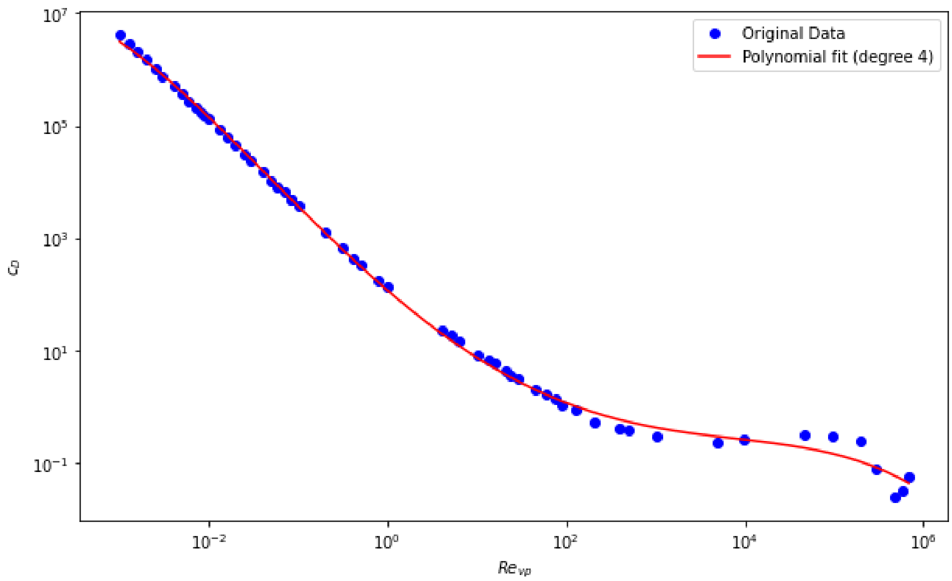

Figure 1: Plot of Re_{vp} vs. c_D with polynomial fit.

3 Derivation of R_f

Using the derived empirical relationship for c_D, I calculated the settling velocity ratio R_f as a function of the particle Reynolds number Re_p. The relationship for R_f is given by:

$$R_f = \left(\frac{4}{3c_D}\right)^{1/2} \tag{2}$$

The following plot shows the calculated R_f as a function of Re_p based on this empirical relationship:

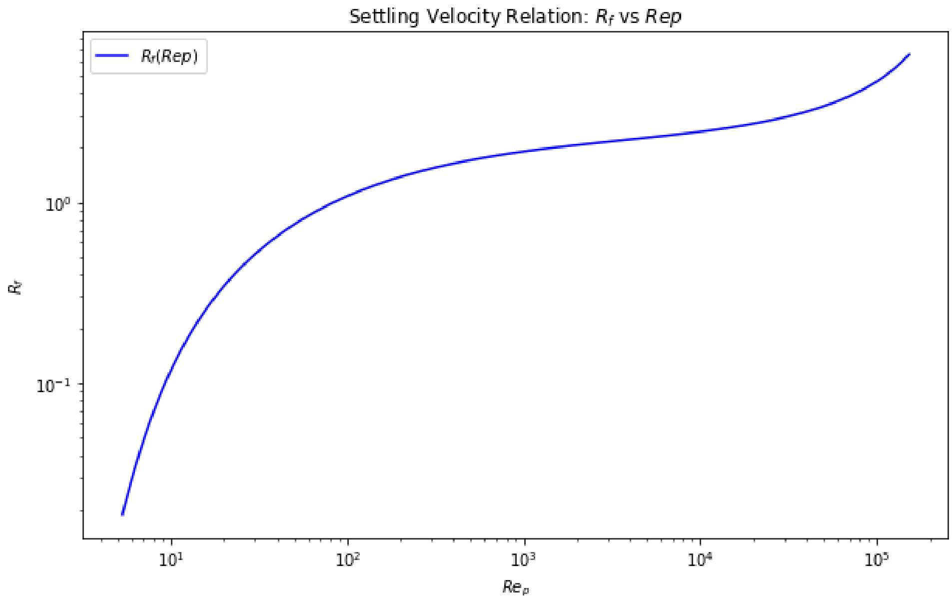

Figure 2: Plot of R_f vs. Re_p calculated using the empirical formula.

Next, we applied a polynomial fit to the calculated R_f values for Re_p. The equation for the polynomial fit of degree 4 is:

$$\log_{10}(R_f) = -4.374 + 5.174\log_{10}(Re_p) - 2.150(\log_{10}(Re_p))^2 + 0.388(\log_{10}(Re_p))^3 - 0.02493(\log_{10}(Re_p))^4 \tag{3}$$

The plot below shows the polynomial fit applied to the R_f vs. Re_p data:

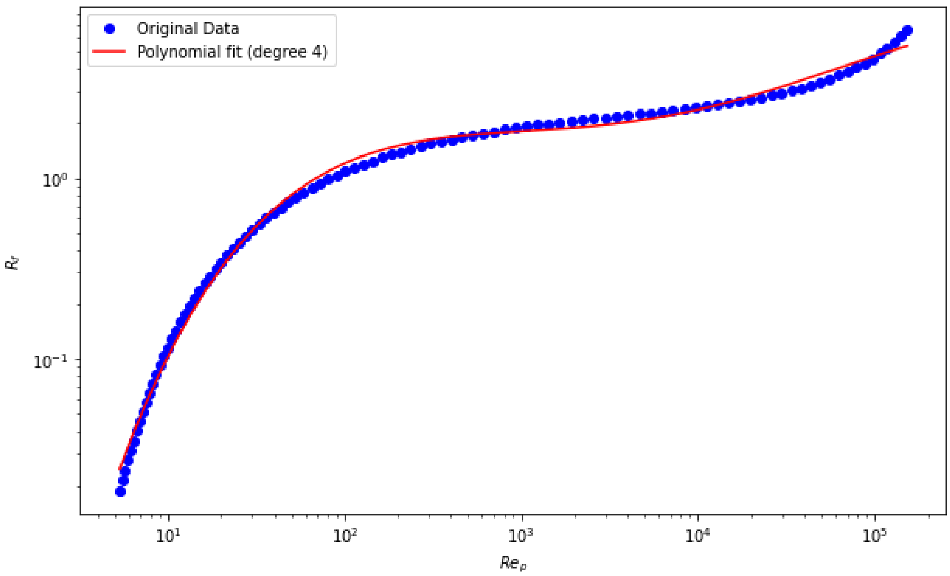

Figure 3: Plot of R_f vs. Re_p with polynomial fit.

The Python code used for this computation is provided in the accompanying Jupyter notebook file *HW1.ipynb*.

4 Comparison with Dietrich (1982) Formula

To validate the derived empirical relationship, I compared the R_f values obtained from the empirical formula with those from the Dietrich (1982) formula. The Dietrich formula is expressed as:

$$R_f = \exp\left(-2.891394 + 0.952696\ln(Re_p) - 0.056835(\ln(Re_p))^2 - 0.002892(\ln(Re_p))^3 + 0.000245(\ln(Re_p))^4\right) \tag{4}$$

Using both the empirical equation and the Dietrich formula, I calculated R_f for a range of Re_p values and compared the results. The empirical equation derived in previous sections is:

$$R_f = \left(\frac{4}{3c_D}\right)^{1/2} \tag{5}$$

where:

$$\log_{10}(c_D) = 2.081 - 1.372\log_{10}(Re_{vp}) + 0.1545(\log_{10}(Re_{vp}))^2 + 0.02525(\log_{10}(Re_{vp}))^3 - 0.004918(\log_{10}(Re_{vp}))^4 \tag{6}$$

Re_p	**Empirical** R_f	**Dietrich** R_f
65.0	0.961853	0.959999
100.0	1.202732	1.125633
250.0	1.575469	1.458016
500.0	1.723461	1.657831
1000.0	1.808387	1.789948
2000.0	1.892705	1.848135
16000.0	2.668133	1.704172

Table 1: Comparison of R_f values from empirical and Dietrich (1982) formulas for different Re_p.

The comparison shows that the empirical formula closely follows the Dietrich (1982) results for larger particle sizes, but there are some deviations at the extreme ends.

5 Analysis of Clay Particles

For clay particles, with a grain size smaller than 2 microns, the Reynolds number Re_p was calculated as:

$$Re_p = \frac{\sqrt{RgD}D}{\nu} \tag{7}$$

Substituting the given values ($R = 1.65$, $g = 9.81\,\mathrm{m/s}^2$, $D = 2 \times 10^{-6}\,\mathrm{m}$, $\nu = 1 \times 10^{-6}\,\mathrm{m^2/s}$), we get:

$$Re_p = 0.0114$$

The empirical formula gave an unrealistic R_f value of zero for clay particles, while the Dietrich formula gave a reasonable value of 3.574×10^{-4}. This indicates that the empirical formula is not valid for very small particles like clay, as it was likely derived from data for larger particles, and it breaks down when extrapolated to smaller sizes.

6 Conclusion

In conclusion, the empirical relationship derived for c_D and R_f works well for particles in the range of sand and gravel but breaks down for fine particles like clay. The Dietrich formula, on the other hand, provides accurate results across a wider range of particle sizes. Therefore, for small particles like clay, it is recommended to use the Dietrich formula.

7 References

- Dietrich, E. W. (1982). Settling velocity of natural particles. *Water Resources Research, 18*(6), 1626-1982.
- Viparelli, E. ECIV 767 Lecture Notes, Fall 2024.