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The metabolic economy

A dynamic model for energy and material flows

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List of Symbols

$\mathbf{A}$	input-output matrix [-]
a	input-output ratio [-]
a	stock of apples [apples]
$\dot{a}$	flow rate of apples [apples/time]
$\dot{B}$	embodied energy flow rate [W]
B	embodied energy [J]
δ_{ij}	Kronecker delta
$\boldsymbol{\varepsilon}$	column vector of sector energy intensities [J/$]
$\dot{E}$	direct energy flow rate [W]
ε	energy intensity [J/$]
$\mathbf{E}_0$	vector of direct energy inputs from the biosphere [W]
E	direct energy [J]
$\hat{\boldsymbol{\gamma}}$	matrix of depreciation rates [$/year]
i	economic sector index
j	economic sector index
K	mass of capital goods [kg]
$\dot{K}$	capital goods mass flow rate [kg/s]
n	number of sectors in the economy
P	mass of products [kg]
$\dot{P}$	product mass flow rate [kg/s]
$\dot{Q}$	waste heat flow rate [W]
R	mass of resources [kg]
$\dot{R}$	resource mass flow rate [kg/s]
S	mass of short-lived goods [kg]
$\dot{S}$	short-lived goods mass flow rate [kg/s]
s	stock of steel [kg]
$\dot{s}$	steel mass flow rate [kg/s]
$\dot{T}$	total energy flow rate [W]
$\mathbf{T}_1$	vector of total energy inputs from society to the economy [W]
T	total energy [J]
t	time [s]

$\dot{X}$	economic value flow rate [$/s]
$\hat{\mathbf{X}}$	matrix of sector outputs [$/year]
$\mathbf{X}_t$	transaction matrix [$/year]
X	stock of economic value [$]
$\dot{X}_{dest}$	rate of destruction of economic value [$/s]
$\dot{X}_{gen}$	rate of generation of economic value [$/s]

Chapter 1
Introduction

DISCUSSION POINTS

- Important to highlight difference between material and financial depreciation.
- Embodied energy flows with material, not with dollars
- Financial depreciation is typically (under normal circumstances) a leading indicator of material depreciation
- Prior to accumulation, embodied energy is passed into products.
- Is ‘society’ the best word ‘final consumption?’
- Do we use energy circuit language?
- For economists, capital is only in the productive sector, $\dot{K}_{i2}$ flows would be ‘consumer durables’
- Housing is ‘residential capital’

1.1 Traditional view of economy

The traditional economic flows accounts for gross payments from the household sector for goods and services that flow from the production sector and payments from the production sector for wages and rents to the household sector.

1.1.1 Economic models that include resource inputs

1.1.2 The metabolic economy

1.2 Brief history of input-output (I-O) modeling

Input-output analysis, developed by Wassilly Leontief in the 1930s as an extension to the work of Quesnay and Walras [1], is of primary importance in national ac-

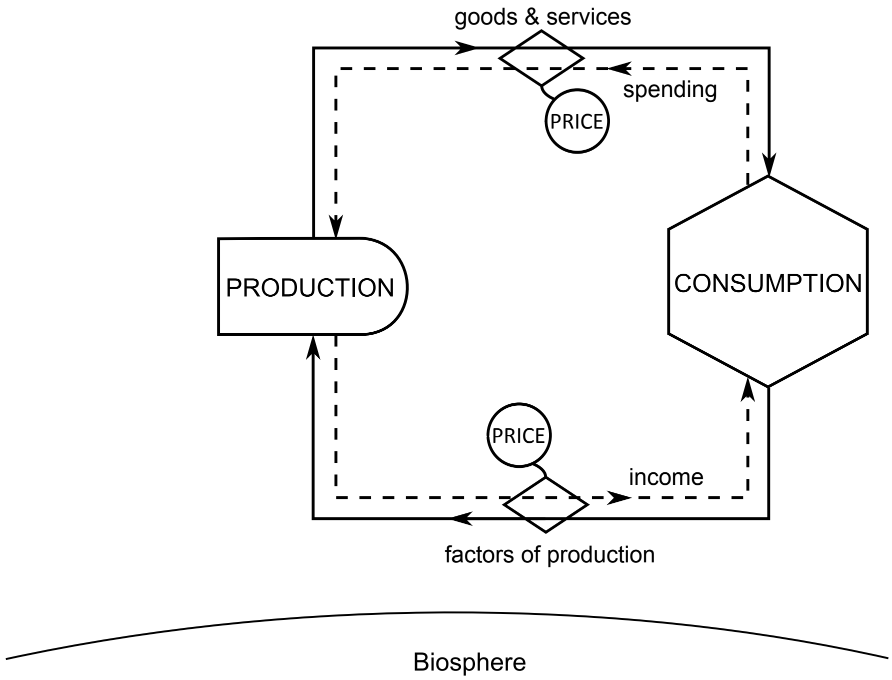

Fig. 1.1 The economy is represented as a circular flow of goods and services between two sectors. The producers manufacture goods and services by taking in labor and capital. Consumers exchange labor for wages which are used to purchase the goods and services of the producers. This may be considered a perpetual motion machine of the first kind.

counting, allowing determination of the structure of an economy as well as, among other things, calculation of a nation's gross domestic product (GDP), the predominant measure of economic activity.

1.3 Basic I-O method

The basic premise of the I-O method, as depicted in Figure 1.4A, is that each economic sector takes in factors of production from other sectors (and possibly itself) to produce an economic good at some rate. For example, the automotive sector takes in steel, rubber, glass, etc. and produces a number of cars per year. In contrast to high-level economic growth models that include only a few factors of production (such as land, capital, and labor), the I-O analysis technique allows many differentiated factors of production and raw material feedstocks.[2] In I-O frameworks, each factor of production is considered to be a output from a sector of the economy. As will be discussed later [MAKE SURE TO DISCUSS THIS LATER!], the traditional primary factors of production (land, capital, and labor) are not *flows* into the

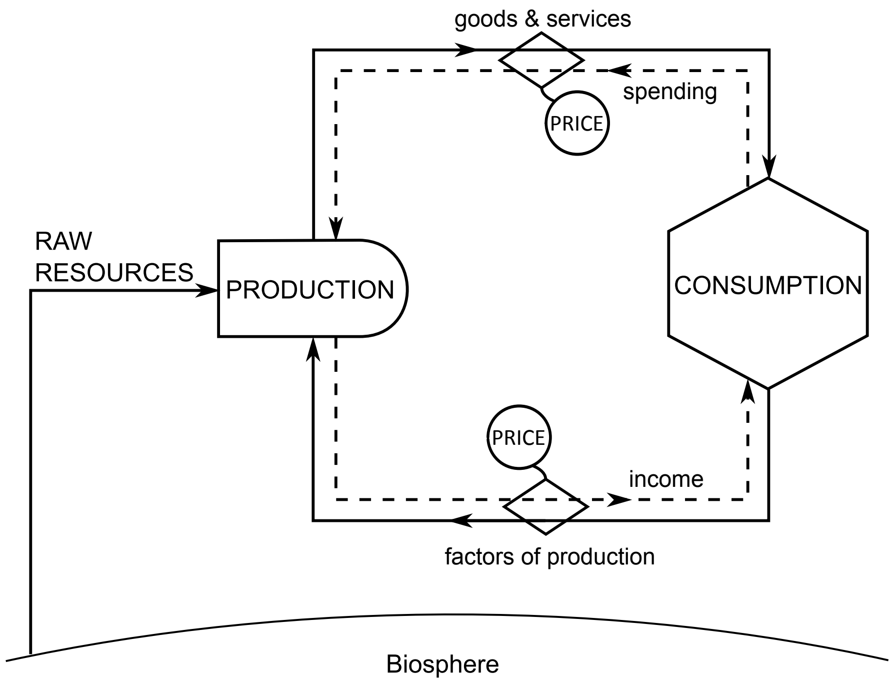

Fig. 1.2 Energy and material input output analysis has included the flows into the economy from the environment. This may be considered a perpetual motion machine of the second kind. [SHOULD FLOW FROM 'RAW RESOURCES' ALSO GO STRAIGHT INTO CONSUMPTION?]

production processes. Rather, they are *stocks* that, when present, allow factors of production (steel, rubber, and glass) to be transformed into final products (automobiles). The quantity and quality of these stocks determine the quantity and quality of their flow of productive services.

In addition to the productive services provided by stocks of land, capital, and labor, a flow of energy[1] is required for economic activity. These energy flows originate from the natural environment, recognition of which has provoked researchers from fields of net energy analysis (NEA), material flow analysis (MFA), industrial ecology (IE), and life-cycle assessment (LCA) to extend the traditional (Leontief) input-output framework to include important material and energy flows to and from the environment, as depicted in Figure 1.4B.[3, 4, 5, 6, 2, 7, 8, 9] While the Leontief I-O approach relies exclusively on monetary units to represent value flows among sectors of an economy, the key insight of these extensions of the Leontief I-O framework is to rely upon physical units (especially energy units of joules) to represent some of the value flows among economic sectors. In doing so, energy and material

[1] Or, more precisely, the degradation of an exergetic gradient/destruction of exergy.

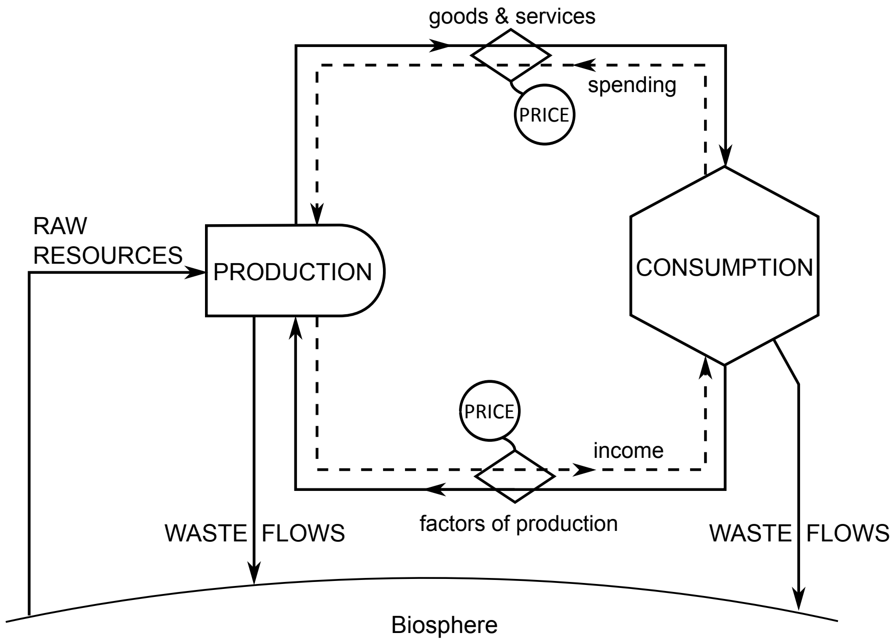

Fig. 1.3 A comprehensive model of the economy, fully consistent with the laws of thermodynamics must include degraded resources (waste) expelled to the environment as a necessary consequence of economic activity.

intensities of value flows can be estimated. This extended approach is depicted in Figure 1.4C.

Both the original Leontief I-O framework and the extensions cited above assume steady-state conditions in an economy, i.e., flows of value and material into and out of each economic sector are in balance. Dynamic or transient behavior of the economic system is not considered. Thus, there is no accumulation of economic factors or embodied energy within any of the sectors. The analysis techniques provide "snapshots" of economic activity at an instant in time.

[MIK'S NEW ADDITION]

Assuming no accumulation of materials, within economic sectors or society itself, is tantamount to assuming that *all* material flows through the economy are directed toward the production of non-durable goods. However, evidence of the durability of goods and the accumulation of materials surrounds us. Furthermore, energy was required to both fabricate and emplace the durable goods and infrastructure of modern economies. (The energy it took to create the durable goods and infrastructure can be considered "embodied" within the built environment, a point to which we will return in detail later). As Georgescu-Roegen notes, "in the everyday world one cannot possibly cross a river only on the flow of maintenance materials of a non-existent bridge."[10]

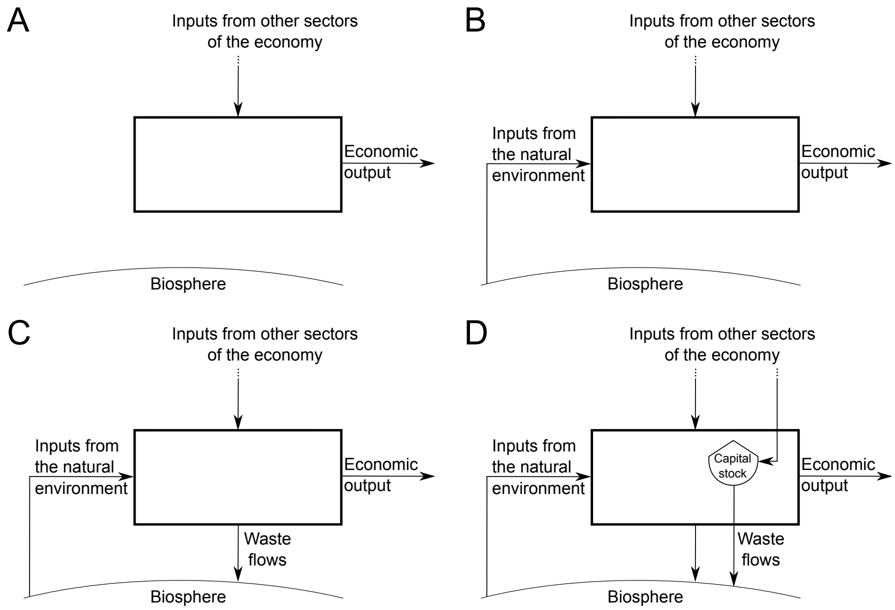

Fig. 1.4 The basic unit of input-output modeling: **A** the standard economic approach includes only transactions among sectors of the economy; **B** the ecological economics approach models inputs from the natural environment outside the economy as factors of production; **C** including waste flows to the environment makes the model physically consistent and; **D** the method presented here accounts also for accumulation in capital stock, *K*, of embodied energy within materials in economic sectors. [SHOULD WE HAVE THE LABEL "K" IN THE ACCUMULATION, OR JUST HAVE THE TANK?]

Analysis methods that neglect the accumulation of materials and embodied energy in the durable goods and infrastructure of the everyday world lack explanatory power. Such models can tell us at what rates materials and energy are required to *use* our built environment. But, such models cannot tell us *how* the built environment came to be (and how much energy was required to construct it) or *why* flows of goods are needed. To use Georgescu-Roegen's imagery, models that neglect accumulation fail to explain why we need any material flows to maintain a non-existent bridge. Stocks of accumulated materials (capital, appliances, even people) are the drivers of demand. It is to service their needs and wants that we put the economy to work.

Because economic activity requires energy, we need to understand the way energy flows through economies. The steady-state I-O techniques of Bullard, Herendeen, and others[4, 6] offer a means to that end. We contend, however, that these techniques need to be extended and modified to include transient effects that arise when durability of goods and infrastructure (and associated embodied energy) are considered. This manuscript attempts to address that need.

1.4 An I-O method for dynamic (transient) economic analysis

In this manuscript, we develop a physical input-output, matrix-based method for modeling multi-sector economies, in the tradition of Georgescu-Roegen's "flow-fund" model.[11, 12] The method presented in this paper takes a decidedly engineering approach **** Need to re-cast in metabolism language!!!! **** to extend the techniques of Bullard, Herendeen, and others to account for durability of goods and embodied energy. This method allows us to see how energy and materials flow through the economy, where embodied energy accumulates in the economy, and how declining resource quality may affect these dynamics. [NEED TO MAKE SURE WE ACHIEVE THIS LAST POINT]

The remainder of this manuscript is organized as follows. Chapter 2 presents a discussion of material flows through economies. Flows of direct energy are discussed in Chapter 3, and a rigorous, thermodynamics-based definition of and accounting for embodied energy is presented in Chapter 4. Flows of economic value and the energy intensity of economic output are discussed in Chapters 5 and 6, respectively. Finally, Chapter 7 looks ahead and draws some implications for the future economic growth.

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Part I
Material and Energy Flows

Chapter 2
Material flows

In the Introduction, we introduced the idea that economies are like organisms. This chapter explores this idea further by observing the interchange of materials *within* an economy, as well as exchanges of materials between an economy and surrounding environment—the biosphere.

There are many easily observable instances of material flow within an economy. I look around my office at my computer screen and coffee cup and myriad other items. I look out my window to the street and building opposite. All of these goods came originally from natural resources, be it paper or petroleum or rock. They were extracted and processed, transported and transformed requiring yet more materials and energy inputs in the form of electricity or fuels.

There are also inumerable material flows caused by an economy that we do not observe. The extraction of raw materials generates additional overburden—earth that must be extracted and processed and ultimately discarded without ever entering the economy proper. Other flows occur around us unseen. The cars outside my window suck in nitrogen and oxygen (without which the engine would not work) and emit water, carbon dioxide and other more harmful substances.

Even services which we tend to think of as non-material, require at least some material infrastructure. The hairdresser requires scissors (and to a greater or lesser extent some hair) with which to work. Even the internet, often lauded as the exemplar of dematerialization of the economic process, requires a whole host of computer infrastructure including electricity, data servers, telephone networks and a computer by which to access it.

In this chapter, we will define a mathematical framework by which to track the flow of materials within an economy, building from a one-sector economy up to examples of both a two- and three-sector economy. We will finally apply this framework to the illustrative example of the US automobile industry that runs through the whole book. First let us outline the basic methodology.

2.1 Methodology

This book is about tracking (accounting) flows through the economy with a focus on counting materials, energy, and value. That an entire academic discipline and industry are focused on counting money ("accounting") is evidence of its importance in today's economies. That energy is required to do *anything* is evidence of its importance in the economic activity of our daily lives. And, we believe that the interplay between money and energy has shaped the past and will continue to influence the future. In this section, we define rigorous "counting" methods that will be applied to money and energy throughout this book.

Everyone counts material (and even non-material) things. Rigorous counting requires precise definition of both what we will be counting and the place (defined in both time and space) in which we will be counting. Engineers often call the definition of space a "control volume." Another way to think of creating a control volume is drawing a boundary. What gets counted is what passes through the boundary. For example, I may wish to count (or "make an accounting of") what happens to the stock of apples in my home over a week-long period. I've drawn a boundary around my home and around a week-long time period. I count the apples that enter and leave my home, apples that are eaten (consumed), and, if I own an apple tree, apples that I grow (produce) during a week. A rigorous apple accounting equation, in units of [apples], is:

$$\Delta \text{apples} = \text{apples in} - \text{apples out} + \text{apples grown} - \text{apples eaten}. \tag{2.1}$$

More generally, we may say:

$$\text{Accumulation} = \text{Transfers in} - \text{Transfers out} + \text{Production} - \text{Consumption}. \tag{2.2}$$

Notice that, when discussing apples we use the specific terms, "grown" and "eaten" instead of the more general terms, "produced" and "consumed." Later, in Chapter 5, when discussing value, we will use the terms "generated" and "destroyed." For our purposes, these terms all have the equivalent sets of meaning and we use them inter-changeably.

Once I account for how much the stock of apples changes in a weekly period, I can reframe my question to ask, "how fast does the stock of apples change." That is, I can examine the rate of change of the apple stock per time unit in relation to the flow of apples per time unit ($\dot{a}$), e.g. in units of apples per day, in which case our accounting equation would become:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \dot{a}_{in} - \dot{a}_{out} + \dot{a}_{grown} - \dot{a}_{eaten} \tag{2.3}$$

where the dot above the variable ($\dot{a}$) indicates the flow rate per unit time [apples/time] and the time derivative $\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)$ is the rate of change of the stock of apples per time unit, or more simply, the accumulation rate.

Notice, that instead of focusing on apples as our unit of accounting, we could track the mass flow, measured in kg, of the main chemical elements within the apples. From this perspective, although an apple may be consumed, the elements within the apple—hydrogen, oxygen (coupled together as water for the overwhelming majority of the mass) and carbon (which, bonded with hydrogen as carbohydrates make up most of the remaining mass)—will not be consumed. They will instead be subsumed within my body, or leave my house as waste or via the air.

Throughout this book, we will illustrate theoretical concepts with a running example of the auto industry. We choose the auto industry, because it remains a large portion of most industrialized economies, because is very resource intensive, because it has been used in the literature (Reference Bullard and Herendeen here) to illustrate input-output accounting methods, because its links with energy are obvious, because its health is sensitive to disruptions in energy supplies, and because it shows evidence of post-industrial decline (shrinking profit margins, etc.).

If we want to account for steel [kg] in the auto industry, we might write an equation like this:

$$\Delta \text{steel} = \text{steel in} - \text{steel out} \tag{2.4}$$

Note that the production and consumption terms are zero since steel is not created or destroyed within the automobile sector. Tracking the rate flows of steel, $\dot{s}$ [kg/s], we would write the following equation:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \dot{s}_{in} - \dot{s}_{out} \tag{2.5}$$

Again, the last two terms are zero. This is in direct contrast with the apple accounting equation outlined in Equation 2.3. Despite the fact that steel is not produced or consumed within the automobile sector, there are sectors of the economy that *do* produce steel. They do this by mixing molten iron with varying amounts of carbon. This illustrates that while certain material products, e.g. steel, may be produced or, in some circumstances destroyed, the *mass* of iron, and other chemical elements, cannot be created or distroyed, though it may change form through the process.[1]

Indeed, within the car industry, inputs of steel, glass, plastic, rubber, etc. are used to produce cars, such that cars are created within the automobile industry. Were we to have an accounting equation tracking cars within the economy, we would have to have terms in the equation for production and destruction of cars. Again, focussing on mass flows of the chemical elements avoids this necessity, since mass

[1] For the sake of absolute rigor, we must point out that, in actuality, iron *is* created within the core of silicon-burning stars. Mass and energy may also be converted in such processes, such that only mass-energy is conserved. However, for the purposes of terrestrial processes, the amount of iron is constant. There are, additionally, some economic processes, within nuclear reactors, that change the atomic structure of elements and thus violate the accounting law presented here. Since the mass flows involved with these nuclear plants is negligible compared with total materials flows, we shall assume that the law holds.

is *conserved* in physical processes. This conservation of mass is a statement of the First Law of Thermodynamics, which says that *mass*, i.e. materials, and *energy* can neither be created nor destroyed.

In the discussion that follows, we will make great use of this law. If I eat an apple, it is no longer an apple, but the materials (i.e. chemical elements) and energy contained within the apple can still be traced via their mass and energy, even if they change form (apples into compost or chemical potential energy to thermal energy). Thus, the apple accounting equation (Equation 2.3) can include terms accounting for the production and consumption of apples, but mass and energy equations applied to sectors of economies will *not* include terms for the production or destruction of materials and energy. Rather, any addition of material or energy *to* the economy or waste of material or energy *from* the economy will occur as an interaction between the economy and the biosphere.

When applying accounting equations to economic sectors, we distinguish among four types of materials flowing into or out of a production sector: products (P) , resources (R), short-lived goods (S), and capital goods (K), as shown in Figure 2.1.

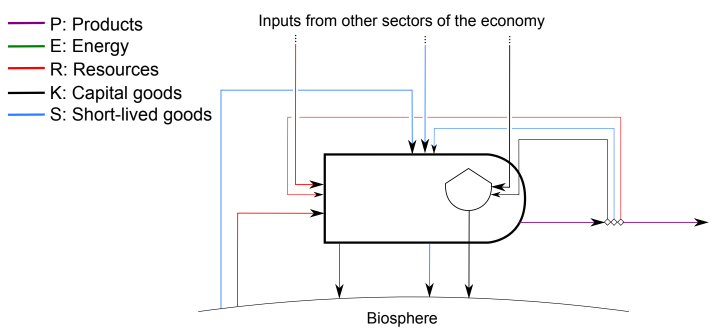

Fig. 2.1 Material flows into and out of a single sector of the economy. Resource flows ($\dot{R}$) enter the sector from the left and are embodied in products ($\dot{P}$) which leave from the right. Some waste resources are leave the sector at the bottom and are returned to the biosphere. Short-lived material flows ($\dot{S}$) enter the sector from above and leave from below to return to the biosphere. Only capital stock ($\dot{K}$) may accumulate within the sector, depicted by the storage tank. These also enter the sector from above. Depreciated capital leaves the sector from below and is returned to the biosphere.

Resource materials ($\dot{R}$) enter the sector on the left and comprise those materials that are destined to be *embodied* in the goods produced by the sector ($\dot{P}$), except for some proportion that are wasted. Wastes depart from the bottom of the sector and are returned to the biosphere. For example, sheet metal, rubber, and glass (as well as many other materials) enter the automobile sector as resources and end up as material parts of the cars that are produced. Some fraction of these resources ($\dot{R}$) may not make it into the final product, such as trimming scrap from metal parts stamping, and may be either recycled internally, or wasted to the biosphere. Resource materials are not accumulated within a sector.

Short-lived goods ($\dot{S}$) include those materials that are necessary for the production processes of a sector, but are neither accumulated within the sector, nor destined to be materially part of the product of the sector. They enter the sector from above and leave the sector from below and return to the biosphere. Examples of these short-lived flows include energy resources, such as the electricity needed to run automobile factories, including any contribution from labor, and water used by the sector.

A number of material flows, such as production equipment, are necessary for the continued operation of a sector but are not counted as short-lived goods, because the operation of the sector is dependent upon the accumulation of these materials within the sector. Such flows are counted as capital goods ($\dot{K}$) . Capital flows also enter from above, but are stored within the sector (represented by storage tanks) and are returned to the biosphere as capital depreciation from below. Examples of these capital flows would be the factory and office buildings or manufacturing equipment within the automobile industry.

All products ($\dot{P}$) leave the right of the sector. Some of this $\dot{P}$ flow is returned to the sector as self-consumption counted as resources ($\dot{R}$), short-lived ($\dot{S}$), or capital goods ($\dot{K}$); the remainder flows to other sectors within the economy or final demand. Within this view, focussing solely on material flows, energy may be accounted as either an $\dot{R}$ flow, when the energy inflow is *literally* embodied within the outflowing product $\dot{P}$, such as crude oil converted into gasoline within a refinery, or as an $\dot{S}$ flow, when the inflowing energy is not literally embodied within the product. Examples include the electricity use by an automobile factor, but also the coal or natural gas flowing into a power plant, since the incoming chemical elements (carbon and hydrogen) *do not* travel through the electricity transmission lines.

2.2 Example A: single-sector economy

Our first example looks at the case where all processes within the economy occur within one sector—society (1)—which exchanges materials with the biosphere (0) as depicted in Figure 2.2. We do not distinguish between production and consumption.

Resources, or perhaps more accurately raw materials, ($\dot{R}_{01}$), such as crude oil or iron ore, and short-lived materials ($\dot{S}_{01}$), such as oxygen or water that flow *through* economic processes but are not *embodied* in the product output, flow into the economy (1) from the biosphere (0). These materials are processed within the economy into products ($\dot{P}_1$) consisting of resource goods ($\dot{R}_{11}$), short-lived goods ($\dot{S}_{11}$) and also capital goods ($\dot{K}_{11}$), which are able to be accumulated at some rate $\frac{\mathrm{d}K_1}{\mathrm{d}t}$. Waste resources ($\dot{R}_{10}$) and used short-lived materials/goods ($\dot{S}_{10}$) are returned to the biosphere without accumulating. Capital goods ($\dot{K}_{10}$) are returned to the biosphere upon depreciation.

Drawing control volumes around both the biosphere (0) and the economy (1), we can consruct our material accounting equations, such that:

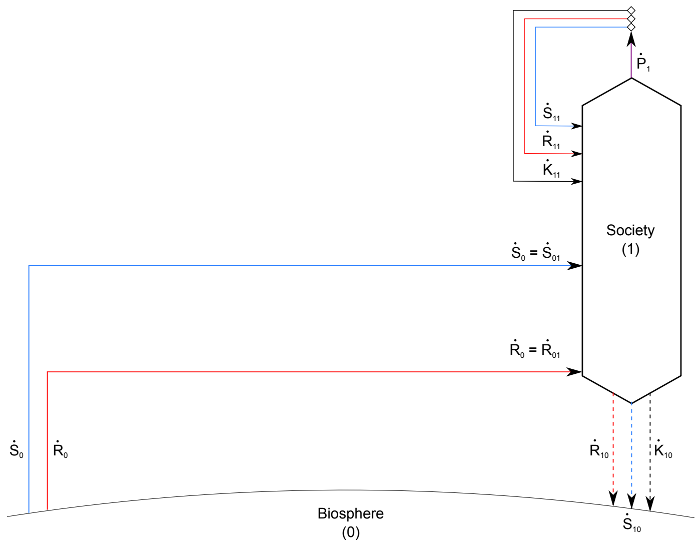

Fig. 2.2 Flows of materials for a one-sector economy
Flows of materials for a one-sector economy. Resources ($\dot{R}_{01}$) and short-lived materials ($\dot{S}_{01}$) flow into the economy (1) from the biosphere (0). Waste resources ($\dot{R}_{10}$) short-lived materials/goods ($\dot{S}_{10}$) and capital goods ($\dot{K}_{10}$) are returned to the biosphere.

$$\frac{dR_0}{dt} + \frac{dS_0}{dt} + \frac{dK_0}{dt} = \dot{R}_{10} + \dot{S}_{10} + \dot{K}_{10} - \dot{R}_0 - \dot{S}_0 \tag{2.6}$$

$$\frac{dR_1}{dt} + \frac{dS_1}{dt} + \frac{dK_1}{dt} = \dot{R}_{01} + \dot{S}_{01} + \dot{R}_{11} + \dot{S}_{11} + \dot{K}_{11} - \dot{P}_1 - \dot{R}_{10} - \dot{S}_{10} - \dot{K}_{10} \tag{2.7}$$

Remembering that neither resources (R) nor short-lived goods (S) accumulate within economic sectors, we may state:

$$\frac{dS_1}{dt} = \frac{dR_1}{dt} = 0. \tag{2.8}$$

Additionally, since mass is conserved, we may also say that:

$$\dot{S}_{11} = \dot{P}_1, \tag{2.9}$$

such that Equations 2.6 and 2.7 become:

$$\frac{dR_0}{dt} + \frac{dS_0}{dt} + \frac{dK_0}{dt} = \dot{R}_{10} + \dot{S}_{10} + \dot{K}_{10} - \dot{R}_0 - \dot{S}_0 \tag{2.10}$$

$$\frac{dK_1}{dt} = \dot{R}_{01} + \dot{S}_{01} - \dot{R}_{10} - \dot{S}_{10} - \dot{K}_{10}. \tag{2.11}$$

Equation 2.11 indicates that the accumulation of capital within society is the imbalance between the flow of materials pulled from the biosphere ($\dot{R}_{01} + \dot{S}_{01}$) and the flow rate of materials disposed back to the biosphere ($\dot{R}_{10} + \dot{S}_{10} + \dot{K}_{10}$). Because the only "capital" that accumulates in the biosphere is that which is a waste flow (capital depreciation) from the economy, e.g. worn-out machines in the scrap yard, we may say that:

$$\frac{\mathrm{d}K_0}{\mathrm{d}t} = \dot{K}_{10} \tag{2.12}$$

2.3 Example B: two-sector economy

In our second example B, we split society into two sectors: a production sector (2) and society (1) which acts as a consumer, as depicted in Figure 2.3. Sector (2) produces all of the goods and services that are delivered to final consumption within society, as well as all of the intermediate goods that are not "consumed" within society but stay within the production sector, e.g. manufacturing equipment.

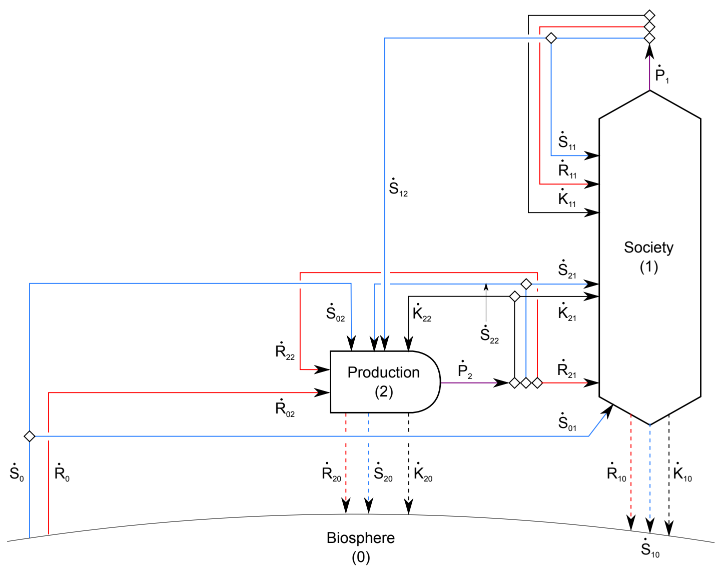

Fig. 2.3 Flows of materials for a two-sector economy
Flows of materials for a two-sector economy

Again, setting control volumes around the biosphere and our two economic sectors, material accounting equations become:

$$\frac{dR_0}{dt}+\frac{dS_0}{dt}+\frac{dK_0}{dt}=\dot{R}_{10}+\dot{R}_{20}+\dot{S}_{10}+\dot{S}_{20}+\dot{K}_{10}+\dot{K}_{20}-\dot{R}_0-\dot{S}_0 \tag{2.13}$$

$$\frac{dR_1}{dt}+\frac{dS_1}{dt}+\frac{dK_1}{dt}=\dot{R}_{21}+\dot{S}_{01}+\dot{S}_{11}+\dot{S}_{21}+\dot{K}_{21}-\dot{S}_1-\dot{R}_{10}-\dot{S}_{10}-\dot{K}_{10}, \tag{2.14}$$

$$\frac{dR_2}{dt}+\frac{dS_2}{dt}+\frac{dK_2}{dt}=\dot{R}_{02}+\dot{R}_{22}+\dot{S}_{02}+\dot{S}_{12}+\dot{S}_{22}+\dot{K}_{22}-\dot{P}_2-\dot{R}_{20}-\dot{S}_{20}-\dot{K}_{20}, \tag{2.15}$$

One point worthy of noting is that what we are here denoting as a flow of 'capital goods' into society ($\dot{K}_{21}$) would more normally be referred to by economist as 'consumer durables', though would also include other products, such as housing. The important concept being that some goods (fridges, televisions, apartment blocks) may accumulate within society and would be represented within flow $\dot{K}_{21}$, whereas other short-lived goods (newspapers, plastic packaging, electricity) do not accumulate and are represented within flow $\dot{S}_{21}$.

Resource flow $\dot{R}_{21}$ into society represents the material flow that will be embodied within the 'product' of society—the human population—i.e. food produced by the agriculture industry. [WE NEED TO DISCUSS IF WE WANT TO ACCOUNT IT IN THIS WAY] Since no resources flow directly into society from the biosphere, we may say:

$$\dot{R}_0=\dot{R}_{02}. \tag{2.16}$$

In contrast, short-lived materials may flow directly into society from the biosphere; e.g. the flow of photons in sunlight or oxygen into car engines and lungs; we can redefine flows $\dot{S}_0$ and $\dot{S}_1$:

$$\dot{S}_0=\dot{S}_{01}+\dot{S}_{02}; \qquad \dot{S}_1=\dot{S}_{11}+\dot{S}_{12} \tag{2.17}$$

Since only capital flows ($\dot{K}$) may be accumulated and are dependent only on flows of capital in and depreciation of capital, we may define capital balance equations:

$$\frac{dK_1}{dt}=\dot{K}_{12}-\dot{K}_{20}, \tag{2.18}$$

[NOT SURE IF THIS IS TRUE IF WE THINK OF $\dot{R}_{12}$ AS FOOD AND K_1 AS INCLUDING HUMANS... YES, I THINK $\dot{R}_{12}$ CAN BE TURNED INTO $\dot{K}_1$ INTERNALLY, AS THE ACCUMULATION OF (LITERAL) HUMAN CAPITAL, I.E. POPULATION]

$$\frac{dK_2}{dt}=\dot{K}_{22}-\dot{K}_{20}, \tag{2.19}$$

We can also redefine product flow $\dot{P}_2$:

$$\dot{P}_2=\dot{R}_{21}+\dot{R}_{22}+\dot{S}_{21}+\dot{S}_{22}+\dot{K}_{21}+\dot{K}_{22}, \tag{2.20}$$

Again, remembering that resources and short-lived goods do not accumulate within sectors of the economy:

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = \frac{\mathrm{d}R_2}{\mathrm{d}t} = \frac{\mathrm{d}S_1}{\mathrm{d}t} = \frac{\mathrm{d}S_2}{\mathrm{d}t} = 0, \tag{2.21}$$

and substituting Equations 2.17, 2.19 and 2.20, our balance equations may now be written:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} + \frac{\mathrm{d}S_0}{\mathrm{d}t} + \frac{\mathrm{d}K_0}{\mathrm{d}t} = \dot{R}_{10} + \dot{R}_{20} + \dot{S}_{10} + \dot{S}_{20} + \dot{K}_{10} + \dot{K}_{20} - \dot{R}_0 - \dot{S}_0 \tag{2.22}$$

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{21} + \dot{S}_{01} + \dot{S}_{21} + \dot{K}_{21} - \dot{S}_{12} - \dot{R}_{10} - \dot{S}_{10} - \dot{K}_{10}, \tag{2.23}$$

$$\dot{K}_{22} = \dot{R}_{02} + \dot{S}_{02} + \dot{S}_{12} - \dot{R}_{21} - \dot{S}_{21} - \dot{K}_{21} - \dot{R}_{20} - \dot{S}_{20} \tag{2.24}$$

[NOT SURE IF THIS IS THE BEST WAY TO WRITE THESE]

2.4 Example C: three-sector economy

In example C, we differentiate between two production sectors, sector (2) produces energy products and sector (3) produces other goods and services, as depicted in Figure 2.4.

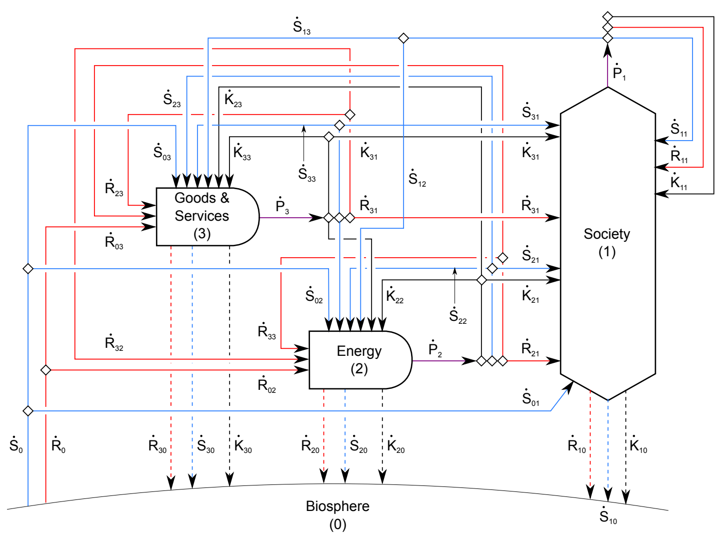

Fig. 2.4 Flows of materials for a three-sector economy
Flows of materials for a three-sector economy

Accounting for the material flows into and out of the biosphere (sector 0) gives the following equation:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t}+\frac{\mathrm{d}S_0}{\mathrm{d}t}+\frac{\mathrm{d}K_0}{\mathrm{d}t}=\dot{R}_{10}+\dot{R}_{20}+\dot{R}_{30}+\dot{S}_{10}+\dot{S}_{20}+\dot{S}_{30}+\dot{K}_{10}+\dot{K}_{20}+\dot{K}_{30}-\dot{R}_0-\dot{S}_0, \tag{2.25}$$

which may be rewritten as:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t}+\frac{\mathrm{d}S_0}{\mathrm{d}t}+\frac{\mathrm{d}K_0}{\mathrm{d}t}=\sum_{i=1}^{3}\dot{R}_{i0}+\sum_{i=1}^{3}\dot{S}_{i0}+\sum_{i=1}^{3}\dot{K}_{i0}-\dot{R}_0-\dot{S}_0, \tag{2.26}$$

Similarly, flows for the other sectors may be written:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t}=\dot{R}_{01}+\dot{S}_{01}+\sum_{i=1}^{3}\dot{R}_{i1}+\sum_{i=1}^{3}\dot{S}_{i1}+\sum_{i=1}^{3}\dot{K}_{i1}-\dot{P}_1-\dot{R}_{10}-\dot{S}_{10}-\dot{K}_{10}, \tag{2.27}$$

$$\frac{\mathrm{d}K_2}{\mathrm{d}t}=\dot{R}_{02}+\dot{S}_{02}+\sum_{i=1}^{3}\dot{R}_{i2}+\sum_{i=1}^{3}\dot{S}_{i2}+\sum_{i=1}^{3}\dot{K}_{i2}-\dot{P}_2-\dot{R}_{20}-\dot{S}_{20}-\dot{K}_{20}, \tag{2.28}$$

$$\frac{\mathrm{d}K_3}{\mathrm{d}t}=\dot{R}_{03}+\dot{S}_{03}+\sum_{i=1}^{3}\dot{R}_{i3}+\sum_{i=1}^{3}\dot{S}_{i3}+\sum_{i=1}^{3}\dot{K}_{i3}-\dot{P}_3-\dot{R}_{30}-\dot{S}_{30}-\dot{K}_{30}. \tag{2.29}$$

Equations 2.27 through 2.29 may be summarized in one single equation as:

$$\frac{\mathrm{d}K_j}{\mathrm{d}t}=\dot{R}_{0j}+\dot{S}_{0j}+\sum_{i=1}^{3}\dot{R}_{ij}+\sum_{i=1}^{3}\dot{S}_{ij}+\sum_{i=1}^{3}\dot{K}_{ij}-\dot{P}_j-\dot{R}_{j0}-\dot{S}_{j0}-\dot{K}_{j0};\quad j\in[1,3]\,. \tag{2.30}$$

Again, we can say that:

$$\frac{\mathrm{d}K_j}{\mathrm{d}t}=\sum_{i=1}^{3}\dot{K}_{ij}-\dot{K}_{j0}; \tag{2.31}$$

[AGAIN, NOT SURE IF THIS HOLDS FOR SECTOR (1), AS DISCUSSED EARLIER]
hence we may say that:

$$\dot{R}_{0j}+\dot{S}_{0j}+\sum_{i=1}^{3}\dot{R}_{ij}+\sum_{i=1}^{3}\dot{S}_{ij}=\dot{P}_j-\dot{R}_{j0}-\dot{S}_{j0}. \tag{2.32}$$

The flows of resources ($\dot{R}$) and short-lived goods ($\dot{S}$) between each of the biosphere and three economic sectors may be thought of as four matrices (as depicted in Figure 2.5 for $\dot{S}$ flows): one 3×3 matrix of flows within the economy, a 3×1 vector of flows from the biosphere into the economy (extraction), a 1×3 vector of flows from the economy into the biosphere (waste) and a 1×1 matrix of flows solely within the biosphere (environment), that do not enter the economy.

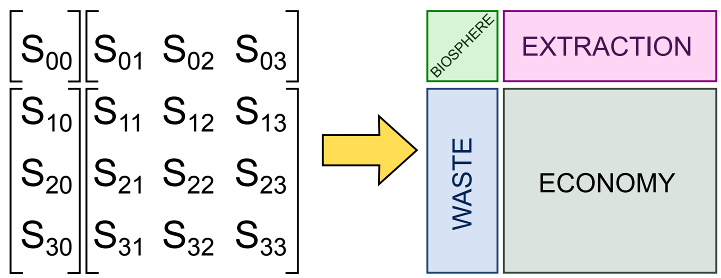

Fig. 2.5 The matrix of biosphere-economy flows.

2.5 Materials in the auto industry

2.6 Summary

References

Chapter 3
Direct energy flows

In Chapter 1, we formulated a model of economies consisting of producers and consumers who exchange goods and services and factors of production while extracting resources and disposing of wastes. In Chapter 2, we established the material basis of economies: economies are analogous to organisms with metabolisms that processes raw resources for the benefit of producers and consumers while generating unavoidable wastes. In this chapter, we describe and analyze the direct energy that is associated with material flows through an economy.

All forms of energy provide the potential to do mechanical work. The quantification of the mechanical work potential of energy is *exergy*. When energy is "consumed" by an economy, exergy (work potential) is destroyed. Energy (as work potential) is an essential aspect of the metabolic economy; with it, materials are refined, shaped, and assembled into useful intermediate and consumption products; food is made available to people in society; jobs are made easier for workers; human ingenuity is multiplied; and complex systems and civilizations are possible. In the absence of high rates of energy availablity at low cost, life becomes much more difficult, even impossible, for many people.

Our analogy is this: energy is to thermodynamics as currency is to financial accounting. In other words, energy is the currency of thermodynamics. Just as an accountant understands a firm by watching how and where currency flows through it, so we can understand an economy by watching how and where energy flows through it. Accounting for energy flows through an economy is essential for developing a dynamic picture of its metabolism.

The purpose of this chapter is to develop a model for energy flows within economies. With an energy model in hand, we will be positioned to assess the rate at which energy becomes embodied within the products and services that an economy provides (Chapter 4).

3.1 Methodology

We begin by noting that direct energy travels with material through an economy. "Direct" energy refers to forms of energy accounted by the First Law of Thermodynamics, including chemical potential energy, nuclear potential energy, gravitational potential energy, thermal energy, and kinetic energy. We use the term "direct" energy distinct from "embodied" energy, which will be discussed in Chapter 4. Examples of direct energy flows include the chemical potential energy of coal inflows to an energy sector, the thermal energy of process steam into a textile plant, and the thermal energy of CO_2 automobile exhaust. In each case, the material (coal, steam, and CO_2) carries direct energy with it.[1] Figure 3.1 shows a corresponding direct energy flow for each material flow of Figure 2.1.

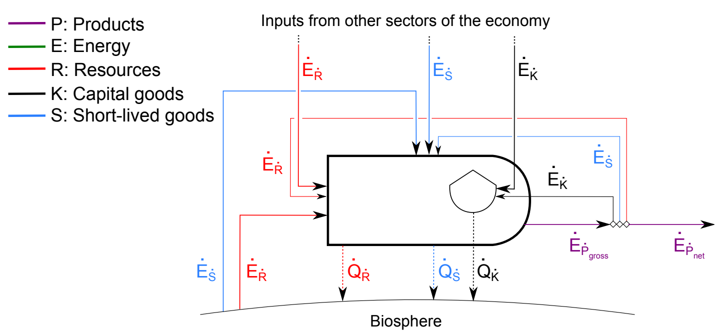

Fig. 3.1 Energy content of material flows for a single sector
Energy content ($\dot{E}$) of material flows ($\dot{R}$, $\dot{S}$, and $\dot{K}$) from Figure 2.1.

For any boundary (around, say, a machine, a plant, a sector of the economy, or the entire economy itself), the First Law of Thermodynamics says that the accumulation rate of direct energy (E) within the boundary is equal to the sum of the signed direct energy flow rates ($\dot{E}$) across the boundary (where inflows are positive and outflows are negative) less outflowing energy carried by wastes ($\dot{Q}_{out}$): energy is conserved.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \sum \dot{E} - \sum \dot{Q}_{out} \tag{3.1}$$

When there is no accumulation of direct energy within the boundary $\left(\frac{\mathrm{d}E}{\mathrm{d}t} = 0\right)$, the sum of all signed direct energy flow rates ($\dot{E}$) and waste heats ($\dot{Q}_{out}$) will be zero.

$$0 = \sum \dot{E} - \sum \dot{Q}_{out} \tag{3.2}$$

It is important to note that the direct energy associated with some material flows can be so small as to be negligible compared to other direct energy flows in the

[1] Even radiative thermal energy can be considered a material flow (photons) that carries direct energy (thermal) with it.

economy. For example, the direct energy of steel into the automobile sector of the economy is almost negligible. (The *embodied* energy of the steel is almost certainly *not* negligible, as will be discussed in Chapter 4.) On the other hand, the direct energy flow rates for fossil fuels (coal, oil, and natural gas) are typically orders of magnitude larger than any other material flows due to large chemical potential energy content.

To simplify the direct energy analysis, we can aggregate the direct energy flows of Figure 3.1 into single arrows when appropriate. For example, the direct energy inputs from other sectors of the economy (labeled as $\dot{E}_{\dot{R}}$, $\dot{E}_{\dot{S}}$, and $\dot{E}_{\dot{K}}$ at the top of Figure 3.1) can be summed to $\dot{E}$ (in Figure 3.2) such that

$$\dot{E}_{\text{Fig. 3.2}} = \dot{E}_{\dot{R},\,\text{Fig. 3.1}} + \dot{E}_{\dot{S},\,\text{Fig. 3.1}} + \dot{E}_{\dot{K},\,\text{Fig. 3.1}}. \tag{3.3}$$

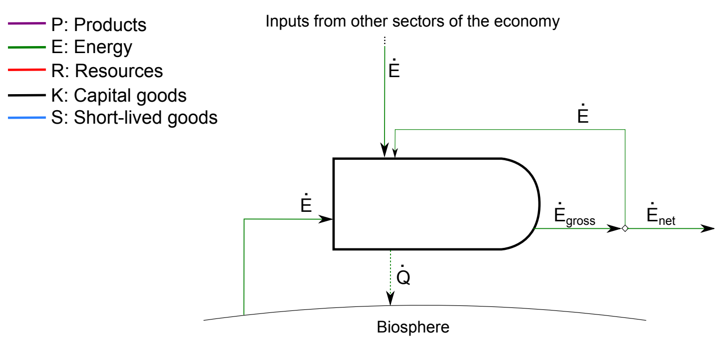

Fig. 3.2 Aggregated direct energy flows for a single sector
Aggregated direct energy flows ($\dot{E}$) around the producer of Figure 3.1.

3.2 Example A: single-sector economy

Aggregated direct energy flows are now applied to Example A, the single-sector economy shown in Figure 2.2. By summing the direct energy flows associated with each material flow of Figure 2.2, we obtain a simplified picture of direct energy flows in the economy, as shown in Figure 3.3.[2]

We distinguish useful direct energy inputs to a sector of the economy ($\dot{E}_{01}$ in Figure 3.3) from wasteful direct energy flows ($\dot{Q}_{10}$ in Figure 3.3), because $\dot{Q}$ typically denotes thermal energy, and many waste energy flows are in the form of thermal

[2] Single subscripts on quantities such as E can mean one of two things: $\dot{E}_i$ indicates the outflow of direct energy from sector i, whereas E_i denotes the direct energy content of sector i. Double subscripts on quantities (e.g., $\dot{E}_{ij}$) indicate a flow from sector i to sector j. The first index always indicates the sector *from* which a quantity flows, and the second index indicates the sector *to* which a quantity flows.

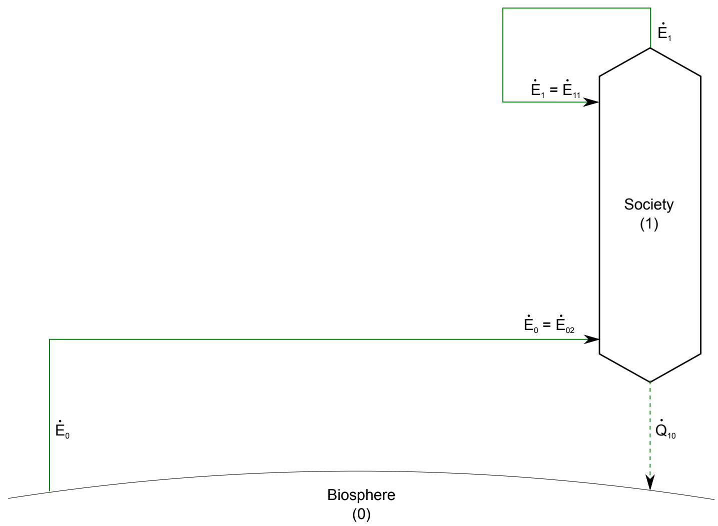

Fig. 3.3 Direct energy flows a one-sector economy
Direct energy flows ($\dot{E}$) a one-sector economy.

energy, i.e., waste heat. In Figure 3.3, direct energy input to the economy ($\dot{E}_{01}$) is shown as being extracted from the biosphere, because the vast majority of direct energy today is derived from fossil fuels. Waste heat from the economy ($\dot{Q}_{10}$) is shown as returning to the biosphere.

As discussed in Section 3.1, both direct energy ($\dot{E}$), and waste heat ($\dot{Q}$) are accounted by the First Law of Thermodynamics. Accounting for possible accumulation of direct energy, the First Law of Thermodynamics for Example A indicates that

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{Q}_{10} - \dot{E}_{01} \tag{3.4}$$

and

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{01} + \dot{E}_{11} - \dot{E}_1 - \dot{Q}_{10}. \tag{3.5}$$

Note that $\dot{E}_1$ is the gross direct energy production rate of society. For example, firms extract crude oil (a component of $\dot{E}_{01}$) and refine it into petroleum products (a component of $\dot{E}_1$) that are consumed by society. The direct energy consumption of extraction and refining firms is a component of $\dot{E}_{11}$.

Aside from, for example, the U.S. Strategic Petroleum Reserve, we are not stockpiling oil and coal at any meaningful rate, i.e. we consume fossil fuels at a rate equal to their extraction rate. Thus, the world is not accumulating direct energy in the economy.[3] (The world *is*, however, accumulating *embodied* energy in the economy as we shall see in Chapter 4.) Thus, the accumulation rates for direct energy $\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)$ in the above equations could be set to zero as follows:

$$0 = \dot{Q}_{10} - \dot{E}_{01} \tag{3.6}$$

and

$$0 = \dot{E}_{01} + \dot{E}_{11} - \dot{E}_{1} - \dot{Q}_{10}. \tag{3.7}$$

However, we shall see later (in Chapter 3) that keeping direct energy accumulation terms $\left(\frac{\mathrm{d}\dot{E}}{\mathrm{d}t}\right)$ provides an advantage when deriving embodied energy accounting equations.

3.3 Example B: two-sector economy

For Example B, we split an economic sector (2) from society (1). Figure 3.4 shows aggregated direct energy flows associated with the material flows of Figure 2.3.

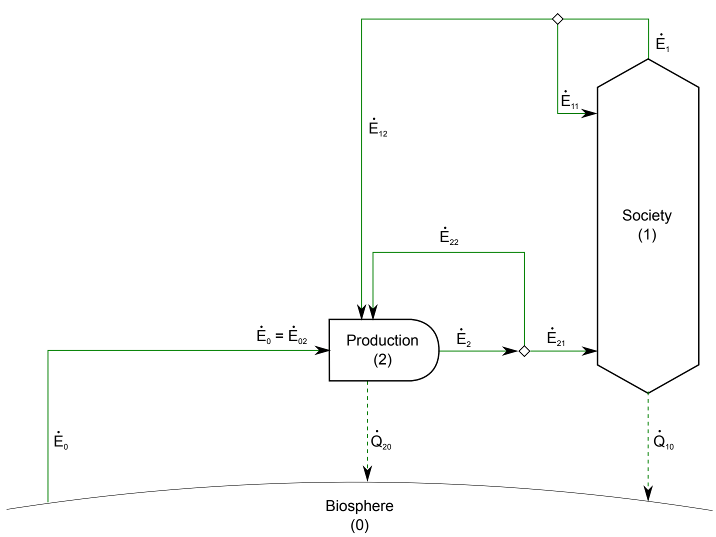

Fig. 3.4 Direct energy flows for a two-sector economy

Direct energy flows ($\dot{E}$) for a two-sector economy

[3] A counter-example could be made for nuclear fuels where "spent" fuel represents a large exergetic stockpile. However, this reserve is not (presently) economically useful.

The First Law of Thermodynamics requires that both direct energy and waste heat be conserved around each entity (1 and 2) as well as around the biosphere (0).

First Law energy accounting around the biosphere (0) and society (1) gives

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{Q}_{10} + \dot{Q}_{20} - \dot{E}_{02}, \tag{3.8}$$

and

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{11} + \dot{E}_{21} - \dot{E}_1 - \dot{Q}_{10}. \tag{3.9}$$

Note that $\dot{E}_{12}$ represents useful work that people and draught animals contribute to Production (2). Ayres and Warr [1, 2] call this "muscle work." $\dot{E}_{11}$ represents the muscle work required for consumption. Direct energy required for consumption by final demand (electricity, oil, natural gas, etc.) is included in $\dot{E}_{21}$.

The First Law around the economy (2), including the accumulation rate of direct energy in the sector $\left(\frac{\mathrm{d}E_2}{\mathrm{d}t}\right)$, yields

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \dot{E}_{02} + \dot{E}_{12} + \dot{E}_{22} - \dot{E}_2 - \dot{Q}_{20}. \tag{3.10}$$

It is notable that the economy (2) consumes ($\dot{E}_{22}$) a portion of its gross energy output ($\dot{E}_2$): it takes energy to make energy.

Equation 3.8 can be generalized with a sum as

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \sum_{i=1}^{n} \left(\dot{Q}_{i0} - \dot{E}_{0i}\right), \tag{3.11}$$

where n is the number of economic sectors in the model (in this example, $n = 2$). Similarly, Equations 3.9 and 3.10 can generalized with a sum as

$$\frac{\mathrm{d}E_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{E}_{ij} - \dot{E}_j - \dot{Q}_{j0}, \tag{3.12}$$

where $j \in [1, n]$.

3.4 Example C: three-sector economy

We can extend Example B, to include an energy sector (2) and a goods and services sector (3), thereby obtaining a fuller picture of direct energy flows among sectors (Figure 3.5).

We note that the gross direct energy production of the energy sector (2) is $\dot{E}_2$, and the direct energy consumption of the energy sector (2) is $\dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32}$. The net direct energy production by the energy sector (2) is given by $\dot{E}_2 - (\dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32})$. The energy return on investment (*EROI*) of the energy sector (2) is given by

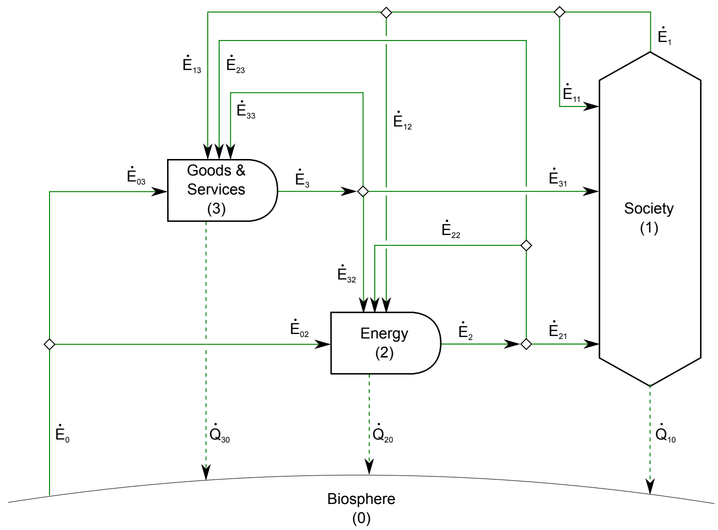

Fig. 3.5 Direct energy flows for a three-sector economy.
Direct energy flows ($\dot{E}$) for a three-sector economy.

$$EROI_2 = \frac{\dot{E}_2}{\dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32}}. \tag{3.13}$$

The First Law of Thermodynamics around the biosphere (0), society (1), and the energy sector (2) gives

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{Q}_{10} + \dot{Q}_{20} + \dot{Q}_{30} - \dot{E}_{02} - \dot{E}_{03}, \tag{3.14}$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{11} + \dot{E}_{21} + \dot{E}_{31} - \dot{E}_1 - \dot{Q}_{10}, \tag{3.15}$$

and

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \dot{E}_{02} + \dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32} - \dot{E}_2 - \dot{Q}_{20}. \tag{3.16}$$

The First Law around the goods and services sector (3) including, for now, the accumulation rate of direct energy in the sector $\left(\frac{\mathrm{d}E_3}{\mathrm{d}t}\right)$ yields

$$\frac{\mathrm{d}E_3}{\mathrm{d}t} = \dot{E}_{03} + \dot{E}_{13} + \dot{E}_{23} + \dot{E}_{33} - \dot{E}_3 - \dot{Q}_{30}. \tag{3.17}$$

Similar to Example B, we can generalize Equations 3.14–3.17 with sums to obtain

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{Q}_{i0} - \sum_{i=1}^{n} \dot{E}_{0i} \tag{3.18}$$

and

$$\frac{\mathrm{d}E_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{E}_{ij} - \dot{E}_j - \dot{Q}_{j0}, \tag{3.19}$$

where $j \in [1,n]$. Equations 3.18 and 3.19 are identical to Equations 3.11 and 3.12, indicating that we have successfully generalized the model to any number of sectors.

In this economy, the purpose of the goods and services sector (3) is to produce goods and provide services, it provides no direct energy to society. The purpose of the energy sector (2) is to make direct energy ($\dot{E}$) available to the economy and society in a useful form. We may simplify the above equations by realizing that (a) $\dot{E}_3 = \dot{E}_{3i} = 0$, because the goods and services sector is assumed to produce no direct energy, and (b) $\dot{E}_{03} = 0$, because the goods and services sector (3) receives no direct energy from the biosphere (0), except via the energy sector (2). Thus, several terms in the sums of Equation 3.19 will be zero.

3.5 Direct energy in the auto industry

**** Becky: Develop an auto industry example using real energy numbers. ****

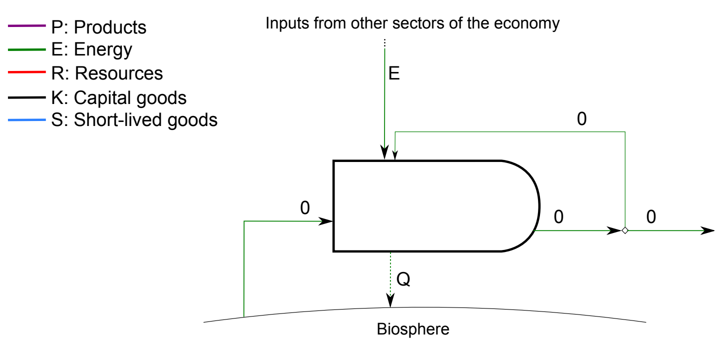

Fig. 3.6 Direct energy flows for the US Automobile Industry using data from XXXX

3.6 Summary

In the next chapter, the above direct energy equations will be used to develop *embodied* energy accounting equations for Examples A-C.

References

[1] Robert U Ayres, Leslie W Ayres, and Benjamin S Warr. Exergy, power and work in the US economy, 1900–1998. *Energy*, 28(3):219–273, mar 2003.
[2] Benjamin S Warr and Robert U Ayres. Useful work and information as drivers of economic growth. *Ecological Economics*, 73(C):93–102, January 2012.

Chapter 4
Embodied energy flows

In Chapter 3, the First Law of Thermodynamics accounted direct energy ($\dot{E}$) flowing among sectors of an economy. In this chapter, we will adapt the First Law to account *embodied* energy in the material flows of an economy.[1]

Energy can become "embodied" in the output of an economic sector and within the material in the sector itself. The energy embodied in the output of an economic sector (e.g., energy embodied in the automobiles produced by the automotive sector) is related to the sum of all direct energy consumed in the manufacture of its products, including all upstream processing stages. Embodied energy gives an indication of the energy demand created by consumption of goods and services within an economy.

Energy that becomes embodied in the materials of an economic sector (such as the machines, factories, and dealerships within the automotive sector itself) is essential for the efficient operation of the sector. The amount of energy embodied in the sector is an indicator of the complexity of the sector; the amount of energy embodied in an entire economy can be an indicator of the level of economic development of the economy.

The purpose of this chapter is to develop a model for embodied energy flows within economies. With an embodied energy model in hand, we will be positioned to develop a model for the energy intensity of goods and services within an economy (Chapter 6).

4.1 Methodology

We begin the derivation of embodied energy accounting equations by defining the concept of *total* energy.

[1] To the authors' knowledge, this is the first appearance in the literature of a systematic, detailed, and mathematically rigorous derivation of embodied energy accounting equations based upon the laws of thermodynamics.

4.1.1 Total Energy Accounting

Total energy (T) is defined as the sum of direct energy (E, see Chapter 3) and embodied energy (B).

$$T \equiv E + B \tag{4.1}$$

The flow rate of total energy ($\dot{T}$) among sectors in the economy, the biosphere, and society is the sum of direct energy ($\dot{E}$) and embodied energy ($\dot{B}$).

$$\dot{T} = \dot{E} + \dot{B} \tag{4.2}$$

Figure 4.1 illustrates that total energy flows are comprised of direct energy ($\dot{E}$) and embodied energy ($\dot{B}$).

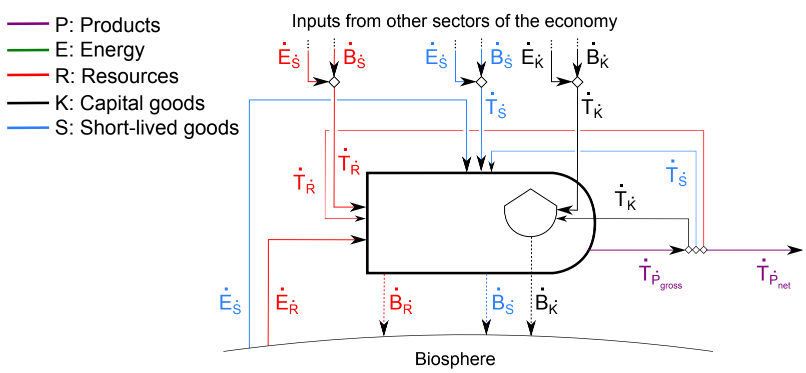

Fig. 4.1 Total energy flows for a single sector
Total energy flows ($\dot{T}$) for a single sector of an economy. For the sake of clarity, embodied ($\dot{B}$) and direct ($\dot{E}$) energy are shown separately for material inflows from other sectors only.

In some cases, a material flow may include either direct energy ($\dot{E}$) or embodied energy ($\dot{B}$), exclusive. For example, the flow of extracted crude oil from the earth consists of direct energy only ($\dot{B} = 0$ and $\dot{T} = \dot{E}$), because, in this method, no embodied energy (B) is added to the crude oil until it reaches the downstream side of the oil rig. The material produced by a non-energy sector of the economy consists of indirect energy only ($\dot{E} \approx 0$, and therefore $\dot{T} \approx \dot{B}$), because direct energy (E) produced by a non-energy sector is negligible in this economy.

In other cases, a material flow may include both direct energy flow ($\dot{E}$) *and* embodied energy flow ($\dot{B}$) components. For example, the outgoing flow of refined petroleum from the energy sector has both a direct energy ($\dot{E}$, the energy content of the oil product, usually represented by chemical potential energy) and embodied energy ($\dot{B}$, which accounts for the energy consumed in upstream processes to extract and refine the crude oil).[2]

Most of the I-O literature [1, 2] applies the following (and often unstated) assumptions:

I. flows of total energy ($\dot{T}$) are *conserved*,[3]
II. steady state conditions exist (i.e., total energy does not accumulate in economic sectors),[4] and
III. the sum of the signed (input is positive, output is negative) total energy inflows of a sector is assigned to the products of the sector (i.e., there is no "waste" of total energy).

Like the I-O literature, we assume that total energy is conserved and never wasted.[5] However, we depart from the I-O literature to allow durability of goods as represented by total energy accumulation in economic sectors. Steady state, this approach is not.

Total energy (T) may accumulate within an economic sector as stocks of direct energy materials (piles of coal or tanks of oil) but also as energy embodied in stocks of capital goods (e.g., machinery or buildings). The rate of accumulation of total energy in a sector of the economy, the biosphere, or society is given by the time derivative of total energy:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t} + \frac{\mathrm{d}B}{\mathrm{d}t}. \tag{4.3}$$

The following equation provides a total energy accounting for a sector of the economy, where the $\dot{T}$ terms are signed: positive for total energy input and negative for total energy output.

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \sum \dot{T} \tag{4.4}$$

By substituting Equations 4.2 and 4.3 into Equation 4.4, we obtain

$$\frac{\mathrm{d}E}{\mathrm{d}t} + \frac{\mathrm{d}B}{\mathrm{d}t} = \sum \left(\dot{E} + \dot{B}\right). \tag{4.5}$$

---

[2] Outputs from agricultural sectors will be similar: both (a) the direct energy component (comprising chemical potential energy) and (b) the embodied energy component (representing upstream energy consumed in food production) will be non-zero.

[3] Total energy can be neither created nor destroyed.

[4] We will see later how the steady-state assumption in the literature can introduce errors into I-O analyses.

[5] Of course, waste heat exists and is accounted by the First Law of Thermodynamics. However, waste heat is ignored when accounting for total energy.

4.1.2 Embodied Energy Accounting

We note that the definition of total energy (Equation 4.1) includes direct energy (E) and embodied energy (B) terms. On the other hand, the First Law of Thermodynamics (Equation 3.1) includes direct energy (E) and waste heat (Q) terms. The consequence of the foregoing difference is that an interesting relationship exists between embodied energy (B) and waste heat (Q), as we shall see below.

To derive an accounting equation for embodied energy, we substitute the First Law of Thermodynamics (Equation 3.1) into the total energy accounting equation (Equation 4.5).

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \sum \dot{B} + \sum \dot{Q}_{out} \tag{4.6}$$

The waste energy terms ($\dot{Q}_{out}$) in Equation 4.6 are *outflows* of energy from the sector. The embodied energy terms ($\dot{B}$) represent embodied energy of inflows and outflows of material. Splitting the $\dot{B}$ term into inflows and outflows and rearranging gives

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \sum \dot{B}_{in} - \sum \dot{B}_{out} + \sum \dot{Q}_{out} \tag{4.7}$$

In words, the rate of accumulation of embodied energy in a sector of the economy $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$ is equal to the sum of the rates of inflow of embodied energy into the sector ($\dot{B}_{in}$) less the rate of output of embodied energy from the sector ($\dot{B}_{out}$) *plus* the rate of waste direct energy from the sector ($\dot{Q}_{out}$). The first two terms on the right side of Equation 4.7 are expected: accumulation is the difference between inflow and outflow rates.

Rearranging Equation 4.7 yields another version of the embodied energy accounting equation: one that illuminates issues related to stages of growth for an economic sector.

$$\frac{\mathrm{d}B}{\mathrm{d}t} + \sum \dot{B}_{out} = \sum \dot{B}_{in} + \sum \dot{Q}_{out} \tag{4.8}$$

From Equation 4.8, we see that incoming embodied energy ($\dot{B}_{in}$) and waste heat[6] ($\dot{Q}_{out}$) can be used to increase either (a) the embodied energy within a sector of the economy $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$ or (b) the embodied energy output of a sector of the economy ($\dot{B}_{out}$), depending on decisions by actors (firms, households, or the government) within the sector. If the sector is "building up" production capacity, much of the incoming embodied energy ($\dot{B}_{in}$) and direct energy consumption (represented by $\dot{Q}_{out}$) will be used to increase infrastructure (and associated embodied energy, B) within the sector, and $\frac{\mathrm{d}B}{\mathrm{d}t}$ will be positive. If, on the other hand, the sector is mature, much of the incoming embodied energy ($\dot{B}_{in}$) and direct energy consumption (represented by $\dot{Q}_{out}$) will be used for production of goods ($\dot{B}_{out}$). $\frac{\mathrm{d}B}{\mathrm{d}t}$ will be close to zero. Equa-

[6] Because we have substituted the First Law of Thermodynamics into the total energy accounting equation, $\dot{Q}_{out}$ becomes a proxy for direct energy consumption by the sector.

tion 4.7 shows that an economic sector in decline may experience an outflow of embodied energy (via products or depreciation) in excess of the sum of its embodied energy inflows ($\dot{B}_{in}$) and direct energy consumption (represented by $\dot{Q}_{out}$), and $\frac{\mathrm{d}B}{\mathrm{d}t}$ will be negative.

Equations 4.7 and 4.8 highlight a contrast between the present dynamic analysis and the I-O literature. The traditional assumption of steady-state conditions in economic sectors is tantamount to assuming that $\frac{\mathrm{d}B}{\mathrm{d}t} = 0$ in Equations 4.7 and 4.8. That assumption precludes analysis of stages of growth and the embodied energy implications thereof.

Equations 4.7 and 4.8 are generalized embodied energy accounting equations that we will see again for Examples A-C in the sections that follow.

4.2 Example A: single-sector economy

Figure 4.2 shows the flows of total energy ($\dot{T}$) through the single-sector economy.

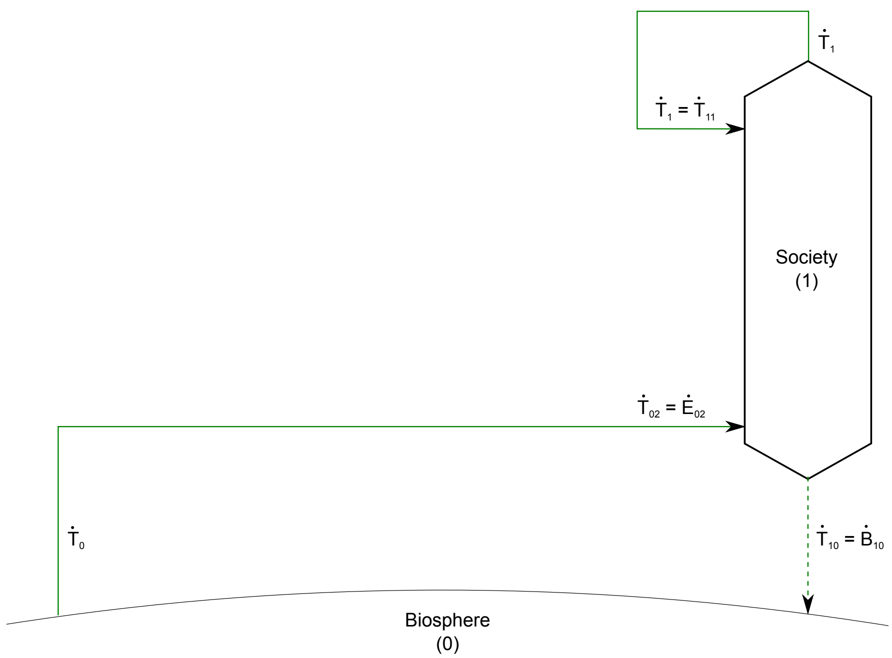

Fig. 4.2 Total energy flows in a one-sector economy
Total energy flows ($\dot{T}$) in a one-sector economy.

As discussed above, we follow the I-O literature in assuming that total energy (T) is conserved. A total energy accounting around the biosphere (0) and the single-sector economy (1) gives

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = \dot{T}_{10} - \dot{T}_{01}, \tag{4.9}$$

and

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \dot{T}_{01} + \dot{T}_{11} - \dot{T}_1 - \dot{T}_{10}. \tag{4.10}$$

Substituting Equations 4.2 and 4.3 into Equations 4.9 and 4.10 yields

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} + \frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} - \dot{E}_{01} - \dot{B}_{01} \tag{4.11}$$

and

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} + \frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{E}_{01} + \dot{B}_{01} + \dot{E}_{11} + \dot{B}_{11} - \dot{E}_1 - \dot{B}_1 - \dot{E}_{10} - \dot{B}_{10}. \tag{4.12}$$

At this point, we can proceed in two directions. The first direction, simplifying Equations 4.11 and 4.12, provides an intuitive result. The second direction, substituting the First Law of Thermodynamics into Equations 4.11 and 4.12, provides the advantage of cancelling most of the direct energy terms. We begin with the first approach: simplification.

4.2.1 Simplification of the embodied energy accounting equation

To simplify Equations 4.11 and 4.12, we first realize that, by definition, no embodied energy flows from the earth with extracted material, so $\dot{B}_{01} = 0$ and $\dot{T}_0 = \dot{E}_{01}$ as shown in Figure 4.2. Second, we can assume that direct energy (E) does not accumulate in the economy such that $\frac{\mathrm{d}E_0}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}E_1}{\mathrm{d}t} = 0$. Finally, we note that $\dot{E}_{10} = 0$, because society does not supply direct energy to the biosphere. Thus, Equations 4.11 and 4.12 become

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{B}_{10} - \dot{E}_{01} \tag{4.13}$$

and

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{E}_{01} + \dot{E}_{11} + \dot{B}_{11} - \dot{E}_1 - \dot{B}_1 - \dot{B}_{10}. \tag{4.14}$$

These equations show that direct energy consumed by a sector ($\dot{E}_{01}$) increases the energy embodied within the sector (B_1), whereas the waste from the sector produces an embodied energy outflow ($\dot{B}_{10}$) that reduces the energy embodied within the sector.

4.2.2 *Substitution of First Law into the embodied energy accounting equation*

The second approach to the derivation of embodied energy accounting equations is to substitute the First Law (Equations 3.4 and 3.5) into the total energy accounting equations (Equations 4.11 and 4.12).

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} - \dot{B}_{01} - \dot{Q}_{10} \tag{4.15}$$

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{B}_{01} + \dot{B}_{11} - \dot{B}_1 - \dot{B}_{10} - \dot{E}_{10} + \dot{Q}_{10} \tag{4.16}$$

This substitution has the advantage of canceling most of the direct energy terms from the embodied energy accounting equations. And, it is no longer necessary to assume that the accumulation rate of direct energy $\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)$ is zero, because the $\frac{\mathrm{d}E}{\mathrm{d}t}$ term is cancelled by the substitution.

Note that the Equation 4.15 includes the term $-\dot{Q}_{10}$, which, at first glance, appears different from Equation 4.7.[7] However, upon realizing that $\dot{Q}_{10}$ is an *inflow* of waste energy into the biosphere, we can use $\dot{Q}_{10} = -\dot{Q}_{01}$ to rewrite Equation 4.15 with an *outflow* term,

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} - \dot{B}_{01} + \dot{Q}_{01}, \tag{4.17}$$

thereby maintining consistency with Equation 4.7.

We can simplify Equations 4.15 and 4.16 using the assumptions of Section 4.2.1 (namely, that $\dot{B}_{01} = 0$ and $\dot{E}_{10} = 0$) to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{B}_{10} - \dot{Q}_{10} \tag{4.18}$$

and

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{B}_{11} - \dot{B}_1 - \dot{B}_{10} + \dot{Q}_{10}, \tag{4.19}$$

the embodied energy accounting equations for Example A.

In Examples B and C following, we will choose the approach of this section, namely substitution of the First Law of Thermodynamics into the total energy accounting equation (instead of simplifying the total energy equation as discussed in Section 4.2.1), because of the benefit of canceling direct energy terms.

[7] Equation 4.7 has a positive sign for the waste energy term (+ $\dot{Q}_{out}$), whereas Equation 4.15 has a negative sign for the waste energy term ($-\dot{Q}_{10}$).

4.2.3 Physical Depreciation

The term $\dot{B}_{10}$ in Equation 4.19 represents the disposal rate of embodied energy from the economy to the biosphere, akin to depreciation of physical assets. This physical depreciation is different from, but related to, financial depreciation: financial depreciation is usually faster than physical depreciation. Embodied energy depreciation ($\dot{B}_{10}$ in Example A) can be represented by a depreciation term such as

$$\dot{B}_{10} = \gamma_1 B_1, \tag{4.20}$$

where γ represents the depreciation rate in units of inverse time (e.g., 1/year) and $\gamma > 0$. The depreciation rate (γ) indicates that a fraction of the total stock of embodied energy is disposed over a period of time (e.g, $\gamma = 0.05$/year). In the absence of other inputs or outputs, this depreciation function provides exponential decay of embodied energy (B) in an economic sector. γ is, in general, a function of time.

Equation 4.20 can be substituted into Equations 4.18 and 4.19 to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \gamma_1 B_1 - \dot{Q}_{10} \tag{4.21}$$

and

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = -\gamma_1 B_1 + \dot{Q}_{10}. \tag{4.22}$$

Equation 4.22 indicates that the accumulation rate of embodied energy in an economic sector $\left(\frac{\mathrm{d}B_1}{\mathrm{d}t}\right)$ is equal to the sum of the waste heat from the economic sector ($\dot{Q}_{10}$) less the rate of disposal of embodied energy ($\gamma_1 B_1$).

We turn now to Example B, a two-sector economy.

4.3 Example B: two-sector economy

For the two-sector economy of Figures 2.3 and 3.4, we again follow the I-O literature by assuming that total energy (T) is conserved. Figure 4.3 shows total energy flows for the two-sector economy.

Accounting for accumulation of total energy and using the assumption that total energy is conserved, we can write the following equations.

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = \dot{T}_{10} + \dot{T}_{20} - \dot{T}_{02}, \tag{4.23}$$

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \dot{T}_{11} + \dot{T}_{21} - \dot{T}_1 - \dot{T}_{10}, \tag{4.24}$$

and

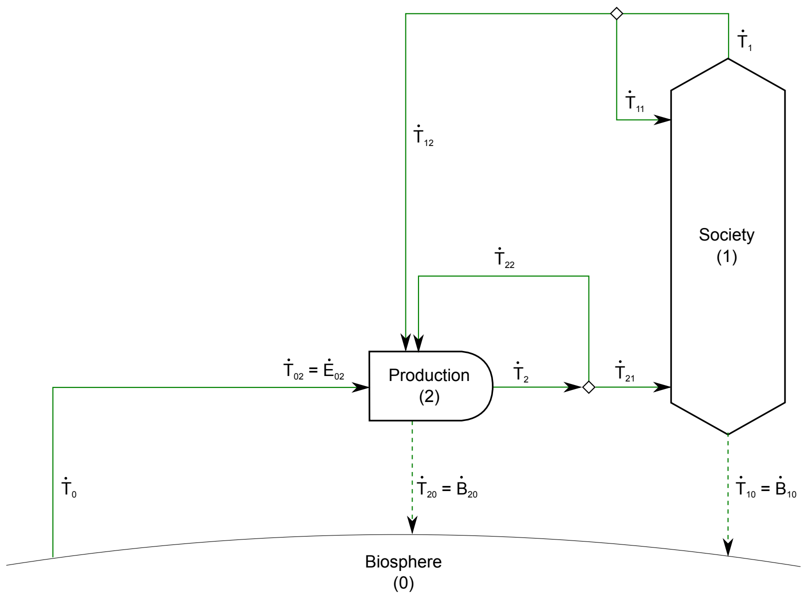

Fig. 4.3 Flows of total energy in a two-sector economy.
Flows of total energy ($\dot{T}$) in a two-sector economy.

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \dot{T}_{02} + \dot{T}_{12} + \dot{T}_{22} - \dot{T}_2 - \dot{T}_{20}. \tag{4.25}$$

Substituting Equations 4.2 and 4.3 into Equations 4.23 through 4.25 gives

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} + \frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} + \dot{E}_{20} + \dot{B}_{20} - \dot{E}_{02} - \dot{B}_{02}, \tag{4.26}$$

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} + \frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{11} + \dot{B}_{11} + \dot{E}_{21} + \dot{B}_{21} - \dot{E}_1 - \dot{B}_1 - \dot{E}_{10} - \dot{B}_{10}, \tag{4.27}$$

and

$$\frac{\mathrm{d}B_2}{\mathrm{d}t} + \frac{\mathrm{d}E_2}{\mathrm{d}t} = \dot{E}_{02} + \dot{B}_{02} + \dot{E}_{12} + \dot{B}_{12} + \dot{E}_{22} + \dot{B}_{22} - \dot{E}_2 - \dot{B}_2 - \dot{E}_{20} - \dot{B}_{20}. \tag{4.28}$$

As in Example A, we can substitute the First Law of Thermodynamics (Equations 3.8–3.10) into the total energy accounting equations (Equations 4.26–4.28) and employ the assumptions that $\dot{E}_{i0} = 0$ and $\dot{B}_{0j} = 0$ to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{B}_{10} + \dot{B}_{20} - \dot{Q}_{10} - \dot{Q}_{20}, \tag{4.29}$$

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{B}_{11} + \dot{B}_{21} - \dot{B}_2 - \dot{B}_{10} + \dot{Q}_{10}, \tag{4.30}$$

and

$$\frac{dB_2}{dt} = \dot{B}_{12} + \dot{B}_{22} - \dot{B}_2 - \dot{B}_{20} + \dot{Q}_{20}. \tag{4.31}$$

Equations 4.29–4.31 can be simplified using sums:

$$\frac{dB_0}{dt} = \sum_{i=1}^{n} \dot{B}_{i0} - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.32}$$

and

$$\frac{dB_j}{dt} = \sum_{i=1}^{n} \dot{B}_{ij} - \dot{B}_j - \dot{B}_{j0} + \dot{Q}_{j0}, \tag{4.33}$$

where $j \in [1, n]$.

Similar to Example A, we observe that the accumulation rate of embodied energy in the economic sectors (1 and 2) is the sum of the rates of waste heat flowing from the sector ($\dot{Q}_{20}$) and embodied energy into the sector ($\dot{B}_{12} + \dot{B}_{22}$) less the rate of embodied energy leaving the sector on its output stream ($\dot{B}_{20}$).

We can replace the waste embodied energy terms in Equations 4.32–4.33 with depreciation terms to obtain

$$\frac{dB_0}{dt} = \sum_{i=1}^{n} \gamma_i B_i - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.34}$$

and

$$\frac{dB_j}{dt} = \sum_{i=1}^{n} \dot{B}_{ij} - \dot{B}_j - \gamma_i B_i + \dot{Q}_{j0}. \tag{4.35}$$

In the next section, we apply embodied energy accounting to Example C, a three-sector economy.

4.4 Example C: three-sector economy

Again, we begin with a diagram showing total energy ($\dot{T}$) flows among the economic sectors of Example C (Figure 4.4).

Accounting for accumulation of total energy and applying the assumption that total energy is conserved, we can write the following equations. We build from the derivation in Section 4.3 and utilize sums for each equation below.

$$\frac{dT_0}{dt} = \sum_{i=1}^{n} \dot{T}_{i0} - \sum_{j=1}^{n} \dot{T}_{0j} \tag{4.36}$$

and

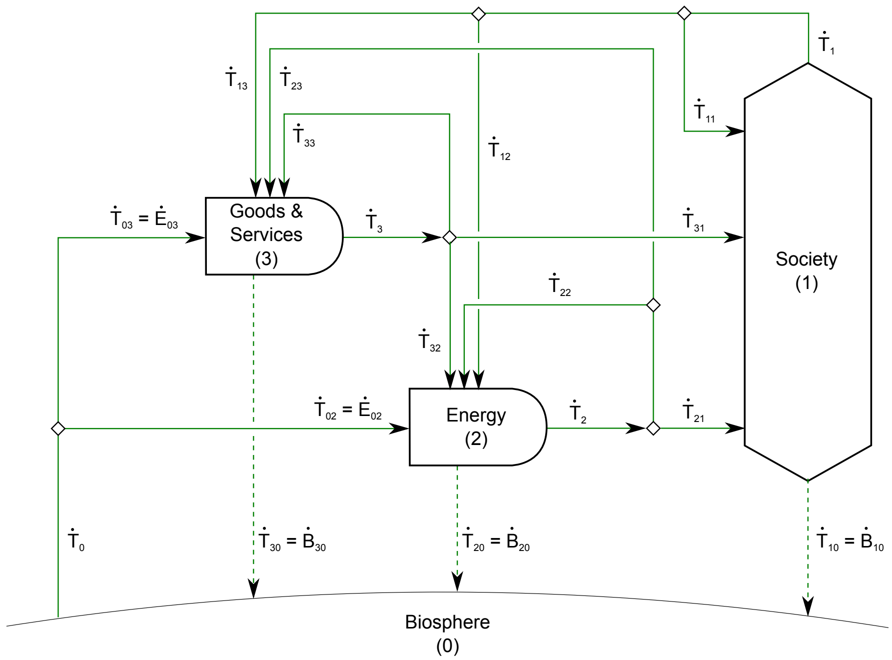

Fig. 4.4 Flows of total energy in a three-sector economy.
Flows of total energy ($\dot{T}$) in a three-sector economy.

$$\frac{\mathrm{d}T_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{T}_{ij} - \dot{T}_j - \dot{T}_{j0}. \tag{4.37}$$

where $j \in [1, n]$.

Substituting Equations 4.2 and 4.3 into Equations 4.36 and 4.37 gives

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} + \frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{E}_{i0} + \sum_{i=1}^{n} \dot{B}_{i0} - \sum_{j=1}^{n} \dot{E}_{0j} - \sum_{j=1}^{n} \dot{B}_{0j} \tag{4.38}$$

and

$$\frac{\mathrm{d}E_j}{\mathrm{d}t} + \frac{\mathrm{d}B_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{E}_{ij} + \sum_{i=0}^{n} \dot{B}_{ij} - \dot{E}_j - \dot{B}_j - \dot{E}_{j0} - \dot{B}_{j0}. \tag{4.39}$$

Substituting the First Law of Thermodynamics (Equations 3.18 and 3.19) into the total energy accounting equations (Equations 4.38 and 4.39) and recognizing that $\dot{B}_{0j} = 0$ for $j \in [1, n]$ and $\dot{E}_{i0} = 0$ for $i \in [1, n]$ gives embodied energy accounting equations for Example C:

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{i0} - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.40}$$

$$\frac{\mathrm{d}B_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{B}_{ij} - \dot{B}_j - \dot{B}_{j0} + \dot{Q}_{j0} \tag{4.41}$$

We can replace the waste embodied energy terms in Equations 4.40 and 4.41 with depreciation terms to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n} \gamma_i B_i - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.42}$$

and

$$\frac{\mathrm{d}B_j}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{ij} - \dot{B}_j - \gamma_j B_j + \dot{Q}_{j0} \tag{4.43}$$

which are the same as Equations 4.34 and 4.35, indicating that we have successfully generalized the embodied energy equations to an arbitrarily-large economy.

To verify the above derivation, we sum Equations 4.42 and 4.43 for sectors of the economy to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} + \sum_{j=1}^{n} \frac{\mathrm{d}B_j}{\mathrm{d}t} = \sum_{i=1}^{n} \gamma_i B_i - \sum_{i=1}^{n} \dot{Q}_{i0} + \sum_{j=1}^{n} \sum_{i=1}^{n} \dot{B}_{ij} - \sum_{j=1}^{n} \dot{B}_j - \sum_{j=1}^{n} \gamma_j B_j + \sum_{j=1}^{n} \dot{Q}_{j0}. \tag{4.44}$$

Using the identities

$$\dot{B}_j = \sum_{k=1}^{n} \dot{B}_{jk} \tag{4.45}$$

and

$$\sum_{j=1}^{n} \dot{B}_j = \sum_{j=1}^{n} \sum_{k=1}^{n} \dot{B}_{jk} = \sum_{i=1}^{n} \sum_{k=1}^{n} \dot{B}_{ik} = \sum_{i=1}^{n} \sum_{j=1}^{n} \dot{B}_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{n} \dot{B}_{ij}, \tag{4.46}$$

Equation 4.44 becomes

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} + \sum_{j=1}^{n} \frac{\mathrm{d}B_j}{\mathrm{d}t} = 0, \tag{4.47}$$

as expected. The total embodied energy content of the system remains constant with respect to time in this model.

4.5 Embodied energy in the auto industry

Develop an example here.

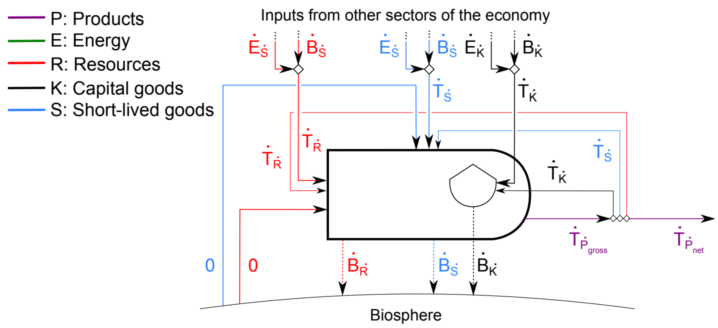

Fig. 4.5 Embodied energy flows for the US Automobile Industry
Embodied energy flows ($\dot{B}$) for the US Automobile Industry using data from XXXX.

4.6 Summary

References

[1] Clark W Bullard and Robert A Herendeen. The energy cost of goods and services. *Energy Policy*, 3(4):268–278, 1975.

[2] Robert A Herendeen. Input-output techniques and energy cost of commodities. *Energy Policy*, 6(2):162–165, 1978.

Part II
Economic Value Flows and Energy Intensity

Chapter 5
Value flows

In Chapters 3 and 4, we noted that energy is the currency of Thermodynamics, and we developed accounting equations for flows of direct ($\dot{E}$) and embodied ($\dot{B}$) energy through an economy. In this chapter, we develop a framework for accounting value flows ($\dot{X}$) through economies. Accounting flows of value is a necessary step along the path to developing equations (in Chapter 6) that describe the energy intensity of intermediate and final products within an economy.

5.1 Methodology

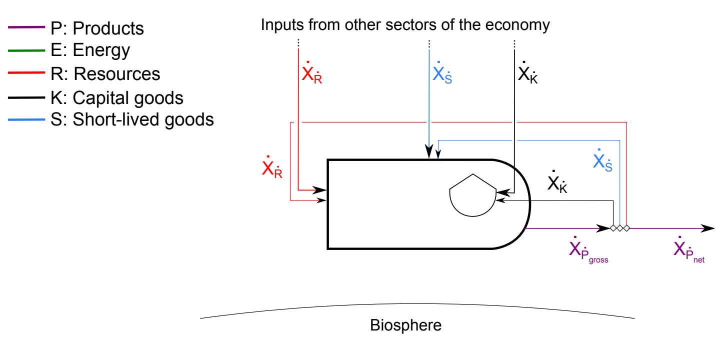

Fig. 5.1 Flows of value ($\dot{X}$) for a single sector. The value flows are associated with each of the different material and energy flows outlined in previous chapters.

*********************** Oct 31, 2013 Second attempt-BRH *************************************
(Outline)

I. What we are not doing: creating a new “energy theory of value”

   A. by the late 70’s the conversation morphed into an “energy theory of value”

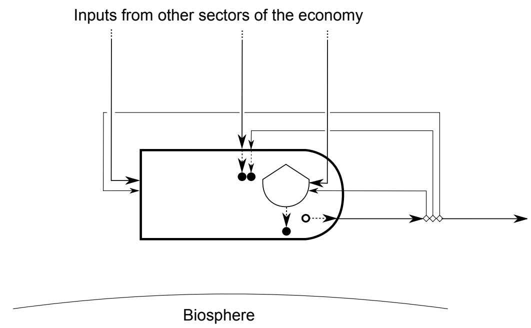

Fig. 5.2 Aggregated flows of value ($\dot{X}$) for a single sector. Distinction is made between value flows that enter the sector and are accumulated (i.e. capital goods) and value flows that are not accumulated. Within the sector there is destruction of value $\dot{X}_{dest}$, represented by the downward arrow flowing into the black sink and generation of value, represented by the arrow flowing out of a source.

B. econmists rejected energy analysis for two reasons:
   i. economists thought that energy analylsis offered no addition useful information
   ii. economists rejected intrinsic theory of value promomted by energy analysts (Costanza, 1980) because it was too reductive and narrow.
C. we agree that an energy theory of value is too reductive and narrow.
D. however, we disagree that enegy analysis offers no useful information.
E. The baby was thrown out with the bathwater.

II. what we are doing: using subjective theory of value combined with a technique that quantifies an intrinsic measure of economic activity in terms of resource use and wastes (i.e., measures impact of of economic activity on the biosphere).

A. the intrinsic measure used in this paper is energy intensity: kJoules/dollar of economic output
B. a variety of intrinsic measures could be used, for example:
   i. gallons of H20/dollar
   ii. tons of steel/dollar
   iii. CO2 emmitted/dollar
   iv. species destroyed/dollar
   v. heavy metals/dollar
   vi. solid waste/dollar
C. used in conjunction with subjective measures of value (e.g., market prices) intrinsic measures help capture the impact of economic development on the biosphere
D. economic growth cannot continue indefinitely in scale, there are environmental constraints

E. currency flows within economies based on subjective measures of value do not a measure the interaction processes between the economy and the biosphere
   i. some exceptions might include Pagovian taxes and other attempts to apply monetary value to intrinsic measures
F. subjective measures of value within an economy alone cannot foretell environmental constraints on the horizon
   i. Our approach will *also* not foretell environmental problems.
   ii. A measure of the external system is needed for control "feedback."
   iii. Even if "feedback" were available, subjective value wouldn't help.

III. To obtain an intrinsic measure of economic activity, you need a bridge between currency flows and physical flows. (previous sentence: Instrinsic measures of economic impact must include some theory of value of economic output.)

A. we use subjective theory of value, the mainstream economic approach to measuring economic output using market prices
B. We acknowledge that subjective theory of value has its problems, though:
   i. externalities
   ii. imperfect information
   iii. tax and subsidy distortions
C. despite these drawbacks, we use subjective theory of value because it is what today's economies are based on.
D. this allows us to use currency flows as a first-approximation of value flows

IV. One outcome of this approach is to highlight the paucity of information contained in current measures of the economy based solely on subjective values

A. An airplane has a portfolio of instruments to measure the status of a wide range of aspects of the flight and its physical status relative to the biosphere; conversely, an economy is much more complicated, yet has relatively few measures of its well-being or physical status
B. economic policy tends to focus on one main gauge, GDP
C. In addition to GDP, alternative measures of an economy's status are vital, such as:
   i. Human Capital accumulated
   ii. Genuine Progress Indicator

5.2 Example A: single-sector economy

Figure 5.3 shows flows of value in the single-sector economy. Following typical assumptions in economic modeling, the economy is *completely isolated* from the biosphere in terms of both material inputs and wastes. In other words, the value flows of an economy are *independent from* material inputs and wastes. Value flows are independent from material inputs, because raw materials have no economic value until they have been removed from the biosphere by the extraction industry. Value flows are independent from wastes, because wastes, by definition, have no economic value after they leave the economy.

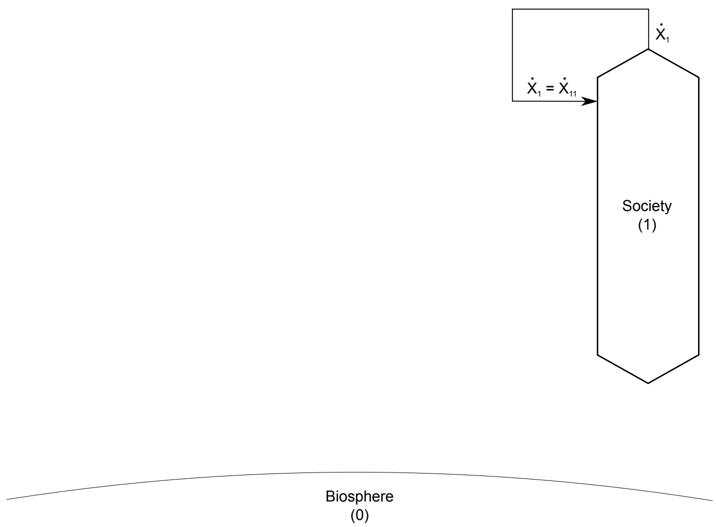

Fig. 5.3 Flows of value ($\dot{X}$) for a one-sector economy.

The contrast between Figures 2.2 and 3.3, on the one hand, and Figure 5.3, on the other, is striking. The picture of material and energy flows in Figures 2.2 and 3.3 indicates a dependence upon the biosphere that is not reflected in the value flows of Figure 5.3. The isloation of the value flows from the biosphere is a consequence of the subjective theory of value that underpins modern economics. The biosphere is akin to a third party with no voice in determining the value of a transaction: it is neither buyer nor seller.

Equation 5.1 describes the accumulation of value (X) in Society (1).

$$\frac{\mathrm{d}X_1}{\mathrm{d}t} = \dot{X}_{11} - \dot{X}_1 + \dot{X}_{gen,1} - \dot{X}_{dest,1}. \tag{5.1}$$

The following subsections discuss the terms in Equation 5.1.

5.2.1 Value Generation ($\dot{X}_{gen}$)

In Equation 5.1, the value generation term ($\dot{X}_{gen}$) is akin to growing apples in Section 2.1: value is generated, seemingly out of nothing. But, in fact, value is not created out of nothing. Rather, value is created from a variety of factors that have no apparent cost to producers, including:

- flow of solar energy into the economy, as in the example of growing apples,
- extraction of resources (e.g., water, minerals, and fossil fuels) or any other unpriced goods from the biosphere,
- exploitation of the unpriced waste assimilation capacity of the biosphere,
- utilization of capital stock, labor, and energy to produce products that are more valuable than inputs,
- demand in excess of supply, **** Discuss this one **** and
- application of human ingenuity and innovation, which lead to increasingly efficient production processes.

The subjective theory of value indicates that there is no economic value associated with these "transactions" that generate value, because no currency is exchanged.

The above factors indicate that the process of value generation has both direct and indirect impacts on the biosphere. The direct impacts are obvious: extraction of non-renewable resources from the biosphere, at rates greater than their natural accretion, represents unsustainable overuse of natural capital. The indirect impacts are less obvious: human ingenuity can lead to increased wealth, leading to increased demand rates for goods and services, whose production requires ever-increasing rates of unsustainable natural resource extraction.

$\dot{X}_{gen}$ is accounted as "value added" to an industry in the BEA tables. **** Becky: is this correct? Becky: look at the BEA tables' definition of "value added" to see if there is another sentence that can be added for clarification. ****

5.2.2 *Value Destruction ($\dot{X}_{dest}$)*

In Equation 5.1, the value destruction term ($\dot{X}_{dest}$) is akin to consuming apples: value is destroyed by a process that consumes, or otherwise renders unusable, previously-valuable things in the economy (see Section 2.1). The factors that lead to value destruction ($\dot{X}_{dest}$) include:

- depreciation, usually associated with disposal of materials and equipment to the biosphere at end of life,
- supply in excess of demand, **** Discuss this one **** and
- natural disasters, such as hurricanes and typhoons, that destroy equipment and property.

$\dot{X}_{dest}$ is accounted as **** what **** to an industry in the BEA tables. Is it simply a negative "value add?" **** Becky: What do we write here? ****

5.2.3 Economic Transactions ($\dot{X}_{11}$ and $\dot{X}_1$)

The returning arrow in Figure 5.3 represents transactions between

- buyers (who receive things of value, $\dot{X}_{11}$, in exchange for currency), and
- sellers (who give up things of value, $\dot{X}_1$, in exchange for currency).

It is interesting to note that when a good is sold for more than the producer paid for its inputs, the seller has created value and sold it into the economy. As a consequence, the seller's stock of currency grows, providing the seller with an increased level of claim on value in the economy.

The subjective theory of value (Section 5.1) posits that buyers and sellers agree on value at the time of the transaction. Thus, $\dot{X}_1 = \dot{X}_{11}$, and Equation 5.1 simplifies to

$$\frac{\mathrm{d}X_1}{\mathrm{d}t} = \dot{X}_{gen,1} - \dot{X}_{dest,1}, \tag{5.2}$$

indicating that value accumulates in the economy $\left(\frac{\mathrm{d}X_1}{\mathrm{d}t}\right)$ due to value generation ($\dot{X}_{gen,1}$) and destruction ($\dot{X}_{dest,1}$) processes.

5.2.4 GDP and the Stock of Value

If Society (1) in Figure 5.3 represents the economy of an entire country, $\dot{X}_1$ is its gross domestic product (GDP) in units of \$/year. The stock of value, X_1, is the total value of everything that is accumulated in society.

**** Becky: anything to add to the above sections? ****

5.3 Example B: two-sector economy

Figure 5.4 shows flows of value ($\dot{X}$) within a two-sector economy. Again, we note the isloation of value from the biosphere.

We can account for value flows by writing the following equations:

$$\frac{\mathrm{d}X_1}{\mathrm{d}t} = \dot{X}_{11} + \dot{X}_{21} - \dot{X}_1 + \dot{X}_{gen,1} - \dot{X}_{dest,1} \tag{5.3}$$

and

$$\frac{\mathrm{d}X_2}{\mathrm{d}t} = \dot{X}_{12} + \dot{X}_{22} - \dot{X}_2 + \dot{X}_{gen,2} - \dot{X}_{dest,2}. \tag{5.4}$$

Equations 5.3 and 5.4 can be generalized as

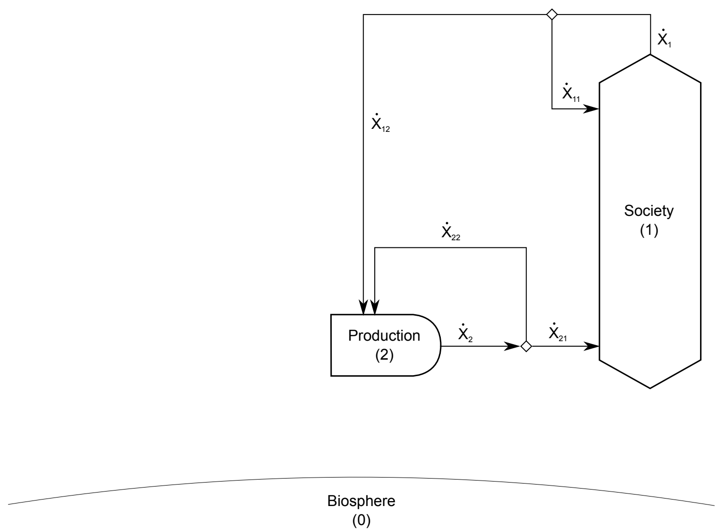

Fig. 5.4 Flows of value within a two-sector economy.
Flows of value ($\dot{X}$) within a two-sector economy.

$$\frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{X}_{ij} - \dot{X}_j + \dot{X}_{gen,j} - \dot{X}_{dest,j}, \tag{5.5}$$

where n is the number of sectors in the economy, and $j \in [1,n]$.

5.4 Example C: three-sector economy

Figure 5.5 shows flows of value ($\dot{X}$) within a three-sector economy.

The equations representing flows of value in Example C are:

$$\frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{X}_{ij} - \dot{X}_j + \dot{X}_{gen,j} - \dot{X}_{dest,j}, \tag{5.6}$$

where n is the number of sectors in the economy, and $j \in [1,n]$. Equation 5.6 is identical to Equation 5.5. If we sum the value accounting equations for the entire economy, we obtain

$$\sum_{j=1}^{n} \frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{j=1}^{n} \sum_{i=1}^{n} \dot{X}_{ij} - \sum_{j=1}^{n} \dot{X}_j + \sum_{j=1}^{n} \dot{X}_{gen,j} - \sum_{j=1}^{n} \dot{X}_{dest,j}. \tag{5.7}$$

With the identities

$$\dot{X}_j = \sum_{k=1}^{n} \dot{X}_{jk} \tag{5.8}$$

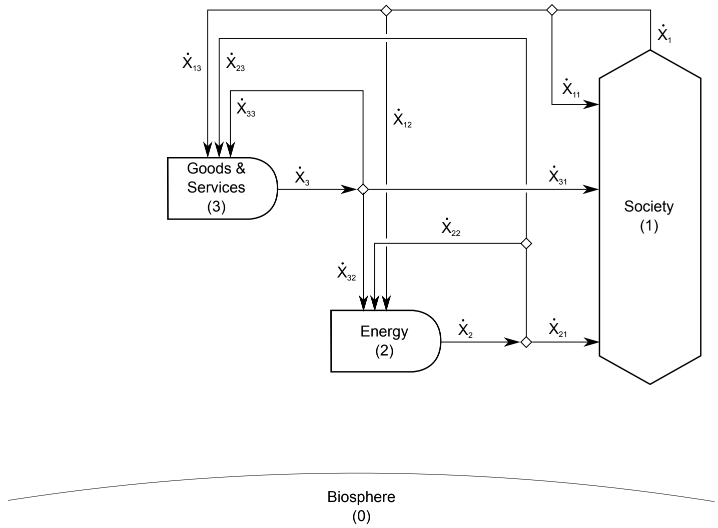

Fig. 5.5 Flows of value ($\dot{X}$) within a three-sector economy.

and

$$\sum_{j=1}^{n} \dot{X}_j = \sum_{j=1}^{n} \sum_{k=1}^{n} \dot{X}_{jk} = \sum_{i=1}^{n} \sum_{k=1}^{n} \dot{X}_{ik} = \sum_{i=1}^{n} \sum_{j=1}^{n} \dot{X}_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{n} \dot{X}_{ij}, \tag{5.9}$$

Equation 5.7 becomes

$$\sum_{j=1}^{n} \frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{j=1}^{n} \dot{X}_{gen,j} - \sum_{j=1}^{n} \dot{X}_{dest,j}, \tag{5.10}$$

for $j \in [1,n]$, indicating that value generation ($\dot{X}_{gen,j}$) and destruction ($\dot{X}_{dest,j}$) are the only mechanisms by which value is accumulated or lost $\left(\frac{\mathrm{d}X_j}{\mathrm{d}t}\right)$ within the economy.

5.5 Value in the auto industry

5.6 Summary

References

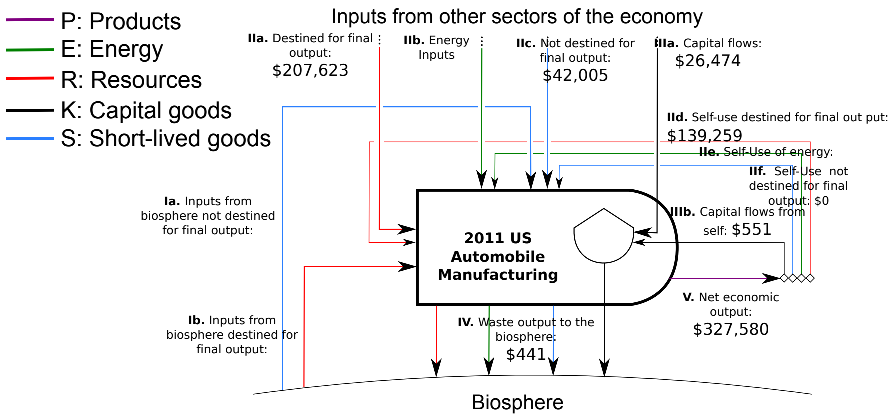

Fig. 5.6 Value of Material and Energy flows into and out of the US Automobile Industry
Value of Material and Energy flows into and out of the US Automobile Industry (2011USD).

Chapter 6
Energy intensity

In Chapters 3, 4, and 5, we defined flows of direct energy, embodied energy, and value in an economy. In this chapter, we merge energy and value together to estimate the energy intensity (ε) of economic sectors.

6.1 Methodology

Energy intensity (ε) is the ratio of total energy ($\dot{T}$) and value ($\dot{X}$) outflow rates from an economic sector, such that for the j^{th} goods and services sector,

$$\varepsilon_j \equiv \frac{\dot{T}_j}{\dot{X}_j}, \tag{6.1}$$

and ε is in units of J/\$. For inter-sector flows, we have

$$\varepsilon_{jk} = \frac{\dot{T}_{jk}}{\dot{X}_{jk}}. \tag{6.2}$$

Furthermore, we note that

$$\varepsilon_j = \varepsilon_{jk} \tag{6.3}$$

for all k, because the energy intensity of sector j's output is independent of its destination (k). I.e., we assume that all goods produced by a sector are produced at the average energy intensity of that sector.[1]

We define the input-ouput ratio (a_{ij}) that represents the input of good i required to produce a unit of output from sector j.

[1] If this approach is unsatisfactory, the sector may be divided into sub-sectors with different energy intensities.

$$a_{ij} \equiv \frac{\dot{X}_{ij}}{\dot{X}_j} \tag{6.4}$$

Input-output ratios are given in mixed units, depending on both the purpose of each sector of the economy and the type of input as shown in Figure 6.1.

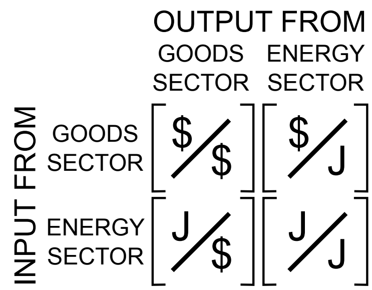

Fig. 6.1 Units for input-output ratios (a). **** Mik: Can we turn this into a matrix where row zero and column zero represent society, row and column 1 represents energy, and the rest represent goods and services? ****

6.2 Example A: single-sector economy

With reference to Figures 3.3, 4.2, and 5.3, the energy intensity (ε_1) of a single-sector economy is calculated by

$$\varepsilon_1 = \frac{\dot{T}_1}{\dot{X}_1} = \frac{\dot{T}_{11}}{\dot{X}_{11}}. \tag{6.5}$$

Appendix A illustrates that the energy intensity of a single-sector economy (ε_1) is comprised of the sum of the infinite recursions of energy consumed during production of output ($\dot{X}_1$).

To estimate energy intensities when more than one economic sector is involved, we move to Examples B and C in the following sections.

6.3 Example B: two-sector economy

With reference to Figures 3.4, 4.3, and 5.4, the energy intensity (ε_2) of the production sector is given by

$$\varepsilon_2 = \frac{\dot{T}_2}{\dot{X}_2} = \frac{\dot{T}_{22}}{\dot{X}_{22}}. \tag{6.6}$$

Thus,

$$\dot{T}_2 = \varepsilon_2 \dot{X}_2, \tag{6.7}$$

The input-output ratio for the production sector's self-use of output (a_{22}) is

$$a_{22} = \frac{\dot{X}_{22}}{\dot{X}_2}, \tag{6.8}$$

thus

$$\dot{T}_{22} = \varepsilon_2 a_{22} \dot{X}_2. \tag{6.9}$$

Realizing that (a) $\frac{\mathrm{d}T_2}{\mathrm{d}t} = \frac{\mathrm{d}B_2}{\mathrm{d}t}$ due to $\frac{\mathrm{d}E_2}{\mathrm{d}t} = 0$, because direct energy does not accumulate within economic sectors and (b) $\dot{T}_{02} = \dot{E}_{02}$ due to $\dot{B}_{02} = 0$, because embodied energy appears only in the *output* of a sector, and substituting Equations 6.7 and 6.9 into Equation 4.25 gives

$$\frac{\mathrm{d}B_2}{\mathrm{d}t} = \dot{E}_{02} + \dot{T}_{12} + \varepsilon_2 a_{22} \dot{X}_2 - \varepsilon_2 \dot{X}_2 - \gamma_2 B_2. \tag{6.10}$$

To extend Equation 6.10 to a matrix formulation, we turn to Example C.

6.4 Example C: three-sector economy

The three-sector economy of Example C affords the opportunity to develop a matrix version of the total energy accounting equation (4.37) and to develop an equation that estimates the energy intensity of economic sectors. We begin with a matrix version of the total energy accounting equation.

6.4.1 Total energy accounting equation

We apply Equation 4.37 to the three-sector economy shown in Figures 3.5, 4.4, and 5.5 to obtain the following total energy accounting equations for the Energy (2) and Goods and Services (3) sectors of the three-sector economy:

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \dot{T}_{02} + \dot{T}_{12} + \dot{T}_{22} + \dot{T}_{32} - \dot{T}_2 - \dot{T}_{20} \tag{6.11}$$

and

$$\frac{\mathrm{d}T_3}{\mathrm{d}t} = \dot{T}_{03} + \dot{T}_{13} + \dot{T}_{23} + \dot{T}_{33} - \dot{T}_3 - \dot{T}_{30}. \tag{6.12}$$

Similar to Example B, we realize that (a) $\frac{\mathrm{d}T_i}{\mathrm{d}t} = \frac{\mathrm{d}B_i}{\mathrm{d}t}$ due to $\frac{\mathrm{d}E_i}{\mathrm{d}t} = 0$, because direct energy does not accumulate within economic sectors and (b) $\dot{T}_{0j} = \dot{E}_{0j}$ due to $\dot{B}_{0j} = 0$, because embodied energy appears only in the *output* of a sector, and we substitute $\dot{T}_j = \varepsilon_j \dot{X}_j$ and $\dot{T}_{jk} = \varepsilon_j \dot{X}_{jk}$ into Equations 6.11 and 6.12 to obtain

$$\frac{\mathrm{d}B_2}{\mathrm{d}t} = \dot{E}_{02} + \dot{T}_{12} + \varepsilon_2 \dot{X}_{22} + \varepsilon_3 \dot{X}_{32} - \varepsilon_2 \dot{X}_2 - \gamma_2 B_2 \tag{6.13}$$

and

$$\frac{\mathrm{d}B_3}{\mathrm{d}t} = \dot{E}_{03} + \dot{T}_{13} + \varepsilon_2 \dot{X}_{23} + \varepsilon_3 \dot{X}_{33} - \varepsilon_3 \dot{X}_3 - \gamma_3 B_3. \tag{6.14}$$

6.4.2 Matrix formulation

Equations 6.13 and 6.14 can be rewritten in matrix and vector notation as

$$\begin{aligned} \begin{Bmatrix} \frac{\mathrm{d}B_2}{\mathrm{d}t} \\ \frac{\mathrm{d}B_3}{\mathrm{d}t} \end{Bmatrix} &= \begin{Bmatrix} \dot{E}_{02} \\ \dot{E}_{03} \end{Bmatrix} + \begin{Bmatrix} \dot{T}_{12} \\ \dot{T}_{13} \end{Bmatrix} + \begin{bmatrix} \dot{X}_{22} & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} \\ &\quad - \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix} \begin{Bmatrix} \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} - \begin{bmatrix} \gamma_2 & 0 \\ 0 & \gamma_3 \end{bmatrix} \begin{Bmatrix} B_2 \\ B_3 \end{Bmatrix} \end{aligned} \tag{6.15}$$

If we define the following matrices and vectors:

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \equiv \begin{Bmatrix} \frac{\mathrm{d}B_2}{\mathrm{d}t} \\ \frac{\mathrm{d}B_3}{\mathrm{d}t} \end{Bmatrix}, \tag{6.16}$$

$$\mathbf{E}_0 \equiv \begin{Bmatrix} \dot{E}_{02} \\ \dot{E}_{03} \end{Bmatrix}, \tag{6.17}$$

$$\mathbf{T}_1 \equiv \begin{Bmatrix} \dot{T}_{12} \\ \dot{T}_{13} \end{Bmatrix}, \tag{6.18}$$

$$\mathbf{X}_t \equiv \begin{bmatrix} \dot{X}_{22} & \dot{X}_{23} \\ \dot{X}_{32} & \dot{X}_{33} \end{bmatrix}, \tag{6.19}$$

$$\boldsymbol{\varepsilon} \equiv \begin{Bmatrix} \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}, \tag{6.20}$$

$$\hat{\mathbf{X}} \equiv \delta_{ij} \dot{X}_j = \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix}, \tag{6.21}$$

$$\hat{\boldsymbol{\gamma}} \equiv \delta_{ij}\gamma_j = \begin{bmatrix} \gamma_2 & 0 \\ 0 & \gamma_3 \end{bmatrix}, \tag{6.22}$$

and

$$\mathbf{B} \equiv \begin{Bmatrix} B_2 \\ B_3 \end{Bmatrix}; \tag{6.23}$$

with the “Kronecker delta”

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}; \tag{6.24}$$

we can rewrite Equation 6.15 compactly as

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \mathbf{E}_0 + \mathbf{T}_1 + \mathbf{X}_t^{\mathrm{T}}\boldsymbol{\varepsilon} - \hat{\mathbf{X}}\boldsymbol{\varepsilon} - \hat{\boldsymbol{\gamma}}\mathbf{B}. \tag{6.25}$$

Equation 6.25 can be simplified to

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \mathbf{E}_0 + \mathbf{T}_1 + (\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}})\boldsymbol{\varepsilon} - \hat{\boldsymbol{\gamma}}\mathbf{B}. \tag{6.26}$$

We can define the input-output matrix ($\mathbf{A}$) as

$$\mathbf{A} \equiv \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}. \tag{6.27}$$

Appendix B shows that

$$\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}} = \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I}), \tag{6.28}$$

which allows Equation 6.26 to be recast as

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \mathbf{E}_0 + \mathbf{T}_1 + \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I})\boldsymbol{\varepsilon} - \hat{\boldsymbol{\gamma}}\mathbf{B}. \tag{6.29}$$

Equation 6.29 is the matrix version of the total energy accounting equation written in terms of energy intensities ($\boldsymbol{\varepsilon}$) and input-output ratios ($\mathbf{A}$). Equation 6.15 applies for the three-sector economy of Example C, but the equivalent matrix formulation (Equation 6.29) can be extended to any desired level of economic and energy sector disaggregation by expanding the vectors and matrices in Equations 6.16–6.23 and 6.27 to include all sectors of the economy. [1, 2]

Equation 6.29 provides a means to estimate the embodied energy accumulation rate in economic sectors $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ knowing only direct energy inputs to the economy from the biosphere ($\mathbf{E}_0$), total energy inputs from society to the economy ($\mathbf{T}_1$), sector outputs ($\hat{\mathbf{X}}$), sector input-output ratios ($\mathbf{A}$), sector energy intensities ($\boldsymbol{\varepsilon}$), and sector physical depreciation rates ($\hat{\boldsymbol{\gamma}}\mathbf{B}$). In theory, the transaction matrix ($\mathbf{X}_t$) is not required if the input-ouput matrix ($\mathbf{A}$) is known, though in practice, knowledge of input-output matrix ($\mathbf{A}$) would be derived from the transaction matrix ($\mathbf{X}_t$), as shown in Appendix C.

6.5 Estimating $\boldsymbol{\varepsilon}$

Equation 6.29 can be rearranged to obtain

$$\hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I})\boldsymbol{\varepsilon} = \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \hat{\boldsymbol{\gamma}}\mathbf{B} - \mathbf{E}_0 - \mathbf{T}_1 \tag{6.30}$$

and

$$\boldsymbol{\varepsilon} = \left[\hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I})\right]^{-1}\left[\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \hat{\boldsymbol{\gamma}}\mathbf{B} - \mathbf{E}_0 - \mathbf{T}_1\right]. \tag{6.31}$$

We apply the matrix identity [3, Formula 6.2, p. 308]

$$(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1} \tag{6.32}$$

to the right side of Equation 6.31 to obtain

$$\boldsymbol{\varepsilon} = (\mathbf{A}^{\mathrm{T}} - \mathbf{I})^{-1}\hat{\mathbf{X}}^{-1}\left[\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \hat{\boldsymbol{\gamma}}\mathbf{B} - \mathbf{E}_0 - \mathbf{T}_1\right]. \tag{6.33}$$

Changing signs on the right side of Equation 6.31 gives

$$\boldsymbol{\varepsilon} = (\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1}\hat{\mathbf{X}}^{-1}\left[\mathbf{E}_0 + \mathbf{T}_1 - \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} - \hat{\boldsymbol{\gamma}}\mathbf{B}\right]. \tag{6.34}$$

6.6 The effect of capital depreciation

There has been some attempt in the literature to account for the effect of capital depreciation on estimates of $\boldsymbol{\varepsilon}$. Casler [**?**] attempted to correct the BEA's input-output tables, but the present analysis provides an opportunity to develop a mathematically-rigorous approach to the effects of depreciation based on fundamentals.

Physical depreciation ($\hat{\boldsymbol{\gamma}}\mathbf{B}$) occurs due to wear and tear, obsolescence, and end-of-life of material and capital stock within economic sectors and society.[2] Equation 6.34 shows that depreciation decreases the energy intensity of economic products, all other things being equal.

But, we note that depreciation ($\hat{\boldsymbol{\gamma}}\mathbf{B}$) and accumulation of embodied energy $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ should be considered in tandem, because the rate of outflow of embodied energy due

[2] Physical depreciation is different from financial depreciation. Financial depreciation occurs during the useful life of material, while physical depreciation occurs at end of life, when material is discarded from an economic sector to the biosphere.

to depreciation ($\hat{\boldsymbol{\gamma}}\mathbf{B}$) will cause an equal reduction of the net accumulation rate of embodied energy $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$. We can split the embodied energy accumulation rate $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ into a portion due to depreciation and a portion due to all other factors (exclusive of physical depreciation) as follows:

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \left.\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right|_{\text{other}} + \left.\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right|_{\text{depreciation}}. \tag{6.35}$$

When physical depreciation occurs, $\hat{\gamma}B > 0$ and $\left.\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right|_{\text{depreciation}} < 0$. In fact,

$$\left.\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right|_{\text{depreciation}} + \hat{\boldsymbol{\gamma}}\mathbf{B} = 0. \tag{6.36}$$

Substituting Equations **??** and **??** into Equation 6.34 yields

$$\boldsymbol{\varepsilon} = \left(\mathbf{I} - \mathbf{A}^{\mathrm{T}}\right)^{-1}\hat{\mathbf{X}}^{-1}\left[\mathbf{E}_0 + \mathbf{T}_1 - \left.\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right|_{\text{other}}\right]. \tag{6.37}$$

Equation **??** allows estimation of the energy intensity of economic sectors ($\boldsymbol{\varepsilon}$) knowing only sector input-output ratios ($\mathbf{A}$), sector outputs ($\hat{\mathbf{X}}$), energy input to the economy from the biosphere ($\mathbf{E}_0$), total energy input from society to the economy ($\mathbf{T}_1$), and sector embodied energy accumulation rates exclusive of the effects of physical depreciation $\left(\left.\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right|_{\text{other}}\right)$.

Again, the transaction matrix ($\mathbf{X}_t$) is not required for estimating the energy intensity of economic sectors ($\boldsymbol{\varepsilon}$) if the input-ouput matrix ($\mathbf{A}$) is known, though in practice, knowledge of input-output matrix ($\mathbf{A}$) would be derived from the transaction matrix ($\mathbf{X}_t$), as shown in Appendix C.

6.7 Energy intensity of the auto industry

***** Becky complete this section. Is there anything to include here? *****

6.8 Summary

References

[1] Stephen Casler and Suzanne Wilbur. Energy input-output analysis : A simple guide. *Resources and Energy*, 6(2):187–201, 1984.
[2] C.W. Bullard, P.S. Penner, and D.A. Pilati. Net energy analysis: Handbook for combining process and input-output analysis. *Resources and energy*, 1(3):267–313, 1978.

[3] William H Beyer. *Standard Mathematical Tables and Formulae*. CRC Press, 29 edition, 1991.

Part III
Implications

Chapter 7
Implications

Several implications can be drawn from the detailed development of the I-O method in Chapters 2–6 that includes energy input from society to the economy, embodied energy accumulation, and depreciation.

7.1 Implications for the I-O method

The first set of implications are for the I-O method itself; specifically, for the process of estimating the energy intensity of economic output ($\boldsymbol{\varepsilon}$).

7.1.1 Estimating $\boldsymbol{\varepsilon}$

Extension of the Leontief Input-Output method for energy analysis has allowed energy analysts to estimate the energy intensity of economic products ($\boldsymbol{\varepsilon}$). As discussed in Section 5.1, we do not take this important result as a license to declare an intrinsic "energy theory of value." Rather, we belive that energy intensity is an important and useful metric that can assess the energy performance of economies, even within the prevailing subjective theory of value that underlies modern economics. Thus, it is important to consider the assumptions behind the literature's presentation of the I-O method for estimating the energy intensity economic output.

In this manuscript, Equation **??** provides the means of estimating the energy intensity:

$$\boldsymbol{\varepsilon} = \left(\mathbf{I} - \mathbf{A}^{\mathrm{T}}\right)^{-1} \hat{\mathbf{X}}^{-1} \left[\mathbf{E}_0 + \mathbf{T}_1 - \left.\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right|_{\mathrm{other}} \right] . \qquad (\mathbf{??})$$

The I-O literature [1, 2], on the other hand, writes Equation **??** as[1]

$$\boldsymbol{\varepsilon} = \left(\mathbf{I} - \mathbf{A}^{\mathrm{T}}\right)^{-1} \hat{\mathbf{X}}^{-1} \mathbf{E}_0. \tag{7.1}$$

The differences between Equations **??** and 7.1 are obvious. We have extended the literature to include:

- energy input to the economy from society ($\mathbf{T}_1$) and
- accumulation of embodied energy in the economy, exclusive of depreciation $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\big|_{\mathrm{other}}\right)$.

Comparison of Equations **??** and 7.1 shows that the literature neglects energy input from society ($\mathbf{T}_1$) and accumulation of embodied energy in the economy, exclusive of physical depreciation $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\big|_{\mathrm{other}}\right)$, when estimating the energy intensity ($\boldsymbol{\varepsilon}$) of economic sectors.[2] In other words, energy analysts using the input-output method have, to date, and perhaps unwittingly, assumed a developed-world ($\mathbf{T}_1 \ll \mathbf{E}_0$), steady state economy $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\big|_{\mathrm{other}} = \mathbf{0}\right)$.

The following subsections discuss the assumptions made by the I-O literature.

7.1.1.1 Negligible energy input from society $(\mathbf{T}_1 = 0)$

Energy input from society to the economy ($\mathbf{T}_1$) is "muscle work" supplied by working humans and draught animals.[3**?** , 4] For developed economies, muscle work is a small fraction of the energy input from fossil fuels ($\mathbf{E}_0$), so neglecting $\mathbf{T}_1$ causes negligible error when estimating energy intensity ($\boldsymbol{\varepsilon}$) by Equation 7.1. However, for some agrarian and developing economies, where $\mathbf{T}_1$ and $\mathbf{E}_0$ could be on the same order of magnitude, neglecting $\mathbf{T}_1$ could cause errors in estimates of $\boldsymbol{\varepsilon}$. To the extent that $\mathbf{T}_1$ is significant relative to $\mathbf{E}_0$, Equation 7.1 will underpredict the energy intensity of the economy.

Accurate estimation of the energy intensity of economic output ($\boldsymbol{\varepsilon}$) requires independent knowledge of the rate at which society supplies energy to the economy ($\mathbf{T}_1$). Ayres and Warr have estimated human and animal muscle work input to the economy for a few developed countries.[**?**] We recommend that more of this work be done in the future for many more countries.

7.1.1.2 Negligible accumulation of embodied energy $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\big|_{\mathrm{other}} = 0\right)$

As discussed in detail below (Section **??**), the accumulation of embodied energy $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ in society and the economy can be considered a marker of economic "development." Equation **??** shows that embodied energy accumulation within economic sectors exclusive of depreciation $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\big|_{\mathrm{other}}\right)$ decreases the energy intensity of products

[1] For a discussion of other, more-subtle, differences between the energy intensity equations in the literature and this manuscript, see Appendix D.

[2] To be precise, the literature effectively assumes $\mathbf{T}_1 - \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\big|_{\mathrm{other}} = \mathbf{0}$

($\boldsymbol{\varepsilon}$), because incoming energy is embodied within capital stock rather than products. Comparison with Equation 7.1 shows that the literature's method will overestimate the energy intensity of economic products ($\boldsymbol{\varepsilon}$) when $\frac{d\mathbf{B}}{dt}\big|_{\text{other}} > 0$.

Rapidly growing economies, such as China and India today, are expected to have rather large values of $\frac{d\mathbf{B}}{dt}\big|_{\text{other}}$, while developed economies, such as the United States and the United Kingdom, are expected to have rather smaller values of $\frac{d\mathbf{B}}{dt}\big|_{\text{other}}$. Applying Equation 7.1 (i.e., assuming a steady-state economy) will tend to overestimate the energy intensity of economic products for countries such as China and India.

Accurate estimation of the energy intensity of economic output ($\boldsymbol{\varepsilon}$) requires independent knowledge of the rate at which embodied energy accumulates in the economy $\left(\frac{d\mathbf{B}}{dt}\big|_{\text{other}}\right)$. To our knowledge, there are no examples of estimating the accumulation of embodied energy in the economy. Further work focused on estimating the relative magnitudes of $\left(\frac{d\mathbf{B}}{dt}\big|_{\text{other}}\right)$ and $\mathbf{T}_1$ will benefit energy analysts who utilize the I-O method.

A second set of implications for the I-O method comes from one of the ways that the energy intensity vector ($\boldsymbol{\varepsilon}$) is often used: to estimate energy demand from the biosphere ($\mathbf{E}_0$).

7.1.2 Estimating $\mathbf{E}_0$

The assumptions of Equation 7.1 may cause another challenge for energy analysts. As discussed in Sections **??** and **??** above, the I-O method can be used to estimate energy intensities for each sector of the economy ($\boldsymbol{\varepsilon}$). With $\boldsymbol{\varepsilon}$ values in hand, and assuming that $\boldsymbol{\varepsilon}$ is constant with respect to time, energy analysts can estimate changes in energy demand from the biosphere ($\mathbf{E}_0$) as the output of economic sectors ($\hat{\mathbf{X}}$) increases or decreases by solving Equation 7.1 for $\mathbf{E}_0$:

$$\mathbf{E}_0 = \hat{\mathbf{X}}(\mathbf{I} - \mathbf{A}^{\mathrm{T}})\boldsymbol{\varepsilon}. \tag{7.2}$$

When Equation 7.1 is modified to account for accumulation of embodied energy in the economy $\frac{d\mathbf{B}}{dt}\big|_{\text{other}}$ and energy supplied by society to the economy ($\mathbf{T}_1$), we see that the energy demands ($\mathbf{E}$) must be calculated differently. Solving Equation **??** for $\mathbf{E}_0$ gives

$$\mathbf{E}_0 = \hat{\mathbf{X}}(\mathbf{I} - \mathbf{A}^{\mathrm{T}})\boldsymbol{\varepsilon} + \left.\frac{d\mathbf{B}}{dt}\right|_{\text{other}} - \mathbf{T}_1. \tag{7.3}$$

Comparing Equations 7.2 and 7.3, shows that to the extent that embodied energy accumulation, exclusive of physical depreciation, $\frac{d\mathbf{B}}{dt}\big|_{\text{other}}$ is non-zero, estimates of energy demand ($\mathbf{E}_0$) using Equation 7.2 are too low. And, if energy input from society to the economy ($\mathbf{T}_1$) is significant, estimates of energy demand ($\mathbf{E}_0$) using Equation 7.2 are too high.

At this time, the relative magnitudes of $\mathbf{E}_0$, $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\Big|_{\text{other}}$, and $\mathbf{T}_1$ are unknown. Further work to clarify these magnitudes will be beneficial for energy analysts who employ the I-O method.

******** Matt stopped here. *********

7.2 Implications for economic "development"

[IT WOULD BE GOOD TO HAVE A COMPARISON BETWEEN $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ AND STANDARD METRIC OF DEVELOPMENT, I.E. GDP WHICH I GUESS WOULD BE SOMETHING LIKE $\sum_i \dot{X}_i$. WE CAN CERTAINLY ENVISION SITUATIONS WHERE $\sum_i \dot{X}_i$ IS INCREASING AND]

One consequence of economic "progress" or "development" is that embodied energy accumulates in economic sectors and society. In fact, accumulation of embodied energy in economic sectors and society could be considered a *proxy* for development. This proxy for development is overly materialistic, one-dimensional, and reductionist, but alternatives such as GDP can be similarly criticized. In fact, GDP could continue to increase whilst accumulation of embodied energy or value actually decreased.

Figure **??** shows that energy extraction from the Earth is what ultimately drives development as measured by the accumulation of embodied energy in the economy and society. Development occurs over time. If embodied energy is the measure, development can be expressed as the integral of $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ for economic sectors

***************** Need to edit the following equations for index changes. ************

$$\mathbf{B}(t) = \mathbf{B}(0) + \int_{t=0}^{t=t} \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \mathrm{d}t, \tag{7.4}$$

or, using Equation **??**, as the integral of $\frac{\mathrm{d}B_2}{\mathrm{d}t}$ for society,

$$B_2(t) = B_2(0) + \int_{t=0}^{t=t} \frac{\mathrm{d}B_2}{\mathrm{d}t} \mathrm{d}t = B_2(0) + \int_{t=0}^{t=t} (Y_{\dot{T}} - \gamma_2 B_2 - \dot{Q}_{21}) \mathrm{d}t. \tag{7.5}$$

Using embodied energy is obviously an incomplete measure of development. We might also use $X(t) = X(0) + \int \frac{dX}{dt} dt$. In fact, B and X are two complimentary factors to the economic process. For capital, B, to be useful, we need direct energy, E (to run the capital) and economic value, X (i.e. currency). Therefore each of these factors are necessary, but insufficient.

Table 7.1 describes some of the dynamics that can be observed from Equation **??**. It is quite possible that, especially for regions like the U.S. and Western Europe, the rate of embodied energy accumulation in the economy $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ will be small relative to the rate of energy extraction from the Earth ($\mathbf{E}$). On the other hand, in rapidly developing countries, like China or India, the rate of embodied energy accumulation in the economy may be significantly higher than in a developed economy.

Table 7.1 Factors from Equation **??** affecting the rate of embodied energy accumulation in the economy.

Term	Implication
$\hat{\mathbf{X}}$	As economic output increases, $\frac{d\mathbf{B}}{dt}$ goes up (as will $\mathbf{E}$)
$\mathbf{A}$	As input-ouput ratios increase, $\frac{d\mathbf{B}}{dt}$ goes up
ε	As the energy intensity of the economy increases, $\frac{d\mathbf{B}}{dt}$ goes up
$\mathbf{E}$	As the rate of energy flow from the Earth increases, $\frac{d\mathbf{B}}{dt}$ goes up
$\hat{\gamma}$	As the depreciation rate increases, $\frac{d\mathbf{B}}{dt}$ goes down
$\mathbf{B}$	As the embodied energy in the economy increases, $\frac{d\mathbf{B}}{dt}$ goes down

The behavior of $\mathbf{B}$ with $\frac{d\mathbf{B}}{dt}$ is vitally important. A developed economy has significantly higher embodied energy ($\mathbf{B}$) than a developing economy, and, thus, the outflow rate of embodied energy due to depreciation ($\hat{\gamma}\mathbf{B}$) will be higher. As increasingly large amounts of energy are embodied in the economy, increasingly large energy extraction rates ($\mathbf{E}$) are required to offset depreciation ($\hat{\gamma}\mathbf{B}$) and maintain positive growth $\left(\frac{d\mathbf{B}}{dt} > 0\right)$ in the sectors of the economy. Depreciation may also be, temporarily, offset by increasing energy efficiency, i.e. by decreasing energy intensity, ε.

In a similar manner, Equation **??** indicates that maintaining a positive rate of societal development $\left(\frac{dB_2}{dt} > 0\right)$ requires ever increasing embodied energy input rates to society ($Y_{\dot{T}}$) as the society "develops." This mechanism provides a natural restraint to the continued growth of physical economies.

7.3 Implications for recycling, reuse, and dematerialization

Dematerialization is the idea that economic activity can be unlinked from material or energy demands (UNEP, 2011) ******** Get real reference ********. One of the primary methods for dematerializing an economy is reuse and recycling of materials. The impact of recycling can be seen in the I-O formulation only when depreciation and accumulation are included.

One effect of recycling is to reduce the magnitude of the disposal rate ($\hat{\gamma}$). Equation **??** indicates that recycling of material in an economy, thereby reducing $\hat{\gamma}$, will slow the effect of depreciation ($\hat{\gamma}\mathbf{B}$) and put upward pressure on growth $\left(\frac{d\mathbf{B}}{dt}\right)$.

Recycling has a mixed effect on energy demand ($\mathbf{E}$). Because recycled material displaces newly-produced material in the economy and society, recycling will tend to reduce energy demand ($\mathbf{E}$). Equation **??** indicates that this displacement effect will put downward pressure on growth $\left(\frac{dB}{dt}\right)$. However, recycling processes require energy to operate, thereby increasing energy demand (**E**). Equation **??** indicates that additional energy demand will put upward pressure on growth $\left(\frac{dB}{dt}\right)$.

If recycling produces a net reduction in energy demand (**E**), that is if the effect of displaced production dominates over the effect of energy consumed in recycling processes, the upward pressure on growth $\left(\frac{dB}{dt}\right)$ from decrease in $\hat{\gamma}$ and the downward pressure on growth from net reduction of **E** offset each other, the growth rate $\left(\frac{dB}{dt}\right)$ will remain near zero, and total embodied energy $(\mathbf{B})$ will remain constant. In that scenario, dematerialization can develop: reduced material and energy input (**E**) can be accompanied by no change in growth $\left(\frac{d\mathbf{B}}{dt}\right)$.

7.4 Comparison to a Steady-state Economy

****** Finish this section. In terms of what a SSE would look like in the I-O framework, at first blush, I would think that dB/dt = 0 is one aspect. Also, with no growth, inflow rates = depreciation rates. The larger that B is for any society, the larger E must be (to overcome depreciation). To minimize E, hyper-recycling is probably useful. Those are at least a place to start. ******

****** In our discussion, we also addressed the attempts at SSE from point of view of society. In order to achieve this goal *without* recycling, the goods and services sector should have to increase extraction to offset decreasing ore grade, the energy sector should have to increase extraction of energy to allow increasing extraction (unless efficiency could make up the gap: unlikely) in which case the SSE would be violated from these two and from the POV of the earth. ******

References

[1] Clark W Bullard and Robert A Herendeen. The energy cost of goods and services. *Energy Policy*, 3(4):268–278, 1975.

[2] Stephen Casler and Suzanne Wilbur. Energy input-output analysis : A simple guide. *Resources and Energy*, 6(2):187–201, 1984.

[3] Robert U Ayres, Leslie W Ayres, and Benjamin S Warr. Exergy, power and work in the US economy, 1900–1998. *Energy*, 28(3):219–273, mar 2003.

[4] Benjamin S Warr and Robert U Ayres. Useful work and information as drivers of economic growth. *Ecological Economics*, 73(C):93–102, January 2012.

Appendix A
Infinite series representation of energy intensity

The single-sector economy of Figures 2.2, 3.3, 4.2, and 5.3 can be re-drawn as shown in Figure A.1.

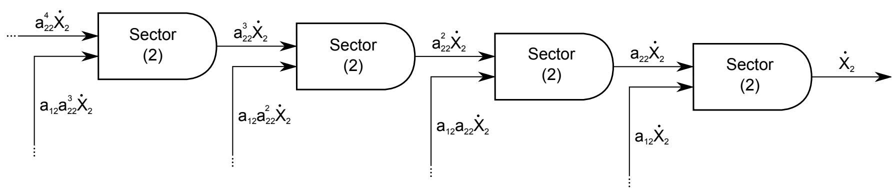

Fig. A.1 Process flows in a single-sector economy. ***** Mik: please update to include Biopshere (0), change "Sector (2)" to "Production (2)", and change all subscripts accordingly. ******

The economy produces output at a rate of $\dot{X}_2$, but it requires energy from the biosphere ($\dot{E}_{02} = a_{02}\dot{X}_2$) to do so. The economy also consumes a fraction of its own gross output ($\dot{X}_{22} = a_{22}\dot{X}_2$). To produce $a_{22}\dot{X}_2$, the economy requires an additional $a_{02}a_{22}\dot{X}_2$ of energy from the biosphere. The sum of all direct energy ($\dot{E}$) required for the economy to produce at a rate of $\dot{X}_2$ is an infinite sum.

$$\dot{E}_{demand,tot} = a_{02}\dot{X}_2 + a_{02}a_{22}\dot{X}_2 + a_{02}a_{22}^2\dot{X}_2 + \dots \tag{A.1}$$

The energy intensity of the economy (ε_2) is

$$\varepsilon_2 = \frac{\dot{E}_{demand,tot}}{\dot{X}_2} = a_{02}(1 + a_{22} + a_{22}^2 + \dots) = a_{02}\sum_{n=0}^{\infty} a_{22}^n. \tag{A.2}$$

Realizing that $\sum_{n=0}^{\infty} a_{22}^n = \frac{1}{1-a_{22}}$ and $a_{02} = \frac{\dot{E}_{02}}{\dot{X}_2}$ (direct energy can be considered the output of the biosphere in this situation) gives

$$\varepsilon_2 = (1 - a_{22})^{-1}\dot{X}^{-1}\dot{E}_{02}. \tag{A.3}$$

Accounting for the differences between scalar and matrix equations and neglecting energy flows from society to the economy ($T_{12} = 0$), accumulation of embodied energy in the economy $\left(\frac{\mathrm{d}B_2}{\mathrm{d}t} = 0\right)$, and physical depreciation ($\gamma_2 B_2 = 0$), Equations 6.34 and A.3 are identical, indicating that the I-O approach accounts for the infinite recursion of energy demand by the economy.

Appendix B
Proof of Equation 6.28

We begin with a restatement of Equation 6.28.

$$\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}} = \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I}) \tag{6.28}$$

We expand the matrices to obtain

$$\begin{bmatrix} \dot{X}_{22} & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} \end{bmatrix} - \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix} \begin{bmatrix} a_{22} - 1 & a_{32} \\ a_{23} & a_{33} - 1 \end{bmatrix}. \tag{B.1}$$

Subtracting and multiplying matrices gives

$$\begin{bmatrix} \dot{X}_{22} - \dot{X}_2 & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} - \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \dot{X}_2 a_{22} - \dot{X}_2 & \dot{X}_2 a_{32} \\ \dot{X}_3 a_{23} & \dot{X}_3 a_{33} - \dot{X}_3 \end{bmatrix}. \tag{B.2}$$

Using $\dot{X}_j a_{ij} = \dot{X}_{ij}$ (see Equation 6.4) gives

$$\begin{bmatrix} \dot{X}_{22} - \dot{X}_2 & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} - \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \dot{X}_{22} - \dot{X}_2 & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} - \dot{X}_3 \end{bmatrix} \tag{B.3}$$

to complete the proof.

Appendix C
Estimating the input-output matrix (A)

Using Equation 6.28 (which is proved in Appendix B), we can derive an expression for estimating the input-output matrix ($\mathbf{A}$) given sector outputs ($\hat{\mathbf{X}}$) and the transaction matrix ($\mathbf{X}_t$). Premultiplying Equation 6.28 by $\hat{\mathbf{X}}^{-1}$ gives

$$\hat{\mathbf{X}}^{-1}\left(\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}}\right) = \mathbf{A}^{\mathrm{T}} - \mathbf{I} \tag{C.1}$$

Further rearranging gives

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\left(\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}}\right) + \mathbf{I}, \tag{C.2}$$

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}}^{-1}\hat{\mathbf{X}} + \mathbf{I}, \tag{C.3}$$

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\mathbf{X}_t^{\mathrm{T}} - \mathbf{I} + \mathbf{I}, \tag{C.4}$$

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\mathbf{X}_t^{\mathrm{T}}, \tag{C.5}$$

and

$$\mathbf{A} = \mathbf{X}_t\left(\hat{\mathbf{X}}^{-1}\right)^{\mathrm{T}}. \tag{C.6}$$

Realizing that $\hat{\mathbf{X}}$ and $\hat{\mathbf{X}}^{-1}$ are diagonal matrices gives

$$\mathbf{A} = \mathbf{X}_t\hat{\mathbf{X}}^{-1}. \tag{C.7}$$

Expanding the matrices of Equation C.7 gives

$$\mathbf{A} = \begin{bmatrix} \dot{X}_{11} & \dot{X}_{12} & \cdots \\ \dot{X}_{21} & \dot{X}_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1/\dot{X}_1 & 0 & \cdots \\ 0 & 1/\dot{X}_2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{C.8}$$

and

$$\mathbf{A} \equiv \begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & a_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} = \begin{bmatrix} \frac{\dot{X}_{11}}{\dot{X}_1} & \frac{\dot{X}_{12}}{\dot{X}_2} & \cdots \\ \frac{\dot{X}_{21}}{\dot{X}_1} & \frac{\dot{X}_{22}}{\dot{X}_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \tag{C.9}$$

as expected given the definition of the input-output ratio (a) in Equation 6.4:

$$a_{ij} \equiv \frac{\dot{X}_{ij}}{\dot{X}_j}. \tag{6.4}$$

Equation C.7 provides a method of estimating the input-output matrix (**A**) using the transaction matrix ($\mathbf{X}_t$) and sector outputs ($\hat{\mathbf{X}}$).

Appendix D
Column vs. row vectors in energy intensity equations

In this manuscript, we choose to define energy intensity ($\boldsymbol{\varepsilon}$) and energy input ($\mathbf{E}_0$ and $\mathbf{T}_1$) as a column vectors (see Equations 6.20, 6.17, and 6.18, respectively), because it natural to solve a system of equations for a column vector rather than a row vector. And, Equation 6.15 could not be written as neatly if $\boldsymbol{\varepsilon}$ and $\mathbf{E}_0$ were row vectors.

In contrast, the I-O literature (see, e.g., [1] and [**?**]) defines energy intensity and energy input as row vectors. The row vs. column difference is manifest in the appearance of the energy intensity matrix equation.

This appendix derives a column vector version of the energy intensity equation that is often found in the literature. The point of comparison is Casler [1]. Casler's energy intensity equation [1, Equation 6] was derived from row vectors as[1]

$$\boldsymbol{\varepsilon} = \mathbf{E}\hat{\dot{\mathbf{X}}}^{-1}(\mathbf{I}-\mathbf{A})^{-1}. \tag{D.1}$$

We begin with Equations 3 and 4 from Casler [1], converted to overdot notation for rates.

$$\varepsilon_1 \dot{X}_{11} + \varepsilon_2 \dot{X}_{21} = \varepsilon_1 \dot{X}_1 \tag{D.2}$$

$$\varepsilon_1 \dot{X}_{12} + \varepsilon_2 \dot{X}_{22} + \dot{E}_{02} = \varepsilon_2 \dot{X}_2 \tag{D.3}$$

Adding an $\dot{E}_{01}$ term[2] and utilizing matrix notation with column vectors (instead of row vectors) gives

$$\begin{bmatrix} \dot{X}_{11} & \dot{X}_{21} \\ \dot{X}_{12} & \dot{X}_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \dot{E}_{01} \\ \dot{E}_{02} \end{Bmatrix} = \begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix}. \tag{D.4}$$

Substituting $\dot{X}_{ij} = a_{ij}\dot{X}_j$ (from Equation 6.4) gives

[1] Equation D.1 is written according to the variable conventions in this manuscript. The literal Equation 6 in Casler [1] is $\varepsilon = E\hat{X}^{-1}(I-A)^{-1}$.

[2] Note that $\dot{E}_{01} = 0$ for Casler [1], so $\dot{E}_0$ can be included without changing the equation.

$$\begin{bmatrix} a_{11}\dot{X}_1 & a_{21}\dot{X}_1 \\ a_{12}\dot{X}_2 & a_{22}\dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \dot{E}_{01} \\ \dot{E}_{02} \end{Bmatrix} = \begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix}. \tag{D.5}$$

Expanding Equation D.5 gives

$$\begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \dot{E}_{01} \\ \dot{E}_{02} \end{Bmatrix} = \begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix}. \tag{D.6}$$

With the definitions of $\hat{\mathbf{X}}$, $\mathbf{A}$, $\boldsymbol{\varepsilon}$, and $\mathbf{E}_0$ from Equations 6.21, 6.27, 6.17, and 6.20, respectively, we can rewrite Equation D.6 as

$$\hat{\mathbf{X}}\mathbf{A}^{\mathrm{T}}\boldsymbol{\varepsilon} + \mathbf{E}_0 = \hat{\mathbf{X}}\boldsymbol{\varepsilon}. \tag{D.7}$$

Solving for $\boldsymbol{\varepsilon}$ gives

$$\boldsymbol{\varepsilon} = \left(\mathbf{I} - \mathbf{A}^{\mathrm{T}}\right)^{-1}\hat{\mathbf{X}}^{-1}\mathbf{E}_0. \tag{D.8}$$

The differences between Equations D.1 and D.8 are due to the use of row vectors (for Equation D.1) or column vectors (for Equation D.8) only. Note that Equation D.8 is similar to Equation 6.34. A detailed discussion of the differences between Equations D.8 and 6.34 can be found in Section 7.1.

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