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The metabolic economy

A dynamic framework for material, energy, and value accounting

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$\mathbf{A}$	input-output matrix [-]
a	input-output ratio [-]
a	stock of apples [apples]
$\dot{a}$	flow rate of apples [apples/time]
$\dot{B}$	embodied energy flow rate [W]
$\mathbf{B}_{waste}$	column vector of waste embodied energy from resource ($\dot{R}$) and short-lived material ($\dot{S}$) flow [J/s]
B	embodied energy [J]
δ_{ij}	Kronecker delta
$\boldsymbol{\varepsilon}$	column vector of sector energy intensities [J/$]
$\dot{E}$	direct energy flow rate [W]
ε	energy intensity [J/$]
$\mathbf{E}_0$	vector of direct energy inputs from the biosphere [W]
E	direct energy [J]
$\hat{\boldsymbol{\gamma}}$	diagonal matrix of depreciation rates [$/year]
$\eta_{\dot{R}}$	resource efficiency [kg/kg]
i	economic sector index
j	economic sector index
K	mass of capital goods [kg]
k	economic sector index
$\dot{K}$	capital goods mass flow rate [kg/s]
n	number of sectors in the economy
P	mass of products [kg]
$\dot{P}$	product mass flow rate [kg/s]
$\dot{Q}$	waste heat flow rate [W]
$\rho_{\dot{S}}$	ratio of short-lived material flow to resource inputs [kg/kg]
R	mass of resources [kg]
$\dot{R}$	resource mass flow rate [kg/s]
S	mass of short-lived goods [kg]
$\dot{S}$	short-lived goods mass flow rate [kg/s]
s	stock of steel [kg]

$\dot{s}$	steel mass flow rate [kg/s]
$\dot{T}$	total energy flow rate [W]
$\mathbf{T}_1$	vector of total energy inputs from society to the economy [W]
T	total energy [J]
t	time [s]
$\dot{X}$	economic value flow rate [\$/s]
$\hat{\mathbf{X}}$	matrix of sector outputs [\$/year]
$\mathbf{X}_t$	transaction matrix [\$/year]
X	stock of economic value [\$]
$\dot{X}_{dest}$	rate of destruction of economic value [\$/s]
$\dot{X}_{gen}$	rate of generation of economic value [\$/s]

Chapter 1
Introduction

> *Where there is no reliable accounting and therefore no competent knowledge of the economic and ecological effects of our lives, we cannot live lives that are economically and ecologically responsible. It is futile to plead and protest and lobby in favor of public ecological responsibility while, in virtually every act of our private lives, we endorse and support an economic system that is by intention, and perhaps by necessity, ecologically irresponsible.* [1, p. 26]
>
> —Wendell Berry

This book is primarily about accounting and change. It is born out of a belief that our economies are in constant flux; changing dynamically in the short-term with human behavior and evolving over longer time frames in response to new inventions [**** technological change? ****] and external pressures. Real-world systems, including economies, are messy and chaotic. They do not march neatly from one stage to the next in an orderly fashion.

This is also a book about models and metaphors. The real world influences the mental models by which we come to frame the world and the metaphors we use to describe it. We perceive the world by the data we collect. We interpret the real world through our models which are influenced by our metaphors. The things that we account (and by extension those things we leave out) indirectly influence our behavior. These models then mold the real world by shaping our interactions with it. Feedback loop - models to accounting to behavior to metaphors to models. Our accounting systems are inextricably linked with our models inform us what is important to measure. In order to better understand the complex dynamics of real-world economies, our model economies (the ones that exist in our minds) need to be able to cope with rapid transience, not just ordered stability.

**** MKH version ****

This is also a book about metaphors and models. We use metaphors to help us simplify and make sense of the world around us. These metaphors inform the mental and empirical models we construct. And, as we collect data to assess the validity of those models, our perception of the world is molded and shaped by our accounting which was informed, in the first place, by the stories we told ourselves about reality. It is easy to collect data that confirms our models, because the models tell us which data to collect. It takes courage to collect data that might be counterfactual and still more courage to modify or let go of established models and metaphors.

For many decades, the "economy is a machine" metaphor has conjured images of *smoothly running*, *well-maintained* systems. But, if we hope to better understand the complex, messy dynamics of real-world economies, if we hope to make sense of real-world events, if we hope to learn where economies can break down, we had better be counting data that informs *dynamic* models guided by metaphors that tell us *more* than “the world is an orderly place.” We need metaphors and models that are able to cope with rapid transience, not just ordered stability.

1.1 Motivation

> The world needs another industrial revolution *in which our sources of energy are affordable, accessible and sustainable. Energy efficiency and conservation, as well as decarbonizing our energy sources, are essential to this revolution.* [2, p. 294]
>
> —Steven Chu, US Secretary for Energy

> *In developed economies we live the good life for now - with an amazing level of comfort and interest created by our astonishing ability to make and transform materials. We've really only done this at scale in the past 150 years, in which time our use of engineered materials has rocketed, literally. However, if we have some concern about ‘sustainability’ we need to anticipate what effects our use might have on future generations - and we're getting some clear indicators that there's a problem.* [3, p. 3]
>
> —Julian Allwood

The original motivation for writing this book came from a belief that our global economy is facing a necessary transition and that in order to adequately manage this transition, we need to better understand our economies. What transition are we facing? The twin dictates of climate change and sustainable development require a transition to a low-emission energy system based on non-depletable resources. Given that our energy system currently runs primarily on non-renewable resources (fossil fuels), we essentially need to rebuild our entire energy system.

Furthermore, our economies are founded on a fundamental principle; *the moral imperative of economic growth*. [4] Growth in production and consumption of goods and services entails the encroachment of the human economy into the biosphere on which it is dependent. One more table means one less tree. As we increase the scale of the economy, we increase the environmental impact of all of our actions. There is good reason to believe that the earth is running low on its ability to handle more growth in human impact.[5]

In the face of such fears, the hopeful vision of “dematerialization” has appeared (materialized). [6–8] The hope is that we can continue to grow our economies while reducing our impact on the environment. Our increasing knowledge and use of technology will allow us to reduce the material consumption of our goods and services. In order to achieve such a dematerialization will require changing the structure of our economies.

The driving motivation behind this book is an attempt to understand the structural elements of our economies, not just the flows; to understand the circulatory system, not just the blood. It is only by understanding this structure that we can begin to understand how our economies change. And it is only by understanding how our economies change that we can understand how best to manage this necessary transition.

1.2 Models-metaphors for the economy

In its most basic form, a model is any "simplified representation of reality". [9, p. 105] Our concept of "reality" is a vastly simplified mental model of the real world. These mental representations are vital to theory consruction in science, since they mediate between human cognition and phenomena. The power of our models lies in explaining some aspect of our universe. The appeal of these explanations lies in their relative simplicity. Interwoven with this simplification are the analogies or metaphors we use to describe or communicate our models. Metaphors contextualize new models and theory with reference to familiar settings and events, often from everyday experience. The archetypal (classic?) example is classical physics representation of the "clockwork" universe. Our models (and subsequently the metaphors we use to talk about them) guide our actions and interactions with the real world. Firstly, a model tells us what aspects of the world are important to value (in the literal sense of making measurements), and also, by extension, which parts of the world (literally) have no value. This evaluative procedure has a deeper normative consequence. That the aspects of the world to which our model places value are valuable and that those that have no value are worthless. Thus metaphors inform our thinking about the real world, but consequently, they may also constrain our ability to frame reality. We interact with reality in the manner with which we interact with the objects of our metaphors. We mistake the model-metaphor for reality. Classical physics tells us the universe is *like* clockwork, hence we begin to interact with it as if it *really were* clockwork.

In the next sections we will explore some of the models and metaphors associated with different views of the economy.

A primer on system theory:

Some concepts of system theory will aid the discussion in the rest of this chapter. Those familiar with thermodynamics or general system theory may skip over the following sections. Systems are generally characterized within three distinct categories: isolated, closed and open systems.

Isolated systems

Isolated systems (Figure 1.2A) are disconnected from their environment such that neither energy nor material crosses the boundary of the system. Inside the boundary, materials and energy may be exchanged between element of the system. The concept of an isolated system is used regularly in thermodynamics to represent an ideal, perfectly insulated container with no heat flow to or from the environment. Within the universe no systems are truly isolated, since all must interact with their environment to at least some degree. The universe itself may be an example of an isolated system.

Closed systems

A closed system (Figure 1.2B) can exchange energy, but not materials, with its environment. The earth is often considered to be a closed system. There is little exchange of material—some meteorites and loss of atmospheric gases—but a large incoming flow of solar radiation balanced (according to the Law of Conservation of Energy) by dissipation of (infra-red) radiation. The incoming radiation, at $\approx$ 5000K, is of much higher quality than the thermal radiation, which leaves at a temperature of $\approx$ 300K. This degradation in energy quality drives practically all of the biological processes on Earth.

Open systems

Open systems (Figure 1.2C), as the name suggests, are open to flows of both energy and materials. The biosphere is a good example of an open system. It exchanges energy and materials with the other systems (atmosphere, hydrosphere, lithosphere) [10] of the earth, of which it is a subset.

Feedback and non-linearity

Feedback occurs when elements of a system interact so as to reinforce or diminish some property of the system. For example, if the rate of growth of a population is dependent on the size of that population. If the rate of growth *increases* as population increases, we say that the feedback is *positive* or *reinforcing*. If the rate of growth *decreases* as population increases, we say that the feedback is *negative* or *correcting*. Often feedback systems involve some method of control or intentional regulation, as in self-regulating systems. Systems containing feedback loops will often display non-linear or even chaotic behavior. Such systems may also display *resilience*, even in the face of large changes in external conditions.

Self-regulating systems

Organisms are open systems. They maintain their internal structure by taking in high quality energy and materials from and expending low quality to their environment. This maintenance requires an ability to regulate internal conditions, such as temperature or chemical composition, known as *self-regulation*. Self-regulation relies on feedback among system elements. Non-living systems may also display self-regulating behavior.

Emergence and hierarchy

Due to the non-linear nature and complexity of natural systems, new *emergent* properties arise which cannot be explained solely in terms of the properties of the elements of which the system is composed. An obvious example is the emergence of life from large molecular structures. Because the properties of the system are *irreducible* to properties of the system elements, we may envision a hierarchy of nested systems. Boulding [11] classifies a number of system levels (outlined in Table 1.1) ranging from simple *structures* (atoms, bridges) up to *transcendental systems* such as religion.

Self-organizing systems

Self-organization involves spontaneous increase in complexity of the internal structure of a system, normally observed in systems far from equilibrium. Examples are the formation of convection cells in heated fluids [12] The self-organizing behavior is an emergent property of the system; it cannot be explained in terms of properties of the system elements.

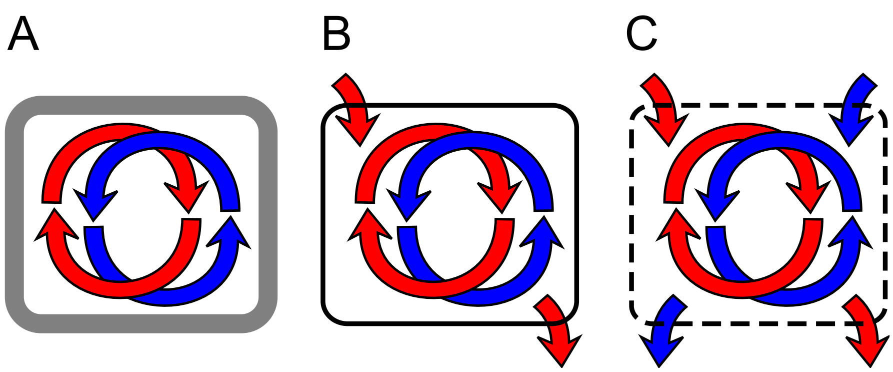

Fig. 1.1 Isolated (A), closed (B) and open (C) systems

1.2.1 Traditional mechanistic view of economy

The classical school of economics flourished during the Enlightenment. Newton's clockwork universe was ticking along nicely. Physicists were beginning to build an understanding of energy and its conservation. Physicians and biologists were beginning to understand the workings of the human body, in particular the circulatory system [REF]. Most cultures were agrarian, depending largely on land for food and muscle power to get things done. Agriculture was still at the whims of nature and

Table 1.1 Hierarchy of systems [11]

Level	Description	Characteristic	Examples
1	Structures	Static, spatial frameworks	Atom, crystal, bridge
2	Clockworks	Predetermined motion	Solar system, clocks, machines
3	Control	Closed-loop feedback control mechanism	Heater with thermostat
4	Open systems	Structurally self-maintaining	Cells
5	Genetic systems	Community of cells	Plants
6	Animals	Nervous system, self-awareness	Birds and beasts
7	Humans	Self-consciousness, knowledge, language	Human beings
8	Socio-cultural systems	Roles, values, communication	Family, community, society
9	Transcendental systems	Beyond our knowledge	Religion

a sense of stewardship was still largely prevalent. Economists were largely preoccupied with delineating the main factors of production (land, labor and capital) and their influence on economic output.

The neo-classical school of economics was born out of the fire of the Industrial Revolution. Thermodynamicists were outlining theories on heat cycles and equilibrium processes allowing the development of more efficient engines. Machines were changing everything. For the first time in history, nature could be bent to the will of humans. New vistas of experience (and riches) were opening up in the New World. The bounties of nature were there for the taking.

Figure 1.2.1 **** MCD—not sure why the reference is not correct **** depicts the traditional model of the economy, represented in mainstream *Principles of Economics* texts. Goods and services flow from the production sector to the household sector (consumption) in exchange for payments. The factors of production (labor, capital) flow from the household sector to the production sector in exchange for wages and rents. Attention is primarily focused on the circular flow of money (dashed line) which is is often described in reference to the "circular" flow of blood around the body. Hence we often speak of money as the "lifeblood" of the economy.

This traditional model of the economy is unashamedly mechanistic. General equilibrium models of the economy [13, 14] borrowed directly from classical physics' models of mechanical equilibrium. [15] Markets specifically, and the economy more generally, are seen as being in "balance".

Additionally, the traditional view of the economy (as depicted in Figure 1.2.1), in much the same fashion as our naive picture of the circulatory system, is represented as separate from its environment. "Blood" circulates around the economy with no need to interact with the rest of the universe. Although the biosphere is represented here in Figure 1.2.1, it is rarely represented in mainstream economic texts. In neoclassical economics, the economy is implicitly and figuratively separate from the material world within which it operates. The traditional economic model is presented as an *isolated* system, independent from flows of materials and energy to and from the biosphere. All of the necessary factors to keep the economy running exist within its confines. Because these physical elements of the biosphere are absent from economic models, the physical constraint they place on the allocation of resources, distribution of outputs, and scale of an economy are outside the scope of traditional economic discussion. Since the economy is independent of any other processes (floating in the void), there is no effect of changing the scale of the economy. Hence, economic growth has no associated costs. If growing the economy can be shown to have any societal benefits whatever, we are delivered to a moral imperative for economic growth. This moral imperative for economic growth combined with the expansionist vision of the pioneers leads to the "cowboy economy" where everything is up for grabs. [16]

The next section will explore some problems with this optimistic view.

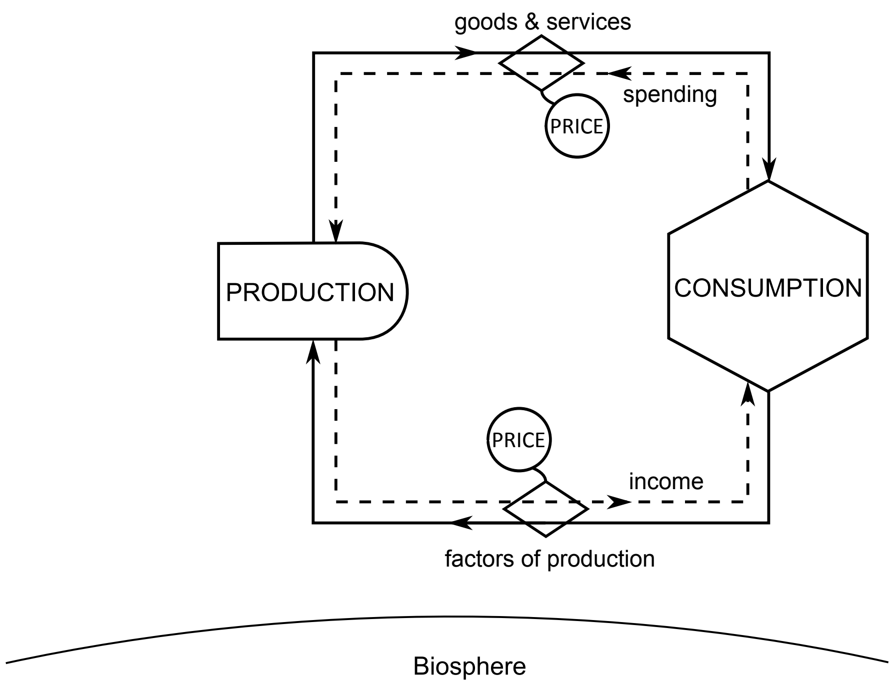

Fig. 1.2 The economy is represented as a circular flow of goods and services between two sectors. The producers manufacture goods and services by taking in labor and capital. Consumers exchange labor for wages which are used to purchase the goods and services of the producers.

1.2.2 Economic models that include resource inputs

Thermodynamics tells us that all physical processes require a transfer of energy. Since Figure 1.2.1 has no flow of energy to the economy, we may consider it a perpetual motion machine of the *first kind*: it produces work without the input of energy and thus violates the first law of thermodynamics—the law of conservation of energy. [17] The real circulatory system is connected to the lungs from where it takes in oxygen and to the digestive system where it takes in processed food which is passed to the cells. This is a major function of the blood. To act as an intermediary between the input of energy and material resources, the food we eat and air we breathe, and the internal working of the body. That is, the energy and material resources, required to run the economy are outside the model. The economy is presented as a "perpetual motion machine," seemingly able to operate indefinitely with no binding constraints.

The traditional, economy-is-all viewpoint fought back by appeals to *substitutability*. A particular material (oil) may be in short supply, but our technological expertise will allow us to substitute another resource (coal) for it. Substitution may continue indefinitely so, goes the story, in the final analysis the only limiting resource is human ingenuity. [18, 19]

This assumption was thrown into stark relief following the oil shocks of the Seventies. Suddenly the global economy was thrown into reverse for lack of one fundamental resource. The necessity of including, at the very least, energy resources into the economic picture spurred the efforts of early (net) energy analysts. [20, 21] Figure 1.3 depicts the traditional economic model updated to account for resource flows from the biosphere into the economy. The economy has changed from an *isolated* system into an *closed* system, since only energy inputs have been accounted.

The metaphor for such a model is still very much a mechanistic one. This mechanistic description lends itself to a view of the economy, much like the engines of the Industrial Revolution, as well-behaved and amenable to control. Machine metaphors abound in our economic discussions. We speak of the "fueling" the "economic engine" lest it should "stall." [22] Like an engine, the economy is assumed to be resilient to small and even quite large perturbations. It can either self-correct, or be corrected with adjustments to a few predictable policy levers. Additionally, the biosphere is relegated to the position of a provider of resources; the larder of the economy, if you will.[23]

1.2.3 The metabolic economy

The cowboy economy soon depletes resources. The consumption of resources also leads to wastes. The question arises as to ''what to do with these wastes?" The traditional economic view sees these wastes as an anomaly to the normal functioning of the economy. [**?**] Thermodynamics takes a different stance.

According to thermodynamics, all real-world processes involve the degradation of material and especially energy resources and the creation of entropy. This is the second law of thermodynamics. High quality (low entropy) resources come in, low quality (high entropy) resources come out. The depiction in Figure 1.3 can be classified as a perpetual motion machine of the *second kind*: it perfectly converts energy resources into work (useful services) without generating any entropy, in violation of the second law of thermodynamics. Since the generation of high entropy (low qual-

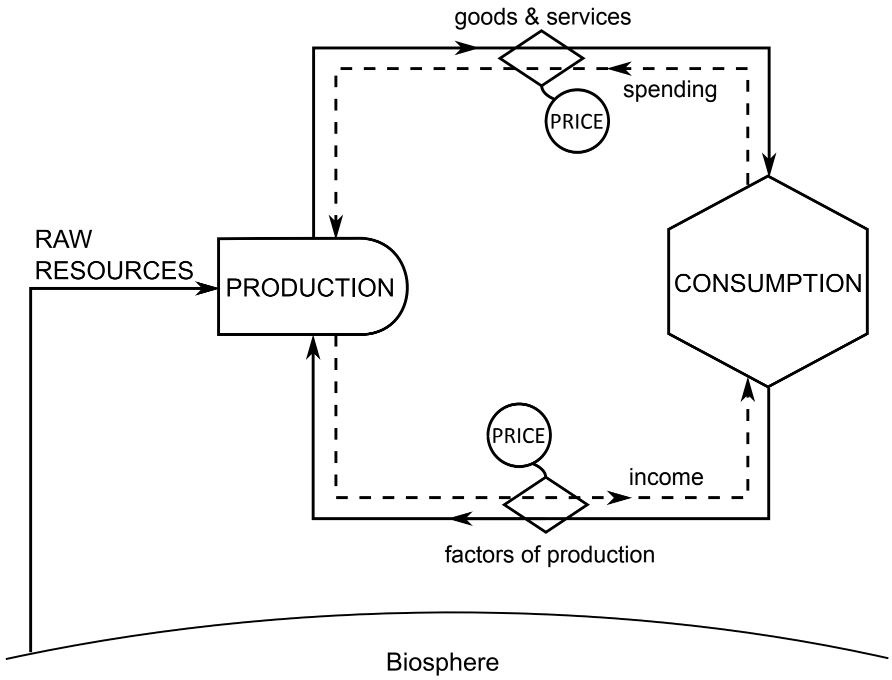

Fig. 1.3 Energy and material input output analysis has included the flows into the economy from the environment. This may be considered a perpetual motion machine of the second kind. *** MCD—SHOULD FLOW FROM 'RAW RESOURCES' ALSO GO STRAIGHT INTO CONSUMPTION?***

ity) output is a *necessary* feature of *all* processes (including economic processes) then the generation of wastes is a normal feature of economic processes, not an anomaly. Within a closed system, such as the earth, these wastes soon accumulate, necessitating the change to a "spaceship" economy, wherein account is made of the waste outflows of the economy.

In Figure 1.4, the production and consumption sectors both produce waste flows which must be assimilated by the biosphere. Our circulatory system also circulates carbon dioxide (waste from the consumption of food-fuel with oxygen) and urea (waste from protein catabolism) for extraction and exchange to the environment.

However, even with this expanded model, the simplifying metaphor, that of an engine, remains intact. Predominantly linear relationships of inputs and outputs rule the day. Successful management of local pollution, such as SO_2, through environmental legislation is a positive result of this expanded model. However, systemic events resulting from reinforcing feedback loops or from events occurring outside historical experience cannot be modeled and are not predictable. The 2007 global financial crisis is an example of one such systemic event. **** BRH cite the economist article *** The 1930's dust bowl is also a result of a systemic event. It was one of the greatest man-made environmental disasters, arising from unexpected, non-linear results of widely accepted traditional farming practices, implemented in a new, and unknown ecosystem. ***BRH cite Lockertz 1978 ***

Ecological economists, in the tradition of Herman Daly, have begun to update the traditional models. [24] In ecological economics texts, the economy is represented as a *open* system. The guiding metaphor for this kind of economic model is an *organism*. The blood serves a self-regulatory, homeostatic role by transporting hormones. Like an organism, the economy metabolizes energy and materials that it receives from natural resources into forms usable for human purposes. The economy's behavior is non-linear and chaotic, but also self-regulating. Models and accounting systems are being developed to describe "society's metabolism." [25–31] The following sections outline attempts to account flows of economic factors through the economy.

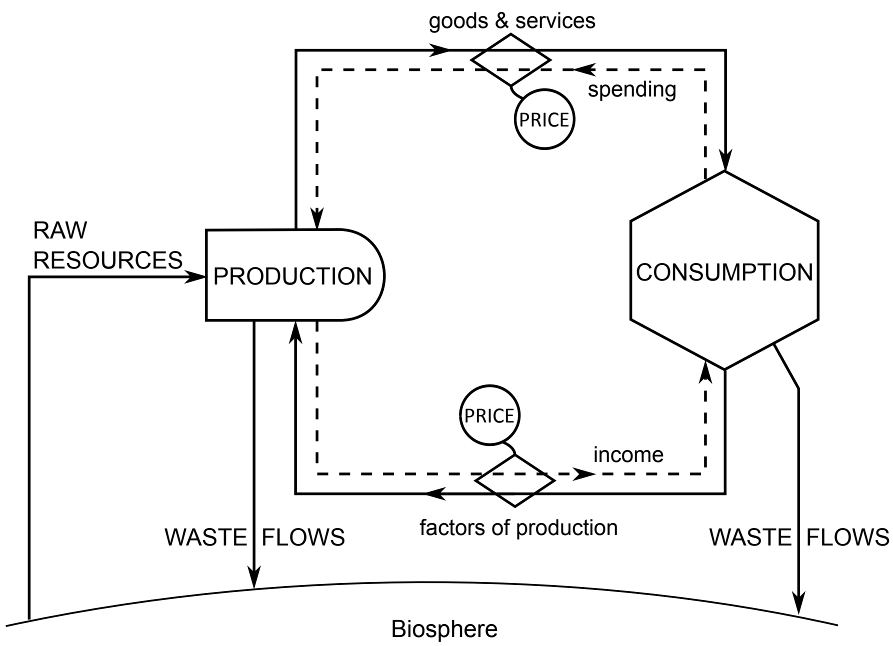

Fig. 1.4 A comprehensive model of the economy, fully consistent with the laws of thermodynamics must include degraded resources (waste) expelled to the environment as a necessary consequence of economic activity.

1.3 Brief history of input-output (I-O) modeling

Input-output analysis, developed by Wassilly Leontief in the 1930s as an extension to the work of Quesnay and Walras [32], is of primary importance in national accounting. The method allows determination of the flows of value through an economy as well as, among other things, calculation of a nation's gross domestic product (GDP), the predominant measure of economic activity.

1.3.1 Basic I-O method

The basic premise of the I-O method, as depicted in Figure 1.5A, is that each economic sector takes in factors of production from other sectors (and possibly itself) to produce an economic good at some rate. For example, the automotive sector takes in steel, rubber, glass, etc. and produces a number of cars per year. In contrast to high-level economic growth models that include only a few factors of production (such as land, capital, and labor), the I-O analysis technique allows many differentiated factors of production and raw material feedstocks.[33] In I-O frameworks, each factor of production is considered to be a output from a sector of the economy. As will be discussed later, the traditional primary factors of production (land, capital, and labor) are not *flows* into the production processes. Rather, they are *stocks* that, when present, allow factors of production (steel, rubber, and glass) to be transformed into final products (automobiles). The quantity and quality of these stocks determine the quantity and quality of their flow of productive services.

1.3.2 I-O method including inputs from the biosphere

Now that the global economy is experiencing the pressure of these physical constraints, the necessity for a more comprehensive model, and collection of data to estimate it, is emerging. Environmental Economics, as a sub-discipline, expands the traditional model to include material exchanges with the biosphere.

In addition to the productive services provided by flows of capital and labor, a flow of energy[1] is required for economic activity. These energy flows originate from the natural environment, recognition of which has provoked researchers from fields of net energy analysis (NEA), to extend the traditional (Leontief) input-output framework to include important energy flows from the environment, to form an energy input-output (EI-O) model as depicted in Figure 1.5B.[34–37, 33, 38–40] While the Leontief I-O approach relies exclusively on monetary units to represent value flows among sectors of an economy, the key insight of the EI-O framework is to rely upon physical units (especially energy units of joules) to represent some of the flows among economic sectors. In doing so, energy intensities of monetary flows can be estimated.

However, as we discussed above, resource inputs are only one side of the coin. According to thermodynamics, all acts of production (requiring inputs from the bio-

[1] Or, more precisely, the degradation of an exergetic gradient/destruction of exergy.

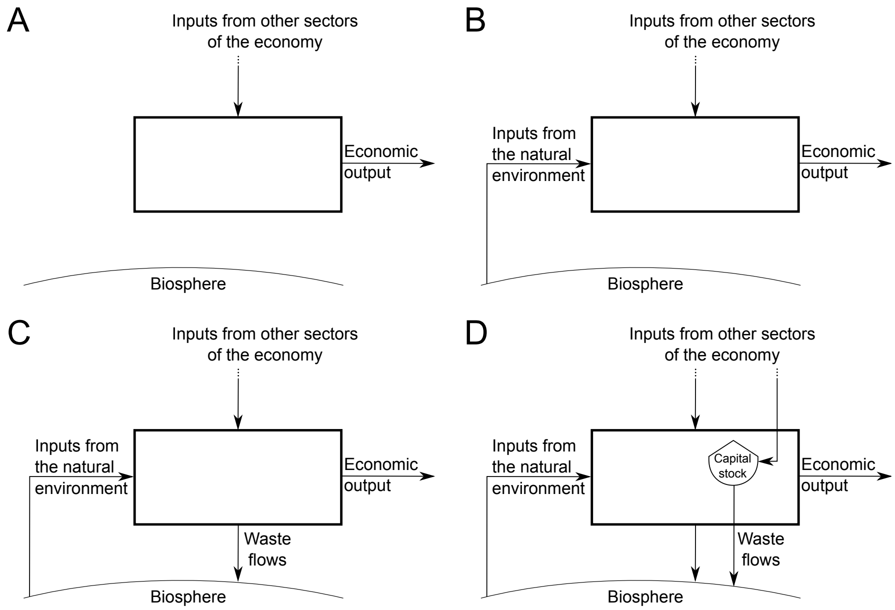

Fig. 1.5 The basic unit of input-output modeling: **A** the standard economic approach includes only transactions among sectors of the economy; **B** the energy input-output approach models inputs from the natural environment outside the economy as factors of production; **C** including waste flows to the environment makes the model physically consistent and; **D** the method presented here accounts also for accumulation in capital stock, K, of embodied energy within materials in economic sectors.

sphere) are simultaneously acts of consumption, degrading high quality inputs into wastes to be discharged back into the biosphere.

1.3.3 I-O methods including resource inputs and waste flows

A number of disciplines including material flow analysis (MFA), industrial ecology (IE), and life-cycle assessment (LCA) have developed further extensions of the EI-O framework to account for energy and material flows both into and out of the economy from the biosphere. Motivated by concerns over environmental impacts (especially climate change), the primary focus of such efforts has been on tracking flows of materials (especially greenhouse gases) into the biosphere rather than on resource depletion. [41, 26, 42**?** –44] This extended approach is depicted in Figure 1.5C.

The making industries related to extraction, refining, and utilities mark the entry point for materials extracted from the biosphere into the economy. These three industries (extraction, refining, and utilities) are the first three industries listed in the NAICS (North American Industry Classification System) that the BEA uses to track economic information. ****BRH add cite (Law- son and et al. 2002, 25). **** MCD—Becky, do you have this ref? *** Conversely, consumption, with associated waste discharge, occur from all sectors of the economy. But why do we have consumption? Boulding tells us that consumption is the real cost of living in a physical world; that there is,

> no particular virtue in consumption. It is, unfortunately, a necessary incident in the business of living. We cannot eat without destroying food; we cannot walk without destroying shoes; we cannot drive without destroying gasoline, tires, and cars; and so on. [45, p.2]

In a sense then, it is the existence of physical objects within our economies (and their subsequent degradation) that is the driver of consumption. As such, to truly understand our economies and the flows of materials through them, we must account the physical stocks within them.

1.3.4 An I-O method for dynamic (transient) economic analysis

- Important to highlight difference between material depreciation (Boulding's 'consumption') and financial depreciation.
- Financial depreciation is typically (under normal circumstances) a leading indicator of material depreciation
- **** MCD—I did not include these here. Have we included them elsewhere?****

Both the original Leontief I-O framework and the extensions cited above assume steady-state conditions in an economy, i.e., flows of value and material into and out of each economic sector are in balance. Dynamic or transient behavior of the economic system is not considered. Thus, there is no accumulation of economic factors or embodied energy within any of the sectors. The analysis techniques provide "snapshots" of economic activity at an instant in time.

Assuming no accumulation of materials, within economic sectors or society itself, is tantamount to assuming that *all* material flows through the economy are directed toward the production of non-durable goods. However, evidence of the durability of goods and the accumulation of materials surrounds us. Furthermore, energy was required to both fabricate and emplace the durable goods and infrastructure of modern economies. (The energy it took to create the durable goods and infrastructure can be considered "embodied" within the built environment, a point to which we will return in detail later). As Georgescu-Roegen notes, "in the everyday world one cannot possibly cross a river only on the flow of maintenance materials of a non-existent bridge."[46]

Analysis methods that neglect the accumulation of materials and embodied energy in the durable goods and infrastructure of the everyday world lack explanatory power. Such models can tell us at what rates materials and energy are required to *use* our built environment. But, such models cannot tell us *how* the built environment came to be (and how much energy was required to construct it) or *why* flows of goods are needed. To use Georgescu-Roegen's imagery, models that neglect accumulation fail to explain why we need any material flows to maintain a non-existent bridge. Stocks of accumulated materials (capital, appliances, even people) are the drivers of demand. It is to service their needs and wants that we put the economy to work.

Georgescu-Roegen distinguishes two types of economic inputs: *flows* which are the resources (such as cloth and thread) to be transformed into final products (shirts) and *funds*—stocks (such as needles) that must be present in order to facilitate the transformation. Confusing the two categories could lead to some painful consequences [47]. Daly makes a similar distinction between *material* and *efficient causes*. [48] A full model of the economy must account the stocks (funds, efficient causes) that give rise to the flows (material causes).

Because economic activity requires energy, we need to understand the way energy flows through economies. The steady-state I-O techniques of Bullard, Herendeen, and others[35, 37] offer a means to that end. We contend, however, that these techniques need to be extended and modified to include transient effects that arise when durability of goods and infrastructure (and associated embodied energy) are considered. This manuscript attempts to address that need.

In this manuscript, we develop a physical input-output, matrix-based method for modeling multi-sector economies, in the tradition of Georgescu-Roegen's "flow-fund" model.[49, 50] The method presented in this paper takes a decidedly engineering approach **** Need to re-cast in metabolism language!!!! **** to extend the techniques of Bullard, Herendeen, and others to account for durability of goods and embodied energy. This method allows us to see how energy and materials flow through the economy, where embodied energy accumulates in the economy, and how declining resource quality may affect these dynamics.

1.4 Structure of the book

The remainder of this book is organized as follows. Part I models flows of physical matter through the economy. Chapter 2 presents a discussion of material flows. Flows of direct energy are discussed in Chapter 3, and a rigorous, thermodynamics-based definition of and accounting for embodied energy is presented in Chapter 4. In Part II we turn to non-physical flows through the economy. Flows of economic value are discussed in Chapter 5. In Chapter 6 we combine the results from Chapters 4 and Chapter 5 to calculate the energy intensity of economic production. Part III gives context to the framework developed in Parts I and **??**. Chapter 7 draws out some of the direct implications of the results. Chapter 8 looks at unfinished business: practical, conceptual, and theoretical issues that arise in the development of this framework. We finish off the book with a summary in Chapter 9.

Throughout the methodological Chapters (2-5) the framework is developed through a series of increasingly-disaggregated models of the economy (Table 1.2). In addition, we use the the US auto industry as a running example for application and discussion.

Table 1.2 Examples used throughout this book

Example	Sector 0	Sector 1	Sector 2	Sector 3
A	Biosphere	Society	NA	NA
B	Biosphere	Final Consumption	Production	NA
C	Biosphere	Final Consumption	Energy	Goods & Services

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Part I
Material and Energy Flows

Chapter 2
Material flows

Well, I have my rights, sir, and I'm telling you I intend to go on doing just what I do! And, for your information, you Lorax, I'm figgering on biggering and BIGGERING and BIGGERING and BIGGERING... [1]

—The Once-ler

In Chapter 1, we put forward the idea that economies are like organisms. This chapter explores this idea further by observing the interchange of materials *within* an economy, as well as exchanges of materials between an economy and surrounding environment—the biosphere.

There are many easily observable instances of material flow within an economy. I look around my office at my computer screen and coffee cup and myriad other items. I look out my window to the street and building opposite. All of these goods came originally from natural resources, be it paper or petroleum or rock. They were extracted and processed, transported and transformed requiring yet more materials and energy inputs in the form of electricity or fuels.

There are also inumerable material flows caused by an economy that we do not observe. The extraction of raw materials generates additional overburden—earth that must be extracted and processed and ultimately discarded without ever entering the economy proper. Other flows occur around us unseen. The cars outside my window suck in nitrogen and oxygen (without which the engine would not work) and emit water, carbon dioxide and other more harmful substances.

Even services which we tend to think of as non-material, require at least some material infrastructure. The hairdresser requires scissors (and to a greater or lesser extent some hair) with which to work. Even the internet, often lauded as the exemplar of dematerialization of the economic process, requires a whole host of computer infrastructure including electricity, data servers, telephone networks and a computer by which to access it.

In this chapter, we will define a mathematical framework by which to track the flow of materials within an economy, building from a one-sector economy up to examples of both a two- and three-sector economy. We will finally apply this framework to the illustrative example of the US automobile industry that runs through the whole book. First let us outline the basic methodology.

2.1 Methodology

This book is about tracking (accounting) flows through the economy with a focus on counting materials, energy, and value. That an entire academic discipline and industry are focused on counting money (“accounting”) is evidence of its importance in today’s economies. That energy is required to do *anything* is evidence of its importance in the economic activity of our daily lives. And, we believe that the interplay between money and energy has shaped the past and will continue to influence the future. In this section, we define rigorous “counting” methods that will be applied to money and energy throughout this book.

**** Include somewhere in here the difference between *flows* and *funds*, between *efficient cause* and *material cause*. Resources are *transformed* into products.****

2.1.1 Accounting in everyday life

Everyone counts material (and even non-material) things. Rigorous counting requires precise definition of both what we will be counting and the place (defined in both time and space) in which we will be counting. Engineers often call the spatial definition a “control volume.” Another way to think of creating a control volume is drawing a boundary. What gets counted is what passes through the boundary. For example, we

**** Do we want to write in the first person? **** **** Good question. We’re using the first person in most of the chapters, but plural, “we”, rather than singular. I’ve changed the “I” to “we”, but we should double-check that we’re happy with it, as it mixes plural and singular first person: “we” with “my” and “I” ****

may wish to count (or “make an accounting of”) the stock of apples in my home over the course of a week. We draw a spatial boundary (control volume) around my home and a temporal boundary “around” the week. We count the apples that enter and leave my home, apples that are eaten (consumed), and, if I own an apple tree, apples that I grow (produce) during a week. A rigorous apple accounting equation, in units of apples, is:

$$\Delta\text{apples} = \text{apples in} - \text{apples out} + \text{apples grown} - \text{apples eaten}. \tag{2.1}$$

More generally, we may say:

$$\text{Accumulation} = \text{Transfers in} - \text{Transfers out} + \text{Production} - \text{Consumption}. \tag{2.2}$$

Notice that, when discussing apples we use the specific terms “grown” and “eaten” instead of the more general terms, “produced” and “consumed.” Later, in Chapter 5, when discussing value, we will use the terms “generated” and “destroyed.” For our purposes, these terms all have the equivalent meanings and we use them interchangeably.

After accounting for the stock of apples in a week, we can reframe the questionL "at what rate does the stock of apples change?" That is, we can examine the rate of change of the apple stock per unit of time relative to the flow of apples ($\dot{a}$), e.g. in units of apples per day, in which case our accounting equation would become:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \dot{a}_{in} - \dot{a}_{out} + \dot{a}_{grown} - \dot{a}_{eaten} \tag{2.3}$$

where the dot above the variable ($\dot{a}$) indicates a flow rate per unit time [apples/time] and the time derivative $\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)$ is the rate of change of the stock of apples per time unit, or more simply, the accumulation rate.

Notice, that instead of focusing on apples as our unit of accounting, we could track the mass flow, e.g. measured in kg/sec, of the main chemical elements within the apples. From this perspective, although an apple may be consumed, the elements within the apple—hydrogen, oxygen (coupled together as water for the overwhelming majority of the mass), and carbon (which, bonded with hydrogen as carbohydrates make up most of the remaining mass)—will not be consumed. The chemical elements will instead be stored within my body, leave the house as waste, or leave the house via the air as exhaled carbon dioxide (CO_2).

If, instead of a home, we drew a spatial control volume around a sector of an economy, similar accounting methods can be applied. In fact, throughout this book, we will illustrate theoretical concepts with a running example of the auto industry. We choose the auto industry, because it remains a large portion of most industrialized economies, because is very resource intensive, because it has been used in the literature [2] to illustrate input-output accounting methods, because its links with energy are obvious, because its health is sensitive to disruptions in energy supplies, and because it shows evidence of post-industrial decline (shrinking profit margins, etc.).

If we account for steel (in units of kg) in the auto industry, we might write an equation like this:

$$\Delta\text{steel} = \text{steel in} - \text{steel out} \tag{2.4}$$

Note that the production and consumption terms are zero since steel is not created or destroyed within the automobile sector. Tracking the rate flows of steel, $\dot{s}$ (in kg/s), we would write the following equation:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \dot{s}_{in} - \dot{s}_{out} \tag{2.5}$$

Again, the last two terms (steel production and consumption) are not present. This is in direct contrast with the apple accounting equation outlined in Equation 2.3. **** Do the previous two sentences make sense? The equation has only 2 terms. **** I have reworded to specify the production and consumption terms. **** Despite the fact that steel is neither produced nor consumed within the automobile sector, there are sectors of the economy that *do* produce steel, by mixing molten iron with varying amounts of carbon. The flow of steel through an economy illustrates that although certain economic products, e.g. steel, may be produced or, in some circumstances, destroyed, the *mass* of iron, and other chemical elements, is constant, even as the mass changes form (e.g. from iron to steel) through many economic processes.[1]

Indeed, within the car industry, inputs of steel, glass, plastic, rubber, etc. are used to produce cars, such that cars are created within the automobile industry. An accounting equation for cars within the economy must include terms for production and destruction[2] of cars. Again, focussing on mass flows of the chemical elements avoids this necessity, because mass is *conserved* in physical processes. Conservation of mass is expressed in equations such as the ones above for apples and steel.

Another important conservation principle is the conservation of energy. Similar to the principle of the conservation of mass, the First Law of Thermodynamics, says that *energy* can neither be created nor destroyed. In the discussion that follows, we will make great use of the First Law. If I eat an apple, it is no longer an apple, but the materials (i.e. chemical elements) and energy contained within the apple can still be traced via their mass and energy, even if they change form (apples into compost or chemical potential energy to thermal energy). Thus, the apple accounting equation (Equation 2.3) can include terms accounting for the production and consumption of apples. However, mass and energy accounting equations applied to sectors of economies will *not* include terms for the production or destruction of mass or energy. Rather, any addition of material or energy added *to* the economy or waste of material or energy *from* the economy will occur as an interaction between the economy and the biosphere. Chapters 2–4 cover mass and energy accounting for economies. Accounting for economic value, in contrast, *will* require terms that both create and destroy economic value, as discussed in Chapter 5.

2.1.2 Product, resource, short-lived, and capital flows

When applying accounting equations to economic sectors, we distinguish among four types of materials flowing into or out of a production sector: products (P) , resources (R), short-lived goods (S), and capital goods (K), as shown in Figure 2.1.

Resource materials ($\dot{R}$) enter the sector on the left and comprise those materials that are destined to be *embodied* in the goods produced by the sector ($\dot{P}$), except

[1] For the sake of absolute rigor, we must point out that, in actuality, iron *is* created within the core of silicon-burning stars. Mass and energy may also be converted in such processes, such that only mass-energy is conserved. However, for the purposes of terrestrial processes, the total mass (in kg) of iron is constant. There are, additionally, some economic processes, within nuclear reactors, that change the atomic structure of elements and thus violate the accounting law presented here. Because the mass flows involved with these nuclear plants is negligible compared with total materials flows, we shall assume that the law holds.

[2] In economic terms, destruction of physical goods is often called "depreciation." We shall explore the importance of and distinctions between physical depreciation and economic depreciation in Chapters 4–7.

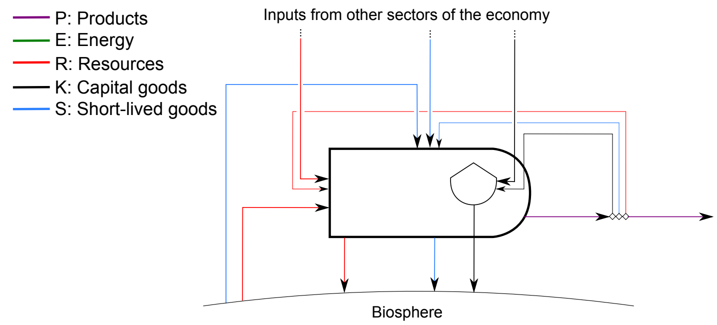

Fig. 2.1 Material flows into and out of a single sector of the economy. Resource flows ($\dot{R}$) enter the sector from the left and are embodied in products ($\dot{P}$) which leave from the right. Some waste resources are leave the sector at the bottom and are returned to the biosphere. Short-lived material flows ($\dot{S}$) enter the sector from above and leave from below to return to the biosphere. Only capital stock ($\dot{K}$) may accumulate within the sector, depicted by the storage tank. These also enter the sector from above. Depreciated capital leaves the sector from below and is returned to the biosphere.

for some proportion that are wasted. Wastes depart from the bottom of the sector and are returned to the biosphere. For example, sheet metal, rubber, and glass (as well as many other materials) enter the automobile sector as resources and end up as material parts of the cars that are produced. Some fraction of these resources ($\dot{R}$) may not make it into the final product, such as trimming scrap from metal parts stamping, and may be either recycled internally, or wasted to the biosphere. In this model, resource materials are not accumulated within a sector.

Short-lived goods ($\dot{S}$) include those materials that are necessary for the production processes of a sector, but are neither accumulated within the sector, nor destined to be materially part of the product of the sector. They enter the sector from above and leave the sector from below and return to the biosphere. Examples of these short-lived flows include energy resources, such as the electricity needed to run automobile factories and water used by the sector.

Many material flows into the sector, such as production equipment, are necessary for the continued operation of a sector but are not counted as short-lived goods, because the operation of the sector is dependent upon the accumulation of these materials within the sector. Such flows are counted as capital goods ($\dot{K}$). Capital flows also enter from above, but are stored within the sector (represented by a storage tank) and are returned to the biosphere as physical capital depreciation from below. Examples of these capital flows would be the factory and office buildings or manufacturing equipment within the automobile industry. We are here assuming that there is no re-use of capital stock within other sectors of an economy, e.g. resale of vehicles or other equipment after depreciation, or recycling of material from capital stock into other goods, e.g. scrap metal. The issue of recycling is in greater detail in Section 7.3.

All products ($\dot{P}$) leave the right of the sector. A fraction of the $\dot{P}$ flow may be returned to the sector as self-consumption counted as resources destined to be embodied in the product ($\dot{R}$), short-lived materials ($\dot{S}$), or capital goods ($\dot{K}$); the remainder flows to other sectors within the economy or final demand. In this model, energy may be accounted as either an $\dot{R}$ flow or an $\dot{S}$ flow. An example of energy as an $\dot{R}$ flow is crude oil converted into gasoline within a refinery: the resource inflow (crude oil) is *literally* embodied within the outflowing product (gasoline). An example of energy as an $\dot{S}$ flow is electricity used by an automobile factory: the resource inflow (electrons) is not embodied *literally* in the outflowing product (automobiles). Similarly, the coal or natural gas flowing into a power plant is accounted as an $\dot{S}$ flow, because the incoming chemical elements (carbon and hydrogen) *do not* depart the plant as the product. (The product of a power plant is electrons that travel through electricity transmission lines.)

Throughout this book, we illustrate concepts with three example economies, with increasing levels of disaggregation. Example A includes a single economic sector (1) and the biosphere (0). Example B includes a production sector (2), society (1, which provides final demand), and the biosphere (0). Example C adds an energy sector (3) to Exammple B. We begin with materials accounting for Example A.

2.2 Example A: single-sector economy

Our first example looks at the case where all processes within the economy occur within one sector—Society (1)—which exchanges materials with the biosphere (0) as depicted in Figure 2.2. We do not distinguish between production and consumption.

Resources, or perhaps more accurately raw materials, ($\dot{R}_{01}$), such as crude oil or iron ore, and short-lived materials ($\dot{S}_{01}$), such as oxygen or water that flow *through* economic processes but are not literally *embodied* within the output, flow into the economy (1) from the biosphere (0). These materials are processed within the economy into products ($\dot{P}_1$) consisting of resource goods ($\dot{R}_{11}$), short-lived goods ($\dot{S}_{11}$) and capital goods ($\dot{K}_{11}$) which are able to be accumulated at some rate $\frac{\mathrm{d}K_1}{\mathrm{d}t}$ within the stock of materials within society[3]. Waste resources ($\dot{R}_{10}$) and used short-lived materials/goods ($\dot{S}_{10}$) are returned to the biosphere without accumulating in society (1). Capital goods ($\dot{K}_{10}$) are returned to the biosphere when they are physically depreciated.

Drawing control volumes around both the biosphere (0) and society (1) in Figure 2.2, we can consruct material accounting equations, such that:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} + \frac{\mathrm{d}S_0}{\mathrm{d}t} + \frac{\mathrm{d}K_0}{\mathrm{d}t} = \dot{R}_{10} + \dot{S}_{10} + \dot{K}_{10} - \dot{R}_0 - \dot{S}_0 \tag{2.6}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} + \frac{\mathrm{d}S_1}{\mathrm{d}t} + \frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{01} + \dot{S}_{01} + \dot{R}_{11} + \dot{S}_{11} + \dot{K}_{11} - \dot{P}_1 - \dot{R}_{10} - \dot{S}_{10} - \dot{K}_{10}. \tag{2.7}$$

[3] See Section!8.7 for more discussion on the inclusion of human beings as societal capital stock.

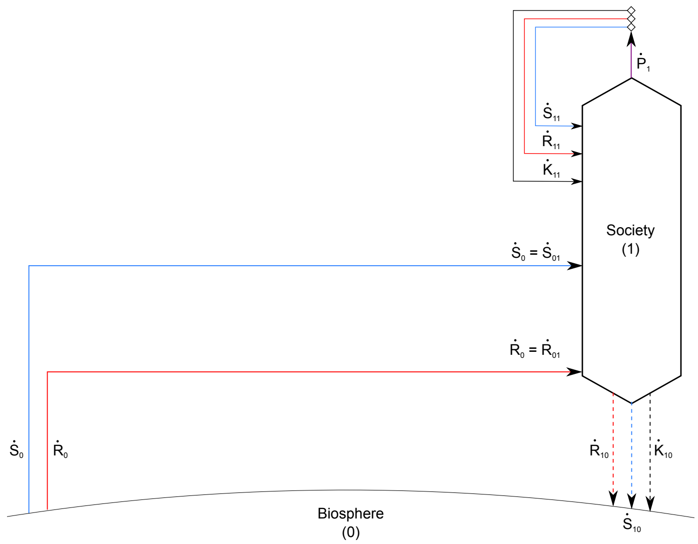

Fig. 2.2 Flows of materials for a one-sector economy. Resources ($\dot{R}_{01}$) and short-lived materials ($\dot{S}_{01}$) flow into the economy (1) from the biosphere (0). Waste resources ($\dot{R}_{10}$) short-lived materials/goods ($\dot{S}_{10}$) and capital goods ($\dot{K}_{10}$) are returned to the biosphere.

Because mass is conserved, we find that:

$$\dot{R}_0 = \dot{R}_{01}, \tag{2.8}$$

$$\dot{S}_0 = \dot{S}_{01}, \tag{2.9}$$

$$\dot{P}_1 = \dot{R}_{11} + \dot{S}_{11} + \dot{K}_{11}, \tag{2.10}$$

Clearly, $\dot{R}_{01} \neq \dot{R}_{10}$ since some resources are converted into short-lived goods ($\dot{S}_{11}$) or man-made capital ($\dot{K}_{11}$) and are only returned to the biosphere as either $\dot{S}_{10}$ or $\dot{K}_{10}$, respectively. Hence, we may say that:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} = \dot{R}_{10} - \dot{R}_{01} \neq 0. \tag{2.11}$$

Additionally, and for similar reasons, we know that $\dot{S}_{01} \neq \dot{S}_{10}$,[4] such that:

$$\frac{\mathrm{d}S_0}{\mathrm{d}t} = \dot{S}_{10} - \dot{S}_{01} \neq 0. \tag{2.12}$$

In this model, neither resources (R) nor short-lived goods (S) accumulate within economic sectors, so we may also state:

[4] While this inequality may be true in theory, it may be that in practice, the large amount of material, e.g. water or oxygen, that passes straight through the economy 'unaffected', i.e. without being embodied in products, is very large compared to the additional flow of short-lived goods produced within the economy, i.e. $\dot{S}_{11} << \dot{S}_{01}$.

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = 0, \tag{2.13}$$

$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = 0. \tag{2.14}$$

Because the only "capital" that accumulates in the biosphere is that which is a waste flow (capital depreciation) from the economy, e.g. worn-out machines in the scrap yard, we may say that:

$$\frac{\mathrm{d}K_0}{\mathrm{d}t} = \dot{K}_{10} \tag{2.15}$$

Looking deeper at flows of resources and short-lived goods, we can make some further observations. Imagine following a kilogram of coal on its journey through the economy. It is pulled out of the earth as part of flow $\dot{R}_{01}$. It enters the economy and is transformed into useful products (part of $\dot{P}_1$) and some is wasted ($\dot{R}_{10}$). Some of the coal is destined for metalurgical processes (such as the production of steel) and so re-enters the economy within flow $\dot{R}_{11}$, since the carbon in the coal ends up physically *embodied* within the steel however, again, some of the coal is wasted, leaving the economy as flow $\dot{R}_{10}$. Some of coal is destined for electricity generation and so re-enters the economy as part of flow $\dot{S}_{11}$, since the coal is *not* physically embodied in the electricity and leaves the economy (in the form of carbon dioxide and ash) as part of flow $\dot{S}_{10}$. In summary, we may say that resources are destined to end up either as products or waste 'resources' and that short-lived materials flow 'straight through' the economy. As such, we may state that:

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = \dot{R}_{01} + \dot{R}_{11} - \dot{P}_1 - \dot{R}_{10} = 0, \tag{2.16}$$

$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = \dot{S}_{01} + \dot{S}_{11} - \dot{S}_{10} = 0, \tag{2.17}$$

We may rearrange these equations in terms of the important variable as:

$$\dot{P}_1 = \dot{R}_{01} + \dot{R}_{11} - \dot{R}_{10}, \tag{2.18}$$

$$\dot{S}_{11} = \dot{S}_{10} - \dot{S}_{01}. \tag{2.19}$$

Substituting equations 2.8, 2.9 and 2.15 into Equation 2.6 we obtain:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} + \frac{\mathrm{d}S_0}{\mathrm{d}t} = \dot{R}_{10} + \dot{S}_{10} - \dot{R}_{01} - \dot{S}_{01}, \tag{2.20}$$

Equation 2.20 states that the rate of "accumulation" (or more accurately depletion) of natural capital (R_0 and S_0) is dependent on the rates at which society extracts these materials from the biosphere ($\dot{R}_{01}$ and $\dot{S}_{01}$) and the rates of disposal of waste materials back to the biosphere ($\dot{R}_{10}$ and $\dot{S}_{10}$).

Notice however, that although Equation 2.20 is true for total mass of materials, it makes no comparison of the *quality* of these materials. Society relies heavily on extracted resources from naturally occurring accumulations that are far from equilibrium with their surroundings, e.g. fossil fuel reservoirs or seams of high-grade ore. As these high quality material reserves are depleted and society must turn to lower grade reserves, more energy and other economic factors (labor, capital, money) *must* be expended in material extraction.[3] It is likely that the quality of flow R_{01} is higher than flow R_{10} (e.g. overburden from mining operations). If this were not the case, $\dot{R}_{10}$ could be easily substituted into the production process (i.e. recycled) thus offsetting the need for primary resource extraction[5]. This relationship between incoming high quality resources and emission of low quality waste is a general truth for any thermodynamic process but is especially true for open systems operating far from thermodynamic equilibrium (of which every living system is an example). Such systems maintain their internal structure by importing high quality energy and materials (negentropy) and exporting low quality energy and materials (entropy) [4].

Substituting Equation 2.17 into Equation 2.7, we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{01} + \dot{R}_{11} + \dot{K}_{11} - \dot{P}_1 - \dot{R}_{10} - \dot{K}_{10}. \tag{2.21}$$

Since we have two different formulations for $\dot{P}_1$, represented by Equations 2.10 and 2.18, we may substitute either into Equation 2.21. Substituting Equation 2.18 into Equation 2.21, we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{K}_{11} - \dot{K}_{10} \tag{2.22}$$

which tells us that accumulation of capital in society (K_1) is dependent only on inflows of capital into society ($\dot{K}_{11}$) and depreciation of capital to the biosphere ($\dot{K}_{10}$).

Substituting instead Equation 2.10 into Equation 2.21, we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{01} - \dot{R}_{10} - \dot{S}_{11} - \dot{K}_{10}. \tag{2.23}$$

The last depreciation term[6] ($\dot{K}_{10}$) may be rewritten as the total stock of man-made capital (K_1) multiplied by some depreciation rate (γ_{K_1}) in units of inverse time (e.g. 1/year), where γ_{K_j} is defined as:

[5] The issue of recycling is discussed in more detail in Section 7.3. The issue of material (and energy) quality is discussed in more detail in Section 8.5.

[6] This depreciation term will be discussed in more depth in Sections 4.2.3 and 7.1.2.2.

$$\gamma_{K_j} = \frac{\dot{B}_{j0}}{B_j}, \tag{2.24}$$

i.e. the depreciation *per unit* of capital stock, such that Equation 2.23 becomes:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{01} - \dot{R}_{10} - \dot{S}_{11} - \gamma_{K_1} K_1. \tag{2.25}$$

It is important to note that γ_{K_1} is inversely proportional to the average lifetime of man-made capital (K_1). This will be discussed in more detail in Section 7.4. We may rearrange Equation 2.25 as:

$$\dot{R}_{01} - \dot{R}_{10} = \frac{\mathrm{d}K_1}{\mathrm{d}t} + \dot{S}_{11} + \gamma_{K_1} K_1. \tag{2.26}$$

Noticing that the left-hand side of Equation 2.26 is the negation of the right-hand side of Equation 2.11, we may rewrite Equation 2.26 in terms of the accumulation (or more accurately, depletion) of natural resources:

$$-\frac{\mathrm{d}R_0}{\mathrm{d}t} = \frac{\mathrm{d}K_1}{\mathrm{d}t} + \dot{S}_{11} + \gamma_{K_1} K_1. \tag{2.27}$$

Equation 2.27 tells us that depletion of natural resources $\left(-\frac{\mathrm{d}R_0}{\mathrm{d}t}\right)$ are used within society in order to:

- build up societal capital stock $\left(\frac{\mathrm{d}K_1}{\mathrm{d}t}\right)$,
- provide short-lived goods and energy to run society ($\dot{S}_{11}$), and
- overcome depreciation ($\gamma_{K_1} K_1$).

2.3 Example B: two-sector economy

In our second example B, we split the economy into two sectors: a production sector (2) and final consumption (1), as depicted in Figure 2.3. Sector (2) produces all of the goods and services that are delivered to final consumption (1), as well as all of the intermediate goods that are not "consumed" as part of final consumption, but stay within the production sector, e.g. manufacturing equipment.

As can be seen in Figure 2.3, Sector (2) resembles very closely the basic unit outlined in Figure 2.1. Resource flows from the biosphere ($\dot{R}_{02}$) and those produced by Sector (2) itself ($\dot{R}_{22}$) are *transformed* into product flow ($\dot{P}_2$). Flows of short-lived goods ($\dot{S}$) and capital ($\dot{K}$) are required to support this transformative process. Much of the product flow from Sector (2) enters final consumption (1) as resource ($\dot{R}_{21}$), short-lived good ($\dot{S}_{21}$) and capital good ($\dot{K}_{21}$) flows.

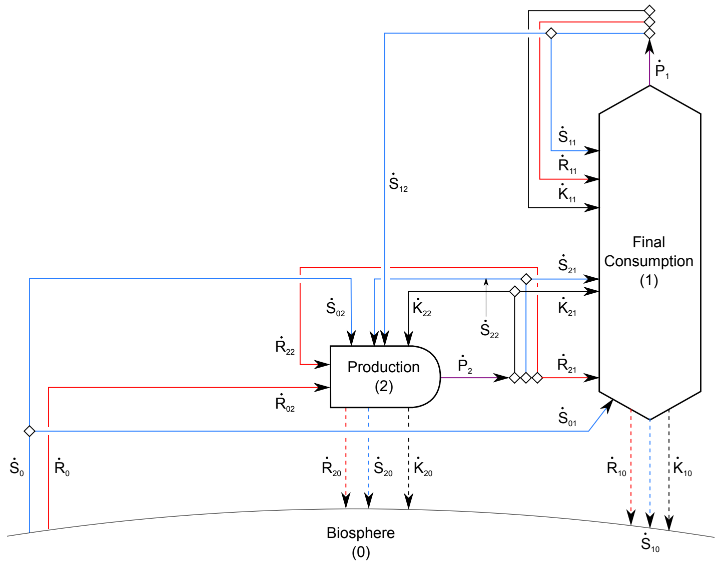

Fig. 2.3 Flows of materials for a two-sector economy.

There is also a product outflow from final consumption ($\dot{P}_1$), some of which is returned to final consumption as resource ($\dot{R}_{11}$), short-lived good ($\dot{S}_{11}$) and capital good ($\dot{K}_{11}$) flows. A flow of short-lived goods ($\dot{S}_{12}$) flows from final consumption (1) to production (2) associated with the flow of labor. There is no resource ($\dot{R}_{12}$) nor capital good ($\dot{K}_{12}$) flow from society (1) to production (2). This is because the 'product' of final consumption is the production of labor services and the "consumption" of final goods. No materials flow from final consumption to be *transformed* into or *embodied* in the production goods of Sector (2), therefore $\dot{R}_{12} = 0$. Additionally, no capital *goods* flow from final consumption to accumulate within the production sector, therefore $\dot{K}_{12} = 0$.

The short-lived goods flow ($\dot{S}_{12}$) from final consumption to the production sector represents labor, specifically the material flow associated with labor's energy which is used within the production sector. [7]

As in Example A, we set control volumes around the biosphere and our two economic sectors, such that the material accounting equations become:

[7] We assume that this flow is the adenosine triphosphate (ATP), used as an energy carrier within the cells of organisms.

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} + \frac{\mathrm{d}S_0}{\mathrm{d}t} + \frac{\mathrm{d}K_0}{\mathrm{d}t} = \dot{R}_{10} + \dot{R}_{20} + \dot{S}_{10} + \dot{S}_{20} + \dot{K}_{10} + \dot{K}_{20} - \dot{R}_0 - \dot{S}_0 \tag{2.28}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} + \frac{\mathrm{d}S_1}{\mathrm{d}t} + \frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{11} + \dot{R}_{21} + \dot{S}_{01} + \dot{S}_{11} + \dot{S}_{21} + \dot{K}_{11} + \dot{K}_{21} - \dot{P}_1 - \dot{R}_{10} - \dot{S}_{10} - \dot{K}_{10}, \tag{2.29}$$

$$\frac{\mathrm{d}R_2}{\mathrm{d}t} + \frac{\mathrm{d}S_2}{\mathrm{d}t} + \frac{\mathrm{d}K_2}{\mathrm{d}t} = \dot{R}_{02} + \dot{R}_{22} + \dot{S}_{02} + \dot{S}_{12} + \dot{S}_{22} + \dot{K}_{22} - \dot{P}_2 - \dot{R}_{20} - \dot{S}_{20} - \dot{K}_{20}, \tag{2.30}$$

One point worth noting is that what we are here denoting as a flow of ‘capital goods’ into final consumption ($\dot{K}_{21}$) would more normally be referred to by economist as ‘consumer durables’, though would also include other products, such as housing. The important concept being that some goods (fridges, televisions, apartment blocks) may accumulate within sector (1) and would be represented within flow $\dot{K}_{21}$, whereas other short-lived goods (newspapers, plastic packaging, electricity) do not accumulate and are represented within flow $\dot{S}_{21}$.

Resource flow $\dot{R}_{21}$ into final consumption represents the material flow that will be embodied within the ‘product’ of final consumption—human labor— i.e. food produced by the agriculture industry Since no resources flow directly to final consumption from the biosphere,[8] we may say:

$$\dot{R}_0 = \dot{R}_{02}. \tag{2.31}$$

In contrast, short-lived materials may flow directly to final consumption from the biosphere, e.g. the flow of photons in sunlight or oxygen into car engines and lungs. We can redefine flows $\dot{S}_0$ and $\dot{S}_1$:

$$\dot{S}_0 = \dot{S}_{01} + \dot{S}_{02}; \tag{2.32}$$

As in Example A, we may easily define the balance of resources ($\dot{R}$), short-lived materials ($\dot{S}$) and capital ($\dot{K}$) within the biosphere:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} = \dot{R}_{10} + \dot{R}_{20} - \dot{R}_{02}, \tag{2.33}$$

$$\frac{\mathrm{d}S_0}{\mathrm{d}t} = \dot{S}_{10} + \dot{S}_{20} - \dot{S}_{01} - \dot{S}_{02}, \tag{2.34}$$

$$\frac{\mathrm{d}K_0}{\mathrm{d}t} = \dot{K}_{10} + \dot{K}_{20}. \tag{2.35}$$

Since we are assuming that only man-made capital is accounted within the physical stock of final consumption[9] (K_1) and that the ‘product’ of final consumption is labor (a short-lived material flow, $\dot{S}$), then we may also state that:

[8] A counter-example to this assumption is the production of food outside of the agricultural industry, i.e. by households. This may be large in agrarian economies.

[9] If we were assuming that the human population was accounted within $\dot{K}_1$, then the ‘product’ of final consumption would be human beings (and the labor they provide), resource flow $\dot{R}_{11}$ would be

$$\dot{R}_{11} = 0, \tag{2.36}$$
$$\dot{K}_{11} = 0, \tag{2.37}$$

since all capital goods are produced within the production sector (2).

From conservation of mass, we can also define product flows $\dot{P}_1$ and $\dot{P}_2$ as:

$$\dot{P}_1 = \dot{S}_{11} + \dot{S}_{12}, \tag{2.38}$$
$$\dot{P}_2 = \dot{R}_{21} + \dot{R}_{22} + \dot{S}_{21} + \dot{S}_{22} + \dot{K}_{21} + \dot{K}_{22}, \tag{2.39}$$

Again, remembering that resources and short-lived goods do not accumulate within any sectors of the economy:

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = 0, \tag{2.40}$$
$$\frac{\mathrm{d}R_2}{\mathrm{d}t} = 0, \tag{2.41}$$
$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = 0, \tag{2.42}$$
$$\frac{\mathrm{d}S_2}{\mathrm{d}t} = 0. \tag{2.43}$$

As in Example A, we may also define the resource-product and short-lived goods flows balances separately for each of the sectors of the economy:

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = \dot{R}_{21} - \dot{P}_1 - \dot{R}_{10} = 0, \tag{2.44}$$
$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = \dot{S}_{01} + \dot{S}_{11} + \dot{S}_{21} - \dot{S}_{10} = 0, \tag{2.45}$$
$$\frac{\mathrm{d}R_2}{\mathrm{d}t} = \dot{R}_{02} + \dot{R}_{22} - \dot{P}_2 - \dot{R}_{20} = 0, \tag{2.46}$$
$$\frac{\mathrm{d}S_2}{\mathrm{d}t} = \dot{S}_{02} + \dot{S}_{12} + \dot{S}_{22} - \dot{S}_{20} = 0, \tag{2.47}$$

We may rearrange these equations in terms of the important variable.

material resources provided to human reproduction and ‘capital goods’ flow $\dot{K}_{11}$ would be material added to the human population stock. Again, this issue is discussed in greater detail in Section 8.7.

$$\dot{P}_1 = \dot{R}_{21} - \dot{R}_{10} \tag{2.48}$$

$$\dot{S}_{11} = \dot{S}_{10} - \dot{S}_{21}, \tag{2.49}$$

$$\dot{P}_2 = \dot{R}_{02} + \dot{R}_{22} - \dot{R}_{20}, \tag{2.50}$$

$$\dot{S}_{22} = \dot{S}_{20} - \dot{S}_{02} - \dot{S}_{12}. \tag{2.51}$$

Substituting Equations 2.31-2.35 into Equation 2.28, we obtain:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} + \frac{\mathrm{d}S_0}{\mathrm{d}t} = \dot{R}_{10} + \dot{R}_{20} + \dot{S}_{10} + \dot{S}_{20} - \dot{R}_{02} - \dot{S}_{01} - \dot{S}_{02}. \tag{2.52}$$

Substituting Equations 2.40, 2.45 and 2.47 into Equations 2.29 and 2.30, respectively, we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{21} + \dot{K}_{21} - \dot{P}_1 - \dot{R}_{10} - \dot{K}_{10}, \tag{2.53}$$

$$\frac{\mathrm{d}K_2}{\mathrm{d}t} = \dot{R}_{02} + \dot{R}_{22} + \dot{K}_{22} - \dot{P}_2 - \dot{R}_{20} - \dot{K}_{20}, \tag{2.54}$$

As in Example A, we again have two definitions for $\dot{P}_1$ (Equations 2.38 and 2.48) and $\dot{P}_2$ (Equations 2.39 and 2.50) which may be substituted into Equations 2.53 and 2.54, respectively. Let us start by substituting Equations 2.48 and 2.50, in which case we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{K}_{21} - \dot{K}_{10}, \tag{2.55}$$

$$\frac{\mathrm{d}K_2}{\mathrm{d}t} = \dot{K}_{22} - \dot{K}_{20}, \tag{2.56}$$

which tells us that accumulation of man-made capital (K) is dependent only on inflows of capital goods into the sector ($\dot{K}_{2j}$) and depreciation of capital to the biosphere ($\dot{K}_{j0}$).

Now, substituting Equations 2.38 and 2.39, we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{21} + \dot{K}_{21} - \dot{S}_{11} - \dot{S}_{12} - \dot{R}_{10} - \dot{K}_{10}, \tag{2.57}$$

$$\frac{\mathrm{d}K_2}{\mathrm{d}t} = \dot{R}_{02} - \dot{R}_{21} - \dot{S}_{21} - \dot{S}_{22} - \dot{K}_{21} - \dot{R}_{20} - \dot{K}_{20}, \tag{2.58}$$

to which we may make the substitution of the depreciation term (as before) and rearrange to obtain:

$$-\dot{R}_{10} = \frac{\mathrm{d}K_1}{\mathrm{d}t} - \dot{R}_{21} - \dot{K}_{21} + \dot{S}_{11} + \dot{S}_{12} + \gamma_{K_1} K_1, \tag{2.59}$$

$$\dot{R}_{02} - \dot{R}_{20} = \frac{\mathrm{d}K_2}{\mathrm{d}t} + \left(\dot{R}_{21} + \dot{S}_{21} + \dot{K}_{21}\right) + \dot{S}_{22} + \gamma_{K_2} K_2. \tag{2.60}$$

Equation 2.60 tells us that the resources extracted and used by the production sector ($\dot{R}_{02} - \dot{R}_{20}$) are for the purpose of:

- building up capital stock in the production sector $\left(\frac{\mathrm{d}K_2}{\mathrm{d}t}\right)$
- manufactured goods for final consumption $\left(\dot{R}_{21} + \dot{S}_{21} + \dot{K}_{21}\right)$,
- providing short-lived goods to support the production sector ($\dot{S}_{22}$), and
- overcoming depreciation of production capital stock ($\gamma_{K_2} K_2$).

Adding these two equations together, we obtain:

$$\begin{aligned} -\frac{\mathrm{d}R_0}{\mathrm{d}t} &= \dot{R}_{02} - \dot{R}_{10} - \dot{R}_{20} \\ &= \frac{\mathrm{d}K_1}{\mathrm{d}t} + \frac{\mathrm{d}K_2}{\mathrm{d}t} + \dot{S}_{11} + \dot{S}_{12} + \dot{S}_{21} + \dot{S}_{22} + \gamma_{K_1} K_1 + \gamma_{K_2} K_2. \end{aligned} \tag{2.61}$$

which tells us that the depletion of natural capital $\left(\frac{\mathrm{d}R_0}{\mathrm{d}t}\right)$ is used within the economy in order to:

- build up capital stock $\left(\frac{\mathrm{d}K_1}{\mathrm{d}t} + \frac{\mathrm{d}K_2}{\mathrm{d}t}\right)$,
- produce short-lived goods ($\dot{S}_{11} + \dot{S}_{12} + \dot{S}_{21} + \dot{S}_{22}$), and
- overcome depreciation $\left(\gamma_{K_1} K_1 + \gamma_{K_2} K_2\right)$.

2.4 Example C: three-sector economy

In example C, we differentiate between two production sectors, sector (2) produces energy products and sector (3) produces other goods and services, as depicted in Figure 2.4.

In this example, we will take a slightly different approach than in the previous two examples. Instead of discerning whether or not certain flows exist—asking for instance, is there a flow of resources ($\dot{R}_{21}$) from the energy sector (2) to final consumption (1)?—-we shall account for all flows, *even if* those flows are zero. In this way, we may build up a completely general framework for material accounting within an economy of any size.

Accounting for the material flows into and out of the biosphere (0) gives the following equation:

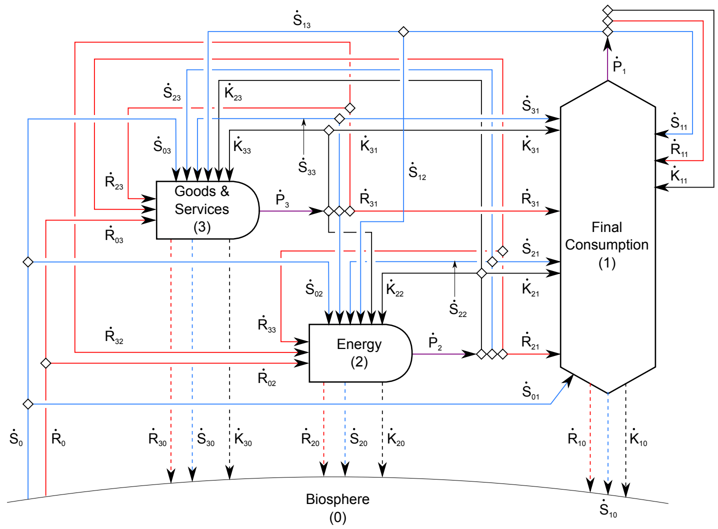

Fig. 2.4 Flows of materials for a three-sector economy.

$$\frac{\mathrm{d}R_0}{\mathrm{d}t}+\frac{\mathrm{d}S_0}{\mathrm{d}t}+\frac{\mathrm{d}K_0}{\mathrm{d}t}=\dot{R}_{10}+\dot{R}_{20}+\dot{R}_{30}+\dot{S}_{10}+\dot{S}_{20}+\dot{S}_{30}+\dot{K}_{10}+\dot{K}_{20}+\dot{K}_{30}-\dot{R}_0-\dot{S}_0, \tag{2.62}$$

which may be rewritten as:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t}+\frac{\mathrm{d}S_0}{\mathrm{d}t}+\frac{\mathrm{d}K_0}{\mathrm{d}t}=\sum_{i=1}^{3}\dot{R}_{i0}+\sum_{i=1}^{3}\dot{S}_{i0}+\sum_{i=1}^{3}\dot{K}_{i0}-\dot{R}_0-\dot{S}_0, \tag{2.63}$$

Similarly, flows for the other sectors may be written:

$$\frac{\mathrm{d}R_1}{\mathrm{d}t}+\frac{\mathrm{d}S_1}{\mathrm{d}t}+\frac{\mathrm{d}K_1}{\mathrm{d}t}=\dot{R}_{01}+\dot{S}_{01}+\sum_{i=1}^{3}\dot{R}_{i1}+\sum_{i=1}^{3}\dot{S}_{i1}+\sum_{i=1}^{3}\dot{K}_{i1}-\dot{P}_1-\dot{R}_{10}-\dot{S}_{10}-\dot{K}_{10}, \tag{2.64}$$

$$\frac{\mathrm{d}R_2}{\mathrm{d}t}+\frac{\mathrm{d}S_2}{\mathrm{d}t}+\frac{\mathrm{d}K_2}{\mathrm{d}t}=\dot{R}_{02}+\dot{S}_{02}+\sum_{i=1}^{3}\dot{R}_{i2}+\sum_{i=1}^{3}\dot{S}_{i2}+\sum_{i=1}^{3}\dot{K}_{i2}-\dot{P}_2-\dot{R}_{20}-\dot{S}_{20}-\dot{K}_{20}, \tag{2.65}$$

$$\frac{\mathrm{d}R_3}{\mathrm{d}t}+\frac{\mathrm{d}S_3}{\mathrm{d}t}+\frac{\mathrm{d}K_3}{\mathrm{d}t}=\dot{R}_{03}+\dot{S}_{03}+\sum_{i=1}^{3}\dot{R}_{i3}+\sum_{i=1}^{3}\dot{S}_{i3}+\sum_{i=1}^{3}\dot{K}_{i3}-\dot{P}_3-\dot{R}_{30}-\dot{S}_{30}-\dot{K}_{30}. \tag{2.66}$$

As in previous examples, we may define the balance of resources ($\dot{R}$), short-lived materials ($\dot{S}$) and capital ($\dot{K}$) within the biosphere:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} = \dot{R}_{10} + \dot{R}_{20} + \dot{R}_{30} - \dot{R}_{01} - \dot{R}_{02} - \dot{R}_{03}, \tag{2.67}$$

$$\frac{\mathrm{d}S_0}{\mathrm{d}t} = \dot{S}_{10} + \dot{S}_{20} + \dot{S}_{30} - \dot{S}_{01} - \dot{S}_{02} - \dot{S}_{03}, \tag{2.68}$$

$$\frac{\mathrm{d}K_0}{\mathrm{d}t} = \dot{K}_{10} + \dot{K}_{20} + \dot{K}_{30}. \tag{2.69}$$

which may be rewritten:

$$\frac{\mathrm{d}R_0}{\mathrm{d}t} = \sum_{i=1}^{3} \dot{R}_{i0} - \sum_{j=1}^{3} \dot{R}_{0j}, \tag{2.70}$$

$$\frac{\mathrm{d}S_0}{\mathrm{d}t} = \sum_{i=1}^{3} \dot{S}_{i0} - \sum_{j=1}^{3} \dot{S}_{0j}, \tag{2.71}$$

$$\frac{\mathrm{d}K_0}{\mathrm{d}t} = \sum_{i=1}^{3} \dot{K}_{i0}. \tag{2.72}$$

Applying conservation of mass allows us to define the product flows ($\dot{P}$) as:

$$\dot{P}_1 = \sum_{j=1}^{3} \dot{R}_{1j} + \sum_{j=1}^{3} \dot{S}_{1j} + \sum_{j=1}^{3} \dot{K}_{1j}, \tag{2.73}$$

$$\dot{P}_2 = \sum_{j=1}^{3} \dot{R}_{2j} + \sum_{j=1}^{3} \dot{S}_{2j} + \sum_{j=1}^{3} \dot{K}_{2j}, \tag{2.74}$$

$$\dot{P}_3 = \sum_{j=1}^{3} \dot{R}_{3j} + \sum_{j=1}^{3} \dot{S}_{3j} + \sum_{j=1}^{3} \dot{K}_{3j} \tag{2.75}$$

As in Example B, final consumption (1) provides only labor (represented by $\dot{S}$ flows) to the other sectors of the economy. The energy sector (2) provides energy products ($\dot{S}_{2j}$) to the other sectors of the economy. It may also provide resources to itself ($\dot{R}_{22}$) and to the goods and services sector (3), as in the case of metallurgical coke or natural gas for fertilizer. The energy sector does not produce capital goods, hence, for $j \in [1,3] : \dot{K}_{2j} = 0$. The goods and services (3) sector does not provide resources for the energy sector (2)[10], hence $\dot{R}_{32} = 0$.

Since we do not allow accumulation of either resources (R) or short-lived capital goods (S) in economic sectors, then we may say:

[10] There may be some exceptions to this, as in the case of energy from industrial waste streams.

$$\frac{\mathrm{d}R_j}{\mathrm{d}t} = 0, \qquad j \in [1,3], \tag{2.76}$$
$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = 0, \qquad j \in [1,3]. \tag{2.77}$$

As before, we may also define the resource-product and short-lived goods flows balances separately for each of the sectors of the economy:[11]

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = \dot{R}_{01} + \dot{R}_{11} + \dot{R}_{21} + \dot{R}_{31} - \dot{P}_1 - \dot{R}_{10} = 0, \tag{2.78}$$
$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = \dot{S}_{01} + \dot{S}_{11} + \dot{S}_{21} + \dot{S}_{31} - \dot{S}_{10} = 0, \tag{2.79}$$
$$\frac{\mathrm{d}R_2}{\mathrm{d}t} = \dot{R}_{02} + \dot{R}_{12} + \dot{R}_{22} + \dot{R}_{32} - \dot{P}_2 - \dot{R}_{20} = 0, \tag{2.80}$$
$$\frac{\mathrm{d}S_2}{\mathrm{d}t} = \dot{S}_{02} + \dot{S}_{12} + \dot{S}_{22} + \dot{S}_{32} - \dot{S}_{20} = 0, \tag{2.81}$$
$$\frac{\mathrm{d}R_3}{\mathrm{d}t} = \dot{R}_{03} + \dot{R}_{13} + \dot{R}_{23} + \dot{R}_{33} - \dot{P}_3 - \dot{R}_{30} = 0, \tag{2.82}$$
$$\frac{\mathrm{d}S_3}{\mathrm{d}t} = \dot{S}_{03} + \dot{S}_{13} + \dot{S}_{23} + \dot{S}_{33} - \dot{S}_{30} = 0, \tag{2.83}$$

We may rearrange these equations in terms of the important variable:

$$\dot{P}_1 = \dot{R}_{01} + \dot{R}_{11} + \dot{R}_{21} + \dot{R}_{31} - \dot{R}_{10} \tag{2.84}$$
$$\dot{S}_{11} = \dot{S}_{10} - \dot{S}_{01} - \dot{S}_{21} - \dot{S}_{31}, \tag{2.85}$$
$$\dot{P}_2 = \dot{R}_{02} + \dot{R}_{12} + \dot{R}_{22} + \dot{R}_{32} - \dot{R}_{20}, \tag{2.86}$$
$$\dot{S}_{22} = \dot{S}_{20} - \dot{S}_{02} - \dot{S}_{12} - \dot{S}_{32}, \tag{2.87}$$
$$\dot{P}_3 = \dot{R}_{03} + \dot{R}_{13} + \dot{R}_{23} + \dot{R}_{33} - \dot{R}_{30}, \tag{2.88}$$
$$\dot{S}_{33} = \dot{S}_{20} - \dot{S}_{03} - \dot{S}_{13} - \dot{S}_{23}. \tag{2.89}$$

We now make use of Equations 2.76, 2.79, 2.81 and 2.83, in simplifying Equations 2.64 - 2.66, to obtain:

[11] It is worth remembering here that $\dot{R}_{01} = \dot{R}_{21} = 0$, since final consumption only takes resources (in the form of food) from the goods and services sector (3) and that $R_{32} = 0$ since the goods and services sector does not provide resources to the energy sector (2).

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{01} + \sum_{i=1}^{3} \dot{R}_{i1} + \sum_{i=1}^{3} \dot{K}_{i1} - \dot{P}_1 - \dot{R}_{10} - \dot{K}_{10}, \tag{2.90}$$

$$\frac{\mathrm{d}K_2}{\mathrm{d}t} = \dot{R}_{02} + \sum_{i=1}^{3} \dot{R}_{i2} + \sum_{i=1}^{3} \dot{K}_{i2} - \dot{P}_2 - \dot{R}_{20} - \dot{K}_{20}, \tag{2.91}$$

$$\frac{\mathrm{d}K_3}{\mathrm{d}t} = \dot{R}_{03} + \sum_{i=1}^{3} \dot{R}_{i3} + \sum_{i=1}^{3} \dot{K}_{i3} - \dot{P}_3 - \dot{R}_{30} - \dot{K}_{30}. \tag{2.92}$$

As in previous examples, we have two different formulations for the $\dot{P}$ terms. Substituting, first, Equations 2.84, 2.86 and 2.88, we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \sum_{i=1}^{3} \dot{K}_{i1} - \dot{K}_{10}, \tag{2.93}$$

$$\frac{\mathrm{d}K_2}{\mathrm{d}t} = \sum_{i=1}^{3} \dot{K}_{i2} - \dot{K}_{20}, \tag{2.94}$$

$$\frac{\mathrm{d}K_3}{\mathrm{d}t} = \sum_{i=1}^{3} \dot{K}_{i3} - \dot{K}_{30}, \tag{2.95}$$

which we may rewrite as the more general result:

$$\frac{\mathrm{d}K_j}{\mathrm{d}t} = \sum_{i} \dot{K}_{ij} - \dot{K}_{j0}. \tag{2.96}$$

Equation 2.96 states that for any economic sector, the accumulation of man-made capital stock is dependent only on inflows of capital stock from other economic sectors ($\dot{K}_{ij}$) and depreciation of capital stock back to the biosphere ($\dot{K}_{j0}$).

Instead, substituting the alternative formulation for $\dot{P}$ from Equations 2.73 - 2.75 into Equations 2.90 - 2.92, respectively, we obtain:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t} = \dot{R}_{01} + \sum_{i=1}^{3} \dot{R}_{i1} + \sum_{i=1}^{3} \dot{K}_{i1} - \sum_{j=1}^{3} \dot{R}_{1j} - \sum_{j=1}^{3} \dot{S}_{1j} - \sum_{j=1}^{3} \dot{K}_{1j} - \dot{R}_{10} - \dot{K}_{10}, \tag{2.97}$$

$$\frac{\mathrm{d}K_2}{\mathrm{d}t} = \dot{R}_{02} + \sum_{i=1}^{3} \dot{R}_{i2} + \sum_{i=1}^{3} \dot{K}_{i2} - \sum_{j=1}^{3} \dot{R}_{2j} - \sum_{j=1}^{3} \dot{S}_{2j} - \sum_{j=1}^{3} \dot{K}_{2j} - \dot{R}_{20} - \dot{K}_{20}, \tag{2.98}$$

$$\frac{\mathrm{d}K_3}{\mathrm{d}t} = \dot{R}_{03} + \sum_{i=1}^{3} \dot{R}_{i3} + \sum_{i=1}^{3} \dot{K}_{i3} - \sum_{j=1}^{3} \dot{R}_{3j} - \sum_{j=1}^{3} \dot{S}_{3j} - \sum_{j=1}^{3} \dot{K}_{3j} - \dot{R}_{30} - \dot{K}_{30}. \tag{2.99}$$

As before, we can rearrange these equations to obtain:

$$\dot{R}_{01} - \dot{R}_{10} = \frac{\mathrm{d}K_1}{\mathrm{d}t} - \sum_{i=1}^{3} \dot{R}_{i1} - \sum_{i=1}^{3} \dot{K}_{i1} + \sum_{j=1}^{3} \dot{R}_{1j} + \sum_{j=1}^{3} \dot{S}_{1j} + \sum_{j=1}^{3} \dot{K}_{1j} + \dot{K}_{10}, \quad (2.100)$$

$$\dot{R}_{02} - \dot{R}_{20} = \frac{\mathrm{d}K_2}{\mathrm{d}t} - \sum_{i=1}^{3} \dot{R}_{i2} - \sum_{i=1}^{3} \dot{K}_{i2} + \sum_{j=1}^{3} \dot{R}_{2j} + \sum_{j=1}^{3} \dot{S}_{2j} + \sum_{j=1}^{3} \dot{K}_{2j} + \dot{K}_{20}, \quad (2.101)$$

$$\dot{R}_{03} - \dot{R}_{30} = \frac{\mathrm{d}K_3}{\mathrm{d}t} - \sum_{i=1}^{3} \dot{R}_{i3} - \sum_{i=1}^{3} \dot{K}_{i3} + \sum_{j=1}^{3} \dot{R}_{3j} + \sum_{j=1}^{3} \dot{S}_{3j} + \sum_{j=1}^{3} \dot{K}_{3j} + \dot{K}_{30}. \quad (2.102)$$

Summing Equations 2.100 - 2.102, we obtain:

$$\begin{aligned} -\frac{\mathrm{d}R_0}{\mathrm{d}t} &= \sum_{j=1}^{3} \dot{R}_{0j} - \sum_{i=1}^{3} \dot{R}_{i0} \\ &= \frac{\mathrm{d}K_1}{\mathrm{d}t} + \frac{\mathrm{d}K_2}{\mathrm{d}t} + \frac{\mathrm{d}K_3}{\mathrm{d}t} - \sum_{j=1}^{3}\sum_{i=1}^{3} \dot{R}_{ij} - \sum_{j=1}^{3}\sum_{i=1}^{3} \dot{K}_{ij} + \sum_{j=1}^{3}\sum_{i=1}^{3} \dot{R}_{ij} \\ &\quad + \sum_{j=1}^{3}\sum_{i=1}^{3} \dot{S}_{ij} + \sum_{j=1}^{3}\sum_{i=1}^{3} \dot{K}_{ij} + \dot{K}_{10} + \dot{K}_{20} + \dot{K}_{30}, \end{aligned} \quad (2.103)$$

which, after substituting for the depreciation term ($\dot{K}_{i0}$), simplifies to:

$$-\frac{\mathrm{d}R_0}{\mathrm{d}t} = \sum_{j=1}^{3} \frac{\mathrm{d}K_j}{\mathrm{d}t} + \sum_{j=1}^{3}\sum_{i=1}^{3} \dot{S}_{ij} + \sum_{j=1}^{3} \gamma_{K_j} K_j \quad (2.104)$$

or, more generally:

$$-\frac{\mathrm{d}R_0}{\mathrm{d}t} = \sum_{j} \frac{\mathrm{d}K_j}{\mathrm{d}t} + \sum_{i,j} \dot{S}_{ij} + \sum_{j} \gamma_{K_j} K_j. \quad (2.105)$$

Similarly to what we saw in Examples A and B, Equation 2.105 tells us that, depletion of natural resources in the biosphere $\left(-\frac{\mathrm{d}R_0}{\mathrm{d}t}\right)$ by the economy are used for the purposes of:

- increasing man-made capital stocks within the economy $\left(\frac{\mathrm{d}K_j}{\mathrm{d}t}\right)$
- providing short-lived goods exchanged within the economy ($\dot{S}_{ij}$), and
- overcoming depreciation of man-made capital stocks ($\sum_j \gamma_{K_j} K_j$).

This issue will be discussed in greater detail in Section 7.4 concerning sustainable scale of the economy and the concept of a steady-state economy.

The exchange of resources ($\dot{R}$) and short-lived goods ($\dot{S}$) between each of the four "sectors" (the biosphere and the three economic sectors) may be thought of as four matrices (as depicted in Figure 2.5 for $\dot{S}$ flows): one 3×3 matrix of flows entirely within the economy, a 3×1 vector of flows from the biosphere into the economy (extraction), a 1×3 vector of flows from the economy into the biosphere (waste) and a 1×1 matrix of flows solely within the biosphere (environment), that do not enter the economy.

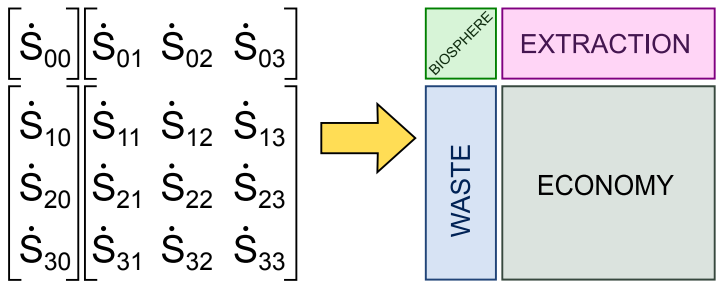

Fig. 2.5 The matrix of biosphere-economy flows.

2.5 Materials in the auto industry

Throughout the book, we shall be applying the methodology that has been outlined through the examples to the real-world case of the US auto industry. In Figure 2.6 we see the flows of resources, short-lived and capital materials into the auto industry. Because the industry does not extract resources directly from the biosphere, the rate of flow of resources ($\dot{R}_{0j}$) from the biosphere to the auto industry has a zero value. Each of the other flows represented in the diagram is, in actuality, a vector of hundreds (or even thousands!) of elemental material flows, each of which must be accounted (and balanced) separately.

There are a number of key material inputs into the production of automobiles, directly as resources ($\dot{R}$) as well as short-lived materials ($\dot{S}$) and capital goods ($\dot{K}$) outlined in Table 2.1.

Data on the rest of the flows at the industry level is very hard to obtain.[12] In Europe, economy-wide material flow accounts (EW-MFA) have been produced for each of the member states.[5] Work is ongoing to characterize the inter-sectoral flows of materials.[6] These material flows could be modeled in much the same manner as traditional energy IO estimates energy flows through the economy, by converting financial data (which is available, as discussed in Section 5.5) into phys-

[12] The issue of lack of physical flow data is discussed in Section 8.3.

ical flow data via knowledge of the entry points of materials into the economy, such as iron or bauxite mining sectors.

A number of studies have looked at the material and energy flows associated with specific or representative vehicle manufacturing processes.[7–14] The US automobile industry is composed of many of these manufacturing processes. In theory, a representation of the industry-level flows could be "built up" by assuming that the results from these process-based models represent average processes within the whole industry and scaling the material flows accordingly, with appropriately wide uncertainty bounds.

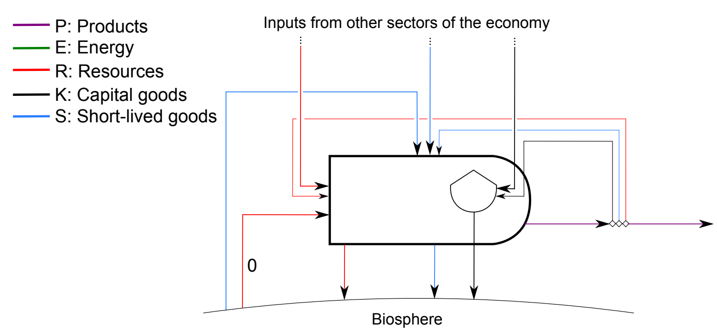

Fig. 2.6 Material flows for the US Automobile Industry using data from [7–14].

2.6 Summary

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Table 2.1 List of material input and output flows for the US auto industry as resources ($\dot{R}$), short-lived materials ($\dot{S}$), and capital goods ($\dot{K}$) using data from [7–14]. This list is illustrative and by no means exhaustive.

Material Flow		**Materials**
Resources from biosphere	$\dot{R}_{0j}$	none
Short-lived from biosphere	$\dot{S}_{0j}$	oxygen, nitrogen, water
Resources from other sectors	$\dot{R}_{ij}$	cast iron (engine block); steel (chassis, panels); aluminum (body parts); copper (wiring); zinc, chromium, carbon (alloying); lead, nickel (battery cells); glass (windows, wind shield); rubber (tires); plastic (bodywork, interiors, seals) petroleum (paints, lubricants)
Short-lived from other sectors	$\dot{S}_{ij}$	coal, oil, natural gas, electricity water (process) petroleum (solvents) paper (towels)
Capital from other sectors	$\dot{K}_{ij}$	steel (buildings, equipment) concrete (buildings) glass (windows, screens) plastic (fixtures, fittings, equipment) petroleum (paints, lubricants)
Product output	$\dot{P}_j$	finished automobile
Resources to biosphere	$\dot{R}_{j0}$	trimmings and dust (metal, plastic, rubber)
Short-lived to biosphere	$\dot{S}_{j0}$	air emissions (GHG, NO_x, SO_x) emissions to water
Capital to biosphere	$\dot{K}_{j0}$	discarded equipment disused buildings

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Chapter 3
Direct energy flows

> *Living organisms need to be open to a constant flow of resources (energy and matter) to stay alive;*
> *human organizations need to be open to a flow of mental resources (information and ideas), as well as to the flows of energy and materials that are part of the production of goods or services.* [1, p.117]
>
> —Fritjof Capra

In Chapter 1, we formulated a model of economies consisting of producers and consumers who exchange goods and services and factors of production while extracting resources and disposing of wastes. In Chapter 2, we established the material basis of economies: economies are analogous to organisms with metabolisms that processes raw resources for the benefit of producers and consumers while generating unavoidable wastes. In this chapter, we describe and analyze the direct energy that is associated with material flows through an economy.

All forms of energy provide the potential to do mechanical work. The quantification of the mechanical work potential of energy is *exergy*. When energy is "consumed" by an economy, exergy (work potential) is destroyed. Energy (as mechanical work) is an essential aspect of the metabolic economy; with it, materials are refined, shaped, and assembled into useful intermediate and consumption products; food is made available to people in society; jobs are made easier for workers; human ingenuity is multiplied; and complex systems and civilizations are possible. In the absence of high rates of energy availablity at low cost, life becomes much more difficult, even impossible, for many people.

Our analogy is this: energy is to thermodynamics as currency is to financial accounting. Or, energy is the currency of thermodynamics. Just as an accountant understands a firm by watching how and where currency flows through it, so we can understand an economy by watching how and where energy flows through it. Accounting for energy flows through an economy is essential for developing a dynamic picture of its metabolism.

The purpose of this chapter is to develop a model for energy flows within economies. With an energy model in hand, we will be positioned to assess the rate at which consumed direct energy becomes embodied within the products and services that an economy provides (Chapter 4).

3.1 Methodology

We begin by noting that direct energy travels with material through an economy. "Direct" energy refers to forms of energy accounted by the First Law of Thermodynamics, including chemical potential energy, nuclear potential energy, gravitational potential energy, thermal energy, and kinetic energy. We use the term "direct" energy distinct from "embodied" energy, which will be discussed in Chapter 4. Examples of direct energy flows include the chemical potential energy of coal inflows to an energy sector, the thermal energy of process steam into a textile plant, and the thermal energy of CO_2 automobile exhaust. In each case, the material (coal, steam, and CO_2) carries direct energy with it. Figure 3.1 shows a corresponding direct energy flow for each material flow of Figure 2.1.

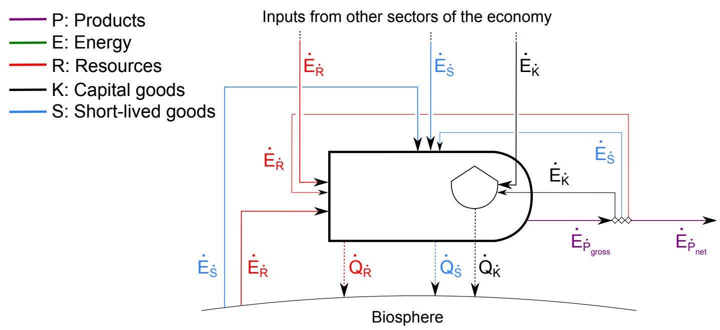

Fig. 3.1 Energy content ($\dot{E}$) of material flows ($\dot{R}$, $\dot{S}$, and $\dot{K}$) from Figure 2.1.

For any boundary (around, say, a machine, a plant, a sector of the economy, or the entire economy itself), the First Law of Thermodynamics says that the accumulation rate of direct energy (E) within the boundary is equal to the sum of the signed direct energy flow rates ($\dot{E}$) across the boundary (where inflows are positive and outflows are negative) less outflowing energy carried by wastes ($\dot{Q}_{out}$): energy is conserved.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \sum \dot{E} - \sum \dot{Q}_{out} \tag{3.1}$$

When there is no accumulation of direct energy within the boundary $\left(\frac{\mathrm{d}E}{\mathrm{d}t} = 0\right)$, the sum of all signed direct energy flow rates ($\dot{E}$) and waste heats ($\dot{Q}_{out}$) will be zero.

$$0 = \sum \dot{E} - \sum \dot{Q}_{out} \tag{3.2}$$

It is important to note that the direct energy associated with some material flows can be so small as to be negligible compared to other direct energy flows in the economy. For example, the direct energy of steel into the automobile sector of the economy is almost negligible. (The *embodied* energy of the steel is almost certainly *not* negligible, as will be discussed in Chapter 4.) On the other hand, the direct energy flow rates for fossil fuels (coal, oil, and natural gas) are typically orders of magnitude larger than any other material flows due to large chemical potential energy content.

To simplify the direct energy analysis, we can aggregate the direct energy flows of Figure 3.1 into single arrows when appropriate. For example, the direct energy inputs from other sectors of the economy (labeled as $\dot{E}_{\dot{R}}$, $\dot{E}_{\dot{S}}$, and $\dot{E}_{\dot{K}}$ at the top of Figure 3.1) can be summed to $\dot{E}$ (in Figure 3.2) such that

$$\dot{E}_{\text{Fig. 3.2}} = \dot{E}_{\dot{R},\,\text{Fig. 3.1}} + \dot{E}_{\dot{S},\,\text{Fig. 3.1}} + \dot{E}_{\dot{K},\,\text{Fig. 3.1}}. \tag{3.3}$$

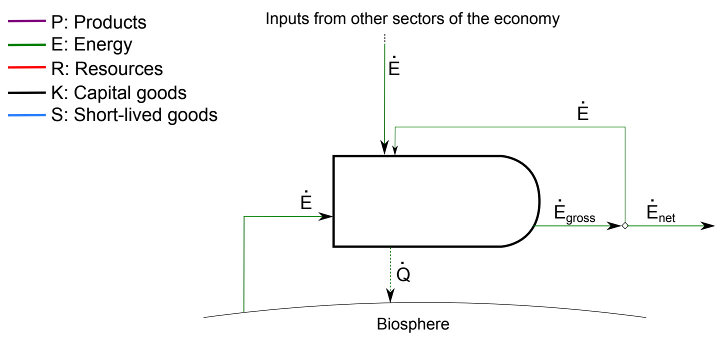

Fig. 3.2 Aggregated direct energy flows ($\dot{E}$) around the producer of Figure 3.1.

3.2 Example A: single-sector economy

Aggregated direct energy flows are now applied to Example A, the single-sector economy shown in Figure 2.2. By summing the direct energy flows associated with each material flow of Figure 2.2, we obtain a simplified picture of direct energy flows in the economy, as shown in Figure 3.3.[1]

We distinguish useful direct energy inputs to a sector of the economy ($\dot{E}_{01}$ in Figure 3.3) from wasteful direct energy flows ($\dot{Q}_{10}$ in Figure 3.3), because $\dot{Q}$ typically denotes thermal energy, and many waste energy flows are in the form of thermal energy, i.e., waste heat. In Figure 3.3, direct energy input to the economy ($\dot{E}_{01}$) is shown as being extracted from the biosphere, because the vast majority of direct en-

[1] Single subscripts on quantities such as E can mean one of two things: $\dot{E}_i$ indicates the outflow of direct energy from sector i, whereas E_i denotes the direct energy content of sector i. Double subscripts on quantities (e.g., $\dot{E}_{ij}$) indicate a flow from sector i to sector j. The first index always indicates the sector *from* which a quantity flows, and the second index indicates the sector *to* which a quantity flows.

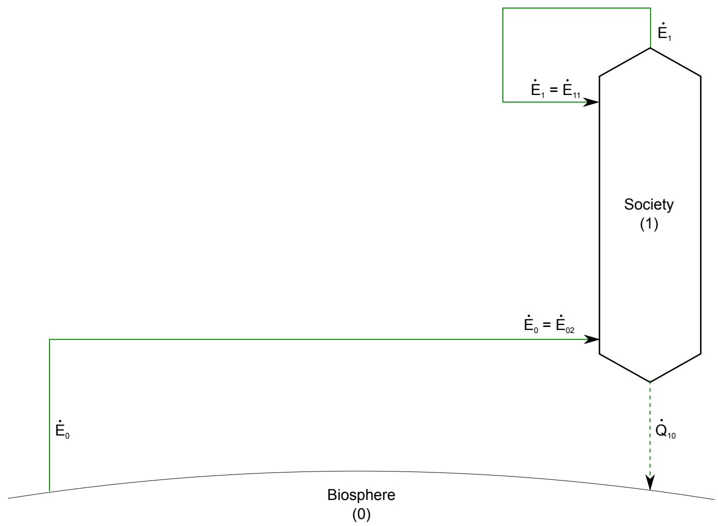

Fig. 3.3 Direct energy flows ($\dot{E}$) a one-sector economy.

ergy today is derived from fossil fuels. Waste heat from the economy ($\dot{Q}_{10}$) is shown as returning to the biosphere.

As discussed in Section 3.1, both direct energy ($\dot{E}$), and waste heat ($\dot{Q}$) are accounted by the First Law of Thermodynamics. Accounting for possible accumulation of direct energy, the First Law of Thermodynamics for Example A indicates that

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{Q}_{10} - \dot{E}_{01} \tag{3.4}$$

and

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{01} + \dot{E}_{11} - \dot{E}_1 - \dot{Q}_{10}. \tag{3.5}$$

Note that $\dot{E}_1$ is the gross direct energy production rate of society. For example, firms extract crude oil (a component of $\dot{E}_{01}$) and refine it into petroleum products (a component of $\dot{E}_1$) that are consumed by society. The direct energy consumption of extraction and refining firms is a component of $\dot{E}_{11}$.

Aside from, for example, the US Strategic Petroleum Reserve, we are not stockpiling oil and coal at any meaningful rate, i.e. we consume fossil fuels at a rate equal to their extraction rate. Thus, the world is not accumulating direct energy in the economy.[2] (The world *is*, however, accumulating *embodied* energy in the econ-

[2] A counter-example could be made for nuclear fuels where "spent" fuel represents a large exergetic stockpile. However, this reserve is not (presently) economically useful.

omy as we shall see in Chapter 4.) Thus, the accumulation rates for direct energy $\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)$ in the above equations could be set to zero as follows:

$$0 = \dot{Q}_{10} - \dot{E}_{01} \tag{3.6}$$

and

$$0 = \dot{E}_{01} + \dot{E}_{11} - \dot{E}_{1} - \dot{Q}_{10}. \tag{3.7}$$

However, we shall see later (in Chapter 3) that keeping direct energy accumulation terms $\left(\frac{\mathrm{d}\dot{E}}{\mathrm{d}t}\right)$ provides an advantage when deriving embodied energy accounting equations.

3.3 Example B: two-sector economy

For Example B, we split an economic sector (2) from society (1). Figure 3.4 shows aggregated direct energy flows associated with the material flows of Figure 2.3.

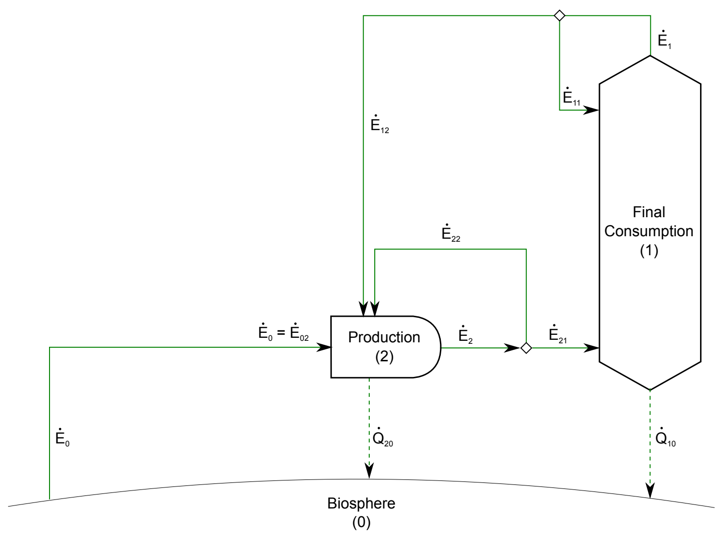

Fig. 3.4 Direct energy flows ($\dot{E}$) for a two-sector economy.

The First Law of Thermodynamics requires that both direct energy and waste heat be conserved around each entity (1 and 2) as well as around the biosphere (0).

First Law energy accounting around the biosphere (0) and society (1) gives

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{Q}_{10} + \dot{Q}_{20} - \dot{E}_{02}, \tag{3.8}$$

and

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{11} + \dot{E}_{21} - \dot{E}_1 - \dot{Q}_{10}. \tag{3.9}$$

Note that $\dot{E}_{12}$ represents useful work that people and draught animals contribute to Production (2). Ayres and Warr [2, 3] call this "muscle work." $\dot{E}_{11}$ represents the muscle work required for consumption. Direct energy required for consumption by final demand (electricity, oil, natural gas, etc.) is included in $\dot{E}_{21}$.

The First Law around the economy (2), including the accumulation rate of direct energy in the sector $\left(\frac{\mathrm{d}E_2}{\mathrm{d}t}\right)$, yields

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \dot{E}_{02} + \dot{E}_{12} + \dot{E}_{22} - \dot{E}_2 - \dot{Q}_{20}. \tag{3.10}$$

It is notable that the economy (2) consumes ($\dot{E}_{22}$) a portion of its gross energy output ($\dot{E}_2$): it takes energy to make energy.

Equation 3.8 can be generalized with a sum as

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \sum_{i=1}^{n} \left(\dot{Q}_{i0} - \dot{E}_{0i}\right), \tag{3.11}$$

where n is the number of economic sectors in the model (in this example, $n = 2$). Similarly, Equations 3.9 and 3.10 can generalized with a sum as

$$\frac{\mathrm{d}E_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{E}_{ij} - \dot{E}_j - \dot{Q}_{j0}, \tag{3.12}$$

where $j \in [1, n]$.

3.4 Example C: three-sector economy

We can extend Example B, to include an energy sector (2) and a goods and services sector (3), thereby obtaining a fuller picture of direct energy flows among sectors (Figure 3.5).

We note that the gross direct energy production of the energy sector (2) is $\dot{E}_2$, and the direct energy consumption of the energy sector (2) is $\dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32}$. The net direct energy production by the energy sector (2) is given by $\dot{E}_2 - (\dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32})$. The energy return on investment (*EROI*) of the energy sector (2) is given by

$$EROI_2 = \frac{\dot{E}_2}{\dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32}}. \tag{3.13}$$

The First Law of Thermodynamics around the biosphere (0), society (1), and the energy sector (2) gives

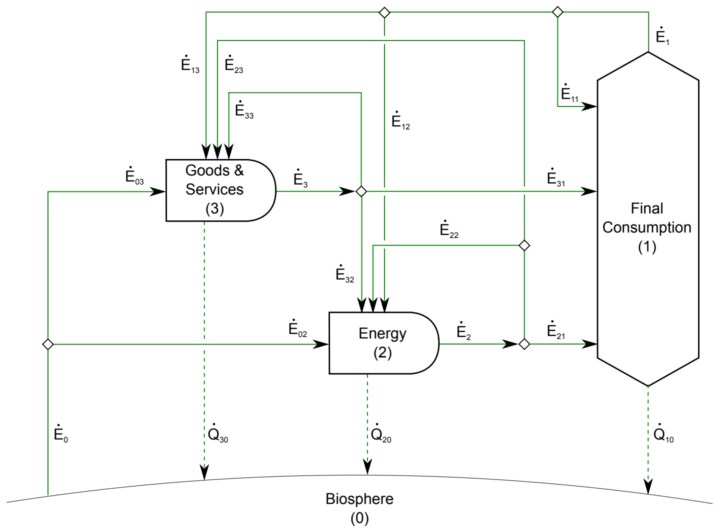

Fig. 3.5 Direct energy flows ($\dot{E}$) for a three-sector economy.

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{Q}_{10} + \dot{Q}_{20} + \dot{Q}_{30} - \dot{E}_{02} - \dot{E}_{03}, \tag{3.14}$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{11} + \dot{E}_{21} + \dot{E}_{31} - \dot{E}_1 - \dot{Q}_{10}, \tag{3.15}$$

and

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \dot{E}_{02} + \dot{E}_{12} + \dot{E}_{22} + \dot{E}_{32} - \dot{E}_2 - \dot{Q}_{20}. \tag{3.16}$$

The First Law around the goods and services sector (3) including, for now, the accumulation rate of direct energy in the sector $\left(\frac{\mathrm{d}E_3}{\mathrm{d}t}\right)$ yields

$$\frac{\mathrm{d}E_3}{\mathrm{d}t} = \dot{E}_{03} + \dot{E}_{13} + \dot{E}_{23} + \dot{E}_{33} - \dot{E}_3 - \dot{Q}_{30}. \tag{3.17}$$

Similar to Example B, we can generalize Equations 3.14–3.17 with sums to obtain

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{Q}_{i0} - \sum_{i=1}^{n} \dot{E}_{0i} \tag{3.18}$$

and

$$\frac{\mathrm{d}E_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{E}_{ij} - \dot{E}_j - \dot{Q}_{j0}, \tag{3.19}$$

where $j \in [1,n]$. Equations 3.18 and 3.19 are identical to Equations 3.11 and 3.12, indicating that we have successfully generalized the model to any number of sectors.

In this economy, the purpose of the goods and services sector (3) is to produce goods and provide services, it provides no direct energy to society. The purpose of the energy sector (2) is to make direct energy ($\dot{E}$) available to the economy and society in a useful form. We may simplify the above equations by realizing that (a) $\dot{E}_3 = \dot{E}_{3i} = 0$, because the goods and services sector is assumed to produce no direct energy, and (b) $\dot{E}_{03} = 0$, because the goods and services sector (3) receives no direct energy from the biosphere (0), except via the energy sector (2). Thus, several terms in the sums of Equation 3.19 will be zero.

3.5 Direct energy in the auto industry

In this section we develop an estimate for the inflow of Energy into the automobile industry, line E in Figure 3.6.

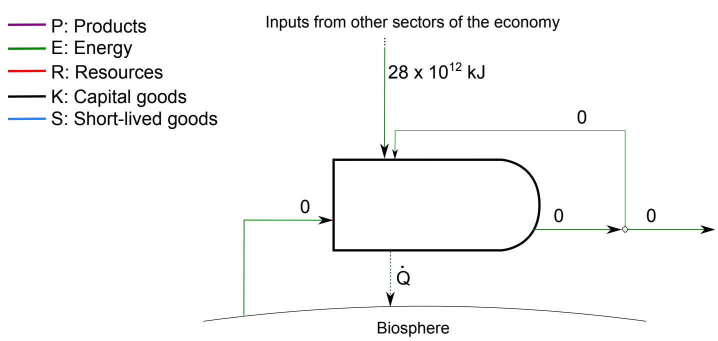

Fig. 3.6 Direct energy flows for the US Automobile Industry using data from US Energy Information Administration. **** Use citation instead. ****

In 2010, the automobile industry purchased 28 trillion kJ of total energy from all sources. Table 3.1 shows the breakdown of energy by source.

Table 3.1 Energy inputs to US auto industry (NAICS Code 336111) in 2010.[4, Table 7.6]

Source	Quantity
Electricity	3.0×10^9 kW-hr
Natural gas	1.5×10^{10} ft^3
Other	1.0×10^{12} BTU
Total	2.8×10^{13} kJ (thermal equivalent)

According to the International Organization of Motor Vehicle Manufacturers (OICA), 2.7 million cars were produced in 2010 in the US.[5] In 2006, prior to the Great Recession, the automobile industry purchased 40 trillion kJ of total energy and produced 4.4 million cars.

Total energy use can also be estimated by summing the energy use of the underlying detailed processes in manufacturing automobiles. Sullivan et al. arrives at an estimate of the "gate-to-gate" energy used in the process of creating one automobile (the direct energy used within the automobile manufacturing process only).[6] This can be mulitplied by the number of vehicles manufactured in a given year to obtain total energy use by the automobile industry. Sullivan estimated a total direct energy use of 34,000 MJ for a generic 1,532 kg vehicle.

3.6 Summary

In this chapter, we have developed equations, assisted by the First Law of Thermodynamics, that describe the flow of direct energy ($\dot{E}$) through economies (Section 3.1). Examples A–C afforded the opportunity to apply the equations to model economies with increasing levels of disaggregation (Sections 3.2–3.4). Finally, the energy flows for our running example, the US auto industry, were discussed in Section 3.5.

In the next chapter, the direct energy equations developed above will be used to develop *embodied* energy accounting equations for Examples A–C.

References

[1] Fritjof Capra. *The Hidden Connections: A Science for Sustainable Living*. Anchor Books, New York, January 2004.

[2] Robert U Ayres, Leslie W Ayres, and Benjamin S Warr. Exergy, Power and Work in the US Economy, 1900–1998. *Energy*, 28(3):219–273, March 2003.

[3] Benjamin S Warr and Robert U Ayres. Useful work and information as drivers of economic growth. *Ecological Economics*, 73(C):93–102, January 2012.

[4] US Energy Information Agency (EIA). 2010 Manufacturing Energy Consumption Survey (MECS) Survey Data. `http://www.eia.gov/consumption/manufacturing/data/2010/#r8`, 2010.

[5] International Organization of Motor Vehicle Manufacturers (OICA). 2010 Production Statistics. `http://www.oica.net/category/production-statistics/2010-statistics/`, 2010.

[6] JL Sullivan, A Burnham, M Wang, et al. Energy-consumption and carbon-emission analysis of vehicle and component manufacturing. Technical report, Argonne National Laboratory (ANL), 2010.

Chapter 4
Embodied energy flows

> *One of the main sinks of energy in the "developed" world is the creation of stuff. In its natural life cycle, stuff passes through three stages. First, a new-born stuff is displayed in shiny packaging on a shelf in a shop. At this stage, stuff is called "goods." As soon as the stuff is taken home and sheds its packaging, it undergoes a transformation from "good" to its second form, "clutter." The clutter lives with its owner for a period of months or years. During this period, the clutter is largely ignored by its owner, who is off at the shops buying more goods. Eventually, by a miracle of modern alchemy, the clutter is transformed into its final form, rubbish. To the untrained eye, it can be difficult to distinguish this "rubbish" from the highly desirable "good" that it used to be. Nonetheless, at this stage the discerning owner pays the dustman to transport the stuff away.* [1, p.88]
>
> —David MacKay

In Chapter 3, the First Law of Thermodynamics accounted direct energy ($\dot{E}$) flowing among sectors of an economy. In this chapter, we will adapt the First Law to account *embodied* energy in the material flows of an economy.[1]

Energy can become "embodied" in the output of an economic sector and within the material in the sector itself. The energy embodied in the output of an economic sector (e.g., energy embodied in the automobiles produced by the automotive sector) is related to the sum of all direct energy consumed in the manufacture of its products, including all upstream processing stages. Embodied energy gives an indication of the energy demand created by consumption of goods and services within an economy.

Energy that becomes embodied in the materials of an economic sector (such as the machines, factories, and dealerships within the automotive sector itself) is essential for the efficient operation of the sector. The amount of energy embodied in the sector is an indicator of the complexity of the sector; the amount of energy embodied in an entire economy can be an indicator of the level of economic development of the economy.

The purpose of this chapter is to develop a model for embodied energy flows within economies. With an embodied energy model in hand, we will be positioned to develop a model for the energy intensity of goods and services within an economy (Chapter 6).

[1] To the authors' knowledge, this is the first appearance in the literature of a systematic, detailed, and mathematically rigorous derivation of embodied energy accounting equations based upon the laws of thermodynamics.

4.1 Methodology

We begin the derivation of embodied energy accounting equations by defining the concept of *total* energy.

4.1.1 Total energy accounting

Total energy (T) is defined as the sum of direct energy (E, see Chapter 3) and embodied energy (B).

$$T \equiv E + B \tag{4.1}$$

The flow rate of total energy ($\dot{T}$) among sectors in the economy, the biosphere, and society is the sum of direct energy ($\dot{E}$) and embodied energy ($\dot{B}$).

$$\dot{T} = \dot{E} + \dot{B} \tag{4.2}$$

Figure 4.1 illustrates that total energy flows are comprised of direct energy ($\dot{E}$) and embodied energy ($\dot{B}$).

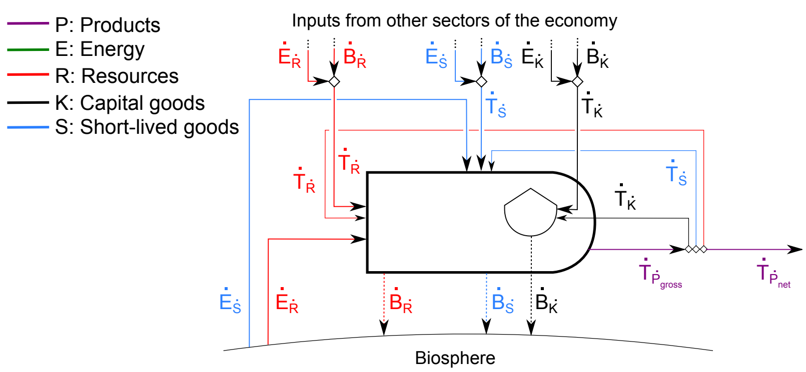

Fig. 4.1 Total energy flows ($\dot{T}$) for a single sector of an economy. For the sake of clarity, direct ($\dot{E}$) and embodied ($\dot{B}$) energy flows are shown separately for material inflows from other sectors only.

In some cases, a material flow may include either direct energy ($\dot{E}$) or embodied energy ($\dot{B}$), exclusive. For example, the flow of extracted crude oil from the earth consists of direct energy only ($\dot{B} = 0$ and $\dot{T} = \dot{E}$), because, in this method, no embodied energy (B) is added to the crude oil until it reaches the downstream side of the oil rig. The material produced by a non-energy sector of the economy consists of indirect energy only ($\dot{E} \approx 0$, and therefore $\dot{T} \approx \dot{B}$), because direct energy (E) produced by a non-energy sector is negligible in this economy.

In other cases, a material flow may include both direct energy flow ($\dot{E}$) *and* embodied energy flow ($\dot{B}$) components. For example, the outgoing flow of refined petroleum from the energy sector has both a direct energy ($\dot{E}$, the energy content of the oil product, usually represented by chemical potential energy) and embodied energy ($\dot{B}$, which accounts for the energy consumed in upstream processes to extract and refine the crude oil).[2]

Most of the I-O literature [2, 3] applies the following (and often unstated) assumptions:

I. flows of total energy ($\dot{T}$) are *conserved*,[3]
II. steady state conditions exist (i.e., total energy does not accumulate in economic sectors),[4] and
III. the sum of the signed (input is positive, output is negative) total energy inflows of a sector is assigned to the products of the sector (i.e., there is no "waste" of total energy).

Like the I-O literature, we assume that total energy is conserved and never wasted.[5] However, we depart from the I-O literature to allow durability of goods as represented by total energy accumulation in economic sectors. Steady state, this approach is not.

Total energy (T) may accumulate within an economic sector as stocks of direct energy materials (piles of coal or tanks of oil) but also as energy embodied in stocks of capital goods (e.g., machinery or buildings). The rate of accumulation of total energy in a sector of the economy, the biosphere, or society is given by the time derivative of total energy:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t} + \frac{\mathrm{d}B}{\mathrm{d}t}. \tag{4.3}$$

The following equation provides a total energy accounting for a sector of the economy, where the $\dot{T}$ terms are signed: positive for total energy input and negative for total energy output.

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \sum \dot{T} \tag{4.4}$$

By substituting Equations 4.2 and 4.3 into Equation 4.4, we obtain

$$\frac{\mathrm{d}E}{\mathrm{d}t} + \frac{\mathrm{d}B}{\mathrm{d}t} = \sum\left(\dot{E} + \dot{B}\right). \tag{4.5}$$

---

[2] Outputs from agricultural sectors will be similar: both (a) the direct energy component (comprising chemical potential energy) and (b) the embodied energy component (representing upstream energy consumed in food production) will be non-zero.

[3] Total energy can be neither created nor destroyed.

[4] We will see later how the steady-state assumption in the literature can introduce errors into I-O analyses.

[5] Of course, waste heat exists and is accounted by the First Law of Thermodynamics. However, waste heat is ignored when accounting for total energy.

4.1.2 Embodied energy accounting

We note that the definition of total energy (Equation 4.1) includes direct energy (E) and embodied energy (B) terms. On the other hand, the First Law of Thermodynamics (Equation 3.1) includes direct energy (E) and waste heat (Q) terms. The consequence of the foregoing difference is that an interesting relationship exists between embodied energy (B) and waste heat (Q), as we shall see below.

To derive an accounting equation for embodied energy, we substitute the First Law of Thermodynamics (Equation 3.1) into the total energy accounting equation (Equation 4.5).

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \sum \dot{B} + \sum \dot{Q}_{out} \tag{4.6}$$

The waste energy terms ($\dot{Q}_{out}$) in Equation 4.6 are *outflows* of energy from the sector. The embodied energy terms ($\dot{B}$) represent embodied energy of inflows and outflows of material. Splitting the $\dot{B}$ term into inflows and outflows and rearranging gives

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \sum \dot{B}_{in} - \sum \dot{B}_{out} + \sum \dot{Q}_{out} \tag{4.7}$$

In words, the rate of accumulation of embodied energy in a sector of the economy $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$ is equal to the sum of the rates of inflow of embodied energy into the sector ($\dot{B}_{in}$) less the rate of output of embodied energy from the sector ($\dot{B}_{out}$) *plus* the rate of waste direct energy from the sector ($\dot{Q}_{out}$). The first two terms on the right side of Equation 4.7 are expected: accumulation is the difference between inflow and outflow rates.

Rearranging Equation 4.7 yields another version of the embodied energy accounting equation: one that illuminates issues related to stages of growth for an economic sector.

$$\frac{\mathrm{d}B}{\mathrm{d}t} + \sum \dot{B}_{out} = \sum \dot{B}_{in} + \sum \dot{Q}_{out} \tag{4.8}$$

From Equation 4.8, we see that incoming embodied energy ($\dot{B}_{in}$) and waste heat[6] ($\dot{Q}_{out}$) can be used to increase either (a) the embodied energy within a sector of the economy $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$ or (b) the embodied energy output of a sector of the economy ($\dot{B}_{out}$), depending on decisions by actors (firms, households, or the government) within the sector. If the sector is "building up" production capacity, much of the incoming embodied energy ($\dot{B}_{in}$) and direct energy consumption (represented by $\dot{Q}_{out}$) will be used to increase infrastructure (and associated embodied energy, B) within the sector, and $\frac{\mathrm{d}B}{\mathrm{d}t}$ will be positive. If, on the other hand, the sector is mature, much of the incoming embodied energy ($\dot{B}_{in}$) and direct energy consumption (represented by

[6] Because we have substituted the First Law of Thermodynamics into the total energy accounting equation, $\dot{Q}_{out}$ becomes a proxy for direct energy consumption by the sector.

$\dot{Q}_{out}$) will be used for production of goods ($\dot{B}_{out}$). $\frac{\mathrm{d}B}{\mathrm{d}t}$ will be close to zero. Equation 4.7 shows that an economic sector in decline may experience an outflow of embodied energy (via products or depreciation) in excess of the sum of its embodied energy inflows ($\dot{B}_{in}$) and direct energy consumption (represented by $\dot{Q}_{out}$), and $\frac{\mathrm{d}B}{\mathrm{d}t}$ will be negative.

Equations 4.7 and 4.8 highlight a contrast between the present dynamic analysis and the I-O literature. The traditional assumption of steady-state conditions in economic sectors is tantamount to assuming that $\frac{\mathrm{d}B}{\mathrm{d}t} = 0$ in Equations 4.7 and 4.8. That assumption precludes analysis of stages of growth and the embodied energy implications thereof.

Equations 4.7 and 4.8 are generalized embodied energy accounting equations that we will see again for Examples A-C in the sections that follow.

4.2 Example A: single-sector economy

Figure 4.2 shows the flows of total energy ($\dot{T}$) through the single-sector economy.

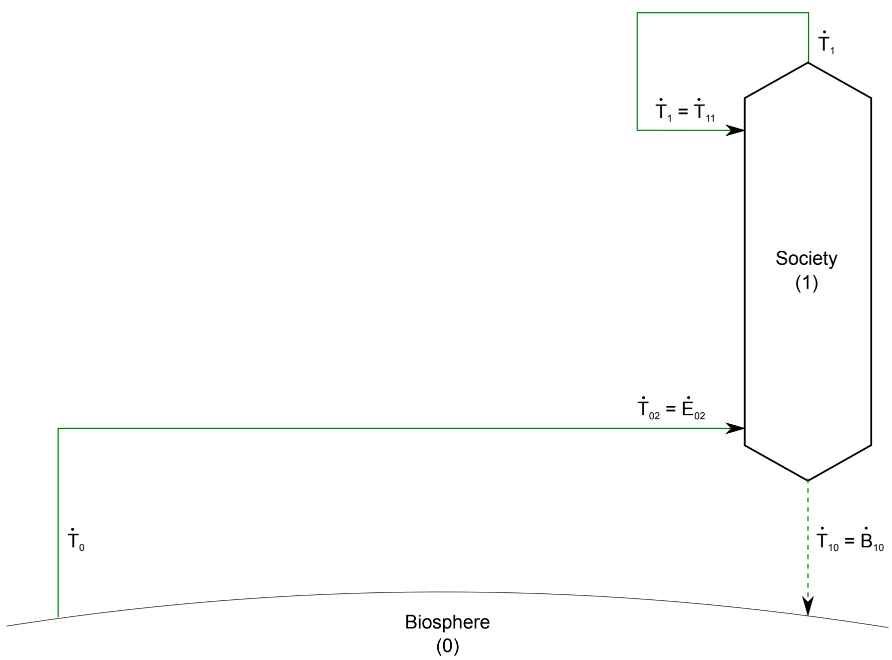

Fig. 4.2 Total energy flows ($\dot{T}$) in a one-sector economy.

As discussed above, we follow the I-O literature in assuming that total energy (T) is conserved. A total energy accounting around the biosphere (0) and the single-sector economy (1) gives

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = \dot{T}_{10} - \dot{T}_{01}, \tag{4.9}$$

and

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \dot{T}_{01} + \dot{T}_{11} - \dot{T}_1 - \dot{T}_{10}. \tag{4.10}$$

Substituting Equations 4.2 and 4.3 into Equations 4.9 and 4.10 yields

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} + \frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} - \dot{E}_{01} - \dot{B}_{01} \tag{4.11}$$

and

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} + \frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{E}_{01} + \dot{B}_{01} + \dot{E}_{11} + \dot{B}_{11} - \dot{E}_1 - \dot{B}_1 - \dot{E}_{10} - \dot{B}_{10}. \tag{4.12}$$

At this point, we can proceed in two directions. The first direction, simplifying Equations 4.11 and 4.12, provides an intuitive result. The second direction, substituting the First Law of Thermodynamics into Equations 4.11 and 4.12, provides the advantage of cancelling most of the direct energy terms. We begin with the first approach: simplification.

4.2.1 Simplification of the embodied energy accounting equation

To simplify Equations 4.11 and 4.12, we first realize that, by definition, no embodied energy flows from the earth with extracted material, so $\dot{B}_{01} = 0$ and $\dot{T}_0 = \dot{E}_{01}$ as shown in Figure 4.2. Second, we can assume that direct energy (E) does not accumulate in the economy such that $\frac{\mathrm{d}E_0}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}E_1}{\mathrm{d}t} = 0$. Finally, we note that $\dot{E}_{10} = 0$, because society does not supply direct energy to the biosphere. Thus, Equations 4.11 and 4.12 become

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{B}_{10} - \dot{E}_{01} \tag{4.13}$$

and

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{E}_{01} + \dot{E}_{11} + \dot{B}_{11} - \dot{E}_1 - \dot{B}_1 - \dot{B}_{10}. \tag{4.14}$$

These equations show that direct energy consumed by a sector ($\dot{E}_{01}$) increases the energy embodied within the sector (B_1), whereas the waste from the sector produces an embodied energy outflow ($\dot{B}_{10}$) that reduces the energy embodied within the sector.

4.2.2 Substitution of First Law into the embodied energy accounting equation

The second approach to the derivation of embodied energy accounting equations is to substitute the First Law (Equations 3.4 and 3.5) into the total energy accounting equations (Equations 4.11 and 4.12).

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} - \dot{B}_{01} - \dot{Q}_{10} \tag{4.15}$$

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{B}_{01} + \dot{B}_{11} - \dot{B}_1 - \dot{B}_{10} - \dot{E}_{10} + \dot{Q}_{10} \tag{4.16}$$

This substitution has the advantage of canceling most of the direct energy terms from the embodied energy accounting equations. And, it is no longer necessary to assume that the accumulation rate of direct energy $\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)$ is zero, because the $\frac{\mathrm{d}E}{\mathrm{d}t}$ term is cancelled by the substitution.

Note that the Equation 4.15 includes the term $-\dot{Q}_{10}$, which, at first glance, appears different from Equation 4.7.[7] However, upon realizing that $\dot{Q}_{10}$ is an *inflow* of waste energy into the biosphere, we can use $\dot{Q}_{10} = -\dot{Q}_{01}$ to rewrite Equation 4.15 with an *outflow* term,

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} - \dot{B}_{01} + \dot{Q}_{01}, \tag{4.17}$$

thereby maintining consistency with Equation 4.7.

We can simplify Equations 4.15 and 4.16 using the assumptions of Section 4.2.1 (namely, that $\dot{B}_{01} = 0$ and $\dot{E}_{10} = 0$) to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{B}_{10} - \dot{Q}_{10} \tag{4.18}$$

and

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \dot{B}_{11} - \dot{B}_1 - \dot{B}_{10} + \dot{Q}_{10}. \tag{4.19}$$

The material model of this framework (see Chapter 2) indicates that materials are comprised of resources (R), short-lived materials (S), and capital (K). Thus, we can write

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \frac{\mathrm{d}B_{R,1}}{\mathrm{d}t} + \frac{\mathrm{d}B_{S,1}}{\mathrm{d}t} + \frac{\mathrm{d}B_{K,1}}{\mathrm{d}t}, \tag{4.20}$$

but neither resources (R) nor short-lived materials (S) accumulate in economic sectors at a significant rate. Thus,

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[7] Equation 4.7 has a positive sign for the waste energy term ($+\dot{Q}_{out}$), whereas Equation 4.15 has a negative sign for the waste energy term ($-\dot{Q}_{10}$).

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \frac{\mathrm{d}B_{K_1}}{\mathrm{d}t}. \tag{4.21}$$

We can substitute Equation 4.21 into Equation 4.19 to obtain

$$\frac{\mathrm{d}B_{K,1}}{\mathrm{d}t} = \dot{B}_{11} - \dot{B}_1 - \dot{B}_{10} + \dot{Q}_{10}. \tag{4.22}$$

Equations 4.18 and 4.22 are the embodied energy accounting equations for Example A.

In Examples B and C following, we will choose the approach of this section, namely substitution of the First Law of Thermodynamics into the total energy accounting equation (instead of simplifying the total energy equation as discussed in Section 4.2.1), because of the benefit of canceling direct energy flow terms ($\dot{E}$).

4.2.3 Physical depreciation

The term $\dot{B}_{10}$ in Equation 4.22 represents the disposal rate of embodied energy from Society (1) to the Biosphere (0). Figure 2.2 shows that the outgoing material flow from Society (1) is comprised of resources ($\dot{R}_{10}$), short-lived materials ($\dot{S}_{10}$), and capital ($\dot{K}_{10}$). Each of these material flows will have associated embodied energy such that

$$\dot{B}_{10} = \dot{B}_{\dot{R}_{10}} + \dot{B}_{\dot{S}_{10}} + \dot{B}_{\dot{K}_{10}}. \tag{4.23}$$

The term $\dot{B}_{\dot{K}_{10}}$ represents the energy embodied in depreciated phyiscal assets. Physical depreciation is counted at the moment when material physically departs an economic sector and enters the biosphere, presumably a landfill, where the material in the wasted assets will decay. Financial depreciation is usually faster than physical depreciation according to rates set by accounting rules. The embodied energy associated with physical depreciation ($\dot{B}_{\dot{K}_{10}}$) can be represented by a depreciation term such as

$$\dot{B}_{\dot{K}_{10}} = \gamma_{B,1} B_{K_1}, \tag{4.24}$$

where γ_B represents the depreciation rate of embodied energy in units of inverse time (e.g., 1/year) with $\gamma_B > 0$.[8] The depreciation rate (γ_B) indicates that a fraction of the energy embodied in capital stock is disposed over a period of time (e.g., $\gamma_B = 0.05$/year). In the absence of other inputs or outputs, this depreciation function provides exponential decay of embodied energy (B) in an economic sector. γ_B is, in general, a function of time.

Equation 4.24 can be substituted into Equation 4.23 to obtain

---

[8] Note that γ_B will, in general, be different from γ_K defined in Section 2.2. γ_B will equal γ_K if and only if the depreciated capital has an embodied energy content that is identical to the average embodied energy content of the sector on a per-unit-mass basis.

$$\dot{B}_{10} = \dot{B}_{\dot{R}_{10}} + \dot{B}_{\dot{S}_{10}} + \gamma_{B,1} B_{K_1}. \tag{4.25}$$

Equation 4.25 can be substituted into Equations 4.18 and 4.22 to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \dot{B}_{\dot{R}_{10}} + \dot{B}_{\dot{S}_{10}} + \gamma_{B,1} B_{K_1} - \dot{Q}_{10} \tag{4.26}$$

and

$$\frac{\mathrm{d}B_{K_1}}{\mathrm{d}t} = \dot{B}_{11} - \dot{B}_1 - \dot{B}_{\dot{R}_{10}} - \dot{B}_{\dot{S}_{10}} - \gamma_{B,1} B_{K_1} + \dot{Q}_{10}. \tag{4.27}$$

Equation 4.27 indicates that the accumulation rate of embodied energy in an economic sector $\left(\frac{\mathrm{d}B_{K_1}}{\mathrm{d}t}\right)$ is equal to the sum of the embodied energy input to the sector ($\dot{B}_{11}$) and waste heat from the economic sector ($\dot{Q}_{10}$), less embodied energy the leaves the sector in its products ($\dot{B}_1$), less the rate of disposal of embodied energy associated with scrap resources ($\dot{B}_{\dot{R}_{10}}$), short-lived material ($\dot{B}_{\dot{S}_{10}}$), and depreciated capital stock ($\gamma_{B,1} B_{K_1}$).

We turn now to Example B, a two-sector economy.

4.3 Example B: two-sector economy

For the two-sector economy of Figures 2.3 and 3.4, we again follow the I-O literature by assuming that total energy (T) is conserved. Figure 4.3 shows total energy flows for the two-sector economy.

Accounting for accumulation of total energy and using the assumption that total energy is conserved, we can write the following equations.

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = \dot{T}_{10} + \dot{T}_{20} - \dot{T}_{02}, \tag{4.28}$$

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \dot{T}_{11} + \dot{T}_{21} - \dot{T}_1 - \dot{T}_{10}, \tag{4.29}$$

and

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \dot{T}_{02} + \dot{T}_{12} + \dot{T}_{22} - \dot{T}_2 - \dot{T}_{20}. \tag{4.30}$$

Substituting Equations 4.2 and 4.3 into Equations 4.28 through 4.30 gives

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} + \frac{\mathrm{d}E_0}{\mathrm{d}t} = \dot{E}_{10} + \dot{B}_{10} + \dot{E}_{20} + \dot{B}_{20} - \dot{E}_{02} - \dot{B}_{02}, \tag{4.31}$$

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} + \frac{\mathrm{d}E_1}{\mathrm{d}t} = \dot{E}_{11} + \dot{B}_{11} + \dot{E}_{21} + \dot{B}_{21} - \dot{E}_1 - \dot{B}_1 - \dot{E}_{10} - \dot{B}_{10}, \tag{4.32}$$

and

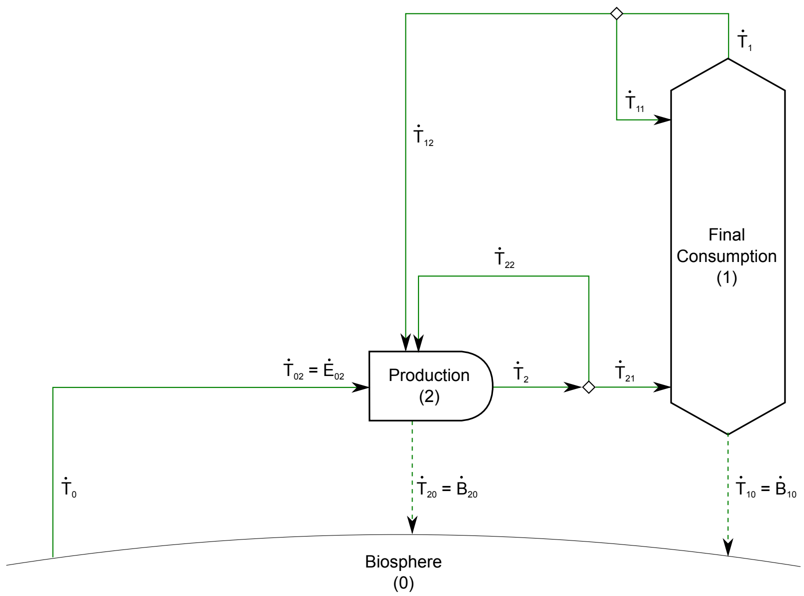

Fig. 4.3 Flows of total energy ($\dot{T}$) in a two-sector economy.

$$\frac{dB_2}{dt} + \frac{dE_2}{dt} = \dot{E}_{02} + \dot{B}_{02} + \dot{E}_{12} + \dot{B}_{12} + \dot{E}_{22} + \dot{B}_{22} - \dot{E}_2 - \dot{B}_2 - \dot{E}_{20} - \dot{B}_{20}. \quad (4.33)$$

As in Example A, we can substitute the First Law of Thermodynamics (Equations 3.8–3.10) into the total energy accounting equations (Equations 4.31–4.33) and employ the assumptions that $\dot{E}_{i0} = 0$ and $\dot{B}_{0j} = 0$ to obtain

$$\frac{dB_0}{dt} = \dot{B}_{10} + \dot{B}_{20} - \dot{Q}_{10} - \dot{Q}_{20}, \quad (4.34)$$

$$\frac{dB_1}{dt} = \dot{B}_{11} + \dot{B}_{21} - \dot{B}_2 - \dot{B}_{10} + \dot{Q}_{10}, \quad (4.35)$$

and

$$\frac{dB_2}{dt} = \dot{B}_{12} + \dot{B}_{22} - \dot{B}_2 - \dot{B}_{20} + \dot{Q}_{20}. \quad (4.36)$$

Similar to Example A, we observe that the accumulation rate of embodied energy in the economic sectors (1 and 2) is the sum of the rates of waste heat flowing from the sector ($\dot{Q}_{20}$) and embodied energy into the sector ($\dot{B}_{12} + \dot{B}_{22}$) less the rate of embodied energy leaving the sector on its output streams ($\dot{B}_2 + \dot{B}_{20}$).

Equations 4.34–4.36 can be simplified using sums:

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{i0} - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.37}$$

and

$$\frac{\mathrm{d}B_j}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{ij} - \dot{B}_j - \dot{B}_{j0} + \dot{Q}_{j0}, \tag{4.38}$$

where $j \in [1,n]$.

As in Example A, we can disaggregate the accumulation and waste embodied energy terms and express physical waste of capital stock as depreciation in Equations 4.37 and 4.38 to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n} \left(\dot{B}_{\dot{R}_{i0}} + \dot{B}_{\dot{S}_{i0}} + \gamma_{B,i} B_{K_i} \right) - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.39}$$

and

$$\frac{\mathrm{d}B_{K,j}}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{ij} - \dot{B}_j - \left(\dot{B}_{\dot{R}_{j0}} + \dot{B}_{\dot{S}_{j0}} + \gamma_{B,j} B_{K_j} \right) + \dot{Q}_{j0}. \tag{4.40}$$

In the next section, we apply embodied energy accounting to Example C, a three-sector economy.

4.4 Example C: three-sector economy

Again, we begin with the diagram showing total energy ($\dot{T}$) flows among the economic sectors of Example C (Figure 4.4).

Accounting for accumulation of total energy and applying the assumption that total energy is conserved, we can write the following equations. We build from the derivation in Section 4.3 and utilize sums for each equation below.

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{T}_{i0} - \sum_{j=1}^{n} \dot{T}_{0j} \tag{4.41}$$

and

$$\frac{\mathrm{d}T_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{T}_{ij} - \dot{T}_j - \dot{T}_{j0}. \tag{4.42}$$

where $j \in [1,n]$.

Substituting Equations 4.2 and 4.3 into Equations 4.41 and 4.42 gives

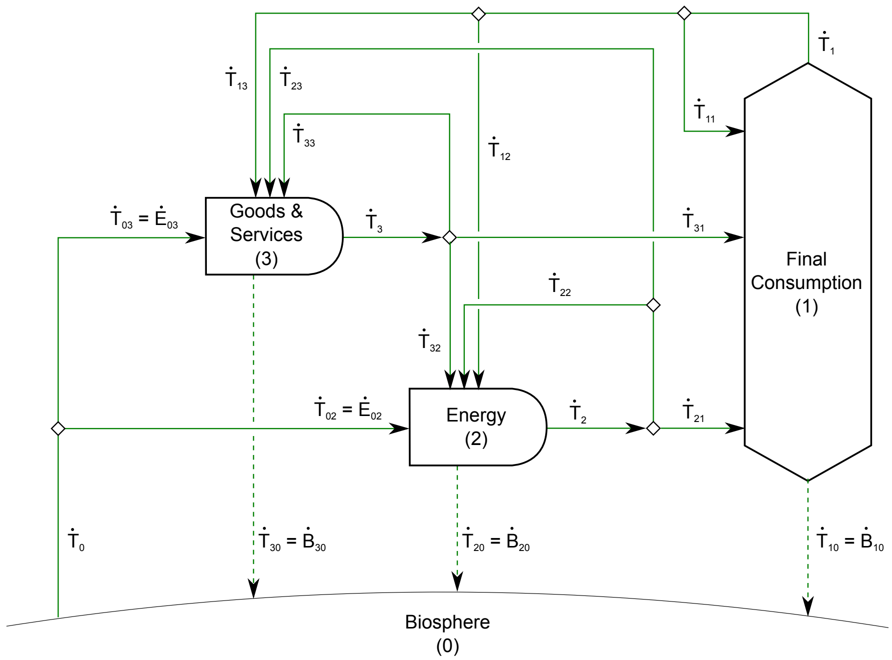

Fig. 4.4 Flows of total energy ($\dot{T}$) in a three-sector economy.

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} + \frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{E}_{i0} + \sum_{i=1}^{n} \dot{B}_{i0} - \sum_{j=1}^{n} \dot{E}_{0j} - \sum_{j=1}^{n} \dot{B}_{0j} \tag{4.43}$$

and

$$\frac{\mathrm{d}E_j}{\mathrm{d}t} + \frac{\mathrm{d}B_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{E}_{ij} + \sum_{i=0}^{n} \dot{B}_{ij} - \dot{E}_j - \dot{B}_j - \dot{E}_{j0} - \dot{B}_{j0}. \tag{4.44}$$

Substituting the First Law of Thermodynamics (Equations 3.18 and 3.19) into the total energy accounting equations (Equations 4.43 and 4.44) and recognizing that $\dot{B}_{0j} = 0$ for $j \in [1,n]$ and $\dot{E}_{i0} = 0$ for $i \in [1,n]$ gives embodied energy accounting equations for Example C:

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{i0} - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.45}$$

$$\frac{\mathrm{d}B_j}{\mathrm{d}t} = \sum_{i=0}^{n} \dot{B}_{ij} - \dot{B}_j - \dot{B}_{j0} + \dot{Q}_{j0} \tag{4.46}$$

As in Example B, we can disaggregate the accumulation and waste embodied energy terms and express physical waste of capital stock as depreciation in Equations 4.45 and 4.46 to obtain

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} = \sum_{i=1}^{n}\left(\dot{B}_{\dot{R}_{i0}} + \dot{B}_{\dot{S}_{i0}} + \gamma_{B,i} B_{K_i}\right) - \sum_{i=1}^{n} \dot{Q}_{i0} \tag{4.47}$$

and

$$\frac{\mathrm{d}B_{K_j}}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{ij} - \dot{B}_j - \left(\dot{B}_{\dot{R}_{j0}} + \dot{B}_{\dot{S}_{j0}} + \gamma_{B,j} B_{K_j}\right) + \dot{Q}_{j0}. \tag{4.48}$$

which are the same as Equations 4.39 and 4.40, indicating that we have successfully generalized the embodied energy equations to an arbitrarily-large economy.

To verify the above derivation, we sum Equations 4.47 and 4.48 for all sectors of the economy ($j \in [1,n]$) to obtain

$$\begin{aligned}\frac{\mathrm{d}B_0}{\mathrm{d}t} + \sum_{j=1}^{n} \frac{\mathrm{d}B_{K_j}}{\mathrm{d}t} = & \sum_{i=1}^{n}\left(\dot{B}_{\dot{R}_{i0}} + \dot{B}_{\dot{S}_{i0}} + \gamma_{B,i} B_{K,i}\right) - \sum_{i=1}^{n} \dot{Q}_{i0} \\ & + \sum_{j=1}^{n}\sum_{i=1}^{n} \dot{B}_{ij} - \sum_{j=1}^{n} \dot{B}_j \\ & - \sum_{j=1}^{n}\left(\dot{B}_{\dot{R}_{j0}} + \dot{B}_{\dot{S}_{j0}} + \gamma_{B,j} B_{K,j}\right) + \sum_{j=1}^{n} \dot{Q}_{j0}.\end{aligned} \tag{4.49}$$

Using the identities

$$\dot{B}_j = \sum_{k=1}^{n} \dot{B}_{jk}, \tag{4.50}$$

and

$$\sum_{j=1}^{n} \dot{B}_j = \sum_{j=1}^{n}\sum_{k=1}^{n} \dot{B}_{jk} = \sum_{i=1}^{n}\sum_{k=1}^{n} \dot{B}_{ik} = \sum_{i=1}^{n}\sum_{j=1}^{n} \dot{B}_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{n} \dot{B}_{ij}, \tag{4.51}$$

Equation 4.49 becomes

$$\frac{\mathrm{d}B_0}{\mathrm{d}t} + \sum_{j=1}^{n} \frac{\mathrm{d}B_j}{\mathrm{d}t} = 0, \tag{4.52}$$

as expected. The total embodied energy content of the system remains constant with respect to time in this model.

We can further simplify the above equations by expressing the embodied energy of the inflowing capital ($\sum_{i=1}^{n} \dot{B}_{ij}$) as a fraction ($\alpha_{B,j}$) of the energy embodied in the capital stock (B_{K_j})

$$\alpha_{B,j} \equiv \frac{\sum_{i=1}^{n} \dot{B}_{ij}}{B_{K_j}} \tag{4.53}$$

and resource and short-lived material flows as waste

$$\dot{B}_{waste,j} \equiv \dot{B}_{\dot{R}_{j0}} + \dot{B}_{\dot{S}_{j0}}. \tag{4.54}$$

With the above definitions, Equation 4.48 can be expressed as

$$\frac{\mathrm{d}B_{K_j}}{\mathrm{d}t} = (\alpha_{B,j} - \gamma_{B,j})B_{K_j} - \dot{B}_{waste,j} - \dot{B}_j + \dot{Q}_{j0}. \tag{4.55}$$

With Equation 4.55, we see that the rate of accumulation of embodied energy in the capital stock of an economic sector $\left(\frac{\mathrm{d}B_{K_j}}{\mathrm{d}t}\right)$ is affected by the balance between the inflow ($\alpha_{B,j}$) and depreciation ($\gamma_{B,j}$) rates, the rate of wasting embodied energy ($\dot{B}_{waste,j}$), the rate at which embodied energy leaves with the products of the sector ($\dot{B}_j$), and the waste heat that leaves the sector ($\dot{Q}_{j0}$).

4.5 Embodied energy in the auto industry

In this section, we apply the methodology tracking flows of total energy to the US auto industry, as depicted in Figure 4.5. As in Section 2.5, we face difficulties due to lack of data. We know that some flows will have zero value, as shown in Figure 4.5. For instance, there is zero energy content (direct or embodied) associated with flows from the biosphere into the auto industry. Furthermore, we may assume that the resource flows ($\dot{R}$, red in Figure 4.5) and capital flows ($\dot{K}$, black in Figure 4.5) will have no direct energy ($\dot{E}$) associated with them.[9] Firstly, because energy products enter the industry as short-lived flows ($\dot{S}$, blue in Figure 4.5) and secondly, because energy products are not stored as capital within the sector. In fact, we can assume that all flows, other than inputs of short-lived goods ($\dot{S}$), will have no direct energy content ($\dot{E}$) associated with them.

Historically, very few estimates of embodied energy of automobiles have been made. In 1973, Berry and Fels, used a process-based analysis (rather an Input-Output analysis), to find that the energy cost of automobile manufacturing[10] in the U.S. was 37,275 kW-hr = 134 GJ (10^9 J) per vehicle.[4, Table 2] Of this, 100 GJ was (upstream) energy embodied in the materials ($\dot{B}_{\dot{R}}$) and the remaining was (di-

[9] Exceptions to this assumption may be the direct energy content of rubber, plastic and other petroleum products, e.g. motor oils which are used as resource inputs to the auto industry.

[10] The "energy cost" estimated by Berry and Fels is the energy embodied in a single automobile. The "energy cost" (in kW-hr/automobile) multiplied by the the production rate (in autmobiles/year) gives the rate of gross embodied energy outflow in the product stream of the auto sector ($\dot{B}_{\dot{P}_{gross}}$). A limitation of the process-based approach employed by Berry and Fels is trucation error for upstream energy demand. See Section 8.4 for details.

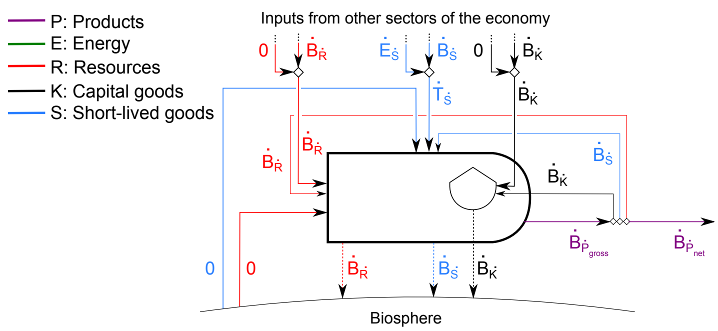

Fig. 4.5 Embodied energy flows ($\dot{B}$) for the US automobile industry using data from **** XXXX ****.

rect) energy used within the auto industry to manufacture, assemble and transport the automobile.

Two decades later in 1995, Stodolsky et al. estimated the energy consumed in materials and manufacturing automobiles to be 79 GJ per vehicle for a conventional automobile and 66 GJ per vehicle for an aluminum intensive vehicle, both under a maximum-recycling scenario.[5, p.11]. Three years later, MacLean and Lave estimated the the embodied energy for an automobile to be 113.6 MBtu = 120 GJ (13 GJ upstream and 107 GJ direct) per vehicle [6, Figure 2], which they compare with contemporaneous estimates from Sullivan of 81 GJ per vehicle [7] and Volkswagen of 62 GJ per vehicle.[8]

Estimates of vehicle embodied energy are related to contemporary debates on whether Electric Vehicles (EVs) reduce CO_2 emissions relative to Internal Combustion Vehicles (ICVs), insofar as embodied energy includes upstream supply chain energy consumption, a major contributor to both EV and ICV lifecycle emissions. Although EVs have no direct emissions during operation, accounting for the upstream energy consumed in generating electricity, the manufacture of batteries, and the production of lightweight materials (employed to offset the weight of EV battery packs) leads to significantly increased lifecycle emissions. Many studies find that negligible or negative emissions savings are achieved by EVs.[9–11]

4.6 Summary

This chapter relies upon the results from Chapter 3 to develop equations that describe the flow of *embodied* energy ($\dot{B}$) through economies (Section 4.1). We found that waste heat from a sector ($\dot{Q}$) is additive to the energy embodied within products of a sector, thereby providing the mechanism for accumulating embodied energy along the manufacturing supply chain. The embodied energy accounting equations were applied to example economies A–C in Sections 4.2–4.4. Finally, we discussed embodied energy in the context of our running example, the US auto industry (Section 4.5). We found that there are a few historical estimates of energy embodied within automobiles.

In Chapter 5, we develop theory and equations to account for value flows though economies, leading (in Chapter 6) to techniques to estimate energy intensity of economic products.

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Part II
Economic Value Flows and Energy Intensity

Chapter 5
Value flows

We try to measure what we value. We come to value what we measure. [1, p. 2]

—Donella Meadows

In Chapters 3 and 4, we noted that energy is the currency of Thermodynamics, and we developed accounting equations for flows of direct ($\dot{E}$) and embodied ($\dot{B}$) energy through an economy. In this chapter, we develop a framework for accounting value flows ($\dot{X}$) through economies. Accounting flows of value is a necessary step along the path to developing equations (in Chapter 6) that describe the energy intensity of intermediate and final products within an economy.

5.1 Methodology

We begin by explicitly describing what we mean by value. We follow the standard, neoclassical approach of using the market price at the time of an exchange to determine the value of the flows of products (goods, services and capital). As materials and energy flow in one direction between sectors, currency flows in the opposite direction. That monetary flow is an easy and logical proxy for the value of the material and energy flows. Market transactions are readily captured and the data to estimate these flows is available in most developed countries. (See for example, `http://www.iioa.org/io-data/io-data.html`) Using the price at the time of exchange to place a value on the flow of material goods and resources is a *subjective* theory of value. It is subjective in that it is based on a price that two trading partners are willing to accept for whatever reason. It is based on its ability to meet the needs of the human trading partners. The market price is not necessarily a good measure of the *intrinsic* values of the goods (e.g., for bio-diversity or ecosystem services). Market price ignores the costs and benefits that accrue other parties (externalities), including the impact of trade on the quality of human relations, just distribution, or sustainable scale. [**?**] (***CITE Daly, Herman E., Beyond Growth, Boston: Beacon Press, 1996, p. 55) Section 8.2 contains further discussion of the benefits and limitations of the neoclassical subjective theory of value.

Thus, that we use the subjective theory of value for pragmatic reasons does not indicate our endorsement of it. Of greatest concern for our framework is that, at this time, market values are not ascribed to flows of natural resources and wastes. Important value flows that accompany material flows to and from the biosphere are not included. Those flows will be conspicuously missing from the figures in this chapter, see for example Figure 5.1. Significant challenges exist to measuring physical flows of materials and energy to and from the biosphere, and even more challenging is trying to place a monetary value on them. ****ADD CITE (Nordhaus, William, The Future of Environmental and Augmented National Accounts An Overview, Survey of Current Business, November 1999, p. 45-65. `http://www.bea.gov/scb/account_articles/national/1199od/nordhaus.htm`.) ***ADD CITE (United Nations, et al., Environmental-Economic Accounting 2012: Experimental Ecosystem Accounting, United Nations, 2013. (`http://unstats.un.org/unsd/envaccounting/eea_white_cover.pdf`) However, an international experimental system for accounting environmental and economic flows is in place. [2][1] As the challenges to measuring values associated with the material resource, energy and waste flows to and from the biosphere begin to be overcome, our framework will be able to incorporate those values easily. However, it is important to be clear that despite our focus on material and energy flows through an economy, our framework does not assume, nor does it lead to, an energy theory of value. Goods and services are valued based on market transactions.

Because the basic unit of analysis in our framework is the economic sector, flows of value within the economy are based on the prices agreed upon in the transactions. The flows of value that accompany the material and energy flows in and out of one sector in an economy are depicted in Figure 5.1.

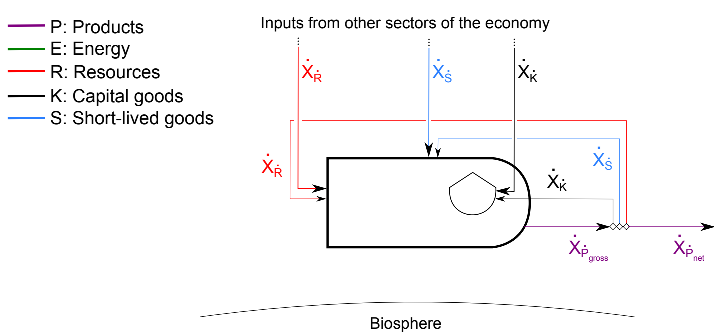

Fig. 5.1 Flows of value ($\dot{X}$) for a single sector. The value flows are associated with each of the different material and energy flows outlined in previous chapters.

We denote creation and destruction of value within a sector using the notion of "source" and "sink." In Figure 5.2, the open circle, "source," inside the economic sector represents the value-added, that is, the value that is created by the economic processes within that sector. The flows of value from a value-source are denoted

[1] As of this printing, the System of Environmental-Economic Accounting (SEEA) is in its third revision using a process of global consultation.

$\dot{X}_{gen}$. Similarly, black circles represent the value "sinks" where value is destroyed by economic processes or natural disasters. The flows of value into the value sinks are denoted $\dot{X}_{dest}$. Although we do not define the value creation and destruction processes any further (mathematically), we discuss what is meant by the underlying processes in more detail below.

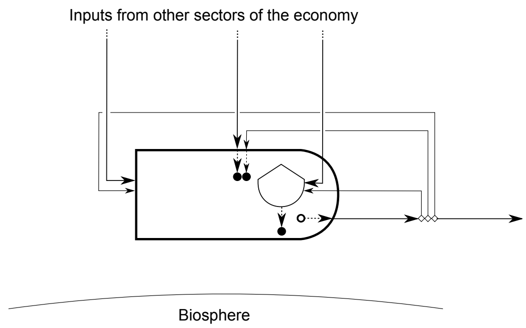

Fig. 5.2 Aggregated flows of value ($\dot{X}$) for a single sector. Distinction is made between value flows that enter the sector and are accumulated (i.e. capital goods) and value flows that are not accumulated. Within the sector there is destruction of value $\dot{X}_{dest}$, represented by the downward arrow flowing into the black sink and generation of value, represented by the arrow flowing out of a source.

5.2 Example A: single-sector economy

Figure 5.3 shows flows of value in the single-sector economy. Following typical assumptions in economic modeling, the economy is *completely isolated* from the biosphere in terms of both material inputs and wastes. In other words, the value flows of an economy are *independent from* material inputs and wastes. Value flows are independent from material inputs, because raw materials have no economic value until they have been removed from the biosphere by the extraction industry. Value flows are independent from wastes, because wastes, by definition, have no economic value after they leave the economy.

The contrast between Figures 2.2 and 3.3, on the one hand, and Figure 5.3, on the other, is striking. The picture of material and energy flows in Figures 2.2 and 3.3 indicates a dependence upon the biosphere that is not reflected in the value flows of Figure 5.3. The isloation of the value flows from the biosphere is a consequence of the subjective theory of value that underpins modern economics. The biosphere is akin to a third party with no voice in determining the value of a transaction: it is neither buyer nor seller.

Equation 5.1 describes the accumulation of value (X) in Society (1).

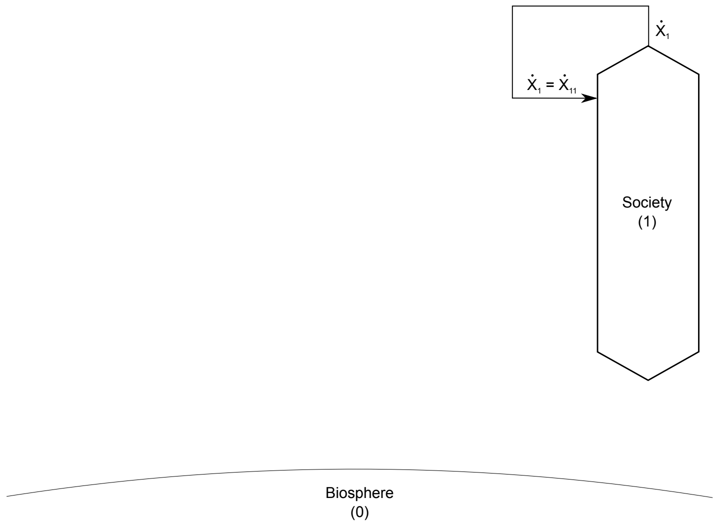

Fig. 5.3 Flows of value ($\dot{X}$) for a one-sector economy.

$$\frac{\mathrm{d}X_1}{\mathrm{d}t} = \dot{X}_{11} - \dot{X}_1 + \dot{X}_{gen,1} - \dot{X}_{dest,1}. \tag{5.1}$$

The following subsections discuss the terms in Equation 5.1.

5.2.1 Economic transactions ($\dot{X}_{11}$ and $\dot{X}_1$)

The returning arrow in Figure 5.3 represents transactions between

- buyers (who receive things of value, $\dot{X}_{11}$, in exchange for currency) and
- sellers (who give up things of value, $\dot{X}_1$, in exchange for currency).

It is interesting to note that when a good is sold for more than the producer paid for its inputs, the seller has created value and sold it into the economy. As a consequence, the seller's stock of currency grows, providing the seller with an increased level of claim on value in the economy.

The subjective theory of value (Section 5.1) posits that buyers and sellers agree on value at the time of the transaction. Thus, $\dot{X}_1 = \dot{X}_{11}$, and Equation 5.1 simplifies to

$$\frac{\mathrm{d}X_1}{\mathrm{d}t} = \dot{X}_{gen,1} - \dot{X}_{dest,1}, \tag{5.2}$$

indicating that value accumulates in the economy $\left(\frac{\mathrm{d}X_1}{\mathrm{d}t}\right)$ due to value generation ($\dot{X}_{gen,1}$) and destruction ($\dot{X}_{dest,1}$) processes only.

5.2.2 *Value generation* ($\dot{X}_{gen}$)

In Equation 5.1, the value generation term ($\dot{X}_{gen}$) is akin to growing apples in Section 2.1: value is generated, seemingly out of nothing. But, in fact, value is not created out of nothing. Rather, value is created from a variety of factors that have no apparent monetary cost to producers, including:

- flow of solar energy into the economy, as in the example of growing apples,
- extraction of resources (e.g., water, minerals, and fossil fuels) or any other unpriced goods from the biosphere,
- exploitation of the unpriced waste assimilation capacity of the biosphere,
- utilization of capital stock, labor, and energy to produce products that are more valuable than inputs, and
- application of human ingenuity and innovation, which lead to increasingly efficient production processes.

The subjective theory of value indicates that there is no economic value associated with these "transactions" that generate value, because no currency is exchanged.

The above factors indicate that the process of value generation has both direct and indirect impacts on the biosphere. The direct impacts are obvious: extraction of non-renewable resources from the biosphere, at rates greater than their natural accretion, represents unsustainable overuse of natural capital. The indirect impacts are less obvious: human ingenuity can lead to increased wealth, leading to increased demand rates for goods and services, whose production requires ever-increasing rates of unsustainable natural resource extraction.

$\dot{X}_{gen}$ is accounted as "value added" to an industry in national accounts. It is calculated as the difference between gross economic output of the industry and the cost of its intermediate inputs. [**?**] ****ADD CITE BEAVA `http://www.bea.gov/faq/index.cfm?faq_id=184` A simple way to think of value added is the increase in value of the raw materials from the work performed on them by workers and manufactured capital.

5.2.3 *Value destruction* ($\dot{X}_{dest}$)

In Equation 5.1, the value destruction term ($\dot{X}_{dest}$) is akin to consuming apples: value is destroyed by a process that consumes, or otherwise renders unusable, previously-valuable things in the economy (see Section 2.1). The factors that lead to value destruction ($\dot{X}_{dest}$) include:

- depreciation, usually associated with disposal of materials and equipment to the biosphere at end of life and
- natural disasters, such as hurricanes and typhoons, that destroy equipment and property.

$\dot{X}_{dest}$ is accounted as depreciation, or "consumpton of fixed capital," to an industry in the BEA tables. It is a monetary estimate of the physical effects on assets from "wear and tear, obsolescence, accidental damage, and aging." [**?**] ****ADD CITE Katz2008 Katz, Arnold. Accounting for Obsolescence: An Evaluation of Current NIPA Practice. Paper prepared for presentation at The 2008 World Congress on National Accounts for Nations in Arlington, VA, May 12-17, 2008, May 2008.

5.2.4 GDP and the stock of value

If Society (1) in Figure 5.3 represents the economy of an entire country, $\dot{X}_1$ is its gross domestic product (GDP) in units of \$/year. The stock of value, X_1, is the total value of everything that is accumulated in society.

5.3 Example B: two-sector economy

Figure 5.4 shows flows of value ($\dot{X}$) within a two-sector economy. Again, we note the isloation of value from the biosphere.

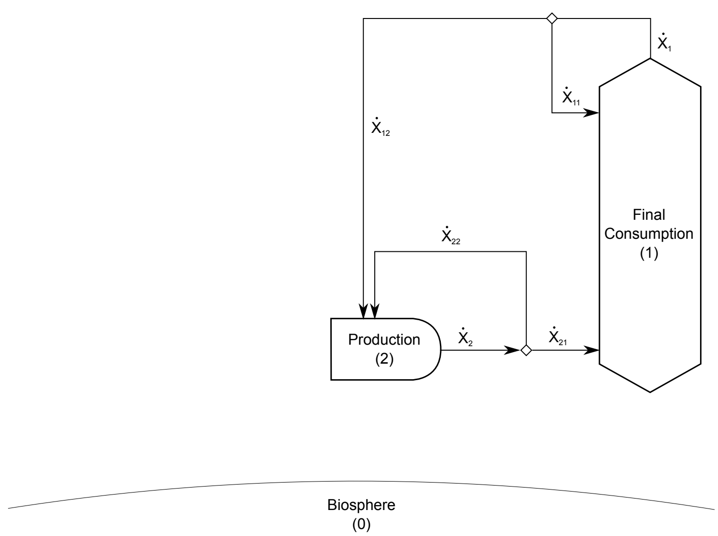

Fig. 5.4 Flows of value ($\dot{X}$) within a two-sector economy.

We can account for value flows by writing the following equations:

$$\frac{\mathrm{d}X_1}{\mathrm{d}t} = \dot{X}_{11} + \dot{X}_{21} - \dot{X}_1 + \dot{X}_{gen,1} - \dot{X}_{dest,1} \tag{5.3}$$

and

$$\frac{\mathrm{d}X_2}{\mathrm{d}t} = \dot{X}_{12} + \dot{X}_{22} - \dot{X}_2 + \dot{X}_{gen,2} - \dot{X}_{dest,2}. \tag{5.4}$$

Equations 5.3 and 5.4 can be generalized as

$$\frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{X}_{ij} - \dot{X}_j + \dot{X}_{gen,j} - \dot{X}_{dest,j}, \tag{5.5}$$

where n is the number of sectors in the economy, and $j \in [1, n]$.

5.4 Example C: three-sector economy

Figure 5.5 shows flows of value ($\dot{X}$) within a three-sector economy.

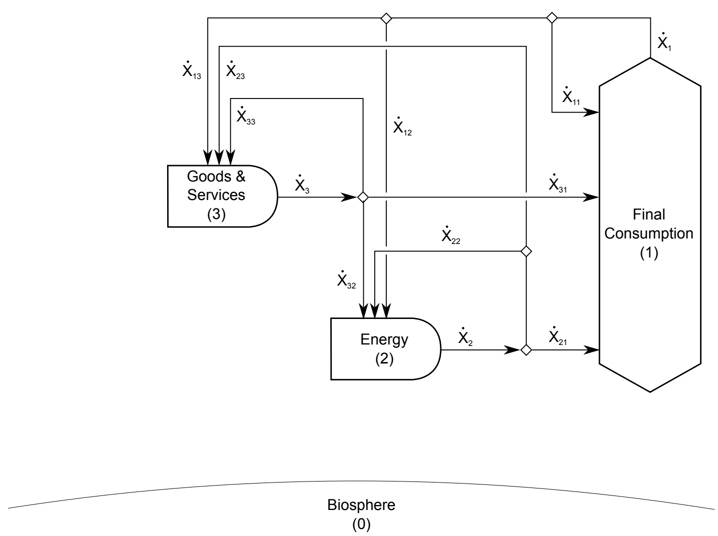

Fig. 5.5 Flows of value ($\dot{X}$) within a three-sector economy.

The equations representing flows of value in Example C are:

$$\frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{X}_{ij} - \dot{X}_j + \dot{X}_{gen,j} - \dot{X}_{dest,j}, \tag{5.6}$$

where n is the number of sectors in the economy, and $j \in [1, n]$. Equation 5.6 is identical to Equation 5.5. If we sum the value accounting equations for the entire economy, we obtain

$$\sum_{j=1}^{n} \frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{j=1}^{n}\sum_{i=1}^{n} \dot{X}_{ij} - \sum_{j=1}^{n} \dot{X}_j + \sum_{j=1}^{n} \dot{X}_{gen,j} - \sum_{j=1}^{n} \dot{X}_{dest,j}. \tag{5.7}$$

With the identities

$$\dot{X}_j = \sum_{k=1}^{n} \dot{X}_{jk} \tag{5.8}$$

and

$$\sum_{j=1}^{n} \dot{X}_j = \sum_{j=1}^{n}\sum_{k=1}^{n} \dot{X}_{jk} = \sum_{i=1}^{n}\sum_{k=1}^{n} \dot{X}_{ik} = \sum_{i=1}^{n}\sum_{j=1}^{n} \dot{X}_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{n} \dot{X}_{ij}, \tag{5.9}$$

Equation 5.7 becomes

$$\sum_{j=1}^{n} \frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_{j=1}^{n} \dot{X}_{gen,j} - \sum_{j=1}^{n} \dot{X}_{dest,j}, \tag{5.10}$$

for $j \in [1,n]$, indicating that value generation ($\dot{X}_{gen,j}$) and destruction ($\dot{X}_{dest,j}$) are the only mechanisms by which value is accumulated or lost $\left(\frac{\mathrm{d}X_j}{\mathrm{d}t}\right)$ within the economy.

5.5 Value in the auto industry

To estimate value flows through the automobile industry, we use data publicly available for download from the US Bureau of Economic Analysis (BEA) (`http://www.bea.gov`).[2]. The two main tables needed to estimate dynamic value flows and capital accumulation within the economy are the Input-Output Tables (I-O) and the Fixed Asset, non-residential detail, table. The I-O tables consist of " make" and use "tables". The use table tracks which *industries* (columns) use which *commodities* (rows) that are made in one industry and then used in another as intermediate inputs ($\dot{R}$, $\dot{S}$). The Use table also tracks the output that is used for final consumption, including Private Fixed Investment (capital in-flows). However, the more detailed Fixed Asset accounts are used to estimate all capital value flows in this example ($\dot{K}$) so that capital from other sectors can be separated from the capital that is produced and used within the automobile industry.

Using these data, Figure 5.6 provides estimates of value flows for the US auto industry.

Table 5.1 contains a brief description of the data sources that were used to obtain the values in Figure 5.6. Appendix A contains the detailed calculations and describes the BEA data sources.

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[2] A "primer" on using the US BEA industry data can be found on the BEA website here `http://bea.gov/industry/pdf/industry_primer.pdf`

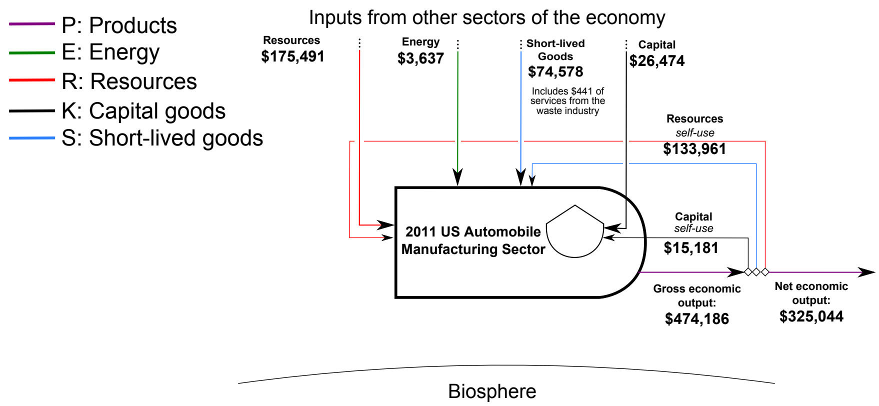

Fig. 5.6 Value of material and energy flows into and out of the US automobile industry (in millions of 2011USD).

Table 5.1 Data sources for auto industry (IOC 3361MV) example.

Value Flow	2011 USD (millions)	BEA Data Source
Resources	$175,491	2011 KLEMS Total Material Inputs
Energy	3,367	2011 KLEMS Total Energy Inputs
Short-lived Goods	42,446	2011 KLEMS Total Services Inputs
Capital	46,079	2011 Fixed Assets 2011 (non-residential detailed estimates)
Gross Economic Output	467,941	2011 Input-Output Use Tables
Resources (self-use)	133,961	2011 KLEMS Total Material Inputs
Capital (self-use)	15,181	2011 Fixed Assets (non-residential detailed estimates)
Net Economic Output	325,044	2011 Input-Output Use Tables

5.6 Summary

In this chapter, we developed techniques to account for flows of economic value ($\dot{X}$) through economies (Section 5.1). We began with a discussion about theories of value and settled on the prevailing subjective theory of value for our framework. Thereafter, value accounting equations were developed and applied to example economies A–C in Sections 5.2–5.4. We noted the need for terms that describe creation and destruction of value ($\dot{X}_{gen}$ and $\dot{X}_{dest}$, respectively) within economic sectors. Finally, we explored value flows to and from the US auto economy (Section 5.5). In contrast to direct and emboded energy, we found that there is no lack of data on value flows to and from the US auto industry.

In Chapter 6, we combine results from Chapter 3 and this chapter to develop techniques the estimate the energy intensity of economic products.

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[2] United Nations, Department of Economic and Social Affairs, Statistics Division. System of Environmental-Economic Accounting (SEEA). `http://unstats.un.org/unsd/envaccounting/seea.asp`.

Chapter 6
Energy intensity

*We have the most crude accounting tools. It's tragic because our accounts and our national arithmetic doesn't [don't?] tell us the things that we need to know. **** Citation? *****

—Susan George

In Chapters 3, 4, and 5, we defined flows of direct energy, embodied energy, and value in an economy. In this chapter, we merge energy and value together to estimate the energy intensity (ε) of economic sectors.[1]

6.1 Methodology

Energy intensity (ε) is the ratio of total energy ($\dot{T}$) and value ($\dot{X}$) outflow rates from an economic sector, such that for the j^{th} goods and services sector,

$$\varepsilon_j \equiv \frac{\dot{T}_j}{\dot{X}_j}, \tag{6.1}$$

and ε is in units of J/\$.[2] Equation 6.1 includes the embodied energy of products in the numerator ($\dot{T}_j$) term. A narrower definition ef energy intensity would be $\varepsilon_j \equiv \frac{\dot{Q}_{j0}}{\dot{X}_j}$, which includes only direct energy consumed by the sector in the numerator and excludes the energy demanded upstream by the resource flows ($\dot{R}$) that comprise the product of the sector ($\dot{P}$). We choose the broader definition of Equation 6.1, because it accounts for upstream energy consumption, thereby providing an estimate of the true energy cost of products. For inter-sector flows, we have

$$\varepsilon_{jk} = \frac{\dot{T}_{jk}}{\dot{X}_{jk}}. \tag{6.2}$$

[1] The literature discusses the energy embodied in *products*, e.g. "The data and methodologies described in this report permit calculation of five types of energy 'embodied' in a particular goods [*sic*] or service," from Bullard [1, p. 268]. It can be meaningful to discuss the energy intensity of *processes*, too, and we switch between these two meanings of the word "embodied."

[2] It may be instructive to consider energy intensity as the quotient of embodied energy (in units of J/kg) and price (in \$/kg).

Furthermore, we note that

$$\varepsilon_j = \varepsilon_{jk} \tag{6.3}$$

for all k, because the energy intensity of sector j's output is independent of its destination (k). All goods produced by a sector are produced at the average energy intensity of that sector.[3]

We define the input-ouput ratio (a_{ij}) that represents the input of good i required to produce a unit of output from sector j.

$$a_{ij} \equiv \frac{\dot{X}_{ij}}{\dot{X}_j} \tag{6.4}$$

We note that the value ($\dot{X}$) of all material flows must be counted such that

$$a_{ij} = \frac{\dot{X}_{\dot{R}_{ij}} + \dot{X}_{\dot{S}_{ij}} + \dot{X}_{\dot{K}_{ij}}}{\dot{X}_{\dot{R}_j} + \dot{X}_{\dot{S}_j} + \dot{X}_{\dot{K}_j}}. \tag{6.5}$$

Input-output ratios (a_{ij}) are given in mixed units, depending on both the purpose of each sector of the economy and the type of input as shown in Figure 6.1.

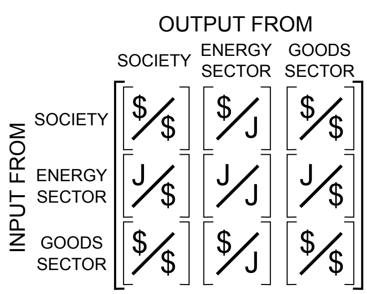

Fig. 6.1 Units for input-output ratios (a).

Equations 6.2, 6.3, and 6.4 can be combined to give

$$\dot{T}_{jk} = \varepsilon_j a_{jk} \dot{X}_k. \tag{6.6}$$

6.2 Example A: single-sector economy

With reference to Figures 3.3, 4.2, and 5.3, the energy intensity (ε_1) of a single-sector economy is calculated by

[3] If this approach is unsatisfactory, the sector may be divided into sub-sectors with different energy intensities.

$$\varepsilon_1 = \frac{\dot{T}_1}{\dot{X}_1} = \frac{\dot{T}_{11}}{\dot{X}_{11}}. \tag{6.7}$$

Appendix B illustrates that the energy intensity of a single-sector economy (ε_1) is comprised of the sum of the infinite recursions of energy consumed during production of output ($\dot{X}_1$).

To estimate energy intensities when more than one economic sector is involved, we move to Examples B and C in the following sections.

6.3 Example B: two-sector economy

With reference to Figures 3.4, 4.3, and 5.4, the energy intensity (ε_2) of the production sector is given by

$$\varepsilon_2 = \frac{\dot{T}_2}{\dot{X}_2} = \frac{\dot{T}_{22}}{\dot{X}_{22}}. \tag{6.8}$$

Thus,

$$\dot{T}_2 = \varepsilon_2 \dot{X}_2, \tag{6.9}$$

The input-output ratio for the production sector's self-use of output (a_{22}) is

$$a_{22} = \frac{\dot{X}_{22}}{\dot{X}_2}, \tag{6.10}$$

thus

$$\dot{T}_{22} = \varepsilon_2 a_{22} \dot{X}_2. \tag{6.11}$$

We can rewrite the total energy accounting equation for the two-sector economy

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \dot{T}_{02} + \dot{T}_{12} + \dot{T}_{22} - \dot{T}_2 - \dot{T}_{20} \tag{4.30}$$

using energy intensity by realizing that

- $\frac{\mathrm{d}E_2}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}T_2}{\mathrm{d}t} = \frac{\mathrm{d}B_2}{\mathrm{d}t}$, because direct energy does not accumulate within economic sectors,
- $\frac{\mathrm{d}B_2}{\mathrm{d}t} = \frac{\mathrm{d}B_{K_2}}{\mathrm{d}t}$, because resources ($R$) and short-lived materials (S) do not accumulate at appreciable rates in economic sectors,
- $\dot{B}_{02} = 0$ and $\dot{T}_{02} = \dot{E}_{02}$, because embodied energy appears only in the *output* of a sector,
- $\dot{E}_{20} = 0$ and $\dot{T}_{20} = \dot{B}_{20}$, because direct energy is not wasted to the biosphere at any significant rate, and
- $\dot{B}_{20} = \left(\dot{B}_{\dot{R}_{20}} + \dot{B}_{\dot{S}_{20}} + \dot{B}_{\dot{K}_{20}}\right) = \left(\dot{B}_{\dot{R}_{20}} + \dot{B}_{\dot{S}_{20}} + \gamma_{B,2} B_{K_2}\right)$, as shown in Section 4.4.

If we substitute Equations 6.9 and 6.11 into Equation 4.30, we obtain

$$\frac{\mathrm{d}B_{K_2}}{\mathrm{d}t} = \dot{E}_{02} + \dot{T}_{12} + \varepsilon_2 a_{22}\dot{X}_2 - \varepsilon_2\dot{X}_2 - \left(\dot{B}_{\dot{R}20} + \dot{B}_{\dot{S}20} + \gamma_{B,2}B_{K_2}\right). \tag{6.12}$$

Equation 6.12 can be solved for energy intensity (ε_2) to obtain

$$\varepsilon_2 = (1-a_{22})^{-1}\dot{X}_2^{-1}\left[\dot{E}_{02} + \dot{T}_{12} - \frac{\mathrm{d}B_{K_2}}{\mathrm{d}t} - \dot{B}_{\dot{R}20} - \dot{B}_{\dot{S}20} - \gamma_{B,2}B_{B_2}\right] \tag{6.13}$$

To extend Equation 6.13 to a matrix formulation, we turn to Example C.

6.4 Example C: three-sector economy

The three-sector economy of Example C affords the opportunity to develop a matrix version of the total energy accounting equation (4.42) and to develop an equation that estimates the energy intensity of economic sectors. We begin with a matrix version of the total energy accounting equation.

6.4.1 Total energy accounting equation

We apply Equation 4.42 to the three-sector economy shown in Figures 3.5, 4.4, and 5.5 to obtain the following total energy accounting equations for the Energy (2) and Goods and Services (3) sectors of the three-sector economy:

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \dot{T}_{02} + \dot{T}_{12} + \dot{T}_{22} + \dot{T}_{32} - \dot{T}_2 - \dot{T}_{20} \tag{6.14}$$

and

$$\frac{\mathrm{d}T_3}{\mathrm{d}t} = \dot{T}_{03} + \dot{T}_{13} + \dot{T}_{23} + \dot{T}_{33} - \dot{T}_3 - \dot{T}_{30}. \tag{6.15}$$

Similar to Example B, we realize that

- $\frac{\mathrm{d}E_i}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}T_i}{\mathrm{d}t} = \frac{\mathrm{d}B_i}{\mathrm{d}t}$, because direct energy does not accumulate within economic sectors,
- $\frac{\mathrm{d}B_i}{\mathrm{d}t} = \frac{\mathrm{d}B_{K_i}}{\mathrm{d}t}$, because resources ($R$) and short-lived materials (S) do not accumulate at appreciable rates in economic sectors,
- $\dot{B}_{0j} = 0$ and $\dot{T}_{0j} = \dot{E}_{0j}$, because embodied energy appears only in the *output* of a sector,
- $\dot{E}_{j0} = 0$ and $\dot{T}_{j0} = \dot{B}_{j0}$, because direct energy is not wasted to the biosphere at any significant rate, and

- $\dot{B}_{j0} = \left(\dot{B}_{\dot{R}_{j0}} + \dot{B}_{\dot{S}_{j0}} + \dot{B}_{\dot{K}_{j0}}\right) = \left(\dot{B}_{\dot{R}_{j0}} + \dot{B}_{\dot{S}_{j0}} + \gamma_{B,j} B_{K_j}\right)$, as shown in Section 4.4.

to obtain

$$\frac{\mathrm{d}B_{K_2}}{\mathrm{d}t} = \dot{E}_{02} + \dot{T}_{12} + \varepsilon_2 \dot{X}_{22} + \varepsilon_3 \dot{X}_{32} - \varepsilon_2 \dot{X}_2 - \left(\dot{B}_{\dot{R}_{20}} + \dot{B}_{\dot{S}_{20}} + \gamma_{B,2} B_{K_2}\right) \tag{6.16}$$

and

$$\frac{\mathrm{d}B_{K_3}}{\mathrm{d}t} = \dot{E}_{03} + \dot{T}_{13} + \varepsilon_2 \dot{X}_{23} + \varepsilon_3 \dot{X}_{33} - \varepsilon_3 \dot{X}_3 - \left(\dot{B}_{\dot{R}_{30}} + \dot{B}_{\dot{S}_{30}} + \gamma_{B,3} B_{K_3}\right). \tag{6.17}$$

6.4.2 Matrix formulation

Equations 6.16 and 6.17 can be rewritten in vector notation as

$$\begin{aligned} \begin{Bmatrix} \frac{\mathrm{d}B_{K_2}}{\mathrm{d}t} \\ \frac{\mathrm{d}B_{K_3}}{\mathrm{d}t} \end{Bmatrix} = \begin{Bmatrix} \dot{E}_{02} \\ \dot{E}_{03} \end{Bmatrix} + \begin{Bmatrix} \dot{T}_{12} \\ \dot{T}_{13} \end{Bmatrix} + \begin{bmatrix} \dot{X}_{22} & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} - \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix} \begin{Bmatrix} \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} \\ - \begin{Bmatrix} \dot{B}_{\dot{R}_{20}} \\ \dot{B}_{\dot{R}_{30}} \end{Bmatrix} - \begin{Bmatrix} \dot{B}_{\dot{S}_{20}} \\ \dot{B}_{\dot{S}_{30}} \end{Bmatrix} - \begin{bmatrix} \gamma_{B,2} & 0 \\ 0 & \gamma_{B,3} \end{bmatrix} \begin{Bmatrix} B_{K_2} \\ B_{K_3} \end{Bmatrix}. \end{aligned} \tag{6.18}$$

If we define the following matrices and vectors:

$$\mathbf{B}_K \equiv \begin{Bmatrix} B_{K_2} \\ B_{K_3} \end{Bmatrix}, \tag{6.19}$$

$$\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} \equiv \begin{Bmatrix} \frac{\mathrm{d}B_{K_2}}{\mathrm{d}t} \\ \frac{\mathrm{d}B_{K_3}}{\mathrm{d}t} \end{Bmatrix}, \tag{6.20}$$

$$\mathbf{E}_0 \equiv \begin{Bmatrix} \dot{E}_{02} \\ \dot{E}_{03} \end{Bmatrix}, \tag{6.21}$$

$$\mathbf{T}_1 \equiv \begin{Bmatrix} \dot{T}_{12} \\ \dot{T}_{13} \end{Bmatrix}, \tag{6.22}$$

$$\mathbf{X}_t \equiv \begin{bmatrix} \dot{X}_{22} & \dot{X}_{23} \\ \dot{X}_{32} & \dot{X}_{33} \end{bmatrix}, \tag{6.23}$$

$$\boldsymbol{\varepsilon} \equiv \begin{Bmatrix} \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}, \tag{6.24}$$

$$\mathbf{B}_{waste} = \begin{Bmatrix} \dot{B}_{waste,20} \\ \dot{B}_{waste,30} \end{Bmatrix} \equiv \begin{Bmatrix} \dot{B}_{\dot{R}20} \\ \dot{B}_{\dot{R}30} \end{Bmatrix} + \begin{Bmatrix} \dot{B}_{\dot{S}\,20} \\ \dot{B}_{\dot{B}30} \end{Bmatrix}, \tag{6.25}$$

$$\hat{\mathbf{X}} \equiv \delta_{ij}\dot{X}_j = \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix}, \tag{6.26}$$

and

$$\hat{\boldsymbol{\gamma}}_B \equiv \delta_{ij}\gamma_{B,j} = \begin{bmatrix} \gamma_{B,2} & 0 \\ 0 & \gamma_{B,3} \end{bmatrix}; \tag{6.27}$$

with the “Kronecker delta”

$$\delta_{ij} \equiv \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}; \tag{6.28}$$

we can rewrite Equation 6.18 compactly as

$$\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} = \mathbf{E}_0 + \mathbf{T}_1 + \mathbf{X}_t^{\mathrm{T}}\boldsymbol{\varepsilon} - \hat{\mathbf{X}}\boldsymbol{\varepsilon} - \mathbf{B}_{waste} - \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K. \tag{6.29}$$

Equation 6.29 can be simplified to

$$\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} = \mathbf{E}_0 + \mathbf{T}_1 + (\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}})\boldsymbol{\varepsilon} - \mathbf{B}_{waste} - \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K. \tag{6.30}$$

We can define the input-output matrix ($\mathbf{A}$) as

$$\mathbf{A} \equiv \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}. \tag{6.31}$$

Appendix C shows that

$$\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}} = \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I}), \tag{6.32}$$

which allows Equation 6.30 to be recast as

$$\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} = \mathbf{E}_0 + \mathbf{T}_1 + \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I})\boldsymbol{\varepsilon} - \mathbf{B}_{waste} - \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K. \tag{6.33}$$

Equation 6.33 is the matrix version of the total energy accounting equation written in terms of embodied energy ($\mathbf{B}$), energy intensities ($\boldsymbol{\varepsilon}$), and input-output ratios ($\mathbf{A}$). Equation 6.18 applies for the three-sector economy of Example C, but the equivalent matrix formulation (Equation 6.33) can be extended to any desired level of economic and energy sector disaggregation by expanding the vectors and matrices in Equations 6.20–6.19 and 6.31 to include all sectors of the economy.[1, 2]

Equation 6.33 provides a means to estimate the embodied energy accumulation rate in economic sectors $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$ knowing only direct energy inputs to the economy from the biosphere ($\mathbf{E}_0$), total energy inputs from society to the economy ($\mathbf{T}_1$), sector outputs ($\hat{\mathbf{X}}$), sector input-output ratios ($\mathbf{A}$), sector energy intensities ($\boldsymbol{\varepsilon}$), energy embodied in wastes from the economy ($\mathbf{B}_{waste}$), and physical depreciation rates of capital stock ($\hat{\boldsymbol{\gamma}}_B\mathbf{B}_K$). In theory, the transaction matrix ($\mathbf{X}_t$) is not required if the input-ouput matrix ($\mathbf{A}$) is known, though in practice, knowledge of input-output matrix ($\mathbf{A}$) would be derived from the transaction matrix ($\mathbf{X}_t$), as shown in Appendix D.

Equation 6.33 can be rearranged to obtain

$$\hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}}-\mathbf{I})\boldsymbol{\epsilon} = \frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} + \mathbf{B}_{waste} + \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K - \mathbf{E}_0 - \mathbf{T}_1 \tag{6.34}$$

and

$$\boldsymbol{\epsilon} = \left[\hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}}-\mathbf{I})\right]^{-1}\left[\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} + \mathbf{B}_{waste} + \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K - \mathbf{E}_0 - \mathbf{T}_1\right]. \tag{6.35}$$

We apply the matrix identity from Formula 6.2, p. 308 in Beyer [3]

$$(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1} \tag{6.36}$$

to the right side of Equation 6.35 to obtain

$$\boldsymbol{\epsilon} = (\mathbf{A}^{\mathrm{T}}-\mathbf{I})^{-1}\hat{\mathbf{X}}^{-1}\left[\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} + \mathbf{B}_{waste} + \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K - \mathbf{E}_0 - \mathbf{T}_1\right]. \tag{6.37}$$

Finally, we can multiply both parenthetical terms[4] on the right side of Equation 6.37 by −1 to obtain

$$\boldsymbol{\varepsilon} = (\mathbf{I}-\mathbf{A}^{\mathrm{T}})^{-1}\hat{\mathbf{X}}^{-1}\left[\mathbf{E}_0 + \mathbf{T}_1 - \frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} - \mathbf{B}_{waste} - \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K\right]. \tag{6.38}$$

Equation 6.38 provides a means to estimate energy intensity ($\boldsymbol{\varepsilon}$) of the sectors of the economy, under the assumption that final consumption (sector 1) is exogenous to the economy (sectors 2–n). We discuss Equation 6.38 further in Section 7.1. But first, the following section examines energy intensity in the context of our running example, the US auto industry.

[4] The parenthetical terms on the right side of Equation 6.37 are $(\mathbf{A}^{\mathrm{T}} - \mathbf{I})$ and $\left[\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} + \mathbf{B}_{waste} + \hat{\boldsymbol{\gamma}}_B\mathbf{B}_K - \mathbf{E}_0 - \mathbf{T}_1\right]$.

6.5 Energy intensity of the US auto industry

Equation 6.38 shows that it is possible to estimate the energy intensity of products of economic sectors using the Input-Output method.[5] Several studies have used similar energy-based, input-output methods to estimate the energetic cost of goods and services produced by various economic sectors.[4–15] We review a few of these studies below.

Using national accounts data for 1967, Bullard and Herendeen calculated the total energy consumption rate ($\dot{T}$) of the US automobile industry as $13{,}240 \times 10^{15}$ joules/year = 13.24 EJ, which was around 20% of the nation's energy consumption in that year.[4] Around half of this energy was directly consumed within the auto industry itself ($\dot{Q}_{j0}$), meaning the rest was upstream consumption in material processing, entering the auto industry as embodied energy ($\sum_i \dot{B}_{ij}$). Given the number of autos produced per year, Bullard and Herendeen calculated that the embodied energy per vehicle was 148 GJ (10^9 J), 11% higher than the estimate obtained via process analysis in a study by Berry and Fels [16] two years earlier.[6]

In 1980, Costanza [5] estimated the energy intensity of all economic sectors of the US economy using the Input-Output method. Unfortunately, the energy intensity of the Motor Vehicles and Equipment sector (63) was not reported in [5]. Later, Costanza and Herendeen [6] re-estimated energy intensity and reported the energy intensity of outputs from all 87 Bureau of Economic Affairs (BEA) sectors. The energy intensity of the Motor Vehicles and Equipment sector (63) and selected other sectors are given in Tables 6.1 and 6.2.[7]

Table 6.1 Motor Vehicles and Equipment sector (63) energy intensity values.[6]

Year	Energy Intensity [kJ/$]
1963	1.16×10^5
1967	1.04×10^5
1972	0.95×10^5

However, the Costanza results [6] (and nearly all I-O results in the literature) assume the following: negligible energy input from society (which is probably a reasonable assumption for the US), steady-state economic conditions with negligible accumulation of embodied energy in economic sectors (which probably didn't exist in the US in the late 1960s), and negligible energy embodied in wastes (which is untrue).

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[5] For a discussion of differences between Equation 6.38 and similar equations in the literature, see Appendix E.

[6] See Section 4.5 for discussion of the Berry and Fels [16] paper.

[7] Values from Costanza and Herendeen's DIRECT method are provided here. See Section 8.10 for discussion of the differences between DIRECT and DEC methods and justification for reporting DIRECT method values only.

Table 6.2 Selected US economic sector energy intensities, 1972.[6]

Sector	Energy Intensity [kJ/$]
Coal Mining (1)	3.23×10^6
Air Transport (73)	1.76×10^5
New Construction (14)	1.03×10^5
Motor Vehicles and Equipment (63)	9.50×10^4
Auto Repair (82)	8.35×10^4

The Economic Input-Output Life Cycle Assessment (EIOLCA) online tool [7] is based on the framework outlined by Hendrickson and Lave [8] and allows computation of the energy flows through the economy based on US national accounts data from 1992, 1997 and 2002.[8] Using the tool with the 2002 producer price model, we find that $1M of output from the automobile manufacturing industry (NAICS sector 336111) generates a total flow of 8.33 TJ (10^{12} J) of energy through the economy, 2.19 TJ from the power generation and supply sector (221100) and 1.25 TJ from the iron and steel mills sector (331110).

Table 6.3 Automobile manufacturing sector (NAICS 33611x) energy intensity values.[7]

Year	Energy Intensity [kJ/$]
1992[a]	1.26×10^4
1997[b]	0.76×10^4
2002[c]	0.83×10^4

[a] Motor vehicles and Passenger Car Bodies (590301)
[b] Automobile and Light Truck Manufacturing (336110)
[c] Automobile Manufacturing (336111)

Table 6.4 Selected US economic sector energy intensities, 1997.[7]

Sector	Energy Intensity [kJ/$]
Coal Mining (212100)	1.11×10^4
Air Transportation (481000)	2.62×10^4
Manufacturing and Industrial Buildings (230210)	0.76×10^4
Automobile and Light Truck Manufacturing (336110)	0.76×10^4
Automotive Repair and Maintenance, except car washes (8111A0)	0.52×10^4

As discussed in Section 4.1.1, all of these methods differ from the framework presented in this Chapter due to the assumption that embodied energy does not ac-

[8] The US national accounts data has not been updated since 2002. The issue of national accounts data is discussed in more detail in Section 8.3.

cumulate within economic sectors. This issue is discussed by Bullard and Herendeen [4, p.273], where they state of capital goods that,

> these are not considered part of the inter-industry transactions but are listed as sales to final demand.

They later offer calculation of industry energy intensity values (which will be discussed in greater depth in Chapter 6) including and excluding these capital goods flows [4, p.489], finding the inclusion of these flows adds on average 3–6% to the sector energy intensity.

It would be interesting to know how the above energy intensity results (a) change if the assumptions above are relaxed,[9] (b) vary with time, and (c) vary across economies at different stages of industrialization. However, we know of no longitudinal estimates of the energy intensity of automobiles using the Input-Output method. In fact, the current account records, upon which the estimates of energy intensity values above are based, are no longer maintained by the US government. So, we could not update these results, even if we wanted to. Furthermore, few countries maintain and publish records with enough detail to perform these analyses. In Section 8.3, we discuss further the need for additional data.

6.6 Summary

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[9] See Section 7.1 for a discussion of the significance of these assumptions.

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Part III
Implications, Issues, and Summary

Chapter 7
Implications

*The economy is a wholly owned subsidiary of the environment, not the reverse. **** Need citation. *****

—Herman E. Daly

Several implications can be drawn from the detailed development of our framework for materials and energy accounting (in Chapters 2–6). In the sections below, we discuss implications for the I-O method itself, implications for economic "development," implications for recycling, reuse, and dematerialization, and comparisons with the idea of a steady-state economy. We begin by examining the I-O method itself through the lens of our framework.

7.1 Implications for the I-O method

Extension of the Leontief Input-Output method for energy analysis has allowed energy analysts to estimate the energy intensity of economic products ($\boldsymbol{\varepsilon}$). As discussed in Section 5.1, we do not take the ability to estimate energy intensity as a license to declare an intrinsic "energy theory of value." Rather, we belive that energy intensity ($\boldsymbol{\varepsilon}$) is an important and useful metric that can assess the energy performance of economies, even within the prevailing subjective theory of value that underlies modern economics. It is important to consider the assumptions behind the literature's presentation of the energy I-O method for estimating the energy intensity of economic output before drawing implications from our framework for the energy I-O method.

As we investigate, we will use the following coordinates of analysis: product-based vs. physical accounting frameworks, whether capital stock is included in the accounting framework, and whether energy input from society to the economy is included. (See Figure 7.1.) We will end with our suggestion for how best to estimate $\boldsymbol{\varepsilon}$ within a materials, energy, and value accounting system.

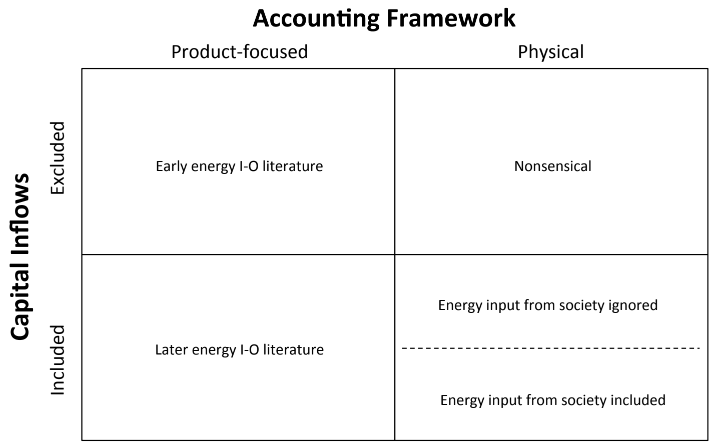

Fig. 7.1 Coordinates of analysis for implications for the I-O method.

7.1.1 Product-based vs. physical approaches

The distinction between product-focused and physical accounting frameworks is located in the columns of Figure 7.1. A physical accounting framework strictly follows materials through the economy. Embodied energy is accounted with the material stock or material flow in which it resides. For example, energy embodied within wastes ($\mathbf{B}_{waste}$) is not assigned to economic products. Rather, the energy embodied in wastes flows from sectors to the biosphere *with the waste material*. In contrast, a product-focused accounting framework assigns energy embodied in wastes to the products of the sector. Both product-based and physical accounting frameworks assign direct energy ($\dot{E}$) consumed by each sector to the products of each sector.

Equation 7.1 describes the outflow of embodied energy from sector j, exclusive of capital stock, for a physical accounting system that neglects capital stock (upper-right quadrant of Figure 7.1).[1]

$$\dot{B}'_j = \sum_{i=1}^{n} \dot{B}'_{ij} - \dot{B}_{waste,j} + \dot{Q}_{j0} \tag{7.1}$$

Variables written with a "prime" (e.g. $\dot{B}'_j$) indicate definitions and terms that include only resource ($\dot{R}$) and short-lived ($\dot{S}$) flows and exclude input capital flows ($\dot{K}$) and stock (K). The term $\dot{B}_{waste,j}$ represents the energy embodied within wasted resource ($\dot{R}_{j0}$) and short-lived ($\dot{S}_{j0}$) material flows. The $\dot{B}_{waste,j}$ term is subtracted, because waste material flows *out of* the sector. In a physical accounting framework, the energy embodied in waste flows ($\dot{B}_{waste,j}$) is not assigned to the product ($\dot{B}'_j$).

[1] Equation 7.1 is used for illustrative purposes only. A physical accounting framework would necessarily include both flows and stocks of capital. Thus, the upper-right quadrant of Figure 7.1 (physical accounting framework that neglects capital) is labeled as nonsensical.

In contrast, Equation 7.2 describes the outflow of embodied energy from sector j, exclusive of capital stock, for a product-focused accounting framework (upper left quadrant of Figure 7.1).

$$\dot{B}'_j = \sum_{i=1}^{n} \dot{B}'_{ij} + \dot{Q}_{j0} \tag{7.2}$$

Notice that Equation 7.2 does not subtract the energy embodied in waste resource and short-lived material flows ($\dot{B}_{waste,j}$) on the right side of the equation, because product-focused accounting systems assign energy embodied in wastes to products.

7.1.2 Capital stock

The rows of Figure 7.1 represent the role of capital stock in an accounting framework. The BEA Industry Accounts include capital flows in the "make" tables for each industry [1, Table 1], but captial inflows are accounted separately from intermediate uses as "Private fixed invest ment".[1, Table 2] During the earliest years of the energy I-O method (prior to the mid-1970s) both capital inflows to economic sectors and the stock of capital were ignored. In essence, the state of the art was located in the upper-left quadrant of Figure 7.1. In time, Kirkpatrick [2], Bullard and Herendeen [3], and Casler [4] attempted to include inflows of capital stock in a product-focused accounting framework, thereby moving the state of the art to the lower-left quadrant of Figure 7.1.

We agree with this move, because of the many ways in which capital stock is important for economies. We can use the ecosystem work of Eugene Odum explain the importance of capital stock. And, we rely upon Herman Daly to connect ecosystems to economies.

In 1969, Odum outlined a number of defining characteristics of both *developmental* (growing) and *mature* ecosystems in terms of key properties of the system.[5] Ecosystems cannot grow indefinitely in their (photosynthetic) production (P) due mainly to the necessity of increasing maintenance demands as the amount of biomass (capital stock, B) increases. Eventually, all production is used in this manner and growth ceases $\left(\frac{dP}{dt} = 0\right)$.

In the early stages of ecosystem development, the the energy production per unit of biomass stock ($\frac{P}{B}$) is high. As the ecosystem approaches maturity, this ratio decreases. Put another way, the biomass stock (maintained) per unit of energy produced (the inverse ratio $\frac{B}{P}$) starts low and asymptotically increases to a maximum when growth (in both P and B) has ceased. The value of $\frac{B}{P}$ at the asymptote may be high or low[2] and may therefore be considered a measure of the "efficiency" to which

[2] The value of $\frac{B}{P}$ at maturity (and the time taken to reach it) "may vary not only with different climatic and physiographic situations but also with different ecosystem attributes in the same physical environment." [5, p.263]

the ecosystem applies energy production toward the goal of maintaining biomass stock.

Turning back to economies, Daly has, in our view, correctly applied this concept to societal patterns of economic consumption.[6] Our framework analogously suggests that as capital stock ($\mathbf{B}_K$) increases, an increasing flow of energy supply ($\mathbf{E}_0$) will be needed to maintain that stock.[3] Thus, it is important to account for capital stock in a material, energy, and value accounting framework.

To see the effect of the move from the upper-left to the lower-left quadrant of Figure 7.1, it is important to understand clearly both the assumptions and data that were used. Energy analysts in the mid-1970s were utilizing the BEA I-O tables, which do not include capital input flows. Thus, this early literature implicitly assumes that

$$a'_{ij} \equiv \frac{\dot{X}_{\dot{R}_{ij}} + \dot{X}_{\dot{S}_{ij}}}{\dot{X}_{\dot{R}_j} + \dot{X}_{\dot{S}_j} + \dot{X}_{\dot{K}_j}} = \frac{\dot{X}'_{ij}}{\dot{X}_j}. \tag{7.3}$$

Comparison between Equations 6.5 and 7.3 highlights the fact that the early literature neglects flows of capital stock ($\dot{X}_{\dot{K}_{ij}}$) on the input. Thus, the the input-output matrix in the early literature ($\mathbf{A}'$) is

$$\mathbf{A}' = \begin{bmatrix} a'_{22} & a'_{23} \\ a'_{32} & a'_{33} \end{bmatrix}. \tag{7.4}$$

The implicit assumptions of the early energy I-O literature are consistent with the upper-left quadrant of Figure 7.1, and the energy intensity equation found in most of the early literature is

$$\boldsymbol{\varepsilon}' = \left(\mathbf{I} - \mathbf{A}'^{\mathrm{T}}\right)^{-1} \left(\hat{\mathbf{X}}\right)^{-1} \mathbf{E}_0. \tag{7.5}$$

Bullard and Herendeen [3], following Kirkpatrick [2], added flows of capital as inputs to each sector [3, Figure 5], and, in so doing, changed Equation 7.5 to Equation 7.6:

$$\boldsymbol{\varepsilon} = \left[\mathbf{I} - \left(\mathbf{A}'^{\mathrm{T}} + \mathbf{A}_{\dot{K}}^{\mathrm{T}}\right)\right]^{-1} \left(\hat{\mathbf{X}}\right)^{-1} \mathbf{E}_0 \tag{7.6}$$

with

$$\mathbf{A}_{\dot{K}} \equiv \begin{bmatrix} a_{\dot{K}_{22}} & a_{\dot{K}_{23}} \\ a_{\dot{K}_{32}} & a_{\dot{K}_{33}} \end{bmatrix} \tag{7.7}$$

and

[3] Today's economies (and economic models and economic assumptions) are still focused on the objective of growth. If energy supply rates ($\mathbf{E}_0$) are constrained, these dynamics provide a possible reason for the difficulty of maintaining high levels of economic growth in mature economies. Eventually, we believe, we must learn to maximize the $\frac{B}{P}$ ratios of our economies $\left(\frac{\mathbf{B}_K}{\mathbf{E}_0}\right)$.

$$a_{\dot{K}_{ij}} \equiv \frac{\dot{X}_{\dot{K}_{ij}}}{\dot{X}_j}. \tag{7.8}$$

Bullard and Herendeen [3] counted embodied energy from incoming capital stock in $\mathbf{A}_{\dot{K}}$ only if it was used for replacement.[3, p. 488] Consequently, they did not count incoming energy embodied in capital if the incoming capital was used to increase the stock of capital within a sector. In fact, Bullard and Herendeen's product-focused accounting system did not include an embodied energy stock for economic sectors ($\mathbf{B}$). They assumed that half of the incoming capital went toward replacement. These early researchers moved from the upper-left quadrant to the lower-left quadrant of Figure 7.1. And, Equation 7.6 represents a partial step toward developing a method for estimating energy intensity ($\boldsymbol{\varepsilon}$) that fully accounts for capital stock.

As stated above, we agree with Kirkpatrick [2], Bullard and Herendeen [3], and Casler [4] that incoming capital is important and should be included in an accounting framework (i.e., we should be on the lower half of Figure 7.1). But, we recommend that inclusion of incoming capital should be done within a *physical* accounting framework, i.e. we should move from the lower-left to the lower-right quadrant of Figure 7.1. Specifically, incoming capital should be included not only on incoming material streams but also as a stock that can accumulate within the economic sector itself.

Our recommendation is informed by the work of Odum [5] and Daly [6] and is based on the belief that accounting for stocks of capital is important for developing a coherent view of the structure of an economy. Stocks of capital are essential to the production process: without machines and factories, cars cannot be produced. And, in industrialized economies maintenance of captial stock becomes an important driver of demand. Thus, the buildup of capital stock (and associated embodied energy) within economic sectors is an essential aspect of industrialization. Carefully tracking (on a physical, as opposed to financial, basis) capital stock in each economic sector is essential for understanding the network effects of upstream energy demand as new industries and products arise (e.g., electric vehicles).

In a physical accounting system that includes capital stock (lower-right quadrant of Figure 7.1), energy embodied within accumulated capital stock is not assigned to products ($\mathbf{P}$); rather, accumulated embodied energy is assigned to a stock of embodied energy for each sector ($\mathbf{B}_K$). And, the stock of embodied energy ($\mathbf{B}_K$) can depreciate.

A physical accounting framework that fully includes capital stock (lower-right quadrant of Figure 7.1) is described by Equation 7.9.

$$\boldsymbol{\varepsilon} = \left(\mathbf{I} - \mathbf{A}^{\mathrm{T}}\right)^{-1}\left(\hat{\mathbf{X}}\right)^{-1}\left[\mathbf{E}_0 - \frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} - \mathbf{B}_{waste} - \hat{\gamma}_B \mathbf{B}_K\right]. \tag{7.9}$$

Differences between Equation 7.9 and Equation 7.6 include:

- Equation 7.9 includes $\mathbf{A}$ while Equation 7.6 splits $\mathbf{A}$ into $\mathbf{A}'$ and $\mathbf{A}_{\dot{K}}$ (this is a difference in appearance only),

- Equation 7.9 subtracts accumulation $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$ and depreciation ($\hat{\boldsymbol{\gamma}}_B \mathbf{B}_K$) of energy embodied in capital stock, because energy embodied in the stock of capital (K) for a sector is not assigned to products of the sector, and
- Equation 7.9 subtracts $\mathbf{B}_{waste}$, because energy embodied in waste products is not assigned to products of the sector.

There are two topics related to Equation 7.9 that are worthy of consideration: waste flows and an accounting equation for capital stock.

7.1.2.1 Waste flows

We are unaware of any estimates of the energy embodied in wasted material in an economy ($\mathbf{B}_{waste}$). But, it may be possible to develop a metric for the resource material effiency of an economic sector ($\eta_{\dot{R}}$) such that

$$\eta_{\dot{R},j} \equiv \frac{\dot{P}_j}{\sum\limits_{i=1}^{n} \dot{R}_{ij}}. \tag{7.10}$$

With the above definition, the scrap rate for resources could be expressed as $(1 - \eta_{\dot{R}}) \sum\limits_{i=1}^{n} \dot{R}_{ij}$. Allwood et. al. [7, p. 193] used a process-based approach to manufacturing efficiencies for metals used in manufacturing. The data are summarized in Table 7.1.

Table 7.1 Manufacturing efficiencies (Equation 7.10) for selected manufactured goods.[7]

Product	Manufacturing Efficiency [%]
Steel I-beam	90
Car Door Panel	50
Aluminium Drink Can	50
Aircraft Wing Skin Panel	10

Furthermore, one could assume that the rate of short-lived materials ($\dot{S}$) used by a sector could be given as a fraction of the resource ($\dot{R}$) use rate such that:

$$\rho_{\dot{S},j} \equiv \frac{\dot{S}_{j0}}{\sum\limits_{i=1}^{n} \dot{R}_{ij}} = \frac{\sum\limits_{i=1}^{n} \dot{S}_{ij}}{\sum\limits_{i=1}^{n} \dot{R}_{ij}}. \tag{7.11}$$

With the above definitions, the waste resource rate from an economic sector can be given as

$$\dot{R}_{j0} + \dot{S}_{j0} = (1 - \eta_{\dot{R}_j} + \rho_{\dot{S},j}) \sum_{i=1}^{n} \dot{R}_{ij}. \tag{7.12}$$

The embodied energy in the waste materials would need to be estimated from the embodied energy of the incoming resource and short-lived material flows as

$$\dot{B}_{waste,j} = \dot{B}_{\dot{R}_{j0}} + \dot{B}_{\dot{S}_{j0}}. \tag{7.13}$$

7.1.2.2 Simplification via capital stock accounting equation

A possible simplification to Equation 7.9 can be obtained from a control volume around the stock of capital in sector j:

$$\frac{\mathrm{d}B_{K_j}}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{B}_{\dot{K}_{ij}} - \gamma_{B,j} B_{K_j}. \tag{7.14}$$

We can express the incoming energy embodied in capital ($\sum_{i=1}^{n} \dot{B}_{ij}$) as a fraction ($\alpha_j$) of the capital stock ($B_{K_j}$) as

$$\alpha_{B,j} \equiv \frac{\sum\limits_{i=1}^{n} \dot{B}_{\dot{K}_{ij}}}{B_{K_j}} \tag{7.15}$$

for $j \in [2,n]$ and the vector $\boldsymbol{\alpha}_B$ as

$$\boldsymbol{\alpha}_{\mathbf{B}} \equiv \begin{Bmatrix} \alpha_{B,2} \\ \alpha_{B,3} \end{Bmatrix}. \tag{7.16}$$

Together with the Kronecker delta (δ_{ij}), we can write

$$\hat{\boldsymbol{\alpha}}_B \equiv \delta_{ij} \boldsymbol{\alpha}_B = \begin{bmatrix} \alpha_{B,2} & 0 \\ 0 & \alpha_{B,3} \end{bmatrix}. \tag{7.17}$$

Thus, the embodied energy accounting equation around the stock of capital in the economy can be written in matrix form as

$$\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} = \hat{\boldsymbol{\alpha}}_B \mathbf{B}_K - \hat{\boldsymbol{\gamma}}_B \mathbf{B}_K. \tag{7.18}$$

Rearranging slightly gives

$$\hat{\boldsymbol{\alpha}}_B \mathbf{B}_K = \frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} + \hat{\boldsymbol{\gamma}}_B \mathbf{B}_K, \tag{7.19}$$

which says that incoming capital ($\hat{\boldsymbol{\alpha}}_B \mathbf{B}_K$) can be used to either increase the stock of capital in the economy $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$ or overcome depreciation ($\hat{\boldsymbol{\gamma}}_B \mathbf{B}_K$). Substituting Equation 7.19 into Equation 7.9 gives

$$\boldsymbol{\varepsilon} = (\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1}\hat{\mathbf{X}}^{-1}\left[\mathbf{E}_0 - \hat{\alpha}_B \mathbf{B}_K - \mathbf{B}_{waste}\right]. \tag{7.20}$$

7.1.3 Energy input from society

In Sections 7.1.1 and 7.1.2 above, we implicitly assumed that society (final consumption, Sector 1 in our model) contributes negligible energy to the economy. Thus, all vectors and matrices in Equation 7.9 involve Sectors 2–n, but not Sector 1.

Energy input from society to the economy ($\mathbf{T}_1$) is "muscle work" supplied by working humans and draught animals.[8–10] This muscle work term ($\mathbf{T}_1$) should include all upstream energy required to make the labor available.[4] Equation 6.38 adds the effect of energy input from society to the economy, effectively moving from the top half to the lower half of the lower-right quadrant in Figure 7.1.

$$\boldsymbol{\varepsilon} = (\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1}\hat{\mathbf{X}}^{-1}\left[\mathbf{E}_0 + \mathbf{T}_1 - \frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} - \mathbf{B}_{waste} - \hat{\boldsymbol{\gamma}}_B \mathbf{B}_K\right]. \tag{6.38}$$

For industrialized economies, the direct energy component ($\mathbf{E}_1$) of muscle work ($\mathbf{T}_1$) is likely to provide only a small fraction of the energy input from fossil fuels ($\mathbf{E}_0$). But, the embodied energy of the muscle work ($\mathbf{B}_1$) is likely to be large. For agrarian and developing economies, $\mathbf{T}_1$ and $\mathbf{E}_0$ could be on the same order of magnitude. For both industrial and agrarian economies, neglecting $\mathbf{T}_1$ could cause errors in estimates of $\boldsymbol{\varepsilon}$. To the extent that $\mathbf{T}_1$ is significant relative to $\mathbf{E}_0$, neglecting $\mathbf{T}_1$ will underpredict the energy intensity of economic output. Energy input from society is discussed further in Section 8.9.

7.1.4 Recommendation

Sections 7.1.1–7.1.3 discussed three factors that affect the form of the energy intensity equation: product-focused vs. physical accounting frameworks, whether capital stock is included, and whether energy input from society is included. The three factors are summarized in Figure 7.1.

At this point, it is instructive to look back at the product-focused vs. physical discussion in Section 7.1.1. We understand the argument for including capital stock in a product-focused accounting framework (lower-left quadrant of Figure 7.1): capi-

[4] It is important to note that $\mathbf{T}_1$ should include all upstream energy, because at this point in the development of our framework, we are assuming that Final Consumption (Sector 1) is exogenous to the economy (Sectors 2–n), and upstream energy consumption needs to be included manually. However, in Section 8.9, we show that Final Consumption can be endogenized. Once endogenized, the energy intensity of Final Consumption (ε_1) will automatically include the upstream energy required to make labor available. (See Appendix B.)

It is important to note, too, that labor can have very high energy intensity, because ε_1 includes the energy required to supply food for and transport to workers.

tal stock and waste exist solely due to product demand, therefore energy embodied in capital and waste should be assigned to products. However, a product-focused framework that includes capital stock (lower-left quadrant of Figure 7.1) masks structural aspects of economies that we believe are essential to fully understanding how and why energy flows through economies, namely the accumulation of capital and associated energy embodied within sectors.

The metabolic metaphor provides guidance here. If we were to create a model of an organism that neglects tissues that accumulate embodied energy, the organism (in the model) has nothing with which to absorb, process, waste, or otherwise exchange material with the biosphere. The organism doesn't physically exist (in the model)! Neglecting to account for the stock of capital (and its embodied energy) is tantamount to assuming that economic production occurs out of nothing! Accounting for capital stock is essential.

For our framework, we chose a physical accounting approach (which puts us in the right column of Figure 7.1). We chose the physical approach primarily because of our belief that capital is an important aspect of economies, and the physical accounting approach properly includes a stock of capital for each sector of the economy. Product-based accounting frameworks mask crucial aspects of why and how energy flows through economies will be masked. We acknowledge that the choice of a physical accounting framework necessitates careful tracking of capital flows (and associated embodied energy) through the economy.

Finally, we suggest that accounting for energy input from society to the economy is important, and we need to be in the lower half of the bottom-right quadrant of Figure 7.1. So, the state of the art has moved from the nascent energy I-O literature located in the upper-left quadrant of Figure 7.1 as represented by Equation 7.5 through the lower-left quadrant of Figure 7.1 as represented by Equation 7.6 to the lower half of the bottom-right quadrant of Figure 7.1 as represented by Equation 7.20.

The implication of this detailed development of a framework for material, energy, and value accounting on the energy I-O method is some suggested enhancements to the energy I-O method, including

- conversion to a physical accounting framework such as the one we propose herein,
- physical (as opposed to financial) tracking of accumulated capital stock within economic sectors,
- redefinition of $\mathbf{A}$ and $\boldsymbol{\varepsilon}$ to include embodied energy on both inflows and outflows of material, and
- use of Equation 7.20 instead of Equations 7.5 or 7.6 for estimating energy intensity ($\boldsymbol{\varepsilon}$) of economic sectors within an economy.

7.2 Implications for economic "development"

Growth of Gross Domestic Product (GDP) is virtually the only gauge that developed countries use to measure the health of an economy. However, it is not the only indicator available; several complementary measures of economic well-being are in use. The Human Development Index is a globally accepted measure that add education and longevity scales to National Income to place countries' relative GDP in context. [**?**] ***ADD CITE HDI:`http://hdr.undp.org/en/statistics/hdi` In the US, the state of Maryland has been tracking well-being using the *Genuine Progress Indicator*. [**?**] ***ADD CITE MDGPI1 `http://www.dnr.maryland.gov/mdgpi/`. This indicator combines measures of economic transactions with environmental and social costs and is closely related to ecological economist Herman Daly's Index of Sustainable Economic Welfare (ISEW). [**? ?**] ***ADD CITE Daly1994 Daly, Herman E. and John B. Cobb, For The Common Good: Redirecting the Economy Toward Community, the Environment, and a Sustainable Future, Boston: Beacon Press, 1994. ****ADD CITE MDGPI2 `http://www.dnr.maryland.gov/mdgpi/whatisthegpi.asp` It allows policy-makers to account for contributions of and impacts on the natural environment. Another example is the Nation of Bhutan's *Gross National Happiness*. ***ADD CITE `http://www.grossnationalhappiness.com/` Despite the superficial sense that some may have about the term "happiness," the index is a systematic, annual compliation of survey and other data related to nine sectors: ecological diversity and resilience, as well as psychological well-being, health, education, culture, time use, good governance, community vitality, and living standards. [**?**] ****ADD CITE ura2012 `http://www.grossnationalhappiness.com/wp-content/uploads/2012/04/Short-GNH-Index-edited.pdf` These and other broad measures of economic well-being are slowly gaining acceptance, particularly as their valuation methods continue to be strengthened. [**?**] ***ADD CITE lawn2003 `http://www.dnr.maryland.gov/mdgpi/pdfs/GPIFoundation-Lawn2003.pdf`

Economic "development"[5] is usually "measured" by Gross Domestic Product (GDP). With reference to Figure 5.5, *GDP* is calculated by

$$GDP = \sum_{j=2}^{n} \dot{X}_j \tag{7.21}$$

where n is the number of sectors in the economy. Equation 7.21 clearly shows that *GDP* is a *flow* of value in units of \$/year.

As a given economy moves along its "development" path, sectors within the economy accumulate capital stock (K, typically expressed in units of dollars) and associated embodied energy (B_K, expressed in units of joules). If we turn this around, accumulation of embodied energy in economic sectors and society could be consid-

[5] We choose to use the word "development" to describe expanding economies, despite significant misgivings about the term. The unambiguously positive connotations of the words "development" and "developed" fail to capture the nuances of travel along economic development paths: there are so many ways in which life experience in "developed" countries is both better and worse than life in "developing" countries. We hope to convey our misgivings by surrounding these words with quotation marks in this text.

ered a *proxy* for “development.”[6] Equation 7.22 indicates how accumulated embodied energy in the capital stock of an economy ($\mathbf{B}_K$) could be calculated:

$$\mathbf{B}_K(t) = \mathbf{B}_K(0) + \int_{t=0}^{t=t} \frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\mathrm{d}t, \tag{7.22}$$

where $\mathbf{B}_K$ is given by Equation 6.19. Equation 7.22 clearly shows that energy embodied energy in capital stock ($\mathbf{B}_K$) is a *stock* (in units of joules), not a flow.

The behavior of $\mathbf{B}_K$ with respect to $\left.\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right|_{\text{other}}$ is vitally important. As an economy transitions from agrarian to industrialized, its capital stock (K) and associated embodied energy (B_K) grows ever larger. The outflow of depreciated capital stock and its associated embodied energy will occur at a faster rate, too. As increasingly large amounts of energy are embodied in the capital stock of an economy (B_K), Equation 6.33 shows that increasingly large energy extraction rates ($\mathbf{E}_0$) are required to maintain capital stock in the sectors of the economy to offset the effects of depreciation ($\hat{\gamma}_K\mathbf{B}_K$), assuming that $\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} \geq 0$ is desired.

During a period of rapid industrialization and infrastructure build-out, we expect both *GDP* and energy embodied in the economy ($\mathbf{B}_K$) to increase. But, there is no guarantee that *GDP* and $\mathbf{B}_K$ move in the same direction at all times. Industrialized economies may experience *GDP* growth while the stock of embodied energy in the economy ($\mathbf{B}_K$) remains nearly constant, because the economy is running circles to overcome the effects of depreciation. There can be a time lag between movements of GDP and $\mathbf{B}_K$, too. At the beginning of an economic downturn (defined as prolonged GDP reduction), capital stock and associated embodied energy ($\mathbf{B}_K$) will remain approximately constant: GDP moves but $\mathbf{B}_K$ doesn’t. But as the GDP decline continues, maintenance flows for capital stock will be reduced and depreciation will overtake maintenance leading to a decline in $\mathbf{B}_K$. If economic decline is associated with significant external infrastructure investment (such as occurred in post-colonial Africa), GDP may decrease while the stock of energy embodied in the economy ($\mathbf{B}_K$) increases due to foreign aid focused on infrastructure enhancement. **** BRH (could this work?) An example of this might be the underdevelopment of the Amazon from 1600 to 1980 that occured due to “extract and export” fueled economic “growth.” Per capita incomes increased by a power of 10 from 1820 to 1900 as the rubber export boom occured. However, even as rubber exports continued to increase, and supposedly capital along with that, per capita incomes dropped precipitously back to original levels. [**?**] ****ADD CITE: bunker1984: Bunker, Stephen G. ”Modes of Extraction, Unequal Exchange, and the Progressive Underdevelopment of an Extreme Periphery: The Brazilian Amazon, 1600-1980. ”American Journal of Sociology 89, no. 5 (March 1, 1984): 10171064.

A second possible measure of economic “development” is another *stock*, wealth:

$$X_j(t) = X_j(0) + \int_{t=0}^{t=t} \frac{\mathrm{d}X_j}{\mathrm{d}t}\mathrm{d}t, \tag{7.23}$$

[6] Embodied energy as a proxy for development may be overly focused on capital stock, therefore one-dimensional, and reductive, but GDP and other measures are open to similar criticism.

where $j = 1$ for societal wealth and $j \in [2,n]$ for corporate wealth, both measured in dollars. In fact, capital (as represented by energy embodied in infrastructure, $\mathbf{B}_K$) and financial resources (as represeted by $X_{2\ldots n}$) are complimentary factors of production for economic processes. But, we can go further than linking capital with financial resources. If capital ($\mathbf{B}_K$) is to be useful, we need financial resources or currency ($\dot{X}$) to

- purchase direct energy ($\dot{E}$) to power the capital,
- purchase resources ($\dot{R}$) to feed the capital, and
- pay workers (represented by societal energy input to the economy, $\mathbf{T}_1$) to operate the capital.

Thus, economic growth could be considered a "fully coupled" problem: understanding it requires breadth of knowledge and appreciation for interactions among many important factors. Each of the factors discussed above ($\dot{X}$, $\mathbf{B}_K$, $\dot{E}$, $\dot{R}$, and $\mathbf{T}_1$) is necessary, but not sufficient, for economic growth.

Our framework serves highlight to several issues in economic "growth." Should it be measured by a stock or a flow? Which measure is most appropriate? What roles do currency, capital stock, energy, resources, and labor play in economic processes? These are overlapping and complementary areas of inquiry. We encourage further research in all of these areas.

7.3 Implications for recycling, reuse, and dematerialization

Dematerialization is the idea that economic activity can be unlinked from material or energy demands.[11] One method for dematerializing an economy is reuse and recycling of materials from both short-lived goods ($\mathbf{B}_{waste}$) and depreciation of capital stock ($\hat{\boldsymbol{\gamma}}_K \mathbf{B}_K$) that would otherwise have been expended to the biosphere.[7]

In Chapter 6, we defined the rate of accumulation of embodied energy within the economy $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$ by the following equation:

$$\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t} = \mathbf{E}_0 + \mathbf{T}_1 + \hat{\mathbf{X}}(\mathbf{A}^\mathrm{T} - \mathbf{I})\boldsymbol{\varepsilon} - \mathbf{B}_{waste} - \hat{\boldsymbol{\gamma}}_K \mathbf{B}_K \tag{6.33}$$

One effect of recycling is to reduce the magnitude of the waste ($\mathbf{B}_{waste}$) and depreciation ($\hat{\boldsymbol{\gamma}}_K$) terms, since for a given level of capital stock ($\mathbf{B}_K$) the amount of depreciation to the biosphere ($\hat{\boldsymbol{\gamma}}_K \mathbf{B}_K$) will be lower due to the fraction that is now recycled. As can be seen by looking at Equation 6.33, this indicates that recycling of material in an economy, by reducing both $\mathbf{B}_{waste}$ and $\hat{\boldsymbol{\gamma}}_K$, puts *upward* pressure on the accumulation of energy embodied in capital stock $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$.

Recycling has a mixed effect on energy demand ($\mathbf{E}_0$). Because recycled materials can displace newly-produced material in the economy and society, recycling will

[7] Another method of dematerialization is substitution away from production of material goods and toward information and services in an economy.

tend to reduce energy demand ($\mathbf{E}_0$). However, recycling processes require energy to operate, thereby putting upward pressure on energy demand ($\mathbf{E}_0$). If the energetic cost of recycling is lower than that of obtaining materials from virgin resources, as is the case for many metals, e.g. aluminum [12], the result is a net reduction of energy demand from the biosphere ($\mathbf{E}_0$). Berry and Fels found that recycling of the material in automobiles would result in 12,640 kW-hr of energy reduction per vehicle.[13, p. 15] Therefore recycling will also put *downward* pressure on the growth of embodied energy in the economy $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$.

If recycling produces a net reduction in energy demand ($\mathbf{E}_0$), the upward pressure on growth $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$ from decrease in depreciation ($\hat{\boldsymbol{\gamma}}_K$) and the downward pressure on growth from net reduction in energy demand ($\mathbf{E}_0$) can offset each other. Under those conditions, the accumulation rate of energy embodied in capital stock $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$ will remain near zero and total embodied energy ($\mathbf{B}_K$) will remain constant. In that scenario, dematerialization can occur: reduced material and energy input ($\mathbf{E}_0$) can be accompanied by no change in the growth of the economy $\left(\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right)$. However, as will be discussed in Section 8.5.2, recycled materials can never entirely replace the need for new materials.

7.4 Comparison to a steady-state economy

As discussed in Chapter 1, the human economy is a subset of the biosphere which is a finite, non-growing system, *open* to flows of solar energy but *closed* to material transfers. Because the biosphere is finite in size, the human economy cannot physically grow indefinitely. The concept of a non-growing or "steady-state" economy has existed for centuries. There are a number of different conditions that may characterize a system as steady-state. In thermodynamics, the condition of a zero rate of accumulation of some stock, x $\left(\frac{\mathrm{d}x}{\mathrm{d}t}=0\right)$ is normally used to define steady state conditions.[8] Other conditions that might define a steady-state economy are: a constant rate of material through-put, constant GDP, or constant population. Our framework, as outlined here, can address the first three steady-state conditions. The fourth condition, constant population, could be accommodated with some adaptation of the framework. The issue of human population as part of society's capital stock is addressed in Section 8.7.

7.4.1 Constant level of capital stock

In Chapter 2, we introduced Equation 2.105:

[8] Or, more generally, a change in any property over time $\left(\frac{\partial p}{\partial t}\right)$.

$$-\frac{\mathrm{d}R_0}{\mathrm{d}t} = \sum_j \frac{\mathrm{d}K_j}{\mathrm{d}t} + \sum_{i,j} \dot{S}_{ij} + \sum_j \gamma_{K_j} K_j. \tag{2.105}$$

which indicates that the depletion of natural resources in the biosphere $\left(-\frac{\mathrm{d}R_0}{\mathrm{d}t}\right)$ by the economy is used for the purposes of:

- increasing man-made capital stocks within the economy $\left(\frac{\mathrm{d}K_j}{\mathrm{d}t}\right)$,
- providing short-lived goods exchanged within the economy $\left(\dot{S}_{ij}\right)$, and
- overcoming depreciation of man-made capital stocks $\left(\sum_j \gamma_{K_j} K_j\right)$.

Assuming, first, that a steady-state economy exists when the level of capital stock remains constant $\left(\sum_j \frac{\mathrm{d}K_j}{\mathrm{d}t} = 0\right)$,[9] we can see that Equation 2.105 reduces to:

$$-\frac{\mathrm{d}R_0}{\mathrm{d}t} = \sum_{i,j} \dot{S}_{ij} + \sum_j \gamma_{K_j} K_j. \tag{7.24}$$

A number of interesting concepts may be understood in relation to Equation 7.24. Firstly, if our steady-state economy is to be supported sustainably, then withdrawal of natural resources from the biosphere $\left(\frac{\mathrm{d}R_0}{\mathrm{d}t}\right)$ had better be at some rate lower than the biosphere can replenish the natural capital stocks. In reality, $\frac{\mathrm{d}R_0}{\mathrm{d}t}$ is really the sum of many different resources (flora and fauna, water) each of which will have its own natural rate of regeneration. As such, the sustainability criterion is a vector of values, one for each natural resource.

Secondly, the steady state condition $\left(\sum_j \frac{\mathrm{d}K_j}{\mathrm{d}t} = 0\right)$ says nothing about the transfer rates of short-lived goods within in the economy $\left(\sum_{i,j} \dot{S}_{ij}\right)$ or the depreciation of capital stock back to the biopshere $\left(\sum_j \gamma_{K_j} K_j\right)$. Equation 7.24 indicates that the higher the rates of these flows, the greater the rate of depletion of natural resources, and the more difficult it will be to meet the sustainability condition (that the withdrawal rate of natural resources from the biosphere is lower than the biosphere replenishment rate). Within industrial society, the flow of short-lived goods (packaging, paper products, disposable tableware, cutlery, and napkins) is large, and, presumably, attaining a sustainable steady-state economy will be difficult. This definition of steady state $\left(\sum_j \frac{\mathrm{d}K_j}{\mathrm{d}t} = 0\right)$ does not necessarily coincide with sustainability.

As discussed in Chapter 2, the rate of depreciation (γ_K) is inversely proportional to the average lifetime of capital stock—as the average lifetime of capital stock decreases, the rate of depreciation of capital stock increases which increases the draw on natural resources (by Equation 7.24). It is likely that the average lifetime of capital stock has decreased over the last century, due to a decrease in durability of capital stock (the average table built today is not as durable as the average table built

[9] Note that the steady-state condition does not preclude expansion of some sectors of the economy, provided that there is equal contraction elsewhere.

in the early twentieth century) and also due to increasing proportions of consumer electronics with short lifetimes (cell phones, laptops, tablets). Decreasing lifetime causes higher rates of flow for replacement materials. In the absence of extreme recycling of materials, these large replacement flows place large demands on natural resources.

Thirdly, the maintenance flows necessary to overcome depreciation $\left(\sum_j \gamma_{K_j} K_j\right)$ are proportional to the magnitude of the capital stock (K_j). As such, a larger stock of capital requires greater draw on natural resources and is thus harder to maintain within any sustainability constraint. These points emphasize that constant capital stock (or analogously constant population) is not a sufficient condition for environmental sustainability.

7.4.2 Constant material throughput

Herman Daly has placed great emphasis on a steady-state economy as having a constant rate of material throughput [14, 15] which, as discussed above, should be below biophysical limits if sustainability is to be achieved. This is often referred to as the "scale" issue—how large is the (currently growing) human economy in relation to the finite, non-growing biosphere of which it is a sub-system? Growth of the human economy must either displace other natural ecosystems (replacing old growth forest for crops) or deplete natural capital stocks, be they renewable (fisheries) or non-renewable (fossil fuels). As shown in Figure 2.4, material throughput is composed of two distinct processes: exchange of material *from* the biosphere *into* the economy (extraction) and exchange of material *from* the economy *into* biosphere (waste and depreciation). We may characterizing constant material throughput as either constant rate of extraction, constant rate of waste disposal, or both. In the language of our framework, we could write:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{R}_0\right) = 0 \tag{7.25}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{S}_0\right) = 0 \tag{7.26}$$

$$\sum_i \left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{R}_{i0}\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{S}_{i0}\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{K}_{i0}\right)\right] = 0. \tag{7.27}$$

The above equations say nothing about the level of man-made capital stock (K) or the flow rate of short-lived goods ($\dot{S}$). Thus, within the constant throughput constraint, increasingly effective use of materials could theoretically allow increasing accumulation of man-made capital (K) and increasing flow of short-lived goods ($\dot{S}$) as society learns to use resources better. Eventually, physical limits would entail that capital stock could no longer be increased. Presumably, society would desire that the throughput of materials would be within levels that could be sustained by the biosphere, both at the input side—natural resources extracted at rates lower than natural regeneration rates—and at the output side—wastes emitted at rates below which the biosphere can assimilate.

7.4.3 Constant GDP

The issue of GDP and economic "development" has already been discussed in Section 7.2. However, the case of zero growth in GDP was not. Within our framework, a condition of constant GDP would be characterized by the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}(GDP) = \sum_j \frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{X}_j\right) = 0 \tag{7.28}$$

Since, under the subjective theory of value, no value is attributed to the flow of materials to or from the biosphere, it is unclear what the impact constant GDP would be on capital stock (K) or material throughput (extraction and waste disposal). If we constrained $\dot{R}_0$ and $\dot{S}_0$, it is likely that economic growth would decrease or even become zero or negative $\left(\frac{\mathrm{d}}{\mathrm{d}t}(GDP) \leq 0\right)$. It is conceivable that constraining economic growth may act to constrain material throughput, though this is certainly not assured. Many authors argue that increasing GDP no longer guarantees increasing welfare [16–20] for two main reasons:

- firstly, that the costs of growth in GDP, some of which are added as benefits into GDP but also externalities (especially environmental) which are not counted at all, outweigh any benefit that comes from increasing GDP; and
- secondly, that increasing GDP serves to increase relative income inequality, despite increasing absolute income, which decreases welfare for both rich and poor alike. [19]

Furthermore, the argument for increasing GDP as a means to alleviate poverty is undermined by this second point. The world's poor are poor because they cannot afford to buy food and other goods due to their *relative* poverty, not because there is an *absolute* lack of these goods. The rising tide raises all boats, but the (growing) luxury cruisers soon capsize the (shrinking) dinghies.

**** MCD—Becky, can you speak to this? This last paragraph is way outside my area of expertise and is a little bit "soap boxy". ****

**** Discuss maintenance vs. accumulation flows? Is this a better place to discuss this issue? Can refer to Section 6.5 and Odum B/P vs. P/B. ****

7.5 Summary

In this chapter, we discussed several implications that arise from the detailed development of our dynamic framework for material, energy, and value accounting.

The first implications are for the energy I-O method itself. We recommend a physical accounting method that fully accounts for capital stock and energy input from society (final consumption) to the economy. We then discussed implications for economic "development," namely that economic growth could be considered a "fully coupled" problem: understanding it requires breadth of knowledge and appreciation for interactions among many important factors, including financial capital, physical capital and associated ambodied energy, direct energy, resources, and societal inputs. Each, alone, is necessary, but not sufficient, for economic development. We discussed implications for recycling and reuse of materials as well as the concept of dematerialziation. Finally, we viewed the concept of a steady-state economy through the lens of our framework. We found that there are many potential definitions of a steady-state economy, none of which are fully satisfying when compared against the ideal of sustainability.

In the next chapter, we point to some unfinished business.

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Chapter 8
Unfinished Business: Practical, Methodological, and Theoretical Issues

Accounting systems change behavior.
—unknown JPL/NASA accountant

With any endeavor of this magnitude, namely development and comprehensive presentation of a framework for economies of the world, unfinished business is inevitable. This chapter discusses several practical, methodological, and theoretical issues that should be addressed in the future. On a practical level, additional data are needed to fully utilize the framework developed herein. In terms of methodological issues, we encourage that economists of all types embrace material, energy, embodied energy, and economic value counting as a valid method of inquiry for modern economics. And, there are issues of co-products and the choice of the energy input vector that need to be addressed. Finally, several theoretical issues, including material and energy quality, model boundaries, and the Sun are addressed. We begin with the topic of data.

8.1 Metaphors and models

**** Mik: Consider moving this section or a version of this section to the introduction when you get back to reviewing the introduction. **** MCD—I'm leaving this here to bookend the metaphors discussion as I think it nicely leads into the reconciliation between neoclassical and ecological economists.

Historically, mainstream economists have used the metaphor of *machine* to describe economies. In fact, much of mathematical economic modeling today relies upon mechanistic conceptual foundations whose roots are in Newtonian physics and The Enlightenment.[1] If your metaphor is a machine, it makes sense to analyze economies with equations that describe machines. In the mainstream, such equations usually, but not exclusively or restrictively, are developed assuming that the economic machine is a *isolated* system.

[1] The subfield of longwave economic growth modeling, in particular, uses mechanistic language to describe their equations. For example, Jones [1, pp. 6 & 18] refers to the ordinary differential equations that describe births, deaths, and the production of ideas as "laws of motion."

In this book, we take the strong position that economies are better-represented as open, organic metabolisms. Many ecological economists, who are predisposed to view the economy as a metabolism, reject the use of mechanistic equations to describe the economy, in part because such equations often assume a machine that operates independently from the biosphere. Other ecological economists dismiss mainstream economics and its mechanistic models out of hand. For example, Söllner pejoratively says "The prime example of physics envy and the desire to emulate the natural sciences is, of course, neoclassical economics which was explicitly and purposefully copied from classical mechanics." [2, p. 178]

But, it doesn't have to be this way. We claim that it is not the use of equations and mathematical models themselves that is the problem. Rather, it is the application of such equations to an assumed closed system that is is worthy of criticism. Thus, our approach herein includes rigorous application of the admittedly mechanistic accounting equations for materials, energy, embodied energy, and economic value *under the assumption of* and *including* vigorous and necessary interactions between the economy and the biosphere.

It is our hope that this book allows

- mainstream economists to see that
  - the metabolism metaphor is apt because interaction between the economy and the biosphere is what drives economic growth and
  - rigorous mechanistic models can and should include interactions between the biosphere and the economy; and
- ecological economists to see that rigorous mathematical modeling is an important tool for understanding interactions between the economy and the biosphere.

In summary, we encourage both mainstream and ecological economists to embrace mathematical modeling of material, energy, embodied energy, and economic value flows both within the economy and between the economy and the biosphere as a valid method of inquiry for modern economics.

8.2 Subjective vs. intrinsic theory of value

As stated in Chapter 5, the neoclassical approach is a *subjective* theory of value. It is determined by the relative ability of the product to satisfy a persons wants. However, a persons wants are malleable and are, in turn, formed within a "constellation of shared goals to which a society aspires." [3] Thoughout history, economists (particularly the classicals) and non-economists have searched for an invariant, objective, *intrinsic* determinant of value.[2] Adam Smith, Karl Marx, David Ricardo and

[2] Following the ecological economics literature, we use the term, intrinsic, in the sense of "objective." Costanza [3] notes that a better term would be objective in order to avoid moral overtones associated with the term intrinsic.

his student Sraffa all proposed alternative determinants of value. Their proposed objective values were based on identifying the primary input into production, such as labor or land and using that input in the sense of a numeraire. That is, a way to measure value across the entire spectrum of goods and services in commensurate units.

Costanza [3] makes a compelling case for energy as the only truly primary input into production and thus an, or rather the, objective determinant of value. On a global scale, he notes, (solar) energy (including that which is stored in fossil fuels) is the only primary input into production, everything else is an intermediate input. Thus, free energy could be seen as not simply an input into production, but the primary input into production, upon which an objective (intrinsic), energy theory of value could be built. In fact, this line of research has yielded some interesting figures on the amount of solar energy required to run the economy. See Section 8.8 for further discussion.

We work within the neoclassical framework for pragmatic, rather than philosophical, reasons. It is currently not an adequate framework for guiding economies situated within a "full" earth. For example, Herman Daly [4] provides important discussion of the focus on "value-added." The neoclassical framework focuses on measuring value-added and ignores the quality of the resources to which the value is being added. This leads to overstatement of economic value flows in the following way. As easily accessible forms of energy (e.g., oil extracted from the Texas panhandle) are used up and more difficult locations must be tapped (e.g., Alaskan north slope) the economy appears to grow. The "value-added" by human and manufactured capital is higher, the more work humans must do to extract domestic energy sources. However, what is actually happening is that the stock of natural resource is diminishing. Or, stated differently, the subsidy that nature provides the economy is declining.

Therefore, as "that to which value is added" diminishes, economic growth begins to reach binding constraints. Daly, Costanza and others ****include other references here**** discuss the importance of identifying the economic scale—amount of materials put through the economy—that is viable for the biosphere to handle. The neoclassical framework is not able to address this larger question without modification.

While we do not attempt to correct this widely utilized approach, we note that our model provides a natural starting place for extending the neoclassical framework. Value flows to and from the biosphere can be estimated and included in the model, once a system of envrionmental-economic accounting is in place.[3] Eventually, the framework will reconnect the economy to the biosphere and the flows that were conspicuously missing from Figure 5.1 will be restored, as in Figure 8.1.

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[3] As of this printing, the System of Environmental-Economic Accounting (SEEA) [5] is in its third revision using a process of global consultation. This system contains internationally agreed-upon standards for quantifying value flows to and from the biosphere. This is a system analogous to the System of National Accounts (SNA) produced by the same United Nations organization which is a framework for measuring economic value creation consistently across nations.

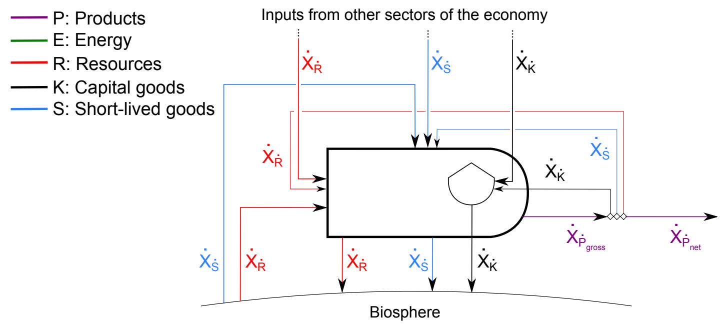

Fig. 8.1 Aggregated flows of value for a single sector including flows to and from the biosphere.

8.3 Data

In Chapter 1, we noted the importance of "counting" materials, energy, and value as they flow through economies. Unfortunately, unless data on these flows is collected and disseminated routinely, such counting is impossible. At several points in this manuscript, we have noted the very practical issue of the need for additional data, information, and analysis.

In Section 2.5, our attempt to model the physical flows of materials through the US auto industry was severely hampered by the lack of any data on material flows. Work to account such flows is starting to be addressed at the level of economies, particularly within Europe [6]. This work needs to continue as well as the development of sub-economy, inter-industry material accounts. Chapter 6 noted the need to collect data on human and draught animal physical work input to sectors of economies, espeically for those economies where muscle work is on the same order of magnitude as fossil fuel energy input.

The need for rigorous and accurate data is all the more pressing in light of the need for an *independent* method to calculate the energy intensity of goods and services, as suggested in Chapter 6. If this independent method relies on process-type analysis, then there is a critical need for systematic collection and public dissemination of such data which is currently available only in proprietary format. The latest version of the ecoinvent database (v3) contains detailed analysis of over 10,000 processes.[7] This is a fantastic effort, but more needs to be done to bring this crucial information into the public arena. Unfortunately, it seems like public agencies are headed in the opposite direction. The US stopped collecting satellite account data in 2002. Beyond the US, few countries collect and disseminate the kind of data required as inputs to the framework developed herein.

The adage "you can't control what you don't measure" seems appropriate here. As the world confronts significant challenges of material and energy supplies in the coming years, it will be impossible to make wise decisions about which materials to use, which energy sources to develop, and which products to incentivize.

Another adage "we count what we value and we value what we count" points out that we are not presently valuing highly enough the important flows of which our economies are comprised. We add our voices to those encouraging governments and institutions to collect high-quality data on material and energy flows. **** Becky, are there references that call for a renewed focus on collecting satellite account data? We should point to those references here. ****

**** Becky: review the above paragraphs. Can you provide additional context on the decision to stop collecting this data? ****

8.4 Hybrids of I-O and process-based methods

In early chapters, we made the distinction between process-based (often called "bottom-up") and input-output (often called "top-down") analyses. The advantages and disadvantages of each type of analysis are outlined in Figure 8.2.

Process analysis is based on detailed technological or engineering models of specific economic processes. Model specification and data collection is arduous, time-consuming, and costly. The aim of process analysis is to calculate the energetic and material flows associated with the process under study by disaggregating the process into several components or sub-processes. In reality, any economic process exists as part of a complex network of interacting processes that encompass the entire economy, as stated by Bullard et al.,

> each step in a process analysis may be viewed as an expansion of the system boundary (around the item being analyzed) into the economic system [8, p.281]

and represented diagrammatically in Figure 8.3. Every process calls on every other process within the economy, even if only minutely and indirectly at many steps removed. Obviously, the time, effort and cost involved with trying to model and measure all of the flows involved become daunting for even low numbers of interacting processes. The decision of where to draw the boundary of a process analysis is known in the lifecycle assessment literature as the *truncation problem* [9]. One method to extend the comprehensiveness of process analyses is to use a hybrid method, utilizing data from an I-O analysis to supplement the missing data from the truncation of the process analysis. The financial cost of goods and services identified by the process analysis are converted to energy (or material) flows via the I-O method. The truncation error is replaced by a smaller aggregation error due to limitations of the I-O method [8]. A variety of other hybrid methods exist which also aim to overcome the limitations of either process or I-O method individually.[8–12]

In Section 7.1, we noted the need for an independent method for estimating the energy intensity or embodied energy of economic sectors. We speculated that a process-based method might provide the independent estimate. However, we realize the limitations of these methods (time, cost, etc.). Perhaps a hybrid method could provide the necessary independent estimate without the need for the cost of a full process-based approach.

	PROS	CONS
TOP-DOWN	* Comprehensiveness * Economy-wide analysis * System-level comparison * Publicly available data * Reproducible results * Assessment of future product development	* Aggregated data * Process analysis difficult * Reliance on financial data * Imports treated as domestic products * Lack of physical data * Data uncertainty
BOTTOM-UP	* Detail and specificity * Comparison of specific products or processes * Identifies process improvements * Assessment of future product development	* Subjective system boundary * Time intensive and costly * Difficult to apply to new product or process * Lack of data or reliance on proprietary data * Reproducibility of results * Data uncertainty

Fig. 8.2 Advantages (pros) and disadvantages (cons) of "top-down," I-O and "bottom-up," process-based analyses, adapted from [13].

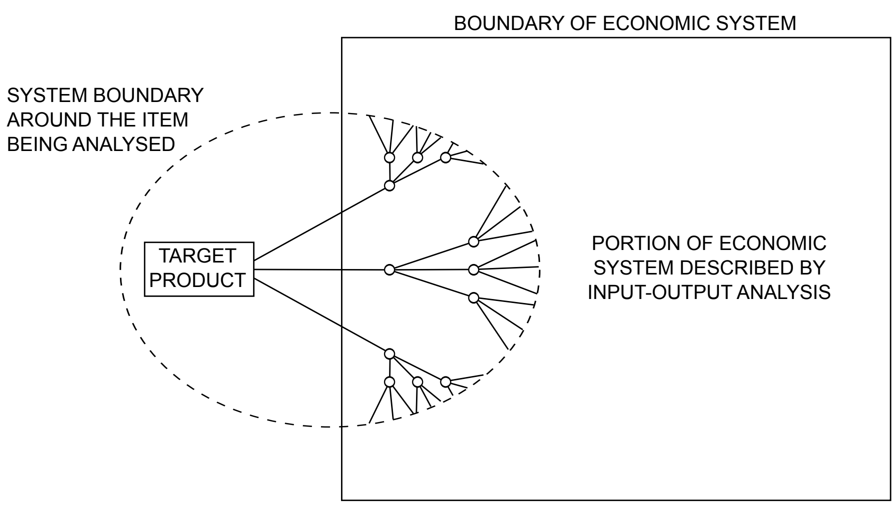

Fig. 8.3 System boundary for process and I-O analyses, adapted from [8].

8.5 Resource quality and irreversibility

The quality of both materials and energy play a role in the efficiency with which economies use energy resources to convert material resources into products.

Raw material and energy resources must first be extracted from the natural environment before they may be utilized in the economy to provide goods and service to society. Despite increasing levels of technological efficiency, for example in consumer goods such as refrigerators and cars, evidence shows that the energy intensity of primary resource extraction, i.e. the energy required to extract raw materials from the environment, has been steadily increasing over the last fifty years.[14–16] This increasing energy requirement for primary extraction means that less *net energy* is available for downstream uses. If this decline in net energy availability outpaces technological advances in energy efficiency, there may be deleterious impacts on the economic output of the economy.

Within our framework, we do not account for either the material or energetic quality of resources that pass through the economy nor the irreversibility of economic processes. The following two sections briefly discuss both. Thereafter, we raise the issue of thermodynamic irreversibility.

8.5.1 Quality of energy

The First Law of Thermodynamics tells us that the quantity of energy is conserved in every process. This conservation, however, belies an important point regarding the quality of energy—not all forms of energy are equally *useful*. For instance, a bath-full of water has as much thermal energy as can be provided by a pint (half liter) of gasoline.[4] However, the gasoline is a much higher quality store of energy than the water. It is much more useful for performing tasks.

There are several ways to assess the quality of energy. The Second Law of Thermodynamics provides, among other things, a framework for discussing the *quality* of energy. Hammond and Winnett [17] reviewed the influence of thermodynamics on ecological economics and noted the importance of the concept of *exergy*, which combines the First and Second Laws of thermodynamics to describe the maximum physical work which can be performed by an energy resource in coming into equilibrium with its environment, stating that exergy:

> represents the thermodynamic 'quality' of an energy carrier, and that of the waste heat or energy lost in the reject stream. Electricity, for instance, may be regarded as an energy carrier having a high quality, or exergy, because it can undertake work. In contrast, low temperature hot water, although also an energy [re]source, can only be used for heating purposes. This distinction between energy (strictly enthalpy) and exergy is very important when considering a switch, for example, from traditional internal combustion engines to electric, hybrid, or fuel cell vehicles. Thus, … it is important to employ exergy analysis alongside a traditional First Law energy analysis in order to illuminate these issues.

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[4] This assumes a 230 liter bath at 40° C, i.e. 20° C above ambient.

As such, exergy is a measure of how far a resource is from thermodynamic equilibrium with its environment. The further a resource is from equilibrium with the environment—the biosphere—the higher the exergy and the higher the quality of the resource.

The quality of energy can be assessed in terms of economic value, too. Some energy resources, such as liquid fuels, are more economically valuable than others, i.e. within society, there is a preference for these resources, such that, "accounting for energy quality reveals a relatively strong relationship between energy use and economic output." [18, p. 313] We see this preference played out on a daily basis when coal is converted to electricity at an average efficiency of around one third. Society is willing to pay a premium for electricity over coal due to its vastly superior usefulness in being put toward a multitude of tasks.

8.5.2 *Quality of materials*

Similarly to energy resources, non-energy resources are conserved in quantity through every process but also have a range of qualities. An intact brick is of higher material quality—is of more use—than after it has been ground to dust and scattered on the wind. Society relies on material stocks or flows that have concentrated by natural biophysical processes, for example mineral seams or water courses. We do not mine material we need from locations with the average abundance of crustal materials. As with energy, we may measure the material quality of a resource in reference to its environment, in this case, the average chemical composition of its environment. The more concentrated the resource, the further it is from chemical equilibrium with the environment and the higher the quality. Again, exergy is a measure of this kind of quality. The further a resource is from chemical equilibrium with its environment, the higher the exergetic content.

Additionally, it takes more energy to process less concentrated resources and more total material must flow through the process (including overburden—the wasted portion that is extracted) and the greater wear and tear on equipment. This additional processing requirement entails that we will likely never mine average crustal abundance for needed materials, or mine gold from seawater. Furthermore, it also entails that recycling—the act of turning low quality materials into high quality resources—requires both energy and degradation of equipment. The lower quality the waste, the more energy and degradation occurs such that one hundred percent recycling of materials is almost certainly practically (and possibly theoretically) impossible. As such, we can deduce that the economy *must always* be a subsidiary of the biosphere *open* to flows of materials both from (resources) and to (wastes) the biosphere. This has direct implications for dematerialization of our economies, which was discussed in reference to our framework in Section 7.3. There are fundamental limits to the amount of material that must be directed to desired end services, for example, in order to transport a human and some means of propulsion, a car must have some minimum embodied materials.[5] Despite the drive to dematerialization and the apparent "unhooking" of the material and energy intensity of GDP much of the progress by "developed" nations has been by exporting manufacturing to other countries. [20] The material footprint of OECD nations, when weighted by consumption, has increased significantly since 1990. [21]

8.5.3 Process irreversibility

Our framework implicitly assumes, since we make no use of the Second Law, that all economic processes could be run in reverse; that products could be unmade into resources and that the energy and other short-lived material inputs would be spontaneously upgraded back into their original form. This of course is not true. Irreversibility is the concept that naturally-occurring processes are uni-directional in time. For example, bouncing a rubber ball heats it up; heating up a rubber ball does not cause it to spontaneously jump off the table.

Considering the production of electricity from coal, we note that two important processes within a power plant are *irreversible*:

- the combustion process that converts coal to CO_2 and ash (with associated thermal energy release, $\dot{Q}$) and
- the heat transfer process wherein $\dot{Q}$ flows from high temperature to low temperature (with associated mechanical work production).[6]

Thermal energy does not spontaneously flow from low temperature to high temperature. Neither do ash and CO_2 spontaneously combine to form coal.

The concepts of resource quality and irreversibility are inextricably linked. One statement of the Second Law indicates that heat flow is irreversible: it flows only from hot to cold. Because lower-temperature heat is less useful, we say that the thermal energy has degraded when it used (come into equilibrium with the environment). Again, exergy is a measure of irreversibility of processes. Exergy cannot be created, it can only be destroyed, hence a process is *irreversible* if exergy is destroyed during the process.[7]

Energy resources, such as coal, are useful since they are far from equilibrium with their environment, energy may be released into the environment by splitting

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[5] Note that this minimum is likely many times lower than the mass of current automobiles, which are driven largely by preference. The Rocky Mountain Institute has done some work on the ultra-light, "hypercar" concept. [19]

[6] Note that the process of generating electricity via mechanical work is (at least in theory) a *reversible* process. The generator could be run as a motor by electricity to produce mechanical work. In theory, electrical work and mechanical work are fully exchangeable, in reality efficiency losses mean this is not quite the case.

[7] The concept of irreversibility is often discussed in terms of *entropy*, which can only be created and cannot be destroyed. A process that generates entropy is said to be irreversible and all real processes generate entropy.

the carbon-carbon bonds to form carbon-oxygen bonds with free oxygen within the atmosphere. The exergy content of the coal is destroyed during the equilibration process.[8]

Similarly, high grade mineral ores, such as bauxite, are useful since they are far from equilibrium with their environment—the average abundance of materials within the earth's crust. The chemical exergy of materials may be upgraded during processing—bauxite is refined into pure aluminum—but only at the expense of a greater amount of exergy destruction elsewhere—the coal burned to generate the electricity—i.e. the process is irreversible.

We recommend that future work be done to incorporate concepts of resource quality and irreversibility into our framework by accounting for flows and destruction of exergy within economic processes.

8.6 Co-products

The intersection between our materials, energy, and value accounting framework and the I-O literature has been developed under the assumption that each economic sector makes a single product ($\dot{P}$). In particular, the matrix mathematics of Chapter 6 relies heavily on this assumption from the early days of the I-O method.[8] However, the I-O method has been extended in the literature to include co-products for each economic sector.[22, 23] To do so, both *make* and *use* data are employed.

We decided to leverage the older, single-product formulation of the I-O method for the purposes of simplicity. The materials, energy, and value accounting framework presented herein is more easily understood without the complexity of the make-use formulation of the I-O method. However, work remains to adapt the materials and energy framework developed herein to the make-use formulation of the I-O method.

8.7 Are people capital stock?

**** Mik: complete this section. ****

There is an open question as to what sort of *stuff* should be included as the capital that accumulates within society. Should the material constituting literal *human capital*, i.e. human bodies, be included. If humans are to be included within K_1, the resource needed to produce the necessary capital flow ($\dot{K}_{11}$) is food. Food itself represents a large "resource" flow and has a large associated energy content. Additionally, within industrial economies, a large amount of energy resources are channeled toward the production of food, meaning that the *embodied energy* of food may actu-

[8] Strictly speaking, the exergy content of the coal is only fully destroyed when the CO_2 and ash have dispersed to their average concentration within the environment since, in theory, this diffusion (which happens spontaneously) could be used to perform work.

ally be several times larger than the actual energy content of the food itself. Further questions arise. What is the "product" of society? A materialistic view might hold that the product of society is human bodies and the labor they can accomplish. If so, should the agriculture industry be accounted as part of the energy sector because its aim is to provide an energy service (labor)? For non-industrial, agrarian societies, the proportion of total energy flow comprised by manual (or draft) energy may be large. In industrialized societies, it may be negligible, however, the energy flows necessary to support agriculture may be many times larger than the food energy (and certainly many times larger than the labor energy) delivered. Agrarian societies are necessarily constrained by the fact that the energy content of the food delivered *must be* greater than the labor (and draft) energy required to produce it. Another view is that societal capital (K_1) includes only man-made capital, i.e. items manufactured by humans, but not humans themselves. For the purposes of the framework outlined in this book, we shall favor the latter view. Other researchers favor the opposing view.[24] However, the framework presented is general enough to encompass either point of view.

8.8 What about the sun?

Costanza [25] included an option to consider solar energy as an input to the economy, thereby significantly increasing the energy intensity of agricultural sectors and other sectors that depend upon agricultural outputs. However later work by Costanza [22, 26] did not include solar input to the economy. Whether solar input to the economy should be considered in a materials, energy, and economic value accounting framework is probably dependent upon the objectives of the analysis.

The motivation for this particular book is primarily the effects of declining energy resource quality due to fossil fuel depletion on industrialized economies. As such, inclusion of solar flows is probably unnecessary. However, expanding the framework to include non-industrialized or agrarian societies may require accounting for solar energy flows.

There are a number of means by which solar flows can be accounted. Solar energy flows could be accounted as short-term flows ($\dot{S}$) for agricultural and forestry sectors, as well as solar thermal, solar photovoltaic, wind, ocean thermal, hydro, and biomass renewable energy production sectors. Doing so would not account for longer-term storage of solar energy used to form fossil fuels, but fossil fuels are already accounted by the $\dot{E}_0$ vector in the framework presented in this book.

A different approach that fully integrates solar into an energy framework is *emergy* accounting. The emergy method counts *all* material flows in terms of *em*bodied solar en*ergy*.[27, 28] The basic unit of measure is the *em*joule which is often given in terms of flows of solar energy embodied in the energy (or material)—the solar emjoule—per unit of resource, abbreviated to seJ/J for energy resources, or seJ/g for materials. As such, even fossil fuels, e.g. coal, extracted from the earth have an embodied energy of around 67,000 seJ/J.[29]

8.9 What is endogenous?

There is debate in the literature about whether government and households (so-called "final consumption" Figure 2.3) should be endogenous to economic models. This debate is a discussion about the appropriate boundary for analysis. Costanza [26] was the first to endogenize government and households, because households provide services to the economy (labor) in exchange for wages and government provides services to the economy in exchange for taxes, both of which require energy. Costanza [26] also demonstrated that energy intensity results are a function of boundary (control volume) selection. By including government and households as sectors in the model, the variation of energy intensity is significantly reduced across all sectors of the economy.

The key energy intensity equation in this book (Equation 6.38) was derived under the assumption that "final consumption" is exogenous to energy intensity calculation. However, Equation 6.38 could be re-derived to endogenize "final consumption."

The total energy accounting equation for final consumption (1) in Figure 4.4 can be written analogously to Equations 6.16 and 6.17 as

$$\frac{\mathrm{d}B_{K_1}}{\mathrm{d}t} = \dot{E}_{01} + \varepsilon_1 \dot{X}_{11} + \varepsilon_2 \dot{X}_{21} + \varepsilon_3 \dot{X}_{31} - \varepsilon_1 \dot{X}_1 - \left(\dot{B}_{\dot{R}_{10}} + \dot{B}_{\dot{S}\,10} + \gamma_{K,1} B_{K_1}\right). \tag{8.1}$$

Furthermore, Equations 6.16 and 6.17 can be rewritten as

$$\frac{\mathrm{d}B_{K_2}}{\mathrm{d}t} = \dot{E}_{02} + \varepsilon_1 \dot{X}_{12} + \varepsilon_2 \dot{X}_{22} + \varepsilon_3 \dot{X}_{32} - \varepsilon_2 \dot{X}_2 - \left(\dot{B}_{\dot{R}_{20}} + \dot{B}_{\dot{S}\,20} + \gamma_{K,2} B_{K_2}\right) \tag{8.2}$$

and

$$\frac{\mathrm{d}B_{K_3}}{\mathrm{d}t} = \dot{E}_{03} + \varepsilon_1 \dot{X}_{13} + \varepsilon_2 \dot{X}_{23} + \varepsilon_3 \dot{X}_{33} - \varepsilon_3 \dot{X}_3 - \left(\dot{B}_{\dot{R}_{30}} + \dot{B}_{\dot{S}\,30} + \gamma_{K,3} B_{K_3}\right). \tag{8.3}$$

Following the derivation of Chapter 6, we can obtain an updated version of Equation 6.38:

$$\boldsymbol{\varepsilon} = (\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1} \hat{\mathbf{X}}^{-1} \left[\mathbf{E}_0 - \left.\frac{\mathrm{d}\mathbf{B}_K}{\mathrm{d}t}\right|_{\text{other}} - \mathbf{B}_{waste} \right], \tag{8.4}$$

wherein

- the vectors and matrices of Equations 6.19–6.27 and 6.31 have been extended to include "final consumption" (1) and
- "final consumption" has been endogenized (the $\mathbf{T}_1$ term of Equation 6.38 has been subsumed into the $(\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1} \hat{\mathbf{X}}^{-1}$ term of Equation 8.4).

Future work could estimate energy intensity ($\boldsymbol{\varepsilon}$) using Equations 6.38 and 8.4 with updated economic data for a wider range of countries.[9] Doing so could provide further insight on Costanza's result [26] that endogenizing "final consumption" reduces variation of energy intensity across all sectors of the economy ($\boldsymbol{\varepsilon}$).

8.10 Choice of energy input vector

There is discussion in the literature about the $\mathbf{E}_0$ vector and how it should be applied to the economy. Costanza and Herendeen [22] counted fossil fuel input from the biosphere to the economy at both

I. the points where direct energy physically enters the economy from the biosphere, typically energy-producing sectors (called the DIRECT method), and
II. the points of conversion to useful work, typically all energy *consuming* sectors (called the DEC method, Direct Energy Conversion).

Costanza and Herendeen justified the Direct Energy Conversion (DEC) approach on both thermodynamic and economic grounds. The thermodynamic justification derived from the purpose of energy consumption in an economy, namely to produce useful work. If direct energy flows *through* a sector, it should not be counted *against* that sector: only energy that is converted to useful work *within* a sector should be counted against that sector. The economic justification derives from the typical treatment of transportation sectors of the economy. Costanza and Herendeen note:

> The primary energy sectors functions [*sic*] ~~are~~ like the transportation sectors, which alse [*sic*] require special treatment in I-O analysis based on the difference between the services they provide and their physical inputs and outputs. If a strictly physical interpretation were applied to the transportation sectors, they would receive almost all goods produced in the whole economy as inputs and redistribute them as output, masking information on transfers of goods between sectors. For this reason, the transportation sectors in I-O analysis are thought of as providing transportation services that are purchased by the producing sector, preserving the connection between the producing and consuming sector but adding a 'transportation margin.' For analogous reasons, the primary energy sectors should be thought of as providing a 'transportation service' in moving primary energy from nature to the consuming sectors. The DEC energy input vector incorporates this interpretation.[22, p. 151]

The derivation of the materials, energy, and value framework presented herein counts energy flows from the biosphere to the economy at the point of physical inflow to the economy. That is, elements of the energy input vector ($\mathbf{E}_0$) are nonzero only for those sectors that receive energy directly from the biosphere. So, for example, in Figure 3.5 from Example C, $\dot{E}_{03} = 0$ and $\dot{E}_{02} \geq 0$. Our approach is equivalent to Costanza's DIRECT method. We believe that the DIRECT approach is correct and that the DEC method is unwarranted.

[9] Costanza's analysis [26] was conducted using US data for 1963, 1967, and 1972.

Justification for our position comes from the detailed derivation of the materials, energy, and value framework presented in Chapters 2–5.

I. Firstly, $\mathbf{E}_0$ was defined as a flow from the biosphere to economic sectors into which the energy *physically* flows. It is inappropriate to route the energy elsewhere.
II. Secondly, Costanza and Herendeen's concern [22, pp. 130 & 138] about flow-through of direct energy is unfounded, because direct energy outflows from a sector are *never* counted against the sector with the DIRECT method. We see this fact in the following terms:
   A. $-\dot{E}_1$ in Equation 4.12,
   B. $-\dot{E}_j$ in Equation 4.44,
   C. $-\begin{bmatrix}\dot{X}_2 & 0\\ 0 & \dot{X}_3\end{bmatrix}\begin{Bmatrix}\varepsilon_2\\ \varepsilon_3\end{Bmatrix}$ in Equation 6.18, and
   D. $-\hat{\mathbf{X}}\boldsymbol{\varepsilon}$ in Equation 6.29.
III. Thirdly, further proof that the DEC approach is unwarranted comes from equations that show waste heat ($\dot{Q}_{j0}$) as counting toward the accumulation of embodied energy within an economic sector. Equation 4.16 of Section 4.2.2 is an example. It is the waste heat ($\dot{Q}_{10}$), i.e. the energy *burned within* the sector, that counts against the sector.

The DIRECT approach *already always* provides the effect that Costanza and Herendeen [22] desired from the DEC approach. Because the DEC approach is unwarranted, we quoted DIRECT energy intensity values only when discussing energy intensities in Section 6.5.

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Chapter 9
Summary

Only a crisis—actual or perceived—produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around. [1, p. ix]

—Milton Friedman

We said at the outset (Chapter 1) that this book would be about counting and change: counting materials, counting energy, and counting economic value as each flows through economies and doing a better job of understanding our economies so that we will be able to manage the upcoming energy transition. Our motivation for counting carefully and well is mounting evidence that scarcity of materials and energy are affecting the economies of our world, thereby affecting us all. We should know precisely *how* and *at what rate* we are using our material and energy resources today if we are to address the challenge of future transitions to new materials and new energy resources. **** Mik: re-work the previous sentence after writing the introduction. ****

When accounting for material flows through economies (Chapter 2), we found that resources extracted from the biosphere are used for (a) build-up of capital stock within society and economic sectros, (b) production of short-term material flows among economic sectors, or (c) overcoming depreciation of capital stock. When accounting for both materials and energy (Chapter 3), we noted that the material and energy resources upon which economies depend are obtained from the biosphere and wastes and depreciated material return to the biosphere. In this sense, economies are "coupled" to the biosphere. When accounting for embodied energy (Chapter 4), we found that the waste heat from each sector is *additive to* the embodied energy of the products of the sector.

We used control volume accounting equations from thermodynamics to develop our accounting models (Chapters 2–4). Similar equations are often applied to thermodynamic machines: engines, refrigerators, heat pumps, and power plants. Our equations are unabashedly mechanical, and, in this regard, we are utilizing (though not embracing) mainstream economics' machine metaphor to describe the economy. And, our mechanistic modeling approach contrasts with many ecological economists who reject, out of hand, the machine metaphor.

Because economies are coupled to the biosphere, we applied the accounting equations to economies that are *open* to their surroundings. Our metaphor for the economy is the metabolic organism, because organisms also exchange material, energy, and wastes with the biosphere throughout their lives. Our use of the metabolic metaphor is anathema to many mainstream economists but embraced by many ecological economists.

As we accounted for economic value (Chapter 5), we found it necessary to develop careful definitions. We pragmatically accepted the subjective theory of value espoused by mainstream economics, despite our misgivings about externalities, intergenerational equity, and mis-measurement of economic growth. Although we emphatically stop short of an "energy theory of value," we agree with many ecological economists who find it important to estimate and communicate the energy intensity of economic products (Chapter 6). With such information, consumers will be able to make better choices about the products and services they consume.

In Chapter 7, we noted several implications that our framework brings to previous methods of analyzing the interactions among materials, energy, and the economy. In particular, we recommend that a physical accounting framework be adopted if we are to accurately determine both the accumulatino rate of energy embodied in capital stock and the energy embodied in economic products.

Along the way, we noted several items of unfinished business that are collected in Chapter 8. Foremost is the paucity of data needed to account well for materials, energy, and economic value flows through our economies. For example, the US no longer maintains the current account data needed for the analyses presented herein. And, few other countries have ever collected and disseminated required national accounts data on material flows that would allow estimation of the energy intensity of goods.

So, in the end, this book is about *more than* counting. It has become a call-to-arms of sorts for data gathering and analysis. If we are to successfully navigate the coming transitions to new materials and energy resources, we simply *must* understand how we are currently using materials and energy. We can't do so without an improved regime of data collection, dissemination, and analysis about the world's economic metabolism.

And, this book has become an attempt to reconcile mainstream and ecological economics. We hope that mainstream economists will see that it is both possible and essential to account for materials and energy flows between the economy and the biosphere. We believe there should be no reason to fear metrics such as energy intensity that will become increasingly important into the future.

Finally, we hope that ecological economists will see that application of mechanistic thermodynamic accounting equations and careful record-keeping can provide significant insights on the ways that economies interact with the biosphere, all the while accepting the mainstream subjective theory of value. If we can find a little common ground, perhaps we can begin to collect the data and develop the analytical tools, metrics, and knowledge needed to make wise choices for the future.

If we apply our minds directly and competently to the needs of the earth, then we will have begun to make fundamental and necessary changes in our minds. We will begin to understand and to mistrust and to change our wasteful economy, which markets not just the produce of the earth, but also the earth's ability to produce. We will see that beauty and utility are alike dependent upon the health of the world. But we will also see through the fads and the fashions of protest. We will see that war and oppression and pollution are not separate issues, but are aspects of the same issue. Amid the outcries for the liberation of this group or that, we will know that no person is free except in the freedom of other persons, and that man's only real freedom is to know and faithfully occupy his place—a much humbler place than we have been taught to think—in the order of creation. [2, p. 89]

—Wendell Berry

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Appendix A
Value flows for the US auto sector

This Appendix describes the calculations used to estimate the value flows to and from the Auto Industry sector in Chapter 5. The details of the calculations and assumptions made to calculate each of the value flows is described in Table A.1. The data sources are are described in Table A.2. These data are free and available for download from the Bureau of Economic Affairs (BEA) website and instructions for downloading them are included in the descriptions.

Footnotes that belong with Table A.1:

a. Two commodities categorized in the KLEMS data as “Material intermediate inputs are “Wholesale Trade (IOC 4200, $26,580) and “Truck Transportation. (IOC 4840, $5,552). For our calculations, these commodities were recategorized as “Services. The value of the flows in the table reflects the fact that these dollar amounts were subtracted from this ‘Resource flow and added to “Short-lived Goods.

b. To confirm that these fixed asset types (particularly “Custom Software” and “Own account software”) actually originated from the Auto Industry (that is, that they are truly self-made capital), the I-O “Make” table was consulted to ensure that these commodities were made by the Auto Industry.

c. Note that this figure for *Output* Resources (self-use) ($133,961) is slightly lower than the one used to calculate *Input* Resources (self-use) ($139,259). The output value flows are obtained from the standard Annual I-O accounts. The input value flows were obtained from the more detailed KLEMS data(in order to seperate Resources, Energy, and Short-lived Goods). The KLEMS data, like the Fixed Asset data, are more detailed than the standard I-O accounts and are less reliable. They may include trend estimates or other judgments. In 2011, the KLEMS total intermediate inputs to the auto industry is higher than the amount from the Use table: $392,965 vs.$368,476.

Table A.1 Data calculations for auto industry (IOC 3361MV) example.

Value Flow	2011 USD (millions)	Data Calculations
Resources	$175,491	2011 KLEMS Total Material Intermediate Inputs into Auto Industry (IOC 3361MV). Total Material Inputs ($346,882), less self-use ($139,259) and inputs recategorized as services ($32,132).[a] Self-use Resources are defined as the two intermediate commodity inputs: Motor Vehicles, Bodies, Trailers & Parts (IOC 3361, $138,077), and Motor Vehicles (IOC 336A, $1,182).
Energy	3,367	2011 KLEMS Total Energy Intermediate Inputs into Auto Industry. The sum of the value of all “Energy” intermediate inputs.
Short-lived Goods	42,446	2011 KLEMS Total Service Intermediate Inputs into Auto Industry. Total Inputs from Service Sector ($42,446) plus Wholesale Trade and Truck Transportation from the KLEMS Material category.[a] The value of waste services that are part of this value flow is the sum of Water & Sewage (IOC 2213, $123) and Waste Management Services (IOC 5620, $381).
Capital	46,079	2011 Fixed Assets (non-residential detailed estimates). The total amount of Capital Investment by the Auto Industry ($61,260) less the purchase of capital made within the Auto Industry ($15,181, see calculation for Capital (self-use) below).
Capital (self-use)	15,181	2011 Fixed Assets (non-residential detailed estimates). Fixed Assets that appear to be capital made from within the Automobile Industry: Autos, Internal combustion engines, Light trucks (including utility vehicles), Other trucks, buses and truck trailers, Custom software, & Own account software.[b]
Resources (self-use)	133,961	2011 KLEMS Material Intermediate Inputs into Auto Industry (IOC 3361MV) that are goods produced by the Auto Industry. The sum of Motor Vehicles, Bodies, Trailers & Parts (IOC 336A, $138,077) and Motor Vehicles (3361, $1,182).
Gross Economic Output	467,941	2011 Input-Output accounts. The Use of Commodities by Industries before Redefinitions. (Producers’ Prices). The sum of Economic Output Destined for Intermediate (e.g. wholesale) and Final (e.g.retail) Uses. Data downloaded from the Bea.gov website for the Automobile Industry (IOC 3361MV).
Net Economic Output	325,044	2011 Input-Output accounts. The Use of Commodities by Industries before Redefinitions. (Producers’ Prices). The sum of Economic Output Destined for Intermediate (e.g. wholesale) and Final (e.g.retail) Uses ($467,896), less Self-Use of Intermediate Output ($140,316).

Table A.2 BEA data sources for calculations.

Dataset	Details
Use Tables	Annual Input-Output (I-O) accounts. These are the primary industry data collected by the BEA. The Use tables present what industries use what commodities as intermediate goods, and the value of the commodities that end up as final goods. The values are computed at Producers prices. That is, the value includes the sales price, plus sales and excise taxes, less any subsidies. This table provides a link from Industry data to National data. The sum of all final output is a measure of National GDP. An introduction to these data is available on line: `https://www.bea.gov/industry/pdf/industry_primer.pdf.` The tables are located at `http://www.bea.gov/industry/io_annual.htm`.
KLEMS	(K-capital, L-labor, E-energy, M-materials, and S-purchased services) refers to broad categories of intermediate inputs that are consumed by industries in their production of goods and services. The detailed estimates of intermediate inputs of an industry are classified into one of three cost categoriesenergy, materials, and purchased services; the labor cost category equals an industrys compensation to labor from value added, and the capital cost category equals the industrys gross operating surplus plus taxes on production and imports less subsidies. Detailed information on KLEMS can be found in the September 2005 *Survey of Current Business* article: `http://www.bea.gov/scb/pdf/2005/09September/0905_Industry.pdf`. The 1998–2011 KLEMS tables are located at `http://www.bea.gov/industry/more.htm`. That page is located at `http://www.bea.gov`, by clicking on the Industry tab then clicking the link to Annual I-O data, then clicking the link to Underlying detail: Additional data from the Industry Economic Accounts.
Fixed Assets	Fixed Assets Table. Detailed Fixed Assets Table. Categorizes capital investment by industry into three categories: equipment, structure, and software. In order to obtain an estimate of self-use of capital, we had to go to the more detailed tables, which are less reliable than the standard tables. The BEA notes on the detailed tables indicates that "the more detailed estimates are more likely to be based on judgmental trends, on trends in the higher level aggregate, or on less reliable source data." `http://www.bea.gov/national/FA2004/Details/Index.html` The table used for our calculations is found at *www.bea.gov*. Clicking on the *National* tab, scroll down to *Fixed Assets Tables*, click on *Interactive Data Tables*; scroll down to *Detailed Data for Fixed Assets and Consumer Durable Goods*. Then select the XLS spreadsheet for Sec. 2.5 Non-Residential Detailed Estimates for Investment.

Appendix B
Infinite series representation of energy intensity

In this appendix, we show that the I-O method accounts for the infinite recursion of energy demands for production.

The single-sector economy of Figures 2.2, 3.3, 4.2, and 5.3 can be re-drawn as shown in Figure B.1.

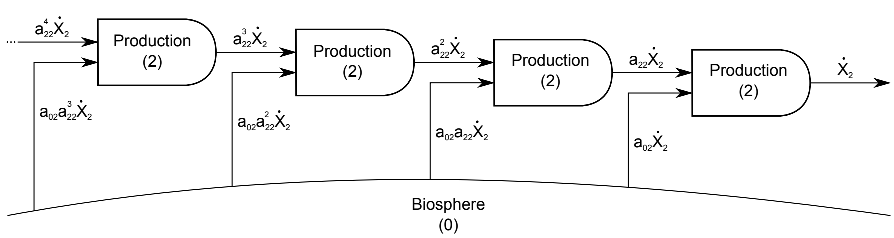

Fig. B.1 Process flows in a single-sector economy.

If we consider the biosphere (0) to be producing a valuable product, namely energy, we can say

$$\dot{X}_{02} = \dot{E}_{02} \tag{B.1}$$

and

$$a_{02} \equiv \frac{\dot{E}_{02}}{\dot{X}_2}. \tag{B.2}$$

The economy produces output at a rate of $\dot{X}_2$, but it requires energy from the biosphere ($\dot{E}_{02} = a_{02}\dot{X}_2$) to do so. The economy also consumes a fraction of its own gross output ($\dot{X}_{22} = a_{22}\dot{X}_2$). To produce $a_{22}\dot{X}_2$, the economy requires an additional $a_{02}a_{22}\dot{X}_2$ of energy from the biosphere. The sum of all direct energy required for the economy to produce at a rate of $\dot{X}_2$ ($\dot{E}_{demand,tot}$) is an infinite sum.

$$\dot{E}_{demand,tot} = a_{02}\dot{X}_2 + a_{02}a_{22}\dot{X}_2 + a_{02}a_{22}^2\dot{X}_2 + \dots \tag{B.3}$$

The energy intensity of the economy (ε_2) is

$$\varepsilon_2 = \frac{\dot{E}_{demand,tot}}{\dot{X}_2} = a_{02}(1 + a_{22} + a_{22}^2 + \ldots) = a_{02} \sum_{n=0}^{\infty} a_{22}^n. \tag{B.4}$$

Realizing that $\sum_{n=0}^{\infty} a_{22}^n = \frac{1}{1-a_{22}}$ and $a_{02} = \frac{\dot{E}_{02}}{\dot{X}_2}$ gives

$$\varepsilon_2 = (1 - a_{22})^{-1} \dot{X}^{-1} \dot{E}_{02}. \tag{B.5}$$

Accounting for the differences between scalar and matrix equations and neglecting energy flows from society to the economy ($\dot{T}_{12} = 0$), accumulation of embodied energy in the economy $\left(\frac{\mathrm{d}B_2}{\mathrm{d}t} = 0\right)$, and physical depreciation ($\gamma_2 B_2 = 0$), Equations 6.38 and B.5 are identical, indicating that the I-O approach accounts for the infinite recursion of energy demand by the economy.

Appendix C
Proof of Equation 6.32

We begin with a restatement of Equation 6.32.

$$\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}} = \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I}) \tag{6.32}$$

We expand the matrices to obtain

$$\begin{bmatrix} \dot{X}_{22} & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} \end{bmatrix} - \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \dot{X}_2 & 0 \\ 0 & \dot{X}_3 \end{bmatrix} \begin{bmatrix} a_{22} - 1 & a_{32} \\ a_{23} & a_{33} - 1 \end{bmatrix}. \tag{C.1}$$

Subtracting and multiplying matrices gives

$$\begin{bmatrix} \dot{X}_{22} - \dot{X}_2 & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} - \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \dot{X}_2 a_{22} - \dot{X}_2 & \dot{X}_2 a_{32} \\ \dot{X}_3 a_{23} & \dot{X}_3 a_{33} - \dot{X}_3 \end{bmatrix}. \tag{C.2}$$

Using $\dot{X}_j a_{ij} = \dot{X}_{ij}$ (see Equation 6.4) gives

$$\begin{bmatrix} \dot{X}_{22} - \dot{X}_2 & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} - \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \dot{X}_{22} - \dot{X}_2 & \dot{X}_{32} \\ \dot{X}_{23} & \dot{X}_{33} - \dot{X}_3 \end{bmatrix} \tag{C.3}$$

to complete the proof.

Appendix D
Estimating the input-output matrix (A)

Using Equation 6.32, which is proved in Appendix C

$$\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}} = \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I}); \tag{6.32}$$

we can derive an expression for estimating the input-output matrix (**A**) given sector outputs ($\hat{\mathbf{X}}$) and the transaction matrix ($\mathbf{X}_t$). Premultiplying both sides of Equation 6.32 by $\hat{\mathbf{X}}^{-1}$ gives

$$\hat{\mathbf{X}}^{-1}\left(\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}}\right) = \mathbf{A}^{\mathrm{T}} - \mathbf{I} \tag{D.1}$$

Further rearranging gives

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\left(\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}}\right) + \mathbf{I}, \tag{D.2}$$

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}}^{-1}\hat{\mathbf{X}} + \mathbf{I}, \tag{D.3}$$

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\mathbf{X}_t^{\mathrm{T}} - \mathbf{I} + \mathbf{I}, \tag{D.4}$$

$$\mathbf{A}^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}\mathbf{X}_t^{\mathrm{T}}, \tag{D.5}$$

and

$$\mathbf{A} = \mathbf{X}_t\left(\hat{\mathbf{X}}^{-1}\right)^{\mathrm{T}}. \tag{D.6}$$

Both $\hat{\mathbf{X}}$ and $\hat{\mathbf{X}}^{-1}$ are diagonal matrices. Therefore, $\left(\hat{\mathbf{X}}^{-1}\right)^{\mathrm{T}} = \hat{\mathbf{X}}^{-1}$, and Equation D.6 becomes

$$\mathbf{A} = \mathbf{X}_t\hat{\mathbf{X}}^{-1}. \tag{D.7}$$

Expanding the matrices of Equation D.7 gives

$$\mathbf{A} = \begin{bmatrix} \dot{X}_{11} & \dot{X}_{12} & \cdots \\ \dot{X}_{21} & \dot{X}_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \frac{1}{\dot{X}_1} & 0 & \cdots \\ 0 & \frac{1}{\dot{X}_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} = \begin{bmatrix} \frac{\dot{X}_{11}}{\dot{X}_1} & \frac{\dot{X}_{12}}{\dot{X}_2} & \cdots \\ \frac{\dot{X}_{21}}{\dot{X}_1} & \frac{\dot{X}_{22}}{\dot{X}_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \tag{D.8}$$

as expected given the definition of the input-output ratio (a) in Equation 6.4:

$$a_{ij} \equiv \frac{\dot{X}_{ij}}{\dot{X}_j}. \tag{6.4}$$

Thus, Equation D.7 provides a method of estimating the input-output matrix (**A**) using the transaction matrix ($\mathbf{X}_t$) and sector outputs ($\hat{\mathbf{X}}$).

Appendix E
Column vs. row vectors in energy intensity equations

In this manuscript, we choose to define energy intensity ($\boldsymbol{\varepsilon}$) and energy input ($\mathbf{E}_0$ and $\mathbf{T}_1$) as a column vectors (see Equations 6.24, 6.21, and 6.22, respectively), because it natural to solve a system of equations for a column vector rather than a row vector. And, Equation 6.18 could not be written as neatly if $\boldsymbol{\varepsilon}$ and $\mathbf{E}_0$ were row vectors.

In contrast, the I-O literature (see, e.g., [1] and [2]) defines energy intensity and energy input as row vectors. The row vs. column difference is manifest in the appearance of the energy intensity matrix equation.

To demonstrate that our column vector formulation is equivalent to the literature's row vector formulation, this appendix derives a column vector version of the energy intensity equation that is often found in the literature. The point of comparison is Casler [1]. Casler's [1] energy intensity (Equation 6) was derived from row vectors as[1]

$$\boldsymbol{\varepsilon} = \mathbf{E}\hat{\mathbf{X}}^{-1}(\mathbf{I} - \mathbf{A})^{-1}. \tag{E.1}$$

We begin with Equations 3 and 4 from Casler [1], converted to overdot notation for rates.

$$\varepsilon_1 \dot{X}_{11} + \varepsilon_2 \dot{X}_{21} = \varepsilon_1 \dot{X}_1 \tag{E.2}$$

$$\varepsilon_1 \dot{X}_{12} + \varepsilon_2 \dot{X}_{22} + \dot{E}_{02} = \varepsilon_2 \dot{X}_2 \tag{E.3}$$

Adding an $\dot{E}_{01}$ term[2] and utilizing matrix notation with column vectors (instead of row vectors) gives

$$\begin{bmatrix} \dot{X}_{11} & \dot{X}_{21} \\ \dot{X}_{12} & \dot{X}_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \dot{E}_{01} \\ \dot{E}_{02} \end{Bmatrix} = \begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix}. \tag{E.4}$$

[1] Equation E.1 is written according to the variable conventions in this manuscript. The literal Equation 6 in Casler [1] is $\varepsilon = E\hat{X}^{-1}(I - A)^{-1}$.

[2] Note that $\dot{E}_{01} = 0$ for Casler [1], so $\dot{E}_{01}$ can be included without changing Equation E.2.

Substituting $\dot{X}_{ij} = a_{ij}\dot{X}_j$ (from Equation 6.4) gives

$$\begin{bmatrix} a_{11}\dot{X}_1 & a_{21}\dot{X}_1 \\ a_{12}\dot{X}_2 & a_{22}\dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \dot{E}_{01} \\ \dot{E}_{02} \end{Bmatrix} = \begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix}. \tag{E.5}$$

Expanding Equation E.5 gives

$$\begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \dot{E}_{01} \\ \dot{E}_{02} \end{Bmatrix} = \begin{bmatrix} \dot{X}_1 & 0 \\ 0 & \dot{X}_2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix}. \tag{E.6}$$

With the definitions of $\hat{\mathbf{X}}$, $\mathbf{A}$, $\boldsymbol{\varepsilon}$, and $\mathbf{E}_0$ from Equations 6.26, 6.31, 6.21, and 6.24, respectively, we can rewrite Equation E.6 as

$$\hat{\mathbf{X}}\mathbf{A}^{\mathrm{T}}\boldsymbol{\varepsilon} + \mathbf{E}_0 = \hat{\mathbf{X}}\boldsymbol{\varepsilon}. \tag{E.7}$$

Solving for $\boldsymbol{\varepsilon}$ gives

$$\boldsymbol{\varepsilon} = \left(\mathbf{I} - \mathbf{A}^{\mathrm{T}}\right)^{-1} \hat{\mathbf{X}}^{-1} \mathbf{E}_0. \tag{E.8}$$

The differences between Equations E.1 and E.8 are due to the choice of row vectors (for Equation E.1) or column vectors (for Equation E.8) only. Note that Equation E.8 is similar to Equation 6.38. A detailed discussion of the differences between Equations E.8 and 6.38 can be found in Section 7.1.

References

[1] Stephen Casler and Suzanne Wilbur. Energy input-output analysis: A simple guide. *Resources and Energy*, 6(2):187–201, 1984.

[2] C.W. Bullard, P.S. Penner, and D.A. Pilati. Net energy analysis: Handbook for combining process and input-output analysis. *Resources and energy*, 1(3):267–313, 1978.

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