Matthew Heun, Michael Dale, Becky Haney

The metabolic economy

A dynamic model for energy and material flows

August 7, 2013

Springer

Contents

Chapter 1
Introduction

1.1 Traditional view of economy

1.2 Brief history of input-output (I-O) modeling

Input-output analysis, developed by Wassilly Leontief in the 1930's as an extension to the work of Quesnay and Walras [**?**], is of primary importance in national accounting, allowing determination of the structure of an economy as well as, among other things, calculation of a nation's gross domestic product (GDP), the predominant measure of economic activity.

1.3 Basic I-O method

The basic premise of the I-O method, as outlined in Figure **??**A, is that each economic sector takes in factors of production from other sectors (and possibly itself) to produce an economic good at some rate. E.g., the automotive sector takes in steel, rubber, glass, etc. and produces a number of cars per year. In contrast to high-level economic growth models that include only a few factors of production (such as land, capital, and labor), the I-O analysis technique allows many differentiated factors of production and raw material feedstocks. [**?**] In I-O frameworks, each factor of production is considered to be the output from a sector of the economy. As will be discussed later [MAKE SURE TO DISCUSS THIS LATER!], the traditional primary factors of production (land, capital, and labor) are not *flows* into the production processes. Rather, they are *stocks* that, when present, allow factors of production (steel, rubber, and glass) to be transformed into final products (automobiles).

In addition to stocks of land, capital, and labor, a flow of energy (or more precisely, the degradation of an exergetic gradient/destruction of exergy) is also required for economic activity. These energy flows originate from the natural environment, recognition of which has provoked researchers from fields of net energy analysis (NEA), material flow analysis (MFA), industrial ecology (IE) and life-cycle assessment (LCA) to extend the traditional (Leontief) input-output framework to include important material and energy flows to and from the environment, as depicted in Figure **??**B [**?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**]. While the Leontief I-O approach relies exclusively on monetary units to represent value flows among sectors of an economy, the key insight of these extensions of the Leontief I-O framework is to rely upon physical units (especially energy units of joules) to represent some of the value flows among economic sectors. In doing so, energy and material intensities of value flows can be estimated. Their approaches are similar to Figure **??**B.

Both the original Leontief I-O framework and the extensions cited above assume steady-state conditions in an economy, i.e., flows of value and material into and out of each economic sector are in balance. Dynamic or transient behavior of the economic system is not considered. Thus, there is no accumulation of economic factors or embodied energy within any of the sectors. The analysis techniques provide "snapshots" of economic activity at an instant in time.

[MIK'S NEW ADDITION]

Assuming no accumulation of materials, within economic sectors or society itself, is tantamount to assuming that *all* material flows through the economy are directed toward the production of non-durable goods. However, evidence of the durability of goods and the accumulation of materials surrounds us. Furthermore, energy was required to both fabricate and emplace the durable goods and infrastructure of modern economies. (The energy it took to create the durable goods and infrastructure can be considered "embodied" within the built environment, a point to which we will return in detail later). As Georgescu-Roegen notes, "in the everyday world one cannot possibly cross a river only on the flow of maintenance materials of a non-existent bridge." [**?**].

Analysis methods that neglect the accumulation of materials and embodied energy in the durable goods and infrastructure of the everyday world lack explanatory power. Such models can tell us how at what rates materials and energy are required to *use* our built environment. But, such models cannot tell us *how* the built environment came to be (and how much energy was required to construct it) or *why* flows of goods are needed. To use Georgescu-Roegen's imagery, models that neglect accumulation fail to explain why we need any material flows to maintain a non-existent bridge. Stocks of accumulated materials (capital, appliances, even people) are the drivers of demand. It is to service their needs and wants that we put the economy to work.

Because economic activity requires energy, we need to understand the way energy flows through economies. The steady-state I-O techniques of Bullard, Herendeen, and others[**?**, **?**] [REFERENCES NEEDED –MKH] offer a means to that end. We contend, however, that these techniques need to be extended and modified to include transient effects that arise when durability of goods and infrastructure (and associated embodied energy) are considered. This paper attempts to address that need.

1.4 An I-O method for dynamic (transient) economic analysis

In this paper, we develop a physical input-output, matrix-based method for modeling multi-sector economies, in the tradition of Georgescu-Roegen's "flow-fund" model [**?**, **?**]. The method presented in this paper takes a decidedly engineering approach to extend the techniques of Bullard, Herendeen, and others to account for durability of goods and embodied energy. This method allows us to see how energy and materials flow through the economy, where embodied energy accumulates in the economy, and how declining resource quality may affect these dynamics. [NEED TO MAKE SURE WE ACHIEVE THIS LAST POINT]

This paper is organized as follows. We first discuss methodology and the model economy. Thereafter, we present three examples, each with increasing levels of disaggregation among society, the energy sector, and goods and services sectors, culminating with a matrix formulation of the new method. The examples leverage the First Law of Thermodynamics, account for total energy ((T), and develop accounting relationships for embodied energy (B). Within the examples, we develop a precise definition for embodied energy and a matrix formulation of the method that can be extended to an arbitrarily large number of economic sectors. Finally, we draw several implications from the development of the new method.

Chapter 2
Material flows

2.1 Methodology

2.2 Example A: one sector economy

2.3 Example B: two sector economy

2.4 Example C: three sector economy

Chapter 3
Direct energy flows

3.1 Methodology

3.2 Example A: one sector economy

3.3 Example B: two sector economy

3.4 Example C: three sector economy

Chapter 4
Embodied energy flows

4.1 Methodology

4.2 Example A: one sector economy

4.3 Example B: two sector economy

4.4 Example C: three sector economy

Chapter 5
Value flows

5.1 Methodology

5.2 Example A: one sector economy

5.3 Example B: two sector economy

5.4 Example C: three sector economy

Chapter 6
Energy intensity

Chapter 7
Implications

Several implications can be drawn from the above detailed development of the I-O method equations in a manner that includes both embodied energy accumulation and depreciation.

7.1 Implications for economic "development"

[IT WOULD BE GOOD TO HAVE A COMPARISON BETWEEN $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ AND STANDARD METRIC OF DEVELOPMENT, I.E. GDP WHICH I GUESS WOULD BE SOMETHING LIKE $\sum_i \dot{X}_i$. WE CAN CERTAINLY ENVISION SITUATIONS WHERE $\sum_i \dot{X}_i$ IS INCREASING AND

One consequence of economic "progress" or "development" is that embodied energy accumulates in economic sectors and society. In fact, accumulation of embodied energy in economic sectors and society could be considered a *proxy* of development. This proxy for development is overly materialistic, one-dimensional, and reductionist, but alternatives such as GDP can be similarly criticized. In fact, GDP could continue to increase whilst accumulation of embodied energy or value actually decreased.

Figure **??** shows that energy extraction from the Earth is what ultimately drives development as measured by the accumulation of embodied energy in the economy and society. Development occurs over time. If embodied energy is the measure, development can be expressed as the integral of $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ for economic sectors

$$\mathbf{B}(t) = \mathbf{B}(0) + \int_{t=0}^{t=t} \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\mathrm{d}t, \tag{7.1}$$

or, using Equation **??**, as the integral of $\frac{\mathrm{d}B_2}{\mathrm{d}t}$ for society,

$$B_2(t) = B_2(0) + \int_{t=0}^{t=t} \frac{\mathrm{d}B_2}{\mathrm{d}t}\mathrm{d}t = B_2(0) + \int_{t=0}^{t=t} (Y_{\dot{T}} - \gamma_2 B_2 - \dot{Q}_{21})\mathrm{d}t. \tag{7.2}$$

Using embodied energy is obviously an incomplete measure of development. We might also use $X(t) = X(0) + \int \frac{dX}{dt} dt$. In fact, B and X are two complimentary factors to the economic process. For capital, B, to be useful, we need direct energy, E (to run the capital) and economic value, X (i.e. money). Therefore each of these factors are necessary, but insufficient.

Table 7.1 describes some of the dynamics that can be observed from Equation **??**. It is quite possible that, especially for regions like the U.S. and Western Europe, the rate of embodied energy accumulation in the economy $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ will be small relative to the rate of energy extraction from the Earth ($\mathbf{E}$). On the other hand, in rapidly developing countries, like China or India, the rate of embodied energy accumulation in the economy may be significantly higher than in a developed economy.

Table 7.1 Factors from Equation **??** affecting the rate of embodied energy accumulation in the economy.

Right-side term	Implication
$\hat{\mathbf{X}}$	As economic output increases, $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ goes up (as will $\mathbf{E}$)
$\mathbf{A}$	As input-ouput ratios increase, $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ goes up
ε	As the energy intensity of the economy increases, $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ goes up
$\mathbf{E}$	As the rate of energy flow from the Earth increases, $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ goes up
$\hat{\gamma}$	As the depreciation rate increases, $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ goes down
$\mathbf{B}$	As the embodied energy in the economy increases, $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ goes down

The behavior of $\mathbf{B}$ with $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ is vitally important. A developed economy has significantly higher embodied energy ($\mathbf{B}$) than a developing economy, and, thus, the outflow rate of embodied energy due to depreciation ($\hat{\gamma}\mathbf{B}$) will be higher. As increasingly large amounts of energy are embodied in the economy, increasingly large energy extraction rates ($\mathbf{E}$) are required to offset depreciation ($\hat{\gamma}\mathbf{B}$) and maintain positive growth $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} > 0\right)$ in the sectors of the economy. Depreciation may also be, temporarily, offset by increasing energy efficiency, i.e. by decreasing energy intensity, ε.

In a similar manner, Equation **??** indicates that maintaining a positive rate of societal development $\left(\frac{\mathrm{d}B_2}{\mathrm{d}t} > 0\right)$ requires ever increasing embodied energy input rates to society ($Y_{\dot{T}}$) as the society "develops." This mechanism provides a natural brake to the continued growth of physical economies.

7.2 Implications for the I-O method

The I-O literature (examples include Bullard (1975) and Cassler (1983)) usually writes Equation **??** as

$$\varepsilon = (\mathbf{I} - \mathbf{A}^T)^{-1} \hat{\mathbf{X}}^{-1} \mathbf{E}. \tag{7.3}$$

It is clear from comparison of Equations **??** and 7.3 that the literature is not accounting for accumulation of energy in the economic sectors $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$, nor does it account for physical depreciation ($\hat{\gamma}\mathbf{B}$). To be precise, the literature assumes

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \hat{\gamma}\mathbf{B} = \mathbf{0}. \tag{7.4}$$

Examining Equation **??**, we see that to the extent that $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \hat{\gamma}\mathbf{B} \ll \mathbf{E}$, estimates of energy intensity (ε) obtained with the assumption of Equation 7.4 contain little error. However, when the sum of the accumulation and depreciation rates $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \hat{\gamma}\mathbf{B}\right)$ becomes significant relative to the rate of energy extracted from the Earth ($\mathbf{E}$), estimates of economic sector energy intensities (ε) using the assumption of Equation 7.4 have a high-side bias (assuming that $\frac{dB}{dt} > 0$ and $\gamma B > 0$). As discussed above, the assumption of Equation 7.4 can be violated in developing economies because accumulation $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ is large or in developed economies because depreciation ($\hat{\gamma}\mathbf{B}$) is large.

The assumption of Equation 7.4 may cause another challenge for energy analysts. The I-O method is often used to estimate energy intensities for each sector of the economy (ε) with Equation 7.4. With ε values in hand, one can estimate changes in energy demand from the Earth ($\mathbf{E}$) as the output of economic sectors ($\hat{\mathbf{X}}$) increases or decreases by solving Equation 7.3 for $\mathbf{E}$.

$$\mathbf{E} = \hat{\mathbf{X}}(\mathbf{I} - \mathbf{A}^{\mathrm{T}})\varepsilon \tag{7.5}$$

When accumulation and depreciation terms are included, we see that the energy demands ($\mathbf{E}$) must be calculated differently. Solving Equation **??** for $\mathbf{E}$ gives

$$\mathbf{E} = \hat{\mathbf{X}}(\mathbf{I} - \mathbf{A}^{\mathrm{T}})\varepsilon + \left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \hat{\gamma}\mathbf{B}\right). \tag{7.6}$$

By comparing Equations 7.5 and 7.6, we see that to the extent that accumulation $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ and depreciation ($\hat{\gamma}\mathbf{B}$) are non-zero, estimates of energy demand are too low. If the sum of accumulation $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ and depreciation ($\hat{\gamma}\mathbf{B}$) are small relative to total energy demand ($\mathbf{E}$), then neglecting these effects causes little error. Economies with fast growth rates $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$ or large sizes ($\mathbf{B}$) are more likely to violate the typical assumptions in the literature.

7.3 Implications for recycling, reuse, and dematerialization

Dematerialization is the idea that economic activity can be unlinked from material or energy demands (UNEP, 2011). One of the primary methods for dematerializing an economy is reuse and recycling of materials. The impact of recycling can be seen in the I-O formulation only when depreciation and accumulation are included.

One effect of recycling is to reduce the magnitude of the disposal rate ($\hat{\gamma}$). Equation **??** indicates that recycling of material in an economy, thereby reducing $\hat{\gamma}$, will slow the effect of depreciation $\left(\hat{\gamma}\mathbf{B}\right)$ and put upward pressure on growth $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$.

Recycling has a mixed effect on energy demand (**E**). Because recycled material displaces newly-produced material in the economy and society, recycling will tend to reduce energy demand (**E**). Equation **??** indicates that this displacement effect will put downward pressure on growth $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$. However, recycling processes require energy to operate, thereby increasing energy demand (**E**). Equation **??** indicates that additional energy demand will put upward pressure on growth $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$.

If recycling produces a net reduction in energy demand (**E**), that is if the effect of displaced production dominates over the effect of energy consumed in recycling processes, the upward pressure on growth $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$ from decrease in $\hat{\gamma}$ and the downward pressure on growth from net reduction of **E** offset each other, the growth rate $\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)$ will remain near zero, and total embodied energy $\left(\mathbf{B}\right)$ will remain constant. In that scenario, dematerialization can develop: reduced material and energy input (**E**) can be accompanied by no change in growth $\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)$.

7.4 Comparison to a Steady-state Economy

****** Finish this section. In terms of what a SSE would look like in the I-O framework, at first blush, I would think that dB/dt = 0 is one aspect. Also, with no growth, inflow rates = depreciation rates. The larger that B is for any society, the larger E must be (to overcome depreciation). To minimize E, hyper-recycling is probably useful. Those are at least a place to start. ******

****** In our discussion, we also addressed the attempts at SSE from point of view of society. In order to achieve this goal *without* recycling, the goods and services sector should have to increase extraction to offset decreasing ore grade, the energy sector should have to increase extraction of energy to allow increasing extraction (unless efficiency could make up the gap - unlikely) in which case the SSE would be violated from these two and from the POV of the earth. ******

Appendix A
Proof of Equation ??

We begin with a restatement of Equation **??**.

$$\mathbf{X}_t^{\mathrm{T}} - \hat{\mathbf{X}} = \hat{\mathbf{X}}(\mathbf{A}^{\mathrm{T}} - \mathbf{I}) \tag{A.1}$$

We expand the matrices to obtain

$$\begin{bmatrix} \dot{X}_{33} & \dot{X}_{43} \\ \dot{X}_{34} & \dot{X}_{44} \end{bmatrix} - \begin{bmatrix} \dot{X}_3 & 0 \\ 0 & \dot{X}_4 \end{bmatrix} = \begin{bmatrix} \dot{X}_3 & 0 \\ 0 & \dot{X}_4 \end{bmatrix} \begin{bmatrix} a_{33} - 1 & a_{43} \\ a_{34} & a_{44} - 1 \end{bmatrix}. \tag{A.2}$$

Multiplication of the matrices provides

$$\begin{bmatrix} \dot{X}_{33} - \dot{X}_3 & \dot{X}_{43} \\ \dot{X}_{34} & \dot{X}_{44} - \dot{X}_4 \end{bmatrix} = \begin{bmatrix} \dot{X}_3 a_{33} - \dot{X}_3 & \dot{X}_3 a_{43} \\ \dot{X}_4 a_{34} & \dot{X}_4 a_{44} - \dot{X}_4 \end{bmatrix}. \tag{A.3}$$

Using $\dot{X}_j a_{ij} = \dot{X}_{ij}$ (see Equation **??**) gives

$$\begin{bmatrix} \dot{X}_{33} - \dot{X}_3 & \dot{X}_{43} \\ \dot{X}_{34} & \dot{X}_{44} - \dot{X}_4 \end{bmatrix} = \begin{bmatrix} \dot{X}_{33} - \dot{X}_3 & \dot{X}_{43} \\ \dot{X}_{34} & \dot{X}_{44} - \dot{X}_4 \end{bmatrix} \tag{A.4}$$

to complete the proof.

Appendix A
Proof of Equation ??

We begin with a restatement of Equation **??**.

$$\hat{\mathbf{X}} - \mathbf{X}_t^{\mathrm{T}} = \hat{\mathbf{X}}(\mathbf{I} - \mathbf{A}^{\mathrm{T}}) \tag{A.1}$$

We expand the matrices to obtain

$$\begin{bmatrix} \dot{X}_3 & 0 \\ 0 & \dot{X}_4 \end{bmatrix} - \begin{bmatrix} \dot{X}_{33} & \dot{X}_{43} \\ \dot{X}_{34} & \dot{X}_{44} \end{bmatrix} = \begin{bmatrix} \dot{X}_3 & 0 \\ 0 & \dot{X}_4 \end{bmatrix} \begin{bmatrix} 1 - a_{33} & -a_{43} \\ -a_{34} & 1 - a_{44} \end{bmatrix}. \tag{A.2}$$

Multiplication of the matrices provides

$$\begin{bmatrix} \dot{X}_3 - \dot{X}_{33} & -\dot{X}_{43} \\ -\dot{X}_{34} & \dot{X}_4 - \dot{X}_{44} \end{bmatrix} = \begin{bmatrix} \dot{X}_3 - \dot{X}_3 a_{33} & -\dot{X}_3 a_{43} \\ -\dot{X}_4 a_{34} & \dot{X}_4 - \dot{X}_4 a_{44} \end{bmatrix}. \tag{A.3}$$

Using $\dot{X}_j a_{ij} = \dot{X}_{ij}$ (see Equation **??**) gives

$$\begin{bmatrix} \dot{X}_3 - \dot{X}_{33} & -\dot{X}_{43} \\ -\dot{X}_{34} & \dot{X}_4 - \dot{X}_{44} \end{bmatrix} = \begin{bmatrix} \dot{X}_3 - \dot{X}_{33} & -\dot{X}_{43} \\ -\dot{X}_{34} & \dot{X}_4 - \dot{X}_{44} \end{bmatrix} \tag{A.4}$$

to complete the proof.

Appendix A
Derivation of Equation ??

We begin with a restatement of Equation **??**.

$$\hat{\mathbf{X}} - \mathbf{X}_t^{\mathrm{T}} = \hat{\mathbf{X}}(\mathbf{I} - \mathbf{A}^{\mathrm{T}}) \tag{A.1}$$

We take the inverse of both sides of the equation to obtain

$$\left(\hat{\mathbf{X}} - \mathbf{X}_t^{\mathrm{T}}\right)^{-1} = \left(\hat{\mathbf{X}}(\mathbf{I} - \mathbf{A}^{\mathrm{T}})\right)^{-1}. \tag{A.2}$$

We now apply the following matrix identity (formula 6.2, pg. 308 from [**?**])

$$(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1} \tag{A.3}$$

to the right side of Equation A.2 to obtain

$$\left(\hat{\mathbf{X}} - \mathbf{X}_t^{\mathrm{T}}\right)^{-1} = (\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1}\hat{\mathbf{X}}^{-1}, \tag{A.4}$$

which is identical to Equation **??**.