1 Mathematical model from Jones (2001) [take 2]

The longwave growth model used as a basis for this project was taken from Jones (2001). The production function that Jones is using to ultimately predict output is given by equation 1 and is a function of the stock of ideas (a_{t}), labor in the goods producing sector ($l_{\mathrm{Y,t}}$), land (t_{t}), and an exogenous shock parameter (ε_{t}).

$$y_{\mathrm{t}} = a_{\mathrm{t}}^{\sigma} l_{\mathrm{Y,t}}^{\beta} t_{\mathrm{t}}^{1-\beta} \varepsilon_{\mathrm{t}} \tag{1}$$

The production function variables are indexed, referenced against some base year to non-dimensionalize them, and given by the following definitions. Where Y represents the output (\$/year), A represents the stock of knowledge and ideas (ideas/year), and t represents the stock of available land (land/year).

$$y \equiv \frac{Y}{Y_0}, \tag{2}$$

$$a \equiv \frac{A}{A_0}, \tag{3}$$

$$t \equiv \frac{T}{T_0}, \tag{4}$$

The utility function from Jones (2001) was the basis for our model and is of the form found in equation 5. The utility function is used to represent an individuals preferences towards consumption or having children at a point in time.

$$u(c_{\mathrm{t}}, b_{\mathrm{t}}) = \frac{1-\mu}{1-\gamma}\left(\frac{k_{\mathrm{c}}\tilde{c}_{\mathrm{t}}}{U_{\mathrm{c},0}}\right)^{1-\gamma} + \frac{\mu}{1-\eta}\left(\frac{k_{\mathrm{b}}\tilde{b}_{\mathrm{t}}}{U_{\mathrm{b},0}}\right)^{1-\eta}, \tag{5}$$

Where μ, γ, and η are dimensionless parameters having values between 0 and 1. k_{c} and k_{b} represent the amount of utility an individual obtains per unit consumption or birth, in units of utils/\$ and utils/birth respectively. The variable $\tilde{c}_{\mathrm{t}}$ is the amount of the individual's consumption in \$/year-person and $\tilde{b}_{\mathrm{t}}$ is the number of births in births/year-person. Both the consumption and birth utility terms are divided by the utility in the first year of the simulation, U_0, which is taken to be the year 25,000 B.C. from Jones. It is also important to note that $\tilde{c}_{\mathrm{t}}$ is the amount of consumption above a certain subsistence level and $\tilde{b}_{\mathrm{t}}$ represents the amount of births above some long-run birth rate, defined by equation 6 and equation 7.

$$\tilde{c}_{\mathrm{t}} \equiv c_{\mathrm{t}} - \bar{c}, \tag{6}$$

$$\tilde{b}_{\mathrm{t}} \equiv b_{\mathrm{t}} - \bar{b}, \tag{7}$$

Where $\bar{c}$ is the subsistence level of consumption and $\bar{b}$ is the long term birth rate.

The utility function is subject to the following first order condition.

$$\frac{\partial u / \partial \tilde{b}}{\partial u / \partial \tilde{c}} = \frac{w_{\mathrm{t}}}{\alpha}, \tag{8}$$

Where w_{t} is the wage rate that a person is paid in \$/year and α is an equivalent "wage rate" on the number of births per year, or an equivalent monetary amount that a person derives from each birth in \$/year. The appearance of these two coefficients are from the time constraints in the economy. These time constraints are given by equations 9, 10, and 11.

$$L = \tau_{\mathrm{t}} N, \tag{9}$$

$$c_{\mathrm{t}} = w_{\mathrm{t}} \tau_{\mathrm{t}}, \tag{10}$$

$$b_{\mathrm{t}} = \alpha(1 - \tau_{\mathrm{t}}), \tag{11}$$

Where τ is a number between 0 and 1 representing the fraction of available time that people spend on labor (unitless), N is the total population, and L is the total amount of available labor in the economy. So, $(1 - \tau)$ represents the fraction of available time that's spent on birth.

The time constraints, in conjunction with the first order condition, reduce the utility function down to a form that directly relates birth and consumption, as shown in equation 12.

$$\frac{k_{\mathrm{b}}}{U_{\mathrm{b},0}} \tilde{b}_{\mathrm{t}} = \left[\frac{k_{\mathrm{b}}}{U_{\mathrm{b},0}} \frac{U_{\mathrm{c},0}}{k_{\mathrm{c}}} \frac{\alpha \mu}{1 - \mu} \frac{\left(\frac{k_{\mathrm{c}}}{U_{\mathrm{c},0}} \tilde{c}_{\mathrm{t}}\right)^{\gamma}}{w_{\mathrm{t}}} \right]^{1/\eta}, \tag{12}$$

Where both sides of this equation are dimensionless (to allow application of the exponential factor).

The labor section of the economy can be split into two different sectors; one devoted to innovation and producing new ideas and one devoted to producing goods. The split between these sectors is based on the wages paid to each, and given by equations 13 and 14.

$$w_{\mathrm{A,t}} L_{\mathrm{A,t}} = \pi_{\mathrm{t}} Y_{\mathrm{t}}, \tag{13}$$

$$w_{\mathrm{Y,t}} L_{\mathrm{Y,t}} = (1-\pi) Y_{\mathrm{t}}, \tag{14}$$

Where $w_{\mathrm{A,t}}$ and $w_{\mathrm{Y,t}}$ are the wages paid, in \$/year-person, to workers in the knowledge and goods producing sectors respectively, the variables $L_{\mathrm{A,t}}$ and $L_{\mathrm{Y,t}}$ are the number of workers employed by each of the sectors, and π_{t} is the fraction of the economy's total output that is devoted to compensating inventors. In equilibrium it is expected that the wages will be equal, as shown in the following conditon.

$$w_{\mathrm{A,t}} = w_{\mathrm{Y,t}} = w_{\mathrm{t}}, \tag{15}$$

Wage equality allows π_{t} to be directly proportional to the fraction of the total employed population working in the knowledge sector.

2 Mathematical model from Jones (2001)

The mathematical model being used for this project was taken from Jones (2001). The production function that Jones uses is defined in Definitions of the production function variables, indexed against some base year.

Equations representing compensation for innovation in the knowledge and labor sector respectively.

$$w_{\mathrm{At}} L_{\mathrm{At}} = \pi_{\mathrm{t}} Y_{\mathrm{t}} \tag{16}$$

$$w_{\mathrm{Yt}} L_{\mathrm{Yt}} = (1-\pi_{\mathrm{t}}) Y_{\mathrm{t}} \tag{17}$$

Also, in static equilibrium we expect:

$$w_{\mathrm{A}} = w_{\mathrm{Y}} = w_{\mathrm{t}} \tag{18}$$

$$\pi = \frac{L_{\mathrm{At}}}{L} \tag{19}$$

Differential equation representing the change in the stock of knowledge over time.

$$\frac{da}{dt} = \delta l_{\mathrm{At}}^{\lambda} a_{\mathrm{t}}^{\phi} \tag{20}$$

The utility function is subject to the first-order condition.

The first order condition, in conjunction with the time constraints, implies that, Indexed working population.

$$n \equiv \frac{N}{N_0} \tag{21}$$

Differential equation representing the change in the population over time.

$$\frac{dn}{dt} = b_{\mathrm{t}} n_{\mathrm{t}} - d_{\mathrm{t}} n_{\mathrm{t}} \tag{22}$$

Death rate equation.

$$d_{\mathrm{t}} = \frac{1}{\omega_1 z_{\mathrm{t}}^{\omega_2} + \omega_3 z_{\mathrm{t}}} + \bar{d} \tag{23}$$

2.1 A second order heading

Some text under the subheading. Paragraphs that follow heads are not indented.

Math should also be set in Times. Use the mathptmx package if you do not have any of the commercially available fonts that are compatible with Times.

$$y^{(n)} = \sum_{i=0}^{n-1} a_i(x) y^{(i)} + r(x) \tag{24}$$

All environments provided by the standard LaTeX document classes are unchanged. Vertical spaces within lists have been altered to comply with De Gruyter requirements.

1. This is the first item within the list. Some more text here in order to display the alignment.
2. Another item in the list.
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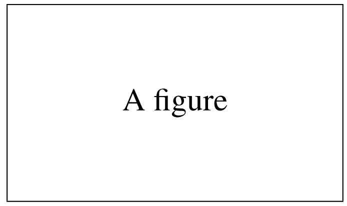
Here is an example of a Figure. It's the same as in standard LaTeX.

An example of a table follows. This is also the same as in standard LaTeX.

Use the `thebibliography` environment for the references. BibTeX users may use the provided BibTeX style file DeGruyter.bst.

References

Lamport, L. (1994): *LaTeX: A Document Preparation System: User's Guide and Reference Manual*, Reading, MA, USA: Addison-Wesley, second edition.



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Symbol	LaTeX Command	Symbol	LaTeX Command
α	`\alpha`	ζ	`\zeta`
β	`\beta`	η	`\eta`
γ	`\gamma`	θ	`\theta`
δ	`\delta`	ϑ	`\vartheta`
ϵ	`\epsilon`	ι	`\iota`
ε	`\varepsilon`	κ	`\kappa`

Mittelbach, F., M. Goossens, J. Braams, D. Carlisle, C. Rowley, C. Detig, and J. Schrod (2004): *The LaTeX Companion*, Tools and Techniques for Computer Typesetting, Reading, MA, USA: Addison-Wesley, second edition.

Oetiker, T. (2008): *The Not So Short Introduction to LaTeX 2_{ε}*, 4.26 edition, URL `http://ctan.tug.org/tex-archive/info/lshort/english/lshort.pdf`.