Toward a comprehensive, consumer-sided energy rebound analysis framework

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Abstract

(242 words at present)
Implementation of widespread energy efficiency measures is an important global strategy to meet Paris greenhouse gas emissions targets. However, energy rebound can "take back" efficiency-based energy reductions via behavior changes and economic effects. Evidence is growing that rebound effects are larger than typically assumed, meaning that efficiency-based efforts to reduce carbon emissions will be less effective than supposed. To date, most work on energy rebound is empirical and microeconomic. Although microeconomic theory is available, it is scattered across papers and lacks rigorous theoretical foundations. Furthermore, there is a paucity of both theoretical and empirical work on macro rebound. Consequently, understanding of rebound mechanisms is inhibited, and macro energy modeling and energy efficiency policy are at risk of being ineffective at efficiency-based emissions reductions. In response, we develop a comprehensive, consumer-sided rebound analysis framework. We build upon microeconomic fundamentals and employ a constant elasticity of substitution utility model to describe direct and indirect channels for emplacement, substitution, and income rebound effects. We utilize a macroeconomic factor to link macro to micro rebound effects. A sequence of graphs shows rebound paths through energy, expenditure, and consumption spaces. The framework is applied to two case studies: upgrades of a car and an electric lamp. We find microeconomic rebounds of 24% and 29% and total rebound of 49% and 86% for the car and lamp examples, respectively. Sensitivity studies are performed, and policy implications are discussed.

Keywords: Energy efficiency, Energy rebound, Energy services, Microeconomic rebound, Substitution and income effects, Macroeconomic rebound
JEL codes: D11, O33, Q41, Q54, Q55

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1. Introduction

1.1. Background

Energy rebound is a phenomenon whereby some expected energy savings fail to materialize after an energy efficiency upgrade (EEU), due to behavior changes and economic effects. Famously, the roots of energy rebound trace back to Jevons who, in his report entitled *The Coal Question*, said "[*i*]*t is wholly a confusion of ideas to suppose that the economical use of fuel is equivalent to a diminished consumption. The very contrary is the truth*" (emphasis in original) Jevons (1865, p. 103). Less famously, the origins of rebound extend backward from Jevons to Williams (1840) who quoted Parkes who wrote "[t]he economy of fuel is the secret of the economy of the steam-engine; it is the fountain of its power, and the adopted measure of its effects. Whatever, therefore, conduces to increase the efficiency of coal, and to diminish the cost of its use, directly tends to augment the value of the steam-engine, and to enlarge the field of its operations" (Parkes, 1838, p. 161). For nearly 200 years, then, it has been understood that efficiency gains may, paradoxically, drive growth in energy consumption.

The oil crises of the 1970s shone a light on energy efficiency, and the modern debate over the magnitude of energy rebound commenced. On one side, scholars including Brookes (1979, 1990) and Khazzoom (1980) suggested rebound could be large. Others, including Lovins (1988) and Grubb (1990, 1992), claimed rebound was likely to be small.

Debate over the size of energy rebound continues today. Advocates of small rebound (less than, say, 50%), suggest "the rebound effect is overplayed" (Gillingham et al., 2013, p. 475), while others, including Saunders (2015) and Stern (2020), claim the evidence for large rebound (greater than 50%) is growing. (Rebound above 100% is known as "backfire.")

Amidst the rebound debate, energy efficiency measures are expected to deliver energy-related CO_2 emissions reductions in support of Paris Agreement targets (Masson-Delmotte et al., 2018), even while the world economy grows. For example, by 2050 energy efficiency is expected to contribute 37% of energy-related carbon emissions reductions globally in the International Energy Agency's Sustainable Development scenario (relative to its Stated Policies scenario, which itself includes planned energy efficiency) (International Energy Agency (IEA), 2019, Fig. 2.1). As Brockway et al. (2021) note, the expected reductions in energy consumption would represent a structural break with the historical record of tight coupling between energy consumption and GDP. Thus, the magnitude of energy rebound is much more than an academic debate, because there is a lot at stake. Large rebound, if it exists, will mean that efficiency-based efforts to reduce carbon emissions will be less effective than supposed, and achieving Paris emissions reduction targets will be at risk.

Sorrell (2007) led an important systematic review of energy rebound estimates, considering over 500 studies and reports from around the world. He found that most studies focused on direct (microeconomic) rebound at the household level, with studies typically estimating small rebound magnitudes (10–30%). In contrast, he found weaker evidence for macroeconomic rebound, but far larger magnitudes, stating "[t]his evidence base is both small and subject to a number of important methodological weaknesses, making

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it difficult to draw any general conclusions. However, the available studies suggest that economy-wide [macroeconomic] rebound effects may frequently exceed 50%" (Sorrell, 2007, p. 9).

Since the Sorrell (2007) review, empirical estimates of (typically) household-level energy rebound at the microeconomic scale (direct and indirect) continue to suggest relatively small microeconomic rebound magnitudes of less than 40%. (See Greene (2012), Thomas and Azevedo (2013b), Antal and van den Bergh (2014), Chitnis et al. (2014), and Stapleton et al. (2016).) Commonly, microeconomic rebound estimates are based on own-sector (direct) and/or cross-sector (indirect, re-spending) price elasticities, as set out by Gillingham et al. (2016). Dahlqvist et al. (2021) provides an example of this price elasticity approach applied to energy-intensive industry sectors.

While estimates of microeconomic rebound are common, estimates of macroeconomic rebound remain very rare. (See Barker et al. (2009) for an exception that proves the rule.) However, a growing number of studies estimate the magnitude of total rebound, the sum of microeconomic and macroeconomic rebound. A recent review of 33 studies by Brockway et al. (2021) found a large group of 21 Computational General Equilibrium (CGE) studies, such as Lecca et al. (2014), Broberg et al. (2015), and Wei and Liu (2017), with a mean total rebound estimate of around 60%. A smaller group of studies used three other methods: non-CGE macroeconomic models (e.g., Barker et al. (2009) and Lemoine (2020)), econometric analysis (e.g., Wei and Liu (2019) and Bruns et al. (2019)), and productivity or growth accounting studies (e.g., Lin and Du (2015)). Like the CGE studies, the average estimate of total rebound magnitude from non-CGE methods was found to be over 60%, leading the review authors to conclude that total rebound magnitudes are "larger than commonly assumed" (Brockway et al., 2021, p. 1).

Comparing one study to the next is complicated by the wide variety of rebound analysis frameworks employed in rebound analysis. So before formally defining the scope and aims of this paper, one of which is consolidation of technical and economic foundations, we survey existing rebound frameworks.

1.2. Rebound analysis frameworks

Rebound analysis frameworks have been developed for microeconomic, macroeconomic, and total energy rebound. At the *microeconomic* level, direct and indirect rebound are often estimated via energy service price elasticities (e.g., Schipper and Grubb (2000)). Recent microeconomic advances include those by Thomas and Azevedo (2013a), who set out a detailed rebound analysis framework of direct and indirect microeconomic effects; Borenstein (2015), who provided a detailed decomposition of rebound into income and substitution effects; Chan and Gillingham (2015), who analyzed welfare implications of the rebound effect; and Sun (2018), who considered direct rebound, including device effects. However, lifecycle and end-of-life considerations are not typically included in microeconomic rebound analysis frameworks, and models of consumer preferences lack robustness. (Babacan et al. (2020) provides a rare example of analysis of life-cycle analysis effects of the energy efficient device.)

The importance of interactions between microeconomic and *macroeconomic* rebound effects is suggested by the so-called *Khazzoom-Brookes Postulate* (Saunders, 1992), defined by Herring as "increased energy-efficiency at the micro-economic level, while leading to a reduction of energy use at this level, leads not to a reduction, but instead to an increase in energy use, at the national, or macroeconomic level" (Herring, 1999, p. 214).

Borenstein described macro effects as follows: "the economy [expands] by more than the private income gain enjoyed by the owner of the upgraded appliance" (Borenstein, 2015, p. 11). To date, frameworks for estimating the magnitude of macroeconomic rebound effects have included an amplification mechanism from micro to macro, the result of energy productivity gains from the energy efficiency upgrade (EEU) (Dimitropoulos, 2007; Sorrell, 2010). These macro effects are hypothesized to operate via a numerical multiplier (Borenstein, 2015; Lange et al., 2019). Discussions of a macro multiplier effect have been purely conceptual, with the magnitude of the macro factor being a central issue. Howarth (1997) and Schipper and Grubb (2000) claim the macroeconomic rebound multiplier effect is small. Aucott and Hall (2014), Enflo et al. (2009), and Bataille and Melton (2017) are among those who claim the macroeconomic multiplier is much larger. Dimitropoulos suggested "[o]ne of the main reasons behind the debate is the lack of a rigorous theoretical framework that can describe the mechanisms and consequences of the rebound effect at the macro-economic level" (Dimitropoulos, 2007, p. 6354). In a recent review on energy productivity, Elkomy et al. found "[w]hile many researchers suggest that energy and productivity are linked, there is relatively little consensus in the empirical literatures either confirming or rejecting these views. In some cases (notably the relationship between capital and energy), we do not appear to have robust methodologies for making empirical assessments" (Elkomy et al., 2020, p. 1).

Saunders (1992) made the earliest attempt at a framework for *total* energy rebound based on neoclassical aggregate production functions (APFs). Later works continued the APF approach, including Saunders (2000, 2008), Wei and Liu (2017), and Brockway et al. (2017). Grepperud and Rasmussen (2004) provide one of the first general equilibrium frameworks for total rebound, overcoming limitations of partial equilibrium frameworks where markets by definition cannot fully close and thus miss some rebound channels (Figus et al., 2016). Further general equilibrium studies of total rebound followed, including Wei (2010), Lemoine (2020), and Fullerton and Ta (2020).

Despite the growth of rebound contributions in micro, macro, and total frameworks, particularly in the last decade, several key issues remain. First, limitations of *microeconomic* rebound analysis frameworks include (i) inseparability of substitution and income effects when applying rebound analysis frameworks to real energy efficiency upgrades and (ii) the lack of an operationalizable link between micro and macro effects.

Second, there is a paucity of both theoretical and empirical work on the *macroeconomic* multiplier. At least two things are unclear: the channel by which a macroeconomic multiplier effect would couple to microeconomic effects and the appropriate magnitude of a macroeconomic multiplier.

Third, results from analysis frameworks which estimate *total* rebound are highly dependent upon two assumptions: the type of production function used (Cobb-Douglas, CES, etc.) and the chosen value of the elasticity of substitution. As Saunders noted, the "choice of production/cost function can pre-determine the results related to rebound ... [and choosing] values for these [elasticity of substitution] parameters is tantamount to assuming rebound results" Saunders (2008, p. 2207-2208). Furthermore, analysis frameworks at the total rebound level often work with aggregate energy and economic variables, rarely discriminating between microeconomic and macroeconomic rebound components.

In summary, most work on energy rebound is empirical and microeconomic. And although microeconomic theory is available, it is scattered across papers, and it has not been developed in a way that allows simultaneous empirical estimation of all microeconomic rebounds. Furthermore, macroeconomic rebound and its links to micro rebound is virtually unexplored territory. This state of affairs inhibits understanding of rebound mechanisms along direct and indirect channels at the device level and throughout the economy. In the absence of integrated theoretical foundations, macro energy modeling and energy efficiency policy are at risk of being ineffective to achieve efficiency-based CO_2 emissions reductions. Adding to that risk, energy rebound is rarely included in either energy models (Brockway et al., 2021) or energy policy-making (Font Vivanco et al., 2016), limiting our ability to assess the interactions among rebound, energy demand, and carbon emissions going forward.

Bringing these concerns together, further theoretical and empirical research is vital to advance our understanding of energy rebound, to formulate effective energy policy, and to take appropriate climate change mitigation actions.

1.3. Need, aim, contributions, and structure

There is a strategic *need* for a comprehensive rebound analysis framework to firm up theoretical foundations, enable empirical analysis, and guide energy policy-making. In particular the framework should be one that (i) builds upon microeconomic foundations and carefully distinguishes among microeconomic rebound mechanisms, (ii) incorporates indicative, plausible macroeconomic multiplier values to provide a better link between micro and macro effects, and (iii) enables analysis of real energy efficiency upgrades. The *aim* of this paper is therefore to build upon the foundations laid by Borenstein (2015)) and others to develop a comprehensive, consumer-sided energy rebound analysis framework. The key *contributions* of this paper are (i) describing a consistent and mathematically-rigorous, consumer-sided energy rebound analysis framework in which direct and indirect dimensions of substitution and income effects are connected, (ii) showing that embodied energy, maintenance and disposal activities, replacements, and macroeconomic effects can be accommodated in a consistent framework, (iii) providing a new way to visualize rebound effects, namely via energy, expenditure, and consumption path graphs, (iv) applying a general CES utility function via empirically-plausible cost shares of other goods consumption, (v) providing two numerical case studies (a car and an electric lamp) to demonstrate the application of the rebound analysis framework, and (vi) an assessment of the rebound implications from policies to enhance energy efficiency.

The remainder of this paper is *structured* as follows. Section 2 describes the comprehensive, consumer-sided rebound analysis framework. Section 3 provides two calculation examples, a car and an electric lamp. Section 4 discusses the results, and Section 5 concludes.

2. Methods: development of the rebound framework

In this section, we develop a comprehensive, consumer-sided energy rebound analysis framework (concisely, "the framework"). We begin with our rebound typology.

2.1. Rebound typology

A challenge to understanding energy rebound is that it comprises many mechanisms and effects. We follow convention to distinguish between microeconomic effects (at the single device or device owner level) and macroeconomic effects (broader, economy-wide responses to a single device upgrade). (See Greening et al. (2000), Barker et al. (2009), and Vivanco et al. (2018).) We follow others, including Jenkins et al. (2011) and Walnum et al. (2014), in splitting microeconomic effects into direct and indirect effects.

Like other authors, we recognize many macroeconomic effects, even if we don't later distinguish among them. Sorrell (2009) sets out five macroeconomic rebound effects: embodied energy effects, respending effects, output effects, energy market effects, and composition effects. Santarius (2016) and Lange et al. (2021) introduce meso (i.e., sectoral) level rebound between the micro and macro levels. van den Bergh (2011) distinguishes 14 types of rebound, providing, perhaps, the greatest complexity.

Table 1 summarizes and reconciles some of the existing typologies and sets out the typology for this paper.

2.2. Rebound mechanisms

We assume an energy conversion device (say, a car) that consumes final energy (say, gasoline) at a rate $\dot{E}^\circ$ (in MJ/year). The final energy is available at price p_E (in \$/MJ). The energy conversion device provides a rate of energy service $\dot{q}_s^\circ$ (in vehicle-km/year) with final-energy-to-service efficiency η° (in km/MJ). An energy efficiency upgrade (EEU) increases final-energy-to-service efficiency[1] such that $\tilde{\eta} > \eta^\circ$, possibly at an increased cost to procure (capital cost), maintain, and dispose of the device such that $\tilde{\dot{C}}_{cap} > \dot{C}_{cap}^\circ$ and $\tilde{\dot{C}}_{md} > \dot{C}_{md}^\circ$. (This is not a costless EEU.) As final-energy-to-service efficiency (η) increases, the price of the energy service (p_s) declines. The final energy price (p_E) is assumed exogenous ($\tilde{p}_E = p_E^\circ$), so the final energy purchaser (the device owner) is a price taker. (Relaxing the exogenous final energy price assumption would require a full-economy model that is beyond the scope of this paper.)

Initially, the device owner spends income ($\dot{M}^\circ$) on final energy for the device ($\dot{C}_s^\circ = p_E \dot{E}_s^\circ$), annualized capital costs for the device ($\dot{C}_{cap}^\circ$), annualized costs for maintenance and disposal of the device ($\dot{C}_{md}^\circ$), and other goods and services ($\dot{C}_o^\circ$). The budget constraint for the device owner is

$$\dot{M}^\circ = \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ + \dot{N}^\circ \ , \tag{1}$$

where original net savings ($\dot{N}^\circ$) is zero. Note that the vocabulary and mathematical notation for rebound effects is extensive; Fig. 1 and Appendix A provide guides to notation elements used throughout the paper, including symbols, Greek letters, decorations, and subscripts. The notation elements can be mixed to comprise a rich and expressive "language" of energy rebound. In several places, we use colored backgrounds on rebound effects for visual convenience.

Later (Sections 2.5.1–2.5.4), we walk through the several conceptual stages of rebound, including the device upgrade and four rebound effects, and we derive rebound expressions for each. But first we define rebound relationships (Section 2.3) and typical energy and cost relationships (Section 2.4).

[1] Note that energy service efficiency (η) improves between the original ($\circ$) and post-emplacement ($*$) stages of Fig. 1, remaining constant thereafter. Thus, $\eta^\circ < \eta^* = \hat{\eta} = \bar{\eta} = \tilde{\eta}$, as shown in Table B.1. We refer to all post-emplacement efficiencies (η^*, $\hat{\eta}$, $\bar{\eta}$, and $\tilde{\eta}$) as $\tilde{\eta}$ to match the nomenclature of Borenstein (2015). When convenient, the same approach to nomenclature is taken with other quantities such as the capital cost rate ($\dot{C}_{cap}$) and maintenance and disposal cost rate ($\dot{C}_{md}$).

Table 1: Rebound typology adopted with comparison to existing literature.

Component of economy-wide energy rebound	**Origin/Mechanism**	
	Existing typologies (Sorrell (2009), Jenkins et al. (2011), Thomas & Azevedo (2013), and Walnum et al. (2014))	This paper (with comparison *in italics* to Sorrell, Jenkins, Thomas & Azevedo, and Walnum)
Microeconomic rebound: these rebound mechanisms occur at the single device level, within a static economy, based on responses to the reduction in implicit price of an energy service.	Direct rebound: describes the direct response to the single energy efficiency improvement. Jenkins et al. (2011) and Walnum (2014) split into two sub-classes: • Substitution effects: captures the substitution of that energy service for other goods or services (consumers) or inputs to production (producers). • Income/output effects: the increasing demand for that energy service by consumers who expand their spending (an "income effect') or by producers who expand their output (an "output effect"). Commonly, direct substitution and income effects are assessed via combined elasticities.	**Emplacement effect** The direct emplacement effect accounts for performance of the Energy Efficiency Upgrade (EEU) only. No behavior changes occur. The direct energy effect of emplacement of the EEU is expected device-level energy savings. By definition, there is no rebound from direct emplacement effects. The direct emplacement effect is also known as expected energy savings. **Substitution effect** Spending of freed cash on more of the energy service. *(Same as other authors.)* **Income effect** Spending of freed cash on more of the energy service. *(Same as other authors.)*
	Indirect rebound: describes the indirect response to the single energy efficiency improvement. Sorrell (2009) and Jenkins et al. (2011) split into two sub-classes: • Embodied energy effects: The energy "embodied" in the efficiency improvements themselves will offset some portion of the energy savings achieved. • Re-spending and re-investment effects: If consumers and firms see net cost savings from energy efficiency improvements, they increase expenditures or investments in production, increasing demand for goods, services, and factors of production, which in turn require energy to produce and use. Commonly, respending or reinvestment effects are assessed via combined cross-price or cross-sector elasticities.	**Emplacement effect** Differential lifecycle energy effects (versus counterfactual) of the EEU, i.e., embodied energy (emb), and implied energy demand from maintenance and disposal (md). (*Other authors include embodied effects (emb) but not effects associated with md.*) **Substitution effect** Decreased spending on other goods and services. (*Other authors typically include indirect substitution effects within re-spending and re-investment effects.)* **Income effect** Increased spending on other goods and services. (*Other authors typically include indirect income effects within re-spending and re-investment effects.*)
Macroeconomic rebound: These mechanisms originate from the dynamic response of the economy to reach a stable equilibrium (between supply and demand for goods and energy services). These mechanisms combine various short and long run effects.	Thomas and Azevedo (2013) split into 5 components: • a lower market price for energy, • changes in economic structure, • economic-competiveness • investment and disinvestment, and • labor market changes. Sorrell (2009), Jenkins et al. (2011), and Walnum et al. (2014) split into three effects: • market price effect, • composition effect, and • economic growth effect.	**Macroeconomic rebound effect** Comprised of numerous components including: • energy market effect, • composition effect, • growth effect, • scale effect, • labor supply effect, and • disinvestment effect (*We have close alignment withThomas and Azevedo (2013). Sorrell (2009), Jenkins et al. (2011), and Walnum et al. (2014) (i) fold the scale and dis/investment effects into economic growth effect and (ii) include labor market effects within the indirect factors of production response.*)

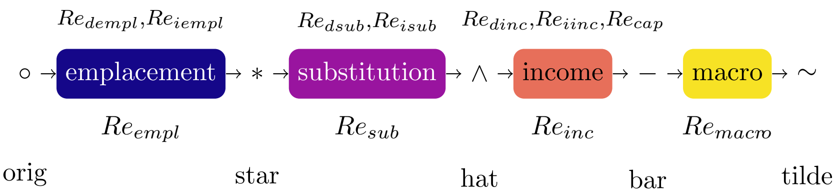

Fig. 1: Flowchart of rebound effects and decorations. Rebound effects are arranged in the order of their discussion in this paper. The left-ro-right order does not necessarily represent the progression of rebound effects through time.

2.3. *Rebound relationships*

Energy rebound is defined as

$$Re \equiv 1 - \frac{\text{actual final energy savings rate}}{\text{expected final energy savings rate}} \,, \tag{2}$$

where both actual and expected final energy savings rates are expected positive. Final energy "takeback" is the rate at which final energy savings are eroded by rebound effects, and the actual final energy savings rate is the expected final energy savings rate less takeback. Note that takeback can be negative, indicating an increase in the final energy savings rate relative to the expected final energy savings rate, sometimes called hyperconservation. Thus,

$$Re = 1 - \frac{\text{expected final energy savings rate} - \text{takeback}}{\text{expected final energy savings rate}} \,. \tag{3}$$

Simplifying gives

$$Re = \frac{\text{takeback}}{\text{expected final energy savings rate}} \,. \tag{4}$$

Thus, energy rebound can be given by either Eq. (2) or Eq. (4).

We define rebound at the final energy stage, because that is the point of energy purchase by the device owner. To simplify derivations, we choose not to apply final-to-primary energy multipliers to final energy rates in the numerators and denominators of rebound expressions derived from Eqs. (2) and (4); they divide out anyway. Henceforth, we drop the adjective "final" unless there is reason to indicate a specific stage of the energy conversion chain.

2.4. *Typical energy and cost relationships*

With the rebound notation of Appendix A, four typical relationships emerge. First, the consumption rate of the energy service ($\dot{q}_s$) is the product of final-to-services efficiency (η) and the rate of energy consumption by the energy conversion machine ($\dot{E}_s$). Typical units for automotive transport and illumination (the examples in Section 3) are shown beneath each equation.

$$\begin{aligned} \dot{q}_s &= \eta \dot{E}_s \qquad (5) \\ \text{pass-km/yr} &= [\text{pass-km/MJ}][\text{MJ/yr}] \\ \text{lm-hr/yr} &= [\text{lm-hr/MJ}][\text{MJ/yr}] \end{aligned}$$

Second, the energy service price (p_s) is the ratio of energy price (p_E) to the final-to-services efficiency (η).

$$
\begin{aligned}
p_s &= \frac{p_E}{\eta} \qquad (6)\\
\$/\text{pass-km} &= \frac{\$/\text{MJ}}{\text{pass-km}/\text{MJ}}\\
\$/\text{lm-hr} &= \frac{\$/\text{MJ}}{\text{lm-hr}/\text{MJ}}
\end{aligned}
$$

Third, energy service expenditure rates ($\dot{C}_s$) are the product of energy price (p_E) and device energy consumption rates ($\dot{E}_s$).

$$
\begin{aligned}
\dot{C}_s &= p_E \dot{E}_s \qquad (7)\\
\$/\text{yr} &= [\$/\text{MJ}][\text{MJ}/\text{yr}]
\end{aligned}
$$

Fourth, indirect energy rates ($\dot{E}_j$) are the product of expenditures rates ($\dot{C}_j$) and energy intensity of the economy (I_E).

$$
\begin{aligned}
\dot{E}_j &= \dot{C}_j I_E \qquad (8)\\
\text{MJ}/\text{yr} &= [\$/\text{yr}][\text{MJ}/\$]
\end{aligned}
$$

The subscript j represents expenditures on non-energy items in this framework: capital costs (subscript *cap*), maintenance and disposal activities (subscript *md*), or other goods (subscript o).

2.5. Rebound effects

The four rebound effects of Fig. 1 (emplacement, substitution, income, and macro) are discussed in subsections below. In each subsection, we (i) define each rebound effect and (ii) show mathematical expressions for rebound. Detailed derivations of all rebound expressions can be found in Appendix B. See, in particular, Tables B.3–B.6 which provide a parallel structure for energy and financial accounting across all rebound effects.

Important quantities for the derivations are (i) direct and indirect energy consumption rates ($\dot{E}_{dir}$, $\dot{E}_{indir}$), (ii) direct and indirect expenditure rates ($\dot{C}_{dir}$ and $\dot{C}_{indir}$), (iii) the consumption rate of the energy service ($\dot{q}_s$), and (iv) the consumption rate of other goods ($\dot{C}_o$). These important rebound quantities can be illustrated via rebound path graphs. (See Appendix C for detailed mathematical descriptions of rebound path graphs.) Figs. 2–4 show notional rebound path graphs through energy, expenditure, and consumption spaces for purposes of illustration. The notional path graphs are not quantified; there is no scale on the axes. Later (Section 3), rebound path graphs (with scales) illustrate the numerical examples.

Fig. 2 shows a notional energy path graph, with the direct energy consumption rate ($\dot{E}_{dir}$) on the abscissa and the indirect energy consumption rate ($\dot{E}_{indir}$) on the ordinate. Lines with negative slope through points $\circ$, a, $*$, $\wedge$, $-$, and $\sim$ indicate energy consumption isoquants (direct and indirect) at key points.

A notional expenditure path graph is shown in Fig. 3, with the direct expenditure rate on the energy service ($\dot{C}_{dir}$) on the abscissa and the indirect expenditure rate ($\dot{C}_{indir}$)

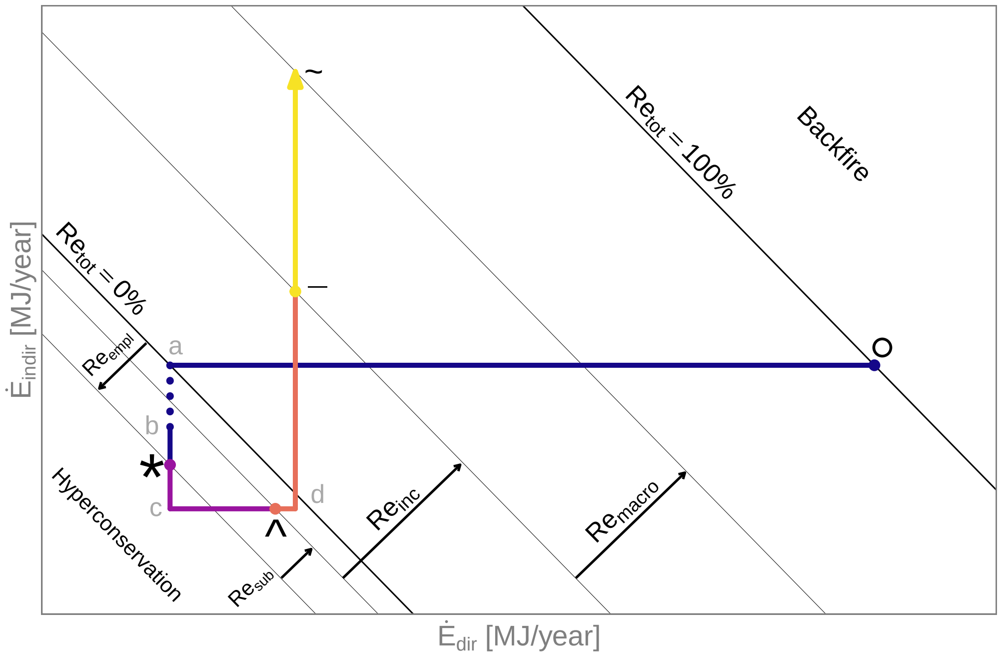

Fig. 2: Notional energy path.

on the ordinate. Lines with negative slope through points $\circ$ and a indicate expenditure isoquants (direct and indirect).

A notional consumption path graph is shown in Fig. 4. The indexed rate of energy service consumption ($\dot{q}_s/\dot{q}_s^\circ$) is shown on the abscissa, and the indexed rate of other goods consumption ($\dot{C}_o/\dot{C}_o^\circ$) is shown on the ordinate. Isoquants of indexed other goods and energy service consumption are shown as lines with negative slope. Indifference curves are denoted by i°—i° and $\bar{\mathrm{i}}$—$\bar{\mathrm{i}}$. A ray from the origin through the $\wedge$ point is denoted r—r.

In the subsections that follow, the notional graphs (Figs. 2–4) serve to illustrate the development of the framework. We begin with the emplacement effect.

2.5.1. Emplacement effect

The emplacement effect (segments $\circ$—a, a⋯b, and b—$*$ in Figs. 2 and 3) accounts for performance of the EEU only; behavior changes are addressed later, in the substitution and income effects. The direct emplacement effects of the EEU include device energy savings ($\dot{S}_{dev}$) and device energy cost savings ($\Delta\dot{C}_s^*$), on segments $\circ$—a in Figs. 2 and 3, respectively. The indirect effects of EEU emplacement are (i) changes[2] in the embodied energy rate ($\Delta\dot{E}_{emb}^*$) (segment a⋯b in Fig. 2), (ii) changes in the capital expenditure rate ($\Delta\dot{C}_{cap}^*$) (segment a⋯b in Fig. 3), and (iii) changes in the maintenance and disposal energy and expenditure rates ($\Delta\dot{E}_{md}^*$ and $\Delta\dot{C}_{md}^*$) (segments b—$*$ in Figs. 2 and 3).

[2] "Changes" means rates *after* the EEU less rates *before* the EEU.

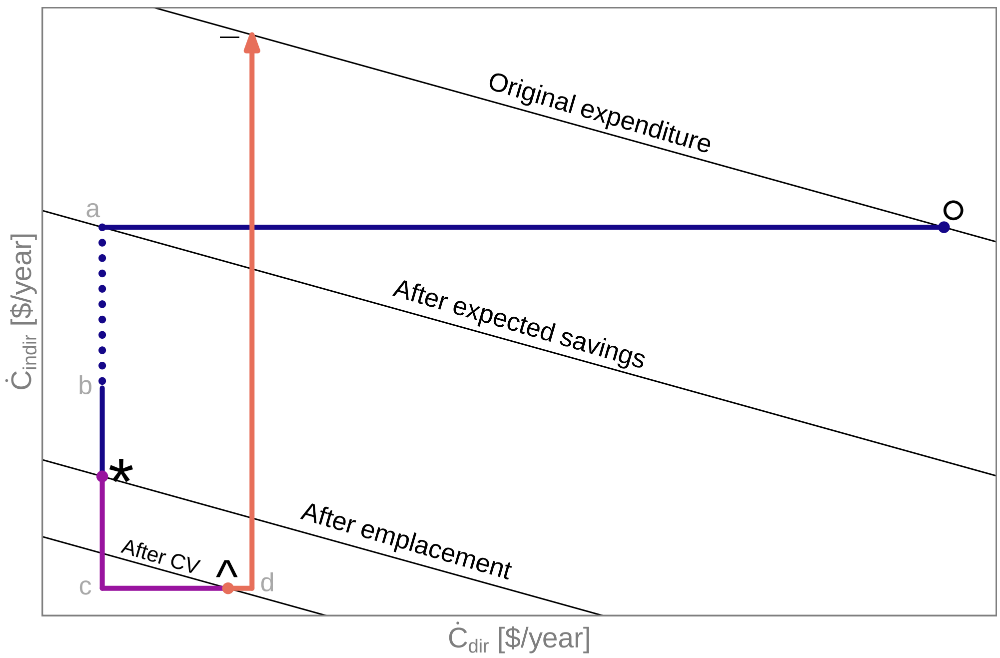

Fig. 3: Notional expenditure path.

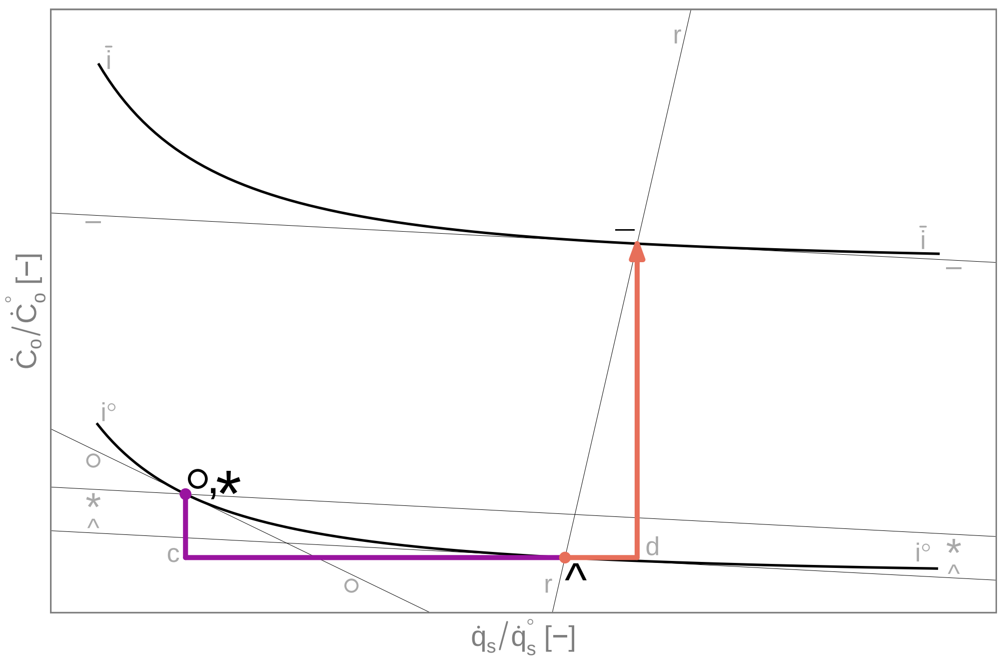

Fig. 4: Notional consumption path.

The rate of energy savings due to the direct emplacement effect ($\dot{S}_{dev}$, segment $\circ$—a in Fig. 2) is given by

$$\dot{S}_{dev} \equiv \dot{E}_s^\circ - \dot{E}_s^* = -\Delta \dot{E}_s^* \ . \tag{9}$$

Appendix B.3.1 shows that $\dot{S}_{dev}$ can be rewritten conveniently as

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^\circ} - 1 \right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ \ . \tag{10}$$

Because behavior changes are not considered in the direct emplacement effect, actual and expected energy savings rates are identical. By definition, then, the direct emplacement effect causes no rebound. Thus,

$$Re_{dempl} = 0 \ . \tag{11}$$

In the notional energy path graph of Fig. 2, point a lies on the $Re_{tot} = 0\%$ line indicating that point a (and the $Re_{tot} = 0\%$ line) is the point from which all rebound effects (Re_{empl}, Re_{sub}, Re_{inc}, and Re_{macro}) are measured. If rebound effects cause total energy demand to return to the original energy consumption level (negative sloping line through the $\circ$ point), all expected energy savings are taken back by rebound effects. Thus, the line of constant energy consumption through the $\circ$ point is labeled $Re_{tot} = 100\%$. Total rebound (Re_{tot}) is measured linearly between and beyond the $Re_{tot} = 0\%$ and $Re_{tot} = 100\%$ lines, with decomposition to direct rebound in the x direction and indirect rebound in the y direction. The region below and to the left of the $Re_{tot} = 0\%$ line in Fig. 2 exhibits negative rebound, indicating hyperconservation. The region above and to the right of the $Re_{tot} = 100\%$ line shows "backfire," i.e. greater energy consumption after the EEU than before it.

Although the direct emplacement effect does not cause rebound, Fig. 2 shows that indirect emplacement effects may cause rebound. Indirect emplacement effects (segments a····b and b—$*$ in Figs. 2 and 3) account for the life cycle of the energy conversion device, including energy embodied by manufacturing processes (subscript emb) and maintenance and disposal activities (subscript md). In the notional energy path graph (Fig. 2), emplacement rebound is negative ($Re_{empl} < 0$), because the upgraded device has a lesser embodied energy rate ($\dot{E}_{emb}^* < \dot{E}_{emb}^\circ$) and a lesser maintenance and disposal expenditure rate ($\dot{C}_{md}^* < \dot{C}_{md}^\circ$) than the original device. Fig. 3 shows segments a····b and b—$*$ moving in the negative direction, consistent with a lower capital expenditure rate ($\dot{C}_{cap}^* < \dot{C}_{cap}^\circ$) and a reduced maintenance and disposal expenditure rate ($\dot{C}_{md}^* < \dot{C}_{md}^\circ$).

One of the unique features of this framework is that independent analyses of embodied energy and capital costs of the EEU are expected. We note that the different terms (embodied energy rate, $\dot{E}_{emb}$, and capital cost rate, $\dot{C}_{cap}$) might seem to imply different processes, but they actually refer to the same emplacement effect, namely energy segment a····b and expenditure segment a····b in energy and expenditure path graphs (Figs. 2 and 3, respectively). Purchasing an upgraded device (which likely leads to $\dot{C}_{cap}^\circ \neq \dot{C}_{cap}^*$) will likely mean a changed embodied energy rate ($\dot{E}_{emb}^\circ \neq \dot{E}_{emb}^*$) to provide the same energy service. Our names for these aspects of rebound (embodied energy and capital cost) reflect common usage in the energy and economics fields, respectively. In the notional graphs of Figs. 2 and 3, embodied energy rates and capital cost rates (represented by energy segment $a \cdots b$ and expenditure segment $a \cdots b$, respectively) move in the same direction (both negative). However, both segments $a \cdots b$ could move in the positive direction, or they could move in opposite directions, depending on the results of the independent analyses for embodied energy and capital cost rates.

Consistent with the energy/exergy analysis literature, we define embodied energy to be the sum of all final energy (direct and indirect) consumed in the production of the energy conversion device. Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. No energy is embodied in the device while in service. The EEU causes the embodied energy of the energy conversion device to change from E°_{emb} to E^{*}_{emb}.

However, for simplicity, we spread all embodied energy over the lifetime of the device, an equal amount assigned to each period. (We later take the same approach to capital costs and maintenance and disposal costs.) A justification for spreading embodied energy purchase costs comes from considering device replacements by many consumers across several years. In the aggregate, evenly-spaced (in time) replacements work out to the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) to obtain embodied energy rates, such that $\dot{E}^{\circ}_{emb} = E^{\circ}_{emb}/t^{\circ}_{life}$ and $\dot{E}^{*}_{emb} = E^{*}_{emb}/t^{*}_{life}$. The change in embodied final energy due to the EEU (expressed as a rate) is given by $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$ and is shown as segment $a \cdots b$ in Fig. 2. The expression for embodied energy rebound is derived in Appendix B.3.2 to be

$$Re_{emb} = \frac{\left(\frac{E^{*}_{emb}}{E^{\circ}_{emb}}\frac{t^{\circ}_{life}}{t^{*}_{life}} - 1\right)\dot{E}^{\circ}_{emb}}{\dot{S}_{dev}} \ . \tag{12}$$

Embodied energy rebound (Re_{emb}) can be either positive or negative, depending on the sign of the term $(E^{*}_{emb}/E^{\circ}_{emb})(t^{\circ}_{life}/t^{*}_{life}) - 1$. Rising energy efficiency can be associated with increased device complexity, additional energy consumption in manufacturing, and more embodied energy, such that $E^{*}_{emb} > E^{\circ}_{emb}$ and $Re_{emb} > 0$, all other things being equal. However, if the upgraded device has longer life than the original device ($t^{*}_{life} > t^{\circ}_{life}$), $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$ could be negative, meaning that the upgraded device has a lower embodied energy rate than the original device, as shown by point *2* being below point *1* in the notional energy path graph of Fig. 2. See Appendix B.3.2 for details.

In addition to embodied energy, indirect emplacement effect rebound accounts for energy demanded by maintenance and disposal (md) activities (segments b — $*$ in Figs. 2 and 3). Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}^{\circ}_{m}$). Disposal costs (e.g., C°_{d}) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal costs across the lifetime of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) to form expenditure rates such that $\dot{C}^{\circ}_{md} = \dot{C}^{\circ}_{m} + C^{\circ}_{d}/t^{\circ}_{life}$ and $\dot{C}^{*}_{md} = \dot{C}^{*}_{m} + C^{*}_{d}/t^{*}_{life}$.

For simplicity, we assume that maintenance and disposal expenditures imply energy consumption elsewhere in the economy at its overall energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in maintenance and disposal expenditures is given by $\Delta\dot{C}^{*}_{md}I_E = (\dot{C}^{*}_{md} - \dot{C}^{\circ}_{md})I_E$, shown as segment b — $*$ in Fig. 2. Rebound from maintenance and disposal activities is given by

$$Re_{md} = \frac{\left(\frac{\dot{C}^{*}_{md}}{\dot{C}^{\circ}_{md}} - 1\right)\dot{C}^{\circ}_{md} I_E}{\dot{S}_{dev}} . \tag{13}$$

See Appendix B.3.2 for details of the derivation.

2.5.2. Substitution effect

Neoclassical consumer theory distinguishes between (i) substituting energy service consumption for other goods consumption due to the lower post-EEU price of the energy service (the substitution effect) and (ii) re-spending the higher real income (the income effect). This section develops mathematical expressions for substitution effect rebound (Re_{sub}). (The next section addresses income effect rebound, Re_{inc}.) The substitution effect alters compensated demand, the expenditure minimizing consumption bundle that maintains utility at the pre-EEU upgrade, given the new prices. It involves (i) an increase in consumption of the energy service, a direct substitution effect (subscript $dsub$) and (ii) a compensating decrease in consumption of other goods, an indirect substitution effect (subscript $isub$). Thus, two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}). The substitution effect is shown by segments $*$—c (indirect component) and c—$\wedge$ (direct component) in Figs. 2–4.

The substitution effect is best described with reference to a consumption path graph (Fig. 4). Prior to the EEU, the consumption basket (of the energy service and other goods) is represented by the $\circ$ point, and the line of constant expenditure, the budget constraint, is $\circ$—$\circ$. The $\circ$—$\circ$ line is tangent to the lower indifference curve (i$^\circ$—i$^\circ$) at point $\circ$, the optimal consumption bundle prior to device upgrade. After emplacement of the more efficient device (but before the substitution effect), the price of the energy service decreases ($p_s^* < p_s^\circ$), pivoting the budget constraint outward, to $*$—$*$. By definition, the substitution effect indicates the optimal consumption bundle at the $\wedge$ point, after a compensating variation translates into a lower budget constraint $\wedge$—$\wedge$ in Fig. 4. The $\wedge$ point shows consumption with new prices yielding utility at the level prior to the EEU, by taking advantage of the lower energy service price, hence favoring energy service consumption (because it is now less expensive) at the expense of other goods consumption (which are now relatively more expensive).

The substitution effect can be decomposed into indirect (the decrease in other goods consumption, segment $*$—c) and direct (the increase in energy service consumption, segment c—$\wedge$) components. The impact of the substitution effect on energy consumption rates and expenditure rates can be seen in Figs. 2 and 3, respectively. In particular, the latter indicates the expenditure after the compensating (CV).

An approximate utility model is often used in the literature (e.g., see Borenstein (2015, p. 17, footnote 43)) for determining the post-substitution effect point and therefore Re_{dsub} and Re_{isub}. The approximate utility model assumes that the compensated energy service price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s, c}$) and the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o, p_s, c}$) are constant along an indifference curve. Whether that is true depends on the utility function used. Constant price elasticities (as in the approximate utility model) are approximations that are only applicable to marginal price changes. Appendix B.3.3 contains details of the approximate utility model.

Here, we present an exact method that allows the compensated energy service price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$) to vary along an indifference curve, allowing analysis of non-marginal price changes ($p_s^* \ll p_s^\circ$). We use a fully specified constant elasticity of substitution (CES) utility model, which allows the exact, direct calculation of the utility maximizing consumption bundle for any constraint. The utility model describes the device owner's behavior as

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_o}{\dot{C}_o^\circ} \right)^\rho \right]^{(1/\rho)} . \tag{14}$$

The device owner's utility (relative to the original condition, $\dot{u}/\dot{u}^\circ$) is determined by the consumption rate of the energy service ($\dot{q}_s$) and the consumption rate of other goods and services ($\dot{C}_o$). The share parameter ($f_{\dot{C}_s}^\circ$) is taken from the orignal (pre-EEU) consumption basket. The exponent ρ is calculated from the (constant) elasticity of substitution (σ) as $\rho \equiv (\sigma - 1)/\sigma$. See Appendix D for further details of the CES utility model.

Direct substitution effect rebound (Re_{dsub}) is given by

$$Re_{dsub} = \frac{\Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} , \tag{15}$$

which can be rearranged to

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} . \tag{16}$$

Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} , \tag{17}$$

which can be rearranged to

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} . \tag{18}$$

To find the post-substitution effect point ($\wedge$), we solve for the location on the i°—i° indifference curve where its slope is equal to the slope of the $*$—$*$ expenditure line. The results are

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} \tag{19}$$

and

$$\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} = \left[\left(1 + f^\circ_{\dot{C}_s} \left\{ \left[\left(\frac{1 - f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} . \tag{20}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an expression for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f^\circ_{\dot{C}_s} + (1 - f^\circ_{\dot{C}_s}) \left[\left(\frac{1 - f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

Eq. (20) can be substituted directly into Eq. (18) to obtain an expression for direct substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left[\left(1 + f^\circ_{\dot{C}_s} \left\{ \left[\left(\frac{1 - f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{22}$$

See Appendix B.3.3 for details of the derivations of Eqs. (16), (18), and (19)–(22).

2.5.3. Income effect

The monetary income rate of the device owner, $\dot{M}^\circ$, remains unchanged across the rebound effects, such that $\dot{M}^\circ = \dot{M}^* = \dot{\hat{M}} = \dot{\bar{M}} = \dot{\tilde{M}}$. Thanks to the energy service price decline, real income rises, and freed cash from the EEU is given by Eq. (B.14) as $\dot{G} = p_E \dot{S}_{dev}$. Emplacement effect expenditures and compensating variation modify freed cash to leave the device owner with *net* income ($\dot{\hat{N}}$) from the EEU, as shown in Eq. (B.24). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\dot{\hat{N}}$) are presented in Tables B.3 and B.4. Rebound from the income effect quantifies the rate of additional energy demand that arises when the energy conversion device owner spends net savings from the EEU (segments $\wedge$—d and d—$-$ in Figs. 2–4).

In this framework, the income effect must satisfy several constraints. First, net savings must be zero after the income effect ($\dot{\bar{N}} = 0$). The monetary budget constraint $\dot{M}^\circ$ holds, so that only net savings ($\dot{\hat{N}}$) can be re-spent. Due to standard non-satiation assumptions on preference (and in particular for the CES utility function), net savings should be completely re-spent. This constraint can be seen in expenditure path graphs (e.g., Fig. 3) where the post-income-effect point ($-$) returns to the original expenditure line. See Appendix E for a mathematical proof that the income preference equations below (Eqs. (23) and (27)) satisfy the first constraint.

Second, net savings should be re-spent on only (i) additional spending on the energy service ($\bar{q}_s > \hat{q}_s$, segment $\wedge$—d) and (ii) additional spending on other goods ($\bar{q}_o > \hat{q}_o$, segment d—$-$). This constraint is satisfied by construction below.

A third constraint follows from the homothetic preferences in a CES utility function but need not hold in general. Additional spending on the energy service and other goods is allocated in the same proportion as post-substitution-effect expenditures on the energy service and other goods. This constraint is also satisfied by construction below, particularly via an effective income ($\hat{\dot{M}}'$). This constraint can be seen in the consumption path graph where the pre- and post-income-effect points ($\wedge$ and $-$, respectively) lie along ray r—r from the origin through point $\wedge$ in Fig. 4. However, this framework could accommodate non-homothetic preferences for spending across the income effect.

We next show expressions for direct and indirect income effect rebound.

Direct income effect. The income elasticity of energy service demand ($\epsilon_{\dot{q}_s,\dot{M}}$) quantifies the amount of net savings spent on more of the energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$). (See Appendix D for additional information about elasticities.) Spending of net savings on the energy service leads to direct income effect rebound (Re_{dinc}).

The ratio of rates of energy service consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} . \tag{23}$$

Homotheticity means that $\epsilon_{\dot{q}_s,\dot{M}} = 1$. The increased consumption rate of the energy service is represented by segments $\wedge$—d in Figs. 2–4.

Effective income ($\hat{\dot{M}}'$) is given by

$$\hat{\dot{M}}' \equiv \dot{M}^{\circ} - \dot{C}^{*}_{cap} - \dot{C}^{*}_{md} - \hat{\dot{N}} . \tag{24}$$

For the purposes of the income effect, the effective income equation (Eq. (24)) adjusts income ($\dot{M}^{\circ}$) to account for sunk costs ($\dot{C}^{*}_{cap}$ and $\dot{C}^{*}_{md}$) and net income ($\hat{\dot{N}}$).

As shown in Table B.5, direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \bar{\dot{E}}_s}{\dot{S}_{dev}} . \tag{25}$$

After much substitution, rearranging, and canceling of terms (see Appendix B.3.4), the expression for direct income rebound is

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^{\circ}} - 1} \left\{ f^{\circ}_{\dot{C}_s} + (1 - f^{\circ}_{\dot{C}_s}) \left[\left(\frac{1 - f^{\circ}_{\dot{C}_s}}{f^{\circ}_{\dot{C}_s}} \right) \frac{\tilde{p}_s \dot{q}^{\circ}_s}{\dot{C}^{\circ}_o} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} . \tag{26}$$

If there is no net savings after the substitution effect ($\hat{\dot{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected. See Appendix B.3.4 for details of the derivation of income effect rebound.

Indirect income effect. The income elasticity of other goods demand ($\epsilon_{\dot{q}_o,\dot{M}}$) quantifies the amount of net savings spent on additional other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$). Spending of net savings on other goods and services leads to indirect income effect rebound (Re_{iinc}).

The ratio of rates of other goods consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} \tag{27}$$

and is represented by segments d–– in Figs. 2–4. Under the assumption that prices of other goods are exogenous (see Appendix F), the ratio of rates of other goods consumption ($\frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ}$) is equal to the ratio of rates of other goods expenditures ($\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ}$) such that

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} . \tag{28}$$

Homotheticity means that $\epsilon_{\dot{q}_o,\dot{M}} = 1$. As shown in Table B.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta \bar{\dot{C}}_o I_E}{\dot{S}_{dev}} . \tag{29}$$

After much substitution, rearranging, and canceling of terms (see Appendix B.3.4), the expression for indirect income rebound is

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left[\left(1 + f_{\dot{C}_s}^\circ \left\{\left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} . \tag{30}$$

See Appendix B.3.4 for details of the derivation of income effect rebound.

2.5.4. *Macro effect*

The previous rebound effects (emplacement effect, substitution effect, and income effect) are microeconomic in nature. However, the changes at the micro level have important implications at the macro level, including economic expansion. As the price level in the economy falls, consumption of goods grows, and the economy grows. Efficiency-driven economic expansion is described by Hulten's theorem, which states that the economy grows at the rate of an efficiency increase in some part of the economy, scaled by that part's expenditure share (Hulten, 1978). Blackburn and Moreno-Cruz (i) show that Hulten's theorem also applies to factor-augmenting technical change and (ii) express rebound as a scale effect (Blackburn and Moreno-Cruz, 2020).

However, the scale effect is static and does not account for possible follow-up technological change induced by the EEU[3]. The framework incorporates annual average effects over the lifetime of the upgraded device, so dynamic effects that take time to unfold matter. To describe all the channels by which these dynamic effects operate, authors distinguish among several macro rebound effects: a scale effect, a structural change effect,

[3] In the first year, the technological improvement could even have contractionary effects (Basu et al., 2006).

employment and investment effects, and further price effects (Colmenares et al., 2020; Brockway et al., 2021). As van Benthem (2015) notes, cheaper energy services facilitate the development and deployment of new products (product innovation), an important driver of rising aggregate demand (Szostack, 1995). Ultimately, these dynamic rebound effects lead to higher incomes and more spending on goods and energy services than otherwise would be the case in the absence of the EEU.

Recently, Borenstein has addressed these macro effects from the demand side, noting that "income effect rebound will be larger economy-wide than would be inferred from evaluating only the direct income gain from the end user's transaction" (Borenstein, 2015, p. 11) and likening it to the Keynesian macroeconomic multiplier. However, the dynamic macro rebound mechanism is not an autonomous expansion of expenditure in an otherwise unchanged economy (like the Keynesian multiplier). Rather, macroeconomic rebound is caused by an energy productivity improvement. That said, Borenstein is right to highlight that supply side and demand side effects both play a role in the macroeconomic rebound. Furthermore, Borenstein's approach has the advantage that it can be directly linked to our income effect and its consequences for macroeconomic rebound.

To operationalize the macro rebound effect, we scale the post-substitution-effect net savings ($\hat{\dot{N}}$) gained by device owners at the micro level. As argued by Antal and van den Bergh (2014), it is reasonable to assume that net savings ($\hat{\dot{N}}$) are spent in the economy at the average energy intensity of the economy (I_E). Scaling from net savings ($\hat{\dot{N}}$) at the device level to productivity-driven growth at the macro level is unexplored territory, as Borenstein (2015) notes. In the absence of better information, we express this scaling simply by a macro factor (k) with $k \geq 0$ expected. $k = 0$ means there is no macroeconomic productivity boost resulting from the energy efficiency upgrade. $k > 0$ means that productivity-driven macroeconomic growth has occurred with consequent implications for additional energy consumption in the economy. For now, we choose $k = 1$ as a placeholder value, with further exploration of its magnitude in Section 4.

As shown in Table B.6, macro rebound is given by

$$Re_{macro} = \frac{k\hat{\dot{N}}I_E}{\dot{S}_{dev}} \ . \tag{31}$$

Macro effects are shown as segment $-$ $\sim$ in Fig. 2. After some algebra (see Appendix B.3.5), we arrive at an expression for macro effect rebound:

$$Re_{macro} = kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} \ . \tag{32}$$

2.6. Rebound sum

The sum of all rebound emerges from the four rebound effects (emplacement effect, substitution effect, income effect, and macro effect). Macro effect rebound (Re_{macro}, Eq. (32)) is expressed in terms of other rebound effects. (Derivation details can be found in Appendix B.3.6.) After algebra and canceling of terms, we find

$$\begin{aligned} Re_{tot} = Re_{emb} &- kRe_{cap} + (1-k)Re_{md} \\ &+ (1 - kp_E I_E)Re_{dsub} + (1-k)Re_{isub} \\ &+ Re_{dinc} + Re_{iinc} + kp_E I_E \ . \end{aligned} \tag{33}$$

3. Calculations: two examples

To demonstrate application of the framework developed in Section 2, we select two case studies: energy efficiency upgrades for a car and an electric lamp. We first collect parameter values for the equations for eight rebound components: Re_{dempl} (Eq. (11)), Re_{emb} (Eq. (12)), Re_{md} (Eq. (13)), Re_{dsub} (Eqs. (16 and 19)), Re_{isub} (Eq. (18 and 20)), Re_{dinc} (Eq. (26)), Re_{iinc} (Eq. (30)), and Re_{macro} (Eq. (32)). The total rebound (Re_{tot}) is given by the sum of the above components, which can also be checked against Eq. (33).

As discussed in Section 2.5.4, the link between microeconomic factors and macro rebound effects is a largely unexplored area. For now, we assume a placeholder value of $k = 1$ for both case studies. We return to the matter of quantifying k in the Discussion (Section 4).

3.1. Case 1: Purchase of new car

For the first example, we consider the purchase of a more fuel efficient car, namely a gasoline-electric Ford Fusion Hybrid car, to replace a conventional gasoline Ford Fusion car. The cars are matched as closely as possible, except for the inclusion of an electric battery in the hybrid car. The car case study features a larger initial capital investment versus the long-term benefit of decreased direct energy costs for the same energy service (mileage/year).

3.1.1. Input parameters

We require four sets of data. First, the basic car parameters are summarized in Table 2. Second, we require several general parameters, mainly relating to the U.S. economy and personal finances of the average U.S. citizen. The parameters and the values used in the study are given in Table 3. Third, we require elasticity parameters, as given in Table 4. Fourth, armed with the parameter values from Tables 2–4, and the equations of Section 2, we calculate important values at each rebound stage, as shown below in Table 5. Note that Table 5 applies to the car owner. Across the macro effect (segment $-$ $\sim$ in Fig. 5), changes occur only in the macroeconomy. For the car owner, no changes are observed across the macro effect. Thus, the $-$ and $\sim$ columns of Table 5 are identical.

Table 2: Car example: Vehicle parameters.

Description Parameters [units]	Ford Fusion (gasoline)	Ford Fusion (hybrid EV)	Data sources and notes
Fuel economy η°, $\tilde{\eta}$ [mpg]	25	42	Combined cycle mpg value taken from Thecarconnection.com (2020), Turbo Premium Unleaded for Titanium FWD model with Intercooled I-4, 2.0 L engine. Combined cycle mpg value taken from Thecarconnection.com (2020), for Titanium FWD model with Gas/Electric I-4, 2.0 L engine.
Capital expenditure rate $\dot{C}^{\circ}_{cap}$, $\dot{C}^{*}_{cap}$ [\$/year]	4,031	3,932	Seven year annual, averaged capital costs = purchase + finance costs. Ford Fusion Hybrid car includes a \$4,000 tax rebate correct at the time of writing, which makes the purchase cost cheaper than the conventional Ford Fusion car. – resale value (purchase – depreciation). Ford Fusion gasoline costs from Edmunds.com (2019a). Ford Fusion Hybrid car costs from Edmunds.com (2019b).
Ownership duration t°_{own}, t^{*}_{own} [years]	7	7	U.S. car ownership (from new) length taken from Businesswire.com (2015), and has risen from 52 months (2005) to 79 months (2015), so assumed as 84 months in 2019 (7 years) for our example.
Lifespan t°_{life}, t^{*}_{life} [years]	14	14	Lifetime taken as 14 years, based on 13–17 years for U.S. cars from Berla.com (2016) and 14 years for UK cars from Society of Motor Manufacturers and Traders (2020).
Embodied energy E°_{emb}, E^{*}_{emb} [MJ]	34,000	40,000	34,000 MJ for conventional Ford Fusion gasoline car taken from Argonne National Laboratory, Energy Systems Division (2010). We assume an additional 6,000 MJ added for Ford Fusion Hybrid Electric Vehicle (battery, as HEV typically adds 10–25% to total LCA energy of vehicle manufacture Onat et al. (2015). Battery lifetime assumed same as car lifetime, based on Nordelöf et al. (2014), and Onat et al. (2015).
Maintenance and disposal expenditure rate $\dot{C}^{\circ}_{md}$, $\dot{C}^{*}_{md}$ [\$/year]	2,861	2,775	Seven year annual, averaged maintenance costs = sum of insurance, maintenance, repairs, taxes, and fees (excluding financing, depreciation, fuel). Ford Fusion maintenance costs from Edmunds.com (2019a). Ford Fusion Hybrid maintenance costs from Edmunds.com (2019b).

Table 3: Car example: Economic parameters.

Description Parameter [units]	Value	Data sources and notes
Distance driven prior to upgrade $\dot{q}_s^\circ$ [miles/year]	14,425	Average U.S. vehicle miles/year, taken from Carinsurance.com (2019). This is slightly higher than the average driver miles/year (13,476) from US Department of Transportation (2018).
Real median personal income U.S., in 2018 [$/year]	33,706	Taken from Federal Reserve Bank of St Louis (2019).
U.S. eisposable income / real income (minus current taxes) [–]	0.88268	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition, US Bureau of Economic Analysis (2020),
Share of savings from disposable income [–]	0.079	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition, US Bureau of Economic Analysis (2020),
Personal consumption $\dot{M}$ [$/year]	27,401.28	Calculation: ($\$33,706$/year)(0.88268)($1 - 0.079$)
Price of gasoline p_E [$/gallon]	2.21	Taken from U.S. Energy Information Agency (EIA) data US Energy Information Administration (2020b)
Fractional spend on energy service $f_{\dot{C}_s}^\circ$ [–]	0.062	Calculation: $1,275 (spend on energy service) / [$19,234 (other goods) + $1,275 (energy service)] = 0.062, where spend on energy service = 14,425 miles / 42 mpg × $2.21/gallon = $1,275.
Macro factor k [–]	1.0	Assumed value.

Table 4: Car example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Price elasticity of car use demand $\epsilon_{\dot{q}_s,p_s}$ [−]	−0.2	We adopt −0.2 as our baseline value, based on U.S. studies including Gillingham (2020), who estimated a value of −0.1, Goetzke and Vance (2018), who estimated values between −0.05 – −0.23, and Parry and Small (2005), who estimated values of −0.1 – −0.3. For comparison, Borenstein (2015) uses a value of −0.1 – −0.4, based on Parry and Small (2005).
Compensated price elasticity of car use demand $\epsilon_{\dot{q}_s,p_s,c}$ [−]	−0.138	Calculated via the Slutsky Equation (Eq. (D.18)).
Compensated cross-price elasticity of demand for other goods $\epsilon_{\dot{q}_o,p_s,c}$ [−]	0.009	Calculated via Eq. (D.24).
Income elasticity of demand for car use $\epsilon_{\dot{q}_s,\dot{M}}$ [−]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\epsilon_{\dot{q}_o,\dot{M}}$ [−]	1.0	Follows from CES utility function.

Table 5: Results for car example with macro factor (k) assumed to be 1.

	$\circ$ (orig)	$*$ (star)	$\wedge$ (hat)	$-$ (bar)	$\sim$ (tilde)
η [mile/gal]	25.0	42.0	42.0	42.0	42.0
η [mile/MJ]	0.197	0.332	0.332	0.332	0.332
p_s [\$/mile]	0.088	0.053	0.053	0.053	0.053
$\dot{q}_s$ [mile/year]	14,425	14,425	15,508	16,066	16,066
$\dot{E}_s$ [MJ/year]	73,061	43,489	46,752	48,435	48,435
$\dot{E}_{emb}$ [MJ/year]	2,429	2,857	2,857	2,857	2,857
$\dot{C}_s$ [\$/year]	1,275	759	816	845	845
$\dot{C}_{cap}$ [\$/year]	4,031	3,932	3,932	3,932	3,932
$\dot{C}_{md}$ [\$/year]	2,861	2,775	2,775	2,775	2,775
$\dot{C}_o$ [\$/year]	19,234	19,234	19,160	19,849	19,849
$\dot{N}$ [\$/year]	0	702	719	0	0
$\dot{M}$ [\$/year]	27,401	27,401	27,401	27,401	27,401

3.1.2. Results

Results are represented graphically in quantified energy, cost, and consumption path graphs in Figs. 5–7. The energy path graph (Fig. 5) shows the size of each rebound effect for the car example.

The base results for the car upgrade are shown in Table 6.

Table 6: Car example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	1.4
Re_{md}	-1.0
Re_{dsub}	11.0
Re_{isub}	-0.9
Re_{dinc}	5.7
Re_{iinc}	7.9
Re_{macro}	8.2
Re_{tot}	32.5

The emplacement effect has three components: the direct emplacement effect, the embodied energy effect, and the maintenance and disposal effect. Rebound from the direct emplacement effect (Re_{dempl}) is 0.0% always, because energy takeback (and, therefore, rebound) occurs after the EEU is emplaced. Indirect rebound due to the embodied energy effect (Re_{emb}) is 1.4%, due to the higher embodied energy rate ($\Delta\dot{E}^*_{emb}$ = 429 MJ/year) stemming from the electric battery in the hybrid EV car. Rebound due to the maintenance and disposal effect (Re_{md}) is small and negative (-1.0%), because of the slightly lower maintenance and disposal costs for the hybrid EV car.

The substitution effect has two components: direct and indirect substitution effect rebound. Rebound from direct substitution (Re_{dsub}) is positive, as expected (11.0%). The car owner will prefer more driving, because of fuel economy enhancements (42 mpg > 25 mpg). In other words, with no other changes, the more fuel efficient car increases

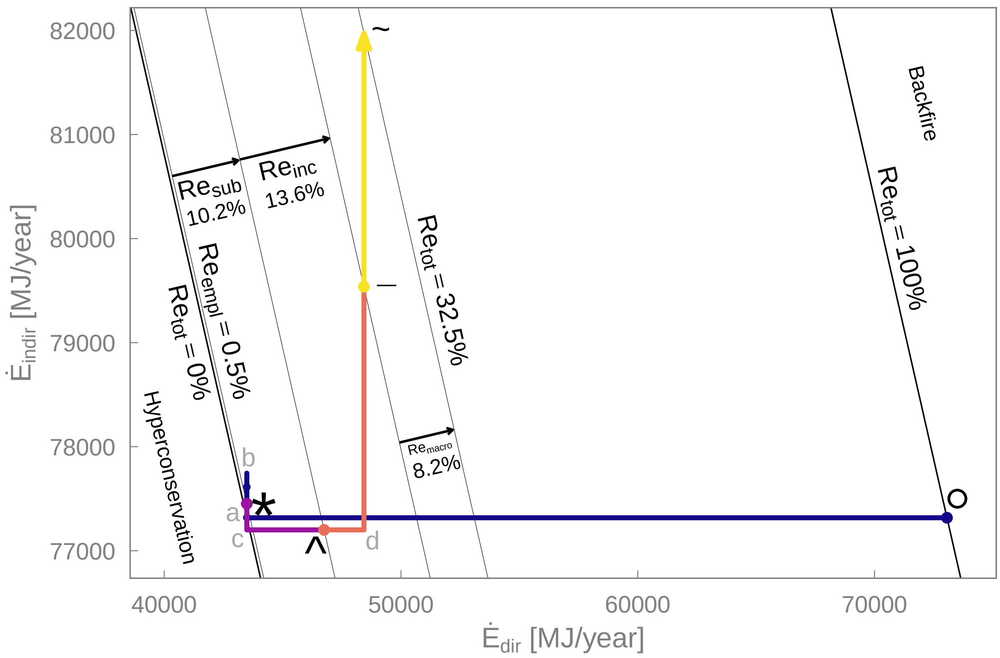

Fig. 5: Energy path, car example with macro factor (k) assumed to be 1.

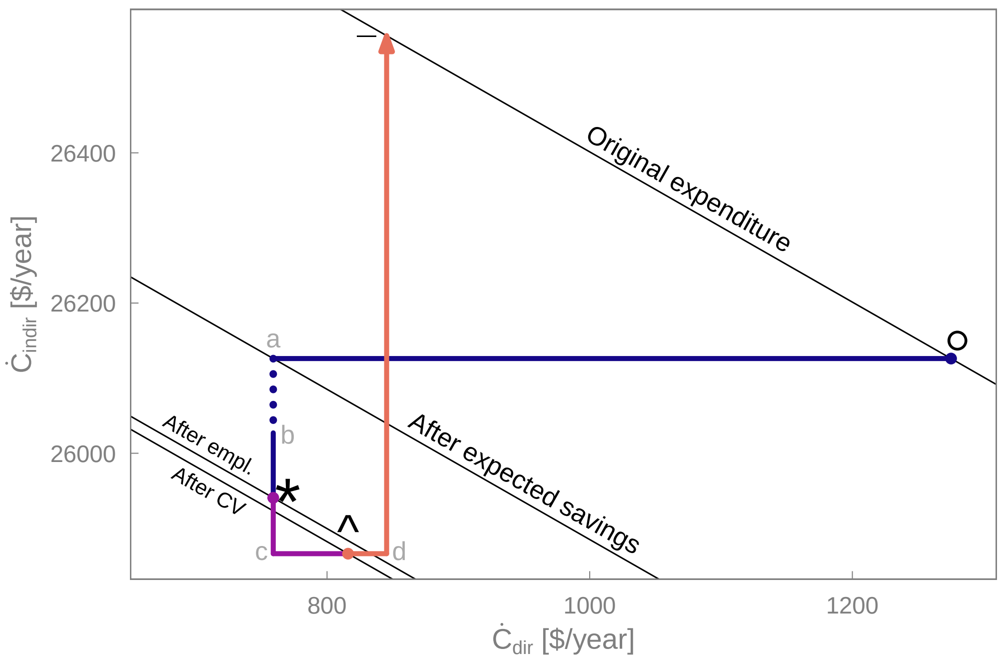

Fig. 6: Expenditure path, car example.

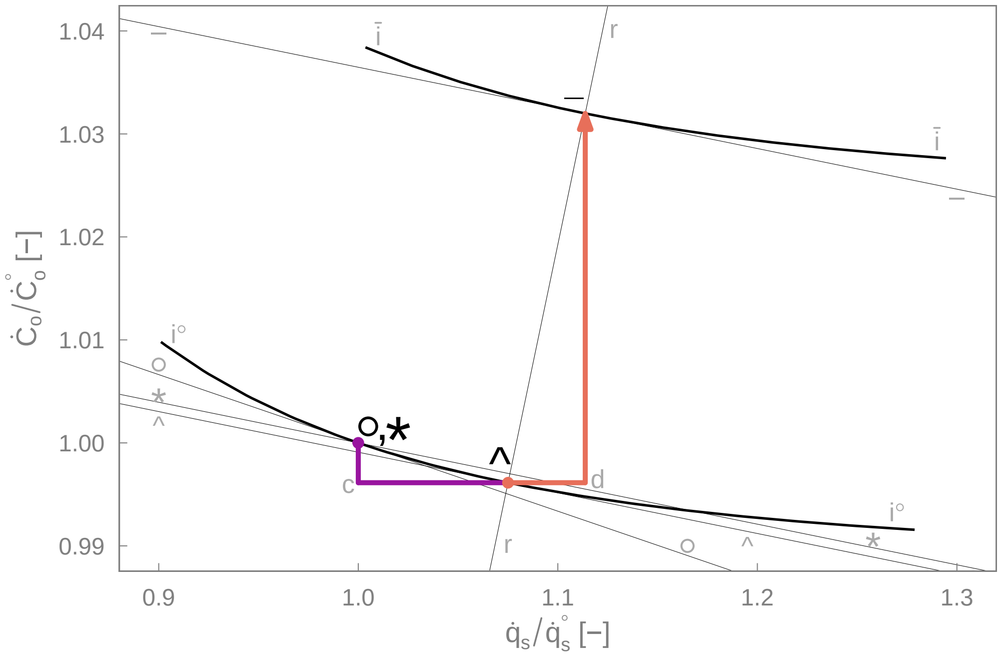

Fig. 7: Consumption path, car example.

driving by 11.0%. Conversely, the indirect substitution effect (Re_{isub}) is slightly negative (-0.9%) to achieve the same level of utility after increased driving. Indeed, less money is spent on other goods ($\Delta\hat{\dot{C}}_o = -74.36$ \$/year).

The income effect also has two components: direct and indirect income effect rebound. The car owner obtains freed cash due to a net monetary savings from the emplacement and substitution effects. The direct income effect (Re_{dinc}) is positive (5.7%), because the car owner allocates some freed cash to additional driving. Rebound from the indirect income effect (Re_{iinc}) is positive (7.9%) due to higher spending on other goods. Thus, the freed cash after the substitution effect ($\hat{\dot{N}} = 718.96$ \$/year) translates into positive direct and indirect income rebound at the micro level. Total micro rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 24.2\%$.

Finally, the macro effect leads to macro rebound (Re_{macro}) of 8.2% with k assumed to be 1, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (transportation).

3.2. Case 2: Purchase of new electric lamp

For our second example, we consider purchasing a light emitting diode (LED) electric lamp to replace a baseline incandescent electric lamp. Both lamps are matched as closely as possible in terms of energy service delivery (measured in lumens), the key difference being the energy required to provide that energy service. The LED lamp has a low initial capital investment rate (negative versus the incumbent incandescent lamp, actually!) and a long-term benefit of decreased direct energy expenditures for the same energy service (lm-hr/year).

3.2.1. Input parameters

Again, we require four sets of data. First, the basic lamp parameters are summarized in Table 7. Second, we require several general parameters, mainly relating to the U.S. economy and personal finances of the average U.S. citizen. The parameters and the values used in the study are given below in Table 8. Third, we require the elasticity parameters, as shown in Table 9. Fourth, with the parameter values from Tables 7–9 and the equations of Section 2 in hand, we calculate important values at each rebound stage, as shown below in Table 10. Similar to Table 5, Table 10 applies to the lamp owner. For the lamp owner, no changes are observed across the macro effect. Thus, the $-$ and $\sim$ columns of Table 10 are identical.

Table 7: Lamp example: Electric lamp parameters.

Description Parameters [units]	Incandescent lamp	LED lamp	Data sources and notes
Lamp efficacy η°, $\tilde{\eta}$ [lm-hr/W-hr]	8.83	81.8	Incandescent: 530 lm output / 60 W energy input. LED: 450 lm output / 5.5 W energy input.
Capital expenditure rate $\dot{C}^{\circ}_{cap}$, $\dot{C}^{*}_{cap}$ [\$/year]	1.044	0.121	Purchase costs: \$1.88 for incandescent lamp from HomeDepot.com (2020b), and \$1.21 for LED lamp from HomeDepot.com (2020a).
Ownership duration t°_{own}, t^{*}_{own} [years]	1.8	10	Assumed same as lamp lifespan
Lifespan t°_{life}, t^{*}_{life} [years]	1.8	10	Based on assumed 3 hours/day from HomeDepot.com (2020b) and HomeDepot.com (2020a).
Life cycle analysis (LCA) embodied energy E°_{emb}, E^{*}_{emb} [MJ]	2.20	6.50	Base document: Table 4.5 Manufacturing Energy Primary energy (MJ/20 Million lumen hours), U.S. DoE Life-cycle assessment of energy and environmental impacts of LED lighting products (US Department of Energy, 2012). Incandescent: LCA energy = 42.2 MJ/20 million lumen-hours. Lifetime output = 530 lumens $\times$ 3 hours/day $\times$ 365 days/year $\times$ 1.8 years = 1,044,630 lumen-hrs. Thus LCA energy / lamp = 42.2 $\times$ 1.0446/20 = 2.21 MJ. LED: LCA energy = 132 MJ/20 Million lumen-hours for pack of 5 LED lamps. Lifetime output = 450 lumens $\times$ 3 hours/day $\times$ 365 days/year $\times$ 10 years = 4,926,405 lumen-hrs. Thus LCA energy / lamp = 132 MJ/5 $\times$ 4.9264/20 = 6.5 MJ.
Maintenance and disposal expenditure rate $\dot{C}^{\circ}_{md}$, $\dot{C}^{*}_{md}$ [\$/year]	0.00	0.00	Assumed negligible.

Table 8: Lamp example: Economic parameters.

Description Parameter [units]	Value	Data sources and notes
Lighting consumption prior to upgrade $\dot{q}_s^\circ$ [lm-hr/year]	580,350	Calculation: (530 lm) (3 hrs/day) (365 days/year).
Real median personal income U.S. in 2018 [$/year]	33,706	Refer to Table 3.
U.S. disposable income / real income (minus current taxes) [–]	0.88268	Refer to Table 3.
Share of savings from disposable income [–]	0.079	Refer to Table 3.
Personal consumption $\dot{M}$ [$/year]	27,401.28	Calculation: ($\$33,706$/year)(0.88268)($1 - 0.079$).
Price of electricity p_E [$/kW-hr]	0.1355	U.S. 2020 average household electricity price (US Energy Information Administration, 2020a).
Fractional spend on energy service $f^\circ_{\dot{C}_s}$ [–]	0.0003249	Calculation: $9/year (spend on energy service) / [$27,391/year (other goods) + $9/year (energy service)] = 0.00032, where spend on energy service = 580,350 lm-hrs/year / 81.8 lm/W / 1000 W/kW $\times$ $0.1355/kW-hr = $9/year.
Macro factor k [–]	1.0	Assumed value.

Table 9: Lamp example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Price elasticity of lighting demand $\epsilon_{\dot{q}_s,p_s}$ [$-$]	-0.4	We adopt -0.4 as our baseline value, as the average of last 50 years from (Fouquet, 2014, Fig. 4) For comparison, Borenstein (2015) uses a range of -0.4 – -0.8, based on Fouquet and Pearson.
Compensated price elasticity of lighting demand $\epsilon_{\dot{q}_s,p_s,c}$ [$-$]	-0.3997	Calculated via the Slutsky Equation (Eq. (D.18)).
Compensated cross-price elasticity of demand for other goods $\epsilon_{\dot{q}_o,p_s,c}$ [$-$]	0.00013	Calculated via Eq. (D.24).
Income elasticity of lighting demand $\epsilon_{\dot{q}_s,\dot{M}}$ [$-$]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\epsilon_{\dot{q}_o,\dot{M}}$ [$-$]	1.0	Follows from CES utility function.

Table 10: Results for lamp example with macro factor (k) assumed to be 1.

	$\circ$ (orig)	$*$ (star)	$\wedge$ (hat)	$-$ (bar)	$\sim$ (tilde)
η [lm-hr/kW-hr]	8,833	81,800	81,800	81,800	81,800
η [lm-hr/MJ]	2,454	22,722	22,722	22,722	22,722
p_s [\$/lm-hr]	0.00001534	0.00000166	0.00000166	0.00000166	0.00000166
$\dot{q}_s$ [lm-hr/year]	580,350	580,350	1,412,810	1,413,421	1,413,421
$\dot{E}_s$ [MJ/year]	236.5	25.5	62.2	62.2	62.2
$\dot{E}_{emb}$ [MJ/year]	1.222	0.650	0.650	0.650	0.650
$\dot{C}_s$ [\$/year]	8.90	0.96	2.34	2.34	2.34
$\dot{C}_{cap}$ [\$/year]	1.04	0.12	0.12	0.12	0.12
$\dot{C}_{md}$ [\$/year]	0.00	0.00	0.00	0.00	0.00
$\dot{C}_o$ [\$/year]	27,391	27,391	27,387	27,399	27,399
$\dot{N}$ [\$/year]	0.00	8.86	11.86	0.00	0.00
$\dot{M}$ [\$/year]	27,401	27,401	27,401	27,401	27,401

3.2.2. Results

Results are represented graphically in energy, cost, and consumption path graphs in Figs. 8–10. The energy path graph (Fig. 8) shows the size of each rebound effect for the lamp example.

The base results for the lamp upgrade are shown in Table 11.

Table 11: Lamp example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	-0.3
Re_{md}	0.0
Re_{dsub}	17.4
Re_{isub}	-7.0
Re_{dinc}	0.0
Re_{iinc}	19.0
Re_{macro}	19.0
Re_{tot}	48.2

The emplacement effect rebound components are as follows: the direct emplacement effect (Re_{dempl}) is always 0.0%. Indirect rebound due to the embodied energy effect (Re_{emb}) is -0.3%. Although the LED lamp has higher embodied energy (E^*_{emb} = 6.50 MJ) than the incandescent lamp (E°_{emb} = 2.20 MJ), the LED lamp has a much longer lifetime, meaning that the LED embodied energy rate ($\dot{E}^*_{emb}$ = 0.65 MJ/year) is less than the incandescent embodied energy rate ($\dot{E}^\circ_{emb}$ = 1.22 MJ/year). Thus, the change in embodied energy rate ($\Delta\dot{E}^*_{emb}$) is -0.57 MJ/year, and embodied energy rebound is negative ($Re_{emb} = -0.3\%$). Rebound due to the maintenance and disposal effect (Re_{md}) is 0.0%, because we assume no difference in maintenance and disposal costs between the incandescent lamp and the LED lamp.

Direct substitution effect rebound (Re_{dsub}) is 17.4% due to the much higher LED lamp efficacy ($\eta^* = 81.8$ lm/W) compared to the incandescent lamp ($\eta^\circ = 8.83$ lm/W),

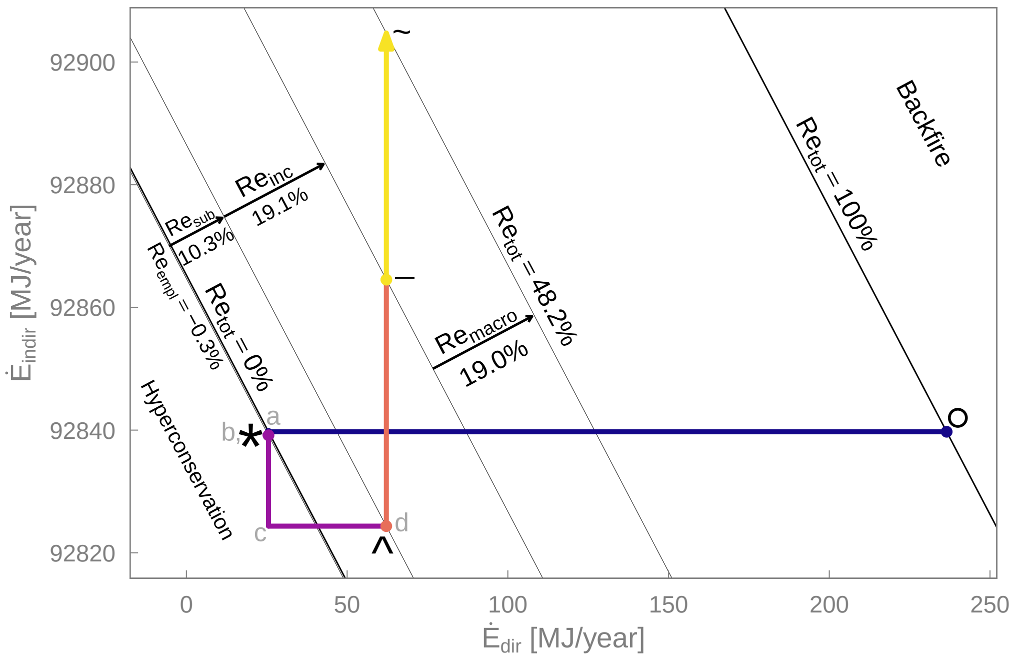

Fig. 8: Energy path, lamp example with macro factor (k) assumed to be 1.

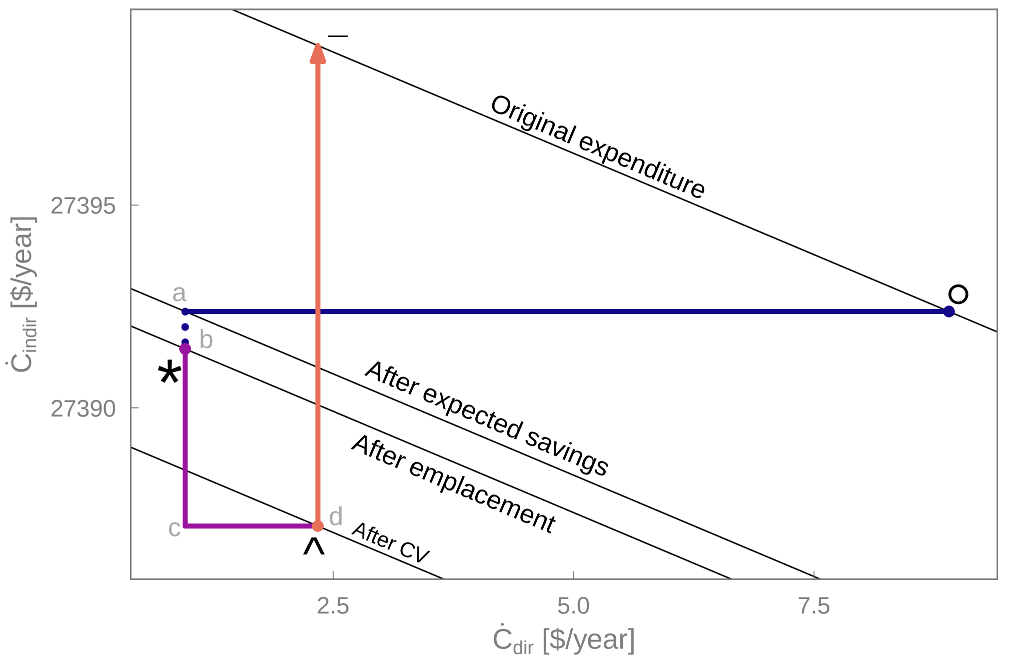

Fig. 9: Expenditure path, lamp example.

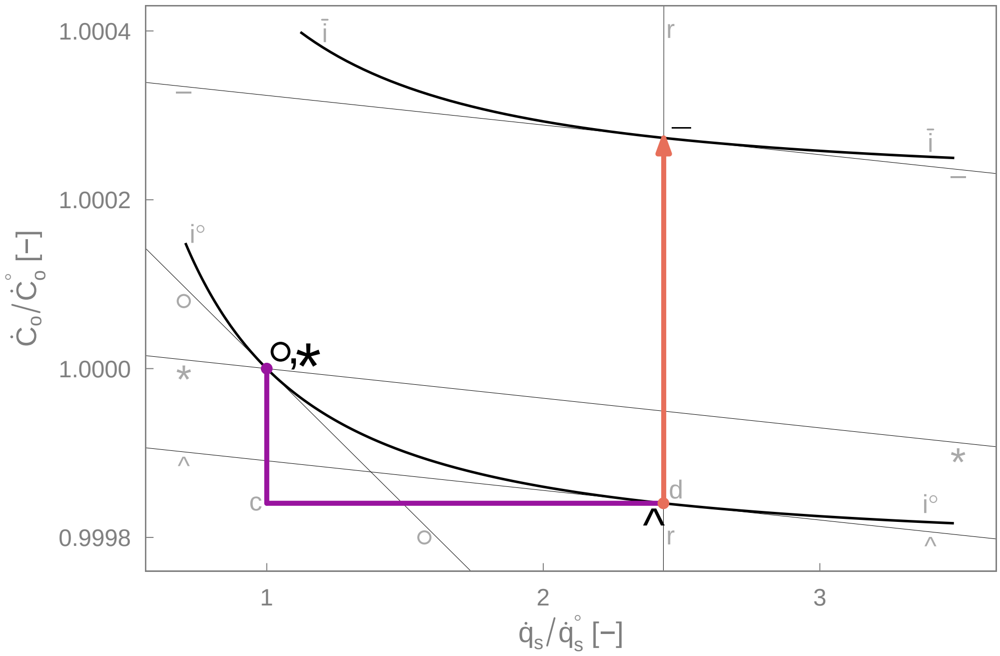

Fig. 10: Consumption path, lamp example.

leading to increased consumption of lighting (from $\dot{q}_s^* = 580,350$ lm-hr/year to $\hat{\dot{q}}_s = 1,412,810$ lm-hr/year) as shown by segment c—$\wedge$ in Fig. 10. To maintain constant utility, consumption of other goods is reduced ($\Delta\hat{\dot{C}}_o = -4.37$ \$/year), as shown by segment $*$—c in Fig. 10, yielding indirect substitution effect rebound (Re_{isub}) of -7.0%.

Income effect rebound arises from respending of net energy cost savings associated with converting from the incandescent lamp to the LED lamp ($\hat{\dot{N}} = 11.86$ \$/year). Direct income effect rebound (Re_{dinc}) is 0.01%), positive but small, as the lamp owner allocates some of the additional income to increased consumption of lighting. The indirect income effect rebound is large ($Re_{iinc} = 19.0\%$), due to the energy implications of increased spending on other goods. Total micro rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 29.1\%$.

Finally, rebound due to the macroeconomic macro effect (Re_{macro}) is coincidentally also 19.0% with k assumed to be 1, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (lighting).

3.3. Sensitivity analyses

Sensitivity analyses show the effect of independently-varied parameters on total rebound and rebound components. In the context of this framework, sensitivity analyses can show important trends, tendencies, and relationships between rebound parameters and rebound magnitudes. Key rebound parameters include post-EEU efficiency ($\tilde{\eta}$), capital cost ($\tilde{C}_{cap}$), energy price (p_E), price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s}$), and the macro factor (k). Sensitivity analyses must be undertaken carefully, because rebound parameters are not expected to be independent.

In each subsection below, a series of graphs shows the effects of varying each of these five key rebound parameters ($\tilde{\eta}$, $\tilde{C}_{cap}$, p_E, $\epsilon_{\dot{q}_s,p_s}$, and k) on energy rebound (Re_{tot}) and its components (Re_{emb}, Re_{md}, Re_{dsub}, Re_{isub}, Re_{dinc}, Re_{iinc}, and Re_{macro}). In the graphs, nominal values for the rebound parameters are shown as points, and sensitivity trends are shown as lines. A limitation of each sensitivity analysis is noted at the end of each subsection.

3.3.1. Effect of post-EEU efficiency ($\tilde{\eta}$)

Fig. 11 shows that both takeback and expected energy savings ($\dot{S}_{dev}$) increase with post-EEU efficiency ($\tilde{\eta}$), but the relationship is asymptotic. Each unit increase of fuel economy or lighting efficacy is less effective than the previous unit increase of fuel economy or lighting efficacy for saving energy. At very high levels of fuel economy economy or lighting efficacy, a unit increase leads to almost no additional energy savings. Thus, we can say there are diminishing returns of fuel economy and lighting efficacy, leading to saturation of energy savings at very high levels of fuel economy and lighting efficacy. A simple example illustrates. A $\eta^{\circ} = 25$ mpg car drives $q_s^{\circ} = 100$ miles using $E_s^{\circ} = 4$ gallons of gasoline. A more-efficient car ($\tilde{\eta} = 30$ mpg) is expected to use $E_s^{*} = 3.33$ gallons to drive the same distance, a savings of $\dot{S}_{dev} = 0.67$ gallons. Another 5 mpg boost in efficiency (to $\tilde{\eta} = 35$ mpg) will use $E_s^{*} = 2.86$ gal to drive 100 miles, a further expected savings of only $\dot{S}_{dev} = 0.47$ gallons. Each successive 5 mpg boost in fuel economy is expected to save less energy than the previous 5 mpg boost in fuel economy

Saturation can be seen mathematically, too. Taking the limit as $\tilde{\eta} \rightarrow \infty$ in Eq. (10) gives $\dot{S}_{dev} = \dot{E}_s^{\circ}$, not ∞. Thus, efficiency saturation must occur. Fig. 11 shows that the framework correctly replicates the expected efficiency saturation trends.

Saturation is especially noticeable in the lamp example compared to the car example, the difference being that the LED lamp is already much more efficient than the incandescent lamp (9.26×), whereas the hybrid car is only 1.68× as efficient as the conventional gasoline car. Thus, at $\tilde{\eta} = 81.8$ lm-hr/W-hr, the energy efficient LED is far closer to efficiency saturation than the hybrid vehicle (at $\tilde{\eta} = 42$ mpg). As a result, further increases in the LED lamp's efficiency are less effective than further increases in the hybrid car's efficiency.

That said, actual savings is the difference between the expected energy savings line (solid line) and the take back line (dashed line) in Fig. 11. Because the gap between the lines grows, higher efficiency yields actual energy savings, even after accounting for rebound effects. But the actual savings are always less than expected savings, due to takebacks.

Fig. 11 shows that expected energy savings ($\dot{S}_{dev}$) increase faster than takeback as $\tilde{\eta}$ increases. Thus, total rebound (Re_{tot}, the ratio of takeback to expected energy savings in Eq. 4), decreases as efficiency grows. The lamp exhibits a relatively smaller rebound decline with efficiency, because the lamp example is closer to saturation than the car example.

Fig. 12 shows the variation of all rebound components with post-EEU efficiency ($\tilde{\eta}$). In the Car and Lamp examples, direct substitution rebound (Re_{dsub}) is the rebound component most sensitive to changes in post-EEU efficiency ($\tilde{\eta}$),

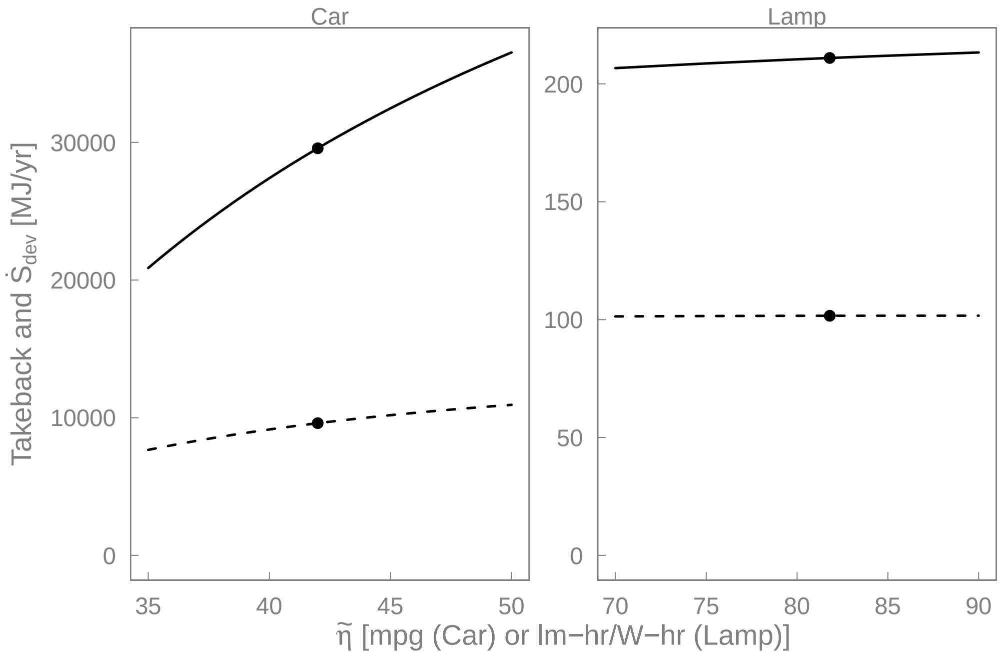

Fig. 11: Expected energy savings ($\dot{S}_{dev}$, solid line) and takeback (dashed line) sensitivity to post-EEU efficiency ($\tilde{\eta}$). (Note different x- and y-axis scales.)

A limitation of the energy efficiency sensitivity analysis is that post-EEU efficiency ($\tilde{\eta}$) is unlikely to be independent of other factors, such as capital cost ($\tilde{C}_{cap}$).

Note that the sensitivity analysis on post-upgrade efficiency ($\tilde{\eta}$, Fig. 12) is the only sensitivity analysis that requires careful explication of both the numerator and denominator of Eq. (4), as in Fig. 11, because both the numerator and denominator of Eq. 4 changes when post-upgrade efficiency ($\tilde{\eta}$) changes. The denominator of Eq. (4) doesn't change for the sensitivity analyses of Figs. 13–16. Thus, for the remaining sensitivity analyses, when the rebound percentage increases (decreases), energy takeback in the numerator of Eq. (4) increases (decreases) proportionally.

3.3.2. Effect of capital cost ($\tilde{C}_{cap}$)

The sensitivity of energy rebound to capital cost ($\tilde{C}_{cap}$) is shown in Fig. 13. All other things being equal, as capital cost of the EEU rises, less cash is freed by the EEU, leading to smaller income, macro, and total rebound. The same effects would be observed with increasing maintenance and disposal cost rate ($\tilde{\dot{C}}_{md}$).

A limitation of the capital cost sensitivity analysis is that capital cost ($\tilde{C}_{cap}$) is unlikely to be independent of $\tilde{\eta}$. Within a given energy efficiency technology (e.g., hybrid cars or LED lamps), greater capital cost ($\tilde{C}_{cap}$) may be associated with greater service efficiency ($\tilde{\eta}$). $\tilde{C}_{cap}$ and $\tilde{\eta}$ should probably be varied jointly, not independently.

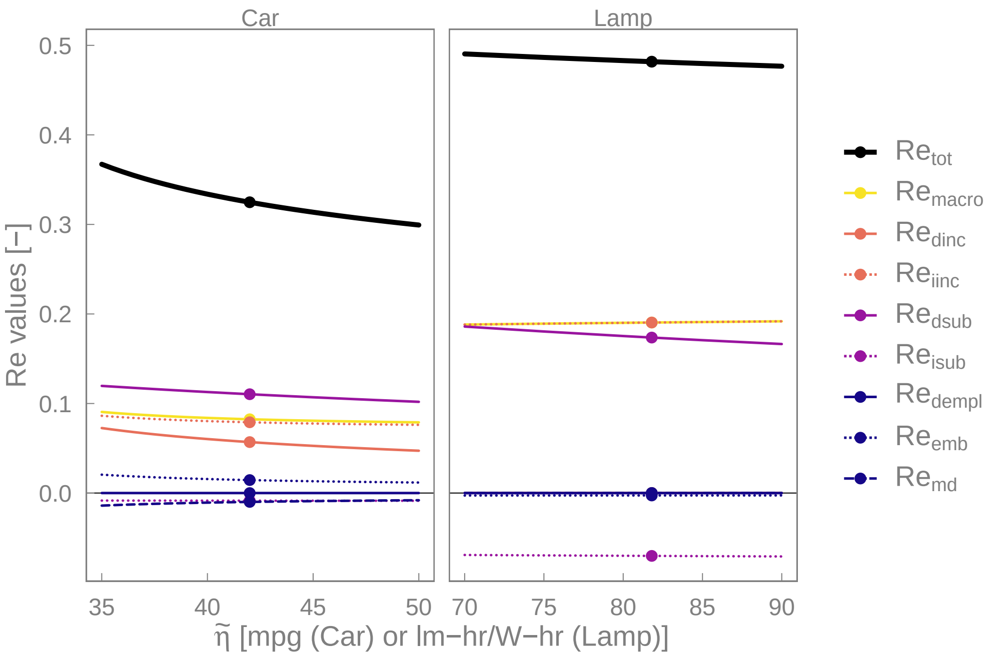

Fig. 12: Sensitivity of rebound components to post-EEU efficiency ($\tilde{\eta}$).

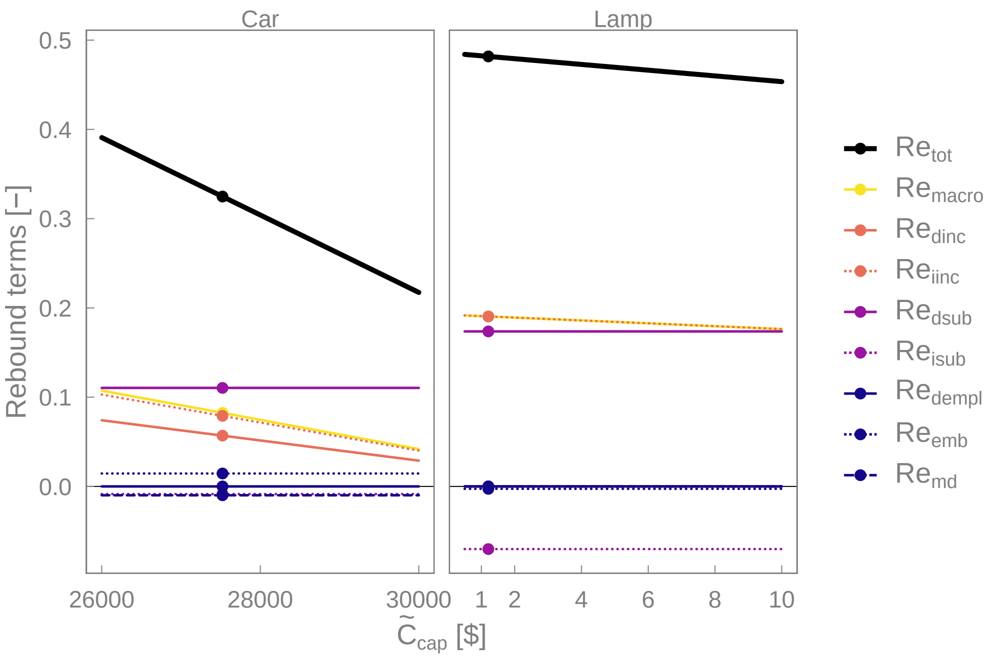

Fig. 13: Sensitivity of rebound components to capital cost ($\tilde{C}_{cap}$).

3.3.3. Effect of energy price (p_E)

The effect of energy price on rebound is shown in Fig. 14. Increasing energy prices lead to larger total rebound (Re_{tot}), because higher energy prices lead to more freed cash to be re-spent by the device owner. All other things being equal, more freed cash leads to more spending on other goods and services that demand energy.

Fig. 14 also shows the effect of energy price (p_E) on all rebound components. Most rebound components increase with energy price, with the examples exhibiting different sensitivities. Substitution effects (Re_{dsub} and Re_{isub}) are the only rebound components that decrease with energy price (p_E). Substitution effects decrease with energy price, because at high energy price, less behavior adjustment is needed to re-equilibrate after emplacement of the energy efficient device.

German energy prices[4] are shown as vertical lines, providing an indication of possible energy price variations. All other things being equal, if U.S. residents paid Germany's energy prices, energy rebound would be 50.4% for the car example and 95.3% for the lamp example.

A limitation of the energy price sensitivity analysis arises from the fact that energy price is not independent of other parameters. Indeed, other rebound parameters would change along with energy price (especially if moving from one country to another), including capital cost ($\tilde{C}_{cap}$), maintenance and disposal expenditure rate ($\tilde{\dot{C}}_{md}$), energy intensity of the economy (I_E), and energy service consumption rate ($\dot{q}_s^\circ$).

3.3.4. Effect of elasticity ($\epsilon_{\dot{q}_s,p_s}$)

Fig. 15 shows the variation of total rebound (Re_{tot}) with the uncompensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s}$). The effect is exponential: total rebound increases with larger negative values of $\epsilon_{\dot{q}_s,p_s}$, as expected. The lamp example shows stronger exponential variation than the car example. Fig. 15 shows that direct substitution rebound (Re_{dsub}) is the most sensitive rebound component to changes in $\epsilon_{\dot{q}_s,p_s}$. For the lamp example, indirect income rebound (Re_{iinc}) also increases substantially with $\epsilon_{\dot{q}_s,p_s}$, because freed cash increases substantially with $\epsilon_{\dot{q}_s,p_s}$ in the lamp example.

A limitation of the elasticity sensitivity study derives from limitations of the CES utility model itself, which constrains price elasticity variation given an elasticity of substitution. Uneconomic conditions should be avoided. For example, negative direct substitution rebound ($Re_{dsub} < 0$) is obtained when $|\epsilon_{\dot{q}_s,p_s}| < f_{\dot{C}_s}^\circ$, because Eq. (D.25) pushes the elasticity of substitution negative ($\sigma < 0$). In reality, smaller negative (closer to 0) price elasticity ($\epsilon_{\dot{q}_s,p_s}$) would correlate with a smaller fraction of expenses spent on the energy service ($f_{\dot{C}_s}^\circ$), thereby avoiding the uneconomic condition. However, a univariate sensitivity study cannot capture this effect and is best used for smaller variations in $\epsilon_{\dot{q}_s,p_s}$.

3.3.5. Effect of macro factor (k)

The sensitivity of energy rebound to the macro factor (k) is shown in Fig. 16. The macro factor has a linear effect on total rebound (Re_{tot}) through the macro rebound component (Re_{macro}). All other rebound components are constant as k varies.

[4]For the car example, the gasoline price in Germany is taken as 1.35 €/liter for "super gasoline" (95 octane), an approximate average price over the last five years (finanzen.net). For the lamp example, the electricity price in Germany is taken as 0.3 €/kW-hr (Bundesministerium für Wirtschaft und Energie). Converting currency (at 1 € = 1.21 USD) and physical units gives 6.18 \$/USgallon and 0.363 \$/kW-hr.

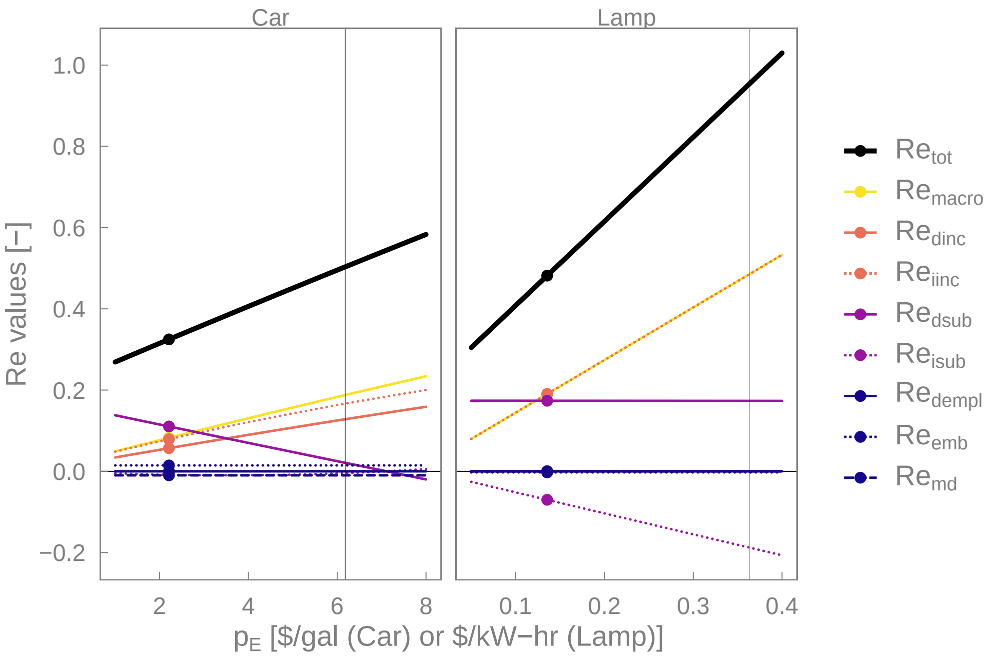

Fig. 14: Sensitivity of rebound components to energy price (p_E).German energy prices denoted by vertical line.

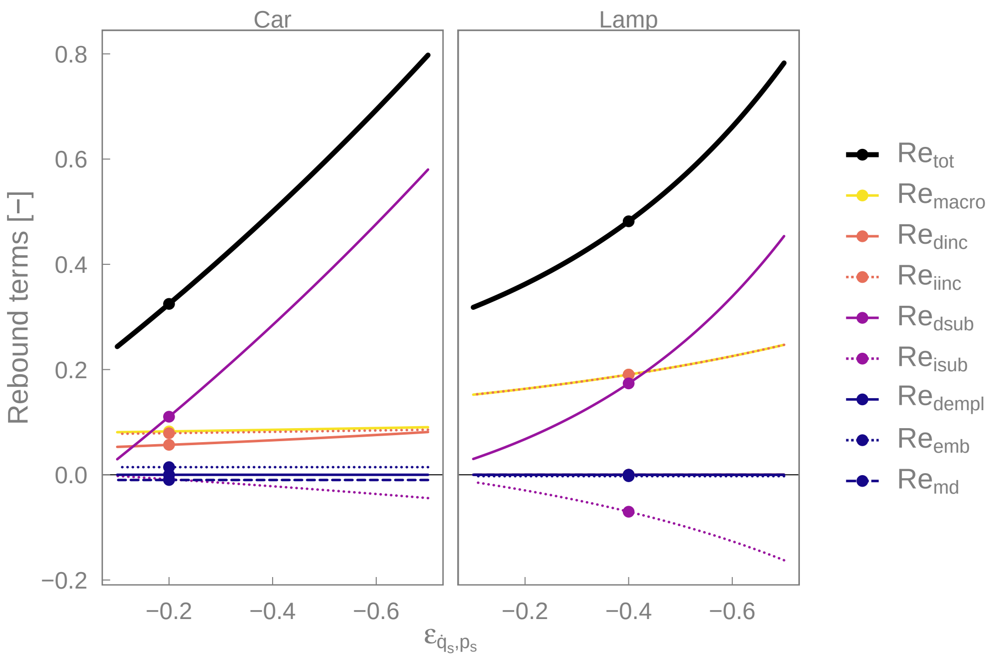

Fig. 15: Sensitivity of rebound components to uncompensated energy service price elasticity of energy demand ($\epsilon_{\dot{q}_s,p_s}$). (Note reversed x-axis scale.)

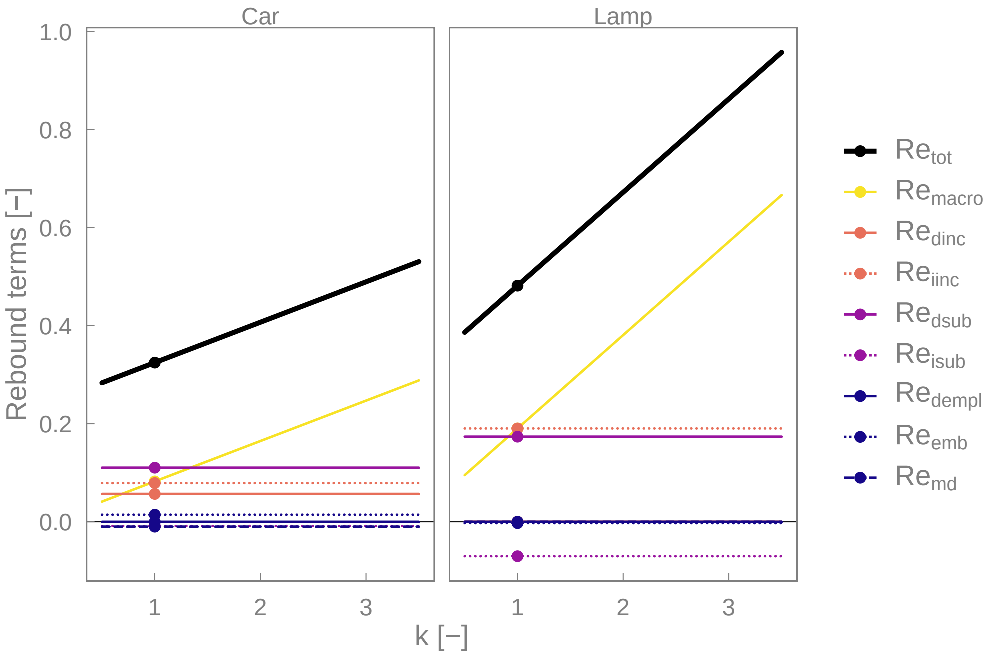

Fig. 16: Sensitivity of rebound components to the macro factor (k).

A limitation of the macro factor sensitivity analysis is that the macro factor (k) is unlikely to be independent of I_E, because different values of k imply a different macroeconomy.

750 ## 4. Discussion

In this section, we highlight several novel characteristics of the comprehensive, consumer-sided energy rebound analysis framework. Next, we show how the framework enables insights on energy policy, especially policies to encourage energy efficiency. Finally, we discuss the macro factor (k), one of the novel features of this paper.

755 ### *4.1. Novel aspects of the framework*

In this section, we highlight four novel characteristics of the rebound analysis framework. First, the framework is, to our knowledge, the first to enable numerically-precise calculation of several microeconomic rebound effects and a macro effect. The two examples show the benefits of the framework for such calculations.

760 Another novelty of our work involves expressing numerically precise income and substitution effects for non-marginal efficiency improvements in numerically precise graphics. The possibility of decomposing direct and indirect rebound effects into substitution and income effects was adumbrated by Khazzoom (1980), spelt out clearly by Greening et al. (2000), and numerical magnitudes approximated by Borenstein (2015). Yet, past investigations of microeconomic rebound typically show only stylized graphs of indifference

curves in the consumption space, even if they are numerically precise in other respects (Thomas and Azevedo, 2013a; Moshiri and Aliyev, 2017; Li et al., 2018). Our precisely drawn rebound path graphs (Figs. 5–10) through energy, expenditure, and consumption spaces aid comprehension of and intuition about the various rebound effects.

The third novel aspect of this framework is its inclusion of embodied energy and maintenance and disposal effects of the EEU. These effects are the energy manifestation of the important difference between costless technology upgrades or (costly) mandated efficiency increases (Gillingham et al., 2013; Fullerton and Ta, 2020). Embodied energy and maintenance and disposal effects are not typically included in microeconomic rebound frameworks (Azevedo, 2014; Gillingham, 2020). Most rebound frameworks assume costless upgrade in both financial and energy terms. This framework shows that it is possible to include such effects.

Finally, this framework is the first, to our knowledge, to operationalize the macroeconomic factor and to identify the place to link macro to micro effects.

4.2. Policy considerations

We take for granted that it is desirable to reduce energy consumption via energy efficiency to reduce CO_2 emissions. For energy efficiency to be as effective as possible, rebound should be kept low. The sensitivity analyses of Section 3.3 enable exploration of several policy options that are often suggested to promote energy efficiency: energy efficiency standards, rebates or subsidies, and carbon taxes. We discuss the rebound implications of each policy option with reference to the sensitivity analysis of Section 3.3.

4.2.1. Energy efficiency standards

Energy efficiency standards mandate minimum energy efficiency levels for selling into a market. One example of energy efficiency standards is the Corporate Average Fuel Economy (CAFE) standards for U.S. cars.

Fig. 12 shows that rebound falls as post-upgrade energy efficiency ($\tilde{\eta}$) rises. This result is driven mainly by a declining direct rebound effect as efficiency rises (Fig. 12), in turn driven by the declining own price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s}$). So raising the minimum efficiency required to participate in a market is beneficial. Indeed, Fig. 11 shows the gap between expected energy savings and takeback grows with efficiency, so absolute energy savings are expected to increase as efficiency grows. However, Fig. 11 shows that takeback increases with efficiency, so caution is merited.

Larger efficiency increases can require higher capital investment within a given technology. To encourage market penetration of higher-efficiency devices, regulators may be tempted to reduce costs to device purchasers via rebates or subsidies, the topic of the next section.

4.2.2. Rebates or subsidies

Rebates or subsidies can be offered to device purchasers to incentivize adoption of energy-efficiency technologies and grow their markets. Doing so effectively reduces the capital cost of energy efficiency devices ($\tilde{C}_{cap}$). However, Fig. 13 shows that reducing capital cost increases energy rebound, the route being, predominantly, the income effect where more net income is available to spend on more of the energy service and other goods. Thus, policymakers should be aware that rebound will reduce the effectiveness of rebates and subsidies to generate actual energy savings. We note in passing that the framework accommodates both costless upgrades and a costly "policy-induced" improvement (Gillingham et al., 2016).

To avoid rebound generated by rebates or subsidies, policymakers may want to implement energy taxes, effectively raising the price of energy, the topic of the next section.

4.2.3. Carbon taxes

Carbon taxes reduce net income earned by device owners after the EEU by raising the effective price of fossil energy and the price of any fossil energy-powered energy service, such as non-electric cars. By raising input prices, consumers are incentivized to use less of the energy service and buy more-efficient devices. However, increasing the price of energy means that subsequent increases in energy efficiency provide increasing net income. Fig. 14 shows that energy rebound rises with energy price. The route to higher rebound from rising energy prices is through the income and macro effects. Thus, policymakers should be aware that raising energy prices will not be as effective as presumed, due to rebound effects. Naturally, higher energy prices are likely to induce structural change towards an overall less energy-intensive economy (e.g. more public transport); yet, rebound effects become more pronounced.

4.3. Macroeconomic rebound

The framework also links to the total or economy-wide rebound literature by relating the size of macroeconomic rebound to that of the microeconomic rebound in a term that corresponds to magnitudes in the microeconomic portion of the framework. We note, again, that most rebound analyses focus on either microeconomic effects or total rebound. Few rebound studies have explored the macro space between micro and total rebound. Inspired by Borenstein (2015) and others, we bridge macro and micro with the macro factor (k), as discussed in Section 2.5.4. For the results presented in Section 3, we assumed a placeholder value of $k = 1$, meaning that every \$1 spending by the device owner in the income effect generates \$1 of additional economic activity in the broader economy. However, the framework presented in Section 2 and the results obtained in Section 3 allow, for the first time, some initial discussion about a value for k.

To calibrate the macro factor, we treat macro rebound (Re_{macro}) as a residual. The macro factor (k) becomes an unknown parameter whose value is to be chosen such that Re_{macro} is sufficient to achieve an expected value for total rebound (Re_{tot}). We take the expected value for Re_{tot} from Brockway et al. (2021) who surveyed 33 recent studies to find average total rebound of $Re_{tot} = 63\%$. The calibrated values of k that give identical $Re_{tot} = 63\%$ for both examples are $k = 4.7$ for the car example and $k = 1.8$ for the lamp example.

Intuition as well as the considerable variance in Re_{tot} in 33 surveyed studies indicates that total rebound from one EEU is likely to be different from total rebound from another EEU. For the purposes of illustration we set $k = 3$, being between the values of k estimated from the car and lamp examples. Note that $k = 3$ implies that every \$1 of net income spent by the device owner generates \$3 of additional economic activity in the broader economy. We multiply $k\hat{\dot{N}}$ by the energy intensity of the economy to find the energy implications of respending in the economy. There are three ways to think of $k = 3$. First, $k = 3$ can be considered to be the average economic growth generated by the owner spending each \$1 of net income, or three times over the growth implied by applying Hulten's theorem. Second, it could be that growth is less than \$3 but that the macroeconomic "energy price effect" (a decline in energy prices due to the fallen demand) induces consumption at a higher energy intensity than that of the pre-EEU economy. Third, from the demand-side perspective entertained by Borenstein (2015), $k = 3$ could be interpreted as a marginal propensity to consume of 75%. (See Appendix G.) $MPC = 0.75$ is a reasonable number, being in the upper half of recent estimates by Carroll et al. (2017). (The analogy with marginal propensity to consume makes a link to Borenstein's approach, even though we consider the macro effect to be driven by supply-side productivity growth.)

After calibrating $k = 3$, we see that choosing a placeholder value of $k = 1$ underestimated Re_{macro} and, therefore, Re_{tot}. In Figs. 5 and 8, the macro effect segments ($-$ ━ $\sim$) should be three times longer than they appear. In Tables 6 and 11, the values of macro rebound (Re_{macro}) should triple to 24.7% and 57.1%, and the values of total rebound (Re_{tot}) should increase to 49.0% and 86.3% for the car and lamp examples, respectively.

5. Conclusions (MKH, 500 words. Currently at 250 words.)

In this paper, we have developed a comprehensive, consumer-sided rebound analysis framework that solidifies theoretical foundations for energy rebound, enables empirical analysis, and can be used to guide energy efficiency policy. The framework is the first to our knowledge to integrate all components of direct and indirect microeconomic rebound for consumers in an analytically consistent yet numerically precise and operationalizable manner. The sets of three rebound path graphs that simultaneously track energy, expenditure, and consumption decisions for every rebound component display and summarize this integration. The framework (i) builds upon microeconomic foundations and carefully distinguishes among microeconomic rebound effects, (ii) identifies and incorporates, for the first time, a plausible value for the macro factor ($k = 3$) to provide a better link between macro and micro effects, and (iii) enables empirical estimation of rebound magnitudes for real energy efficiency upgrades.

From the development and application of the framework, we can draw four important conclusions. First, the car and lamp examples (Section 3) show that the framework enables quantification of magnitudes of all microeconomic rebound mechanisms, including direct and indirect channels for emplacement, substitution, and income effects. Second, the examples show that magnitudes of rebound effects vary with the type of energy efficiency upgrade performed. Third, the sensitivity studies (Section 3.3) enable evaluation of rebound sensitivies to imporant parameters. For the examples in this paper, total rebound is more sensitive to the price of energy (p_E), the elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s}$), and the macro factor (k) than either efficiency ($\tilde{\eta}$) or capital cost ($\widetilde{C}_{cap}$). Last, the discussion (Section 4) shows that rebound is a headwind for efficiency-led energy reduction and CO_2 mitigation policies. Quantification of rebound effect magnitudes is an important precursor to devising energy efficiency policies that would both encourage efficiency and limit rebound effects.

Future work could be pursued in several areas. (i) Other utility models could be explored for the substitution effect. (ii) Further empirical studies could be performed to estimate the magnitude of different rebound mechanisms in a range of real-life case studies. (iii) Deeper study of macro rebound is needed, including improved estimates for the macro factor (k) and its relation to the marginal propensity to consume. (iv) The framework could be extended to producer-sided energy rebound mechanisms. (v) The framework presented here could be embeded in energy-economy models to better include rebound effects in macro energy modeling and energy policy discussions.

Competing interests

Declarations of interest: none.

Author contributions

Matthew Kuperus Heun: Conceptualization, Formal analysis, Resources, Software, Supervision, Validation, Visualization, Writing–original draft, Writing–review & editing. **Gregor Semieniuk**: Conceptualization, Formal analysis, Investigation, Writing–original draft, Writing–review & editing. **Paul Brockway**: Conceptualization, Data curation, Formal analysis, Investigation, Validation, Writing–original draft, Writing–review & editing.

Data repository

All data and calculations are stored at **** Leeds data repository URL. ****. An `R` package for performing all rebound calculations can be found at `https://github.com/MatthewHeun/ReboundTools`.

Acknowledgements

Paul Brockway's time was funded by the UK Research and Innovation (UKRI) Council, supported under EPSRC Fellowship award EP/R024254/1. The authors are grateful for comments from internal reviewers Fred Haan, Becky Haney, and Jeremy Van Antwerp (Calvin University); Nathan Chan (University of Massachusetts at Amherst); and Zeke Marshall (University of Leeds). The authors benefitted from discussions with Daniele Girardi (University of Massachusetts at Amherst) and Christopher Blackburn (Bureau of Economic Analysis). Finally, the authors thank the students of MKH's Fall 2019 Thermal Systems Design course (ENGR333) who studied energy rebound of many energy conversion devices using an early version of this framework.

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Table A.1: Symbols and abbreviations.

Symbol	Meaning [example units]
a	a point in the emplacement effect on rebound path graphs or share parameter in the CES utility function [–]
b	a point in the emplacement effect on rebound path graphs
C	cost [\$]
c	a point in the substitution effect on rebound path graphs
d	a point in the income effect on rebound path graphs
E	final energy [MJ]
e	inverse of final-energy-to-services efficiency (η); used by Borenstein (2015) [MJ/vehicle-km]
f	expenditure share [–]
G	freed cash [\$]
I	energy intensity of economic activity [MJ/\$]
k	macro factor [–]
m	temporary variable for $1 - \epsilon_{\dot{q}_o, p_s, c}$ [–]
M	income [\$]
N	net savings [\$]
p	price [\$]
q	quantity [–]
Re	rebound [–]
S	energy cost savings [\$]
t	energy conversion device lifetime [years]
u	utility [utils]

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Appendix

A. Nomenclature

Presentation of the comprehensive, consumer-sided rebound analysis framework is aided by a nomenclature that describes energy stages and rebound effects. Table A.1 shows symbols and abbreviations, their meanings, and example units. Table A.2 shows Greek letters, their meanings, and example units. Table A.4 shows symbol decorations and their meanings. Table A.5 shows subscripts and their meanings.

Differences are indicated by the Greek letter Δ and always signify subtraction of a quantity at an earlier stage of Fig. 1 from the same quantity at a later stage of Fig. 1. E.g., $\Delta\bar{X} \equiv \bar{X} - \hat{X}$, and $\Delta\tilde{X} \equiv \tilde{X} - \bar{X}$. Lack of decoration on a difference term indicates a difference that spans all stages of Fig. 1. E.g., $\Delta X \equiv \tilde{X} - X^{\circ}$. ΔX is also the sum of

Table A.2: Greek letters.

Greek letter	Meaning [example units]
Δ	difference (later quantity − earlier quantity, see Table B.1)
ϵ	elasticity [–]
$\epsilon_{\dot{q}_s,\dot{M}}$	income ($\dot{M}$) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,\dot{M}}$	income ($\dot{M}$) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s}$	energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s}$	energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s,c}$	compensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s,c}$	compensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
η	final-energy-to-service efficiency [vehicle-km/MJ]
σ	elasticity of substitution between the energy service ($\dot{q}_s^\circ$) and other goods ($\dot{q}_o^\circ$) [–]

Table A.3: Abbreviations.

Abbreviation	Meaning
APF	aggregate production function
CV	compensating variation
EEU	energy efficiency upgrade
GDP	gross domestic product
mpg	miles per U.S. gallon
U.S.	United States
USD	U.S. dollar

Table A.4: Decorations.

Decoration	Meaning [example units]
X°	X originally (before the emplacement effect)
X^*	X after the emplacement effect (before the substitution effect)
$\hat{X}$	X after the substitution effect (before the income effect)
$\bar{X}$	X after the income effect (before the macro effect)
$\tilde{X}$	X after the macro effect
$\dot{X}$	rate of X [units of X/year]
M'	effective income [$]

Table A.5: Subscripts.

Subscript	Meaning
0	quantity at an initial time
c	compensated
cap	capital costs
dev	device
$dempl$	direct emplacement effect
d	disposal
$dinc$	direct income effect
dir	direct effects (at the energy conversion device)
$dsub$	direct substitution effect
emb	embodied
i	index for other goods purchased in the economy
j	one of cap, md, or o in Eq. (8)
$iempl$	indirect emplacement effects
$iinc$	indirect income effect
$indir$	indirect effects (beyond the energy conversion device)
$isub$	indirect substitution effect
$life$	lifetime
m	maintenance
md	maintenance and disposal costs
o	other expenditures (besides energy) by the device owner
own	ownership duration
$macro$	macro effect
s	service stage of the energy conversion chain
tot	sum of all rebound effects in the framework

differences across each stage in Fig. 1, as shown below.

$$\begin{aligned}
\Delta X &= \Delta \tilde{X} + \Delta \bar{X} + \Delta \hat{X} + \Delta X^{*} \\
\Delta X &= (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^{*}) + (X^{*} - X^{\circ}) \\
\Delta X &= (\tilde{X} - \cancel{\bar{X}}) + (\cancel{\bar{X}} - \cancel{\hat{X}}) + (\cancel{\hat{X}} - \cancel{X^{*}}) + (\cancel{X^{*}} - X^{\circ}) \\
\Delta X &= \tilde{X} - X^{\circ}
\end{aligned} \tag{A.1}$$

B. Derivation of comprehensive, consumer-sided rebound analysis framework

**** We may want to delete (or comment out) paragraphs of this appendix that are duplicated in the body of the paper.—MKH ****

This appendix provides a detailed derivation of the comprehensive, consumer-sided rebound analysis framework, beginning with relationships for each rebound effect.

B.1. Relationships for rebound effects

For each energy rebound effect in Fig. 1, energy and financial analysis must be performed. The purposes of the analyses are to determine for each effect (i) an expression for energy rebound (Re) for the effect and (ii) an equation for net savings ($\dot{N}$) remaining after the effect.

Analysis of each rebound effect involves a set of assumptions and constraints as shown in Table B.1. In Table B.1, relationships for emplacement effect embodied energy rates ($\dot{E}^{\circ}_{emb}$ and $\dot{E}^{*}_{emb}$), capital expenditure rates ($\dot{C}^{\circ}_{cap}$ and $\dot{C}^{*}_{cap}$), and maintenance and disposal expenditure rates ($\dot{C}^{\circ}_{md}$ and $\dot{C}^{*}_{md}$) are typical, and inequalities could switch direction for a specific EEU. Macro effect relationships are given for a single device only. If the EEU is deployed at scale across the economy, the energy service consumption rate ($\tilde{\dot{q}}_s$), device energy consumption rate ($\tilde{\dot{E}}_s$), embodied energy rate ($\tilde{\dot{E}}_{emb}$), capital expenditure rate ($\tilde{\dot{C}}_{cap}$), and maintenance and disposal expenditure rate ($\tilde{\dot{C}}_{md}$) will all increase in proportion to the number of devices emplaced.

Table B.1: Assumptions and constraints for analysis of rebound effects.

Parameter	Emplacement Effect	Substitution Effect	Income Effect	Macro Effect
Energy price	$p_E^\circ = p_E^*$	$p_E^* = \hat{p}_E$	$\hat{p}_E = \bar{p}_E$	$\bar{p}_E = \tilde{p}_E$
Energy service efficiency	$\eta^\circ < \eta^*$	$\eta^* = \hat{\eta}$	$\hat{\eta} = \bar{\eta}$	$\bar{\eta} = \tilde{\eta}$
Energy service price	$p_s^\circ > p_s^*$	$p_s^* = \hat{p}_s$	$\hat{p}_s = \bar{p}_s$	$\bar{p}_s = \tilde{p}_s$
Other goods price	$p_o^\circ = p_o^*$	$p_o^* = \hat{p}_o$	$\hat{p}_o = \bar{p}_o$	$\bar{p}_o = \tilde{p}_o$
Energy service consumption rate	$\dot{q}_s^\circ = \dot{q}_s^*$	$\dot{q}_s^* < \dot{\hat{q}}_s$	$\dot{\hat{q}}_s < \dot{\bar{q}}_s$	$\dot{\bar{q}}_s = \dot{\tilde{q}}_s$
Other goods consumption rate	$\dot{q}_o^\circ = \dot{q}_o^*$	$\dot{q}_o^* > \dot{\hat{q}}_o$	$\dot{\hat{q}}_o < \dot{\bar{q}}_o$	$\dot{\bar{q}}_o = \dot{\tilde{q}}_o$
Device energy consumption rate	$\dot{E}_s^\circ > \dot{E}_s^*$	$\dot{E}_s^* < \dot{\hat{E}}_s$	$\dot{\hat{E}}_s < \dot{\bar{E}}_s$	$\dot{\bar{E}}_s = \dot{\tilde{E}}_s$
Embodied energy rate	$\dot{E}_{emb}^\circ < \dot{E}_{emb}^*$	$\dot{E}_{emb}^* = \dot{\hat{E}}_{emb}$	$\dot{\hat{E}}_{emb} = \dot{\bar{E}}_{emb}$	$\dot{\bar{E}}_{emb} = \dot{\tilde{E}}_{emb}$
Capital expenditure rate	$\dot{C}_{cap}^\circ < \dot{C}_{cap}^*$	$\dot{C}_{cap}^* = \dot{\hat{C}}_{cap}$	$\dot{\hat{C}}_{cap} = \dot{\bar{C}}_{cap}$	$\dot{\bar{C}}_{cap} = \dot{\tilde{C}}_{cap}$
Maint. and disp. expenditure rate	$\dot{C}_{md}^\circ < \dot{C}_{md}^*$	$\dot{C}_{md}^* = \dot{\hat{C}}_{md}$	$\dot{\hat{C}}_{md} = \dot{\bar{C}}_{md}$	$\dot{\bar{C}}_{md} = \dot{\tilde{C}}_{md}$
Energy service expenditure rate	$\dot{C}_s^\circ > \dot{C}_s^*$	$\dot{C}_s^* < \dot{\hat{C}}_s$	$\dot{\hat{C}}_s < \dot{\bar{C}}_s$	$\dot{\bar{C}}_s = \dot{\tilde{C}}_s$
Other goods expenditure rate	$\dot{C}_o^\circ = \dot{C}_o^*$	$\dot{C}_o^* > \dot{\hat{C}}_o$	$\dot{\hat{C}}_o < \dot{\bar{C}}_o$	$\dot{\bar{C}}_o = \dot{\tilde{C}}_o$
Income	$\dot{M}^\circ = \dot{M}^*$	$\dot{M}^* = \dot{\hat{M}}$	$\dot{\hat{M}} = \dot{\bar{M}}$	$\dot{\bar{M}} = \dot{\tilde{M}}$
Net savings	$0 = \dot{N}^\circ < \dot{N}^*$	$\dot{N}^* < \dot{\hat{N}}$	$\dot{\hat{N}} > \dot{\bar{N}} = 0$	$\dot{\bar{N}} = \dot{\tilde{N}} = 0$

Table B.2: Sources for zeroed terms in Tables B.3–B.6.

Zeroed term	Justification (from Table B.1).
$\cancelto{0}{\Delta \dot{C}_o^*}$	$\dot{C}_o^\circ = \dot{C}_o^*$ ($\dot{C}_o$ unchanged across emplacement effect.)
$\cancelto{0}{\dot{N}^\circ}$	$0 = \dot{N}^\circ$ (Net savings is zero prior to the EEU.)
$\cancelto{0}{\Delta \dot{\hat{E}}_{emb}}$	$\dot{E}_{emb}^* = \dot{\hat{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \dot{\hat{C}}_{md}}$	$\dot{C}_{md}^* = \dot{\hat{C}}_{md}$ ($\dot{C}_{md}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \dot{\bar{E}}_{emb}}$	$\dot{\hat{E}}_{emb} = \dot{\bar{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across income effect.)
$\cancelto{0}{\Delta \dot{\bar{C}}_{md}}$	$\dot{\hat{C}}_{md} = \dot{\bar{C}}_{md}$ ($\dot{C}_{md}$ unchanged across income effect.)
$\cancelto{0}{\dot{\bar{N}}}$	$\dot{\bar{N}} = 0$ (All net savings is re-spent in the income effect.)

B.2. Derivations

Derivations for rebound definitions and net savings equations are presented in Tables B.3–B.6, one for each rebound effect in Fig. 1. Energy and financial analyses are shown side by side, because each informs the other.

Several terms in Tables B.3–B.6 are zeroed, e.g., $\cancelto{0}{\Delta \dot{C}_o^*}$. These zeroes can be traced back to Table B.1. Table B.2 highlights the equations in Table B.1 that justify zeroing each term.

Table B.3. **Emplacement Effect**

	Energy analysis		*Financial analysis*	
before ($\circ$)	$\dot{E}^\circ = \dot{E}^\circ_s + \dot{E}^\circ_{emb} + (\dot{C}^\circ_{md} + \dot{C}^\circ_o) I_E$	(B.1)	$\dot{M}^\circ = p_E \dot{E}^\circ_s + \dot{C}^\circ_{cap} + \dot{C}^\circ_{md} + \dot{C}^\circ_o + \dot{N}^\circ$	(B.2)
after ($*$)	$\dot{E}^* = \dot{E}^*_s + \dot{E}^*_{emb} + (\dot{C}^*_{md} + \dot{C}^*_o) I_E$	(B.3)	$\dot{M}^* = p_E \dot{E}^*_s + \dot{C}^*_{cap} + \dot{C}^*_{md} + \dot{C}^*_o + \dot{N}^*$	(B.4)

Take differences to obtain the change in energy consumption, $\Delta\dot{E}^* \equiv \dot{E}^* - \dot{E}^\circ$.

$$\Delta\dot{E}^* = \Delta\dot{E}^*_s + \Delta\dot{E}^*_{emb} + (\Delta\dot{C}^*_{md} + \cancelto{0}{\Delta\dot{C}^*_o}) I_E \tag{B.5}$$

Thus,

$$\Delta\dot{E}^* = \Delta\dot{E}^*_s + \Delta\dot{E}^*_{emb} + \Delta\dot{C}^*_{md} I_E \; . \tag{B.6}$$

Define

$$\dot{S}_{dev} \equiv -\Delta\dot{E}^*_s \tag{B.7}$$

(Also see Eqs. (9) and (10)). Use Eq. (2) to obtain

$$Re_{empl} = 1 - \frac{-\Delta\dot{E}^*}{\dot{S}_{dev}} = 1 - \frac{-\Delta\dot{E}^*_s}{\dot{S}_{dev}} - \frac{-\Delta\dot{E}^*_{emb}}{\dot{S}_{dev}} - \frac{-\Delta\dot{C}^*_{md} I_E}{\dot{S}_{dev}} \; . \tag{B.8}$$

Define $Re_{dempl} \equiv 1 - \frac{-\Delta\dot{E}^*_s}{\dot{S}_{dev}} (= 0)$, $Re_{iempl} \equiv Re_{emb} + Re_{md}$, $Re_{emb} \equiv \frac{\Delta\dot{E}^*_{emb}}{\dot{S}_{dev}}$, and $Re_{md} \equiv \frac{\Delta\dot{C}^*_{md} I_E}{\dot{S}_{dev}}$, such that

$$Re_{empl} = Re_{dempl} + Re_{iempl} \; . \tag{B.9}$$

Use the monetary constraint ($\dot{M}^\circ = \dot{M}^*$) and constant spending on other items ($\dot{C}^\circ_o = \dot{C}^*_o$) to cancel terms to obtain

$$\begin{aligned} &p_E \dot{E}^\circ_s + \dot{C}^\circ_{cap} + \dot{C}^\circ_{md} + \cancel{\dot{C}^\circ_o} + \cancelto{0}{\dot{N}^\circ} \\ &\quad = p_E \dot{E}^*_s + \dot{C}^*_{cap} + \dot{C}^*_{md} + \cancel{\dot{C}^*_o} + \dot{N}^* \; . \end{aligned} \tag{B.10}$$

Solving for $\Delta\dot{N}^* \equiv \dot{N}^* - \cancelto{0}{\dot{N}^\circ}$ gives

$$\Delta\dot{N}^* = p_E (\dot{E}^\circ_s - \dot{E}^*_s) + \dot{C}^\circ_{cap} - \dot{C}^*_{cap} + \dot{C}^\circ_{md} - \dot{C}^*_{md} \; . \tag{B.11}$$

Rewriting with Δ terms gives

$$\Delta\dot{N}^* = -p_E \Delta\dot{E}^*_s - \Delta\dot{C}^*_{cap} - \Delta\dot{C}^*_{md} \; . \tag{B.12}$$

Substituting Eq. (B.7) gives

$$\Delta\dot{N}^* = \dot{N}^* = p_E \dot{S}_{dev} - \Delta\dot{C}^*_{cap} - \Delta\dot{C}^*_{md} \; . \tag{B.13}$$

Freed cash ($\dot{G}$) resulting from the EEU, before any energy takeback, is given by

$$\dot{G} = p_E \dot{S}_{dev} \; . \tag{B.14}$$

Note that Eq. (B.2) and $\dot{N}^\circ = 0$ can be used to calculate $\dot{C}^\circ_o$ as

$$\dot{C}^\circ_o = \dot{M}^\circ - p_E \dot{E}^\circ_s - \dot{C}^\circ_{cap} - \dot{C}^\circ_{md} \; . \tag{B.15}$$

Table B.4. **Substitution Effect**

	Energy analysis		*Financial analysis*	
before ($*$)	$\dot{E}^* = \dot{E}_s^* + \dot{E}_{emb}^* + (\dot{C}_{md}^* + \dot{C}_o^*)I_E$	(B.3)	$\dot{M}^* = p_E\dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o^* + \dot{N}^*$	(B.4)
after ($\wedge$)	$\hat{\dot{E}} = \hat{\dot{E}}_s + \hat{\dot{E}}_{emb} + (\hat{\dot{C}}_{md} + \hat{\dot{C}}_o)I_E$	(B.16)	$\hat{\dot{M}} = p_E\hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}}$	(B.17)

Take differences to obtain the change in energy consumption, $\Delta\hat{\dot{E}} \equiv \hat{\dot{E}} - \dot{E}^*$.

$$\Delta\hat{\dot{E}} = \Delta\hat{\dot{E}}_s + \cancelto{0}{\Delta\hat{\dot{E}}_{emb}} + (\cancelto{0}{\Delta\hat{\dot{C}}_{md}} + \Delta\hat{\dot{C}}_o)I_E \tag{B.18}$$

Thus,

$$\Delta\hat{\dot{E}} = \Delta\hat{\dot{E}}_s + \Delta\hat{\dot{C}}_o I_E \; . \tag{B.19}$$

All terms are energy takeback terms. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\hat{\dot{E}}}{\dot{S}_{dev}} = \frac{\Delta\hat{\dot{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \tag{B.20}$$

Define $Re_{sub} \equiv \frac{\Delta\hat{\dot{E}}}{\dot{S}_{dev}}$, $Re_{dsub} \equiv \frac{\Delta\hat{\dot{E}}_s}{\dot{S}_{dev}}$, and $Re_{isub} \equiv \frac{\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{sub} = Re_{dsub} + Re_{isub} \; . \tag{B.21}$$

Use the monetary constraint ($\dot{M}^* = \hat{\dot{M}}$) to obtain

$$\begin{aligned} p_E\dot{E}_s^* &+ \cancel{\dot{C}_{cap}^*} + \cancel{\dot{C}_{md}^*} + \dot{C}_o^* + \dot{N}^* \\ &= p_E\hat{\dot{E}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o + \hat{\dot{N}} \; . \end{aligned} \tag{B.22}$$

For the substitution effect, there is no change in capital or maintenance and disposal costs ($\hat{\dot{C}}_{cap} = \dot{C}_{cap}^*$ and $\hat{\dot{C}}_{md} = \dot{C}_{md}^*$). Solving for $\Delta\hat{\dot{N}} \equiv \hat{\dot{N}} - \dot{N}^*$ gives

$$\Delta\hat{\dot{N}} = -p_E\Delta\hat{\dot{E}}_s - \Delta\hat{\dot{C}}_o \; . \tag{B.23}$$

The substitution effect adjusts net savings relative to $\dot{N}^*$ by $\Delta\hat{\dot{N}}$. Thus, $\hat{\dot{N}} = \dot{N}^* + \Delta\hat{\dot{N}}$. Substituting Eqs. (B.13), (B.14), and (B.23) yields

$$\hat{\dot{N}} = \dot{G} - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^* - p_E\Delta\hat{\dot{E}}_s - \Delta\hat{\dot{C}}_o \; . \tag{B.24}$$

Table B.5. **Income Effect**

	Energy analysis		*Financial analysis*	
before ($\wedge$)	$\hat{\dot{E}} = \hat{\dot{E}}_s + \hat{\dot{E}}_{emb} + (\hat{\dot{C}}_{md} + \hat{\dot{C}}_o) I_E$	(B.16)	$\hat{\dot{M}} = p_E \hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}}$	(B.17)
after ($-$)	$\bar{\dot{E}} = \bar{\dot{E}}_s + \bar{\dot{E}}_{emb} + (\bar{\dot{C}}_{md} + \bar{\dot{C}}_o) I_E$	(B.25)	$\bar{\dot{M}} = p_E \bar{\dot{E}}_s + \bar{\dot{C}}_{cap} + \bar{\dot{C}}_{md} + \bar{\dot{C}}_o + \bar{\dot{N}}$	(B.26)

Take differences to obtain the change in energy consumption, $\Delta\dot{E} \equiv \bar{\dot{E}} - \hat{\dot{E}}$.

$$\Delta\dot{E} = \Delta\dot{E}_s + \cancelto{0}{\Delta\dot{E}_{emb}} + (\cancelto{0}{\Delta\dot{C}_{md}} + \Delta\dot{C}_o) I_E \tag{B.27}$$

Thus,

$$\Delta\dot{E} = \Delta\dot{E}_s + \Delta\dot{C}_o I_E \tag{B.28}$$

All terms are energy takeback terms. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\dot{E}}{\dot{S}_{dev}} = \frac{\Delta\dot{E}_s}{\dot{S}_{dev}} + \frac{\Delta\dot{C}_o I_E}{\dot{S}_{dev}} \tag{B.29}$$

Define $Re_{inc} \equiv \frac{\Delta\dot{E}}{\dot{S}_{dev}}$, $Re_{dinc} \equiv \frac{\Delta\dot{E}_s}{\dot{S}_{dev}}$, and $Re_{iinc} \equiv \frac{\Delta\dot{C}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{inc} = Re_{dinc} + Re_{iinc} \; . \tag{B.30}$$

Use the monetary constraint ($\hat{\dot{M}} = \bar{\dot{M}}$) to obtain

$$\begin{aligned} & p_E \hat{\dot{E}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o + \hat{\dot{N}} \\ & = p_E \bar{\dot{E}}_s + \cancel{\bar{\dot{C}}_{cap}} + \cancel{\bar{\dot{C}}_{md}} + \bar{\dot{C}}_o + \cancelto{0}{\bar{\dot{N}}} \; . \end{aligned} \tag{B.31}$$

For the income effect, there is no change in capital or maintainance and disposal costs ($\hat{\dot{C}}_{cap} = \dot{C}^*_{cap}$ and $\hat{\dot{C}}_{md} = \dot{C}^*_{md}$). Notably, $\bar{\dot{N}} = 0$, because it is assumed that all net monetary savings ($\hat{\dot{N}}$) are spent on more energy service ($\bar{\dot{E}}_s > \hat{\dot{E}}_s$) and additional purchases in the economy ($\bar{\dot{C}}_o > \hat{\dot{C}}_o$). Solving for $\hat{\dot{N}}$ gives

$$\hat{\dot{N}} = p_E \Delta\dot{E}_s + \Delta\dot{C}_o \; , \tag{B.32}$$

the budget constraint for the income effect. By construction, Eq. (B.32) ensures spending of net savings ($\hat{\dot{N}}$) on (i) additional energy services ($\Delta\dot{E}_s$) and (ii) additional purchases of other goods in the economy ($\Delta\dot{C}_o$) only.

Table B.6. **Macro Effect**

	Energy analysis	*Financial analysis*
before (−)	$\bar{\dot{E}}$ (B.33)	
after (∼)	$\tilde{\dot{E}}$ (B.34)	

Take differences to obtain the change in energy consumption,

$$\Delta\tilde{\dot{E}} \equiv \tilde{\dot{E}} - \bar{\dot{E}} \ . \tag{B.35}$$

The energy change due to the macro effect ($\Delta\tilde{\dot{E}}$) is a scalar multiple (k) of net savings ($\hat{\dot{N}}$), assumed to be spent at the energy intensity of the economy (I_E).

$$\Delta\tilde{\dot{E}} = k\hat{\dot{N}} I_E \tag{B.36}$$

All terms are energy takeback terms. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}} = \frac{k\hat{\dot{N}} I_E}{\dot{S}_{dev}} \tag{B.37}$$

Define $Re_{macro} \equiv \frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}}$, such that

$$Re_{macro} = \frac{k\hat{\dot{N}} I_E}{\dot{S}_{dev}} \ . \tag{31}$$

N/A

B.3. Rebound expressions

All that remains is to determine expressions for each rebound effect. We begin with the device-level expected energy savings rate ($\dot{S}_{dev}$), which appears in the denominator of all rebound expressions.

B.3.1. Expected energy savings ($\dot{S}_{dev}$)

$\dot{S}_{dev}$ is the reduction of energy consumption rate by the device due to the EEU. No other effects are considered.

$$\dot{S}_{dev} \equiv \dot{E}_s^\circ - \dot{E}_s^* \tag{9}$$

The final energy consumption rates ($\dot{E}_s^\circ$ and $\dot{E}_s^*$) can be written as Eq. (5) in the forms $\dot{E}_s^\circ = \frac{\dot{q}_s^\circ}{\eta^\circ}$ and $\dot{E}_s^* = \frac{\dot{q}_s^*}{\eta^*}$.

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^*}{\eta^*} \tag{B.38}$$

With reference to Table B.1, we use $\dot{q}_s^* = \dot{q}_s^\circ$ and $\eta^* = \tilde{\eta}$ to obtain

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^\circ}{\tilde{\eta}} \; . \tag{B.39}$$

When the EEU increases efficiency such that $\tilde{\eta} > \eta^\circ$, expected energy savings grows ($\dot{S}_{dev} > 0$) as the rate of final energy consumption declines, as expected. As $\tilde{\eta} \rightarrow \infty$, all final energy consumption is eliminated ($\dot{E}_s^* \rightarrow 0$), and $\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} = \dot{E}_s^\circ$. (Of course, $\tilde{\eta} \rightarrow \infty$ is impossible. See Paoli and Cullen (2020) for a recent discussion of upper limits to device efficiencies.)

After rearrangement and using $\dot{E}_s^\circ = \frac{\dot{q}_s^\circ}{\eta^\circ}$, we obtain a convenient form

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^\circ} - 1 \right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ \; . \tag{10}$$

B.3.2. Emplacement effect

The emplacement effect accounts for performance of the EEU only. No behavior changes occur. The direct emplacement effect of the EEU is device energy savings and energy cost savings. The indirect emplacement effects of the EEU produce changes in the embodied energy rate and the maintenance and disposal expenditure rates. By definition, the direct emplacement effect has no rebound. However, indirect emplacement effects may cause energy rebound. Both direct and indirect emplacement effects are discussed below.

Re_{dempl}. As shown in Table B.3, the direct rebound from the emplacement effect is $Re_{dempl} = 0$. This result is expected, because, in the absence of behavior changes, there is no takeback of energy savings at the upgraded device.

Re_{iempl}. Indirect emplacement rebound effects can occur at any point in the life cycle of an energy conversion device, from manufacturing and distribution to the use phase (maintenance), and finally to disposal. For simplicity, we group maintenance with disposal to form two distinct indirect emplacement rebound effects: (i) an embodied energy effect (Re_{emb}) and (ii) a maintenance and disposal effect (Re_{md}).

Re_{emb}. The first component of indirect emplacement effect rebound involves embodied energy. We define embodied energy consistent with the energy/exergy analysis literature to be the sum of all final energy consumed in the production of the energy conversion device. The EEU causes the embodied final energy of the device to change from $\dot{E}^{\circ}_{emb}$ to $\dot{E}^{*}_{emb}$.

Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. No energy is embodied in the device while in service. However, for simplicity, we spread all embodied energy over the lifetime of the device, an equal amount assigned to each period. We later take the same approach to capital costs and maintenance and disposal costs. A justification for spreading embodied energy purchase costs comes from considering staggered device replacements by many consumers across several years. In the aggregate, staggered replacements work out to about the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) to obtain embodied energy rates, such that $\dot{E}^{\circ}_{emb} = E^{\circ}_{emb}/t^{\circ}_{life}$ and $\dot{E}^{*}_{emb} = E^{*}_{emb}/t^{*}_{life}$. The change in embodied final energy due to the EEU (expressed as a rate) is given by $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$. After substitution and algebraic rearrangement, the change in embodied energy rate due to the EEU can be expressed as $\left(\frac{E^{*}_{emb}}{E^{\circ}_{emb}}\frac{t^{\circ}_{life}}{t^{*}_{life}} - 1\right)\dot{E}^{\circ}_{emb}$, a term that represents energy savings taken back due to embodied energy effects. Thus, Eq. (4) can be employed to write embodied energy rebound as

$$Re_{emb} = \frac{\left(\frac{E^{*}_{emb}}{E^{\circ}_{emb}}\frac{t^{\circ}_{life}}{t^{*}_{life}} - 1\right)\dot{E}^{\circ}_{emb}}{\dot{S}_{dev}} \, . \tag{12}$$

Embodied energy rebound can be either positive or negative, depending on the sign of the term $(E^{*}_{emb}/E^{\circ}_{emb})(t^{\circ}_{life}/t^{*}_{life}) - 1$. Rising energy efficiency can be associated with increased device complexity and more embodied energy, such that $E^{*}_{emb} > E^{\circ}_{emb}$ and $Re_{emb} > 0$. However, if the upgraded device has longer life than the original device ($t^{*}_{life} > t^{\circ}_{life}$), $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$ can be negative, meaning that the upgraded device has a lower embodied energy rate than the original device.

Re_{md}. In addition to embodied energy effects, indirect emplacement rebound can be associated with energy demanded by maintenance and disposal (md) expenditures. Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}^{\circ}_{m}$). Disposal costs (e.g., C°_{d}) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal expenditures across the lifetime of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) to form expenditure rates such that $\dot{C}^{\circ}_{md} = \dot{C}^{\circ}_{m} + C^{\circ}_{d}/t^{\circ}_{life}$ and $\dot{C}^{*}_{md} = \dot{C}^{*}_{m} + C^{*}_{d}/t^{*}_{life}$.

We assume, for simplicity, that md expenditures indicate energy consumption elsewhere in the economy at its energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in md expenditures is given by $\Delta\dot{C}^{*}_{md}I_E$. This term represents energy takeback, so maintenance and disposal rebound is given by

$$Re_{md} = \frac{\Delta\dot{C}^{*}_{md}I_E}{\dot{S}_{dev}} \, , \tag{B.40}$$

as shown in Table B.3. Slight rearrangement gives

$$Re_{md} = \frac{\left(\frac{\dot{C}^*_{md}}{\dot{C}^\circ_{md}} - 1\right) \dot{C}^\circ_{md} I_E}{\dot{S}_{dev}} \; . \tag{13}$$

Rebound from maintenance and disposal can be positive or negative, depending on the sign of the term $\dot{C}^*_{md}/\dot{C}^\circ_{md} - 1$.

B.3.3. Substitution effect

This section derives expressions for substitution effect rebound. Two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}). Assuming that conditions after the emplacement effect ($*$) are known, both the rate of energy service consumption ($\hat{\dot{q}}_s$) and the rate of other goods consumption ($\hat{\dot{C}}_o$) must be determined as a result of the substitution effect (the $\wedge$ point). The EEU's energy efficiency increase ($\tilde{\eta} > \eta^\circ$) causes the price of the energy service provided by the device to fall ($\tilde{p}_s < p^\circ_s$). The substitution effect quantifies the amount by which the device owner, in response, increases the consumption rate of the energy service ($\hat{\dot{q}}_s > \dot{q}^*_s$) and decreases the consumption rate of other goods ($\hat{\dot{q}}_o < \dot{q}^*_o$).

The increase in consumption of the energy service substitutes for consumption of other goods in the economy, subject to a utility constraint. The reduction in spending on other goods in the economy is captured by indirect substitution rebound (Re_{isub}).

We begin by deriving an expression for direct and indirect substitution effect rebound (Re_{dsub} and Re_{isub}, respectively). Thereafter, we develop an approximate model and a CES utility model for determining the post-substitution point ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$).

Direct substitution effect rebound expression. Direct substitution effect rebound (Re_{dsub}) is given by

$$Re_{dsub} = \frac{\Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} = \frac{\hat{\dot{E}}_s - \dot{E}^*_s}{\dot{S}_{dev}} \; . \tag{15}$$

Substituting the typical relationship of Eq. (5) in the form $\dot{E}_s = \frac{\dot{q}_s}{\eta}$ gives

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\tilde{\eta}} - \frac{\dot{q}^*_s}{\tilde{\eta}}}{\dot{S}_{dev}} \; . \tag{B.41}$$

Rearranging produces

$$Re_{dsub} = \frac{\left(\frac{\hat{\dot{q}}_s}{\dot{q}^\circ_s} - \frac{\dot{q}^*_s}{\dot{q}^\circ_s}\right) \frac{\dot{q}^\circ_s}{\tilde{\eta}}}{\dot{S}_{dev}} \; . \tag{B.42}$$

Recognizing that the rate of energy service consumption ($\dot{q}_s$) is unchanged across the emplacement effect leads to $\frac{\dot{q}^*_s}{\dot{q}^\circ_s} = 1$. Furthermore, $\frac{\dot{q}^\circ_s}{\tilde{\eta}} = \frac{\dot{q}^\circ_s}{\eta^\circ} \frac{\eta^\circ}{\tilde{\eta}} = \dot{E}^\circ_s \frac{\eta^\circ}{\tilde{\eta}}$, such that

$$Re_{dsub} = \left(\frac{\hat{\dot{q}}_s}{\dot{q}^\circ_s} - 1\right) \frac{\dot{E}^\circ_s \frac{\eta^\circ}{\tilde{\eta}}}{\dot{S}_{dev}} \; . \tag{B.43}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\cancel{\dot{E}_s^\circ} \, \cancel{\frac{\eta^\circ}{\tilde{\eta}}}}{\cancel{\frac{\eta^\circ}{\tilde{\eta}}} \, \cancel{\dot{E}_s^\circ}} \right) . \tag{B.44}$$

Canceling terms yields

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} . \tag{16}$$

Eq. (16) is the basis for developing both approximate and CES models of determining direct substitution rebound.

Indirect substitution effect rebound expression. Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} = \frac{(\hat{\dot{C}}_o - \dot{C}_o^*) I_E}{\dot{S}_{dev}} . \tag{17}$$

Rearranging gives

$$Re_{isub} = \frac{\left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - \frac{\dot{C}_o^*}{\dot{C}_o^\circ} \right) \dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{B.45}$$

Recognizing that expenditures on other goods are constant across the emplacement effect gives $\frac{\dot{C}_o^*}{\dot{C}_o^\circ} = 1$ and

$$Re_{isub} = \left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1 \right) \frac{\dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{B.46}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} . \tag{18}$$

Eq. (18) is the basis for developing both approximate and CES models of determining indirect substitution rebound.

Determining the post-substitution effect conditions requires reference to a consumer utility model. We first show an approximate model, often used in the literature. Later, we use a constant elasticity of substitution (CES) utility model. The CES model is used for all calculations and graphs in this paper.

Approximate utility model. In the literature, an approximate utility model is often used (Borenstein, 2015, p. 17, footnote 43). In the approximate model, the relationship between energy service price and energy service consumption rate is given by the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\epsilon_{\dot{q}_s,p_s,c}} . \tag{B.47}$$

Note that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) is assumed constant along an indifference curve in the approximate model. A negative value for the compensated price elasticity of energy service demand is expected ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that when the energy service price decreases ($\tilde{p}_s < p_s^\circ$), the rate of energy service consumption increases ($\hat{\dot{q}}_s > \dot{q}_s^*$).

Substituting Eq. (6) in the form $p_s^\circ = \frac{p_E^\circ}{\eta^\circ}$ and $\tilde{p}_s = \frac{p_E^\circ}{\tilde{\eta}}$ and noting that $\dot{q}_s^\circ = \dot{q}_s^*$ gives gives

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} . \tag{B.48}$$

Again, note that the compensated price elasticity of energy service demand is negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), so that as energy service efficiency increases ($\tilde{\eta} > \eta^\circ$), the energy service consumption rate increases ($\hat{\dot{q}}_s > \dot{q}_s^* = \dot{q}_s^\circ$).

Substituting Eq. (B.48) into Eq. (16) yields the approximate expression for direct substitution rebound.

$$Re_{dsub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{B.49}$$

The compensated price elasticity of energy service demand is expected to be negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that, e.g., $\epsilon_{\dot{q}_s,p_s,c} = -0.2$ and $\frac{\tilde{\eta}}{\eta^\circ} = 2$ yields $Re_{dsub} = 0.15$.

With $\epsilon_{\dot{q}_s,p_s,c} \in (-1, 0)$ expected, the approximate model indicates that direct substitution rebound will never be larger than 1. I.e., the direct substitution effect alone can never cause backfire.

To quantify the substitution effect on other purchases in the approximate model, we introduce another elasticity, the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\epsilon_{\dot{q}_o,p_s,c}} . \tag{B.50}$$

Because the compensated cross-price elasticity of other goods demand is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$), an energy service price decrease ($\tilde{p}_s < p_s^\circ$) implies a reduction in the rate of consumption of other goods ($\hat{\dot{q}}_o < \dot{q}_o^*$).

The energy service price is inversely proportional to efficiency, yielding

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{B.51}$$

Assuming that the average price is unchanged across the substitution effect such that $\hat{p}_o = \dot{p}_o^* = p_o^\circ$ (Appendix F), and noting that $\dot{q}_s^* = \dot{q}_s^\circ$ and $\dot{C}_o^* = \dot{C}_o^\circ$, we can write

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{B.52}$$

Note that Eq. (B.52) can be used to determine the rate of expenditures on other goods in the economy ($\hat{\dot{C}}_o$) by

$$\hat{\dot{C}}_o = \dot{C}_o^\circ \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{B.53}$$

Substituting Eq. (B.53) into Eq. (18) gives the expression for indirect substitution rebound for the approximate utility model.

$$Re_{isub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \; \frac{\tilde{\eta}}{\eta^\circ} \; \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{B.54}$$

Because the compensated cross-elasticity of other goods consumption is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$) and the energy service efficiency ratio is greater than 1 ($\tilde{\eta} > \eta^\circ$), indirect substitution rebound will be negative always ($Re_{isub} < 0$), as expected. Negative rebound indicates that indirect substitution effects reduce energy takeback by direct substitution effects.

CES utility model. The approximate utility model assumes that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$) are constant along an indifference curve. These assumptions are approximations that hold only for infinitesimally small energy service price changes ($\Delta p_s^* \equiv p_s^* - p_s^\circ \approx 0$). However, in the case of an energy efficiency upgrade (EEU), the energy service price change is not infinitesimal. Rather, Δp_s^* is finite and may be large.

To determine the new consumption bundle after the substitution effect ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$) and, ultimately, to quantify the direct and indirect substitution rebound effects (Re_{dsub} and Re_{isub}) exactly, we remove the restriction that energy service price elasticities ($\epsilon_{\dot{q}_s,p_s,c}$ and $\epsilon_{\dot{q}_o,p_s,c}$) must be constant along an indifference curve. Instead, we require constancy of only the elasticity of substitution (σ) between the consumption rate of the energy service ($\dot{q}_s$) and the expenditure rate for other goods ($\dot{C}_o$) across the substitution effect. Thus, we employ a constant elasticity of substitution (CES) utility function.

Figure 4 (especially segments $*$—c and c—$\wedge$) illustrates features of the CES utility model for determining the new consumption bundle. Two equations are helpful for this analysis. First, the slope at any point on indifference curve i°—i° is given by Eq. (D.5) with $\dot{u}/\dot{u}^\circ = 1$ and the share parameter (a) replaced by $f_{\dot{C}_s}^\circ$, as discussed in Appendix D.

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} \left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \times \left[\left(\frac{1}{1 - f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho} . \tag{B.55}$$

Second, the equation of the $*$—$*$ expenditure line is

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right) + \frac{1}{\dot{C}_o^\circ}(\dot{M} - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G})\ . \tag{B.56}$$

To find the rate of energy service consumption after the substitution effect ($\hat{\dot{q}}_s$), we set the slope of the $*$—$*$ expenditure line (Eq. B.56) equal to the slope of the i$^\circ$—i$^\circ$ indifference curve at the original utility rate of $\dot{u}/\dot{u}^\circ = 1$ (Eq. (B.55)).

$$-\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)}\left[\left(\frac{1}{1-f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^{\rho}\right]^{(1-\rho)/\rho} \tag{B.57}$$

Solving for $\dot{q}_s/\dot{q}_s^\circ$ gives $\hat{\dot{q}}_s/\dot{q}_s^\circ$ as

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1-f_{\dot{C}_s}^\circ)\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\frac{\rho}{1-\rho}}\right\}^{-1/\rho} . \tag{19}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an estimate for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1-f_{\dot{C}_s}^\circ)\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\frac{\rho}{1-\rho}}\right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

The rate of other goods consumption after the substitution effect ($\hat{\dot{C}}_o$) can be found by substituting Eq. (19) and $\dot{u}/\dot{u}^\circ = 1$ into the functional form of the utility model (Eq. (D.4)) to obtain

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(\left(\frac{1}{1-f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}\right)\left\{ f_{\dot{C}_s}^\circ + (1-f_{\dot{C}_s}^\circ)\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\frac{\rho}{1-\rho}}\right\}^{-1}\right)^{1/\rho} . \tag{B.58}$$

Simplifying gives

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left[\left(1 + f_{\dot{C}_s}^\circ\left\{\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} . \tag{20}$$

Eq. (20) can be substituted into Eq. (18) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left[\left(1 + f_{\dot{C}_s}^\circ\left\{\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\ \frac{\tilde{\eta}}{\eta^\circ}\ \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{22}$$

B.3.4. Income effect

Rebound from the income effect rebound quantifies the rate of additional energy demand that arises because the owner of the energy conversion device spends net savings from the EEU. The income rate of the device owner is $\dot{M}^{\circ}$, which remains unchanged across the rebound effects, such that $\dot{M}^{\circ} = \dot{M}^{*} = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Freed cash from the EEU is given by Eq. (B.14) as $\dot{G} = p_E \dot{S}_{dev}$. In combination, the emplacement effect and the substitution effect leave the device owner with *net* savings ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (B.24). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables B.3 and B.4.

In this framework, all net savings ($\hat{\dot{N}}$) is spent either (i) on additional energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) or (ii) on additional other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$). The income elasticity of energy service demand and the income elasticity of other goods demand ($\epsilon_{\dot{q}_s,\dot{M}}$ and $\epsilon_{\dot{q}_o,\dot{M}}$, respectively) quantify the income preferences of the device owner according to the following expressions:

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} \tag{23}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} , \tag{27}$$

where effective income ($\hat{\dot{M}}'$) is

$$\hat{\dot{M}}' \equiv \dot{M}^{\circ} - \dot{C}^{*}_{cap} - \dot{C}^{*}_{md} - \hat{\dot{N}} . \tag{24}$$

Homotheticity means that $\epsilon_{\dot{q}_s,\dot{M}} = 1$ and $\epsilon_{\dot{q}_o,\dot{M}} = 1$.

The budget constraint across the income effect (Eq. (B.32)) ensures that all net savings available after the substitution effect ($\hat{\dot{N}}$) is re-spent across the income effect, such that $\bar{\dot{N}} = 0$. Appendix E proves that the income preference equations (Eqs. (23) and (27)) satsify the budget constraint (Eq. B.32).

The purpose of this section is derivation of expressions for (i) direct income rebound (Re_{dinc}) arising from increased consumption of the energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) and (ii) indirect income rebound (Re_{iinc}) arising from increased consumption of other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$).

But first, we derive a helpful expression to be used later.

Expression for $\hat{\dot{E}}_s$. An expression for $\hat{\dot{E}}_s$ that will be helpful later begins with

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{E}}_s}{\dot{E}^{*}_s}\right)\left(\frac{\dot{E}^{*}_s}{\dot{E}^{\circ}_s}\right)\dot{E}^{\circ}_s . \tag{B.59}$$

Substituting Eq. (5) and noting efficiency (η) equalities from Table B.1 gives

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{q}}_s/\cancel{\tilde{\eta}}}{\dot{q}_s^*/\cancel{\tilde{\eta}}} \right) \left(\frac{\dot{q}_s^*/\tilde{\eta}}{\dot{q}_s^\circ/\eta^\circ} \right) \dot{E}_s^\circ \ . \tag{B.60}$$

Canceling terms yields

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{q}}_s}{\dot{q}_s^*} \right) \left(\frac{\cancel{\dot{q}_s^*}}{\cancel{\dot{q}_s^\circ}} \right) \left(\frac{\eta^\circ}{\tilde{\eta}} \right) \dot{E}_s^\circ \ . \tag{B.61}$$

Noting energy service consumption rate equalities from Table B.1 ($\dot{q}_s^* = \dot{q}_s^\circ$) gives

$$\hat{\dot{E}}_s = \frac{\hat{\dot{q}}_s}{\dot{q}_s^*} \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ \ . \tag{B.62}$$

The next step is to develop an expression for Re_{dinc} using the income preference for energy service consumption.

Expression for Re_{dinc}. As shown in Table B.5, direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \bar{\dot{E}}_s}{\dot{S}_{dev}} \ . \tag{25}$$

Expanding the difference and rearranging gives

$$Re_{dinc} = \frac{\bar{\dot{E}}_s - \hat{\dot{E}}_s}{\dot{S}_{dev}} \ , \tag{B.63}$$

and

$$Re_{dinc} = \frac{\left(\frac{\bar{\dot{E}}_s}{\hat{\dot{E}}_s} - 1 \right) \hat{\dot{E}}_s}{\dot{S}_{dev}} \ . \tag{B.64}$$

Substituting the Eq. (5) as $\bar{\dot{E}}_s = \frac{\bar{\dot{q}}_s}{\tilde{\eta}}$ and $\hat{\dot{E}}_s = \frac{\hat{\dot{q}}_s}{\tilde{\eta}}$ gives

$$Re_{dinc} = \frac{\left(\frac{\bar{\dot{q}}_s/\cancel{\tilde{\eta}}}{\hat{\dot{q}}_s/\cancel{\tilde{\eta}}} - 1 \right) \hat{\dot{E}}_s}{\dot{S}_{dev}} \ . \tag{B.65}$$

Eliminating terms and substituting Eq. (10) for $\dot{S}_{dev}$ and Eq. (23) for $\bar{\dot{q}}_s/\hat{\dot{q}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1 \right] \hat{\dot{E}}_s}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1 \right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ} \ . \tag{B.66}$$

Substituting Eq. (B.62) for $\hat{\dot{E}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1 \right] \frac{\hat{\dot{q}}_s}{\dot{q}_s^*} \cancel{\frac{\eta^\circ}{\tilde{\eta}}} \cancel{\dot{E}_s^\circ}}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1 \right) \cancel{\frac{\eta^\circ}{\tilde{\eta}}} \cancel{\dot{E}_s^\circ}} \ . \tag{B.67}$$

Eliminating terms, recognizing that $\dot{q}_s^\circ = \dot{q}_s^*$, and substituting Eq. (19), which assumes the CES utility model, gives

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} . \tag{26}$$

If there is no net savings ($\hat{\dot{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected.

The next step is to develop an expression for Re_{iinc} using the income preference for other goods consumption.

Expression for Re_{iinc}. As shown in Table B.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta \bar{\dot{C}}_o I_E}{\dot{S}_{dev}} . \tag{29}$$

Expanding the difference and rearranging gives

$$Re_{iinc} = \frac{(\bar{\dot{C}}_o - \hat{\dot{C}}_o) I_E}{\dot{S}_{dev}} , \tag{B.68}$$

and

$$Re_{iinc} = \frac{\left(\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} - 1\right) \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} . \tag{B.69}$$

Substituting $\bar{\dot{C}}_o = p_o \bar{\dot{q}}_o$ and $\hat{\dot{C}}_o = p_o \hat{\dot{q}}_o$ and cancelling terms gives

$$Re_{iinc} = \frac{\left(\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} - 1\right) \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} . \tag{B.70}$$

Substituting the income preference equation for other goods consumption (Eq. (27) for $\bar{\dot{q}}_o / \hat{\dot{q}}_o$ and Eq. (10) for $\dot{S}_{dev}$ yields

$$Re_{iinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} - 1\right] \hat{\dot{C}}_o I_E}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ} . \tag{B.71}$$

Sutstituting $\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} \dot{C}_o^\circ$ for $\hat{\dot{C}}_o$, recognizing that $\dot{C}_o^* = \dot{C}_o^\circ$, and simplifying gives

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ}\right) . \tag{B.72}$$

Substituting Eq. (20) for $\hat{\dot{C}}_o / \dot{C}_o^\circ$, thereby assuming the CES utility model, gives the final form of the indirect income rebound expression:

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left[\left(1 + f_{\dot{C}_s}^\circ \left\{\left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} . \tag{30}$$

If there is no net savings ($\hat{\dot{N}} = 0$), indirect income effect rebound is zero ($Re_{iinc} = 0$), as expected.

B.3.5. Macro effect

Macro rebound (Re_{macro}) is given by Eq. (31). Substituting Eq. (B.24) for net savings ($\hat{\dot{N}}$) gives

$$Re_{macro} = \frac{k\dot{G}I_E}{\dot{S}_{dev}} - \frac{k\Delta\dot{C}_{cap}^* I_E}{\dot{S}_{dev}} - \frac{k\Delta\dot{C}_{md}^* I_E}{\dot{S}_{dev}} - \frac{kp_E I_E \Delta\hat{\dot{E}}_s}{\dot{S}_{dev}} - \frac{k\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}} . \tag{B.73}$$

Substituting Eq. (B.14) for $\dot{G}$ and Eqs. (B.40), (15), and (17) for rebound terms gives

$$Re_{macro} = \frac{kp_E \cancel{\dot{S}_{dev}} I_E}{\cancel{\dot{S}_{dev}}} - \frac{k\Delta\dot{C}_{cap}^* I_E}{\dot{S}_{dev}} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} . \tag{B.74}$$

Canceling terms and defining Re_{cap} as

$$Re_{cap} \equiv \frac{\Delta\dot{C}_{cap}^* I_E}{\dot{S}_{dev}} \tag{B.75}$$

gives

$$Re_{macro} = kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} . \tag{32}$$

B.3.6. Rebound sum

The sum of four rebound effects is

$$Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro} . \tag{B.76}$$

Substituting Eqs. (B.9), (B.21), and (B.30) gives

$$\begin{aligned} Re_{tot} = {} & Re_{emb} + Re_{md} && \text{emplacement effect} \\ & + Re_{dsub} + Re_{isub} && \text{substitution effect} \\ & + Re_{dinc} + Re_{iinc} && \text{income effect} \\ & + Re_{macro} && \text{macro effect} \end{aligned} \tag{B.77}$$

Macro effect rebound (Re_{macro}, Eq. (32)) can be expressed in terms of other rebound effects. Substituting Eq. (32) gives

$$
\begin{aligned}
Re_{tot} = Re_{emb} &+ Re_{md} && \text{emplacement effect} \\
&+ Re_{dsub} + Re_{isub} && \text{substitution effect} \\
&+ Re_{dinc} + Re_{iinc} && \text{income effect} \\
&+ kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub}\ . && \text{macro effect}
\end{aligned}
\tag{B.78}
$$

Rearranging distributes macro effect terms to emplacement and substitution effect terms. This last rearrangement gives the final expression for total rebound.

$$
\begin{aligned}
Re_{tot} = Re_{emb} &- kRe_{cap} + (1-k)Re_{md} \\
&+ (1 - kp_E I_E)Re_{dsub} + (1-k)Re_{isub} \\
&+ Re_{dinc} + Re_{iinc} + kp_E I_E
\end{aligned}
\tag{33}
$$

Eq. (33) shows that determining seven rebound values,

- Re_{emb} (Eq. (12)),
- Re_{cap} (Eq. (B.75)),
- Re_{md} (Eq. (13)),
- Re_{dsub} (Eq. (21)),
- Re_{isub} (Eqs. (22)),
- Re_{dinc} (Eqs. (26)), and
- Re_{iinc} (Eqs. (30)),

is sufficient to calculate total rebound, provided that the macro factor (k), the price of energy (p_E), and the energy intensity of the economy (I_E) are known.

C. Mathematical details of rebound path graphs

Rebound path graphs show the impact of direct and indirect rebound effects in energy space, expenditure space, and consumption space. Notional rebound path graphs can be found in Figs. 2–4. Rebound path graphs for the car example can be found in Figs. 5–7. Graphs for the lamp example can be found in Figs. 8–10.

This appendix shows the mathematical details of rebound path graphs, specifically derivations of equations for lines and curves shown in Table C.1. The lines and curves enable construction of numerically accurate rebound path graphs as shown in Figs. 5–10.

Table C.1: Lines and curves for rebound path graphs.

Rebound path graph	Lines and curves
Energy	Constant total energy consumption lines 0% and 100% rebound lines
Expenditure	Constant expenditure lines
Consumption	Constant expenditure lines Rays from origin to $\wedge$ point Indifference curves

C.1. Energy path graphs

Energy path graphs show direct (on the abscissa) and indirect (on the ordinate) energy consumption associated with the energy conversion device and the device owner. Lines of constant total energy consumption comprise a scale for total rebound. E.g., the 0% and 100% rebound lines are constant total energy consumption lines which pass through the original point ($\circ$) and the post-direct-emplacement-effect point (a) on an energy path graph.

The equation of a constant total energy consumption line is derived from

$$\dot{E}_{tot} = \dot{E}_{dir} + \dot{E}_{indir} \tag{C.1}$$

at any rebound stage. (See Fig. 1.) Direct energy consumption is energy consumed by the energy conversion device ($\dot{E}_s$), and indirect energy consumption is the sum of embodied energy, energy associated with maintenanace and disposal, and energy associated with expenditures on other goods ($\dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$).

For the energy path graph, direct energy consumption is placed on the abscissa and indirect energy consumption is placed on the ordinate. To derive the equation of a constant energy consumption line, we first rearrange to put the y coordinate on the left of the equation:

$$\dot{E}_{indir} = -\dot{E}_{dir} + \dot{E}_{tot} \ . \tag{C.2}$$

Next, we substitute y for $\dot{E}_{indir}$, x for $\dot{E}_{dir}$, and $\dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$ for $\dot{E}_{tot}$ to obtain

$$y = -x + \dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E \ , \tag{C.3}$$

where all of $\dot{E}_s$, $\dot{E}_{emb}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage of Fig. 1.

The constant total energy consumption line that passes through the original point ($\circ$) shows 100% rebound:

$$y = -x + \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ)I_E \ . \tag{C.4}$$

The 0% rebound line is the constant total energy consumption line that accounts for expected energy savings ($\dot{S}_{dev}$) only:

$$y = -x + (\dot{E}_s^\circ - \dot{S}_{dev}) + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ)I_E \ . \tag{C.5}$$

The above line passes through the a point on an energy path graph.

C.2. Expenditure path graphs

Expenditure path graphs show direct (on the abscissa) and indirect (on the ordinate) expenses associated with the energy conversion device and the device owner. Lines of constant expenditure are important, because they provide budget constraints for the device owner.

The equation of a constant total expenditure line is derived from the budget constraint

$$\dot{C}_{tot} = \dot{C}_{dir} + \dot{C}_{indir} \tag{C.6}$$

at any rebound stage. For the expenditure path graph, indirect expenditures are placed on the ordinate and direct expenditures on energy for the energy conversion device are place on the abscissa. Direct expenditure is the cost of energy consumed by the energy conversion device ($\dot{C}_s = p_E \dot{E}_s$), and indirect expenses are the sum of capital costs, maintenanace and disposal costs, and expenditures on other goods ($\dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$). Rearranging to put the y-axis variable on the left side of the equation gives

$$\dot{C}_{indir} = -\dot{C}_{dir} + \dot{C}_{tot} \; . \tag{C.7}$$

Substituting y for $\dot{C}_{indir}$, x for $\dot{C}_{dir}$, and $\dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$ for $\dot{C}_{tot}$ gives

$$y = -x + \dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o \; , \tag{C.8}$$

where all of $\dot{C}_s$, $\dot{C}_{cap}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage of Fig. 1.

The constant total expenditure line that passes through the original point ($\circ$) shows the budget constraint for the device owner:

$$y = -x + \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \; , \tag{C.9}$$

into which Eq. (B.2) can be substituted with $\dot{C}_s^\circ = p_E \dot{E}_s^\circ$ and $\dot{N}^\circ = 0$ to obtain

$$y = -x + \dot{M}^\circ \; . \tag{C.10}$$

The constant total expenditure line that accounts for expected energy savings ($\dot{S}_{dev}$) and freed cash ($\dot{G} = p_E \dot{S}_{dev}$) only is given by:

$$y = -x + (\dot{C}_s^\circ - \dot{G}) + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \; , \tag{C.11}$$

or

$$y = -x + \dot{M}^\circ - \dot{G} \; . \tag{C.12}$$

The line given by the above equation passes through the a point on an expenditure path graph.

C.3. Consumption path graphs

Consumption path graphs show expenditures in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space to accord with the utility model. (See Appendix D.) Consumption path graphs include (i) constant expenditure lines given prices, (ii) a ray from the origin through the $\wedge$ point, and (iii) indifference curves. Derivations for each are shown in the following subsections.

C.3.1. Constant expenditure lines

There are four constant expenditure lines on the consumption path graphs of Figs. 4, 7, and 10. The constant expenditure lines pass through the original point (line $\circ$—$\circ$), the post-emplacement point (line $*$—$*$), the post-substitution point (line $\wedge$—$\wedge$), and the post-income point (line $-$—$-$). Like the expenditure path graph, lines of constant expenditure on a consumption path graph are derived from the budget constraint of the device owner at each of the four points.

Prior to the EEU, the budget constraint is given by Eq. (B.2). Substituting $p_s^\circ \dot{q}_s^\circ$ for $p_E \dot{E}_s^\circ$ and recognizing that there is no net savings before the EEU ($\dot{N}^\circ = 0$) gives

$$\dot{M}^\circ = p_s^\circ \dot{q}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \; . \tag{C.13}$$

To create the line of constant expenditure on the consumption path graph, we allow $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to vary in a compensatory manner: when one increases, the other must decrease. To show that variation along the constant expenditure line, we remove the notation that ties $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to the original point ($\circ$) to obtain

$$\dot{M}^\circ = p_s^\circ \dot{q}_s + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o \; , \tag{C.14}$$

where all of $\dot{M}^\circ$, p_s°, $\dot{C}_{cap}^\circ$, and $\dot{C}_{md}^\circ$ apply at the same rebound stage of Fig. 1, namely the original point ($\circ$).

To derive the equation of the line representing the original budget constraint in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space (the $\circ$—$\circ$ line through the $\circ$ point in consumption path graphs), we solve for $\dot{C}_o$ to obtain

$$\dot{C}_o = -p_s^\circ \dot{q}_s + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{C.15}$$

Multiplying judiciously by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ}\dot{C}_o^\circ = -p_s^\circ \frac{\dot{q}_s}{\dot{q}_s^\circ}\dot{q}_s^\circ + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{C.16}$$

Dividing both sides by $\dot{C}_o^\circ$ yields

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}\frac{\dot{q}_s}{\dot{q}_s^\circ} + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \; . \tag{C.17}$$

Noting that $\frac{\dot{q}_s}{\dot{q}_s^\circ}$ and $\frac{\dot{C}_o}{\dot{C}_o^\circ}$ are the abscissa and ordinate, respectively, on a consumption path graph gives

$$y = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \; . \tag{C.18}$$

A similar procedure can be employed to derive the equation of the $*$—$*$ line through the $*$ point after the emplacement effect. The starting point is the budget constraint at the $*$ point (Eq. (B.4)) with $\dot{M}^\circ$ replacing $\dot{M}^*$, $\tilde{p}_s \dot{q}_s$ replacing $p_E \dot{E}_s^*$, and $\dot{C}_o$ replacing $\dot{C}_o^*$.

$$\dot{M}^\circ = \tilde{p}_s \dot{q}_s + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o + \dot{N}^* \; . \tag{C.19}$$

Substituting Eq. (B.13) for $\dot{N}^*$, substituting Eq. (B.14) for $\dot{G}$, multiplying judiciously by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$, rearranging, and noting that $\frac{\dot{q}_s}{\dot{q}_s^\circ}$ is the abscissa and $\frac{\dot{C}_o}{\dot{C}_o^\circ}$ is the ordinate gives

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G}) \ . \tag{C.20}$$

Note that the slope of Eq. (C.20) is less negative than the slope of Eq. (C.18), because $\tilde{p}_s < p_s^\circ$. The y-intercept of Eq. (C.20) is less than the y-intercept of Eq. (C.18), reflecting freed cash. Both effects are seen in consumption path graphs (Figs. 4, 7, and 10). The $\circ$—$\circ$ and $*$—$*$ lines intersect at the coincident $\circ$ and $*$ points.

A similar derivation process can be used to find the equation of line representing the budget constraint after the substitution effect (the $\wedge$—$\wedge$ line through the $\wedge$ point). The starting point is Eq. (B.17), and the equation for the constant expenditure line is

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G} + \tilde{p}_s \Delta\hat{\dot{q}}_s + \Delta\hat{\dot{C}}_o) \ . \tag{C.21}$$

Note that the $\wedge$—$\wedge$ line (Eq. (C.21)) has the same slope as the $*$—$*$ line (Eq. (C.20)) but a lower y-intercept.

Finally, the corresponding derivation for the equation of the constant expenditure line through the $-$ point (line $-$—$-$) starts with Eq. (B.26) and ends with

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^*) \ . \tag{C.22}$$

C.3.2. Ray from the origin to the $\wedge$ point

On consumption path graphs, the ray from the origin to the $\wedge$ point (line r—r) defines the path along which the income effect (lines $\wedge$—d and d—$-$) operates. The ray from the origin to the $\wedge$ point has slope $(\hat{\dot{C}}_o/\dot{C}_o^\circ)/(\hat{\dot{q}}_s/\dot{q}_s^\circ)$ and a y-intercept of 0. Therefore, the equation of line r—r is

$$y = \frac{\hat{\dot{C}}_o/\dot{C}_o^\circ}{\hat{\dot{q}}_s/\dot{q}_s^\circ} x \ . \tag{C.23}$$

C.3.3. Indifference curves

On a consumption path graph, indifference curves represent lines of constant utility for the energy conversion device owner. In $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, any indifference curve is given by Eq. (D.4) with $f_{\dot{C}_s}^\circ$ replacing the share parameter a, as shown in Appendix D. Recognizing that $\frac{\dot{C}_o}{\dot{C}_o^\circ}$ is on the ordinate and $\frac{\dot{q}_s}{\dot{q}_s^\circ}$ is on the abscissa leads to substitution of y for $\frac{\dot{C}_o}{\dot{C}_o^\circ}$ and x for $\frac{\dot{q}_s}{\dot{q}_s^\circ}$ to obtain

$$y = \left[\frac{1}{1 - f_{\dot{C}_s}^\circ}\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}(x)^\rho\right]^{(1/\rho)} \ . \tag{C.24}$$

At any point in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, namely $(\dot{q}_{s,1}/\dot{q}_s^\circ,\ \dot{C}_{o,1}/\dot{C}_o^\circ)$, indexed utility $(\dot{u}_1/\dot{u}^\circ)$ is given by Eq. (14) as

$$\frac{\dot{u}_1}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ} \right)^\rho \right]^{(1/\rho)} . \tag{C.25}$$

Substituting Eq. (C.25) into Eq. (C.24) for $\dot{u}/\dot{u}^\circ$ and simplifying exponents gives

$$y = \left\{ \frac{1}{1 - f_{\dot{C}_s}^\circ} \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ} \right)^\rho \right] - \frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} (x)^\rho \right\}^{(1/\rho)} . \tag{C.26}$$

Simplifying further yields the equation of an indifference curve passing through point $(\dot{q}_{s,1}/\dot{q}_s^\circ,\ \dot{C}_{o,1}/\dot{C}_o^\circ)$:

$$y = \left\{ \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} \right) \left[\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ} \right)^\rho - (x)^\rho \right] + \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ} \right)^\rho \right\}^{(1/\rho)} . \tag{C.27}$$

Note that if x is $\dot{q}_{s,1}/\dot{q}_s^\circ$, y becomes $\dot{C}_{o,1}/\dot{C}_o^\circ$, as expected.

D. Utility models and elasticities

As discussed in Section 2.5.2 and Appendix B.3.3, the substitution effect requires a model for device owner utility that compares the perceived benefits of consuming the energy service ($\dot{q}_s$) to consuming other goods and services ($\dot{q}_o$). In this appendix, we describe three utility models. The first utility model is an approximate model that applies only for small and marginal changes in energy efficiency and energy service price, such that $\Delta\eta^* \approx 0$ and $\Delta p_s^* \approx 0$. The approximate utility model is discussed for continuity with the literature only. (See, for example, Borenstein (2015).)

We note that larger and non-marginal efficiency gains cause greater rebound (measured in joules) than small and marginal efficiency gains. Thus, any rebound analysis framework needs to accommodate large, non-marginal efficiency changes. The second utility model discussed in this appendix is the Constant Elasticity of Substitution (CES) utility model which does, in fact, accommodate large, non-marginal energy efficiency and energy service price changes. The CES utility model underlies the substitution effect in this framework. (See Section 2.5.2.) Furthermore, the CES utility model is needed for the example EEUs in this paper, which have very large, non-marginal percentage increases in energy efficiency. For the car in Section 3.1, energy efficiency increases from $\eta^\circ = 25$ mpg to $\tilde{\eta} = 42$ mpg, a 68 % improvement. For the lamp example in Section 3.2, energy efficiency increases by 826 %.

For completeness, we briefly mention a third utility model with Cobb-Douglas form. The Cobb-Douglas utility model is a commonly-used special case of the CES utility model with elasticity of substitution (σ) equal to 1. The Cobb-Douglas utility model simplifies many derivations due to a number of strong assumptions on behavior. However, the simplicity of Cobb-Douglas utility model has side effects: especially that it always gives 100% direct substitution rebound. That characteristic disqualifies its use in a rebound analysis framework that purports to estimate the magnitude of substitution effect rebound. So, we reject the special case Cobb-Douglas utility model in favor of the generalized CES utility model.

Both the substitution effect and the income effect use elasticities to model consumer preferences. In this appendix, the approximate, CES, and Cobb-Douglas utility models are described. Then, elasticities for the substitution effect are discussed. Finally, elasticities for the income effect are discussed.

D.1. Utility models for the substitution effect

A utility model gives the ratio of energy service consumption rate and other goods consumption rates across the substitution effect ($\frac{\hat{\dot{q}}_s}{\dot{q}_s^*}$ and $\frac{\hat{\dot{q}}_o}{\dot{q}_o^*}$, respectively). In so doing, utility models quantify the decrease in other goods consumption ($\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} < 1$) caused by the increase of energy service consumption ($\frac{\hat{\dot{q}}_s}{\dot{q}_s^*} > 1$) resulting from the decrease of the energy service price ($p_s^* < p_s^\circ$). Across the substitution effect, the utility increase of the larger energy service consumption rate must be exactly offset by the utility decrease of the smaller other goods consumption rate.

Before proceeding with the utility models, we note that the rate of other goods consumption ($\dot{q}_o$) is not known independently from the prices of other goods (p_o). With the assumption that the prices of other goods do not change across rebound effects (i.e., p_o is exogenous), the ratio of other goods consumption is equal to the ratio of other goods spending, such that

$$\frac{\dot{q}_o}{\dot{q}_o^\circ} = \frac{\dot{C}_o/\cancel{p_o}}{\dot{C}_o^\circ/\cancel{p_o}} = \frac{\dot{C}_o}{\dot{C}_o^\circ} \tag{D.1}$$

at all rebound stages. (See Appendix F for details.)

D.1.1. Approximate utility model

The approximate utility model is given by Eqs. (B.48) and (B.52). The equations for the approximate utility model are repeated here for convenience.

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} \tag{B.48}$$

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} \tag{B.52}$$

D.1.2. CES utility model

The CES utility model is given by Eq. (14). Here, its derivation is shown.

The constant elasticity of substitution (CES) model for utility ($\dot{u}$) is normalized by (indexed to) parameters before emplacement:

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^\rho + (1-a)\left(\frac{\dot{q}_o}{\dot{q}_o^\circ}\right)^\rho\right]^{(1/\rho)} , \tag{D.2}$$

where $\rho = \frac{\sigma-1}{\sigma}$, a is a share parameter (determined below), and σ is the elasticity of substitution between the normalized consumption rate of the energy service and the normalized consumption rate of other goods.

With the assumption of exogenous other goods prices in Eq. (D.1), we find

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right)^\rho + (1-a) \left(\frac{\dot{C}_o}{\dot{C}_o^\circ} \right)^\rho \right]^{(1/\rho)} . \tag{D.3}$$

Eq. (D.3) is the functional form of the CES utility model, whose share parameter (a) is yet to be determined. The correct expression for the share parameter (a) is found from the equilibrium requirement, namely that the slope of the indifference curve is tangent to (has the same slope as) the expenditure curve in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space prior to the EEU. For example, line ○—○ is tangent to preference curve i°—i° at point ○ in Figs. 4, 7, and 10.

To find the slope of the indifference curve, Eq. (D.3) can be rearranged to find the normalized consumption rate of other goods ($\dot{C}_o/\dot{C}_o^\circ$) as a function of the normalized consumption rate of the energy service ($\dot{q}_s/\dot{q}_s^\circ$) and the normalized utility rate ($\dot{u}/\dot{u}^\circ$):

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = \left[\frac{1}{1-a} \left(\frac{\dot{u}}{\dot{u}^\circ} \right)^\rho - \frac{a}{1-a} \left(\frac{\dot{q}}{\dot{q}_s^\circ} \right)^\rho \right]^{(1/\rho)} , \tag{D.4}$$

a form convenient for drawing constant utility rate ($\dot{u}/\dot{u}^\circ$) indifference curves in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space. (See Figs. 4, 7, and 10.) In $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, the slope of an indifference curve is found by taking the first partial derivative of $\dot{C}_o/\dot{C}_o^\circ$ with respect to $\dot{q}_s/\dot{q}_s^\circ$, starting from Eq. (D.4) and using the chain rule repeatedly. The result is

$$\begin{aligned} \frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = &- \frac{a}{1-a} \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right)^{(\rho-1)} \\ &\times \left[\left(\frac{1}{1-a} \right) \left(\frac{\dot{u}}{\dot{u}^\circ} \right)^\rho - \left(\frac{a}{1-a} \right) \left(\frac{\dot{q}}{\dot{q}_s^\circ} \right)^\rho \right]^{(1-\rho)/\rho} . \end{aligned} \tag{D.5}$$

The budget constraint is the starting point for finding the slope of the expenditure line in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space:

$$\dot{M} = p_s \dot{q}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o + \dot{N} , \tag{D.6}$$

a generic version of Eqs. (B.2), (B.4), (B.17), and (B.26) with $p_s \dot{q}_s$ substituted for $p_E \dot{E}_s$. In a manner similar to derivations in Appendix C.3.1, we solve for $\dot{C}_o$ and judiciously multiplying by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ to obtain

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} \dot{C}_o^\circ = -p_s \frac{\dot{q}_s}{\dot{q}_s^\circ} \dot{q}_s^\circ + \dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N} . \tag{D.7}$$

Solving for $\dot{C}_o/\dot{C}_o^\circ$ and rearranging gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = - \frac{p_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right) + \frac{1}{\dot{C}_o^\circ} (\dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N}) , \tag{D.8}$$

from which the slope in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space is taken by inspection to be

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{p_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \ . \tag{D.9}$$

At any equilibrium point, the expenditure line must be tangent to its indifference curve. Applying the tangency (equal slope) requirement before emplacement enables solving for the correct expression for a. Setting the slope of the expenditure line (Eq. (D.9)) equal to the slope of the indifference curve (Eq. (D.5)) gives

$$\begin{aligned} -\frac{p_s \dot{q}_s^\circ}{\dot{C}_o^\circ} = &- \frac{a}{1-a}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \\ &\times \left[\left(\frac{1}{1-a}\right)\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \left(\frac{a}{1-a}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho} \ . \end{aligned} \tag{D.10}$$

For the equilibrium point prior to emplacement, $\dot{q}_s/\dot{q}_s^\circ = 1$, $\dot{u}/\dot{u}^\circ = 1$, and $p_s = p_s^\circ$, which reduces Eq. (D.10) to

$$-\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{a}{1-a}(1)^{(\rho-1)}\left[\left(\frac{1}{1-a}\right)(1)^\rho - \left(\frac{a}{1-a}\right)(1)^\rho\right]^{(1-\rho)/\rho} \ . \tag{D.11}$$

Simplifying gives

$$\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ} = \frac{a}{1-a} \ . \tag{D.12}$$

Recognizing that $p_s^\circ \dot{q}_s^\circ = \dot{C}_s^\circ$ and solving for a gives

$$a = \frac{\dot{C}_s^\circ}{\dot{C}_s^\circ + \dot{C}_o^\circ} \ , \tag{D.13}$$

which is called $f_{\dot{C}_s}^\circ$, the share of energy service expenditure ($\dot{C}_s^\circ$) relative to the sum of energy service and other goods expenditures ($\dot{C}_s^\circ + \dot{C}_o^\circ$) before emplacement. Thus, the CES utility equation (Eq. (D.3)) becomes

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^\rho + (1 - f_{\dot{C}_s}^\circ)\left(\frac{\dot{C}_o}{\dot{C}_o^\circ}\right)^\rho\right]^{(1/\rho)} \ , \tag{14}$$

with

$$f_{\dot{C}_s}^\circ \equiv \frac{\dot{C}_s^\circ}{\dot{C}_s^\circ + \dot{C}_o^\circ} \ . \tag{D.14}$$

We note briefly (because we don't use it elsewhere in this paper), that a common utility model takes the form of a Cobb-Douglas function

$$\frac{\dot{u}}{\dot{u}^\circ} = A\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^\alpha\left(\frac{\dot{q}_o}{\dot{q}_o^\circ}\right)^{(1-\alpha)} \ , \tag{D.15}$$

where A is a proportionality constant and $\alpha \in (0, 1)$ is an weighing parameter which describes the utility of consuming the energy service ($\dot{q}_s$) relative to consuming other goods ($\dot{q}_o$). The Cobb-Douglas model is a special case of the general CES utility model, with the elasticity of substitution (σ) equal to 1. However, the uncompensated own-price elasticity ($\epsilon_{\dot{q}_s,p_s}$) in the Cobb-Douglas utility model is -1, causing $Re_{dsub} = 1$ always.

To see this, note that the Cobb-Douglas Marshallian demand of any good (illustrated for the energy service here) is of the form

$$\dot{q}_{s,m} = \alpha \dot{M} / p_s \ , \tag{D.16}$$

where $\dot{M}$ is the income rate and the subscript m denotes Marshallian demand. The price elasticity of demand is therefore

$$\epsilon_{\dot{q}_s,p_s} = \frac{\partial \dot{q}_{s,m}}{\partial p_s} \frac{p_s}{\dot{q}_{s,m}} = -\frac{\alpha M}{p_s^2} \frac{p_s^2}{\alpha M} = -1 \ . \tag{D.17}$$

I.e., a Cobb-Douglas model of consumer utility already presumes 100% direct rebound, which is inappropriate for a rebound analysis framework that purports to estimate the magnitude of rebound effects. Saunders (2008) discusses similar problems in a rebound analysis context with the Cobb-Douglas production function.

D.2. Elasticities for the substitution effect ($\epsilon_{\dot{q}_s,p_s,c}$, $\epsilon_{\dot{q}_o,p_s,c}$, and σ)

The substitution effect requires a utility model. In this paper, we describe two utiliity models in detail: an approximate model and a CES model. All utility models require elasticities to describe consumer preferences. In fact, there are three elasticies across the two utility models we discuss in detail: $\epsilon_{\dot{q}_s,p_s,c}$, $\epsilon_{\dot{q}_o,p_s,c}$, and σ. The approximate method requires the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated cross-price elasticity of energy service demand ($\epsilon_{\dot{q}_o,p_s,c}$). The CES utility model requires the elasticity of substitution between energy service consumption and other goods consumption (σ). Since the *un*compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s}$), tends to be more widely estimated, e.g. in studies about driver behavior (Gillingham, 2020), the usual approach is to use it and the Slutsky equation to derive the compensated elasticity. The CES model then imposes enough restrictions to determine the elasticity of substitution.

D.2.1. Elasticities for the approximate utility model ($\epsilon_{\dot{q}_s,p_s,c}$ and $\epsilon_{\dot{q}_o,p_s,c}$)

Analytical expressions for the compensated elasticities in the approximate utility model can be derived using the Slutsky equation, whereby the price elasticity of the energy service ($\epsilon_{\dot{q}_s,p_s}$) is decomposed into the compensated price elasticity ($\epsilon_{\dot{q}_s,p_s,c}$) and the income elasticity ($\epsilon_{\dot{q}_s,\dot{M}}$) as follows:

$$\epsilon_{\dot{q}_s,p_s} = \epsilon_{\dot{q}_s,p_s,c} - f^{\circ}_{\dot{C}_s} \epsilon_{\dot{q}_s,\dot{M}} \ , \tag{D.18}$$

where $f^{\circ}_{\dot{C}_s}$ is given by Eq. (D.14), and $\epsilon_{\dot{q}_s,\dot{M}}$ is given in Section D.3. Solving for the compensated price elasticity ($\epsilon_{\dot{q}_s,p_s,c}$) gives

$$\epsilon_{\dot{q}_s,p_s,c} = \epsilon_{\dot{q}_s,p_s} + f^{\circ}_{\dot{C}_s} \epsilon_{\dot{q}_s,\dot{M}} \ . \tag{D.19}$$

A similar argument allows straightforward derivation of the compensated cross-price elasticity ($\epsilon_{\dot{q}_o,p_s,c}$). With Hicks and Allen (1934), we note that the uncompensated cross-price elasticity ($\epsilon_{\dot{q}_o,p_s}$) can generally be expressed as

$$\epsilon_{\dot{q}_o,p_s} = f^{\circ}_{\dot{C}_s}(\sigma - \epsilon_{\dot{q}_o,\dot{M}}) \ , \tag{D.20}$$

where σ is the elasticity of substitution between the consumption rate of the energy service ($\dot{q}_s$) and the consumption rate of other goods ($\dot{q}_o$). We set Eq. (D.20) equal to the cross-price version of the Slutsky equation

$$\epsilon_{\dot{q}_o,p_s} = \epsilon_{\dot{q}_o,p_s,c} - f^{\circ}_{\dot{C}_s}\epsilon_{\dot{q}_o,\dot{M}} \ , \tag{D.21}$$

to obtain

$$f^{\circ}_{\dot{C}_s}(\sigma - \epsilon_{\dot{q}_o,\dot{M}}) = \epsilon_{\dot{q}_o,p_s,c} - f^{\circ}_{\dot{C}_s}\epsilon_{\dot{q}_o,\dot{M}} \ . \tag{D.22}$$

Solving for $\epsilon_{\dot{q}_o,p_s,c}$ gives

$$\epsilon_{\dot{q}_o,p_s,c} = f^{\circ}_{\dot{C}_s}\sigma \ . \tag{D.23}$$

Substituting σ from Section D.2.2 (Eq. (D.25)) gives

$$\epsilon_{\dot{q}_o,p_s,c} = \frac{f^{\circ}_{\dot{C}_s}(f^{\circ}_{\dot{C}_s} + \epsilon_{\dot{q}_s,p_s})}{f^{\circ}_{\dot{C}_s} - 1} \ . \tag{D.24}$$

D.2.2. Elasticity for the CES utility model (σ)

Gørtz (1977) shows that the elasticity of substitution (σ) in the CES utility model is given by

$$\sigma = \frac{f^{\circ}_{\dot{C}_s} + \epsilon_{\dot{q}_s,p_s}}{f^{\circ}_{\dot{C}_s} - 1} \ . \tag{D.25}$$

Thus, the elasticity of substitution (σ) can be determined from two pieces of readily-available information: (i) the uncompensated own price elasticity ($\epsilon_{\dot{q}_s,p_s}$) and (ii) the share of income spent on the energy service ($f^{\circ}_{\dot{C}_s}$ from Eq. (D.14)).

D.3. Elasticities for the income effect ($\epsilon_{\dot{q}_s,\dot{M}}$ and $\epsilon_{\dot{q}_o,\dot{M}}$)

The income effect requires two elasticities to estimate the respending of net savings: the income elasticity of energy service consumption ($\epsilon_{\dot{q}_s,\dot{M}}$) and the income elasticity of other goods consumption ($\epsilon_{\dot{q}_o,\dot{M}}$). Due to the homotheticity assumption, both income elasticities are unitary. Thus,

$$\epsilon_{\dot{q}_s,\dot{M}} = 1 \ , \tag{D.26}$$

and

$$\epsilon_{\dot{q}_o,\dot{M}} = 1 \ . \tag{D.27}$$

E. Proof: Income preference equations satisfy the budget constraint

After the substitution effect, a rate of net savings is available ($\hat{\dot{N}}$), all of which is spent on additional energy service ($\Delta\bar{\dot{q}}_s$, $\Delta\bar{\dot{C}}_s = p_E \Delta\bar{\dot{E}}_s$) or additional other goods ($\Delta\bar{\dot{q}}_o$, $\Delta\bar{\dot{C}}_o$). The income effect must satisfy the budget constraint such that net savings is zero afterward ($\bar{\dot{N}} = 0$). The budget constraint across the income effect is represented by Eq. (B.32):

$$\hat{\dot{N}} = p_E \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o \ . \tag{B.32}$$

The additional spending due to the income effect is given by income preference equations

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} \tag{23}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} \ , \tag{27}$$

where

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \ . \tag{24}$$

This appendix proves that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (B.32)).

The first step in the proof is to convert the income preference equations to $\dot{C}^\circ_s$ and $\dot{C}^\circ_o$ ratios. For the energy service income preference equation (Eq. (23)), multiply numerator and denominator of the left-hand side by $\tilde{p}_s = p_E/\tilde{\eta}$ (Eq. (6)) to obtain $\bar{\dot{C}}_s/\hat{\dot{C}}_s$. For the other goods income preference equation (Eq. (27)), multiply numerator and denominator of the left-hand side by p_o to obtain $\bar{\dot{C}}_o/\hat{\dot{C}}_o$. Then invoke homotheticity to set $\epsilon_{\dot{q}_s,\dot{M}} = 1$ and $\epsilon_{\dot{q}_o,\dot{M}} = 1$ to obtain

$$\frac{\bar{\dot{C}}_s}{\hat{\dot{C}}_s} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \tag{E.1}$$

and

$$\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \ . \tag{E.2}$$

The second step in the proof is to obtain expressions for $\Delta\bar{\dot{C}}_s$ and $\Delta\bar{\dot{C}}_o$. Multiply the income preference equations above by $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively. Then subtract $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively, to obtain

$$\Delta\bar{\dot{C}}_s = \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} \tag{E.3}$$

and

$$\Delta\bar{\dot{C}}_o = \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \ . \tag{E.4}$$

The above versions of the income preference equations can be substituted into the budget constraint (Eq. (B.32)) to obtain

$$\hat{\dot{N}} \overset{?}{=} \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} + \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \ . \tag{E.5}$$

If equality is demonstrated, the income preference equations satisfy the budget constraint. The remainder of the proof shows the equality of Eq. (E.5).

Dividing by $\hat{\dot{N}}$ and multiplying by $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \hat{\dot{M}}' \ . \tag{E.6}$$

Substituting Eq. (24) for $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \ . \tag{E.7}$$

Substituting Eq. (B.17) for $\dot{M}^\circ$, because $\dot{M}^\circ = \hat{\dot{M}}$, gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} p_E\hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \cancel{\hat{\dot{N}}} - \dot{C}^*_{cap} - \dot{C}^*_{md} - \cancel{\hat{\dot{N}}} \ . \tag{E.8}$$

Cancelling terms and recognizing that $\dot{C}^*_{cap} = \hat{\dot{C}}_{cap}$, $\dot{C}^*_{md} = \hat{\dot{C}}_{md}$, and $\hat{\dot{C}}_s = p_E\hat{\dot{E}}_s$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \hat{\dot{C}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o - \cancel{\hat{\dot{C}}_{cap}} - \cancel{\hat{\dot{C}}_{md}} \ . \tag{E.9}$$

Cancelling terms gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{\checkmark}{=} \hat{\dot{C}}_s + \hat{\dot{C}}_o \ , \tag{E.10}$$

thereby completing the proof that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (B.32)).

F. Other goods expenditures and constant p_o

This framework utilizes a partial equilibrium analysis in which we account for the change of the energy service price due to the EEU ($p_s^\circ \neq p_s^*$), but we do not track the effect of the EEU on prices of other goods. These assumptions have important implications for the relationship between the rate of consumption of other goods ($\dot{q}_o$) and the rate of expenditure on other goods ($\dot{C}_o$).

We assume a basket of other goods (besides the energy service) purchased in the economy, each (i) with its own price ($p_{o,i}$) and rate of consumption ($\dot{q}_{o,i}$), such that the average price of all other goods purchased in the economy prior to the EEU (p_o°) is given by

$$p_o^\circ = \frac{\sum_i p_{o,i}^\circ q_{o,i}^\circ}{\sum_i q_{o,i}^\circ} \; . \tag{F.1}$$

Then, the expenditure rate of other purchases in the economy can be given as

$$\dot{C}_o^\circ = p_o^\circ \dot{q}_o^\circ \tag{F.2}$$

before the EEU and

$$\dot{\hat{C}}_o = \hat{p}_o \dot{\hat{q}}_o \tag{F.3}$$

after the substitution effect, for example.

We assume that any effects (emplacement, substitution, or income) for a single device are not so large that they cause a measurable change in prices of other goods. Thus,

$$p_o^\circ = p_o^* = \hat{p}_o = \bar{p}_o = \tilde{p}_o \; . \tag{F.4}$$

In the partial equilibrium analysis, two other goods prices can be equated across any rebound effect to obtain (for the example of the original conditions ($\circ$) and the post-substitution state ($\wedge$))

$$\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} = \frac{\dot{\hat{q}}_o}{\dot{q}_o^\circ} \; . \tag{F.5}$$

Thus, a ratio of other goods expenditure rates is always equal to a ratio of other goods consumption rates.

G. Income and macro effects and relation to marginal propensity to consume

Borenstein (2015) has postulated a demand-side argument that macro effects can be prepresented by a multiplier, which we call the macro factor (k). Borenstein's formulation and our implementation are reminiscent of the marginal propensity to consume (MPC). In this appendix, we show the relationship between the macro factor (k) and MPC.

The relationship between the macro factor (k) and MPC spans the income and macro effects. In this framework, the device owner's net income ($\dot{\hat{N}}$) is respent completely. One may assume that firms and other consumers who receive the net income have a marginal propensity to re-spend of MPC. The total spending throughout the economy of each year's net income ($\dot{\hat{N}}$) is given by the infinite series

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\dot{\hat{N}} \; , \tag{G.1}$$

where the first term $(1 \times \hat{\dot{N}})$ represents spending of net income by the device owner in the direct and indirect income effects, and the remaining terms represent macro-effect spending in the broader economy.

The macro effect portion of the spending can be represented by the macro factor (k).

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\hat{\dot{N}} = (1 + k)\hat{\dot{N}} \tag{G.2}$$

Cancelling terms and simplifying the infinite series to its convergent fraction gives

$$\frac{1}{\frac{1}{MPC} - 1} = k \; . \tag{G.3}$$

If $k = 3$, as in Section 4.3, $MPC = 0.75$ is implied.