Table A.1: Symbols and abbreviations.

Symbol	Meaning [example units]
a	the share parameter in the CES utility model [–]
C	cost [\$]
E	final energy [MJ]
f	expenditure share [–]
G	freed cash [\$]
g	a constant in the derivation of $\varepsilon_{\dot{q}_s,p_s,c}$ and $\varepsilon_{\dot{q}_o,p_s,c}$ [–]
h	a constant in the derivation of $\varepsilon_{\dot{q}_s,p_s,c}$ and $\varepsilon_{\dot{q}_o,p_s,c}$ [–]
I	energy intensity of economic activity [MJ/\$]
k	macro factor [–]
M	income [\$]
m	mass [kg] an exponent in the derivation of $\varepsilon_{\dot{q}_s,p_s,c}$ and $\varepsilon_{\dot{q}_o,p_s,c}$ [–]
N	net savings [\$]
n	an exponent in the derivation of $\varepsilon_{\dot{q}_s,p_s,c}$ and $\varepsilon_{\dot{q}_o,p_s,c}$ [–]
p	price [\$]
q	quantity [–]
Re	rebound [–]
S	energy cost savings [\$]
t	energy conversion device lifetime [yr]
u	utility [utils]
x	position [m]
z	a constant in the derivation of $\varepsilon_{\dot{q}_s,p_s,c}$ and $\varepsilon_{\dot{q}_o,p_s,c}$ [–]

Appendices

A Nomenclature

Presentation of the rigorous analytical framework is aided by a nomenclature that describes energy stages and rebound effects. Table A.1 shows symbols and abbreviations, their meanings, and example units. Table A.2 shows Greek letters, their meanings, and example units. Table A.3 shows abbreviations and acronyms. Table A.4 shows symbol decorations and their meanings. Table A.5 shows subscripts and their meanings.

Differences are indicated by the Greek letter Δ and always signify subtraction of a quantity at an earlier stage of Fig. 1 from the same quantity at the next later stage of Fig. 1. E.g., $\Delta\bar{X} \equiv \bar{X} - \hat{X}$, and $\Delta\tilde{X} \equiv \tilde{X} - \bar{X}$. Lack of decoration on a difference term indicates a difference that spans all stages of Fig. 1. E.g., $\Delta X \equiv \tilde{X} - X^{\circ}$. ΔX is also the sum of differences across each stage in Fig. 1, as shown below.

Table A.2: Greek letters.

Greek letter	Meaning [example units]
Δ	difference (later quantity less earlier quantity, see Fig. 1)
ε	price or income elasticity [–]
$\varepsilon_{\dot{q}_s,\dot{M}}$	income ($\dot{M}$) elasticity of energy service demand ($\dot{q}_s$) [–]
$\varepsilon_{\dot{q}_o,\dot{M}}$	income ($\dot{M}$) elasticity of other goods demand ($\dot{q}_o$) [–]
$\varepsilon_{\dot{q}_s,p_s}$	uncompensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\varepsilon_{\dot{q}_o,p_s}$	uncompensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
$\varepsilon_{\dot{q}_s,p_s,c}$	compensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\varepsilon_{\dot{q}_o,p_s,c}$	compensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
η	final-energy-to-service efficiency [vehicle-km/MJ]
ρ	exponent in the CES utility function, $\rho \equiv (\sigma - 1)/\sigma$ [–]
σ	elasticity of substitution between the energy service ($\dot{q}_s^\circ$) and other goods ($\dot{q}_o^\circ$) [–]

Table A.3: Abbreviations.

Abbreviation	Meaning
CES	constant elasticity of substitution
CPE	constant price elasticity
CV	compensating variation
EEU	energy efficiency upgrade
EPSRC	engineering and physical sciences research council
GDP	gross domestic product
MPC	marginal propensity to consume
UK	United Kingdom
UKRI	UK research and innovation
U.S.	United States

Table A.4: Decorations.

Decoration	Meaning [example units]
X°	X originally (before the emplacement effect)
X^*	X after the emplacement effect (before the substitution effect)
$\hat{X}$	X after the substitution effect (before the income effect)
$\bar{X}$	X after the income effect (before the macro effect)
$\tilde{X}$	X after the macro effect
$\dot{X}$	rate of X [units of X/yr]
M'	effective income [$]

Table A.5: Subscripts.

Subscript	Meaning
c	compensated
cap	capital costs
dev	device
$dempl$	direct emplacement effect
d	disposal
$dinc$	direct income effect
$dsub$	direct substitution effect
E	energy
emb	embodied
$empl$	emplacement effect
$iempl$	indirect emplacement effects
$iinc$	indirect income effect
inc	income effect
$isub$	indirect substitution effect
$life$	lifetime
m	maintenance
$macro$	macro effect
md	maintenance and disposal
o	other expenditures (besides energy) by the device user
s	service stage of the energy conversion chain
sub	substitution effect
tot	sum of all rebound effects in the framework

$$
\begin{aligned}
\Delta X &= \Delta\tilde{X} + \Delta\bar{X} + \Delta\hat{X} + \Delta X^{*} \\
\Delta X &= (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^{*}) + (X^{*} - X^{\circ}) \\
\Delta X &= (\tilde{X} - \cancel{\bar{X}}) + (\cancel{\bar{X}} - \cancel{\hat{X}}) + (\cancel{\hat{X}} - \cancel{X^{*}}) + (\cancel{X^{*}} - X^{\circ}) \\
\Delta X &= \tilde{X} - X^{\circ} \qquad (35)
\end{aligned}
$$

B Derivation of the rigorous analytical framework

This appendix provides a detailed derivation of the rigorous analytical framework, beginning with relationships for each rebound effect.

B.1 Relationships for rebound effects

For each energy rebound effect in Fig. 1, energy and financial analysis must be performed. The purposes of the analyses are to determine for each effect (i) an expression for energy rebound (Re) for the effect and (ii) an equation for net savings ($\dot{N}$) remaining after the effect.

Analysis of each rebound effect involves a set of assumptions and constraints as shown in

Table B.1. In Table B.1, relationships for emplacement effect embodied energy rates ($\dot{E}^{\circ}_{emb}$ and $\dot{E}^{*}_{emb}$), capital expenditure rates ($\dot{C}^{\circ}_{cap}$ and $\dot{C}^{*}_{cap}$), and maintenance and disposal expenditure rates ($\dot{C}^{\circ}_{md}$ and $\dot{C}^{*}_{md}$) are typical, and inequalities could switch direction for a specific EEU. Macro effect relationships are given for a single device only. If the EEU is deployed at scale across the economy, the energy service consumption rate ($\dot{\tilde{q}}_s$), device energy consumption rate ($\dot{\tilde{E}}_s$), embodied energy rate ($\dot{\tilde{E}}_{emb}$), capital expenditure rate ($\dot{\tilde{C}}_{cap}$), and maintenance and disposal expenditure rate ($\dot{\tilde{C}}_{md}$) will all increase in proportion to the number of devices emplaced.

Table B.1: Assumptions and constraints for analysis of rebound effects.

Parameter	Emplacement Effect	Substitution Effect	Income Effect	Macro Effect
Energy price	$p_E^\circ = p_E^*$	$p_E^* = \hat{p}_E$	$\hat{p}_E = \bar{p}_E$	$\bar{p}_E = \tilde{p}_E$
Energy service efficiency	$\eta^\circ < \eta^*$	$\eta^* = \hat{\eta}$	$\hat{\eta} = \bar{\eta}$	$\bar{\eta} = \tilde{\eta}$
Energy service price	$p_s^\circ > p_s^*$	$p_s^* = \hat{p}_s$	$\hat{p}_s = \bar{p}_s$	$\bar{p}_s = \tilde{p}_s$
Other goods price	$p_o^\circ = p_o^*$	$p_o^* = \hat{p}_o$	$\hat{p}_o = \bar{p}_o$	$\bar{p}_o = \tilde{p}_o$
Energy service consumption rate	$\dot{q}_s^\circ = \dot{q}_s^*$	$\dot{q}_s^* < \dot{\hat{q}}_s$	$\dot{\hat{q}}_s < \dot{\bar{q}}_s$	$\dot{\bar{q}}_s = \dot{\tilde{q}}_s$
Other goods consumption rate	$\dot{q}_o^\circ = \dot{q}_o^*$	$\dot{q}_o^* > \dot{\hat{q}}_o$	$\dot{\hat{q}}_o < \dot{\bar{q}}_o$	$\dot{\bar{q}}_o = \dot{\tilde{q}}_o$
Device energy consumption rate	$\dot{E}_s^\circ > \dot{E}_s^*$	$\dot{E}_s^* < \dot{\hat{E}}_s$	$\dot{\hat{E}}_s < \dot{\bar{E}}_s$	$\dot{\bar{E}}_s = \dot{\tilde{E}}_s$
Embodied energy rate	$\dot{E}_{emb}^\circ < \dot{E}_{emb}^*$	$\dot{E}_{emb}^* = \dot{\hat{E}}_{emb}$	$\dot{\hat{E}}_{emb} = \dot{\bar{E}}_{emb}$	$\dot{\bar{E}}_{emb} = \dot{\tilde{E}}_{emb}$
Capital expenditure rate	$\dot{C}_{cap}^\circ < \dot{C}_{cap}^*$	$\dot{C}_{cap}^* = \dot{\hat{C}}_{cap}$	$\dot{\hat{C}}_{cap} = \dot{\bar{C}}_{cap}$	$\dot{\bar{C}}_{cap} = \dot{\tilde{C}}_{cap}$
Maint. and disp. expenditure rate	$\dot{C}_{md}^\circ < \dot{C}_{md}^*$	$\dot{C}_{md}^* = \dot{\hat{C}}_{md}$	$\dot{\hat{C}}_{md} = \dot{\bar{C}}_{md}$	$\dot{\bar{C}}_{md} = \dot{\tilde{C}}_{md}$
Energy service expenditure rate	$\dot{C}_s^\circ > \dot{C}_s^*$	$\dot{C}_s^* < \dot{\hat{C}}_s$	$\dot{\hat{C}}_s < \dot{\bar{C}}_s$	$\dot{\bar{C}}_s = \dot{\tilde{C}}_s$
Other goods expenditure rate	$\dot{C}_o^\circ = \dot{C}_o^*$	$\dot{C}_o^* > \dot{\hat{C}}_o$	$\dot{\hat{C}}_o < \dot{\bar{C}}_o$	$\dot{\bar{C}}_o = \dot{\tilde{C}}_o$
Income	$\dot{M}^\circ = \dot{M}^*$	$\dot{M}^* = \dot{\hat{M}}$	$\dot{\hat{M}} = \dot{\bar{M}}$	$\dot{\bar{M}} = \dot{\tilde{M}}$
Net savings	$0 = \dot{N}^\circ < \dot{N}^*$	$\dot{N}^* < \dot{\hat{N}}$	$\dot{\hat{N}} > \dot{\bar{N}} = 0$	$\dot{\bar{N}} = \dot{\tilde{N}} = 0$

Table B.2: Justification for zeroed terms in Tables B.3–B.6.

Zeroed term	Justification (from Table B.1).
$\cancelto{0}{\Delta \dot{C}_o^*}$	$\dot{C}_o^\circ = \dot{C}_o^*$ ($\dot{C}_o$ unchanged across emplacement effect.)
$\cancelto{0}{\dot{N}^\circ}$	$0 = \dot{N}^\circ$ (Net savings are zero prior to the EEU.)
$\cancelto{0}{\Delta \hat{\dot{E}}_{emb}}$	$\dot{E}^*_{emb} = \hat{\dot{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \hat{\dot{C}}_{md}}$	$\dot{C}^*_{md} = \hat{\dot{C}}_{md}$ ($\dot{C}_{md}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \bar{\dot{E}}_{emb}}$	$\hat{\dot{E}}_{emb} = \bar{\dot{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across income effect.)
$\cancelto{0}{\Delta \bar{\dot{C}}_{md}}$	$\hat{\dot{C}}_{md} = \bar{\dot{C}}_{md}$ ($\dot{C}_{md}$ unchanged across income effect.)
$\cancelto{0}{\bar{\dot{N}}}$	$\bar{\dot{N}} = 0$ (All net savings are spent in the income effect.)

B.2 Derivations

Derivations for rebound definitions and net savings equations are presented in Tables B.3–B.6, one for each rebound effect in Fig. 1. Energy and financial analyses are shown side by side, because each informs the other.

Several terms in Tables B.3–B.6 are zeroed, e.g. $\cancelto{0}{\Delta \dot{C}_o^*}$. These zeroes can be traced back to Table B.1. Table B.2 highlights the equations in Table B.1 that justify zeroing each term.

Table B.3. Emplacement Effect

	Energy analysis		*Financial analysis*	
before ($\circ$)	$\dot{E}^\circ = \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ) I_E$	(36)	$\dot{M}^\circ = p_E \dot{E}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ + \dot{N}^\circ$	(37)
after ($*$)	$\dot{E}^* = \dot{E}_s^* + \dot{E}_{emb}^* + (\dot{C}_{md}^* + \dot{C}_o^*) I_E$	(38)	$\dot{M}^* = p_E \dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o^* + \dot{N}^*$	(39)

Take differences to obtain the change in energy consumption, $\Delta\dot{E}^* \equiv \dot{E}^* - \dot{E}^\circ$.

$$\Delta\dot{E}^* = \Delta\dot{E}_s^* + \Delta\dot{E}_{emb}^* + (\Delta\dot{C}_{md}^* + \cancelto{0}{\Delta\dot{C}_o^*}) I_E \tag{40}$$

Thus,

$$\Delta\dot{E}^* = \Delta\dot{E}_s^* + \Delta\dot{E}_{emb}^* + \Delta\dot{C}_{md}^* I_E \; . \tag{41}$$

Define

$$\dot{S}_{dev} \equiv -\Delta\dot{E}_s^* \tag{42}$$

(Also see Eqs. (73) and (10)). Use Eq. (1) to obtain

$$Re_{empl} = 1 - \frac{-\Delta\dot{E}^*}{\dot{S}_{dev}} = 1 - \frac{-\Delta\dot{E}_s^*}{\dot{S}_{dev}} - \frac{-\Delta\dot{E}_{emb}^*}{\dot{S}_{dev}} - \frac{-\Delta\dot{C}_{md}^* I_E}{\dot{S}_{dev}} \; . \tag{43}$$

Define $Re_{dempl} \equiv 1 - \frac{-\Delta\dot{E}_s^*}{\dot{S}_{dev}} (= 0)$, $Re_{iempl} \equiv Re_{emb} + Re_{md}$, $Re_{emb} \equiv \frac{\Delta\dot{E}_{emb}^*}{\dot{S}_{dev}}$, and $Re_{md} \equiv \frac{\Delta\dot{C}_{md}^* I_E}{\dot{S}_{dev}}$, such that

$$Re_{empl} = Re_{dempl} + Re_{iempl} \; . \tag{44}$$

Use the monetary constraint ($\dot{M}^\circ = \dot{M}^*$) and constant spending on other items ($\dot{C}_o^\circ = \dot{C}_o^*$) to cancel terms to obtain

$$\begin{aligned} p_E \dot{E}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \cancel{\dot{C}_o^\circ} + \cancelto{0}{\dot{N}^\circ} \\ = p_E \dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \cancel{\dot{C}_o^*} + \dot{N}^* \; . \end{aligned} \tag{45}$$

Solving for $\Delta\dot{N}^* \equiv \dot{N}^* - \cancelto{0}{\dot{N}^\circ}$ gives

$$\Delta\dot{N}^* = p_E(\dot{E}_s^\circ - \dot{E}_s^*) + \dot{C}_{cap}^\circ - \dot{C}_{cap}^* + \dot{C}_{md}^\circ - \dot{C}_{md}^* \; . \tag{46}$$

Rewriting with Δ terms gives

$$\Delta\dot{N}^* = -p_E \Delta\dot{E}_s^* - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^* \; . \tag{47}$$

Substituting Eq. (42) gives

$$\Delta\dot{N}^* = \dot{N}^* = p_E \dot{S}_{dev} - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^* \; . \tag{48}$$

Freed cash ($\dot{G}$) resulting from the EEU, before any energy takeback, is given by

$$\dot{G} = p_E \dot{S}_{dev} \; . \tag{49}$$

Note that Eq. (37) and $\dot{N}^\circ = 0$ can be used to calculate $\dot{C}_o^\circ$ as

$$\dot{C}_o^\circ = \dot{M}^\circ - p_E \dot{E}_s^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{50}$$

Table B.4. Substitution Effect

	Energy analysis		*Financial analysis*	
before $(*)$	$\dot{E}^* = \dot{E}_s^* + \dot{E}_{emb}^* + (\dot{C}_{md}^* + \dot{C}_o^*)I_E$	(38)	$\dot{M}^* = p_E\dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o^* + \dot{N}^*$	(39)
after $(\wedge)$	$\hat{\dot{E}} = \hat{\dot{E}}_s + \hat{\dot{E}}_{emb} + (\hat{\dot{C}}_{md} + \hat{\dot{C}}_o)I_E$	(51)	$\hat{\dot{M}} = p_E\hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}}$	(52)

Take differences to obtain the change in energy consumption, $\Delta\hat{\dot{E}} \equiv \hat{\dot{E}} - \dot{E}^*$.

$$\Delta\hat{\dot{E}} = \Delta\hat{\dot{E}}_s + \cancelto{0}{\Delta\hat{\dot{E}}_{emb}} + (\cancelto{0}{\Delta\hat{\dot{C}}_{md}} + \Delta\hat{\dot{C}}_o)I_E \tag{53}$$

Thus,

$$\Delta\hat{\dot{E}} = \Delta\hat{\dot{E}}_s + \Delta\hat{\dot{C}}_o I_E \ . \tag{54}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\hat{\dot{E}}}{\dot{S}_{dev}} = \frac{\Delta\hat{\dot{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \tag{55}$$

Define $Re_{sub} \equiv \frac{\Delta\hat{\dot{E}}}{\dot{S}_{dev}}$, $Re_{dsub} \equiv \frac{\Delta\hat{\dot{E}}_s}{\dot{S}_{dev}}$, and $Re_{isub} \equiv \frac{\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{sub} = Re_{dsub} + Re_{isub} \ . \tag{56}$$

Use the monetary constraint ($\dot{M}^* = \hat{\dot{M}}$) to obtain

$$\begin{aligned} & p_E\dot{E}_s^* + \cancel{\dot{C}_{cap}^*} + \cancel{\dot{C}_{md}^*} + \dot{C}_o^* + \dot{N}^* \\ & = p_E\hat{\dot{E}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o + \hat{\dot{N}} \ . \end{aligned} \tag{57}$$

For the substitution effect, there is no change in capital or maintenance and disposal costs ($\hat{\dot{C}}_{cap} = \dot{C}_{cap}^*$ and $\hat{\dot{C}}_{md} = \dot{C}_{md}^*$). Solving for $\Delta\hat{\dot{N}} \equiv \hat{\dot{N}} - \dot{N}^*$ gives

$$\Delta\hat{\dot{N}} = -p_E\Delta\hat{\dot{E}}_s - \Delta\hat{\dot{C}}_o \ . \tag{58}$$

The substitution effect adjusts net savings relative to $\dot{N}^*$ by $\Delta\hat{\dot{N}}$. Thus, $\hat{\dot{N}} = \dot{N}^* + \Delta\hat{\dot{N}}$. Substituting Eqs. (48), (49), and (58) yields

$$\hat{\dot{N}} = \dot{G} - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^* - p_E\Delta\hat{\dot{E}}_s - \Delta\hat{\dot{C}}_o \ . \tag{59}$$

Table B.5. **Income Effect**

	Energy analysis		*Financial analysis*	
before ($\wedge$)	$\hat{\dot{E}} = \hat{\dot{E}}_s + \hat{\dot{E}}_{emb} + (\hat{\dot{C}}_{md} + \hat{\dot{C}}_o) I_E$	(51)	$\hat{\dot{M}} = p_E \hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}}$	(52)
after ($-$)	$\bar{\dot{E}} = \bar{\dot{E}}_s + \bar{\dot{E}}_{emb} + (\bar{\dot{C}}_{md} + \bar{\dot{C}}_o) I_E$	(60)	$\bar{\dot{M}} = p_E \bar{\dot{E}}_s + \bar{\dot{C}}_{cap} + \bar{\dot{C}}_{md} + \bar{\dot{C}}_o + \bar{\dot{N}}$	(61)

Take differences to obtain the change in energy consumption, $\Delta\bar{\dot{E}} \equiv \bar{\dot{E}} - \hat{\dot{E}}$.

$$\Delta\bar{\dot{E}} = \Delta\bar{\dot{E}}_s + \cancelto{0}{\Delta\bar{\dot{E}}_{emb}} + (\cancelto{0}{\Delta\bar{\dot{C}}_{md}} + \Delta\bar{\dot{C}}_o) I_E \tag{62}$$

Thus,

$$\Delta\bar{\dot{E}} = \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o I_E \tag{63}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\bar{\dot{E}}}{\dot{S}_{dev}} = \frac{\Delta\bar{\dot{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}} \tag{64}$$

Define $Re_{inc} \equiv \frac{\Delta\bar{\dot{E}}}{\dot{S}_{dev}}$, $Re_{dinc} \equiv \frac{\Delta\bar{\dot{E}}_s}{\dot{S}_{dev}}$, and $Re_{iinc} \equiv \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{inc} = Re_{dinc} + Re_{iinc} \ . \tag{65}$$

Use the monetary constraint ($\hat{\dot{M}} = \bar{\dot{M}}$) to obtain

$$\begin{aligned} & p_E \hat{\dot{E}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o + \hat{\dot{N}} \\ & \quad = p_E \bar{\dot{E}}_s + \cancel{\bar{\dot{C}}_{cap}} + \cancel{\bar{\dot{C}}_{md}} + \bar{\dot{C}}_o + \cancelto{0}{\bar{\dot{N}}} \ . \end{aligned} \tag{66}$$

For the income effect, there is no change in capital or maintainance and disposal costs ($\hat{\dot{C}}_{cap} = \dot{C}^*_{cap}$ and $\hat{\dot{C}}_{md} = \dot{C}^*_{md}$). Notably, $\bar{\dot{N}} = 0$, because it is assumed that all net monetary savings ($\hat{\dot{N}}$) are spent on more energy service ($\bar{\dot{E}}_s > \hat{\dot{E}}_s$) and additional purchases in the economy ($\bar{\dot{C}}_o > \hat{\dot{C}}_o$). Solving for $\hat{\dot{N}}$ gives

$$\hat{\dot{N}} = p_E \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o \ , \tag{67}$$

the budget constraint for the income effect. By construction, Eq. (67) ensures spending of net savings ($\hat{\dot{N}}$) on (i) additional energy services ($\Delta\bar{\dot{E}}_s$) and (ii) additional purchases of other goods in the economy ($\Delta\bar{\dot{C}}_o$) only.

Table B.6. **Macro Effect**

	Energy analysis		*Financial analysis*
before (−)	$\bar{\dot{E}}$	(68)	
after (∼)	$\tilde{\dot{E}}$	(69)	

Take differences to obtain the change in energy consumption,

$$\Delta\tilde{\dot{E}} \equiv \tilde{\dot{E}} - \bar{\dot{E}} \; . \tag{70}$$

The energy change due to the macro effect ($\Delta\tilde{\dot{E}}$) is a scalar multiple (k) of net savings ($\dot{N}^*$), assumed to be spent at the energy intensity of the economy (I_E).

$$\Delta\tilde{\dot{E}} = k\dot{N}^* I_E \tag{71}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}} = \frac{k\dot{N}^* I_E}{\dot{S}_{dev}} \tag{72}$$

Define $Re_{macro} \equiv \frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}}$, such that

$$Re_{macro} = \frac{k\dot{N}^* I_E}{\dot{S}_{dev}} \; . \tag{32}$$

Financial analysis: N/A

B.3 Rebound expressions

All that remains is to determine expressions for each rebound effect. We begin with the device-level expected energy savings rate ($\dot{S}_{dev}$), which appears in the denominator of all rebound expressions.

B.3.1 Expected energy savings ($\dot{S}_{dev}$)

$\dot{S}_{dev}$ is the reduction of energy consumption rate by the device due to the EEU. No other effects are considered.

$$\dot{S}_{dev} \equiv \dot{E}_s^\circ - \dot{E}_s^* \tag{73}$$

The final energy consumption rates ($\dot{E}_s^\circ$ and $\dot{E}_s^*$) can be written as Eq. (5) in the forms $\dot{E}_s^\circ = \dot{q}_s^\circ/\eta^\circ$ and $\dot{E}_s^* = \dot{q}_s^*/\eta^*$.

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^*}{\eta^*} \tag{74}$$

With reference to Table B.1, we use $\dot{q}_s^* = \dot{q}_s^\circ$ and $\eta^* = \tilde{\eta}$ to obtain

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^\circ}{\tilde{\eta}} \; . \tag{75}$$

When the EEU increases efficiency such that $\tilde{\eta} > \eta^\circ$, expected energy savings grows ($\dot{S}_{dev} > 0$) as the rate of final energy consumption declines, as expected. As $\tilde{\eta} \to \infty$, all final energy consumption is eliminated ($\dot{E}_s^* \to 0$), and $\dot{S}_{dev} = \dot{q}_s^\circ/\eta^\circ = \dot{E}_s^\circ$. (Of course, $\tilde{\eta} \to \infty$ is impossible. See Paoli & Cullen (2020) for a recent discussion of upper limits to device efficiencies.)

After rearrangement and using $\dot{E}_s^\circ = \dot{q}_s^\circ/\eta^\circ$, we obtain a convenient form

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right)\frac{\eta^\circ}{\tilde{\eta}}\dot{E}_s^\circ \; . \tag{10}$$

B.3.2 Emplacement effect

The emplacement effect accounts for performance of the EEU only. No behavior changes occur. The direct emplacement effect of the EEU is device energy savings and energy cost savings. The indirect emplacement effects of the EEU produce changes in the embodied energy rate and the maintenance and disposal expenditure rates. By definition, the direct emplacement effect has no rebound. However, indirect emplacement effects may cause energy rebound. Both direct and indirect emplacement effects are discussed below.

Direct emplacement effect rebound expression (Re_{dempl}) As shown in Table B.3, the direct rebound from the emplacement effect is $Re_{dempl} \equiv 0$. This result is expected, because in the absence of embodied energy, maintenance and disposal cost, or behavioral changes, there is no takeback of energy savings at the upgraded device.

Indirect emplacement effect rebound expression (Re_{iempl}) Indirect emplacement rebound effects can occur at any point in the life cycle of an energy conversion device, from manufacturing and distribution to the use phase (maintenance), and finally to disposal. For simplicity, we group maintenance with disposal to form two distinct indirect emplacement rebound effects: (i) an embodied energy effect (Re_{emb}) and (ii) a maintenance and disposal effect (Re_{md}).

Embodied energy effect rebound expression (Re_{emb}) The first component of indirect emplacement effect rebound involves embodied energy. We define embodied energy consistent with the energy analysis literature to be the sum of all final energy consumed in the production of the energy conversion device. The EEU causes the embodied final energy of the device to change from $\dot{E}^{\circ}_{emb}$ to $\dot{E}^{*}_{emb}$.

Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. For simplicity, we spread all embodied energy over the lifetime of the device, an equal amount assigned to each period.

Thus, we allocate embodied energy over the life of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) without discounting to obtain embodied energy rates, such that $\dot{E}^{\circ}_{emb} = E^{\circ}_{emb}/t^{\circ}_{life}$ and $\dot{E}^{*}_{emb} = E^{*}_{emb}/t^{*}_{life}$. The change in embodied final energy due to the EEU (expressed as a rate) is given by $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$. After substitution and algebraic rearrangement, the change in embodied energy rate due to the EEU can be expressed as $[(E^{*}_{emb}/E^{\circ}_{emb})(t^{\circ}_{life}/t^{*}_{life}) - 1]\dot{E}^{\circ}_{emb}$, a term that represents energy savings taken back due to embodied energy effects. Thus, Eq. (3) can be employed to write embodied energy rebound as

$$Re_{emb} = \frac{\left(\frac{E^*_{emb}}{E^\circ_{emb}} \frac{t^\circ_{life}}{t^*_{life}} - 1\right) \dot{E}^\circ_{emb}}{\dot{S}_{dev}} \; . \tag{12}$$

Embodied energy rebound can be either positive or negative, depending on the sign of the term $(E^*_{emb}/E^\circ_{emb})(t^\circ_{life}/t^*_{life}) - 1$. Rising energy efficiency can be associated with increased device complexity and more embodied energy, such that $E^*_{emb} > E^\circ_{emb}$ and $Re_{emb} > 0$. However, if the upgraded device has longer life than the original device ($t^*_{life} > t^\circ_{life}$), $\dot{E}^*_{emb} - \dot{E}^\circ_{emb}$ can be negative, meaning that the upgraded device has a lower embodied energy rate than the original device.

Maintenance and disposal effect rebound expression (Re_{md}) In addition to embodied energy effects, indirect emplacement rebound can be associated with energy demanded by maintenance and disposal (md) expenditures. Like embodied energy, we spread disposal expenditures across the lifetime of the original and upgraded devices (t°_{life} and t^*_{life}, respectively) to form expenditure rates such that $\dot{C}^\circ_{md} = \dot{C}^\circ_m + C^\circ_d/t^\circ_{life}$ and $\dot{C}^*_{md} = \dot{C}^*_m + C^*_d/t^*_{life}$.

We assume, for simplicity, that md expenditures indicate energy consumption elsewhere in the economy at its energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in md expenditures is given by $\Delta\dot{C}^*_{md} I_E$. This term is an energy takeback rate, so maintenance and disposal rebound is given by

$$Re_{md} = \frac{\Delta\dot{C}^*_{md} I_E}{\dot{S}_{dev}} \; , \tag{76}$$

as shown in Table B.3. Slight rearrangement gives

$$Re_{md} = \frac{\left(\frac{\dot{C}^*_{md}}{\dot{C}^\circ_{md}} - 1\right) \dot{C}^\circ_{md} I_E}{\dot{S}_{dev}} \; . \tag{13}$$

Rebound from maintenance and disposal can be positive or negative, depending on the sign of the term $\dot{C}^*_{md}/\dot{C}^\circ_{md} - 1$.

B.3.3 Substitution effect

This section derives expressions for substitution effect rebound. Two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}).

Assuming that conditions after the emplacement effect ($*$) are known, both the rate of energy service consumption ($\hat{\dot{q}}_s$) and the rate of other goods consumption ($\hat{\dot{C}}_o$) must be determined as a result of the substitution effect (the $\wedge$ point).

The EEU's energy efficiency increase ($\tilde{\eta} > \eta^\circ$) causes the price of the energy service provided by the device to fall ($\tilde{p}_s < p_s^\circ$). The substitution effect quantifies the amount by which the device user, in response, increases the consumption rate of the energy service ($\hat{\dot{q}}_s > \dot{q}_s^*$) and decreases the consumption rate of other goods ($\hat{\dot{q}}_o < \dot{q}_o^*$).

The increase in consumption of the energy service substitutes for consumption of other goods in the economy, subject to a utility constraint. The reduction in spending on other goods in the economy is captured by indirect substitution rebound (Re_{isub}).

We begin by deriving an expression for direct and indirect substitution effect rebound (Re_{dsub} and Re_{isub}, respectively). Thereafter, we develop a constant price elasticity (CPE) utility model and a constant elasticity of substitution (CES) utility model for determining the post-substitution point ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$).

Direct substitution effect rebound expression Direct substitution effect rebound (Re_{dsub}) is given by

$$Re_{dsub} = \frac{\Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} = \frac{\hat{\dot{E}}_s - \dot{E}_s^*}{\dot{S}_{dev}} \ . \tag{15}$$

Substituting the typical relationship of Eq. (5) in the form $\dot{E}_s = \dot{q}_s/\eta$ gives

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\tilde{\eta}} - \frac{\dot{q}_s^*}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{77}$$

Rearranging produces

$$Re_{dsub} = \frac{\left(\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - \frac{\dot{q}_s^*}{\dot{q}_s^\circ}\right)\frac{\dot{q}_s^\circ}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{78}$$

Recognizing that the rate of energy service consumption ($\dot{q}_s$) is unchanged across the emplacement effect leads to $\dot{q}_s^*/\dot{q}_s^\circ = 1$. Furthermore, $\dot{q}_s^\circ/\tilde{\eta} = (\dot{q}_s^\circ/\eta^\circ)(\eta^\circ/\tilde{\eta}) = \dot{E}_s^\circ(\eta^\circ/\tilde{\eta})$, such that

$$Re_{dsub} = \left(\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1\right)\frac{\dot{E}_s^\circ \frac{\eta^\circ}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{79}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{dsub} = \frac{\frac{\dot{\hat{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\cancel{\dot{E}_s^\circ} \frac{\cancel{\eta^\circ}}{\cancel{\tilde{\eta}}}}{\frac{\cancel{\eta^\circ}}{\cancel{\tilde{\eta}}} \cancel{\dot{E}_s^\circ}} \right) . \tag{80}$$

Canceling terms yields

$$Re_{dsub} = \frac{\frac{\dot{\hat{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} . \tag{16}$$

Eq. (16) is the basis for developing expressions for Re_{dsub} under both the CPE and the CES utility models.

Indirect substitution effect rebound expression Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \dot{\hat{C}}_o I_E}{\dot{S}_{dev}} = \frac{(\dot{\hat{C}}_o - \dot{C}_o^*) I_E}{\dot{S}_{dev}} . \tag{17}$$

Rearranging gives

$$Re_{isub} = \frac{\left(\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} - \frac{\dot{C}_o^*}{\dot{C}_o^\circ} \right) \dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{81}$$

Recognizing that expenditures on other goods are constant across the emplacement effect gives $\dot{C}_o^* / \dot{C}_o^\circ = 1$ and

$$Re_{isub} = \left(\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} - 1 \right) \frac{\dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{82}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{isub} = \frac{\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, \frac{\tilde{\eta}}{\eta^\circ} \, \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} . \tag{18}$$

Eq. (18) is the basis for developing expressions for Re_{isub} under both the CPE and the CES utility models.

Determining the post-substitution effect conditions requires reference to a consumer utility model. We first show the CPE utility model, often used in the literature. Second, we use a constant elasticity of substitution (CES) utility model. The CES model is used for all calculations and graphs in this paper.

Constant price elasticity (CPE) utility model In the literature, a constant price elasticity (CPE) utility model is often used (Borenstein, 2015, p. 17, footnote 43). However, the CPE model does not produce precisely utility-preserving preferences, thus we do not recommend its use. We discuss the CPE utility model here for completeness only.

In the CPE utility model, the relationship between energy service price and energy service consumption rate is given by the compensated own price elasticity of energy service demand ($\varepsilon_{\dot{q}_s,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\varepsilon_{\dot{q}_s,p_s,c}} . \tag{83}$$

Note that the compensated own price elasticity of energy service demand ($\varepsilon_{\dot{q}_s,p_s,c}$) is assumed constant along an indifference curve in the CPE utility model. A negative value for the compensated own price elasticity of energy service demand is expected ($\varepsilon_{\dot{q}_s,p_s,c} < 0$), such that when the energy service price decreases ($\tilde{p}_s < p_s^\circ$), the rate of energy service consumption increases ($\hat{\dot{q}}_s > \dot{q}_s^*$).

Substituting Eq. (6) in the form $p_s^\circ = p_E^\circ/\eta^\circ$ and $\tilde{p}_s = p_E^\circ/\tilde{\eta}$ and noting that $\dot{q}_s^\circ = \dot{q}_s^*$ gives

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_s,p_s,c}} . \tag{84}$$

Again, note that the compensated own price elasticity of energy service demand is negative ($\varepsilon_{\dot{q}_s,p_s,c} < 0$), so that as energy service efficiency increases ($\tilde{\eta} > \eta^\circ$), the energy service consumption rate increases ($\hat{\dot{q}}_s > \dot{q}_s^* = \dot{q}_s^\circ$).

Substituting Eq. (84) into Eq. (16) yields the CPE model's expression for direct substitution rebound.

$$Re_{dsub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_s,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{85}$$

such that, e.g. $\varepsilon_{\dot{q}_s,p_s,c} = -0.2$ and $\tilde{\eta}/\eta^\circ = 2$ yields $Re_{dsub} = 0.15$.

As long as $\varepsilon_{\dot{q}_s,p_s,c} \in (-1, 0)$, the CPE utility model indicates that direct substitution rebound will be below 1. I.e., the direct substitution effect alone will not cause backfire.

To quantify the substitution effect on other purchases in the CPE utility model, we use another elasticity, the compensated cross price elasticity of other goods demand ($\varepsilon_{\dot{q}_o,p_s,c}$), such that

$$\frac{\dot{\hat{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\varepsilon_{\dot{q}_o,p_s,c}} . \tag{86}$$

For substitution to take place, the compensated cross price elasticity of other goods demand must be positive ($\varepsilon_{\dot{q}_o,p_s,c} > 0$). Thus, an energy service price decrease ($\tilde{p}_s < p_s^\circ$) implies a reduction in the rate of consumption of other goods ($\dot{\hat{q}}_o < \dot{q}_o^*$).

The energy service price is inversely proportional to efficiency, yielding

$$\frac{\dot{\hat{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_o,p_s,c}} . \tag{87}$$

Assuming that the average price is unchanged across the substitution effect such that $\hat{p}_o = \dot{p}_o^* = p_o^\circ$ (Appendix E), and noting that $\dot{q}_s^* = \dot{q}_s^\circ$ and $\dot{C}_o^* = \dot{C}_o^\circ$, we can write

$$\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} = \frac{\dot{\hat{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_o,p_s,c}} . \tag{88}$$

Note that Eq. (88) can be used to determine the rate of expenditures on other goods in the economy ($\dot{\hat{C}}_o$) by

$$\dot{\hat{C}}_o = \dot{C}_o^\circ \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_o,p_s,c}} . \tag{89}$$

Substituting Eq. (89) into Eq. (18) gives the expression for indirect substitution rebound for the CPE utility model.

$$Re_{isub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_o,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, \frac{\tilde{\eta}}{\eta^\circ} \, \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{90}$$

Because the compensated cross price elasticity of other goods consumption is positive ($\varepsilon_{\dot{q}_o,p_s,c} > 0$) and the energy service efficiency ratio is greater than 1 ($\tilde{\eta} > \eta^\circ$), indirect substitution rebound will be negative always ($Re_{isub} < 0$), as expected. Negative rebound indicates that indirect substitution effects reduce the energy takeback rate by direct substitution effects.

CES utility model The CPE utility model assumes that the compensated own price elasticity of energy service demand ($\varepsilon_{\dot{q}_s,p_s,c}$) and the compensated cross price elasticity of other goods demand ($\varepsilon_{\dot{q}_o,p_s,c}$) are constant along an indifference curve. These assumptions hold only for infinitesimally small energy service price changes ($\Delta p_s^* \equiv p_s^* - p_s^\circ \approx 0$). They also provide reasonable approximations for a 1–2% change. However, in the case of an energy efficiency upgrade (EEU), the energy service price change is neither infinitesimal nor confined to single-digit percentages. Rather, Δp_s^* is finite and may be very large in percentage terms.

To determine the new consumption bundle after the substitution effect ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$) and, ultimately, to quantify the direct and indirect substitution rebound effects (Re_{dsub} and Re_{isub}) exactly, we remove the restriction that energy service price elasticities ($\varepsilon_{\dot{q}_s,p_s,c}$ and $\varepsilon_{\dot{q}_o,p_s,c}$) must be constant along an indifference curve (as in the CPE utility model). Instead, we require constancy of only the elasticity of substitution (σ) between the consumption rate of the energy service ($\dot{q}_s$) and the expenditure rate for other goods ($\dot{C}_o$) across the substitution effect. Thus, we employ a CES utility model in our framework. Fig. 4 in Part II (especially segments $*$—c and c—$\wedge$) illustrates features of the CES utility model for determining the new consumption bundle.

Two equations are helpful for this analysis. First, the slope at any point on indifference curve (the i°—i° curve in Fig. 4 of Part II) is given by Eq. (119) with $\dot{u}/\dot{u}^\circ = 1$ and the share parameter (a) replaced by $f_{\dot{C}_s}^\circ$, as discussed in Appendix C.

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \times \left[\left(\frac{1}{1-f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^{\rho}\right]^{(1-\rho)/\rho} . \tag{91}$$

Second, the equation of the pre-substitution-effect expenditure line ($*$—$*$ in Fig. 4 of Part II) is

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right) + \frac{1}{\dot{C}_o^\circ}(\dot{M} - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G}) . \tag{92}$$

To find the rate of energy service consumption after the substitution effect ($\hat{\dot{q}}_s$), we set the slope of the expenditure line (Eq. (92) and line $*$—$*$ in Fig. 4 of Part II) equal to the slope of the indifference curve (i°—i° in Fig. 4 of Part II) at the original utility rate of $\dot{u}/\dot{u}^\circ = 1$ (Eq. (91)).

$$-\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} \left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \left[\left(\frac{1}{1 - f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^{\rho}\right]^{(1-\rho)/\rho} \tag{93}$$

Solving for $\dot{q}_s/\dot{q}_s^\circ$ gives $\hat{\dot{q}}_s/\dot{q}_s^\circ$ as

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\rho/(1-\rho)} \right\}^{-1/\rho} . \tag{19}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an estimate for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\rho/(1-\rho)} \right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

The rate of other goods consumption after the substitution effect ($\hat{\dot{C}}_o$) can be found by substituting Eq. (19) and $\dot{u}/\dot{u}^\circ = 1$ into the functional form of the CES utility model (Eq. (118)) to obtain

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(\left(\frac{1}{1 - f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right) \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\frac{\rho}{1-\rho}} \right\}^{-1}\right)^{1/\rho} . \tag{94}$$

Simplifying gives

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\rho/(\rho-1)} - 1 \right\}\right)^{-1/\rho} . \tag{20}$$

Eq. (20) can be substituted into Eq. (18) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\rho/(\rho-1)} - 1 \right\}\right)^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{22}$$

B.3.4 Income effect

Rebound from the income effect rebound quantifies the rate of additional energy demand that arises because the user of the energy conversion device spends net savings from the EEU. The income rate of the device user is $\dot{M}^\circ$, which remains unchanged across the rebound effects, such that $\dot{M}^\circ = \dot{M}^* = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Freed cash from the EEU is given by Eq. (49) as $\dot{G} = p_E \dot{S}_{dev}$. In combination, the emplacement effect and the substitution effect leave the device user with *net* savings ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (59). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables B.3 and B.4.

In this framework, all net savings ($\hat{\dot{N}}$) are spent on either (i) additional energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) or (ii) additional other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$). The income elasticity of energy service demand and the income elasticity of other goods demand ($\varepsilon_{\dot{q}_s,\dot{M}}$ and $\varepsilon_{\dot{q}_o,\dot{M}}$, respectively) quantify the income preferences of the device user according to the following expressions:

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_s,\dot{M}}} \tag{23}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} , \tag{27}$$

where effective income ($\hat{\dot{M}}'$) is

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} . \tag{24}$$

Homotheticity means that $\varepsilon_{\dot{q}_s,\dot{M}} = 1$ and $\varepsilon_{\dot{q}_o,\dot{M}} = 1$.

The budget constraint across the income effect (Eq. (67)) ensures that all net savings available after the substitution effect ($\hat{\dot{N}}$) is re-spent across the income effect, such that $\bar{\dot{N}} = 0$. Appendix D proves that the income preference equations (Eqs. (23) and (27)) satsify the budget constraint (Eq. (67)).

The purpose of this section is derivation of expressions for (i) direct income rebound (Re_{dinc}) arising from increased consumption of the energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) and (ii) indirect income rebound (Re_{iinc}) arising from increased consumption of other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$).

But first, we derive an expression for device energy consumption rate prior to the income effect ($\hat{\dot{E}}_s$). This expression will be helpful later.

Derivation of expression for $\dot{\hat{E}}_s$ An expression for $\dot{\hat{E}}_s$ that will be helpful later begins with

$$\dot{\hat{E}}_s = \left(\frac{\dot{\hat{E}}_s}{\dot{E}_s^*} \right) \left(\frac{\dot{E}_s^*}{\dot{E}_s^\circ} \right) \dot{E}_s^\circ \ . \tag{95}$$

Substituting Eq. (5) and noting efficiency (η) equalities from Table B.1 gives

$$\dot{\hat{E}}_s = \left(\frac{\dot{\hat{q}}_s / \cancel{\tilde{\eta}}}{\dot{q}_s^* / \cancel{\tilde{\eta}}} \right) \left(\frac{\dot{q}_s^* / \tilde{\eta}}{\dot{q}_s^\circ / \eta^\circ} \right) \dot{E}_s^\circ \ . \tag{96}$$

Canceling terms yields

$$\dot{\hat{E}}_s = \left(\frac{\dot{\hat{q}}_s}{\dot{q}_s^*} \right) \left(\frac{\cancel{\dot{q}_s^*}}{\cancel{\dot{q}_s^\circ}} \right) \left(\frac{\eta^\circ}{\tilde{\eta}} \right) \dot{E}_s^\circ \ . \tag{97}$$

Noting energy service consumption rate equalities from Table B.1 ($\dot{q}_s^* = \dot{q}_s^\circ$) gives

$$\dot{\hat{E}}_s = \frac{\dot{\hat{q}}_s}{\dot{q}_s^*} \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ \ . \tag{98}$$

The next step is to develop an expression for Re_{dinc} using the income preference for energy service consumption.

Derivation of expression for Re_{dinc} As shown in Table B.5, direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \dot{\bar{E}}_s}{\dot{S}_{dev}} \ . \tag{25}$$

Expanding the difference and rearranging gives

$$Re_{dinc} = \frac{\dot{\bar{E}}_s - \dot{\hat{E}}_s}{\dot{S}_{dev}} \ , \tag{99}$$

and

$$Re_{dinc} = \frac{\left(\frac{\dot{\bar{E}}_s}{\dot{\hat{E}}_s} - 1 \right) \dot{\hat{E}}_s}{\dot{S}_{dev}} \ . \tag{100}$$

Substituting Eq. (5) as $\dot{\bar{E}}_s = \frac{\dot{\bar{q}}_s}{\tilde{\eta}}$ and $\dot{\hat{E}}_s = \frac{\dot{\hat{q}}_s}{\tilde{\eta}}$ gives

$$Re_{dinc} = \frac{\left(\frac{\dot{\bar{q}}_s / \cancel{\tilde{\eta}}}{\dot{\hat{q}}_s / \cancel{\tilde{\eta}}} - 1 \right) \dot{\hat{E}}_s}{\dot{S}_{dev}} \ . \tag{101}$$

Eliminating terms and substituting Eq. (10) for $\dot{S}_{dev}$ and Eq. (23) for $\bar{\dot{q}}_s/\hat{\dot{q}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_s,\dot{M}}} - 1\right]\hat{\dot{E}}_s}{\left(\frac{\tilde{\eta}}{\eta^{\circ}} - 1\right)\frac{\eta^{\circ}}{\tilde{\eta}}\dot{E}_s^{\circ}} \; . \tag{102}$$

Substituting Eq. (98) for $\hat{\dot{E}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_s,\dot{M}}} - 1\right]\frac{\hat{\dot{q}}_s}{\dot{q}_s^{*}}\cancel{\frac{\eta^{\circ}}{\tilde{\eta}}}\cancel{\dot{E}_s^{\circ}}}{\left(\frac{\tilde{\eta}}{\eta^{\circ}} - 1\right)\cancel{\frac{\eta^{\circ}}{\tilde{\eta}}}\cancel{\dot{E}_s^{\circ}}} \; . \tag{103}$$

Eliminating terms, recognizing that $\dot{q}_s^{\circ} = \dot{q}_s^{*}$, and substituting Eq. (19), which assumes the CES utility model, gives

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^{\circ}} - 1}\left\{f_{\dot{C}_s}^{\circ} + (1 - f_{\dot{C}_s}^{\circ})\left[\left(\frac{1 - f_{\dot{C}_s}^{\circ}}{f_{\dot{C}_s}^{\circ}}\right)\frac{\tilde{p}_s\dot{q}_s^{\circ}}{\dot{C}_o^{\circ}}\right]^{\rho/(1-\rho)}\right\}^{-1/\rho} \; . \tag{26}$$

If there is no net savings ($\hat{\dot{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected.

The next step is to develop an expression for Re_{iinc} using the income preference for other goods consumption.

Derivation of expression for Re_{iinc} As shown in Table B.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}} \; . \tag{29}$$

Expanding the difference and rearranging gives

$$Re_{iinc} = \frac{(\bar{\dot{C}}_o - \hat{\dot{C}}_o)I_E}{\dot{S}_{dev}} \; , \tag{104}$$

and

$$Re_{iinc} = \frac{\left(\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} - 1\right)\hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \; . \tag{105}$$

Substituting $\bar{\dot{C}}_o = p_o\bar{\dot{q}}_o$ and $\hat{\dot{C}}_o = p_o\hat{\dot{q}}_o$ and cancelling terms gives

$$Re_{iinc} = \frac{\left(\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} - 1\right)\hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \ . \tag{106}$$

Substituting the income preference equation for other goods consumption (Eq. (27) for $\bar{\dot{q}}_o/\hat{\dot{q}}_o$ and Eq. (10) for $\dot{S}_{dev}$ yields

$$Re_{iinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} - 1\right]\hat{\dot{C}}_o I_E}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right)\frac{\eta^\circ}{\tilde{\eta}}\dot{E}_s^\circ} \ . \tag{107}$$

Sutstituting $(\hat{\dot{C}}_o/\dot{C}_o^\circ)\dot{C}_o^\circ$ for $\hat{\dot{C}}_o$, recognizing that $\dot{C}_o^* = \dot{C}_o^\circ$, and simplifying gives

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\left(\frac{\tilde{\eta}}{\eta^\circ}\right)\frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ}\left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ}\right) \ . \tag{108}$$

Substituting Eq. (20) for $\hat{\dot{C}}_o/\dot{C}_o^\circ$, thereby assuming the CES utility model, gives the final form of the indirect income rebound expression:

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\left(\frac{\tilde{\eta}}{\eta^\circ}\right)\frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ}\left(1 + f_{\dot{C}_s}^\circ\left\{\left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\rho/(\rho-1)} - 1\right\}\right)^{-1/\rho} \ . \tag{30}$$

If there is no net savings ($\hat{\dot{N}} = 0$), indirect income effect rebound is zero ($Re_{iinc} = 0$), as expected.

B.3.5 Macro effect

Macro rebound (Re_{macro}) is given by Eq. (32). Substituting Eq. (48) for net savings ($\dot{N}^*$) gives

$$Re_{macro} = \frac{k(p_E\dot{S}_{dev} - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^*)I_E}{\dot{S}_{dev}} \ . \tag{109}$$

Separating terms gives

$$Re_{macro} = \frac{kp_E\cancel{\dot{S}_{dev}}I_E}{\cancel{\dot{S}_{dev}}} - \frac{k\Delta\dot{C}_{cap}^* I_E}{\dot{S}_{dev}} - \frac{k\Delta\dot{C}_{md}^* I_E}{\dot{S}_{dev}} \ . \tag{110}$$

Canceling terms, substituting Eq. (76) to obtain Re_{md}, and defining Re_{cap} as

$$Re_{cap} \equiv \frac{\Delta\dot{C}_{cap}^* I_E}{\dot{S}_{dev}} \tag{111}$$

gives

$$Re_{macro} = k(p_E I_E - Re_{cap} - Re_{md}) \ . \tag{33}$$

B.3.6 Rebound sum

The sum of the four rebound effects is

$$Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro} \ . \tag{112}$$

Substituting Eqs. (44), (56), and (65) gives

$$\begin{aligned} Re_{tot} = Re_{emb} &+ Re_{md} && \text{emplacement effect} \\ &+ Re_{dsub} + Re_{isub} && \text{substitution effect} \\ &+ Re_{dinc} + Re_{iinc} && \text{income effect} \\ &+ Re_{macro} && \text{macro effect} \end{aligned} \tag{113}$$

Macro effect rebound (Re_{macro}, Eq. (33)) can be expressed in terms of other rebound effects. Substituting Eq. (33) gives

$$\begin{aligned} Re_{tot} = Re_{emb} &+ Re_{md} && \text{emplacement effect} \\ &+ Re_{dsub} + Re_{isub} && \text{substitution effect} \\ &+ Re_{dinc} + Re_{iinc} && \text{income effect} \\ &+ k p_E I_E - k Re_{cap} - k Re_{md} \ . && \text{macro effect} \end{aligned} \tag{114}$$

Rearranging distributes macro effect terms to emplacement and substitution effect terms. This last rearrangement gives the final expression for total rebound.

$$Re_{tot} = \ Re_{emb} + k(p_E I_E - Re_{cap}) + (1-k)Re_{md} + Re_{dsub} + Re_{isub} + Re_{dinc} + Re_{iinc} \tag{34}$$

Eq. (34) shows that determining seven rebound values,

- Re_{emb} (Eq. (12)),

- Re_{cap} (Eq. (111)),
- Re_{md} (Eq. (13)),
- Re_{dsub} (Eq. (21)),
- Re_{isub} (Eq. (22)),
- Re_{dinc} (Eq. (26)), and
- Re_{iinc} (Eq. (30)),

is sufficient to calculate total rebound, provided that the macro factor (k), the price of energy (p_E), and the energy intensity of the economy (I_E) are known.

C Utility models and elasticities

As discussed in Section 2.5.2 and Appendix B.3.3, the substitution effect requires a model for device user behavior. Behavior is typically represented by a model of utility that is maximized with arguments of consuming the energy service ($\dot{q}_s$) and other goods and services ($\dot{q}_o$) and subject to income and price constraints. In this appendix, we describe two utility models. The first utility model is a constant price elasticity (CPE) utility model, which allows an easy calculation of price-demand relationships as Appendix B.3.3 illustrates. It gives a good approximation of the behavioral response for very small changes in energy efficiency and energy service price, such that $\Delta\eta^* \approx 0$ and $\Delta p_s^* \approx 0$. The CPE utility model is discussed for continuity with the literature only. (See, for example, Borenstein (2015, p. 17, footnote 43).)

We note that larger and non-marginal efficiency gains cause greater rebound (measured in joules) than small and marginal efficiency gains. Thus, any rebound analysis framework needs to accommodate large, non-marginal efficiency changes. Since price elasticities are point-measures in analytical utility models, a version of the framework amenable to empirical applications should account for the changing price elasticity along an indifference curve.[22] The second utility model

[22] In principle, calculated arc elasticities could describe the relationship between price and quantity changes for any EEU by representing the percentage price and quantity changes between any two known consumption bundles (Allen & Lerner, 1934). However, we do not know the new consumption bundle and instead determine it with the CES utility function whose price elasticities vary along the indifference curve.

discussed in this appendix is the Constant Elasticity of Substitution (CES) utility model which does, in fact, accommodate large, non-marginal energy efficiency and energy service price changes. The CES utility model underlies the substitution effect in this framework. (See Section 2.5.2.) Furthermore, the CES utility model is needed for the example energy efficiency upgrades (EEUs) in Part II, which have large, non-marginal percentage increases in energy efficiency.

In addition to the substitution effect, the income effect requires income elasticities to describe consumer behavior. Elasticities for both the substitution effect and the income effect are discussed below, after we lay out the CPE and CES utility models.

Before proceeding with the utility models and elasticities, we note briefly that the rate of other goods consumption ($\dot{q}_o$) is not known independently from the prices of other goods (p_o). With the assumption that the prices of other goods do not change across rebound effects (i.e., p_o is exogenous), the ratio of other goods consumption is equal to the ratio of other goods spending, such that

$$\frac{\dot{q}_o}{\dot{q}_o^\circ} = \frac{\dot{C}_o/\cancel{p_o}}{\dot{C}_o^\circ/\cancel{p_o}} = \frac{\dot{C}_o}{\dot{C}_o^\circ} \tag{115}$$

at all rebound stages. (See Appendix E for details.)

C.1 Utility models for the substitution effect

A utility model gives the ratio of energy service consumption rate and other goods consumption rates across the substitution effect ($\hat{\dot{q}}_s/\dot{q}_s^*$ and $\hat{\dot{q}}_o/\dot{q}_o^*$, respectively). In so doing, utility models quantify the decrease in other goods consumption ($\hat{\dot{q}}_o/\dot{q}_o^* < 1$) caused by the increase of energy service consumption ($\hat{\dot{q}}_s/\dot{q}_s^* > 1$) resulting from the decrease of the energy service price ($p_s^* < p_s^\circ$) under the constraint of constant device user utility. Across the substitution effect, the utility increase of the larger energy service consumption rate must be exactly offset by the utility decrease of the smaller other goods consumption rate.

C.1.1 Constant price elasticity (CPE) utility model

The constant price elasticity (CPE) utility model is given by Eqs. (84) and (88). The equations for the approximate utility model are repeated here for convenience.

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_s,p_s,c}} \tag{84}$$

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\varepsilon_{\dot{q}_o,p_s,c}} \tag{88}$$

C.1.2 CES utility model

The CES utility model is given by Eq. (14). Here, its derivation is shown. Throughout the derivation, references to Part II are provided for visual representations of several important concepts. Those concepts (equilibrium tangency requirements, e.g.) are best visualized in rebound planes that are introduced in Section 2.2 of Part II.

The CES utility model is normalized by (indexed to) conditions prior to emplacement:

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^\rho + (1-a)\left(\frac{\dot{q}_o}{\dot{q}_o^\circ}\right)^\rho\right]^{(1/\rho)} , \tag{116}$$

where $\rho \equiv (\sigma - 1)/\sigma$, a is a share parameter (determined below), and σ is the elasticity of substitution between the normalized consumption rate of the energy service ($\dot{q}_s$) and the normalized consumption rate of other goods ($\dot{q}_o$).[23] By definition, σ is assumed constant such that $\sigma^\circ = \sigma^* = \hat{\sigma} = \bar{\sigma} = \tilde{\sigma} = \sigma$.

With the assumption of exogenous other goods prices in Eq. (115), we find

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^\rho + (1-a)\left(\frac{\dot{C}_o}{\dot{C}_o^\circ}\right)^\rho\right]^{(1/\rho)} . \tag{117}$$

Eq. (117) is the functional form of the CES utility model, whose share parameter (a) is yet to be determined. The correct expression for the share parameter (a) is found from the equilibrium requirement, namely that the expenditure curve is tangent to the indifference curve in the $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ plane (the "consumption plane" in Part II) prior to the EEU. For example, the ∘—∘ line is tangent to constant-utility indifference curve i°—i° at point ∘ in Fig. 4 of Part II.

To find the slope at any point on the indifference curve (i°—i° in Fig. 4 of Part II), Eq. (117) can be rearranged to give the normalized consumption rate of other goods ($\dot{C}_o/\dot{C}_o^\circ$) as a function of the normalized consumption rate of the energy service ($\dot{q}_s/\dot{q}_s^\circ$) and the normalized utility rate ($\dot{u}/\dot{u}^\circ$):

[23] In the international trade literature, where the CES utility model is often used, the elasticity of substitution is also called the Armington elasticity (Feenstra et al., 2018).

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = \left[\frac{1}{1-a}\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \frac{a}{1-a}\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1/\rho)} , \tag{118}$$

a form convenient for drawing constant utility rate ($\dot{u}/\dot{u}^\circ$) indifference curves on a graph of $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ (the consumption plane of Fig. 4 in Part II). In the consumption plane, the slope of an indifference curve is found by taking the first partial derivative of $\dot{C}_o/\dot{C}_o^\circ$ with respect to $\dot{q}_s/\dot{q}_s^\circ$, starting from Eq. (118) and using the chain rule repeatedly. The result is

$$\begin{aligned}\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = &- \frac{a}{1-a}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \\ &\times \left[\left(\frac{1}{1-a}\right)\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \left(\frac{a}{1-a}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho} .\end{aligned} \tag{119}$$

The budget constraint is the starting point for finding the slope of an expenditure line in the consumption plane. (Example expenditure lines include the ○—○, ∗—∗, ∧—∧, and − — − lines in Fig. 4 of Part II.) The following equation is a generic version of Eqs. (37), (39), (52), and (61) with $p_s\dot{q}_s$ substituted for $p_E\dot{E}_s$.

$$\dot{M} = p_s\dot{q}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o + \dot{N} \tag{120}$$

In a manner similar to derivations in Appendix B.3.1 of Part II, we solve for $\dot{C}_o$ and judiciously multiply by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ to obtain

$$\frac{\dot{C}_o}{\dot{C}_o^\circ}\dot{C}_o^\circ = -p_s\frac{\dot{q}_s}{\dot{q}_s^\circ}\dot{q}_s^\circ + \dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N} . \tag{121}$$

Solving for $\dot{C}_o/\dot{C}_o^\circ$ and rearranging gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right) + \frac{1}{\dot{C}_o^\circ}(\dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N}) , \tag{122}$$

from which the slope of the indifference curve in the consumption plane is taken by inspection to be

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ} . \tag{123}$$

At any equilibrium point, the expenditure line must be tangent to its indifference curve, or, as economists say, the ratio of prices must be equal to the marginal rate of substitution. Applying the tangency requirement before emplacement enables solving for the correct expression for a, the share parameter in the CES utility model. Setting the slope of the expenditure line (Eq. (123)) equal to the slope of the indifference curve (Eq. (119)) gives

$$
\begin{aligned}
-\frac{p_s \dot{q}_s^\circ}{\dot{C}_o^\circ} = &-\frac{a}{1-a}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \\
&\times \left[\left(\frac{1}{1-a}\right)\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^{\rho} - \left(\frac{a}{1-a}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^{\rho}\right]^{(1-\rho)/\rho} .
\end{aligned} \tag{124}
$$

For the equilibrium point prior to emplacement (point $\circ$ in Fig. 4 of Part II), $\dot{q}_s/\dot{q}_s^\circ = 1$, $\dot{u}/\dot{u}^\circ = 1$, and $p_s = p_s^\circ$, which reduces Eq. (124) to

$$
-\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{a}{1-a}(1)^{(\rho-1)}\left[\left(\frac{1}{1-a}\right)(1)^{\rho} - \left(\frac{a}{1-a}\right)(1)^{\rho}\right]^{(1-\rho)/\rho} . \tag{125}
$$

Simplifying gives

$$
\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ} = \frac{a}{1-a} . \tag{126}
$$

Recognizing that $p_s^\circ \dot{q}_s^\circ = \dot{C}_s^\circ$ and solving for a gives

$$
a = \frac{\dot{C}_s^\circ}{\dot{C}_s^\circ + \dot{C}_o^\circ} , \tag{127}
$$

which is called $f_{\dot{C}_s}^\circ$, the share of energy service expenditure ($\dot{C}_s^\circ$) relative to the sum of energy service and other goods expenditures ($\dot{C}_s^\circ + \dot{C}_o^\circ$) before emplacement of the EEU. Thus, the CES utility equation (Eq. (117)) becomes

$$
\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{\rho} + (1 - f_{\dot{C}_s}^\circ)\left(\frac{\dot{C}_o}{\dot{C}_o^\circ}\right)^{\rho}\right]^{(1/\rho)} , \tag{14}
$$

with

$$
f_{\dot{C}_s}^\circ \equiv \frac{\dot{C}_s^\circ}{\dot{C}_s^\circ + \dot{C}_o^\circ} . \tag{128}
$$

C.2 Elasticities for the substitution effect

Calculating the change in consumer preferences across the substitution effect requires a utility model, two of which are described in the section above: the constant price elasticity (CPE) model and the constant elasticity of substitution (CES) model. Within those utility models, price (ε) and substitution (σ) elasticities describe consumer preferences.

Own and cross price elasticities describe consumer preferences for consumption of the energy service ($\dot{q}_s$) and other goods ($\dot{q}_o$) as the price of the energy service (p_s) changes due to the EEU. Thus, there are four price elasticities: (i) the uncompensated own price elasticity of energy service consumption ($\varepsilon_{\dot{q}_s,p_s}$), (ii) the uncompensated cross price elasticity of other goods consumption ($\varepsilon_{\dot{q}_o,p_s}$), (iii) the compensated own price elasticity of energy service consumption ($\varepsilon_{\dot{q}_s,p_s,c}$), and (iv) the compensated cross price elasticity of other goods consumption ($\varepsilon_{\dot{q}_o,p_s,c}$).

The elasticity of substitution (σ) describes the willingness of consumers to substitute one good for another. In the context of rebound from an EEU, substitution is considered between consumption of the energy service ($\dot{q}_s$) and comsumption of the basket of other goods ($\dot{q}_o$).

C.2.1 Original, pre-EEU ($\circ$) elasticities

Economists use surveys, statistical data, and other means to estimate values for the uncompensated own price price elasticity of energy service consumption ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$) prior to the EEU. With $\varepsilon^{\circ}_{\dot{q}_s,p_s}$ in hand, calculation of all other elasticities is possible.

Elasticity of substitution (σ) For the constant price elasticity (CPE) utility model, there is no analytical expression for the elasticity of substitution (σ) and values are most likely taken from estimation, if they are obtained at all. As we show in Tables 12 and 13 of Part II, not all rebounds are typically calculated, so not all elasticities are needed.

For the constant elasticity of substitution (CES) utility model, Gørtz (1977) shows that the elasticity of substitution prior to the EEU (σ°) can be computed by

$$\sigma^{\circ} = \frac{f^{\circ}_{\dot{C}_s} + \varepsilon^{\circ}_{\dot{q}_s,p_s}}{f^{\circ}_{\dot{C}_s} - 1} \, . \tag{129}$$

Thus, the original elasticity of substitution (σ°) can be determined from two pieces of readily available information: (i) the original uncompensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$) and (ii) the share of income spent on the energy service prior to the EEU ($f^{\circ}_{\dot{C}_s}$ from Eq. (128)). In the CES utility model, σ° is assumed invariant and given the undecorated symbol σ to indicate that it applies across all rebound effects.

For the rest of the pre-EEU elasticities ($\varepsilon^{\circ}_{\dot{q}_o,p_s}$, $\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$, and $\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$), there is no difference for the CPE utility model or the CES utility model.

Uncompensated cross price elasticity ($\varepsilon^{\circ}_{\dot{q}_o,p_s}$) From Hicks & Allen (1934), we note that the pre-EEU uncompensated cross price elasticity ($\varepsilon^{\circ}_{\dot{q}_o,p_s}$) can be expressed as

$$\varepsilon^{\circ}_{\dot{q}_o,p_s} = f^{\circ}_{\dot{C}_s}(\sigma - \varepsilon_{\dot{q}_o,\dot{M}}) \ . \tag{130}$$

Compensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$) An expression for the pre-EEU compensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$) can be derived using the Slutsky equation, whereby the uncompensated own price elasticity of the energy service ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$) is decomposed into the compensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$) and the income elasticity ($\varepsilon_{\dot{q}_s,\dot{M}}$) as follows:

$$\varepsilon^{\circ}_{\dot{q}_s,p_s} = \varepsilon^{\circ}_{\dot{q}_s,p_s,c} - f^{\circ}_{\dot{C}_s}\varepsilon_{\dot{q}_s,\dot{M}} \ , \tag{131}$$

where $f^{\circ}_{\dot{C}_s}$ is given by Eq. (128), and the income elasticity ($\varepsilon_{\dot{q}_s,\dot{M}}$) is given in Section C.3. Solving for the compensated price elasticity prior to the EEU ($\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$) gives

$$\varepsilon^{\circ}_{\dot{q}_s,p_s,c} = \varepsilon^{\circ}_{\dot{q}_s,p_s} + f^{\circ}_{\dot{C}_s}\varepsilon_{\dot{q}_s,\dot{M}} \ . \tag{132}$$

Compensated cross price elasticity ($\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$) The cross price version of the Slutsky equation is the starting point for deriving the pre-EEU compensated cross price elasticity ($\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$):

$$\varepsilon^{\circ}_{\dot{q}_o,p_s} = \varepsilon^{\circ}_{\dot{q}_o,p_s,c} - f^{\circ}_{\dot{C}_s}\varepsilon_{\dot{q}_o,\dot{M}} \ . \tag{133}$$

The income elasticity of other goods consumption ($\varepsilon_{\dot{q}_o,\dot{M}}$) is given in Section C.3. Solving for $\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$ gives

$$\varepsilon^{\circ}_{\dot{q}_o,p_s,c} = \varepsilon^{\circ}_{\dot{q}_o,p_s} + f^{\circ}_{\dot{C}_s}\varepsilon_{\dot{q}_o,\dot{M}} \ . \tag{134}$$

An alternative formulation can be derived by setting Eq. (130) equal to Eq. (133) to obtain

$$f^{\circ}_{\dot{C}_s}(\sigma - \varepsilon_{\dot{q}_o,\dot{M}}) = \varepsilon^{\circ}_{\dot{q}_o,p_s,c} - f^{\circ}_{\dot{C}_s}\varepsilon_{\dot{q}_o,\dot{M}} \ . \tag{135}$$

Solving for $\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$ gives

$$\varepsilon^{\circ}_{\dot{q}_o,p_s,c} = f^{\circ}_{\dot{C}_s}\sigma \; . \tag{136}$$

Substituting σ from Eq. (129) gives

$$\varepsilon^{\circ}_{\dot{q}_o,p_s,c} = \frac{f^{\circ}_{\dot{C}_s}(f^{\circ}_{\dot{C}_s} + \varepsilon^{\circ}_{\dot{q}_s,p_s})}{f^{\circ}_{\dot{C}_s} - 1} \; . \tag{137}$$

Assuming a known value for the original uncompensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$), all other pre-EEU elasticities can be calculated from Eqs. (129), (130), (132), and (134) or (137).

Note that the rebound framework in this paper uses the CES utility model and needs only the uncompensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$) and the derived elasticity of substitution (σ) to calculate rebound values. The other price elasticities ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$, $\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$, and $\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$) are not necessary for the model. However, they are helpful for elucidating results derived from the framework, a task left for Part II.

C.2.2 Post substitution effect ($\wedge$) elasticities

The stage after the substitution effect ($\wedge$) represents utility-maximizing behavior after the energy service price drop caused by the EEU and the compensating variation. Post-EEU, elasticities may be different from the original condition, because the consumption bundle has changed (due to a move along the indifference curve). This section derives expressions for elasticities at the $\wedge$ stage. Elasticities at the $\wedge$ stage are different for the CPE utility model and the CES utility model.

CPE utility model By definition, all price elasticities are assumed unchanged from their original values across the substitution effect in the constant price elasticity (CPE) utility model. Thus,

$$\hat{\varepsilon}_{\dot{q}_s,p_s} = \varepsilon^{\circ}_{\dot{q}_s,p_s} \; , \tag{138}$$

$$\hat{\varepsilon}_{\dot{q}_o,p_s} = \varepsilon^{\circ}_{\dot{q}_o,p_s} \; , \tag{139}$$

$$\hat{\varepsilon}_{\dot{q}_s,p_s,c} = \varepsilon^{\circ}_{\dot{q}_s,p_s,c} \; , \text{and} \tag{140}$$

$$\hat{\varepsilon}_{\dot{q}_o,p_s,c} = \varepsilon^{\circ}_{\dot{q}_o,p_s,c} \; . \tag{141}$$

Under the CPE approximation, the post-EEU elasticity of substitution will be different from its original value ($\hat{\sigma} \neq \sigma^{\circ}$). However, there is no analytical expression for σ and values are most likely taken from estimation, if they are found at all.

CES utility model The CES utility model is rather different to the CPE model with respect to the behavior of elasticities across the substitution effect. In the CES utility model, price elasticities (ε) are different after the substitution effect ($\wedge$) compared to the original ($\circ$).

Elasticity of substitution (σ) Be definition, the elasticity of substitution (σ) is constant across the substitution effect for the CES utility model. Thus,

$$\hat{\sigma} = \sigma^{\circ} \ . \tag{142}$$

Because the elasticity of substitution is unchanged, we refer to σ without decoration for the CES utility model. The constancy of σ means that the price elasticities (ε) will vary with the energy service price ($\tilde{p}_s$) across the substitution effect.

Compensated own price elasticity ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$) The compensated own price elasticity of energy service demand ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$) gives the percentage change of the consumption rate of the energy service ($\dot{q}_s$) across the substitution effect due to a unit percentage change in the energy service price ($\tilde{p}_s$) resulting from the EEU under the constraint that utility is unchanged ($\hat{\dot{u}} = \dot{u}^*$). In contrast to the CPE utility model above, the compensated own price elasticity of energy service demand ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$) is not constant in the CES utility model. Rather, $\hat{\varepsilon}_{\dot{q}_s,p_s,c}$ is a function of the post-EEU energy service price ($\tilde{p}_s$). The definition of $\hat{\varepsilon}_{\dot{q}_s,p_s,c}$ is

$$\hat{\varepsilon}_{\dot{q}_s,p_s,c} \equiv \frac{\tilde{p}_s}{\hat{\dot{q}}_s} \left. \frac{\partial \hat{\dot{q}}_s}{\partial \tilde{p}_s} \right|_{\dot{u} = \dot{u}^* = \hat{\dot{u}}} \ . \tag{143}$$

To find an expression for $\hat{\varepsilon}_{\dot{q}_s,p_s,c}$ for the CES utility function, we need to first find the partial derivative of the rate of energy service consumption ($\hat{\dot{q}}_s$) with respect to the post-EEU energy service price $\tilde{p}_s$ at constant utility ($\dot{u} = \dot{u}^* = \hat{\dot{u}}$) across the substitution effect. This derivation of an expression for $\hat{\varepsilon}_{\dot{q}_s,p_s,c}$ for the CES utility model commences with Eq. (19), which was derived for constant utility across the substitution effect.

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\rho/(1-\rho)} \right\}^{-1/\rho} \tag{19}$$

In Eq. (19), all terms on the right side except $\tilde{p}_s$ are constant for the purposes of the partial derivative. Finding the partial derivative of $\hat{\dot{q}}_s$ with respect to $\tilde{p}_s$ amounts to applying the chain rule repeatedly. To simplify the derivation, we can define the following constants

$$f \equiv f_{\dot{C}_s}^\circ \;, \tag{144}$$

$$g \equiv 1 - f_{\dot{C}_s}^\circ \;, \tag{145}$$

$$h \equiv \frac{\dot{q}_s^\circ}{\dot{C}_o^\circ} \;, \tag{146}$$

$$m_s \equiv \rho/(1-\rho) \;, \tag{147}$$

$$n \equiv -1/\rho \;, \text{ and} \tag{148}$$

$$z \equiv \frac{g}{f} h = \frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \frac{\dot{q}_s^\circ}{\dot{C}_o^\circ} \tag{149}$$

and rearrange slightly to obtain

$$\hat{\dot{q}}_s = \dot{q}_s^\circ \left[f + g \left(z \tilde{p}_s \right)^{m_s} \right]^n \;. \tag{150}$$

Taking the partial derivative of $\hat{\dot{q}}_s$ with respect to $\tilde{p}_s$, via repeated application of the chain rule, gives

$$\frac{\partial \hat{\dot{q}}_s}{\partial \tilde{p}_s} = \dot{q}_s^\circ m_s n g z^{m_s} \tilde{p}_s^{m_s - 1} \left\{ \left[f + g \left(z \tilde{p}_s \right)^{m_s} \right]^{n-1} \right\} \;. \tag{151}$$

Forming the elasticity via its definition (Eq. (143)) gives

$$\hat{\varepsilon}_{\dot{q}_s, p_s, c} \equiv \frac{\tilde{p}_s}{\hat{\dot{q}}_s} \left. \frac{\partial \hat{\dot{q}}_s}{\partial \tilde{p}_s} \right|_{\dot{u} = \dot{u}^* = \hat{\dot{u}}} = \frac{\tilde{p}_s}{\cancel{\dot{q}_s^\circ} \left[f + g \left(z \tilde{p}_s \right)^{m_s} \right]^n} \cancel{\dot{q}_s^\circ} m_s n g z^{m_s} \tilde{p}_s^{m_s - 1} \left\{ \left[f + g \left(z \tilde{p}_s \right)^{m_s} \right]^{n-1} \right\} \;. \tag{152}$$

Cancelling terms and combining $\tilde{p}_s$ and $\left[f + g \left(z \tilde{p}_s \right)^{m_s} \right]$ terms with different exponents gives

$$\hat{\varepsilon}_{\dot{q}_s, p_s, c} = \frac{m_s n g (z \tilde{p}_s)^{m_s}}{f + g (z \tilde{p}_s)^{m_s}} \;. \tag{153}$$

Back-substituting the constants and simplifying where possible yields

$$\hat{\varepsilon}_{\dot{q}_s,p_s,c} = -\frac{\frac{1}{1-\rho}\left(1-f^\circ_{\dot{C}_s}\right)\left[\frac{1-f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}}\frac{\tilde{p}_s\dot{q}^\circ_s}{\dot{C}^\circ_o}\right]^{\rho/(1-\rho)}}{f^\circ_{\dot{C}_s}+\left(1-f^\circ_{\dot{C}_s}\right)\left[\frac{1-f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}}\frac{\tilde{p}_s\dot{q}^\circ_s}{\dot{C}^\circ_o}\right]^{\rho/(1-\rho)}} \; . \tag{154}$$

Eq. (154) shows that the compensated energy service price elasticity of energy service consumption ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$) under the CES utility model is a function of the energy service price after the EEU ($\tilde{p}_s$). It is negative, as it should be, because all terms are positive, with ρ and $f^\circ_{\dot{C}_s}$ being bounded above by 1.

Of interest is how the elasticity changes as $\tilde{p}_s$ changes. Taking the derivative of 153 and simplifying gives

$$\frac{\partial\hat{\varepsilon}_{\dot{q}_s,p_s,c}}{\partial\tilde{p}_s} = \frac{m_s^2 n g (z\tilde{p}_s)^{m_s}}{\tilde{p}_s(f+g(z\tilde{p}_s)^{m_s})^2} \; . \tag{155}$$

All terms taken to their power are positive with the exception of n. For $\sigma < 1$, n is positive; for $\sigma > 1$, n is negative. Since we expect $\sigma < 1$ (otherwise we have backfire rebound conditions), the derivative is positive: the compensated own price elasticity becomes less negative as $\tilde{p}_s$ increases.[24] Since the share of income spent on the energy service declines for $\sigma < 1$, it is not immediately clear in which direction $\hat{\varepsilon}_{\dot{q}_s,p_s}$ moves according to equation 130. See Appendix C.6 of Part II for a graph of the sensitivity of price elasticities ($\hat{\varepsilon}$) to energy service price ($\tilde{p}_s$) for concrete examples.

Compensated cross price elasticity ($\hat{\varepsilon}_{\dot{q}_o,p_s,c}$) The compensated cross price elasticity of other goods demand ($\hat{\varepsilon}_{\dot{q}_o,p_s,c}$) gives the percentage change of the consumption rate of other goods ($\dot{q}_o$) across the substitution effect due to a unit percentage change in the energy service price ($\tilde{p}_s$) resulting from the EEU under the constraint that utility is unchanged ($\hat{\dot{u}} = \dot{u}^*$). To find the compensated cross price elasticity of other goods consumption ($\hat{\varepsilon}_{\dot{q}_o,p_s,c}$), we follow a similar procedure as for deriving the own price elasticity of energy service consumption ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$), with two differences being (i) the elasticity definition and (ii) the equation from which the partial derivative is derived.

The first difference is the definition of the compensated cross price elasticity of other goods consumption($\hat{\varepsilon}_{\dot{q}_o,p_s,c}$).

$$\hat{\varepsilon}_{\dot{q}_o,p_s,c} \equiv \frac{\tilde{p}_s}{\hat{\dot{q}}_o}\frac{\partial\hat{\dot{q}}_o}{\partial\tilde{p}_s}\bigg|_{\dot{u}=\dot{u}^*=\hat{\dot{u}}} \tag{156}$$

[24] For $\sigma = 1$, $m_s = 0$ and the derivative is zero: the Cobb-Douglas special case.

Again, we need to find the partial derivative of the rate of other goods consumption ($\dot{q}_o$) with respect to the energy service price ($\tilde{p}_s$) at constant utility ($\dot{u}^* = \hat{u}$) across the substitution effect. The second difference is the starting point for this derivation, Eq. (20) (instead of Eq. (19)).

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\rho/(\rho-1)} - 1 \right\} \right)^{-1/\rho} . \tag{20}$$

In Eq. (20), all terms on the right side except $\tilde{p}_s$ are constant for the purposes of the partial derivative. So finding the derivative amounts to applying the chain rule repeatedly. To simplify the derivation, we can define

$$m_o \equiv \rho/(\rho - 1) \;, \tag{157}$$

invoke the constancy of other prices ($p_o^\circ = \hat{p}_o$) from Appendix E, and rearrange slightly to obtain

$$\hat{\dot{q}}_o = \dot{q}_o^\circ \left\{1 + f\left[(z\tilde{p}_s)^{m_o} - 1\right]\right\}^n \;, \tag{158}$$

with f, n, and z being constants defined in the derivation of $\hat{\varepsilon}_{\dot{q}_s, p_s, c}$ above.

Taking the partial derivative of $\hat{\dot{q}}_o$ with respect to $\tilde{p}_s$, via repeated application of the chain rule, gives

$$\frac{\partial \hat{\dot{q}}_o}{\partial \tilde{p}_s} = \dot{q}_o^\circ m_o n f z^{m_o} \tilde{p}_s^{m_o - 1} \left\{1 + \left[f\left(z\tilde{p}_s\right)^{m_o} - 1\right]\right\}^{n-1} \;. \tag{159}$$

Forming the elasticity via its definition (Eq. (156)) gives

$$\begin{aligned} \hat{\varepsilon}_{\dot{q}_o, p_s, c} &\equiv \left. \frac{\tilde{p}_s}{\hat{\dot{q}}_o} \frac{\partial \hat{\dot{q}}_o}{\partial \tilde{p}_s} \right|_{\dot{u} = \dot{u}^* = \hat{u}} \\ &= \frac{\tilde{p}_s}{\cancel{\dot{q}_o^\circ} \left\{1 + f\left[(z\tilde{p}_s)^{m_o} - 1\right]\right\}^n} \cancel{\dot{q}_o^\circ} m_o n f z^{m_o} \tilde{p}_s^{m_o - 1} \left\{1 + f\left[(z\tilde{p}_s)^{m_o} - 1\right]\right\}^{n-1} \;. \end{aligned} \tag{160}$$

Cancelling terms and combining $\tilde{p}_s$ and $\left\{1 + f\left[(z\tilde{p}_s)^{m_o} - 1\right]\right\}$ terms with different exponents gives

$$\hat{\varepsilon}_{\dot{q}_o, p_s, c} = \frac{m_o n f (z\tilde{p}_s)^{m_o}}{1 + f\left[(z\tilde{p}_s)^{m_o} - 1\right]} \;. \tag{161}$$

Back-substituting the constants and simplifying where possible yields

$$\hat{\varepsilon}_{\dot{q}_o,p_s,c} = -\frac{\frac{1}{\rho-1} f^\circ_{\dot{C}_s} \left(\frac{1-f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \frac{\tilde{p}_s \dot{q}^\circ_s}{\dot{C}^\circ_o} \right)^{\rho/(\rho-1)}}{1 + f^\circ_{\dot{C}_s} \left[\left(\frac{1-f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \frac{\tilde{p}_s \dot{q}^\circ_s}{\dot{C}^\circ_o} \right)^{\rho/(\rho-1)} - 1 \right]} \ . \tag{162}$$

Eq. (162) shows that the compensated energy service price elasticity of other goods consumption ($\hat{\varepsilon}_{\dot{q}_o,p_s,c}$) under the CES utility model is a function of the energy service price after the EEU ($\tilde{p}_s$). It is positive, because all terms except $\frac{1}{\rho-1}$ are positive, with ρ and $f^\circ_{\dot{C}_s}$ being bounded above by 1.

Of interest is how the elasticity changes as $\tilde{p}_s$ changes. Taking the derivative of 161 and simplifying gives

$$\frac{\partial \hat{\varepsilon}_{\dot{q}_o,p_s,c}}{\partial \tilde{p}_s} = \frac{m_o^2 n f (z\tilde{p}_s)^{m_o}}{\tilde{p}_s (1 + f\left[(z\tilde{p}_s)^{m_o} - 1\right])^2} \ . \tag{163}$$

All terms taken to their power are positive with the exception of n, analogous to the derivative of the own price elasticity in equation 155. Thus, with $\sigma < 1$ and n positive, the compensated cross price elasticity becomes more positive as $\tilde{p}_s$ increases.

See Appendix C.6 of Part II for a graph of the sensitivity of price elasticities ($\hat{\varepsilon}$) to energy service price ($\tilde{p}_s$) for concrete examples.

Uncompensated own price elasticity ($\hat{\varepsilon}_{\dot{q}_s,p_s}$) After finding the compensated own price elasticity ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$), the Slutsky equation can be used directly to find the uncompensated own price elasticity ($\hat{\varepsilon}_{\dot{q}_s,p_s}$) after the substitution effect for the CES utility model.

$$\hat{\varepsilon}_{\dot{q}_s,p_s} = \hat{\varepsilon}_{\dot{q}_s,p_s,c} - \hat{f}_{\dot{C}_s} \varepsilon_{\dot{q}_s,\dot{M}} \tag{164}$$

Uncompensated cross price elasticity ($\hat{\varepsilon}_{\dot{q}_o,p_s}$) The result from Hicks & Allen (1934) can be used to calculate the uncompensated cross price elasticity ($\hat{\varepsilon}_{\dot{q}_o,p_s}$) for the CES utility model.

$$\hat{\varepsilon}_{\dot{q}_o,p_s} = \hat{f}_{\dot{C}_s} (\sigma - \varepsilon_{\dot{q}_o,\dot{M}}) \ . \tag{165}$$

C.3 Elasticities for the income effect ($\varepsilon_{\dot{q}_s,\dot{M}}$ and $\varepsilon_{\dot{q}_o,\dot{M}}$)

The income effect requires two elasticities to estimate the spending of net savings: the income elasticity of energy service consumption ($\varepsilon_{\dot{q}_s,\dot{M}}$) and the income elasticity of other goods consumption

($\varepsilon_{\dot{q}_o,\dot{M}}$). Due to the homotheticity assumption, both income elasticities are unitary. Thus,

$$\varepsilon_{\dot{q}_s,\dot{M}} = 1 \,, \tag{166}$$

and

$$\varepsilon_{\dot{q}_o,\dot{M}} = 1 \,. \tag{167}$$

D Proof: Income preference equations satisfy the budget constraint

After the substitution effect, a rate of net savings is available ($\hat{\dot{N}}$), all of which is spent on additional energy service ($\Delta\bar{\dot{q}}_s$, $\Delta\bar{\dot{C}}_s = p_E \Delta\bar{\dot{E}}_s$) or additional other goods ($\Delta\bar{\dot{q}}_o$, $\Delta\bar{\dot{C}}_o$). The income effect must satisfy the budget constraint such that net savings is zero afterward ($\bar{\dot{N}} = 0$). The budget constraint across the income effect is represented by Eq. (67):

$$\hat{\dot{N}} = p_E \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o \,. \tag{67}$$

The additional spending due to the income effect is given by income preference equations

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_s,\dot{M}}} \tag{23}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} \,, \tag{27}$$

where

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \,. \tag{24}$$

This appendix proves that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (67)).

The first step in the proof is to convert the income preference equations to $\dot{C}_s^\circ$ and $\dot{C}_o^\circ$ ratios. For the energy service income preference equation (Eq. (23)), multiply numerator and denominator of the left-hand side by $\tilde{p}_s = p_E/\tilde{\eta}$ (Eq. (6)) to obtain $\bar{\dot{C}}_s/\hat{\dot{C}}_s$. For the other goods income preference equation (Eq. (27)), multiply numerator and denominator of the left-hand side by p_o to obtain $\bar{\dot{C}}_o/\hat{\dot{C}}_o$. Then, invoke homotheticity to set $\varepsilon_{\dot{q}_s,\dot{M}} = 1$ and $\varepsilon_{\dot{q}_o,\dot{M}} = 1$ to obtain

$$\frac{\bar{\dot{C}}_s}{\hat{\dot{C}}_s} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \tag{168}$$

and

$$\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \; . \tag{169}$$

The second step in the proof is to obtain expressions for $\Delta\bar{\dot{C}}_s$ and $\Delta\bar{\dot{C}}_o$. Multiply the income preference equations above by $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively. Then, subtract $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively, to obtain

$$\Delta\bar{\dot{C}}_s = \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} \tag{170}$$

and

$$\Delta\bar{\dot{C}}_o = \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \; . \tag{171}$$

The above versions of the income preference equations can be substituted into the budget constraint (Eq. (67)) to obtain

$$\hat{\dot{N}} \stackrel{?}{=} \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} + \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \; . \tag{172}$$

If equality is demonstrated, the income preference equations satisfy the budget constraint. The remainder of the proof shows the equality of Eq. (172).

Dividing by $\hat{\dot{N}}$ and multiplying by $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \stackrel{?}{=} \hat{\dot{M}}' \; . \tag{173}$$

Substituting Eq. (24) for $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \stackrel{?}{=} \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \ . \tag{174}$$

Substituting Eq. (52) for $\dot{M}^\circ$, because $\dot{M}^\circ = \hat{\dot{M}}$, gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \stackrel{?}{=} p_E \hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \cancel{\hat{\dot{N}}} - \dot{C}^*_{cap} - \dot{C}^*_{md} - \cancel{\hat{\dot{N}}} \ . \tag{175}$$

Cancelling terms and recognizing that $\dot{C}^*_{cap} = \hat{\dot{C}}_{cap}$, $\dot{C}^*_{md} = \hat{\dot{C}}_{md}$, and $\hat{\dot{C}}_s = p_E \hat{\dot{E}}_s$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \stackrel{?}{=} \hat{\dot{C}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o - \cancel{\hat{\dot{C}}_{cap}} - \cancel{\hat{\dot{C}}_{md}} \ . \tag{176}$$

Cancelling terms gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \stackrel{\checkmark}{=} \hat{\dot{C}}_s + \hat{\dot{C}}_o \ , \tag{177}$$

thereby completing the proof that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (67)).

E Other goods expenditures and constant p_o

This framework utilizes a partial equilibrium analysis (at the microeconomic level) in which we account for the change of the energy service price due to the EEU ($p_s^\circ \neq p_s^*$), but we do not track the effect of the EEU on prices of other goods. These assumptions have important implications for the relationship between the rate of consumption of other goods ($\dot{q}_o$) and the rate of expenditure on other goods ($\dot{C}_o$).

We assume a basket of other goods (besides the energy service) purchased in the economy, each (i) with its own price ($p_{o,i}$) and rate of consumption ($\dot{q}_{o,i}$), such that the average price of all other goods purchased in the economy prior to the EEU (p_o°) is given by

$$p_o^\circ = \frac{\sum_i p_{o,i}^\circ q_{o,i}^\circ}{\sum_i q_{o,i}^\circ} \ . \tag{178}$$

Then, the expenditure rate of other purchases in the economy can be given as

$$\dot{C}_o^\circ = p_o^\circ \dot{q}_o^\circ \tag{179}$$

before the EEU and

$$\hat{\dot{C}}_o = \hat{p}_o \hat{\dot{q}}_o \tag{180}$$

after the substitution effect, for example.

We assume that any microeconomic effects (emplacement, substitution, or income) for a single device are not so large that they cause a measurable change in prices of other goods. Thus,

$$p_o^\circ = p_o^* = \hat{p}_o = \bar{p}_o = \tilde{p}_o\ . \tag{181}$$

In the partial equilibrium analysis, any two other goods prices can be equated across any rebound effect to obtain (for the example of the original conditions ($\circ$) and the post-substitution state ($\wedge$))

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ}\ . \tag{182}$$

Thus, a ratio of other goods expenditure rates is always equal to a ratio of other goods consumption rates.

F Responding and the marginal propensity to consume (MPC)

Borenstein (2015) has postulated a demand-side argument that macro effects can be represented by a multiplier, which we call the macro factor (k). Borenstein's formulation and our implementation rely on the marginal propensity to consume (MPC). In this appendix, we show the relationship between the macro factor (k) and MPC.

The relationship between the macro factor (k) and MPC spans the substitution, income, and macro effects. In this framework, the device user's net savings after the emplacement effect ($\dot{N}^*$) is respent completely. One may assume that firms and other consumers who receive the net savings have a marginal propensity to re-spend of MPC. The total spending throughout the economy of each year's net savings ($\dot{N}^*$) is given by the infinite series

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\dot{N}^* \ , \tag{183}$$

where the first term $(1 \times \dot{N}^*)$ represents spending of net savings after emplacement by the device user and the remaining terms $[(MPC + MPC^2 + MPC^3 + \ldots)\dot{N}^*]$ represent macro-effect spending in the broader economy.

The macro effect portion of the spending can be represented by the macro factor (k).

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\dot{N}^* = (1 + k)\dot{N}^* \tag{184}$$

Canceling $\dot{N}^*$ and simplifying the infinite series to its converged fraction (assuming $MPC < 1$) gives

$$\frac{1}{1 - MPC} = 1 + k \ . \tag{185}$$

Solving for k yields

$$k = \frac{1}{\frac{1}{MPC} - 1} \ . \tag{31}$$

With $k = 1$, as assumed early in Part II, $MPC = 0.5$ is implied. If $k = 3$, as calibrated later in Part II, $MPC = 0.75$ is implied. The relationship between k and MPC is given in Fig. F.1.

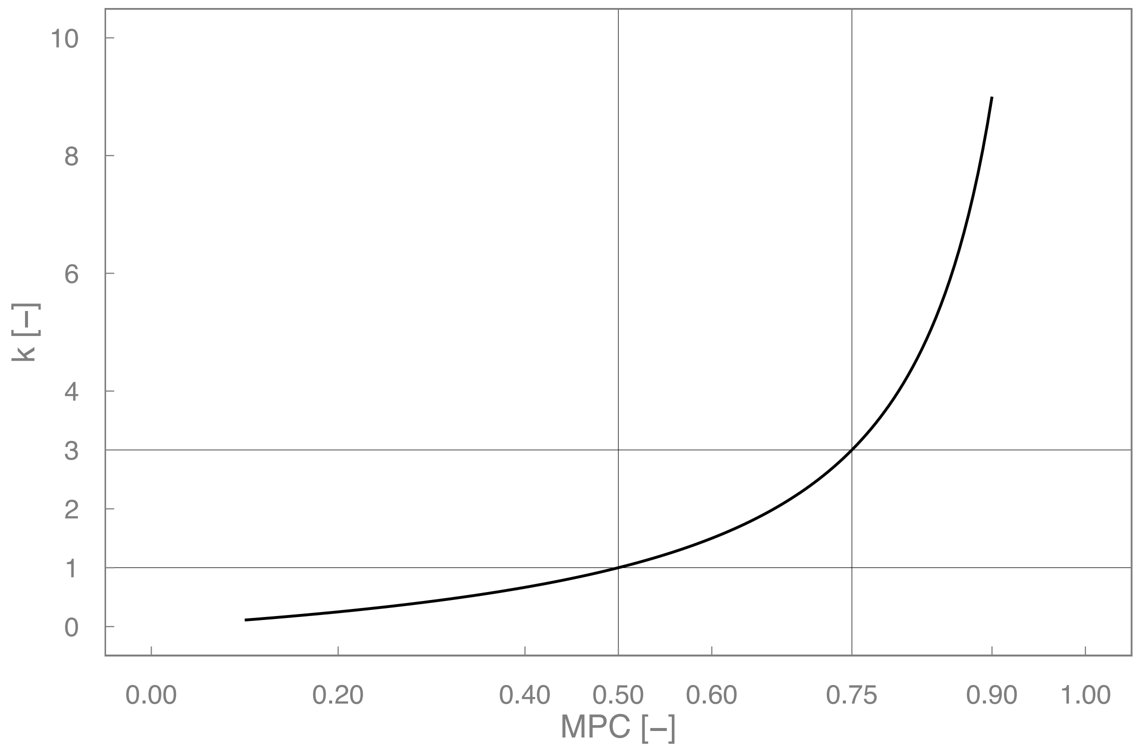

Fig. F.1: The relationship between MPC and k in Eq. (31).

Energy, expenditure, and consumption aspects of rebound, Part II: Applications of the framework

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Abstract

Widespread implementation of energy efficiency is a key greenhouse gas emissions mitigation measure, but rebound can "take back" energy savings. However, the absence of solid analytical foundations hinders empirical determination of the size of rebound. In Part I, we developed a rigorous analytical framework that is approachable for both energy analysts and economists. In this paper (Part II), we develop energy, expenditure, and consumption planes, a novel, mutually consistent, and numerically precise way to visualize and illustrate rebound. Further, we perform the first calibration of the macro factor for macroeconomic rebound, finding $k \approx 3$. Using the framework and rebound planes, we calculate and show total rebound for two examples: energy efficiency upgrades of a car (47%) and an electric lamp (67%). Comparison of our rebound values to previous values is provided. Finally, we provide information about new open source software tools for calculating magnitudes and visualizing rebound effects using the framework.

Keywords: Energy efficiency, Energy rebound, Energy services, Microeconomic rebound, Substitution and income effects, Macroeconomic rebound

JEL codes: O13, Q40, Q43

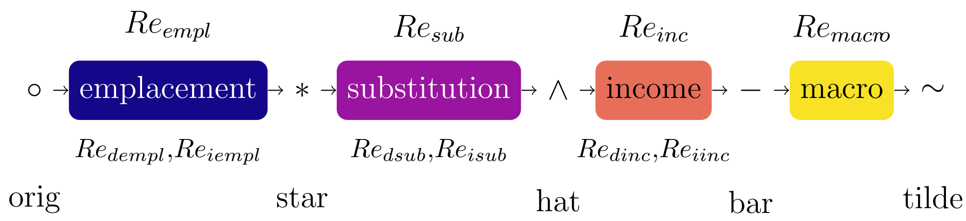

Fig. 1: Flowchart of rebound effects and decorations.

1 Introduction

In Part I of this two-part paper, we argued that improved clarity is needed about energy rebound. We said that

> [a] description of rebound [is needed] that is (i) consistent across energy, expenditure, and consumption aspects, (ii) technically rigorous and (iii) approachable from both sides (economics and energy analysis). ... In other words, the finance and human behavior aspects of rebound need to be presented in ways energy analysts can understand. And the energy aspects of rebound need to be presented in ways economists can understand.

To move improve clarity in the rebound field, we developed in Part I a rigorous analytical framework for energy rebound, one that is tractable for both energy analysts and economists. Three aspects of rebound are analyzed in the framework: energy, expenditure, and consumption. The framework contains both direct and indirect rebound and four rebound effects (emplacement, substitution, income, and macro) between five stages ($\circ$, $*$, $\wedge$, $-$, and $\sim$). Rebound terms and symbol decorations are shown in Fig. 1. (See Table 1 in Part I for details. See Appendix A for nomenclature.)

In this paper (Part II), we make further progress toward the goal of clarity in four ways. First, we develop a new way to visualize components and mechanisms of rebound (rebound planes). Second, we make a first attempt at calibrating the macro rebound effect via a macro factor (k). Third, we apply the framework to two energy efficiency upgrades (EEUs) (a car and an electric lamp) with detailed explication of the examples. Finally, we show calculations of rebound magnitudes for both examples.

The key contributions of this paper are (i) development of the first (to our knowledge) mutually consistent and numerically precise visualizations of rebound effects in energy, expenditure, and consumption planes, (ii) calibration of the macro factor (k) to link microeconomic and macroeconomic rebound levels, (iii) presentation of new rebound values for car and electric lamp upgrades based on a calibrated version of our framework from Part I, and (iv) documentation of new open source software tools to calculate and visualize rebound for any EEU.

The remainder of this paper is structured as follows. Section 2 describes data for the examples, our method of visualizing rebound, and open source software tools for calculating and visualizing rebound. Section 3 provides results for two examples: energy efficiency upgrades to a car and an electric lamp. Section 4 calibrates the macro factor (k) and discusses results, and Section 5 concludes.

2 Data and methods

This section contains data for the examples (Section 2.1), an explication of our new method for visualizing rebound effects and magnitudes (Section 2.2), and a description of new open source software tools for rebound calculations and visualization (Section 2.3).

2.1 Data

To demonstrate application of the rebound analysis framework developed in Part I, we analyze two examples: energy efficiency upgrades to a car and an electric lamp. The examples are presented with much detail to support our goal of bringing clarity to the process of calculating the magnitude of rebound effects. Here, we collect parameter values for the equations to calculate eight rebound components: Re_{dempl}, Re_{emb}, Re_{md}, Re_{dsub}, Re_{isub}, Re_{dinc}, Re_{iinc}, and Re_{macro}. Total rebound (Re_{tot}) is given by the sum of the above components or equivalently by Eq. (34) of Part I.

2.1.1 Data for car example

For the first example, we consider the purchase of a more fuel efficient car, namely a gasoline-electric Ford Fusion Hybrid car, to replace a conventional gasoline Ford Fusion car. The cars are matched as closely as possible, except for the inclusion of an electric battery in the hybrid car. The car case study features a larger initial capital investment ($C^{\circ}_{cap} < \tilde{C}_{cap}$) for the long-term benefit of decreased energy service costs ($\dot{C}^{\circ}_{s} > \tilde{\dot{C}}_{s}$).

We require three sets of data. First, basic car parameters are summarized in Table 1. Second, we require several general economic parameters, mainly relating to the U.S. economy and personal finances of a representative U.S.-based user shown in Table 2. Third, we require elasticity parameters, as given in Table 3.

Table 1: Car example: Vehicle parameters.

Description Parameters [units]	Ford Fusion (gasoline)	Ford Fusion (hybrid EV)	Data sources and notes
Fuel economy η°, $\tilde{\eta}$ [mpg]	25	42	Combined cycle mpg value taken from Thecarconnection.com (2020), for Titanium FWD 2020 model with Intercooled I-4, 2.0 L engine. Combined cycle mpg value taken from Thecarconnection.com (2020), for Titanium FWD 2020 model with Gas/Electric I-4, 2.0 L engine.
Capital expenditure rate $\dot{C}^{\circ}_{cap}$, $\dot{C}^{*}_{cap}$ [$/yr]	4,778	4,720	Seven year annual, averaged capital costs = purchase + finance costs – resale value (purchase – depreciation). Ford Fusion gasoline costs from Edmunds.com (2020a). Ford Fusion Hybrid car costs from Edmunds.com (2020b).
Ownership duration t°_{own}, t^{*}_{own} [yr]	7	7	U.S. car ownership (from new) duration from Businesswire.com (2015), and has risen from 52 months (2005) to 79 months (2015), so taken as 84 months in 2018 (7 years) for our example.
Lifespan t°_{life}, t^{*}_{life} [yr]	14	14	Lifetime taken as 14 years, based on 13–17 years for U.S. cars from Berla.com (2016) and 14 years for UK cars from Society of Motor Manufacturers and Traders (2020).
Embodied energy E°_{emb}, E^{*}_{emb} [MJ]	34,000	40,000	34,000 MJ for conventional Ford Fusion gasoline car taken from Argonne National Laboratory, Energy Systems Division (2010). We assume an additional 6,000 MJ added for Ford Fusion Hybrid Electric Vehicle (HEV) battery, as HEV typically adds 10–25% to total LCA energy of vehicle manufacture (Onat et al., 2015). Battery lifetime assumed same as car lifetime, based on Nordelöf et al. (2014) and Onat et al. (2015).
Maintenance and disposal expenditure rate $\dot{C}^{\circ}_{md}$, $\dot{C}^{*}_{md}$ [$/yr]	2,731	2,710	Seven year annual, averaged maintenance costs = sum of insurance, maintenance, repairs, taxes, and fees (excluding financing, depreciation, fuel). Ford Fusion maintenance costs from Edmunds.com (2020a). Ford Fusion Hybrid maintenance costs from Edmunds.com (2020b).

Table 2: Car example: Economic parameters (2020).

Description Parameter [units]	Value	Data sources and notes
Distance driven prior to upgrade $\dot{q}_s^\circ$ [miles/yr]	12,416	Average U.S. vehicle miles/yr, calculated from Carinsurance.com (2019). This is slightly lower than the average driver miles/yr (13,476) (US Department of Transportation, 2018), as there are more registered U.S. vehicles than drivers.
Real median personal income U.S., in 2018 [$/yr]	34,317	Taken from Federal Reserve Bank of St Louis (2019).
U.S. 2018 disposable income / real income (minus current taxes) [–]	0.88319	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition (US Bureau of Economic Analysis, 2020).
Share of savings from 2018 disposable income [–]	0.07848	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition (US Bureau of Economic Analysis, 2020).
Personal consumption in 2018 $\dot{M}$ [$/yr]	27,929.83	Calculation: $(\$34,317/\text{yr})(0.88319)(1 - 0.07848)$
Price of gasoline p_E [$/gallon]	2.63	Source: US Energy Information Administration (2020b)
Fractional spend on original energy service $f^\circ_{\dot{C}_s}$ [–]	0.064	Calculation: $1,306 (spend on energy service) / [$19,115 (other goods) + $1,306 (energy service)] = 0.064, where spend on energy service = 12,416 miles / 25 mpg × $2.63/gallon = $1,306.
Macro factor k [–]	1.0	An initial value. See Section 4.1 for additional details.

Table 3: Car example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Uncompensated own price elasticity of car use demand $\varepsilon^{\circ}_{\dot{q}_s,p_s}$ [–]	-0.2	We adopt -0.2 as our baseline value, based on U.S. studies including Gillingham (2020) who estimated a value of -0.1, Goetzke & Vance (2018) who estimated values between -0.05 and -0.23, and Parry & Small (2005) who estimated values between -0.1 and -0.3. For comparison, Borenstein (2015) uses values of -0.1 to -0.4 based on Parry & Small (2005).
Compensated price elasticity of car use demand $\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$ [–]	-0.136	Calculated via the Slutsky Equation (Eq. (131) in Part I).
Compensated cross price elasticity of demand for other goods $\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$ [–]	0.009	Calculated via Eq. (137) in Part I.
Income elasticity of demand for car use $\varepsilon_{\dot{q}_s,\dot{M}}$ [–]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\varepsilon_{\dot{q}_o,\dot{M}}$ [–]	1.0	Follows from CES utility function.

2.1.2 Data for lamp example

For the second example, we consider purchasing a Light Emitting Diode (LED) electric lamp to replace a baseline incandescent electric lamp. Both lamps are matched as closely as possible in terms of energy service delivery (measured in lumen output per lamp), the key difference being the energy required to provide that service. The LED lamp has a low initial capital investment rate when spread out over the lifetime of the lamp (less than the incumbent incandescent lamp, actually) and a long-term benefit of decreased direct energy expenditures at approximately the same energy service delivery rate (lm·hr/yr).

Again, three sets of data are required. First, basic lamp parameters are summarized in Table 4. Second, several general economic parameters, mainly relating to the U.S. economy and personal finances of a representative U.S.-based user are given in Table 5. Third, we require the elasticity parameters, as shown in Table 6.

Table 4: Lamp example: Electric lamp parameters.

Description Parameters [units]	Incandescent lamp	LED lamp	Data sources and notes
Lamp efficiency η°, $\tilde{\eta}$ [lm·hr/W·hr]	8.83	81.8	Incandescent: 530 lm output / 60 W energy input. LED: 450 lm output / 5.5 W energy input.
Capital expenditure rate $\dot{C}^{\circ}_{cap}$, $\dot{C}^{*}_{cap}$ [$/yr]	1.044	0.121	Purchase costs: $1.88 for incandescent lamp from HomeDepot.com (2020b), and $1.21 for LED lamp from HomeDepot.com (2020a).
Ownership duration t°_{own}, t^{*}_{own} [yr]	1.8	10	Assumed same as lamp lifespan
Lifespan t°_{life}, t^{*}_{life} [yr]	1.8	10	Based on assumed 3 hr/day from HomeDepot.com (2020b) and HomeDepot.com (2020a).
Life cycle analysis (LCA) embodied energy E°_{emb}, E^{*}_{emb} [MJ]	2.20	6.50	Base document: Table 4.5 Manufacturing Phase Primary Energy (MJ/20 million lm·hr), contained in U.S. DoE Life-cycle assessment of energy and environmental impacts of LED lighting products (US Department of Energy, 2012). Incandescent lamp: LCA energy = 42.2 MJ/20 million lm·hr. Lifetime output = 530 lm $\times$ 3 hr/day $\times$ 365 days/yr $\times$ 1.8 yr = 1,044,630 lm·hr. Thus LCA energy / lamp = 42.2 $\times$ 1.0446/20 = 2.21 MJ. LED lamp: LCA energy = 132 MJ/20 Million lm·hr for pack of 5 LED lamps. Lifetime output = 450 lm $\times$ 3 hr/day $\times$ 365 days/yr $\times$ 10 yr = 4,926,405 lm·hr. Thus LCA energy / lamp = 132 MJ/5 $\times$ 4.9264/20 = 6.5 MJ.
Maintenance and disposal expenditure rate $\dot{C}^{\circ}_{md}$, $\dot{C}^{*}_{md}$ [$/yr]	0.00	0.00	Assumed negligible.

Table 5: Lamp example: Economic parameters (2020).

Description Parameter [units]	Value	Data sources and notes
Lighting consumption prior to upgrade $\dot{q}_s^\circ$ [lm·hr/yr]	580,350	Calculation: (530 lm) (3 hrs/day) (365 days/yr).
Real median personal income U.S. in 2018 [$/yr]	34,317	Refer to Table 2.
U.S. 2018 disposable income / real income (minus current taxes) [–]	0.88319	Refer to Table 2.
Share of savings from 2018 disposable income [–]	0.07848	Refer to Table 2.
Personal consumption in 2018 $\dot{M}$ [$/yr]	27,929.83	Calculation: $(\$34,317/\text{yr})(0.88319)(1 - 0.07848)$.
Price of electricity p_E [$/kW·hr]	0.1287	U.S. 2018 average U.S. household electricity price (US Energy Information Administration, 2020a).
Fractional spend on original energy service $f_{\dot{C}_s}^\circ$ [–]	0.0003028	Calculation: $8.5/yr (spend on energy service) / [$27,920/yr (other goods) + $8.5/yr (energy service)] = 0.00030, where spend on energy service = 580,350 lm·hr/yr / 8.83 lm/W / 1000 W/kW × $0.1287/kW·hr= $8.5/yr. Note: this is energy service from a single lamp.
Macro factor k [–]	1.0	An initial value. See Section 4.1 for additional details.

Table 6: Lamp example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Uncompensated own price elasticity of lighting demand $\varepsilon^{\circ}_{\dot{q}_s,p_s}$ [–]	-0.4	We adopt -0.4 as our baseline value, as the average of last 50 years from Fouquet (2014, Fig. 4). For comparison, Borenstein (2015) uses a range of -0.4 to -0.8, based on Fouquet & Pearson (2011).
Compensated own price elasticity of lighting demand $\varepsilon^{\circ}_{\dot{q}_s,p_s,c}$ [–]	-0.3997	Calculated via the Slutsky Equation (Eq. (131) in Part I).
Compensated cross price elasticity of demand for other goods $\varepsilon^{\circ}_{\dot{q}_o,p_s,c}$ [–]	0.00012	Calculated via Eq. (137) in Part I.
Income elasticity of lighting demand $\varepsilon_{\dot{q}_s,\dot{M}}$ [–]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\varepsilon_{\dot{q}_o,\dot{M}}$ [–]	1.0	Follows from CES utility function.

2.2 Visualization

A rigorous rebound analysis should track energy, expenditure, and consumption aspects of rebound at the device (direct rebound) and elsewhere in the economy (indirect rebound) across adjustments for all rebound effects (emplacement, substitution, income, and macro). Doing so involves many terms and much complexity. To date, visualizing the energy, expenditure, and consumption aspects of rebound phenomena has not been done in a numerically precise manner with a set of mutually consistent graphs.

We introduce rebound planes to bring clarity to (direct and indirect) rebound and adjustments (via emplacement, substitution, income, and macro effects) across all aspects (energy, expenditure, and consumption). Each aspect is represented by a path in its own plane, showing adjustments in response to the EEU.

Axes of the rebound planes represent direct and indirect effects, with direct effects shown on the x-axis, and indirect effects shown on the y-axis. Specifically, (i) direct and indirect energy consumption rates ($\dot{E}_{dir}$, $\dot{E}_{indir}$) are placed on the x- and y-axes of the energy plane, respectively; (ii) direct and indirect expenditure rates ($\dot{C}_{dir}$ and $\dot{C}_{indir}$) are placed on the x- and y-axes of the expenditure plane, respectively; and (iii) the indexed consumption rate of the energy service ($\dot{q}_s$) and the indexed expenditure rate of other consumption goods ($\dot{C}_o$) are placed on the x- and y-axes of the consumption plane, respectively. Paths through energy, expenditure, and consumption planes consist of segments that represent changes due to the various rebound effects. Table 7 provides the key for rebound path segments. Effects that include both direct and indirect rebound will show displacement along both axes and create a path in the x-y plane. (See Appendix B for detailed mathematical descriptions for constructing paths on the rebound planes.)

Figs. 2–4 show notional energy, expenditure, and consumption planes, respectively. The notional planes are not quantified, i.e., there are no scales on the axes. Later (Section 3), rebound planes with numerical scales illustrate the car and lamp examples.

Table 7: Segments in rebound planes.

Segment	Rebound effect	Symbol
$\circ$—a	Direct emplacement	Re_{dempl}
$a \cdots b$	Embodied energy	Re_{emb}
b—$*$	Maintenance and disposal	Re_{md}
$*$—c	Indirect substitution	Re_{isub}
c—$\wedge$	Direct substitution	Re_{dsub}
$\wedge$—d	Direct income	Re_{dinc}
d—$-$	Indirect income	Re_{iinc}
$-$—$\sim$	Macro	Re_{macro}

2.2.1 The energy plane

Fig. 2 shows a notional energy plane, with the direct energy consumption rate ($\dot{E}_{dir}$) on the x-axis and the indirect energy consumption rate ($\dot{E}_{indir}$) on the y-axis.[1] Points $\circ$, $*$, $\wedge$, $-$, and $\sim$ represent the rebound stages between the rebound effects. Points a, b, c, and d represent intermediate stages. Lines with negative slope through points $\circ$, a, $*$, $\wedge$, $-$, and $\sim$ indicate energy consumption isoquants at key points. Table 7 shows segments and rebound effects for all rebound planes. Note that segment $-$—$\sim$ appears only in the energy plane, because the framework tracks energy consumption but not expenditures or consumption for the macro effect.

In the notional energy plane of Fig. 2, point a lies on the $Re_{tot} = 0\%$ line indicating that point a (and the $Re_{tot} = 0\%$ line) is the point from which all rebound effects (Re_{empl}, Re_{sub}, Re_{inc}, and Re_{macro}) are measured. If rebound effects cause total energy demand to return to the original energy consumption level (negative sloping line through the $\circ$ point), all expected energy savings are taken back by rebound effects. Thus, the line of constant energy consumption through the $\circ$ point is labeled $Re_{tot} = 100\%$. The contribution of each rebound effect to total rebound is represented by the distance that each component's segment moves across the rebound isoquants. Total rebound (Re_{tot}) is measured linearly between and beyond the $Re_{tot} = 0\%$ and $Re_{tot} = 100\%$ lines, with direct rebound in the x direction and indirect rebound in the y direction. The region below and to the left of the $Re_{tot} = 0\%$ line in Fig. 2 exhibits negative rebound, indicating hyperconservation. The region above and to the right of the $Re_{tot} = 100\%$ line shows backfire, i.e., greater total energy

[1] A related, notional-only (not quantified as in Section 3), one-dimensional visualization of direct and indirect energy rebound (but not on expenditure or consumption planes) can be found in Fig. 1 of Exadaktylos & van den Bergh (2021).

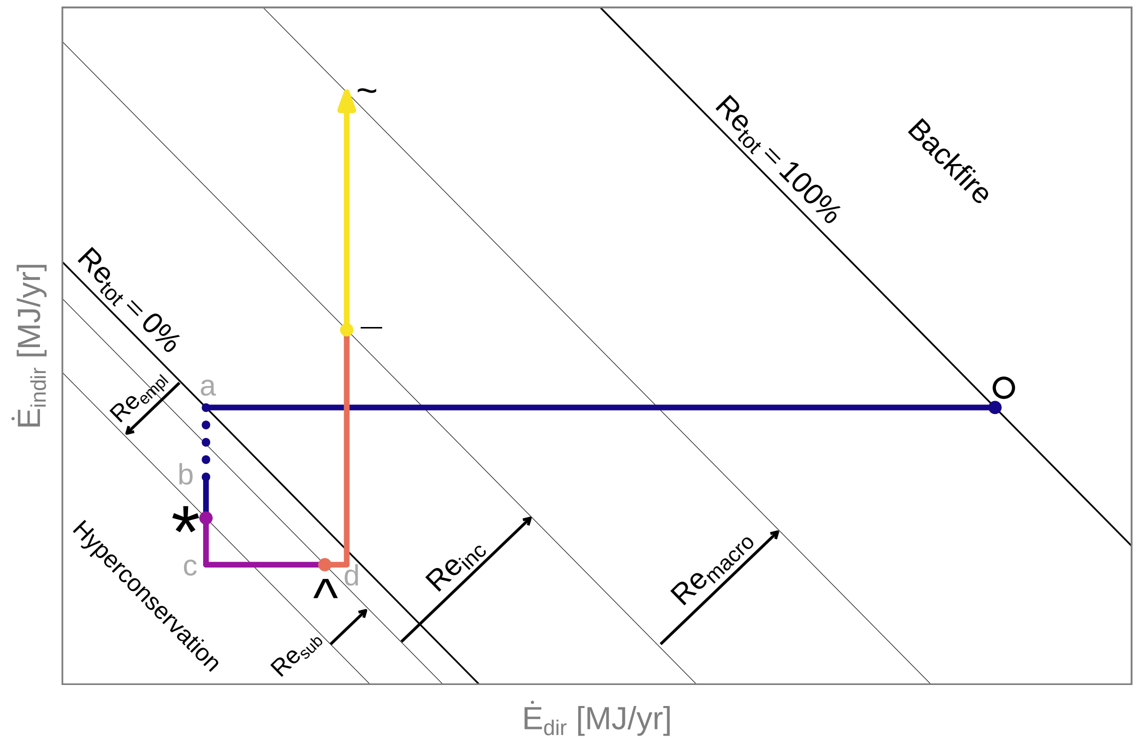

Fig. 2: Notional energy plane. See Table 7 for meanings of path segments.

consumption after the EEU than before it.

In the notional energy plane (Fig. 2), emplacement rebound is negative ($Re_{empl} < 0$), because the upgraded device has a lesser embodied energy rate ($\dot{E}^{\circ}_{emb} > \dot{E}^{*}_{emb}$, as shown by point b being below point a) and has a lesser energy consumption rate for maintenance and disposal ($\dot{E}^{\circ}_{md} > \dot{E}^{*}_{md}$, as shown by point $*$ being below point b) due to lower expenditure rates on these two categories compared to the original device.

In Fig. 3 segments $a \cdots b$ and b — $*$ move in the negative y direction, consistent with the description above. Segment $*$ — c moves in the negative y direction by definition of the indirect substitution effect, and segment c — $\wedge$ moves in the positive x direction by the definition of the direct substitution effect. Both income effect segments ($\wedge$ — d and d — $-$) show more energy consumption, because net savings are spent on goods and services that rely on at least some energy consumption.[2] Segment $-$ — $\sim$ always moves in the positive y direction, because macro effects lead to additional indirect energy consumption.

[2] We exclude the case of an inferior good, whose consumption decreases as real income increases, but we note here the possibility of such behavior. This behavior would however require a different utility model besides the CES utility model, which we use throughout this analysis.

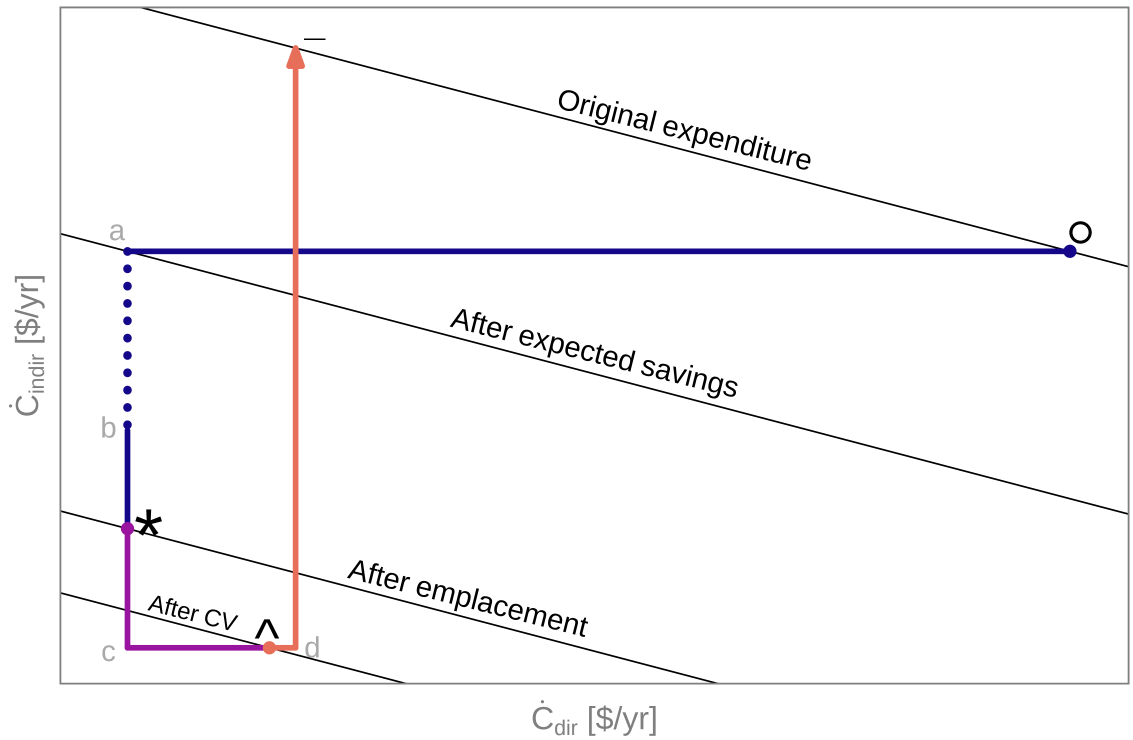

Fig. 3: Notional expenditure plane. CV is compensating variation, the increase in consumption of the energy service and decrease in consumption of other goods and services to maintain constant utility. See Table 7 for meanings of path segments.

2.2.2 The expenditure plane

A notional expenditure plane is shown in Fig. 3, with the direct expenditure rate on the energy service ($\dot{C}_{dir}$) on the x-axis and the indirect expenditure rate ($\dot{C}_{indir}$) on the y-axis. Lines with negative slope through points $\circ$, a, $*$, and $\wedge$ indicate expenditure isoquants. The line through the $\circ$ point is an isoquant for the cost of purchasing the original consumption bundle at the original prices. The line through the $*$ point is an isoquant for the cost of purchasing the original consumption bundle at the new prices.

In the notional planes of Figs. 2 and 3, embodied energy rates and capital cost rates (represented by segments $a \cdots b$ and $b — *$) move in the same direction (both in the negative y direction). However, segments $a \cdots b$ and $b — *$ could both move in the positive y direction, or they could move in opposite directions, depending on the results of the independent analyses for embodied energy and capital cost rates. The substitution effect along segments $* — c$ and $c — \wedge$ will together, by definition, lead to lower expenditure due to the energy service price decline and the budget reducing compensating variation (CV). The income effect (segments $\wedge$—d and d—$-$) must bring expenditure back to the original expenditure line (equal to the budget constraint set by income in dollar or nominal terms) by assumptions about non-satiation and utility maximization in the device user's decision function.

2.2.3 The consumption plane

A notional consumption plane is shown in Fig. 4. The indexed rate of energy service consumption ($\dot{q}_s/\dot{q}_s^\circ$) is shown on the x-axis, and the indexed rate of other goods consumption ($\dot{C}_o/\dot{C}_o^\circ$) is shown on the y-axis. Iso-expenditure loci of indexed energy service and other goods demand, i.e. budget constraints, are shown as lines with negative slope (lines $\circ$—$\circ$, $*$—$*$, $\wedge$—$\wedge$, and $-$—$-$). Note that budget constraints $\circ$—$\circ$ and $-$—$-$ intersect at the y-axis, because the prices of other goods and services don't change as a result of the EEU. (The x-axis of Fig. 4 does not extend to 0 at the left, so the intersection is not visible.) Emplacement (by itself) does not alter consumption patterns, so the rate of energy service consumption and the rate of other goods consumption are unchanged across the emplacement effect ($\dot{q}_s^\circ = \dot{q}_s^*$ and $\dot{C}_o^\circ = \dot{C}_o^*$, respectively). Thus, only movements after the $*$ point are visible as a path in the consumption plane, and points $\circ$, a, b, and $*$ collapse to the same location in the consumption plane.

Indifference curves are denoted by i°—i° and $\bar{\mathrm{i}}$—$\bar{\mathrm{i}}$ and represent lines of constant utility through the $\circ$ and $-$ points. Prior to the EEU, the consumption basket (of the energy service and other goods) is represented by the $\circ$ point. The budget constraint, here in real terms, i.e., the capacity to purchase either the energy service or other goods and services, is shown as isoquant $\circ$—$\circ$. The original budget constraint line ($\circ$—$\circ$) is tangent to the original indifference curve (i°—i°) at point $\circ$, the optimal consumption bundle prior to the EEU. The real budget line $*$—$*$ indicates the (higher) capacity to purchase combinations of energy services and other goods and services using the same money needed to purchase the old consumption bundle but at the new, lower price for the energy service, thanks to the EEU.

The substitution effect leads to the cheaper, optimal utility-preserving consumption bundle at the $\wedge$ point. The substitution effect is shown by segments $*$—c (the indirect component, which represents the decrease in other goods consumption) and c—$\wedge$ (the direct component, which represents the increase in energy service consumption). Although the substitution effect is calculated on the

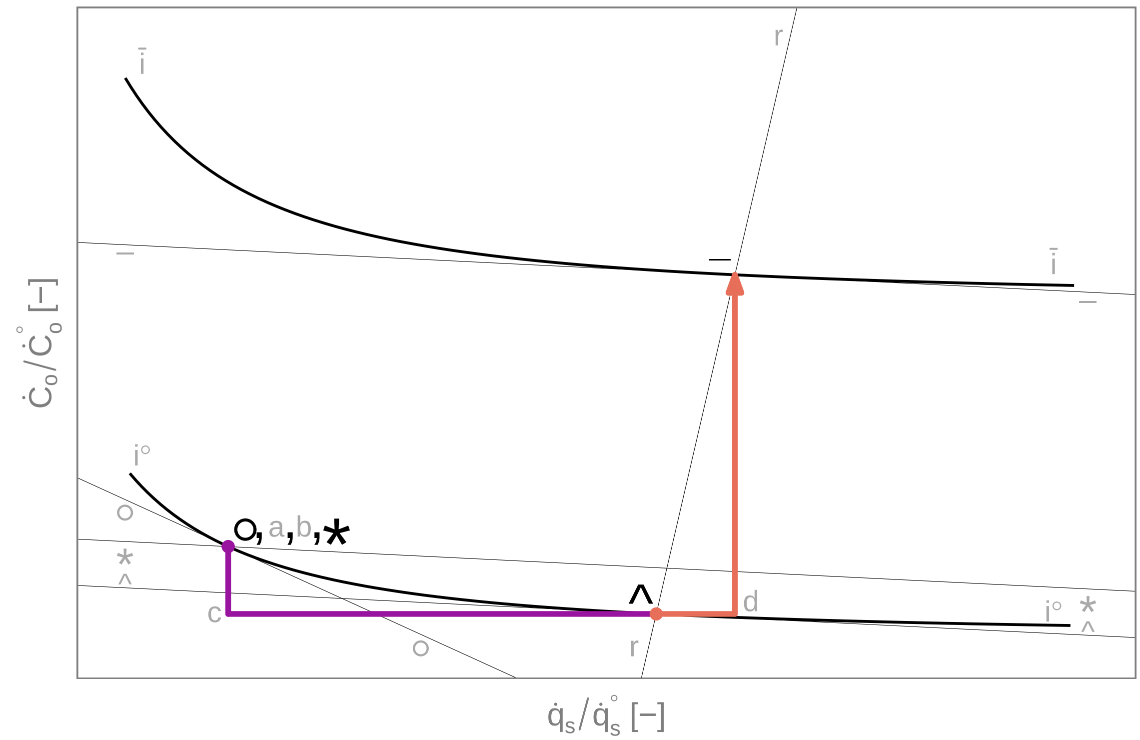

Fig. 4: Notional consumption plane. See Table 7 for meanings of path segments.

164 consumption plane, its impact can be seen in the energy and expenditures planes (Figs. 2 and 3,
165 respectively).

166 The income expansion path is a ray (r—r) from the origin through the $\wedge$ point in the consumption
167 plane. In the consumption plane, the pre- and post-income-effect points ($\wedge$ and $-$, respectively)
168 lie along the r—r ray, due to homotheticity. The increased consumption rate of the energy service
169 is represented by segment $\wedge$— d in Figs. 2–4. The increased consumption rate of other goods and
170 services is represented by segments d— $-$ in Figs. 2–4.

171 2.3 Software tools

172 We developed an open source `R` package called `ReboundTools` to standardize and distribute the
173 methods for calculating rebound magnitudes in our framework. `ReboundTools` can be found at
174 `https://github.com/MatthewHeun/ReboundTools`. (See Heun (2023).) `ReboundTools` provides
175 functions for (i) reading input data from a spreadsheet, (ii) performing rebound calculations, and
176 (iii) generating rebound tables and rebound planes. `ReboundTools` was used for all calculations and

all rebound planes in this paper.

To find the path in storage to an example spreadsheet bundled with the package, users of `ReboundTools` can call the function `ReboundTools::sample_eeu_data_path()`. After filling the example spreadsheet with parameters for an EEU, users can call two functions (`ReboundTools::load_eeu_data()` and `ReboundTools::rebound_analysis()`) to perform all rebound calculations described in this paper. The function `ReboundTools::path_graphs()` creates rebound paths in the energy, expenditure, and consumption planes. Extensive documentation for `ReboundTools` can be found at `https://matthewheun.github.io/ReboundTools/`.

In addition, an Excel workbook that performs identical rebound calculations using the framework of this paper **** will be stored at the Research Data Leeds Repository if this submission is accepted. The spreadsheet file is included with the submission of this paper. **** (See Brockway et al. (2023).)

3 Results

In this section we present rebound calculation results for two examples: energy efficiency upgrades of a car (Section 3.1) and an electric lamp (Section 3.2). Univariate sensitivity studies for both examples (car and lamp) can be found in Appendix C.

3.1 Example 1: Purchase of a new car

Armed with the parameter values from Tables 1–3, and the equations in Section 2 of Part I, we calculate important values at each rebound stage, as shown in Table 8. Note that Table 8 applies to the car user. Across the macro effect (segment $-$ — $\sim$ in Fig. 5), changes occur only in the macroeconomy. For the car user, no changes are recorded across the macro effect. Thus, the $-$ (bar) and $\sim$ (tilde) columns of Table 8 are identical.

Results are represented graphically on energy, expenditure, and consumption planes in Figs. 5–7. The energy plane (Fig. 5) shows the size of each rebound effect for the car example. Rebound components for the car upgrade are shown in Table 9.

The emplacement effect has three components: the direct emplacement effect, the embodied energy effect, and the maintenance and disposal effect. Rebound from the direct emplacement

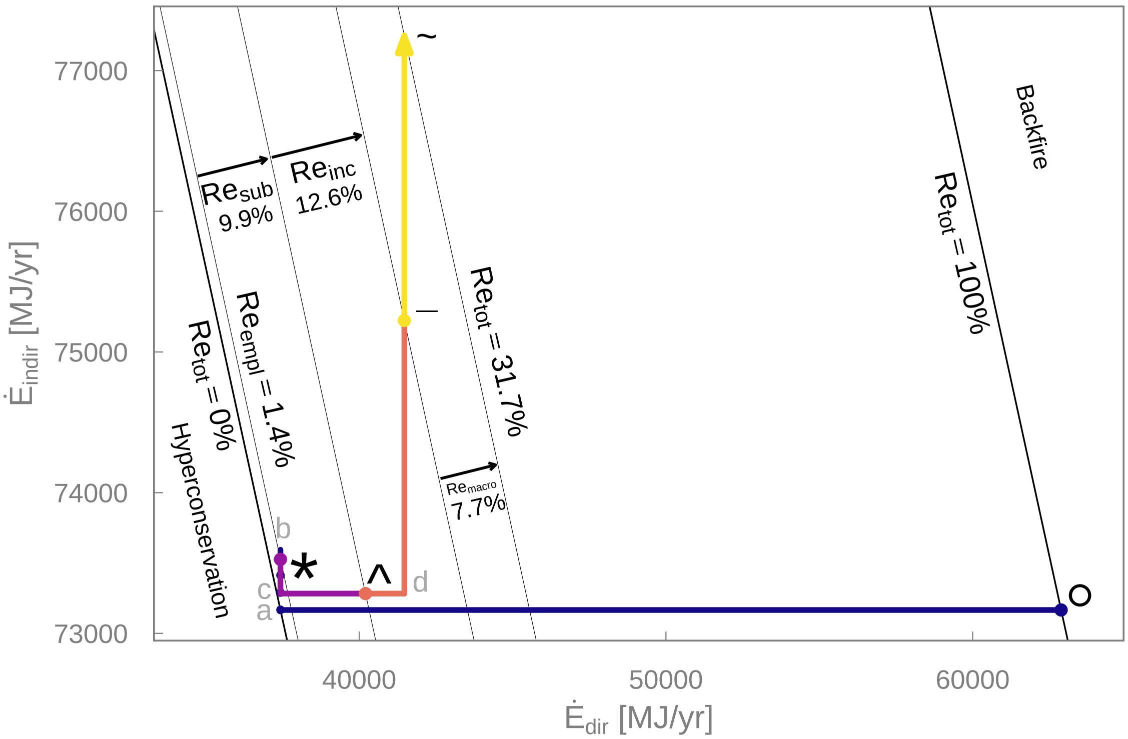

Fig. 5: The energy plane for the car example. Macro factor (k) is assumed to be 1. See Table 7 for meanings of path segments.

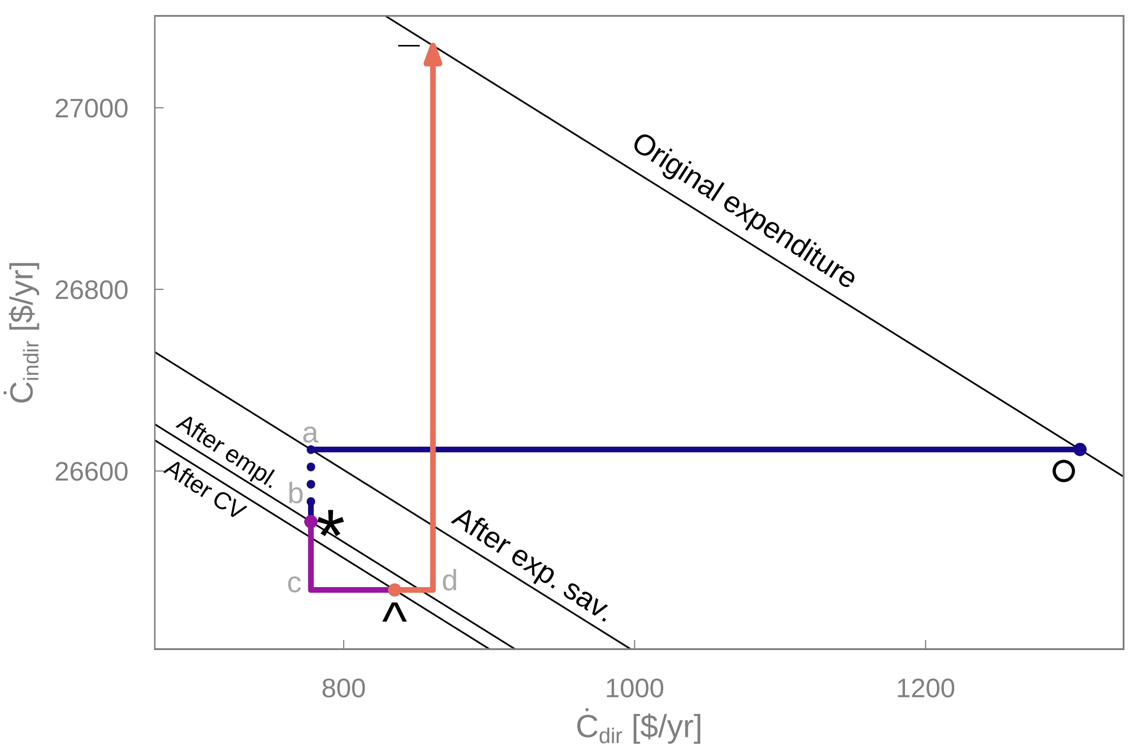

Fig. 6: The expenditure plane for the car example. CV is compensating variation, the increase in consumption of the energy service and decrease in consumption of other goods and services to maintain constant utility. See Table 7 for meanings of path segments.

Table 8: Results for car example with macro factor (k) assumed to be 1.

	$\circ$ (orig)	$*$ (star)	$\wedge$ (hat)	$-$ (bar)	$\sim$ (tilde)
η [mile/gal]	25.0	42.0	42.0	42.0	42.0
η [mile/MJ]	0.197	0.332	0.332	0.332	0.332
p_s [\$/mile]	0.105	0.063	0.063	0.063	0.063
$\dot{q}_s$ [mile/yr]	12,416	12,416	13,336	13,756	13,756
$\dot{E}_s$ [MJ/yr]	62,885	37,432	40,204	41,470	41,470
$\dot{E}_{emb}$ [MJ/yr]	2,429	2,857	2,857	2,857	2,857
$\dot{C}_s$ [\$/yr]	1,306	777	835	861	861
$\dot{C}_{cap}$ [\$/yr]	4,778	4,720	4,720	4,720	4,720
$\dot{C}_{md}$ [\$/yr]	2,731	2,710	2,710	2,710	2,710
$\dot{C}_o$ [\$/yr]	19,115	19,115	19,040	19,639	19,639
$\dot{N}$ [\$/yr]	0	608	626	0	0
$\dot{M}$ [\$/yr]	27,930	27,930	27,930	27,930	27,930

Table 9: Car example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	1.7
Re_{md}	-0.3
Re_{dsub}	10.9
Re_{isub}	-1.0
Re_{dinc}	5.0
Re_{iinc}	7.6
Re_{macro}	7.7
Re_{tot}	31.7

effect (Re_{dempl}) is 0.0% always, because energy takeback (and, therefore, rebound) occurs after the upgraded device is emplaced. Indirect rebound due to the embodied energy effect (Re_{emb}) is 1.7%, due to the higher embodied energy rate ($\Delta\dot{E}^*_{emb} = 429$ MJ/yr) stemming from the electric battery in the hybrid EV car. Rebound due to the maintenance and disposal effect (Re_{md}) is small and negative (-0.3%), because of the slightly lower maintenance and disposal costs for the hybrid EV car.

The substitution effect has two components: direct and indirect substitution effect rebound. Rebound from direct substitution (Re_{dsub}) is positive, as expected (10.9%). The car user will, on average, prefer more driving purely from the change in relative prices because of the fuel economy enhancements (42 mpg > 25 mpg). In other words, due the relative price change, the more fuel-efficient car is driven 7.4% further each year. Conversely, the indirect substitution effect (Re_{isub}) is slightly negative (-1.0%) to achieve the same level of utility after increased driving. Indeed, across

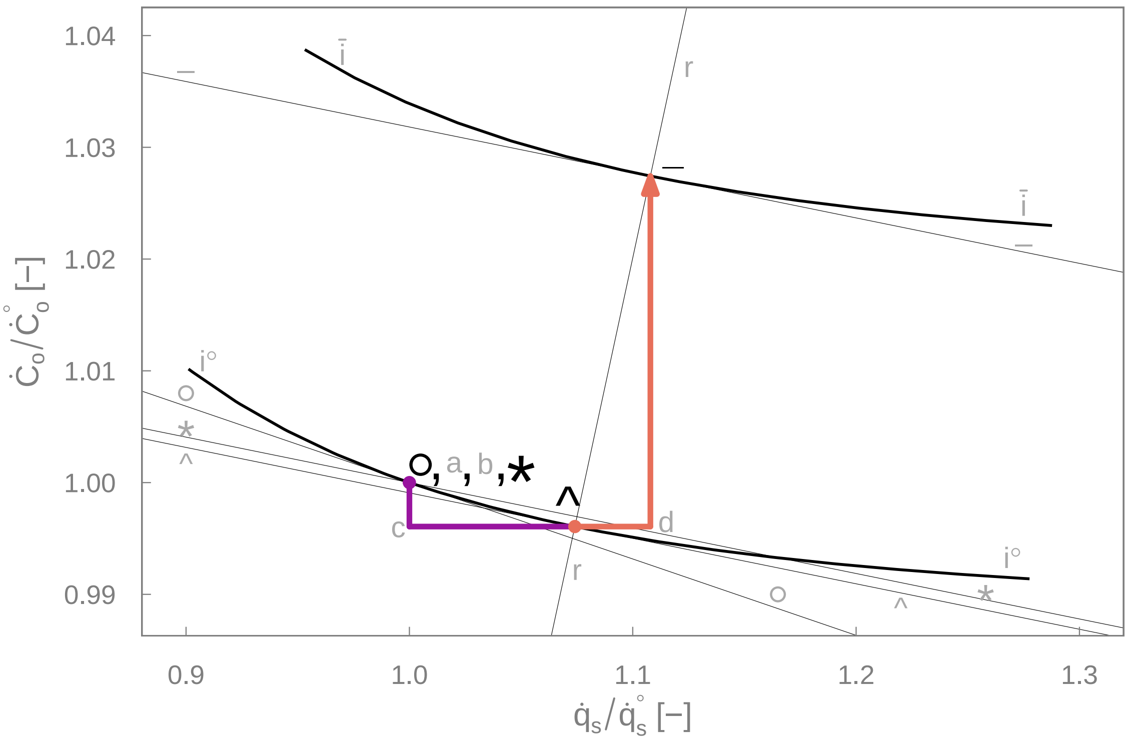

Fig. 7: The consumption plane for the car example. See Table 7 for meanings of path segments.

the substitution effect, less money is spent on other goods ($\Delta\hat{\dot{C}}_o = -75.18$ \$/yr). In Appendix C.6 we show how the displacement along an indifference curve alters the price elasticities, and in particular, that the uncompensated own price elasticity declines in magnitude. The decline slows the rate of additional consumption of energy intensive driving, and attenuates the microeconomic rebound relative to assuming constant price elasticities.

The income effect also has two components: direct and indirect income effect rebound. The direct income effect (Re_{dinc}) is positive (5.0%), because the car user allocates some net savings to additional driving. Rebound from the indirect income effect (Re_{iinc}) is positive (7.6%) due to higher spending on other goods. Thus, the net savings after the substitution effect ($\hat{\dot{N}} = 625.79$ \$/yr) translates into positive direct and indirect income rebound at the microeconomic level. Total microeconomic rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 24.0\%$.

Finally, in Part I we noted that the link between macroeconomic and microeconomic rebound is largely unexplored, so we assume a value of $k = 1$ for both examples, initially. We return to the matter of calibrating k in the Discussion (Section 4.1). With k assumed to be 1, the macro effect leads to macroeconomic rebound (Re_{macro}) of 7.7% for the car example, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (transportation).

3.2 Example 2: Purchase of a new electric lamp

With the parameter values from Tables 4–6 and the equations in Section 2 of Part I in hand, we calculate important values at each rebound stage, as shown in Table 10. Similar to Table 8, Table 10 applies to the lamp user, so no changes are recorded across the macro effect, and the $-$ (bar) and $\sim$ (tilde) columns of Table 10 are identical.

Table 10: Results for lamp example with macro factor (k) assumed to be 1.

	$\circ$ (orig)	$*$ (star)	$\wedge$ (hat)	$-$ (bar)	$\sim$ (tilde)
η [lm·hr/kW·hr]	8,833	81,800	81,800	81,800	81,800
η [lm·hr/MJ]	2,454	22,722	22,722	22,722	22,722
p_s [\$/lm·hr]	0.00001457	0.00000157	0.00000157	0.00000157	0.00000157
$\dot{q}_s$ [lm·hr/yr]	580,350	580,350	1,412,867	1,413,439	1,413,439
$\dot{E}_s$ [MJ/yr]	236.5	25.5	62.2	62.2	62.2
$\dot{E}_{emb}$ [MJ/yr]	1.222	0.650	0.650	0.650	0.650
$\dot{C}_s$ [\$/yr]	8.46	0.91	2.22	2.22	2.22
$\dot{C}_{cap}$ [\$/yr]	1.04	0.12	0.12	0.12	0.12
$\dot{C}_{md}$ [\$/yr]	0.00	0.00	0.00	0.00	0.00
$\dot{C}_o$ [\$/yr]	27,920	27,920	27,916	27,927	27,927
$\dot{N}$ [\$/yr]	0.00	8.47	11.31	0.00	0.00
$\dot{M}$ [\$/yr]	27,930	27,930	27,930	27,930	27,930

Results are represented graphically on energy, expenditure, and consumption planes in Figs. 8–10. The energy plane (Fig. 8) shows the size of each rebound effect for the lamp example. Rebound components for the lamp upgrade are shown in Table 11.

The emplacement effect rebound components start with the direct emplacement effect (Re_{dempl}), which is always 0.0%. Indirect rebound due to the embodied energy effect (Re_{emb}) is -0.3%. Although the LED lamp has higher embodied energy ($E^*_{emb} = 6.50$ MJ) than the incandescent lamp ($E^\circ_{emb} = 2.20$ MJ), the LED lamp has a much longer lifetime, meaning that the LED embodied energy *rate* ($\dot{E}^*_{emb} = 0.65$ MJ/yr) is less than the incandescent embodied energy rate ($\dot{E}^\circ_{emb} = 1.22$ MJ/yr). Thus, the change in embodied energy rate ($\Delta\dot{E}^*_{emb}$) is -0.57 MJ/yr, and embodied energy rebound is negative ($Re_{emb} = -0.3\%$). Rebound due to the maintenance and disposal effect (Re_{md}) is 0.0%, because we assume no difference in maintenance and disposal costs between the incandescent lamp and the LED lamp.[3]

Direct substitution effect rebound (Re_{dsub}) is 17.4% due to the much higher LED lamp efficiency ($\tilde{\eta} = 81.8$ lm/W) compared to the incandescent lamp ($\eta^{\circ} = 8.83$ lm/W), leading to increased demand for lighting (from $\dot{q}_s^* = 580,350$ lm·hr/yr to $\hat{\dot{q}}_s = 1,412,867$ lm·hr/yr) as shown by segment c—$\wedge$ in Fig. 10. To maintain constant utility, consumption of other goods is reduced ($\Delta\hat{\dot{C}}_o = -4.15$ \$/yr), as shown by segment $*$—c in Fig. 10, yielding indirect substitution effect rebound (Re_{isub}) of -6.4%.

Income effect rebound arises from spending net energy cost savings associated with converting from the incandescent lamp to the LED lamp ($\hat{\dot{N}} = 11.31$ \$/yr). Direct income effect rebound (Re_{dinc}) is 0.01%, positive but small, as the lamp user allocates some of the net savings to additional demand for lighting. The indirect income effect rebound is large ($Re_{iinc} = 17.4\%$), due to the energy implications of increased spending on other goods. Total microeconomic level rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 28.1\%$.

Finally, macro effect rebound (Re_{macro}) is 13.0% with k assumed to be 1, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (lighting).

Table 11: Lamp example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	-0.3
Re_{md}	0.0
Re_{dsub}	17.4
Re_{isub}	-6.4
Re_{dinc}	0.0
Re_{iinc}	17.4
Re_{macro}	13.0
Re_{tot}	41.1

[3]Maintenance cost rates for both incandescent and LED lamps are likely to be equal and negligible; lamps are usually installed and forgotten. Real-world disposal cost differences between the incandescent and LED technologies are also likely to be negligible. However, if "disposal" includes recycling processes, cost rates may be different between the two technologies due to the wide variety of materials in LED lamps compared to incandescent lamps.

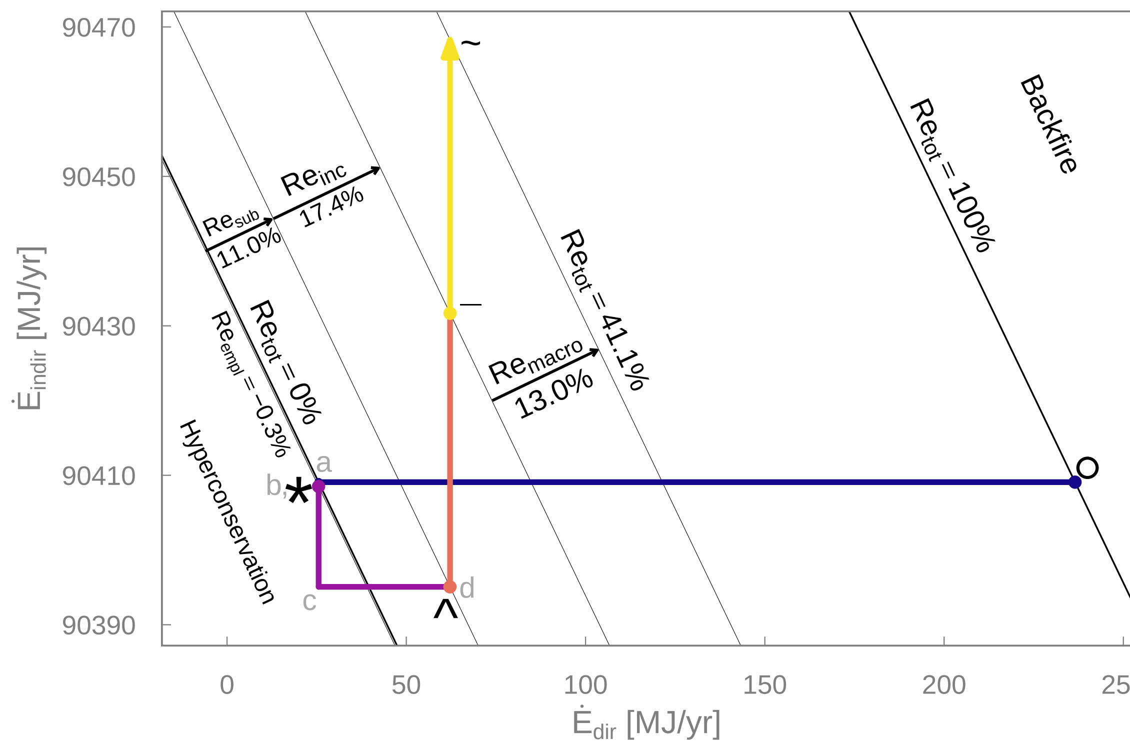

Fig. 8: The energy plane for the lamp example. Macro factor (k) is assumed to be 1. See Table 7 for meanings of path segments.

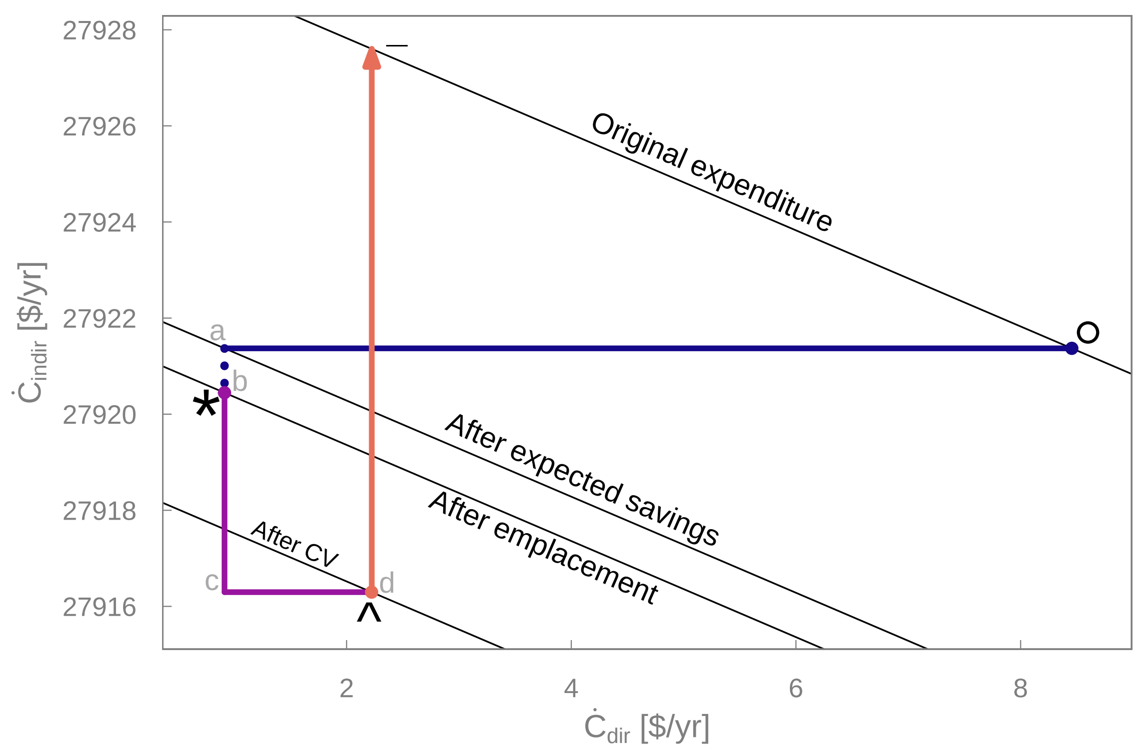

Fig. 9: Expenditure plane for the lamp example. CV is compensating variation, the increase in consumption of the energy service and decrease in consumption of other goods and services to maintain constant utility. See Table 7 for meanings of path segments.

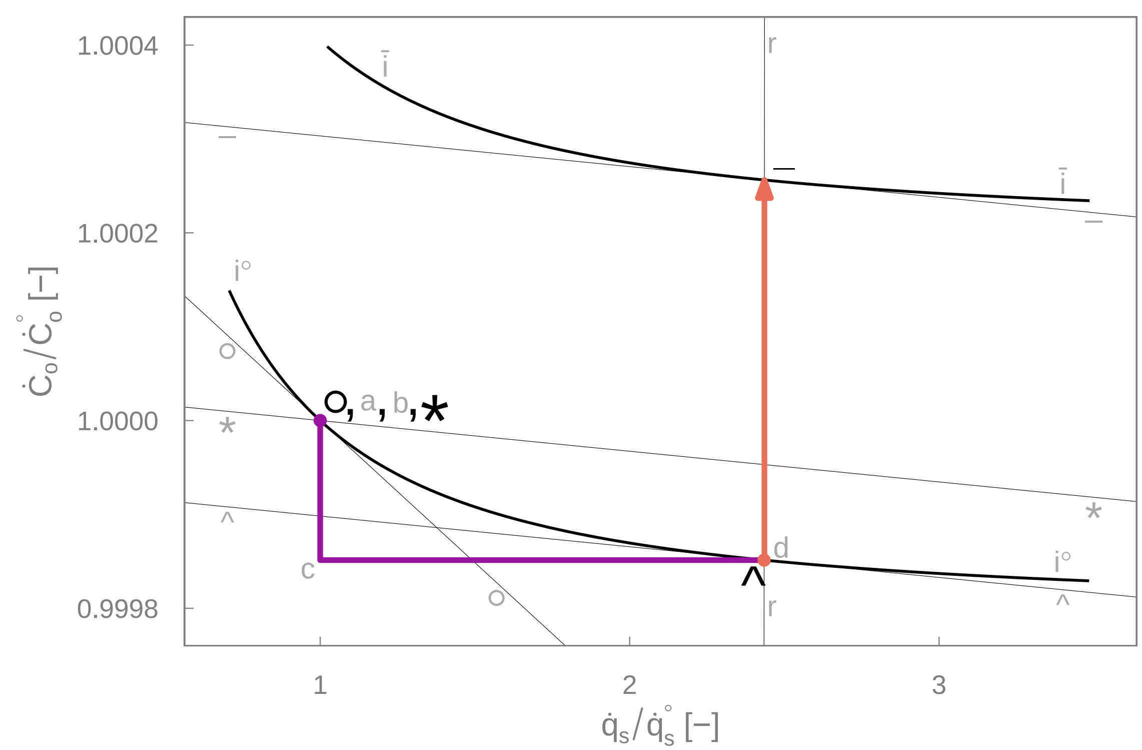

Fig. 10: Consumption plane for the lamp example. See Table 7 for meanings of path segments.

4 Discussion

4.1 A first attempt at calibrating k

Few previous studies explored link between microeconomic and macroeconomic rebound. Inspired by Borenstein (2015) and others, the framework developed in Section 2 of Part I links macroeconomic rebound to microeconomic rebound via the macro factor (k) that scales magnitudes in the microeconomic portion of the framework. (See Section 2.5.4 of Part I.)

For the results presented in Section 3 above, we assumed a placeholder value of $k = 1$, meaning that every \$1 of spending by the device user in the income effect generates only \$1 of additional economic activity in the broader economy. In combination, the framework presented in Section 2 of Part I, the results obtained in Section 3 of this paper, and recent calculations of total rebound in general equilibrium frameworks allow, for the first time, a discussion about calibrating k. After calibrating k, macro rebound and total rebound can be calculated.

To calibrate the macro factor (k), we treat macro rebound (Re_{macro}) as a residual. The macro factor (k) becomes an unknown parameter whose value is to be chosen such that Re_{macro} is sufficient to achieve an expected value for total rebound (Re_{tot}).[4] We take the expected value for Re_{tot} from Brockway et al. (2021). Four of 33 studies reviewed by Brockway et al. (2021) examined total rebound from only consumer EEUs in a computable general equilibrium (CGE) framework. The average total rebound (Re_{tot}) for the four consumer studies is 54%.[5] The calibrated values of k that give identical $Re_{tot} = 54\%$ for both examples are $k = 3.9$ for the car example and $k = 2.0$ for the lamp example.

Qualitative differences in benefits from EEUs as well as the considerable variance in Re_{tot} in 33 surveyed studies (Brockway et al., 2021) indicates that total rebound from one EEU is likely to be different from total rebound from another EEU. For a first approximation of a calibration for k, we take $k \approx 3$, being between the values of k calculated from the car and lamp examples. Note that $k \approx 3$ implies that every \$1 of net savings spent by the device user ($\dot{N}^*$) generates \$3 of additional economic activity in the broader economy. We multiply $k\dot{N}^*$ by the energy intensity of the economy (I_E) to find the energy implications of macro-effect respending throughout the economy.

There are three ways to interpret $k \approx 3$. First, $k \approx 3$ can be considered the average long-run economic growth generated by the productivity increase implied by the EEU and subsequent productivity increases benefitting from the EEU. Efficiency increases in equipment drive a significant part of long-run productivity growth (Greenwood et al., 1997), therefore a large long-run effect is plausible, even if the initial productivity change occurred in household production which is not accounted in GDP. (See Section 2.5.4 of Part I for further discussion of this point.) Second, it could be that growth is less than \$3 for every \$1 of respending, but that the macroeconomic "energy price effect" (a decline in energy prices due to the fallen demand) induces consumption at a higher energy intensity than that of the pre-EEU economy. Third, from the demand-side perspective entertained by Borenstein (2015), $k \approx 3$ could be interpreted as growth induced by the device user's spending of net savings with a marginal propensity to consume (MPC) of approximately 0.75 that translates into a multiplier of 3. (See Fig. F.1 in Appendix F of Part I.) $MPC \approx 0.75$ is a reasonable value, being in the upper half of recent estimates from Carroll et al. (2017). Although the cause of the

[4]This approach means that the calibrated value of k incorporates all macroeconomic rebound sub-effects included in the studies whose total rebound value we calibrate against.

[5]The average total rebound among all 33 studies stood at 63%, supporting the claim by Turner (2013) that consumer and producer rebounds vary.

growth in economic activity and energy consumption from an EEU is a supply-side productivity shock, the subsequent demand-side effects may well be interpreted as a multiplier effect, caused by higher real income instead of by higher monetary income.

After calibrating $k \approx 3$, we can recalculate all rebound components in our framework. Emplacement (Re_{empl}), substitution (Re_{sub}), and income (Re_{inc}) rebound magnitudes are unchanged after calibrating $k \approx 3$. However, we see that choosing a placeholder value of $k = 1$ resulted in a low value for Re_{macro} and, therefore, Re_{tot} in Section 3. In Figs. 5 and 8, the macro effect segments ($-$ $\sim$) should be three times longer than they appear. In Tables 9 and 11, the values of macro rebound (Re_{macro}) should triple to 23.2% and 39.0%, and the values of total rebound (Re_{tot}) should increase to 47.2% and 67.1% for the car and lamp examples, respectively. For the remainder of this paper, we use the calibrated value of $k \approx 3$.

4.2 Comparison between the car and lamp case studies

Tables 9 and 11 and our calibration of $k \approx 3$ in Section 4.1 enable fuller comparisons between the car and lamp examples. Several points can be made.

First, the magnitude of every rebound effect is different between the two examples, the exception being direct emplacement rebound (Re_{dempl}) which is always 0.0 by definition. The implication is that every EEU needs to be analyzed separately. Values for rebound effects for one EEU should never be assumed to apply to a different EEU.

Second, one cannot know *a-priori* which rebound effects will be large and which will be small for a given EEU. Furthermore, some rebound effects are dependent upon economic parameters, such as energy intensity (I_E). Thus, it is important to calculate the magnitude of all rebound effects for each EEU in each economy.

Third, the two examples illustrate the fact that embodied energy rebound (Re_{emb}) can be positive or negative, as discussed in Section 2.5.1 of Part I. The car's embodied energy rebound is positive (1.7%) because of the high embodied energy of the EV's battery relative to the internal combustion engine vehicle. Although the LED lamp's embodied energy is larger than the incandescent lamp's embodied energy, the LED lamp's embodied energy rebound is negative (-0.3%), due to the longer life of the LED lamp compared to the incandescent lamp. Thus, each EEU should be analyzed independently for its embodied energy rebound.

Fourth, macro effect rebound is different between the two examples, owing to differences in net income ($\dot{N}^*$) relative to expected savings ($\dot{S}_{dev}$). (For the car, Re_{macro} is 23.2%. For the lamp, Re_{macro} is 39.0%.) The efficiency gain for the lamp is far greater than the efficiency gain for the car, leading to much different rates of net income ($\dot{N}^*$) and different macro rebound values.

4.3 Comparison to previous rebound estimates

Tables 12 and 13 compare car and lamp results (with $k \approx 3$) to results from previous studies. The comparison studies are neither comprehensive nor definitive of car and lamp EEUs; rather, they are examples that show the sort of calculations and estimations carried out in the general literature using a variety of methods. That said, many of the studies are highly cited, thereby carrying sufficient academic weight for our purposes. Tables 12 and 13 and their associated references enable two types of observations, comparing (i) coverage of rebound components and (ii) magnitudes and associated calculation or estimation methods.

Table 12: Rebound magnitude comparisons for the car example. All numbers in %. Note that $Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro}$, $Re_{tot} = Re_{micro} + Re_{macro}$, and $Re_{tot} = Re_{dir} + Re_{indir}$.

	Rebound study	Coverage	Analysis method	Re_{micro}			Re_{macro}	Re_{dir}	Re_{indir}	Re_{tot}
				Re_{empl}	Re_{sub}	Re_{inc}				
	This paper (2023)	U.S., 2020	Energy, expenditure, and consumption planes	1.4	9.9	12.6	23.2	15.9	31.3	47.2
1	Small & Van Dender (2007)	U.S., 1967–2001	Elasticity of VMT w.r.t. fuel cost per mile					4.5 (short run, 1967–2001) 22.2 (long run, 1967–2001) 2.2 (short run, 1997–2001) 10.7 (long run, 1997–2001)		
2	Greene (2012)	U.S., 1966–2007	Elasticities of transport fuel w.r.t. price & efficiency					4 (short run) 16 (long run)		
3	Koesler (2013)	Germany, 2009	Static CGE model, 10% efficiency shock					≤ 64	≤ 16	56
4	Thomas & Azevedo (2013)	U.S., 2004	Expenditure/cross price elasticities of personal transport fuels, using household spending survey data					10	6	
5	Borenstein (2015)	U.S., 2012	Microeconomic framework		13 (6–28)	11				
6	Chitnis & Sorrell (2015)	UK, 1964–2014	Estimated own/cross price elasticities of transport fuels, uses household spending survey data		72	5		55	23	86
7	Gillingham et al. (2015)	Pennsylvania, 2000–2010	Estimation of gasoline price elasticity of driving demand, from dataset of 75 million vehicle inspection records, including odometer data					10 (short run)		
8	Stapleton et al. (2016)	UK 1970–2011	Elasticity of VMT w.r.t. fuel cost/prices					9–36		
9	Moshiri & Aliyev (2017)	Canada, 1997–2009	Price elasticity of transport fuel, using household spending survey data					82–88		
10	Duarte et al. (2018)	Spain, 2010–2030	Dynamic CGE model, efficiency shock							26 (short run) 52 (long run)

Table 13: Rebound magnitude comparisons for the lamp example. All numbers in %. Note that $Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro}$, $Re_{tot} = Re_{micro} + Re_{macro}$, and $Re_{tot} = Re_{dir} + Re_{indir}$.

	Rebound study	Coverage	Analysis method		Re_{micro}		Re_{macro}	Re_{dir}	Re_{indir}	Re_{tot}
				Re_{empl}	Re_{sub}	Re_{inc}				
	This paper (2023)	U.S., 2020	Energy, expenditure, and consumption planes	−0.3	11.0	17.4	39.0	17.4	49.7	67.1
1	Guertin et al. (2003)	Canada, 1993	Econometric residential energy demand model based on Canadian household data					32–49		
2	Freire-González (2011)	Catalonia, Spain, 2000–2008	Input-output based energy model, utilising expenditure/cross price elasticities					49	16	
3	Thomas & Azevedo (2013)	U.S., 2004	Expenditure/cross price elasticities of home electricity use, using household spending survey data					10	10	
4	Schleich et al. (2014)	Germany, 2012	Survey of electricity consumption in 6409 German households					6		
5	Borenstein (2015)	U.S., 2012	Microeconomic framework		14 (6–37)	6				
6	Chitnis & Sorrell (2015)	UK, 1964–2014	Estimated own/cross price elasticities of transport fuels, uses household spending survey data		14	35		41	8	49
7	Duarte et al. (2018)	Spain, 2010–2030	Dynamic CGE model, efficiency shock							12 (short run) 51 (long run)
8	Barkhordar (2019)	Iran, 2018–2040	Dynamic CGE model					28 (average)		43 (average)
9	Chitnis et al. (2020)	UK, 1964–2015	Household demand analysis via Linear approximation to the Almost Ideal Demand System (LAIDS)					95	−41	54
10	Shojaeddini & Gilbert (2022)	U.S., 2009	Price elasticity of lighting from cross sectional data from the 2009 Residential Energy Consumption Survey (RECS)					18–29		

First, we see that none of the comparison studies report all rebound effects, as we have done in Sections 3.1, 3.2, and 4.1. Also, no previous studies report either emplacement rebound ($Re_{empl} = Re_{emb} + Re_{md}$) or include all of direct and indirect, substitution and income microeconomic rebound effect combinations. In addition, none of the other studies report macro rebound (Re_{macro}) by itself. In fact, only 4 or 5 of the 10 studies in each category (car and lamp, respectively) report total rebound (Re_{tot}). Therefore, by carefully including all rebound components in the framework and elucidating all rebound components in Part II, we are (i) adding conceptual clarity to the field of energy rebound, which (ii) may enable future studies to estimate a broader range of rebound components.

We also observe that studies which provide total rebound are based on a top-down calculation of overall, economy-wide rebound, rather than the bottom-up "sum-of-components" approach that we employ. That finding is instructive. It supports the view that a rigorous analysis framework that sets out individual rebound components has been missing, which informed the objective for Part I of this paper. Further, the finding means that comparisons between top-down estimations or calculations of total, economy-wide rebound may also be of limited value, because the rebound effects included or excluded may not be clear, giving an appearance of a "black box" calculation approach.[6]

Second, helpful insights can be gained from comparison of rebound magnitudes and calculation methods. Greatest alignment between our values and earlier values appears within the direct (microeconomic) rebound (Re_{dir}) column in Tables 12 and 13. Our car (15.9%) and lamp (17.4%) values are in the lower half of the comparison studies for both cases (10% to 49% for the car and 10% to 55% for the lamp). This alignment may be due to the easier determination of direct rebound, from either empirical data (e.g., Small & Van Dender (2007)) or via own price elasticities (e.g., Chitnis & Sorrell (2015)).[7]

For indirect rebound (Re_{indir}), there is little agreement on the magnitude of rebound effects. Our values for car (31.3%) and lamp (49.7%) indirect rebound magnitudes are higher than those found

[6]That said, without the top-down approaches, we would not have the information needed to calibrate the macro factor (k) in Section 4.1.

[7]Also worthy of note is that direct (microeconomic) rebound of personal transport may be the most-studied subfield in the rebound literature and likely the only topic with enough studies to enable meta-reviews such as Sorrell et al. (2009), Dimitropoulos et al. (2018), and Gillingham (2020).

in the comparison studies for either the car (6% to 23%) or the lamp (8% to 16%) cases. The most likely cause of our larger indirect rebound values is that we include both micro and macro rebound levels, whereas the comparison studies focus mainly on microeconomic rebound only (commonly via cross price elasticities). In other words, comparisons of our indirect rebound values with the studies in Tables 12 and 13 may be too simple and not very meaningful, as we (alone) include macro-level effects in indirect rebound. If we exclude Re_{macro} from Re_{indir}, our indirect microeconomic rebound values become 8.1% (car) and 10.7% (lamp), which fit within the ranges reported by the car (6% to 23%) and lamp (−41% to 16%) comparison studies.

For total rebound (Re_{tot}), our values of 47.2% (car) and 67.1% (lamp) are very close to those in the comparison studies for both the car (49% to 51%) and lamp (43% to 51%) examples. Beyond that, comparisons (as noted earlier) are inhibited by methodological differences between previous studies (top-down methods) and our bottom-up approach for calculating total rebound.

5 Conclusions

In this paper (Part II), we advance clarity to the field of energy rebound by (i) developing of the first (to our knowledge) mutually consistent and numerically precise visualizations of rebound effects in energy, expenditure, and consumption planes, (ii) calibrating the macro factor ($k \approx 3$), (iii) documenting in detail new calculations of rebound for car and lighting upgrades, and (iv) providing information about new open source software tools for calculating and visualizing rebound for any energy efficiency upgrade. We encourage energy analysts and economists to use visualizations like the energy, expenditure, and consumption planes to document rebound calculations going forward. Our hope is that additional clarity will (i) narrow the gap between economists and energy analysts, (ii) lead to deeper interdisciplinary understanding of rebound phenomena, and (iii) enable energy and climate policy that takes full account of rebound.

From the development and application of the framework in Part II, we draw two important conclusions. First, the car and lamp examples (Section 3) show that the framework enables quantification of rebound magnitudes at microeconomic and macroeconomic levels, including energy, expenditure, and consumption aspects of direct and indirect rebound for emplacement, substitution, income, and macro effects. Second, the examples show that magnitudes of all rebound effects vary with the type of EEU performed. Thus, values for rebound effects for one EEU should never be assumed to apply to a different EEU, and it is important to calculate the magnitude of all rebound effects for each EEU in each economy.

Further work could be pursued in several areas. (i) Additional empirical studies could be performed to calculate the magnitude of different rebound effects for a variety of real-life EEUs. (ii) Deeper study of macro rebound is needed, including improved determination of the value of the macro factor (k) and its relation to the MPC. (iii) The rebound implications of the distribution of MPC values across socioeconomic groups (Carroll et al., 2017) could be explored. (iv) The rebound effects of fossil-energy taxes could be studied, especially for the web of interconnected dynamic effects among rebound components that are functions of the energy intensity of the economy (I_E). (v) Sensitivities of rebound components to model parameters could be investigated more fully than in Appendix C, although this will be challenging work because many rebound parameters are covariant. For example, post-EEU efficiency ($\tilde{\eta}$) is unlikely to be independent of post-EEU capital cost ($\tilde{C}_{cap}$). (vi) This framework could be embedded in energy-economy models to better include rebound effects in discussions of macro energy modeling, energy policy, and CO_2 emissions mitigation.

Competing interests

Declarations of interest: none.

Author contributions

Author contributions for this paper (Part II of the two-part paper) are shown in Table 14.

Data repository

**** Data and example calculations in spreadsheet format will be stored at the Research Data Leeds Repository if this submission is accepted. The spreadsheet file is included with the submission of

Table 14: Author contributions.

	MKH	GS	PEB
Conceptualization	●	●	
Methodology	●	●	●
Software	●		●
Validation	●		●
Formal analysis	●	●	
Investigation	●	●	●
Resources	●	●	●
Data curation			●
Writing–original draft	●	●	
Writing–review & editing	●	●	●
Visualization	●	●	
Supervision	●		
Project administration	●		
Funding acquisition			●

this paper. ****

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Table A.1: Symbols and abbreviations.

Symbol	Meaning [example units]
a	a point in the emplacement effect in rebound planes or the share parameter in the CES utility model [–]
b	a point in the emplacement effect in rebound planes
C	cost [$]
c	a point in the substitution effect in rebound planes
d	a point in the income effect on rebound planes
E	final energy [MJ]
f	expenditure share [–]
G	freed cash [$]
I	energy intensity of economic activity [MJ/$]
k	macro factor [–]
M	income [$]
N	net savings [$]
p	price [$]
q	quantity [–]
Re	rebound [–]
S	energy cost savings [$]
t	energy conversion device lifetime [yr]
u	utility [utils]
x	the abscissa coordinate
y	the ordinate coordinate

Appendices

A Nomenclature

Presentation of the rigorous analytical framework is aided by a nomenclature that describes energy stages and rebound effects. Table A.1 shows symbols and abbreviations, their meanings, and example units. Table A.2 shows Greek letters, their meanings, and example units. Table A.3 shows abbreviations and acronyms. Table A.4 shows symbol decorations and their meanings. Table A.5 shows subscripts and their meanings.

Differences are indicated by the Greek letter Δ and always signify subtraction of a quantity at an earlier stage of Fig. 1 from the same quantity at the next later stage of Fig. 1. E.g., $\Delta\bar{X} \equiv \bar{X} - \hat{X}$, and $\Delta\tilde{X} \equiv \tilde{X} - \bar{X}$. Lack of decoration on a difference term indicates a difference that spans all stages of Fig. 1. E.g., $\Delta X \equiv \tilde{X} - X^{\circ}$. ΔX is also the sum of differences across each stage in Fig. 1, as shown below.

Table A.2: Greek letters.

Greek letter	Meaning [example units]
Δ	difference (later quantity less earlier quantity, see Fig. 1)
ε	price or income elasticity [–]
$\varepsilon_{\dot{q}_s,\dot{M}}$	income ($\dot{M}$) elasticity of energy service demand ($\dot{q}_s$) [–]
$\varepsilon_{\dot{q}_o,\dot{M}}$	income ($\dot{M}$) elasticity of other goods demand ($\dot{q}_o$) [–]
$\varepsilon_{\dot{q}_s,p_s}$	uncompensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\varepsilon_{\dot{q}_o,p_s}$	uncompensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
$\varepsilon_{\dot{q}_s,p_s,c}$	compensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\varepsilon_{\dot{q}_o,p_s,c}$	compensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
η	final-energy-to-service efficiency [vehicle-km/MJ]
ρ	exponent in the CES utility function, $\rho \equiv (\sigma - 1)/\sigma$ [–]
σ	elasticity of substitution between the energy service ($\dot{q}_s^\circ$) and other goods ($\dot{q}_o^\circ$) [–]

Table A.3: Abbreviations.

Abbreviation	Meaning
APF	aggregate production function
CES	constant elasticity of substitution
CGE	computable general equilibrium
CPE	constant price elasticity
CV	compensating variation
EEU	energy efficiency upgrade
EPSRC	engineering and physical sciences research council
EV	electric vehicle
GDP	gross domestic product
LAIDS	linear approximation to almost ideal demand system
LED	light emitting diode
MPC	marginal propensity to consume
mpg	miles per U.S. gallon
RECS	residential energy consumption survey
UK	United Kingdom
UKRI	UK research and innovation
U.S.	United States
VMT	vehicle miles traveled
w.r.t.	with respect to

Table A.4: Decorations.

Decoration	Meaning [example units]
X°	X originally (before the emplacement effect)
X^*	X after the emplacement effect (before the substitution effect)
$\hat{X}$	X after the substitution effect (before the income effect)
$\bar{X}$	X after the income effect (before the macro effect)
$\tilde{X}$	X after the macro effect
$\dot{X}$	rate of X [units of X/yr]
M'	effective income [$]

Table A.5: Subscripts.

Subscript	Meaning
0	quantity at an initial time
1	a specific point on the consumption plane
c	compensated
cap	capital costs
dev	device
$dempl$	direct emplacement effect
$dinc$	direct income effect
dir	direct effects (at the energy conversion device)
$dsub$	direct substitution effect
E	energy
emb	embodied
$empl$	emplacement effect
$iempl$	indirect emplacement effects
$iinc$	indirect income effect
inc	income effect
$indir$	indirect effects (beyond the energy conversion device)
$isub$	indirect substitution effect
$life$	lifetime
$macro$	macro effect
md	maintenance and disposal
o	other expenditures (besides energy) by the device user
own	ownership duration
s	service stage of the energy conversion chain
sub	substitution effect
tot	sum of all rebound effects in the framework

$$
\begin{aligned}
\Delta X &= \Delta\tilde{X} + \Delta\bar{X} + \Delta\hat{X} + \Delta X^* \\
\Delta X &= (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^*) + (X^* - X^\circ) \\
\Delta X &= (\tilde{X} - \cancel{\bar{X}}) + (\cancel{\bar{X}} - \cancel{\hat{X}}) + (\cancel{\hat{X}} - \cancel{X^*}) + (\cancel{X^*} - X^\circ) \\
\Delta X &= \tilde{X} - X^\circ
\end{aligned} \tag{1}
$$

B Mathematical details of rebound planes

Rebound planes show the impact of direct and indirect rebound effects for energy, expenditure, and consumption aspects. Notional rebound planes can be found in Figs. 2–4. Rebound planes for the car example can be found in Figs. 5–7. Rebound planes for the lamp example can be found in Figs. 8–10.

This appendix shows the mathematical details of rebound planes, specifically derivations of equations for lines and curves shown in Table B.1. The lines and curves enable construction of numerically precise and accurate paths in rebound planes as shown in Figs. 5–10.

Table B.1: Lines and curves for rebound planes.

Rebound plane	Lines and curves
Energy	Constant total energy consumption lines 0% and 100% rebound lines
Expenditure	Constant expenditure lines
Consumption	Constant expenditure lines Rays from origin to $\wedge$ point Indifference curves

B.1 Energy planes

The energy plane shows direct (on the x-axis) and indirect (on the y-axis) energy consumption associated with the energy conversion device and the device user. Lines of total energy consumption isoquants provide a scale for total rebound. For example, the 0% and 100% rebound lines are constant total energy consumption lines which pass through the original point ($\circ$) and the post-direct-emplacement-effect point (a) in the energy plane.

The equation of a constant total energy consumption line is derived from

$$\dot{E}_{tot} = \dot{E}_{dir} + \dot{E}_{indir} \tag{2}$$

at any rebound stage. Direct energy consumption is energy consumed by the energy conversion device ($\dot{E}_s$), and indirect energy consumption is the sum of embodied energy, energy associated with maintenanace and disposal, and energy associated with expenditures on other goods ($\dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$).

For the energy plane, direct energy consumption is placed on the x-axis and indirect energy consumption is placed on the y-axis. To derive the equation of a constant energy consumption line, we first rearrange to put the y coordinate on the left of the equation:

$$\dot{E}_{indir} = -\dot{E}_{dir} + \dot{E}_{tot} \ . \tag{3}$$

Next, we substitute y for $\dot{E}_{indir}$, x for $\dot{E}_{dir}$, and $\dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$ for $\dot{E}_{tot}$ to obtain

$$y = -x + \dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E \ , \tag{4}$$

where all of $\dot{E}_s$, $\dot{E}_{emb}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage.

The constant total energy consumption line that passes through the original point (∘) shows 100% rebound:

$$y = -x + \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ) I_E \ . \tag{5}$$

The 0% rebound line is the constant total energy consumption line that accounts for expected energy savings ($\dot{S}_{dev}$) only:

$$y = -x + (\dot{E}_s^\circ - \dot{S}_{dev}) + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ) I_E \ . \tag{6}$$

The above line passes through the a point in the energy plane.

B.2 Expenditure planes

The expenditure plane shows direct (on the x-axis) and indirect (on the y-axis) expenses associated with the energy conversion device and the device user. Lines of constant expenditure are important, because they provide budget constraints for the device user.

The equation of a constant total expenditure line is derived from the budget constraint

$$\dot{C}_{tot} = \dot{C}_{dir} + \dot{C}_{indir} \tag{7}$$

at any rebound stage. In the expenditure plane, indirect expenditures are placed on the y-axis and direct expenditures on energy for the energy conversion device are place on the x-axis. Direct expenditure is the cost of energy consumed by the energy conversion device ($\dot{C}_s = p_E \dot{E}_s$), and indirect expenses are the sum of capital costs, maintenanace and disposal costs, and expenditures on other goods ($\dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$). Rearranging to put the y-axis variable on the left side of the equation gives

$$\dot{C}_{indir} = -\dot{C}_{dir} + \dot{C}_{tot} \ . \tag{8}$$

Substituting y for $\dot{C}_{indir}$, x for $\dot{C}_{dir}$, and $\dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$ for $\dot{C}_{tot}$ gives

$$y = -x + \dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o \ , \tag{9}$$

where all of $\dot{C}_s$, $\dot{C}_{cap}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage.

The constant total expenditure line that passes through the original point ($\circ$) shows the budget constraint for the device user:

$$y = -x + \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \ , \tag{10}$$

into which Eq. (37) of Part I can be substituted with $\dot{C}_s^\circ = p_E \dot{E}_s^\circ$ and $\dot{N}^\circ = 0$ to obtain

$$y = -x + \dot{M}^\circ \ . \tag{11}$$

The constant total expenditure line that accounts for expected energy savings ($\dot{S}_{dev}$) and freed cash ($\dot{G} = p_E \dot{S}_{dev}$) only is given by:

$$y = -x + (\dot{C}_s^\circ - \dot{G}) + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \ , \tag{12}$$

or

$$y = -x + \dot{M}^\circ - \dot{G} \ . \tag{13}$$

The line given by the equation above passes through the a point in the expenditure plane.

B.3 Consumption planes

The consumption plane shows expenditures in the $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ plane, according to the utility model. (See Appendix C of Part I.) Consumption planes include (i) constant expenditure lines given prices, (ii) a ray from the origin through the $\wedge$ point, and (iii) indifference curves. Derivations for each are shown in the following subsections.

B.3.1 Constant expenditure lines

There are four constant expenditure lines in the consumption planes of Figs. 4, 7, and 10. The constant expenditure lines pass through the original point (line $\circ$—$\circ$), the post-emplacement point (line $*$—$*$), the post-substitution point (line $\wedge$—$\wedge$), and the post-income point (line $-$—$-$). Similar to the expenditure plane, lines of constant expenditure in the consumption plane are derived from the budget constraint of the device user at each of the four points.

Prior to the EEU, the budget constraint is given by Eq. (37) of Part I. Substituting $p_s^\circ \dot{q}_s^\circ$ for $p_E \dot{E}_s^\circ$ and recognizing that there is no net savings before the EEU ($\dot{N}^\circ = 0$) gives

$$\dot{M}^\circ = p_s^\circ \dot{q}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \; . \tag{14}$$

To create the line of constant expenditure in the consumption plane, we allow $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to vary in a compensatory manner: when one increases, the other must decrease. To show that variation along the constant expenditure line, we remove the notation that ties $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to the original point ($\circ$) to obtain

$$\dot{M}^\circ = p_s^\circ \dot{q}_s + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o \; , \tag{15}$$

where all of $\dot{M}^\circ$, p_s°, $\dot{C}_{cap}^\circ$, and $\dot{C}_{md}^\circ$ apply at the same rebound stage, namely the original point ($\circ$) in this instance.

To derive the equation of the line representing the original budget constraint in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space (the $\circ$—$\circ$ line through the $\circ$ point in consumption planes), we solve for $\dot{C}_o$ to obtain

$$\dot{C}_o = -p_s^\circ \dot{q}_s + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{16}$$

Multiplying judiciously by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ}\dot{C}_o^\circ = -p_s^\circ \frac{\dot{q}_s}{\dot{q}_s^\circ}\dot{q}_s^\circ + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{17}$$

Dividing both sides by $\dot{C}_o^\circ$ yields

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}\frac{\dot{q}_s}{\dot{q}_s^\circ} + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \; . \tag{18}$$

Noting that $\dot{q}_s/\dot{q}_s^\circ$ and $\dot{C}_o/\dot{C}_o^\circ$ are the x-axis and y-axis, respectively, of the consumption plane gives

$$y = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \; . \tag{19}$$

A similar procedure can be employed to derive the equation of the $*$—$*$ line through the $*$ point after the emplacement effect. The starting point is the budget constraint at the $*$ point (Eq. (39) of Part I) with $\dot{M}^\circ$ replacing $\dot{M}^*$, $\tilde{p}_s\dot{q}_s$ replacing $p_E\dot{E}_s^*$, and $\dot{C}_o$ replacing $\dot{C}_o^*$.

$$\dot{M}^\circ = \tilde{p}_s\dot{q}_s + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o + \dot{N}^* \tag{20}$$

Substituting Eq. (48) of Part I for $\dot{N}^*$, substituting Eq. (49) of Part I to obtain $\dot{G}$, multiplying judiciously by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$, rearranging, and noting that $\dot{q}_s/\dot{q}_s^\circ$ is the x-axis and $\dot{C}_o/\dot{C}_o^\circ$ is the y-axis gives

$$y = -\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G}) \ . \tag{21}$$

Note that the slope of Eq. (21) is less negative than the slope of Eq. (19), because $\tilde{p}_s < p_s^\circ$. The y-intercept of Eq. (21) is less than the y-intercept of Eq. (19), reflecting freed cash. Both effects are seen in the consumption planes (Figs. 4, 7, and 10). The $\circ$—$\circ$ and $*$—$*$ lines intersect at the coincident $\circ$ and $*$ points.

A similar derivation process can be used to find the equation of the line representing the budget constraint after the substitution effect (the $\wedge$—$\wedge$ line through the $\wedge$ point). The starting point is Eq. (52) of Part I, and the equation for the constant expenditure line is

$$y = -\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G} + \tilde{p}_s\Delta\hat{\dot{q}}_s + \Delta\hat{\dot{C}}_o) \ . \tag{22}$$

Note that the $\wedge$—$\wedge$ line (Eq. (22)) has the same slope as the $*$—$*$ line (Eq. (21)) but a lower y-intercept.

Finally, the corresponding derivation for the equation of the constant expenditure line through the $-$ point (line $-$—$-$) starts with Eq. (61) of Part I and ends with

$$y = -\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^*) \ . \tag{23}$$

B.3.2 Ray from the origin to the $\wedge$ point

In the consumption plane, the ray from the origin to the $\wedge$ point (line r—r) defines the path along which the income effect (lines $\wedge$—d and d—$-$) operates. The ray from the origin to the $\wedge$ point has slope $(\hat{\dot{C}}_o/\dot{C}_o^\circ)/(\hat{\dot{q}}_s/\dot{q}_s^\circ)$ and a y-intercept of 0. Therefore, the equation of line r—r is

$$y = \frac{\hat{\dot{C}}_o/\dot{C}_o^\circ}{\hat{\dot{q}}_s/\dot{q}_s^\circ}\, x \, . \tag{24}$$

B.3.3 Indifference curves

In the consumption plane, indifference curves represent lines of constant utility for the energy conversion device user. In the consumption plane ($\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$), any indifference curve is given by Eq. (118) of Part I with $f_{\dot{C}_s}^\circ$ replacing the share parameter a, as shown in Appendix C of Part I. Recognizing that $\dot{C}_o/\dot{C}_o^\circ$ is on the y-axis and $\dot{q}_s/\dot{q}_s^\circ$ is on the x-axis leads to substitution of y for $\dot{C}_o/\dot{C}_o^\circ$ and x for $\dot{q}_s/\dot{q}_s^\circ$ to obtain

$$y = \left[\frac{1}{1-f_{\dot{C}_s}^\circ}\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}(x)^\rho\right]^{(1/\rho)} \, . \tag{25}$$

At any point on the $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ plane, namely ($\dot{q}_{s,1}/\dot{q}_s^\circ$, $\dot{C}_{o,1}/\dot{C}_o^\circ$), indexed utility ($\dot{u}_1/\dot{u}^\circ$) is given by Eq. (14) of Part I as

$$\frac{\dot{u}_1}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ}\right)^\rho + (1-f_{\dot{C}_s}^\circ)\left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ}\right)^\rho\right]^{(1/\rho)} \, . \tag{26}$$

Substituting Eq. (26) into Eq. (25) for $\dot{u}/\dot{u}^\circ$ and simplifying exponents gives

$$y = \left\{\frac{1}{1-f_{\dot{C}_s}^\circ}\left[f_{\dot{C}_s}^\circ\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ}\right)^\rho + (1-f_{\dot{C}_s}^\circ)\left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ}\right)^\rho\right] - \frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}(x)^\rho\right\}^{(1/\rho)} \, . \tag{27}$$

Simplifying further yields the equation of an indifference curve passing through point ($\dot{q}_{s,1}/\dot{q}_s^\circ$, $\dot{C}_{o,1}/\dot{C}_o^\circ$):

$$y = \left\{\left(\frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}\right)\left[\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ}\right)^\rho - (x)^\rho\right] + \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ}\right)^\rho\right\}^{(1/\rho)} \, . \tag{28}$$

Note that if x is $\dot{q}_{s,1}/\dot{q}_s^\circ$, y becomes $\dot{C}_{o,1}/\dot{C}_o^\circ$, as expected.

C Univariate sensitivity analyses

Sensitivity analyses show the effect of independently varied parameters on total rebound and rebound components. In the context of this framework, sensitivity analyses can show important trends, tendencies, and relationships between rebound parameters and rebound magnitudes. Key rebound parameters include post-EEU efficiency ($\tilde{\eta}$), post-EEU capital cost ($\tilde{C}_{cap}$), energy price (p_E), pre-EEU uncompensated price elasticity of energy service demand ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$), the macro factor ($k$), and post-EEU energy service price ($\tilde{p}_s$). Univariate sensitivity analyses (the kind shown here) should be interpreted carefully, because some rebound parameters are not expected to be independent from others.

C.1 Effect of post-EEU efficiency ($\tilde{\eta}$) on rebound terms

Fig. C.1 shows that both the energy takeback rate and expected energy savings ($\dot{S}_{dev}$) increase with post-EEU efficiency ($\tilde{\eta}$), but the relationship is asymptotic. Each unit increase of fuel economy or lighting efficiency is less effective than the previous unit increase of fuel economy or lighting efficiency for saving energy. At very high levels of fuel economy or lighting efficiency, a unit increase leads to almost no additional energy savings. Thus, we can say there are diminishing returns of fuel economy and lighting efficiency, leading to saturation of energy savings at very high levels of fuel economy and lighting efficiency. A simple example illustrates. A $\eta^{\circ} = 25$ mpg car drives $q_s^{\circ} = 100$ miles using $E_s^{\circ} = 4$ gallons of gasoline. A more-efficient car ($\tilde{\eta} = 30$ mpg) is expected to use $E_s^{*} = 3.33$ gallons to drive the same distance, a savings of $\dot{S}_{dev} = 0.67$ gallons. Another 5 mpg boost in efficiency (to $\tilde{\eta} = 35$ mpg) will use $E_s^{*} = 2.86$ gal to drive 100 miles, a further expected savings of only $\dot{S}_{dev} = 0.47$ gallons. Each successive 5 mpg boost in fuel economy saves less energy than the previous 5 mpg boost in fuel economy.

Saturation can be seen mathematically, too. Taking the limit as $\tilde{\eta} \to \infty$ in Eq. (10) of Part I gives $\dot{S}_{dev} = \dot{E}_s^{\circ}$, not ∞. Thus, efficiency saturation must occur. Fig. C.1 shows that this framework correctly replicates expected efficiency saturation trends.

Saturation is especially noticeable in the lamp example compared to the car example, the difference being that the LED lamp is already much more efficient than the incandescent lamp (9.26×), whereas the hybrid car is only 1.68× more efficient than the conventional gasoline car.

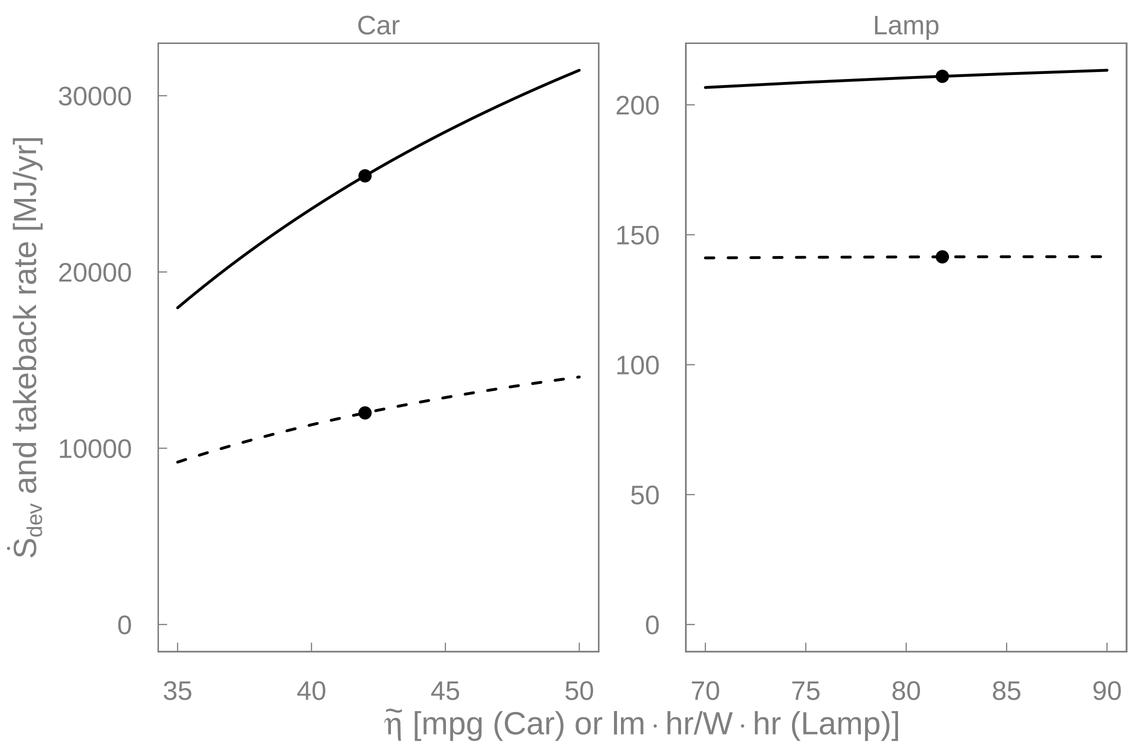

Fig. C.1: Expected energy savings rate ($\dot{S}_{dev}$, solid line) and takeback rate (dashed line) sensitivity to post-EEU efficiency ($\tilde{\eta}$). The macro factor is set to its calibrated value ($k = 3$). (Note different x- and y-axis scales.)

Thus, at $\tilde{\eta} = 81.8$ lm·hr/W·hr, the energy efficient LED is far closer to efficiency saturation than the hybrid vehicle (at $\tilde{\eta} = 42$ mpg). As a result, further increases in the LED lamp's efficiency are less effective than further increases in the hybrid car's efficiency.

That said, actual savings is the difference between the expected energy savings line (solid line) and the takeback line (dashed line) in Fig. C.1. Because the gap between the lines grows, higher efficiency yields greater energy savings, even after accounting for rebound effects. But the actual savings are always less than expected savings, due to takeback.

Fig. C.1 shows that expected energy savings ($\dot{S}_{dev}$) increase faster than takeback as $\tilde{\eta}$ increases. Thus, total rebound (Re_{tot}, the ratio of takeback rate to expected energy savings rate in Eq. (3) of Part I), decreases as efficiency grows. The lamp exhibits a relatively smaller rebound decline with efficiency, because the lamp example is closer to saturation than the car example.

Fig. C.2 shows the variation of all rebound components with post-EEU efficiency ($\tilde{\eta}$). In the car and lamp examples, direct substitution rebound (Re_{dsub}) is the rebound component most sensitive to changes in post-EEU efficiency ($\tilde{\eta}$).

Note that the sensitivity analysis on post-upgrade efficiency ($\tilde{\eta}$, Fig. C.2) is the only sensitivity analysis that requires careful explication of both the numerator and denominator of Eq. (3) in Part I, as in Fig. C.1, because both the numerator and denominator of Eq. (3) in Part I change when post-upgrade efficiency ($\tilde{\eta}$) changes. The denominator of Eq. (3) in Part I doesn't change for the sensitivity analyses of Figs. C.3–C.6. Thus, for the remaining sensitivity analyses, when the rebound percentage increases (decreases), the energy takeback rate in the numerator of Eq. (3) in Part I increases (decreases) proportionally, and the actual energy savings rate decreases (increases) accordingly.

C.2 Effect of capital cost ($\tilde{C}_{cap}$) on rebound terms

The sensitivity of energy rebound to capital cost ($\tilde{C}_{cap}$) is shown in Fig. C.3. All other things being equal, as capital cost of the EEU rises, less net savings result from the emplacement effect, leading to smaller income, macro, and total rebound. The same effects would be observed with increasing maintenance and disposal cost rate ($\tilde{\dot{C}}_{md}$).

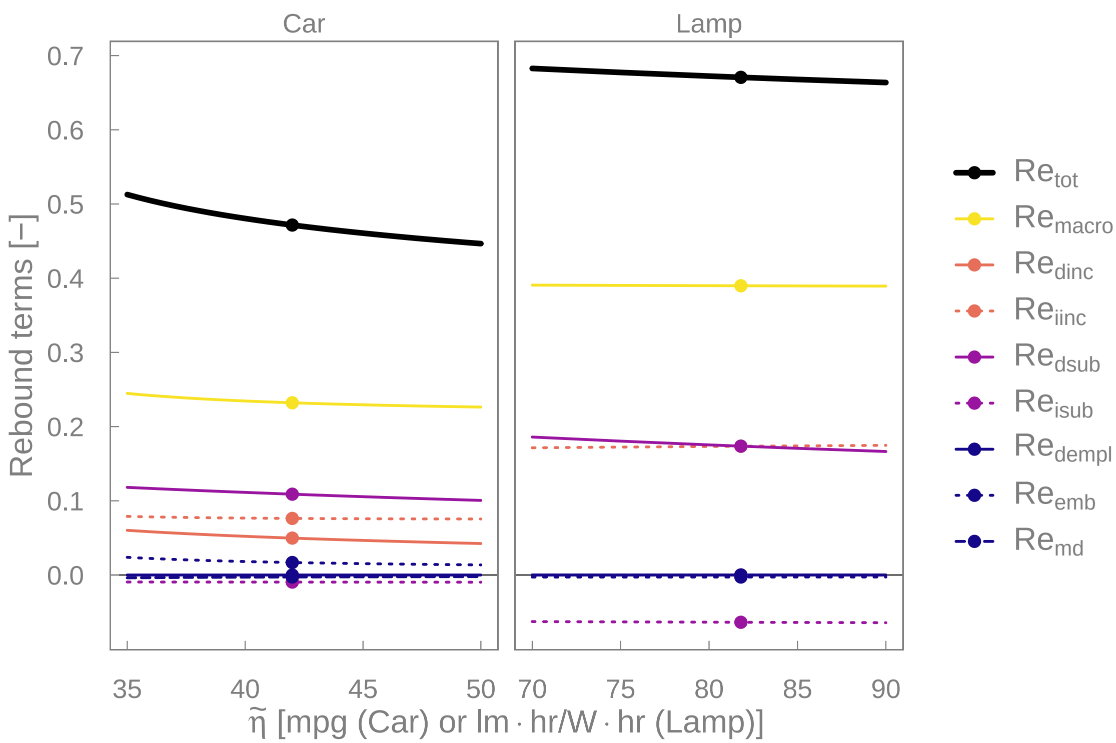

Fig. C.2: Sensitivity of rebound components to post-EEU efficiency ($\tilde{\eta}$). The macro factor is set to its calibrated value ($k = 3$).

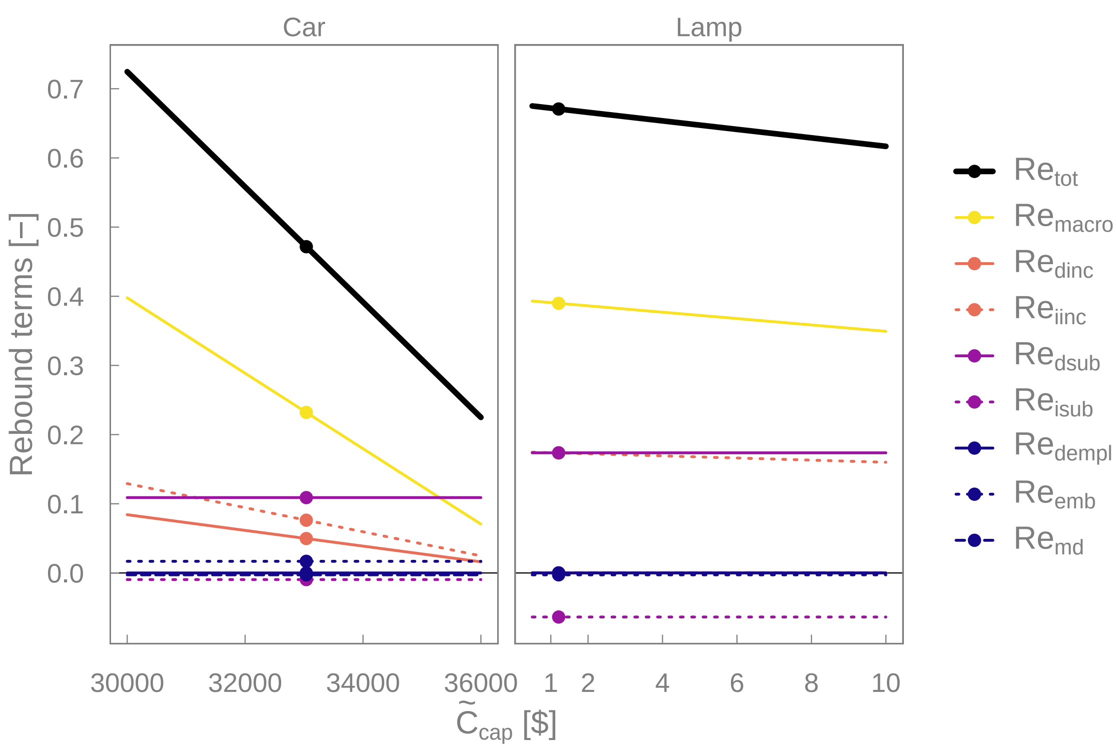

Fig. C.3: Sensitivity of rebound components to capital cost ($\tilde{C}_{cap}$). The macro factor is set to its calibrated value ($k = 3$).

C.3 Effect of energy price (p_E) on rebound terms

The effect of energy price on rebound is shown in Fig. C.4. Increasing energy prices lead to larger total rebound (Re_{tot}), because higher energy prices lead to more net savings ($\dot{\hat{N}}$) to be spent by the device user. All other things being equal, more net savings leads to more spending on other goods and services that demand energy.

Fig. C.4 also shows the effect of energy price (p_E) on all rebound components. Most rebound components increase with energy price, with the car and lamp examples exhibiting different sensitivities. Substitution effects (Re_{dsub} and Re_{isub}) are the only rebound components that decrease with energy price (p_E). Substitution effects decrease with energy price, because at high energy price, less behavior adjustment is needed to re-equilibrate after emplacement of the efficient device.

In Fig. C.4, German energy prices[8] are shown as vertical lines, providing an indication of possible energy price variations. All other things being equal, if U.S. residents paid Germany's energy prices, total energy rebound (Re_{tot}) would be 84.0% for the car example and 148.0% for the lamp example.

C.4 Effect of original uncompensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$) on rebound terms

Fig. C.5 shows the variation of total rebound (Re_{tot}) with the original uncompensated price elasticity of energy service demand ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$). The effect is exponential, and total rebound increases with larger negative values of $\varepsilon^{\circ}_{\dot{q}_s,p_s}$, as expected. The lamp example also shows stronger exponential variation than the car example. The main reason that total rebound values are different between the two examples is the larger absolute value of original uncompensated own price elasticity ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$) for the lamp ($-0.4$) compared to the car ($-0.2$). Were the car to have the same original uncompensated own price elasticity as the lamp (i.e., -0.4), total rebound would be closer for both examples (64.1% for the car and 67.1% for the lamp). Fig. C.5 shows that direct substitution rebound (Re_{dsub}) is the most sensitive rebound component to changes in $\varepsilon^{\circ}_{\dot{q}_s,p_s}$. For the lamp example, indirect income

[8]For the car example, the gasoline price in Germany is taken as 1.42 €/liter for the average "super gasoline" (95 octane) price in 2018 (finanzen.net, 2021). For the lamp example, the electricity price in Germany is taken as 0.3 €/kW·hrfor the 2018 price of a household using 3.5 MWh/yr, an average value for German households (Bundesministerium für Wirtschaft und Energie, 2018). Converting currency (at 1 € = \$1.21) and physical units gives 6.5 \$/US gallon and 0.363 \$/kW·hr.

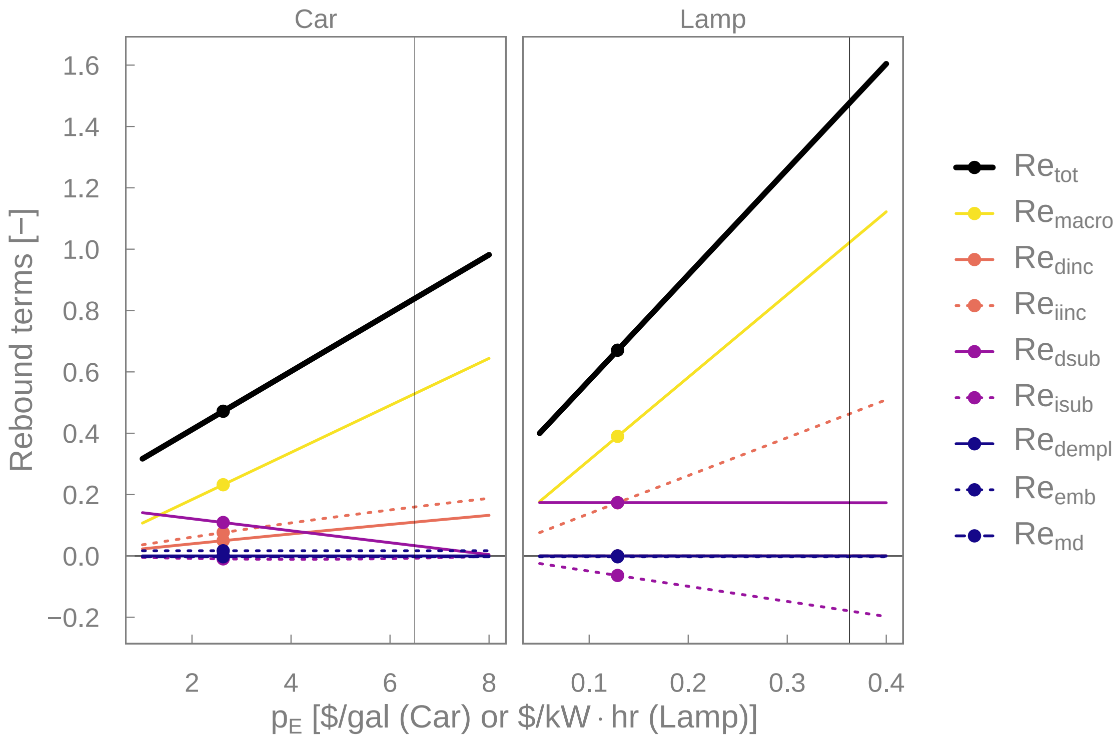

Fig. C.4: Sensitivity of rebound components to energy price (p_E). German energy prices denoted by vertical lines. The macro factor is set to its calibrated value ($k = 3$).

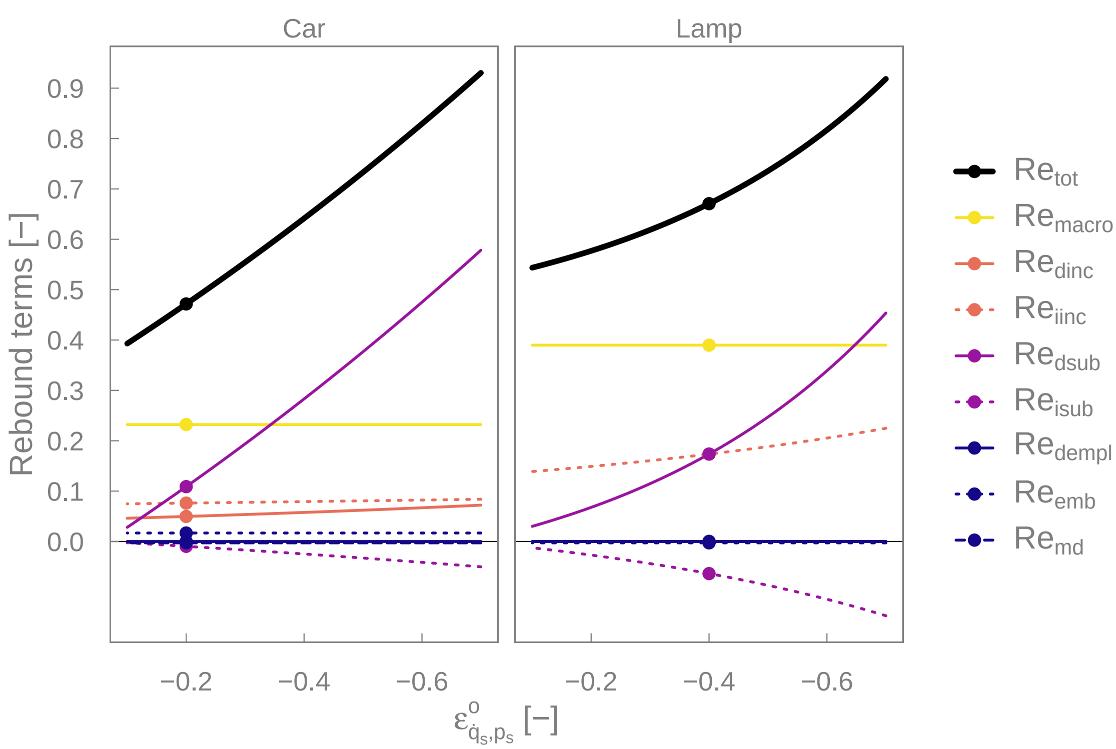

Fig. C.5: Sensitivity of rebound components to uncompensated own price elasticity of energy service demand ($\varepsilon^{\circ}_{\dot{q}_s,p_s}$). The macro factor is set to its calibrated value ($k = 3$). (Note reversed x-axis scale.)

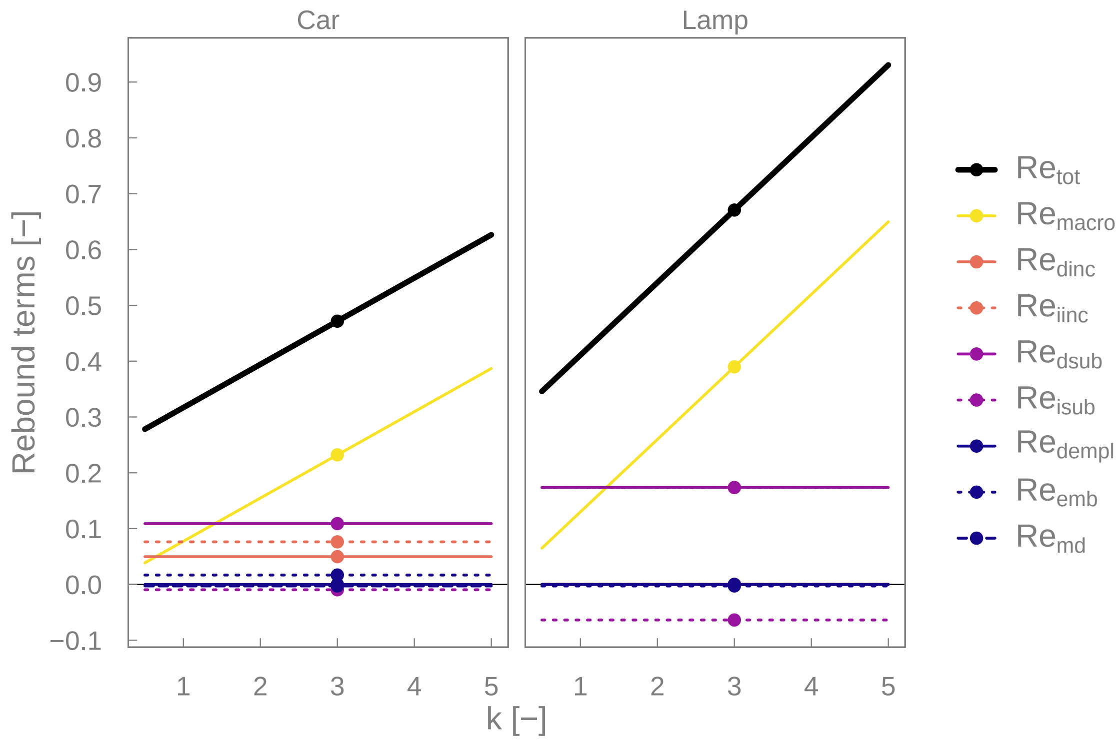

Fig. C.6: Sensitivity of rebound components to the macro factor (k).

rebound (Re_{iinc}) also increases substantially with $\varepsilon^{\circ}_{\dot{q}_s,p_s}$, because net savings increases substantially with $\varepsilon^{\circ}_{\dot{q}_s,p_s}$.

C.5 Effect of macro factor (k) on rebound terms

The sensitivity of energy rebound to the macro factor (k) is shown in Fig. C.6. The macro factor has a linear effect on total rebound (Re_{tot}) through the macro rebound component (Re_{macro}). All other rebound components are constant when k is varied independently.

C.6 Effect of energy service price ($\tilde{p}_s$) on price elasticities ($\hat{\varepsilon}$)

The sensitivity of post-substitution effect price elasticities ($\hat{\varepsilon}$) to post-upgrade energy service price ($\tilde{p}_s$) is shown in Fig. C.7 for the CES utility model described in Section 2.5.2 and Appendix C of Part I. Note that the left side of each graph ($\tilde{p}_s = 0$) represents unattainable infinite efficiency ($\tilde{\eta}_s \rightarrow \infty$), i.e., delivery of the energy service without energy consumption.

First, note the sign of the elasticities. As expected, both of the uncompensated price elasticities

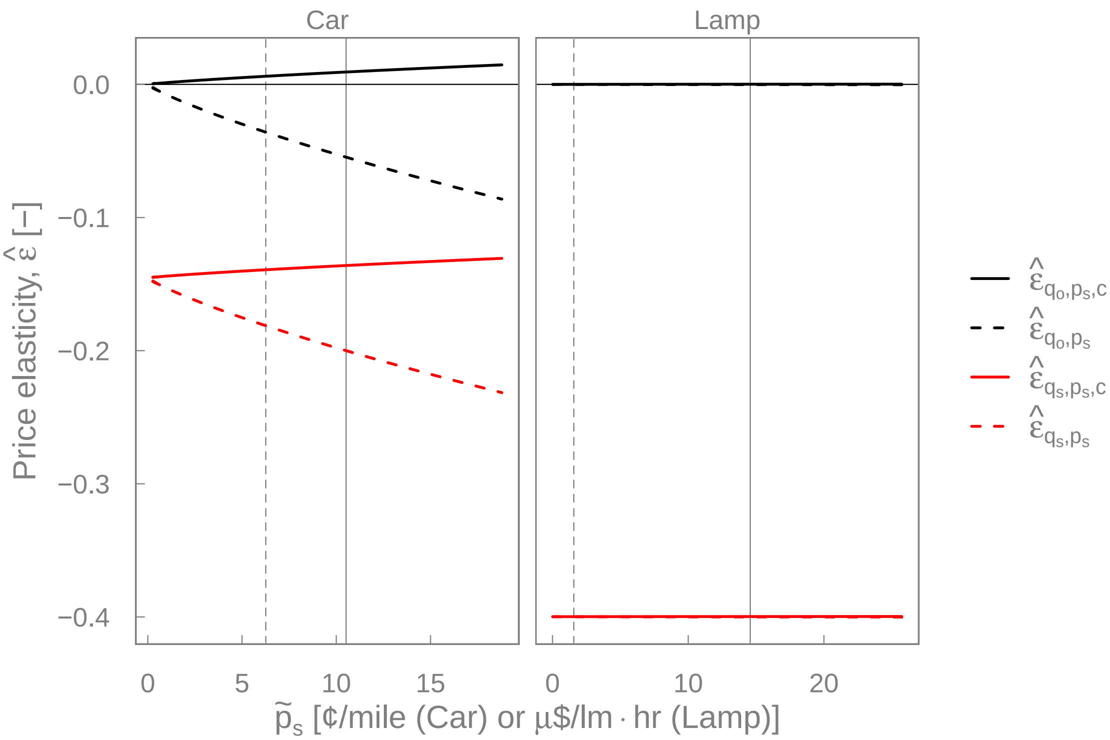

Fig. C.7: Sensitivity of post substitution effect price elasticities ($\hat{\varepsilon}$) to post-EEU energy service price ($\tilde{p}_s$) for the CES utility model. The solid vertical line indicates the original energy service price (p_s°), and the dashed vertical line indicates the upgraded energy service price ($\tilde{p}_s = \bar{p}_s = \hat{p}_s = p_s^*$) for the two examples. See Tables 8 and 10 for p_s in different units.

($\hat{\varepsilon}_{\dot{q}_s,p_s}$ and $\hat{\varepsilon}_{\dot{q}_o,p_s}$, dashed lines in Fig. C.7) are negative, regardless of the energy service price ($\tilde{p}_s$): a lower price means more consumption of both goods, all other things being equal. The compensated own price elasticity ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$) is negative and the compensated cross price elasticity ($\hat{\varepsilon}_{\dot{q}_o,p_s,c}$) is positive. As $\tilde{p}_s$ declines, the consumers substitutes the energy service for other goods.

Second, the magnitude of price elasticities varies. Fig. C.7 shows that the car example exhibits more variation of price elasticities ($\hat{\varepsilon}$) with energy service price ($\tilde{p}_s$) than the lamp example, because the expenditure share ($f^\circ_{\dot{C}_s}$) for the lamp example is very small compared to the car example. Using the constant price elasticity (CPE) utility model may be a good enough approximation in the lamp example. However, for the car example, using the CES utility function will be necessary to eliminate errors that will be present in the CPE approximation. This result is an important finding that should encourage analysts implementing analytical rebound calculations with substitution and income effects to prefer the CES utility model over the CPE approximation.

Fig. C.7 shows that as efficiency increases (and $\tilde{p}_s$ decreases), the absolute value of the uncompensated price elasticities ($\hat{\varepsilon}_{\dot{q}_s,p_s}$ and $\hat{\varepsilon}_{\dot{q}_o,p_s}$) decreases, a change that exceeds the slightly increasing (in absolute value terms) compensated own price elasticity ($\hat{\varepsilon}_{\dot{q}_s,p_s,c}$). Thus, direct rebound is attenuated as efficiency increases, relative to a constant price elasticity model. (See also the patterns of lines of Fig. C.2, which show a declining trend.)