Advancing the necessary foundations
for empirical energy rebound estimates:
A partial equilibrium analysis framework

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Abstract

Widespread implementation of energy efficiency is a key greenhouse gas emissions mitigation measure, but rebound can "take back" energy savings. However, conceptual foundations lag behind empirical estimates of the size of rebound. We posit that development of solid analytical frameworks for rebound is hampered by the interdisciplinary nature of the topic, involving both economics and energy analysis. In this paper, we help advance a rigorous analytical framework that starts at the microeconomics of rebound and is approachable for both energy analysts and economists. We include emplacement, substitution, and income rebound effects and link them to macro rebound. Novel graphs show rebound paths through energy, expenditure, and consumption spaces. Application of the framework provides estimates of total rebound in two case studies: upgrades of a car (48%) and an electric lamp (80%). Sensitivities are evaluated, and implications for common policies are considered.

Keywords: Energy efficiency, Energy rebound, Energy services, Microeconomic rebound, Substitution and income effects, Macroeconomic rebound

1   Introduction

Energy rebound is an interdisciplinary challenge that threatens a low-carbon future (Brockway et al., 2017). Rebound makes energy efficiency less effective at decreasing energy consumption and reducing carbon emissions by taking back (or reversing, in the case of "backfire") energy savings expected from an energy efficiency improvement (Sorrell, 2009). Rebound is an explanation for why energy consumption and carbon emissions have never been absolutely decoupled from economic growth (Haberl et al., 2020; Brockway et al., 2021).

Recent evidence shows that rebound is both (i) larger than commonly assumed (Stern, 2020) and (ii) mostly missing from large energy and climate models (Brockway et al., 2021), despite a long history of research on the topic. Famously, the roots of energy rebound trace back to Jevons who said "[i]t is wholly a confusion of ideas to suppose that the economical use of fuel is equivalent to a diminished consumption. The very contrary is the truth" (Jevons, 1865, p. 103, emphasis in original). Less famously, the origins of rebound extend further backward from Jevons to Williams

(1840) and Parkes (1838).[1] For nearly 200 years, then, it has been understood that efficiency gains may be taken back or, paradoxically even, cause growth in energy consumption.

The oil crises of the 1970s shone a light back onto energy efficiency, and research into rebound appeared late in the decade (Madlener & Turner, 2016; Saunders et al., 2021). A modern debate over the magnitude of energy rebound commenced. On one side, scholars including Brookes (1979, 1990) and Khazzoom (1980) suggested rebound could be large. Others, including Lovins (1988) and Grubb (1990, 1992), claimed rebound was likely to be small. Debate over the size of energy rebound continues today. Advocates of small rebound (less than, say, 50%), suggest "the rebound effect is overplayed" (Gillingham et al., 2013, p. 475), while others, including Saunders (2015), claim the evidence for large rebound (greater than 50%) is growing.

Turner contends that the lack of consensus on the magnitude of rebound in the modern empirical literature is caused by "a rush to empirical estimation in the absence of solid analytical foundations" (Turner, 2013, p. 25). Borenstein (2015) made some progress toward solidifying the analytical foundations for microeconomic rebound, but more is needed to support empirical estimation efforts. In the absence of solid analytical foundations, the wide variety of rebound estimation approaches yields a wide range of rebound estimates, giving the appearance of uncertainty and leading energy and climate modelers to either (i) use questionable numbers, or (ii) ignore rebound altogether. We suggest that improving the conceptual space around rebound and solidifying the analytical frameworks will (i) help generate more robust estimates of rebound, (ii) lead to rebound being better included in more energy and climate models, and (iii) provide improved policymaking around energy efficiency.

But why is there an "absence of solid analytical foundations?" We propose that development of solid analytical frameworks for rebound is hampered by the fact that rebound is a decidedly interdisciplinary topic, involving both economics and energy analysis. Birol & Keppler (2000, p. 458) note that "different implicit and explicit assumptions of different research communities ('economists', 'engineers') … have in the past led to vastly differing points of view."[2] Turner states that "[d]ifferent definitions of energy efficiency will be appropriate in different circumstances. However, … it is often not clear what different authors mean by energy efficiency" (Turner, 2013, p. 237–38). If authors from the two disciplines cannot even agree on the terms, it is unsurprising that only modest progress has been made on analytical foundations. To fully understand rebound, economists need to have an energy analyst's understanding of energy, and energy analysts need to have an economist's understanding of finance and human behavior. That's a tall order, indeed.

We contend that new clarity is needed. A description of rebound is needed that is both (i) technically rigorous and (ii) approachable from both sides (economics and energy analysis). In other words, the finance and human behavior aspects of rebound need to be presented in ways energy analysts can understand. And the energy aspects of rebound need to be presented in ways economists can understand.

Summarizing, we surmise that reducing global carbon emissions has been hampered, in part, by the fact that rebound is not sufficiently included in energy and climate models. We suspect that one reason rebound is not sufficiently included in energy and climate models is the lack of consensus

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[1] Parkes wrote, "[t]he economy of fuel is the secret of the economy of the steam-engine; it is the fountain of its power, and the adopted measure of its effects. Whatever, therefore, conduces to increase the efficiency of coal, and to diminish the cost of its use, directly tends to augment the value of the steam-engine, and to enlarge the field of its operations" (Parkes, 1838, p. 161).

[2] We prefer the term "energy analysts" over "engineers," because "energy analysts" better describes the group of people engaged in "energy analysis". For this paper, we define "energy analysis" as the study of energy transformations from stocks to flows to wastes and energy services along society's energy conversion chain for the purpose of generating energy services, economic activity, and human well-being.

on rebound estimation methods and, hence, rebound magnitude. We agree with Turner that lack of consensus on rebound magnitude is a symptom of the absence of solid analytical foundations for rebound. We posit that developing solid analytical frameworks for rebound is difficult because it is an inherently interdisciplinary topic.

The *objective* of this paper, then, is to support the development of a rigorous analytical framework, one that (i) starts at the microeconomics of rebound (building upon Borenstein (2015)) and (ii) is approachable for both energy analysts and economists.

The key contributions of this paper are development of the first (to our knowledge) (i) rebound analysis framework that combines embodied energy effects, maintenance and disposal effects, non–marginal energy efficiency increases, and non-marginal energy service price decreases, (ii) visualizations of rebound effects in energy, expenditure, and consumption spaces to assist both energy analysts and economists in understanding nuances of rebound effects, (iii) operationalized link between rebound effects on microeconomic and macroeconomic scales, and (iv) open source tools to calculate rebound for other energy efficiency upgrades.

The remainder of this paper is structured as follows. Section 2 lays the groundwork for our framework and reviews existing literature. Section 3 describes the comprehensive rebound analysis framework. Section 4 provides two examples: energy efficiency upgrades to a car and an electric lamp. Section 5 discusses the results, and Section 6 concludes.

2   Background

Rebound analysis frameworks are used to estimate the magnitude of rebound effects and total rebound from an EEU (energy efficiency upgrade). We use the word "framework" to mean the basic structure for analysis of a system or concept, in this case energy rebound. We define a rebound framework to be a rebound typology applied to the task of estimating rebound for a particular EEU. A rebound typology is a description and explication of rebound effects, locations, and scales. Rebound frameworks are operationalized by a series of equations, typically ending with a summation equation for total rebound across all effects, locations, and scales. For accurate analysis of the size of rebound from a given EEU, rebound frameworks should be comprehensive, accounting for all rebound effects in energy, expenditure, and consumption spaces with a detailed model of consumer responses to non-marginal energy efficiency changes.[3]

In the subsections that follow, rebound frameworks are further elucidated, starting with features of rebound typologies and analysis spaces. Thereafter, characteristics of rebound frameworks are discussed, including the importance of detailed models of consumer preferences, the importance of non-marginal energy service price changes, and the need for operationality.

2.1   Typologies: effects, locations, scales

Any comprehensive rebound framework should include in its typology emplacement, substitution, income, and macro effects. For a given EEU, it cannot be known *a priori* which rebound effects will be large and which will be small, so all effects should be analyzed. The emplacement effect of an EEU includes (i) expected energy savings, assuming no behavioral changes; (ii) embodied energy effects, i.e. changes across the EEU of the embodied energy rate of the energy conversion device; and (iii) maintenance and disposal effects (changes in the energy implications of these

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[3]Note that non-marginal (i.e., large) energy efficiency gains are quite common but missing from previous rebound analysis frameworks. See Section 2.4.

activities caused by the EEU). The substitution effect accounts for behavior changes across the EEU, typically increased use of the upgraded device and decreased consumption of other goods due to relative price changes, assuming no change in device owner utility. The emplacement and substitution effects create net financial savings which are spent by the device owner in the income effect on more of the energy service and more other goods and services. Finally, the macro effect accounts for economic expansion (and subsequent changing energy consumption), because the EEU is a cost-saving technological change.

Rebound can occur at direct and indirect locations. Direct rebound involves takeback of expected savings of energy consumed by the upgraded device. Indirect rebound arises from increased energy consumption in other locations and in other economic sectors as a result of the EEU.

Rebound scales include microeconomic and macroeconomic. Rebound at the microeconomic scale arises from the series of adjustments to the reduction in the implicit price of the energy service. A static economy is assumed. Microeconomic rebound includes the emplacement, substitution, and income effects. Rebound at the macroeconomic scale arises from the dynamic response of the economy to reach a new stable equilibrium (between supply and demand for the energy service and other goods) in response to the EEU. Macroeconomic rebound involves various short- and long-run effects.

Because a lot is at stake when analyzing energy rebound, it is important that rebound analysis frameworks account for all effects, locations, and scales; everything is interconnected. The emplacement effect frees the device owner's cash to be spent in the income effect, with energy implications. The substitution effect describes behavior adjustments that provide further net savings to be spent in the income effect, again with energy implications. And greater efficiency gains and behavior changes at the microeconomic scale are likely to lead to larger macroeconomic rebound. (We note briefly that the link between microeconomic and macroeconomic scales is largely unexplored territory. See Section 3.5.4 for our approach to the macro effect.)

2.2 Rebound spaces

A comprehensive rebound framework will enable analyses in several conceptual "spaces" in which changes occur because of the EEU. Energy, expenditure, and consumption spaces are all linked together, as illustrated by two examples. (i) Net savings from the EEU are determined in the expenditure space. Which goods and services are purchased with net savings is determined by consumer preferences and described in the consumption space. The energy implications of purchases vary by the types of goods and services purchased and is described in the energy space. (ii) The amount of embodied energy in the upgraded device is described in the energy space and may be correlated with the cost of device upgrades, which is described in the expenditure space. The cost of the upgrade, in turn, has implications for what can be consumed, which is described in the consumption space. Tracking changes across all spaces increases understanding of causal linkages across rebound effects, locations, and scales and clarifies terms and concepts across disciplines.

2.3 Behavior and preferences

A comprehensive rebound framework will include a detailed description of device owner behavior and preferences. By "detailed," we mean a payoff specification that translates energy service price and device purchase cost changes into choices, simultaneously decomposing into substitution and income effects. These two effects allow for nuanced understanding of behavior for different commodities and payoff specifications. Decomposition into substitution and income effects has become the standard

approach to analyzing direct and indirect rebound locations at the microeconomic scale (Greening et al., 2000).

2.4 Non-marginal energy service price changes

A comprehensive rebound analysis framework will allow non-marginal energy service price changes, because EEUs often involve a large percentage increase in energy efficiency. A good example of non-marginal energy service price change is replacing incandescent lamps with Light Emitting Diode (LED) lamps, as discussed in Section 4.2. For such EEUs, energy efficiency can increase by nearly a factor of 10 and energy service price can decrease similarly.

2.5 Operationality

An operational rebound analysis framework is one that includes sufficient descriptions of energy rebound for all effects, locations, and scales in all spaces (energy, expenditure, and consumption) to calculate quantitative estimates of rebound effects with available real-world data. This is particularly important for microeconomically grounded, consumer-sided rebound frameworks that involve unobserved utility functions and, by extension, substitution and income effects.

2.6 Previous frameworks

Bringing together the importance of energy efficiency *vis-à-vis* emissions reduction goals and the desirable characteristics of rebound frameworks discussed above, we see that comprehensive frameworks are needed to help develop energy efficiency policy to meet emissions targets. I.e., effective policy should be informed by rebound analyses conducted within a comprehensive rebound framework.

We develop below (Section 3) a comprehensive partial equilibrium rebound framework for consumers. We note that many of its component are similar to one for a producer-sided framework due to the symmetry between neoclassical microeconomic producer and consumer theory. Partial equilibrium frameworks provide detailed assessment of individual EEUs with tractable, easy-to-understand mathematics. Partial equilibrium frameworks are easier to understand, in part, because they constrain price variation to the energy service only; all other prices remain constant. In our partial equilibrium framework, general equilibrium effects are captured by a simplified, one-dimensional, macroeconomic-scale rebound effect discussed in Section 3.5.4.

General equilibrium frameworks allow prices of all goods and services in an economy to adjust to the EEU. Recent examples include Hart (2018), Lemoine (2020), Fullerton & Ta (2020), and Blackburn & Moreno-Cruz (2020). General equilibrium frameworks provide detail and precision on economy-wide price adjustments, but they give up specificity about individual device upgrades, make assumptions during calibration, and lose simplicity of exposition.

We are not the first to develop a partial equilibrium rebound analysis framework, so it is worthwhile to assess previous frameworks for the key features discussed above: analysis at all effects, locations, and scales; analysis in energy, expenditure, and consumption spaces; level of detail in the consumer preference model; allowance for non-marginal energy efficiency changes; and operationality. Only when all of the above characteristics are present can a comprehensive picture of rebound emerge. Table 1 shows our assessment of selected recent partial equilibrium frameworks (in columns) relative to the characteristics discussed above (in rows).

Table 1: Comparison among relevant partial equilibrium rebound analysis frameworks. Empty (white) circles indicate no treatment of a subject by a framework. Partially and fully filled circles indicate partial and comprehensive treatment of a subject by a framework.

	Nässén & Holmberg (2009)	Thomas & Azevedo (2013a,b)	Borenstein (2015)	Chan & Gillingham (2015)	Wang et al. (2021)	This paper
Effects, locations, and scales						
Direct emplacement effect	●	●	●	●	●	●
Capital cost and embodied energy effect	◐	◐	◐	○	○	●
Maintenance and disposal effect	○	○	◐	○	○	●
Direct and indirect substitution effects	◐	◐	●	●	●	●
Direct and indirect income effects	◐	◐	●	●	●	●
Macro effect	○	○	◐	○	○	●
Other characteristics						
Presentation of energy, expenditure, and consumption spaces	◐	●	◐	●	◐	●
Detailed model of device owner behavior and preferences	◐	◐	◐	●	●	●
Non-marginal energy service price changes	○	○	○	○	○	●
Operationality	●	●	◐	○	○	●

Because all frameworks evaluate the expected decrease in direct energy consumption from the EEU, the "Direct emplacement effect" row contains ● in all columns. Three early papers (Nässén & Holmberg, 2009; Thomas & Azevedo, 2013a,b) estimate rebound quantitatively, earning high marks (●) in the "Operationality" row. Both motivate their frameworks at least partially with microeconomic theory (consumer preferences and substitution and income effects) but use simple linear demand functions in their empirical analyses. Thus, the connection between economic theory and empirics is tenuous, leading to intermediate ratings (◒ or less) in the "substitution effects," "income effects," and "Detailed model of consumer preferences" rows. More recently, Chan & Gillingham (2015) and Wang et al. (2021) anchor the rebound effect firmly in consumer theory, earning high ratings (●) in the "substitution effects," "income effects," and "Detailed model of consumer preferences" rows. They extend their frameworks to advanced topics such as multiple fuels, energy services, and nested utility functions with intermediate inputs. However, neither framework exhibits operationality, earning ○ in the last row of Table 1. In the middle of the table (and between the other studies in time), the framework by Borenstein (2015) touches on nearly all important characteristics. However, the Borenstein framework cannot separate substitution and income effects cleanly in empirical analysis, reverting to partial analyses of both, leading to a ◓ rating in the "Detailed model of consumer preferences" row.

No previous framework engages fully with either the differential financial effects or the differential energetic effects of the upfront purchase of the upgraded device, leading to low ratings across all previous frameworks in the "Capital cost and embodied energy effect" row. In fact, except for Nässén & Holmberg (2009), no framework engages with capital costs, although all note its importance. (Nässén & Holmberg note that capital costs and embodied energy can have very strong effects on rebound.) Thomas & Azevedo (2013a,b) provide the only framework that traces embodied energy effects of every consumer good using input-output methods, but they do not analyze embodied energy of the upgraded device. Borenstein (2015) notes the embodied energy of the upgraded device and the embodied energy of other goods but does not integrate embodied energy or financing costs into the framework for empirical analysis. Borenstein is the only author to treat the financial side of embodied energy or maintenance and disposal effects. Borenstein (2015) postulates the macro effect, but does not develop the idea into an operational theory, earning ◒ in the "Macro effect" row. No other frameworks even discuss the link between macro and micro rebound effects, leading to ○ in the "Macro effect" row for all other previous frameworks. Finally, all previous frameworks assume constant price elasticities and implicitly "marginal" or small improvements in efficiency, excluding the analysis of important non-incremental upgrades where price elasticities are likely to vary. Therefore, all previous frameworks earn ○ in the "Non-marginal energy service price changes" row.

Table 1 shows that previous frameworks contain many key pieces, providing a substantial base from which to develop a comprehensive and operational rebound analysis framework. A left-to-right reading of the table demonstrates that previous frameworks start from microeconomic consumer theory and move towards more rigorous theoretical treatment over time, with recent frameworks making important advanced theoretical contributions at the expense of operationality. In the end, no previous rebound analysis framework combines all rebound effects, locations, and scales across energy, expenditure, and consumption spaces with a detailed model of consumer preferences and non-marginal energy service price changes in an operational manner for the simplest case (understandable across disciplines) of a single fuel and a single energy service. In particular, non-marginal price changes and the macro effect require conceptual development and operationalization. This paper aims to address the gaps in Table 1, completing the "This paper" column with filled circles (●).
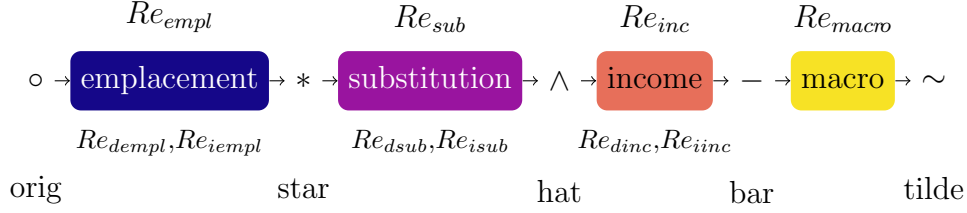
Fig. 1: Flowchart of rebound effects and decorations.

3   Methods: development of the comprehensive rebound framework

In this section, we develop a comprehensive, partial equilibrium, energy rebound analysis framework (concisely, "this framework"). We follow the rebound typology discussed in Section 2.1, distinguishing between microeconomic rebound (at the single device or for the device owner) and macroeconomic rebound (broader, economy-wide responses to a single device upgrade). (See Greening et al. (2000), Barker et al. (2009), and Vivanco et al. (2018).) We follow others, including Jenkins et al. (2011) and Walnum et al. (2014), in splitting microeconomic rebound into direct and indirect locations. Like other authors, we recognize many macroeconomic effects, even if we don't later distinguish among them.[4]

Fig. 1 shows rebound effects arranged in left-to-right order of their discussion in this paper. Rebound symbols are shown above each effect (Re_{empl}, etc.). Nomenclature for direct and indirect locations are shown beneath each effect (Re_{dempl}, etc.). Decorations for each stage are shown between rebound effects ($\circ$, $*$, etc.). Names for the decorations are given at the bottom of the figure ("orig," "star," etc.).[5] The left-to-right order does not necessarily represent the progression of rebound effects through time.

3.1   Rebound effects

We assume an energy conversion device (say, a car) that consumes final energy[6] (say, gasoline) at a rate $\dot{E}^\circ$ (in MJ/year). The final energy is available at price p_E (in \$/MJ). The original energy conversion device provides a rate of energy service $\dot{q}_s^\circ$ (in vehicle-km/year) with final-to-service efficiency η° (in vehicle-km/MJ). An energy efficiency upgrade (EEU) increases final-to-service efficiency[7] such that $\tilde{\eta} > \eta^\circ$, possibly at an increased cost to emplace (capital cost), maintain, and

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[4]For example, Sorrell (2009) sets out five macroeconomic rebound effects: embodied energy effects, respending effects, output effects, energy market effects, and composition effects. (We place the embodied energy effect at the microeconomic scale.) Santarius (2016) and Lange et al. (2021) introduce meso (i.e., sectoral) level rebound between the micro and macro levels. van den Bergh (2011) distinguishes 14 types of rebound, providing, perhaps, the greatest complexity.

[5]Note that the vocabulary and mathematical notation for rebound effects is important; Fig. 1 and Appendix A provide guides to notational elements used throughout this paper, including symbols, Greek letters, abbreviations, decorations, and subscripts. The notational elements can be mixed to provide a rich and expressive "language" of energy rebound. In several places, we use colored backgrounds on rebound effects for visual convenience.

[6]Conventionally, stages of the energy conversion chain are primary energy (e.g., coal, oil, natural gas, wind, and solar), final energy (e.g., electricity and refined petroleum), useful energy (e.g., heat, light, and mechanical drive), and energy services (e.g., transport, illumination, and space heating).

[7]Note that energy service efficiency (η) improves between the original ($\circ$) and post-emplacement ($*$) stages of Fig. 1, remaining constant thereafter. Thus, $\eta^\circ < \eta^* = \hat{\eta} = \bar{\eta} = \tilde{\eta}$, as shown in Table C.1. We refer to all post-emplacement efficiencies (η^*, $\hat{\eta}$, $\bar{\eta}$, and $\tilde{\eta}$) as $\tilde{\eta}$ to match the nomenclature of Borenstein (2015). When convenient, the

dispose of the device such that $C_{cap}^\circ < \tilde{C}_{cap}$ and $\dot{C}_{md}^\circ < \dot{\tilde{C}}_{md}$. (This is not a costless EEU.) As final-to-service efficiency increases $(\eta^\circ < \tilde{\eta})$, the price of the energy service declines $(p_s^\circ > \tilde{p}_s)$. The final energy price (p_E) is assumed exogenous $(p_E^\circ = p_E^* = \hat{p}_E = \bar{p}_E = \tilde{p}_E)$, so the final energy purchaser (the device owner) is a price taker.[8] Initially, the device owner spends income $(\dot{M}^\circ)$ on final energy for the device $(\dot{C}_s^\circ = p_E \dot{E}_s^\circ)$, annualized capital costs for the device $(\dot{C}_{cap}^\circ)$, annualized costs for maintenance and disposal of the device $(\dot{C}_{md}^\circ)$, and other goods and services $(\dot{C}_o^\circ)$. The budget constraint for the device owner is

$$\dot{M}^\circ = \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ + \dot{N}^{\circ\nearrow^0} , \qquad (1)$$

where original net savings $(\dot{N}^\circ)$ is zero.

Later (Sections 3.5.1–3.5.4), we walk through four rebound effects, deriving rebound expressions for each. But first we define rebound relationships (Section 3.2), show typical energy and cost relationships (Section 3.3), and introduce rebound path graphs (Section 3.4). In developing the framework, we endeavor provide sufficient detail to assist energy analysts to understand the economics and economists to understand the energy analysis.

3.2 Rebound relationships

Energy rebound is defined as

$$Re \equiv 1 - \frac{\text{actual final energy savings rate}}{\text{expected final energy savings rate}} , \qquad (2)$$

where both actual and expected final energy savings rates are in \$/year and expected positive. Final energy "takeback" rate is defined as expected final energy savings rate less actual final energy savings rate. Note that the takeback rate can be negative, indicating that the actual final energy savings rate is greater than the expected final energy savings rate, a condition called hyperconservation. Thus,

$$Re = 1 - \frac{\text{expected final energy savings rate} - \text{takeback rate}}{\text{expected final energy savings rate}} . \qquad (3)$$

Simplifying gives

$$Re = \frac{\text{takeback rate}}{\text{expected final energy savings rate}} . \qquad (4)$$

Energy rebound can be given by either Eq. (2) or Eq. (4).

We define rebound at the final energy stage of the energy conversion chain, because that is the point of energy purchase by the device owner. To simplify derivations, we choose not to apply final-to-primary energy multipliers to final energy rates in the numerators and denominators of rebound expressions derived from Eqs. (2) and (4); they divide out anyway. Henceforth, we drop the adjective "final" from the noun "energy," unless there is reason to indicate a specific stage of the energy conversion chain.

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same approach to nomenclature is taken with other quantities such as the capital cost rate $(\dot{C}_{cap})$ and maintenance and disposal cost rate $(\dot{C}_{md})$.

[8]Relaxing the exogenous final energy price assumption would require a general equilibrium model that is beyond the scope of this paper. Rebound due to energy price change is included in the macro effect.

3.3 Typical energy and cost relationships

With the rebound notation of Appendix A, four typical relationships emerge. First, the consumption rate of the energy service ($\dot{q}_s$) is the product of final-to-service efficiency (η) and the rate of energy consumption by the energy conversion device ($\dot{E}_s$). Typical units for automotive transport and illumination (the examples in Section 4) are shown beneath each equation.[9]

$$\dot{q}_s = \eta \dot{E}_s \tag{5}$$
$$[\text{pass-km/yr}] = [\text{pass-km/MJ}][\text{MJ/yr}]$$
$$[\text{lm-hr/yr}] = [\text{lm-hr/MJ}][\text{MJ/yr}]$$

Second, the energy service price (p_s) is the ratio of energy price (p_E) to the final-to-service efficiency (η).

$$p_s = \frac{p_E}{\eta} \tag{6}$$
$$[\$/\text{pass-km}] = \frac{[\$/\text{MJ}]}{[\text{pass-km/MJ}]}$$
$$[\$/\text{lm-hr}] = \frac{[\$/\text{MJ}]}{[\text{lm-hr/MJ}]}$$

Third, energy service expenditure rates ($\dot{C}_s$) are the product of energy price (p_E) and device energy consumption rates ($\dot{E}_s$).

$$\dot{C}_s = p_E \dot{E}_s \tag{7}$$
$$[\$/\text{yr}] = [\$/\text{MJ}][\text{MJ/yr}]$$

Fourth, indirect energy rates ($\dot{E}_j$) for maintenance and disposal ($j = md$) and other goods expenditures ($j = o$) are the product of expenditures rates ($\dot{C}_j$) and energy intensity of the economy (I_E).

$$\dot{E}_j = \dot{C}_j I_E \tag{8}$$
$$[\text{MJ/yr}] = [\$/\text{yr}][\text{MJ/\$}]$$

3.4 Rebound path graphs

Any comprehensive rebound analysis must track energy, expenditure, and consumption at the device (direct location) and elsewhere in the economy (indirect location) across all adjustments for all rebound effects. One novel contribution of this paper is visualization of rebound effects in three spaces: the energy space, the expenditure space, and the consumption space. Each space is represented by a plane that contains a path graph, illustrating adjustments to energy, expenditure, or consumption in response to the EEU. (See Appendix B for detailed mathematical descriptions of rebound path graphs.) Important quantities for the path graphs are (i) direct and indirect energy consumption rates ($\dot{E}_{dir}$, $\dot{E}_{indir}$), (ii) direct and indirect expenditure rates ($\dot{C}_{dir}$ and $\dot{C}_{indir}$), (iii) the consumption rate of the energy service ($\dot{q}_s$), and (iv) the expenditure rate on other consumption goods ($\dot{C}_o$). In rebound path graphs, effects at the direct location are placed on the x-axis, and effects at the indirect location are placed on the y-axis.

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[9]Note that "pass" is short for "passenger," and "lm" is the SI notation for the lumen, a unit of lighting energy rate.

Table 2: Rebound path graph segments.

Segment	Rebound effect	Symbol	Equation
○—a	Direct emplacement	Re_{dempl}	(11)
a····b	Embodied energy	Re_{emb}	(12)
b—∗	Maintenance and disposal	Re_{md}	(13)
∗—c	Indirect substitution	Re_{isub}	(22)
c—∧	Direct substitution	Re_{dsub}	(21)
∧—d	Direct income	Re_{dinc}	(26)
d—−	Indirect income	Re_{iinc}	(30)
−—∼	Macro	Re_{macro}	(32)

Figs. 2–4 show notional rebound path graphs in energy, expenditure, and consumption spaces. The notional path graphs are not quantified, i.e. there are no scales on the axes. Later (Section 4), rebound path graphs (with scales) illustrate the numerical examples.

Fig. 2 shows a notional energy path graph, with the direct energy consumption rate ($\dot{E}_{dir}$) on the x-axis and the indirect energy consumption rate ($\dot{E}_{indir}$) on the y-axis. Points ○, ∗, ∧, −, and ∼ represent the rebound stages between the rebound effects, as shown in Fig. 1. Points a, b, c, and d represent intermediate stages. Lines with negative slope through points ○, a, ∗, ∧, −, and ∼ indicate energy consumption isoquants (sum of direct and indirect components) at key points. Table 2 shows segments and rebound effects for all rebound path graphs. Note that segment −—∼ appears only on energy path graphs, because the framework tracks energy consumption but not expenditures or consumption in the macro effect.

A notional expenditure path graph is shown in Fig. 3, with the direct expenditure rate on the energy service ($\dot{C}_{dir}$) on the x-axis and the indirect expenditure rate ($\dot{C}_{indir}$) on the y-axis. Lines with negative slope through points ○, a, ∗, and ∧ indicate expenditure isoquants (sum of direct and indirect components).

A notional consumption path graph is shown in Fig. 4. The indexed rate of energy service demand ($\dot{q}_s/\dot{q}_s^\circ$) is shown on the x-axis, and the indexed rate of other goods demand ($\dot{C}_o/\dot{C}_o^\circ$) is shown on the y-axis. Iso-cost loci of energy service and other goods demand are shown as lines with negative slope. Indifference curves are denoted by i°—i° and ī—ī. A ray from the origin through the ∧ point is denoted r—r. Note that points ○, a, b, and ∗ collapse together on consumption path graphs, because both the rate of energy service consumption and the rate of other goods consumption are unchanged across the emplacement effect ($\dot{q}_s^\circ = \dot{q}_s^*$ and $\dot{C}_o^\circ = \dot{C}_o^*$, respectively).

3.5 Rebound effects

The four rebound effects (emplacement, substitution, income, and macro) shown in the rebound path graphs are discussed in subsections below. In each subsection, we define each effect and show mathematical expressions for rebound (Re) caused by each effect. The notional rebound path graphs (Figs. 2–4) serve to illustrate the development of the framework. Detailed derivations of all rebound expressions can be found in Appendix C. See, in particular, Tables C.3–C.6, which provide a parallel structure for energy and financial accounting across all rebound effects. We begin with the emplacement effect.
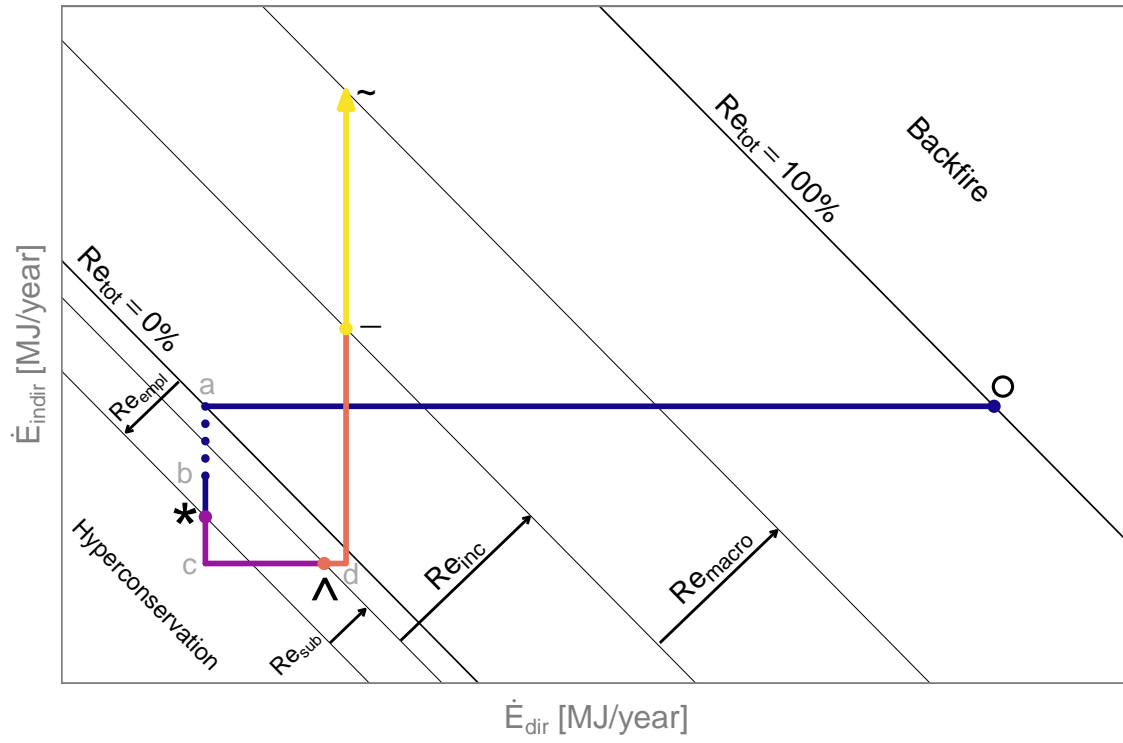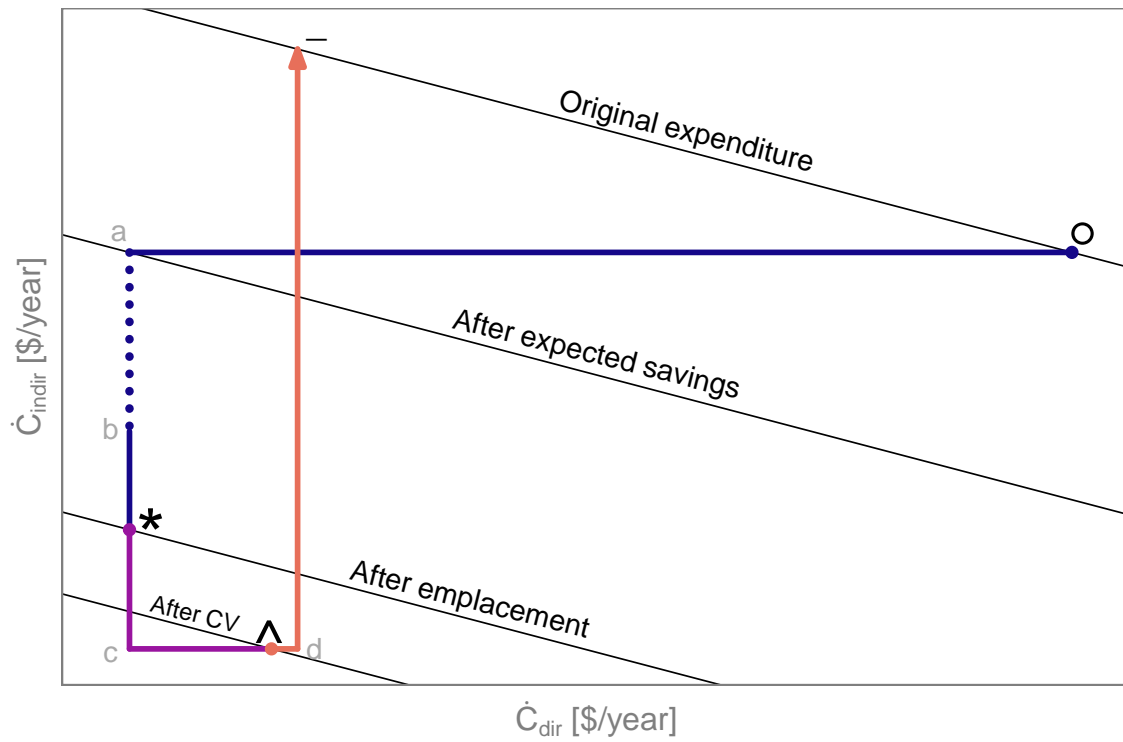
Fig. 2: Notional energy path graph.

Fig. 3: Notional expenditure path graph. CV is compensating variation.

Fig. 4: Notional consumption path graph.

3.5.1 Emplacement effect

The emplacement effect (segments $\circ\!\!-\!\!a$, $a\cdots b$, and $b\!\!-\!\!*$ in Figs. 2 and 3) accounts for performance of the EEU only; behavior changes are addressed later, in the substitution and income effects. The direct emplacement effects of the EEU include device energy savings ($\dot{S}_{dev}$) and device energy cost savings ($\Delta \dot{C}_s^*$), on segments $\circ\!\!-\!\!a$ in Figs. 2 and 3, respectively. The indirect effects of EEU emplacement are (i) changes in the embodied energy rate ($\Delta \dot{E}_{emb}^*$) (segment $a\cdots b$ in Fig. 2), (ii) changes in the capital expenditure rate ($\Delta \dot{C}_{cap}^*$) (segment $a\cdots b$ in Fig. 3), and (iii) changes in the maintenance and disposal energy and expenditure rates ($\Delta \dot{E}_{md}^*$ and $\Delta \dot{C}_{md}^*$) (segments $b\!\!-\!\!*$ in Figs. 2 and 3).

The rate of energy savings due to the direct emplacement effect ($\dot{S}_{dev}$, segment $\circ\!\!-\!\!a$ in Fig. 2) is given by

$$\dot{S}_{dev} \equiv \dot{E}_s^\circ - \dot{E}_s^* = -\Delta \dot{E}_s^* . \tag{9}$$

$\dot{S}_{dev}$ can be rewritten conveniently as

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^\circ} - 1 \right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ . \tag{10}$$

(See Appendix C.3.1 for the derivation.)

Because behavior changes are not considered in the direct emplacement effect, actual and expected energy savings rates are identical, and there is no takeback. By definition, then, the direct emplacement effect causes no rebound. Thus,

$$Re_{dempl} = 0 . \tag{11}$$

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In the notional energy path graph of Fig. 2, point a lies on the $Re_{tot} = 0\%$ line indicating that point a (and the $Re_{tot} = 0\%$ line) is the point from which all rebound effects (Re_{empl}, Re_{sub}, Re_{inc}, and Re_{macro}) are measured. If rebound effects cause total energy demand to return to the original energy consumption level (negative sloping line through the ∘ point), all expected energy savings are taken back by rebound effects. Thus, the line of constant energy consumption through the ∘ point is labeled $Re_{tot} = 100\%$. The contribution of each rebound effect to total rebound is represented by the distance that each component's segment moves across the rebound isoquants. Total rebound (Re_{tot}) is measured linearly between and beyond the $Re_{tot} = 0\%$ and $Re_{tot} = 100\%$ lines, with direct rebound in the x direction and indirect rebound in the y direction. The region below and to the left of the $Re_{tot} = 0\%$ line in Fig. 2 exhibits negative rebound, indicating hyperconservation. The region above and to the right of the $Re_{tot} = 100\%$ line shows backfire, i.e. greater total energy consumption after the EEU than before it.

Although the direct emplacement effect does not cause rebound, Fig. 2 shows that indirect emplacement effects may indeed cause rebound. Indirect emplacement effects (segments $a\cdots b$ and $b\!\!-\!\!*$ in Figs. 2 and 3) account for the life cycle of the energy conversion device, including energy embodied by manufacturing processes (subscript emb) and maintenance and disposal activities (subscript md). In the notional energy path graph (Fig. 2), emplacement rebound is negative ($Re_{empl} < 0$), because the upgraded device has a lesser embodied energy rate ($\dot{E}_{emb}^{\circ} > \dot{E}_{emb}^{*}$) and a lesser maintenance and disposal expenditure rate ($\dot{C}_{md}^{\circ} > \dot{C}_{md}^{*}$) than the original device. Fig. 3 shows segments $a\cdots b$ and $b\!\!-\!\!*$ moving in the negative y direction, consistent with a lower capital expenditure rate ($\dot{C}_{cap}^{\circ} > \dot{C}_{cap}^{*}$) and a reduced maintenance and disposal expenditure rate ($\dot{C}_{md}^{\circ} > \dot{C}_{md}^{*}$).

One of the unique features of this framework is that independent analyses of embodied energy and capital costs of the EEU are expected. We note that the different terms (embodied energy rate, $\dot{E}_{emb}$, and capital cost rate, $\dot{C}_{cap}$) might seem to imply different processes, but they actually refer to the same emplacement effect, namely segment $a\cdots b$ in energy and expenditure path graphs (Figs. 2 and 3, respectively). Purchasing an upgraded device (which likely leads to $\dot{C}_{cap}^{\circ} \neq \dot{C}_{cap}^{*}$) will likely mean a changed embodied energy rate ($\dot{E}_{emb}^{\circ} \neq \dot{E}_{emb}^{*}$) to provide the same energy service. Our names for these aspects of rebound (embodied energy and capital cost) reflect common usage in the energy and economics fields, respectively. In the notional graphs of Figs. 2 and 3, embodied energy rates and capital cost rates (represented by segments $a\cdots b$) move in the same direction (both in the negative y direction). However, both segments $a\cdots b$ could move in the positive y direction, or they could move in opposite directions, depending on the results of the independent analyses for embodied energy and capital cost rates.

Consistent with the energy analysis literature, we define embodied energy to be the sum of all energy consumed in the production of the energy conversion device, all the way back to resource extraction. Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. No energy is embodied in the device while in service. The EEU causes the embodied energy of the energy conversion device to change from E_{emb}° to E_{emb}^{*}.

For simplicity, we spread all embodied energy over the lifetime of the device to provide a constant embodied energy rate ($\dot{E}_{emb}$). (We later take the same approach to capital costs ($\dot{C}_{cap}$) and maintenance and disposal costs ($\dot{C}_{md}$).) A justification for spreading embodied energy purchase costs comes from considering device replacements by many consumers across several years. In the aggregate, evenly spaced (in time) replacements work out to the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices (t_{life}° and t_{life}^{*}, respectively) to obtain embodied energy rates, such that $\dot{E}_{emb}^{\circ} = E_{emb}^{\circ}/t_{life}^{\circ}$ and $\dot{E}_{emb}^{*} = E_{emb}^{*}/t_{life}^{*}$.

The change in embodied final energy due to the EEU (expressed as a rate) is given by $\Delta \dot{E}^*_{emb} = \dot{E}^*_{emb} - \dot{E}^\circ_{emb}$ and is shown as segment $a\cdots b$ in Fig. 2. The expression for embodied energy rebound is

$$Re_{emb} = \frac{\left(\frac{E^*_{emb}}{E^\circ_{emb}} \frac{t^\circ_{life}}{t^*_{life}} - 1\right) \dot{E}^\circ_{emb}}{\dot{S}_{dev}} \ . \tag{12}$$

(See Appendix C.3.2 for details of the derivation.)

Embodied energy rebound (Re_{emb}) can be either positive or negative, depending on the sign of the term $(E^*_{emb}/E^\circ_{emb})(t^\circ_{life}/t^*_{life}) - 1$. Rising energy efficiency can be associated with increased device complexity, additional energy consumption in manufacturing, and more embodied energy, such that $E^\circ_{emb} < E^*_{emb}$ and $Re_{emb} > 0$, all other things being equal. However, if the upgraded device has longer life than the original device ($t^*_{life} > t^\circ_{life}$), $\dot{E}^*_{emb} - \dot{E}^\circ_{emb}$ could be negative, meaning that the upgraded device has a lower embodied energy rate than the original device, as shown by point b being below point a in the notional energy path graph of Fig. 2.

In addition to embodied energy, indirect emplacement effect rebound accounts for energy demanded by maintenance and disposal (md) activities (segments b—$*$ in Figs. 2 and 3). Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}^\circ_m$). Disposal costs (e.g., C°_d) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal costs across the lifetime of the original and upgraded devices (t°_{life} and t^*_{life}, respectively) to form expenditure rates such that $\dot{C}^\circ_{md} = \dot{C}^\circ_m + C^\circ_d/t^\circ_{life}$ and $\dot{C}^*_{md} = \dot{C}^*_m + C^*_d/t^*_{life}$.

For simplicity, we assume that maintenance and disposal expenditures imply energy consumption elsewhere in the economy at its overall energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in maintenance and disposal expenditures is given by $\Delta \dot{C}^*_{md} I_E = (\dot{C}^*_{md} - \dot{C}^\circ_{md}) I_E$, shown as segment b—$*$ in Fig. 2. Rebound from maintenance and disposal activities is given by

$$Re_{md} = \frac{\left(\frac{\dot{C}^*_{md}}{\dot{C}^\circ_{md}} - 1\right) \dot{C}^\circ_{md} I_E}{\dot{S}_{dev}} \ . \tag{13}$$

(See Appendix C.3.2 for details of the derivation.)

3.5.2 Substitution effect

Neoclassical consumer theory decomposes price-induced behavior change into (i) substituting energy service consumption for other goods consumption due to the lower post-EEU price of the energy service (the substitution effect) and (ii) spending the higher real income (the income effect).[10] This section develops mathematical expressions for substitution effect rebound (Re_{sub}), thereby accepting the standard neoclassical microeconomic assumptions about consumer behavior. (The next section addresses income effect rebound, Re_{inc}.) The substitution effect alters compensated demand, which is the demand for the expenditure-minimizing consumption bundle that maintains utility at the pre-EEU level, given the new prices. Compensated demand is a technical term for a thought experiment from welfare economics: the device owner's budget is altered so that the owner is "compensated" for the change in price while maintaining the same level of utility as before. The change in the

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[10]For the original development of the decomposition see Slutsky (1915) and Allen (1936). For a modern introduction see Nicholson & Snyder (2017).

budget is called "compensating variation" (CV). The substitution effect involves (i) an increase in consumption of the energy service, the direct substitution effect (subscript $dsub$) and (ii) a decrease in consumption of other goods, the indirect substitution effect (subscript $isub$). Thus, two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}). The substitution effect is shown by segments $*\!\!-\!\!c$ (the indirect component) and $c\!\!-\!\!\wedge$ (the direct component) in Figs. 2–4.

The substitution effect is best described with reference to a consumption path graph (e.g., Fig. 4). Prior to the EEU, the consumption basket (of the energy service and other goods) is represented by the $\circ$ point. The budget constraint, i.e. the amount of money available for final goods purchases, is shown as isoquant $\circ\!\!-\!\!\circ$. The $\circ\!\!-\!\!\circ$ line is tangent to the lower indifference curve (i$^\circ\!\!-\!\!$i$^\circ$) at point $\circ$, the optimal consumption bundle prior to the EEU. After emplacement of the more efficient device (but before the substitution effect), the price of the energy service decreases ($p_s^\circ > p_s^*$), pivoting the budget constraint counterclockwise about the $\circ$ point. The budget line $*\!\!-\!\!*$ indicates the cost of purchasing the original consumption bundle at the new prices. The substitution effect indicates the cheaper, optimal consumption bundle at the $\wedge$ point. This compensating variation translates into a lower budget constraint $\wedge\!\!-\!\!\wedge$ in Fig. 4. The $\wedge$ point shows consumption with new prices yielding utility at the same level as prior to the EEU and minimizing expenditure, by consuming more of the now-lower-cost energy service and less of the now-relatively-more-expensive other goods.

Rebound from the substitution effect is typically decomposed into indirect (the decrease in other goods consumption, segment $*\!\!-\!\!c$) and direct (the increase in energy service consumption, segment $c\!\!-\!\!\wedge$) components. The impact of the substitution effect on energy consumption rates and expenditure rates can be seen in Figs. 2 and 3, respectively.

An approximate utility model is often used in the literature (e.g., see Borenstein (2015, p. 17, footnote 43)) for determining the post-substitution effect point and therefore Re_{dsub} and Re_{isub}. The approximate utility model assumes that the compensated energy service price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s, c}$) and the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o, p_s, c}$) are constant along an indifference curve. Whether that is true depends on the utility function used. Constant price elasticities (as in the approximate utility model) are approximations that are in general only applicable to marginal price changes. Appendix C.3.3 contains details of the approximate utility model.

Here, we present an exact utility model that allows the compensated energy service price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s, c}$) and the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o, p_s, c}$) to vary along an indifference curve, allowing analysis of non-marginal energy service price changes ($p_s^\circ \gg p_s^*$). We employ a fully specified CES utility model, which allows the direct calculation of the utility-maximizing consumption bundle for any constraint. The utility model describes the device owner's behavior as

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_o}{\dot{C}_o^\circ} \right)^\rho \right]^{(1/\rho)}. \tag{14}$$

The device owner's utility rate (relative to the original condition, $\dot{u}/\dot{u}^\circ$) is determined by the consumption rate of the energy service ($\dot{q}_s$) and the consumption rate of other goods and services ($\dot{C}_o$). The share parameter ($f_{\dot{C}_s}^\circ$) between $\dot{q}_s$ and $\dot{C}_o$ is taken from the original (pre-EEU) consumption basket. The exponent ρ is calculated from the (constant) elasticity of substitution (σ) as $\rho \equiv (\sigma - 1)/\sigma$. All quantities are normalized to pre-EEU values so that the cost share of other goods can be used straightforwardly in empirical applications rather than having to construct quantity and price indices. The normalized specification is commonly used in empirical CES *production*

function applications (Klump et al., 2012; Temple, 2012; Gechert et al., 2021). See Appendix D for further details of the CES utility model.

Direct substitution effect rebound (Re_{dsub}) is

$$Re_{dsub} = \frac{\Delta \dot{\hat{E}}_s}{\dot{S}_{dev}} \, , \tag{15}$$

which can be rearranged to

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, . \tag{16}$$

Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \dot{\hat{C}}_o I_E}{\dot{S}_{dev}} \, , \tag{17}$$

which can be rearranged to

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, \frac{\tilde{\eta}}{\eta^\circ} \, \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \, . \tag{18}$$

To find the post-substitution effect point ($\wedge$), we solve for the location on the i°—i° indifference curve where its slope is equal to the slope of the ∗—∗ expenditure line, assuming the CES utility model.[11] The results are

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} \tag{19}$$

and

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left[\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} \, . \tag{20}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an expression for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

Eq. (20) can be substituted directly into Eq. (18) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

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[11]Other utility models could be used. However, the Cobb-Douglas utility model is inappropriate for this framework, because it assumes that the sum of substitution and income rebound is 100% *always*. Regardless of the utility model, expressions for $\hat{\dot{q}}_s/\dot{q}_s^\circ$ and $\hat{\dot{C}}_o/\dot{C}_o^\circ$ must be determined and substituted into Eqs. 16 and 18, respectively.

$$Re_{isub} = \frac{\left[\left(1 + f^{\circ}_{\dot{C}_s}\left\{\left[\left(\frac{1-f^{\circ}_{\dot{C}_s}}{f^{\circ}_{\dot{C}_s}}\right)\frac{\tilde{p}_s\dot{q}^{\circ}_s}{\dot{C}^{\circ}_o}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^{\circ}} - 1} \frac{\tilde{\eta}}{\eta^{\circ}} \frac{\dot{C}^{\circ}_o I_E}{\dot{E}^{\circ}_s} \tag{22}$$

(See Appendix C.3.3 for details of the derivations of Eqs. (16), (18), and (19)–(22).)

3.5.3 Income effect

The monetary income rate of the device owner ($\dot{M}^{\circ}$) remains unchanged across the rebound effects, such that $\dot{M}^{\circ} = \dot{M}^* = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Thanks to the energy service price decline, real income rises, and freed cash from the EEU is given by Eq. (76) as $\dot{G} = p_E \dot{S}_{dev}$. Emplacement effect adjustments and compensating variation modify freed cash to leave the device owner with *net* savings ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (86). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables C.3 and C.4. Rebound from the income effect quantifies the rate of additional energy demand that arises when the energy conversion device owner spends net savings from the EEU (segments $\wedge$—d and d—$-$ in Figs. 2–4),

Additional energy demand from the income effect is determined by several constraints. The income effect under utility maximization satisfies the budget constraint, so that net savings are zero after the income effect ($\bar{\dot{N}} = 0$). The budget constraint can be seen in expenditure path graphs (e.g., Fig. 3) where the post-income-effect point ($-$) returns to the original expenditure isoquant. See Appendix E for a mathematical proof that the income preference equations below (Eqs. (23) and (27)) satisfy the budget constraint.

A second constraint is that net savings are spent completely on (i) additional spending on the energy service ($\hat{\dot{q}}_s < \bar{\dot{q}}_s$, segment $\wedge$—d) and (ii) additional spending on other goods ($\hat{\dot{q}}_o < \bar{\dot{q}}_o$, segment d—$-$). This constraint is satisfied by construction below.

The proportions in which income-effect spending is allocated depends on preferences, which prescribe an income expansion path. The CES utility function allocates spending on the energy service and other goods in the same proportion as post-substitution-effect expenditures on the energy service and other goods, due to its homotheticity, so that the income expansion path is a ray (r—r) on consumption path graphs. This constraint is satisfied by construction below, particularly via an effective income term ($\hat{\dot{M}}'$). In the consumption path graph, the pre- and post-income-effect points ($\wedge$ and $-$, respectively) lie along ray r—r from the origin through point $\wedge$ in Fig. 4. However, this framework could accommodate non-homothetic preferences for spending across the income effect (turning the income expansion path into a more general curve).

We next show expressions for direct and indirect income effect rebound.

Direct income effect The income elasticity of energy service demand ($\epsilon_{\dot{q}_s, \dot{M}}$) quantifies the amount of net savings spent on more of the energy service ($\hat{\dot{q}}_s < \bar{\dot{q}}_s$). (See Appendix D for additional information about elasticities.) Spending of net savings on additional energy service leads to direct income effect rebound (Re_{dinc}).

The ratio of rates of energy service consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s, \dot{M}}}. \tag{23}$$

18

Homotheticity means that $\epsilon_{\dot{q}_s, \dot{M}} = 1$. The increased consumption rate of the energy service is represented by segments $\wedge$—$\hat{d}$ in Figs. 2–4.

Effective income $(\hat{\dot{M}}')$ is given by

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \ . \tag{24}$$

For the purposes of the income effect, the effective income equation (Eq. (24)) adjusts income $(\dot{M}^\circ)$ to account for sunk costs $(\dot{C}^*_{cap}$ and $\dot{C}^*_{md})$ and net savings $(\hat{\dot{N}})$.

Direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \bar{\dot{E}}_s}{\dot{S}_{dev}} \ . \tag{25}$$

(See Table C.5.) After substitution, rearranging, and canceling of terms (Appendix C.3.4), the expression for direct income rebound is

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s, \dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left\{ f^\circ_{\dot{C}_s} + (1 - f^\circ_{\dot{C}_s}) \left[\left(\frac{1 - f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \right) \frac{\tilde{p}_s \dot{q}^\circ_s}{\dot{C}^\circ_o} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} \ . \tag{26}$$

If there are no net savings after the substitution effect $(\hat{\dot{N}} = 0)$, direct income effect rebound is zero $(Re_{dinc} = 0)$, as expected.[12]

Indirect income effect But not all net savings $(\hat{\dot{N}})$ are spent on more energy for the energy conversion device. The income elasticity of other goods demand $(\epsilon_{\dot{q}_o, \dot{M}})$ quantifies the amount of net savings spent on additional other goods $(\hat{\dot{q}}_o < \bar{\dot{q}}_o)$. Spending of net savings on additional other goods and services leads to indirect income effect rebound (Re_{iinc}).

The ratio of rates of other goods consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \right)^{\epsilon_{\dot{q}_o, \dot{M}}} \tag{27}$$

and is represented by segments $\hat{d}$—$-$ in Figs. 2–4. Under the assumption that prices of other goods are exogenous (see Appendix F), the ratio of rates of other goods consumption $(\bar{\dot{q}}_o/\hat{\dot{q}}_o)$ is equal to the ratio of rates of other goods expenditures $(\bar{\dot{C}}_o/\hat{\dot{C}}_o)$ such that

$$\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \right)^{\epsilon_{\dot{q}_o, \dot{M}}} \ . \tag{28}$$

Homotheticity means that $\epsilon_{\dot{q}_o, \dot{M}} = 1$. As shown in Table C.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta \bar{\dot{C}}_o I_E}{\dot{S}_{dev}} \ . \tag{29}$$

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[12]Zero net savings $(\hat{\dot{N}} = 0)$ could occur if increases in the capital cost rate $(\Delta \dot{C}^*_{cap})$ and/or the maintenance and disposal cost rate $(\Delta \dot{C}^*_{md})$ consume all freed cash $(\dot{G})$ plus savings from the compensating variation.

After substitution, rearranging, and canceling of terms, the expression for indirect income rebound is

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M'}}}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1}{\frac{\hat{\tilde{\eta}}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left[\left(1 + f_{\dot{C}_s}^\circ \left\{\left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho}. \quad (30)$$

(See Appendix C.3.4 for details of the derivation of direct and indirect income effect rebound.)

3.5.4 Macro effect

The previous rebound effects (emplacement effect, substitution effect, and income effect) occur at the microeconomic scale. However, changes at the microeconomic scale can have important implications at the macroeconomic scale, including via aggregate economic growth from the energy-augmenting productivity increase.

Turner (2013) cautions that when households see the productivity of their non-market activities increase, GDP remains unchanged. That may be true in the short run, but the question over longer periods is whether the higher productivity of household production of energy services does not affect also their capacity to contribute to economic growth. One channel could be time-saving. If the EEU saves time, then this time could be spent on more work or on increasing human capital (Sorrell & Dimitropoulos, 2008). If the EEU saves money (but no time), then the freed cash could be spent on human capital-increasing activities or even be used to start a venture. In all cases, it would be rash to conclude that just because some EEUs lead to productivity increases not captured by GDP, it does not eventually lead to additional economic growth.[13]

Borenstein also addressed these macro effects from consumer behavior noting that "income effect rebound will be larger economy-wide than would be inferred from evaluating only the direct income gain from the end user's transaction" (Borenstein, 2015, p. 11) and likening it to the Keynesian macroeconomic multiplier. However, the dynamic macro rebound effect is not an autonomous expansion of expenditure in an otherwise unchanged economy (like the Keynesian multiplier). Rather, macroeconomic rebound is caused by an energy productivity improvement, a supply-side shock. After the EEU, it takes less energy (and therefore less energy cost) to generate the same economic activity, because energy efficiency has improved. That said, Borenstein is right to highlight that supply-side and demand-side effects both play a role as the consequences of the technology shock play themselves out. Furthermore, his approach has the advantage that it can be directly linked to our income effect and its consequences for macroeconomic rebound. Borenstein also notes that scaling from net savings ($\hat{\dot{N}}$) at the device level to productivity-driven growth at the macro level is unexplored territory.

Another novel contribution of this paper (in addition to the framework itself and rebound path graphs) is the first operationalization of the macro rebound multiplier idea. To operationalize the macro rebound effect, we scale the net savings gained by the device owner at the microeconomic scale ($\hat{\dot{N}}$) by a macro factor (k) that represents the economic activity generated by the infinite series of

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[13]To appreciate the difference, consider one case where an increased mileage leads to the household saving on energy per car trip. The household takes more trips (direct rebound), without effect on GDP. In the other case, the household buys the energy service directly from a taxi company. Here, the taxi company lowers the price but gains more customers, leading immediately to growth in inflation-adjusted, i.e. real, GDP. Yet, the physical change of more car trips is the same in both cases.

responding of net savings ($\hat{\dot{N}}$) throughout the economy.[14] The macro factor (k) can be likened to the total respending described by a marginal propensity to consume (MPC). k represents respending in the broader economy after the income effect has occurred and is not tied to any particular EEU or economic sector. $k \geq 0$ is expected. $k = 0$ means there is no dynamic effect resulting from the energy efficiency upgrade. $k > 0$ means that productivity-driven macroeconomic growth has occurred with consequent implications for additional energy consumption in the wider economy. The relationship between k and MPC is given by

$$k = \frac{1}{\frac{1}{MPC} - 1} \ . \tag{31}$$

(See Appendix G for the derivation of Eq. (31).)

A further advantage of using the MPC approach is that there are many estimates of its magnitude, though we stress again that MPC is a *representation* of the effect, while the cause is higher energy efficiency on the supply side. A recent review by Carroll et al. (2017) reports that most empirical estimates show MPC between 0.2 and 0.6, with the full range estimates spanning zero to 0.9. For now, we choose $MPC = 0.5$ and $k = 1$ as a placeholder value, with further exploration of the macro effect in Sections 4.3.5 and 5.3. We assume as a first approximation (following Antal & van den Bergh (2014) and Borenstein (2015)) that macro effect respending implies energy consumption according to the average energy intensity of the economy (I_E). Macro rebound is given by

$$Re_{macro} = \frac{k\hat{\dot{N}}I_E}{\dot{S}_{dev}} \ . \tag{32}$$

(See Table C.6.) The macro effect is shown as segment $-\!\!-\!\!\sim$ in Fig. 2. After some algebra (Appendix C.3.5), we arrive at an expression for macro effect rebound:

$$Re_{macro} = kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} \ . \tag{33}$$

3.6 Rebound sum

The sum of all rebound emerges from the four rebound effects (emplacement effect, substitution effect, income effect, and macro effect). Macro effect rebound (Re_{macro}, Eq. (33)) is expressed in terms of other rebound effects. (Derivation details can be found in Appendix C.3.6.) After algebra and canceling of terms, we find

$$\begin{aligned} Re_{tot} = \ &Re_{emb} - kRe_{cap} + (1-k)Re_{md} \\ &+ (1 - kp_E I_E)Re_{dsub} + (1-k)Re_{isub} \\ &+ Re_{dinc} + Re_{iinc} + kp_E I_E \ . \end{aligned} \tag{34}$$

4 Results: Two applications of the rebound framework

To demonstrate application of the comprehensive rebound analysis framework developed in Section 3, we select two case studies: energy efficiency upgrades to a car and an electric lamp. We first collect parameter values for the equations for eight rebound components: Re_{dempl} (Eq. (11)), Re_{emb}

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[14]The macro factor (k) appears unitless, but its units are actually \$ of economic growth created per \$ of net savings spent by the device owner in the income effect ($\hat{\dot{N}}$).

(Eq. (12)), Re_{md} (Eq. (13)), Re_{dsub} (Eqs. (16 and 19)), Re_{isub} (Eq. (18 and 20)), Re_{dinc} (Eq. (26)), Re_{iinc} (Eq. (30)), and Re_{macro} (Eq. (33)). The total rebound (Re_{tot}) is given by the sum of the above components, which can also be verified against Eq. (34).

As discussed in Section 3.5.4, the link between macroeconomic and microeconomic rebound is largely unexplored, and we assume a placeholder value of $k = 1$ for both case studies. We return to the matter of calibrating k in the Discussion (Section 5.3).

4.1   Case 1: Purchase of a new car

For the first example, we consider the purchase of a more fuel efficient car, namely a gasoline-electric Ford Fusion Hybrid car, to replace a conventional gasoline Ford Fusion car. The cars are matched as closely as possible, except for the inclusion of an electric battery in the hybrid car. The car case study features a larger initial capital investment ($C_{cap}^{\circ} < \tilde{C}_{cap}$) versus the long-term benefit of decreased energy service costs ($\dot{C}_{s}^{\circ} > \tilde{\dot{C}}_{s}$).

4.1.1   Input parameters

We require four sets of data. First, basic car parameters are summarized in Table 3. Second, we require several general parameters, mainly relating to the U.S. economy and personal finances of the average U.S. citizen, shown in Table 4. Third, we require elasticity parameters, as given in Table 5. Fourth, armed with the parameter values from Tables 3–5, and the equations of Section 3, we calculate important values at each rebound stage, as shown in Table 6. Note that Table 6 applies to the car owner. Across the macro effect (segment $-\!\!-\!\!\sim$ in Fig. 5), changes occur only in the macroeconomy. For the car owner, no changes are observed across the macro effect. Thus, the $-$ (bar) and $\sim$ (tilde) columns of Table 6 are identical.

Table 3: Car example: Vehicle parameters.

Description Parameters [units]	Ford Fusion (gasoline)	Ford Fusion (hybrid EV)	Data sources and notes
Fuel economy η°, $\tilde{\eta}$ [mpg]	25	42	Combined cycle mpg value taken from Thecarconnection.com (2020), for Titanium FWD 2020 model with Intercooled I-4, 2.0 L engine. Combined cycle mpg value taken from Thecarconnection.com (2020), for Titanium FWD 2020 model with Gas/Electric I-4, 2.0 L engine.
Capital expenditure rate $\dot{C}_{cap}^\circ$, $\dot{C}_{cap}^*$ [\$/year]	4,778	4,720	Seven year annual, averaged capital costs = purchase + finance costs – resale value (purchase – depreciation). Ford Fusion gasoline costs from Edmunds.com (2020a). Ford Fusion Hybrid car costs from Edmunds.com (2020b).
Ownership duration t_{own}°, t_{own}^* [years]	7	7	U.S. car ownership (from new) length from Businesswire.com (2015), and has risen from 52 months (2005) to 79 months (2015), so taken as 84 months in 2020 (7 years) for our example.
Lifespan t_{life}°, t_{life}^* [years]	14	14	Lifetime taken as 14 years, based on 13–17 years for U.S. cars from Berla.com (2016) and 14 years for UK cars from Society of Motor Manufacturers and Traders (2020).
Embodied energy E_{emb}°, E_{emb}^* [MJ]	34,000	40,000	34,000 MJ for conventional Ford Fusion gasoline car taken from Argonne National Laboratory, Energy Systems Division (2010). We assume an additional 6,000 MJ added for Ford Fusion Hybrid Electric Vehicle (HEV) battery, as HEV typically adds 10–25% to total LCA energy of vehicle manufacture (Onat et al., 2015). Battery lifetime assumed same as car lifetime, based on Nordelöf et al. (2014) and Onat et al. (2015).
Maintenance and disposal expenditure rate $\dot{C}_{md}^\circ$, $\dot{C}_{md}^*$ [\$/year]	2,731	2,710	Seven year annual, averaged maintenance costs = sum of insurance, maintenance, repairs, taxes, and fees (excluding financing, depreciation, fuel). Ford Fusion maintenance costs from Edmunds.com (2020a). Ford Fusion Hybrid maintenance costs from Edmunds.com (2020b).

Table 4: Car example: Economic parameters (2020).

Description Parameter [units]	Value	Data sources and notes
Distance driven prior to upgrade $\dot{q}_s^\circ$ [miles/year]	12,416	Average U.S. vehicle miles/year, calculated from Carinsurance.com (2019). This is slightly lower than the average driver miles/year (13,476) (US Department of Transportation, 2018), as there are more registered U.S. vehicles than drivers.
Real median personal income U.S., in 2018 [\$/year]	34,317	Taken from Federal Reserve Bank of St Louis (2019).
U.S. 2018 disposable income / real income (minus current taxes) [–]	0.88319	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition (US Bureau of Economic Analysis, 2020).
Share of savings from 2018 disposable income [–]	0.07848	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition (US Bureau of Economic Analysis, 2020).
Personal consumption in 2018 $\dot{M}$ [\$/year]	27,929.83	Calculation: $(\$34,317/\text{year})(0.88319)(1 - 0.07848)$
Price of gasoline p_E [\$/gallon]	2.63	Source: US Energy Information Administration (2020b)
Fractional spend on original energy service $f_{\dot{C}_s}^\circ$ [–]	0.064	Calculation: \$1,306 (spend on energy service) / [\$19,115 (other goods) + \$1,306 (energy service)] = 0.064, where spend on energy service = 12,416 miles / 25 mpg × \$2.63/gallon = \$1,306.
Macro factor k [–]	1.0	Assumed value.

Table 5: Car example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Price elasticity of car use demand $\epsilon_{\dot{q}_s,p_s}$ [−]	−0.2	We adopt −0.2 as our baseline value, based on U.S. studies including Gillingham (2020) who estimated a value of −0.1, Goetzke & Vance (2018) who estimated values between −0.05 and −0.23, and Parry & Small (2005) who estimated values between −0.1 and −0.3. For comparison, Borenstein (2015) uses values of −0.1 to −0.4 based on Parry & Small (2005).
Compensated price elasticity of car use demand $\epsilon_{\dot{q}_s,p_s,c}$ [−]	−0.136	Calculated via the Slutsky Equation (Eq. (155)).
Compensated cross-price elasticity of demand for other goods $\epsilon_{\dot{q}_o,p_s,c}$ [−]	0.009	Calculated via Eq. (161).
Income elasticity of demand for car use $\epsilon_{\dot{q}_s,\dot{M}}$ [−]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\epsilon_{\dot{q}_o,\dot{M}}$ [−]	1.0	Follows from CES utility function.

Table 6: Results for car example with macro factor (k) assumed to be 1.

	∘ (orig)	∗ (star)	∧ (hat)	− (bar)	∼ (tilde)
η [mile/gal]	25.0	42.0	42.0	42.0	42.0
η [mile/MJ]	0.197	0.332	0.332	0.332	0.332
p_s [$/mile]	0.105	0.063	0.063	0.063	0.063
$\dot{q}_s$ [mile/year]	12,416	12,416	13,336	13,756	13,756
$\dot{E}_s$ [MJ/year]	62,885	37,432	40,204	41,470	41,470
$\dot{E}_{emb}$ [MJ/year]	2,429	2,857	2,857	2,857	2,857
$\dot{C}_s$ [$/year]	1,306	777	835	861	861
$\dot{C}_{cap}$ [$/year]	4,778	4,720	4,720	4,720	4,720
$\dot{C}_{md}$ [$/year]	2,731	2,710	2,710	2,710	2,710
$\dot{C}_o$ [$/year]	19,115	19,115	19,040	19,639	19,639
$\dot{N}$ [$/year]	0	608	626	0	0
$\dot{M}$ [$/year]	27,930	27,930	27,930	27,930	27,930

4.1.2 Results

Results are represented graphically in quantified energy, expenditure, and consumption path graphs in Figs. 5–7. The energy path graph (Fig. 5) shows the size of each rebound effect for the car example.

Rebound components for the car upgrade are shown in Table 7.

Table 7: Car example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	1.7
Re_{md}	−0.3
Re_{dsub}	10.9
Re_{isub}	−1.0
Re_{dinc}	5.0
Re_{iinc}	7.6
Re_{macro}	8.0
Re_{tot}	31.9

The emplacement effect has three components: the direct emplacement effect, the embodied energy effect, and the maintenance and disposal effect. Rebound from the direct emplacement effect (Re_{dempl}) is 0.0% always, because energy takeback (and, therefore, rebound) occurs after the EEU is emplaced. Indirect rebound due to the embodied energy effect (Re_{emb}) is 1.7%, due to the higher embodied energy rate ($\Delta \dot{E}^*_{emb} = 429$ MJ/year) stemming from the electric battery in the hybrid EV car. Rebound due to the maintenance and disposal effect (Re_{md}) is small and negative (−0.3%), because of the slightly lower maintenance and disposal costs for the hybrid EV car.

The substitution effect has two components: direct and indirect substitution effect rebound. Rebound from direct substitution (Re_{dsub}) is positive, as expected (10.9%). The car owner will, on average, prefer more driving, because of fuel economy enhancements (42 mpg > 25 mpg). In other words, with no other changes, the more fuel-efficient car is driven 10.9% further per year. Conversely, the indirect substitution effect (Re_{isub}) is slightly negative (−1.0%) to achieve the same level of utility after increased driving. Indeed, less money is spent on other goods ($\Delta \hat{C}_o = -75.18$ $/year).
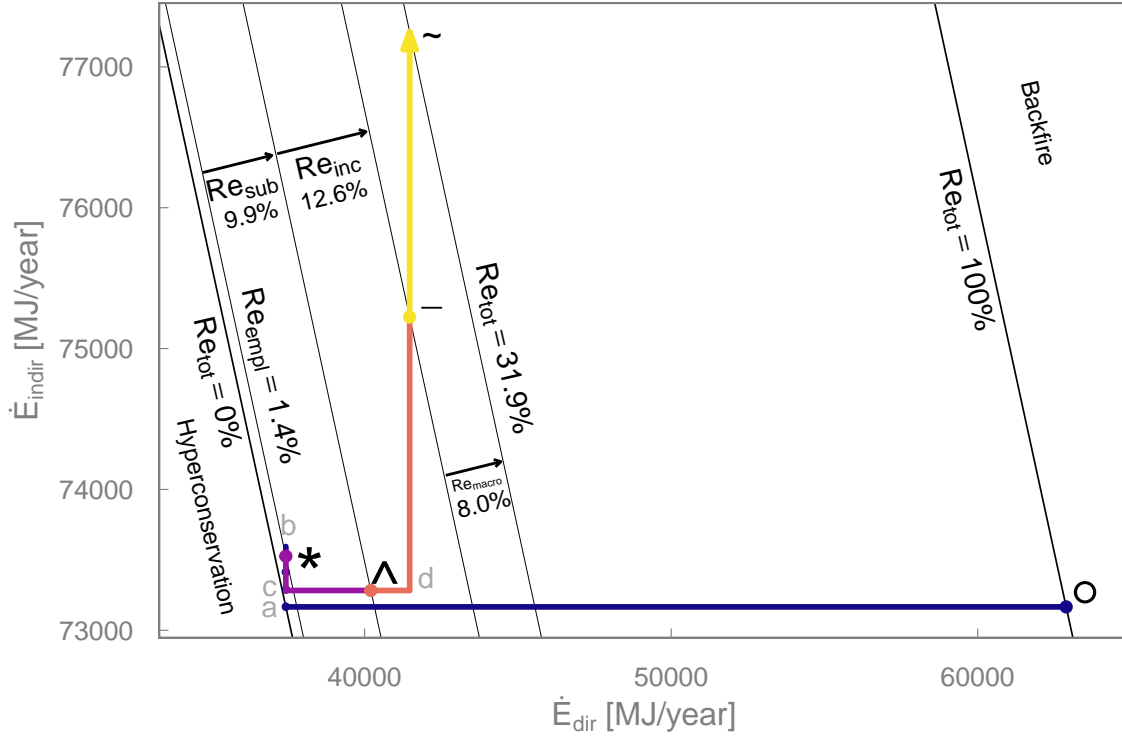
Fig. 5: Energy path graph for the car example. Macro factor (k) is assumed to be 1.

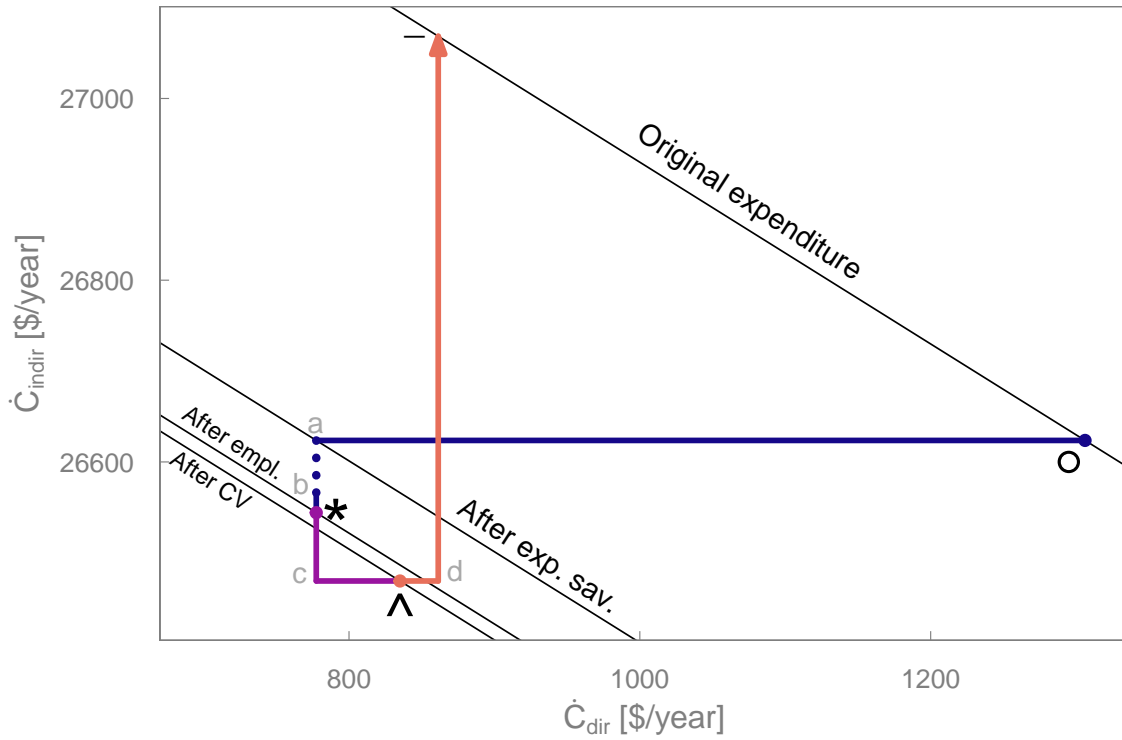
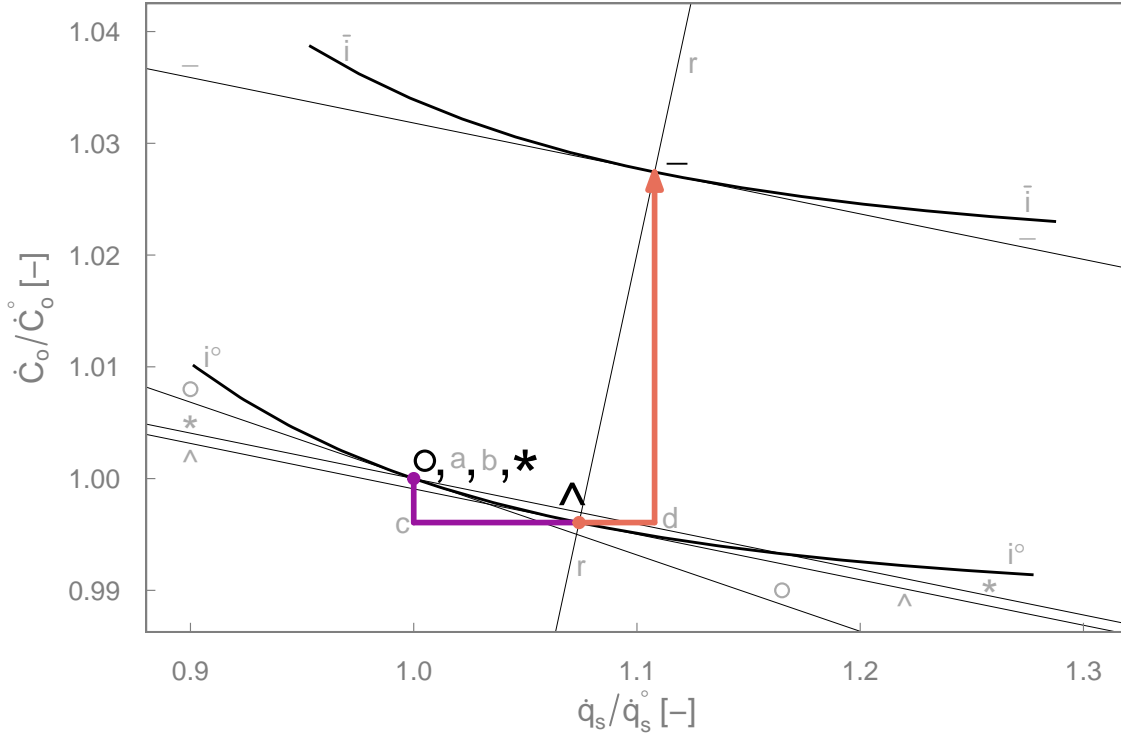Fig. 6: Expenditure path graph for the car example. CV is compensating variation.

Fig. 7: Consumption path graph for the car example.

The income effect also has two components: direct and indirect income effect rebound. The direct income effect (Re_{dinc}) is positive (5.0%), because the car owner allocates some net savings to additional driving. Rebound from the indirect income effect (Re_{iinc}) is positive (7.6%) due to higher spending on other goods. Thus, the net savings after the substitution effect ($\hat{N} = 625.79$ \$/year) translates into positive direct and indirect income rebound at the microeconomic scale. Total microeconomic rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 24.0\%$.

Finally, the macro effect leads to macroeconomic rebound (Re_{macro}) of 8.0%, with k assumed to be 1, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (transportation).

4.2 Case 2: Purchase of a new electric lamp

For the second example, we consider purchasing a Light Emitting Diode (LED) electric lamp to replace a baseline incandescent electric lamp. Both lamps are matched as closely as possible in terms of energy service delivery (measured in lumen output per lamp), the key difference being the energy required to provide that energy service. The LED lamp has a low initial capital investment rate (less than the incumbent incandescent lamp, actually) and a long-term benefit of decreased direct energy expenditures at the same energy service delivery rate (lm-hr/year).

4.2.1 Input parameters

Again, four sets of data are required. First, basic lamp parameters are summarized in Table 8. Second, several general parameters, mainly relating to the U.S. economy and personal finances of the average U.S. citizen are given in Table 9. Third, we require the elasticity parameters, as shown

in Table 10. Fourth, with the parameter values from Tables 8–10 and the equations of Section 3 in hand, we calculate important values at each rebound stage, as shown in Table 11. Similar to Table 6, Table 11 applies to the lamp owner, so no changes are observed across the macro effect, and the $-$ (bar) and $\sim$ (tilde) columns of Table 11 are identical.

4.2.2   Results

Results are represented graphically in energy, expenditure, and consumption path graphs in Figs. 8–10. The energy path graph (Fig. 8) shows the size of each rebound effect for the lamp example. Rebound components for the lamp upgrade are shown in Table 12.

The emplacement effect rebound components start with the direct emplacement effect (Re_{dempl}), which is always 0.0%. Indirect rebound due to the embodied energy effect (Re_{emb}) is -0.3%. Although the LED lamp has higher embodied energy ($E^*_{emb} = 6.50$ MJ) than the incandescent lamp ($E^\circ_{emb} = 2.20$ MJ), the LED lamp has a much longer lifetime, meaning that the LED embodied energy *rate* ($\dot{E}^*_{emb} = 0.65$ MJ/year) is less than the incandescent embodied energy rate ($\dot{E}^\circ_{emb} = 1.22$ MJ/year). Thus, the change in embodied energy rate ($\Delta\dot{E}^*_{emb}$) is -0.57 MJ/year, and embodied energy rebound is negative ($Re_{emb} = -0.3\%$). Rebound due to the maintenance and disposal effect (Re_{md}) is 0.0%, because we assume no difference in maintenance and disposal costs between the incandescent lamp and the LED lamp.[15]

Direct substitution effect rebound (Re_{dsub}) is 17.4% due to the much higher LED lamp efficacy ($\tilde{\eta} = 81.8$ lm/W) compared to the incandescent lamp ($\eta^\circ = 8.83$ lm/W), leading to increased demand for lighting (from $\dot{q}^*_s = 580,350$ lm-hr/year to $\hat{q}_s = 1,412,867$ lm-hr/year) as shown by segment c—$\wedge$ in Fig. 10. To maintain constant utility, consumption of other goods is reduced ($\Delta\hat{C}_o = -4.15$ \$/year), as shown by segment $*$—c in Fig. 10, yielding indirect substitution effect rebound (Re_{isub}) of -6.4%.

Income effect rebound arises from spending net energy cost savings associated with converting from the incandescent lamp to the LED lamp ($\hat{N} = 11.31$ \$/year). Direct income effect rebound (Re_{dinc}) is 0.01%, positive but small, as the lamp owner allocates some of the net savings to increased demand for lighting. The indirect income effect rebound is large ($Re_{iinc} = 17.4\%$), due to the energy implications of increased spending on other goods. Total microeconomic scale rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 28.1\%$.

Finally, macro effect rebound (Re_{macro}) is coincidentally also 17.4% with k assumed to be 1, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (lighting).

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[15]Maintenance cost rates for both incandescent and LED lamps are likely to be equal and negligible; lamps are usually installed and forgotten. Real-world disposal cost differences between the incandescent and LED technologies are also likely to be negligible. However, if "disposal" includes recycling processes, cost rates may be different between the two technologies due to the wide variety of materials in LED lamps compared to incandescent lamps.

Table 8: Lamp example: Electric lamp parameters.

Description Parameters [units]	Incandescent lamp	LED lamp	Data sources and notes
Lamp efficacy η°, $\tilde{\eta}$ [lm-hr/W-hr]	8.83	81.8	Incandescent: 530 lm output / 60 W energy input. LED: 450 lm output / 5.5 W energy input.
Capital expenditure rate $\dot{C}^\circ_{cap}$, $\dot{C}^*_{cap}$ [\$/year]	1.044	0.121	Purchase costs: \$1.88 for incandescent lamp from HomeDepot.com (2020b), and \$1.21 for LED lamp from HomeDepot.com (2020a).
Ownership duration t°_{own}, t^*_{own} [years]	1.8	10	Assumed same as lamp lifespan
Lifespan t°_{life}, t^*_{life} [years]	1.8	10	Based on assumed 3 hours/day from HomeDepot.com (2020b) and HomeDepot.com (2020a).
Life cycle analysis (LCA) embodied energy E°_{emb}, E^*_{emb} [MJ]	2.20	6.50	Base document: Table 4.5 Manufacturing Phase Primary Energy (MJ/20 million lumen-hours), contained in U.S. DoE Life-cycle assessment of energy and environmental impacts of LED lighting products (US Department of Energy, 2012). Incandescent lamp: LCA energy = 42.2 MJ/20 million lumen-hours. Lifetime output = 530 lumens × 3 hours/day × 365 days/year × 1.8 years = 1,044,630 lumen-hrs. Thus LCA energy / lamp = 42.2 × 1.0446/20 = 2.21 MJ. LED lamp: LCA energy = 132 MJ/20 Million lumen-hours for pack of 5 LED lamps. Lifetime output = 450 lumens × 3 hours/day × 365 days/year × 10 years = 4,926,405 lumen-hrs. Thus LCA energy / lamp = 132 MJ/5 × 4.9264/20 = 6.5 MJ.
Maintenance and disposal expenditure rate $\dot{C}^\circ_{md}$, $\dot{C}^*_{md}$ [\$/year]	0.00	0.00	Assumed negligible.

Table 9: Lamp example: Economic parameters (2020).

Description Parameter [units]	Value	Data sources and notes
Lighting consumption prior to upgrade $\dot{q}_s^\circ$ [lm-hr/year]	580,350	Calculation: (530 lm) (3 hrs/day) (365 days/year).
Real median personal income U.S. in 2018 [\$/year]	34,317	Refer to Table 4.
U.S. 2018 disposable income / real income (minus current taxes) [–]	0.88319	Refer to Table 4.
Share of savings from 2018 disposable income [–]	0.07848	Refer to Table 4.
Personal consumption in 2018 $\dot{M}$ [\$/year]	27,929.83	Calculation: (\$34,317/year)(0.88319)(1 − 0.07848).
Price of electricity p_E [\$/kW-hr]	0.1287	U.S. 2018 average U.S. household electricity price (US Energy Information Administration, 2020a).
Fractional spend on original energy service $f_{\dot{C}_s}^\circ$ [–]	0.0003028	Calculation: \$8.5/year (spend on energy service) / [\$27,920/year (other goods) + \$8.5/year (energy service)] = 0.00030, where spend on energy service = 580,350 lm-hrs/year / 8.83 lm/W / 1000 W/kW × \$0.1287/kW-hr = \$8.5/year.
Macro factor k [–]	1.0	Assumed value.

Table 10: Lamp example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Price elasticity of lighting demand $\epsilon_{\dot{q}_s,p_s}$ [−]	−0.4	We adopt −0.4 as our baseline value, as the average of last 50 years from Fouquet (2014, Fig. 4). For comparison, Borenstein (2015) uses a range of −0.4 to −0.8, based on Fouquet & Pearson (2011).
Compensated price elasticity of lighting demand $\epsilon_{\dot{q}_s,p_s,c}$ [−]	−0.3997	Calculated via the Slutsky Equation (Eq. (155)).
Compensated cross-price elasticity of demand for other goods $\epsilon_{\dot{q}_o,p_s,c}$ [−]	0.00012	Calculated via Eq. (161).
Income elasticity of lighting demand $\epsilon_{\dot{q}_s,\dot{M}}$ [−]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\epsilon_{\dot{q}_o,\dot{M}}$ [−]	1.0	Follows from CES utility function.

Table 11: Results for lamp example with macro factor (k) assumed to be 1.

	○ (orig)	∗ (star)	∧ (hat)	− (bar)	∼ (tilde)
η [lm-hr/kW-hr]	8,833	81,800	81,800	81,800	81,800
η [lm-hr/MJ]	2,454	22,722	22,722	22,722	22,722
p_s [\$/lm-hr]	0.00001457	0.00000157	0.00000157	0.00000157	0.00000157
$\dot{q}_s$ [lm-hr/year]	580,350	580,350	1,412,867	1,413,439	1,413,439
$\dot{E}_s$ [MJ/year]	236.5	25.5	62.2	62.2	62.2
$\dot{E}_{emb}$ [MJ/year]	1.222	0.650	0.650	0.650	0.650
$\dot{C}_s$ [\$/year]	8.46	0.91	2.22	2.22	2.22
$\dot{C}_{cap}$ [\$/year]	1.04	0.12	0.12	0.12	0.12
$\dot{C}_{md}$ [\$/year]	0.00	0.00	0.00	0.00	0.00
$\dot{C}_o$ [\$/year]	27,920	27,920	27,916	27,927	27,927
$\dot{N}$ [\$/year]	0.00	8.47	11.31	0.00	0.00
$\dot{M}$ [\$/year]	27,930	27,930	27,930	27,930	27,930

Table 12: Lamp example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	−0.3
Re_{md}	0.0
Re_{dsub}	17.4
Re_{isub}	−6.4
Re_{dinc}	0.0
Re_{iinc}	17.4
Re_{macro}	17.4
Re_{tot}	45.4

4.3 Sensitivity analyses

Sensitivity analyses show the effect of independently varied parameters on total rebound and rebound components. In the context of this framework, sensitivity analyses can show important trends, tendencies, and relationships between rebound parameters and rebound magnitudes. Key rebound parameters include post-EEU efficiency ($\tilde{\eta}$), capital cost ($\tilde{C}_{cap}$), energy price (p_E), uncompensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s}$), and the macro factor (k). Sensitivity analyses must be interpreted carefully, because rebound parameters are not expected to be independent from each other.

In each subsection below, a series of graphs shows the effects of varying each of these five key rebound parameters ($\tilde{\eta}$, $\tilde{C}_{cap}$, p_E, $\epsilon_{\dot{q}_s,p_s}$, and k) on total energy rebound (Re_{tot}) and its components (Re_{emb}, Re_{md}, Re_{dsub}, Re_{isub}, Re_{dinc}, Re_{iinc}, and Re_{macro}). In the graphs, nominal values for the rebound parameters are shown as points, and sensitivity trends are shown as lines. A limitation of each sensitivity analysis is noted at the end of each subsection.

4.3.1 Effect of post-EEU efficiency ($\tilde{\eta}$)

Fig. 11 shows that both the energy takeback rate and expected energy savings ($\dot{S}_{dev}$) increase with post-EEU efficiency ($\tilde{\eta}$), but the relationship is asymptotic. Each unit increase of fuel economy or
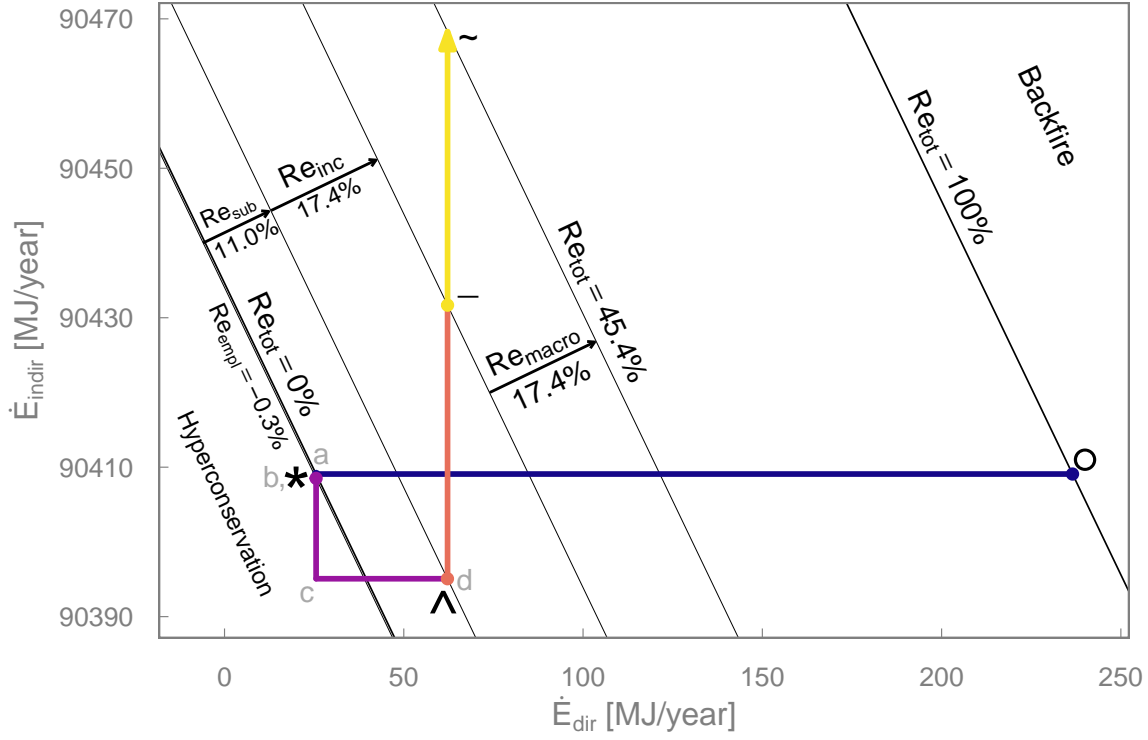
Fig. 8: Energy path graph for the lamp example. Macro factor (k) is assumed to be 1.
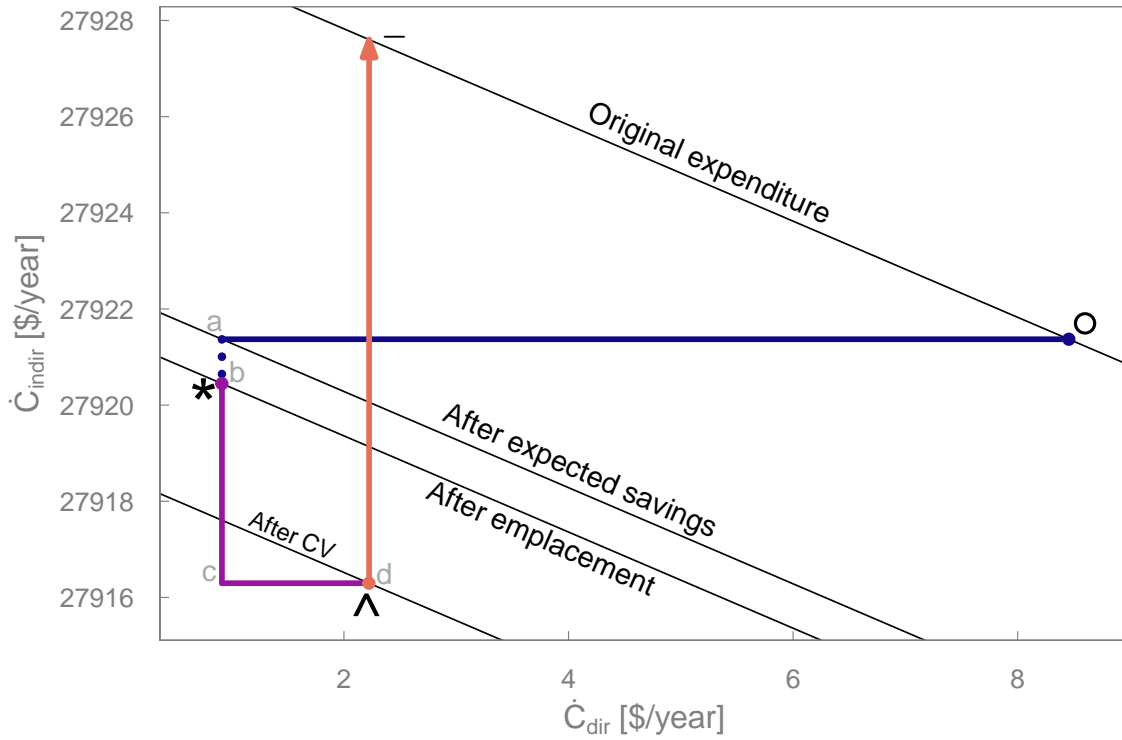

Fig. 9: Expenditure pathgraph for the lamp example. CV is compensating variation.
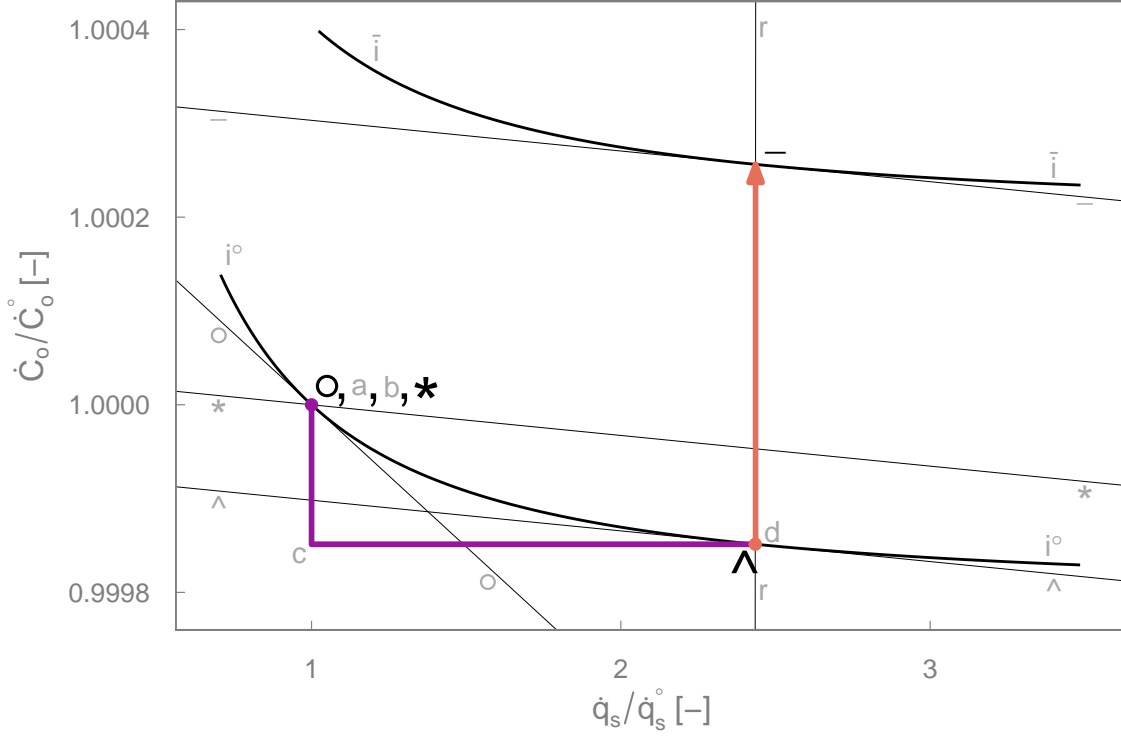
Fig. 10: Consumption path for the lamp example.

lighting efficacy is less effective than the previous unit increase of fuel economy or lighting efficacy for saving energy. At very high levels of fuel economy or lighting efficacy, a unit increase leads to almost no additional energy savings. Thus, we can say there are diminishing returns of fuel economy and lighting efficacy, leading to saturation of energy savings at very high levels of fuel economy and lighting efficacy. A simple example illustrates. A $\eta^\circ = 25$ mpg car drives $q_s^\circ = 100$ miles using $E_s^\circ = 4$ gallons of gasoline. A more-efficient car ($\tilde{\eta} = 30$ mpg) is expected to use $E_s^* = 3.33$ gallons to drive the same distance, a savings of $\dot{S}_{dev} = 0.67$ gallons. Another 5 mpg boost in efficiency (to $\tilde{\eta} = 35$ mpg) will use $E_s^* = 2.86$ gal to drive 100 miles, a further expected savings of only $\dot{S}_{dev} = 0.47$ gallons. Each successive 5 mpg boost in fuel economy saves less energy than the previous 5 mpg boost in fuel economy.

Saturation can be seen mathematically, too. Taking the limit as $\tilde{\eta} \to \infty$ in Eq. (10) gives $\dot{S}_{dev} = \dot{E}_s^\circ$, not ∞. Thus, efficiency saturation must occur. Fig. 11 shows that this framework correctly replicates expected efficiency saturation trends.

Saturation is especially noticeable in the lamp example compared to the car example, the difference being that the LED lamp is already much more efficient than the incandescent lamp (9.26×), whereas the hybrid car is only 1.68× more efficient than the conventional gasoline car. Thus, at $\tilde{\eta} = 81.8$ lm-hr/W-hr, the energy efficient LED is far closer to efficiency saturation than the hybrid vehicle (at $\tilde{\eta} = 42$ mpg). As a result, further increases in the LED lamp's efficiency are less effective than further increases in the hybrid car's efficiency.

That said, actual savings is the difference between the expected energy savings line (solid line) and the takeback line (dashed line) in Fig. 11. Because the gap between the lines grows, higher efficiency yields greater energy savings, even after accounting for rebound effects. But the actual savings are always less than expected savings, due to takeback.

Fig. 11 shows that expected energy savings ($\dot{S}_{dev}$) increase faster than takeback as $\tilde{\eta}$ increases.
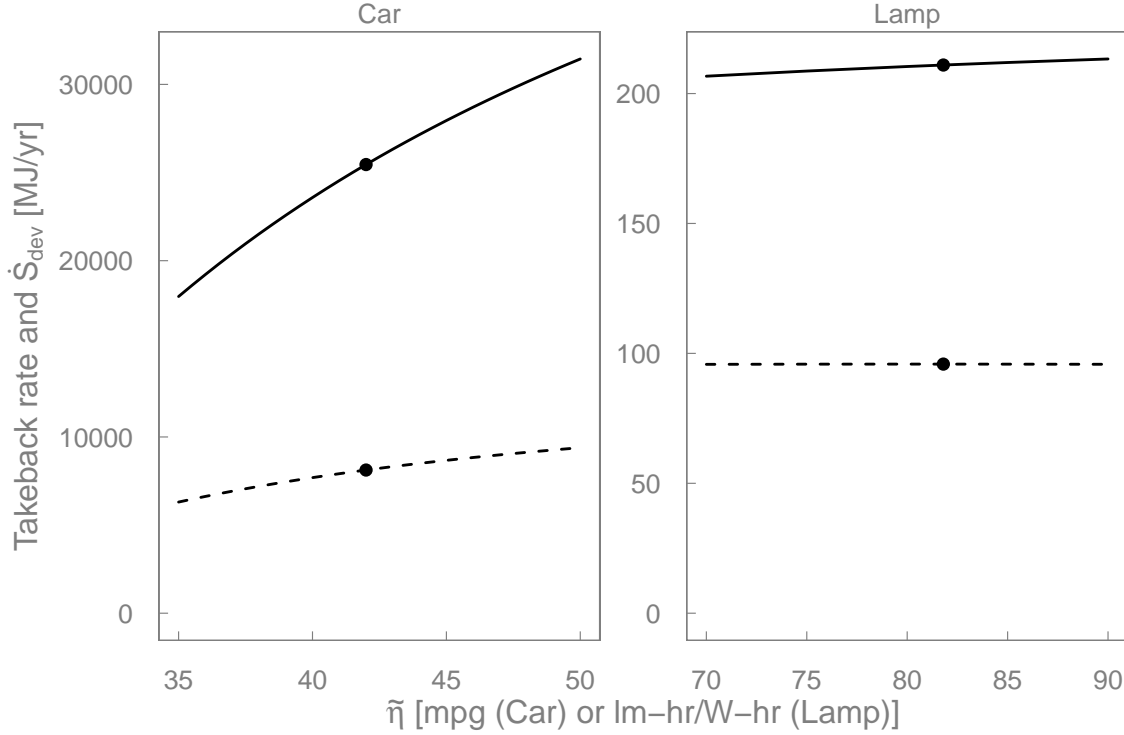
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Fig. 11: Expected energy savings rate ($\dot{S}_{dev}$, solid line) and takeback rate (dashed line) sensitivity to post-EEU efficiency ($\tilde{\eta}$). (Note different x- and y-axis scales.)

Thus, total rebound (Re_{tot}, the ratio of takeback rate to expected energy savings rate in Eq. (4)), decreases as efficiency grows. The lamp exhibits a relatively smaller rebound decline with efficiency, because the lamp example is closer to saturation than the car example.

Fig. 12 shows the variation of all rebound components with post-EEU efficiency ($\tilde{\eta}$). In the car and lamp examples, direct substitution rebound (Re_{dsub}) is the rebound component most sensitive to changes in post-EEU efficiency ($\tilde{\eta}$).

A limitation of the energy efficiency sensitivity analysis is that post-EEU efficiency ($\tilde{\eta}$) is unlikely to be independent of other factors, such as capital cost ($\tilde{C}_{cap}$).

Note that the sensitivity analysis on post-upgrade efficiency ($\tilde{\eta}$, Fig. 12) is the only sensitivity analysis that requires careful explication of both the numerator and denominator of Eq. (4), as in Fig. 11, because both the numerator and denominator of Eq. (4) change when post-upgrade efficiency ($\tilde{\eta}$) changes. The denominator of Eq. (4) doesn't change for the sensitivity analyses of Figs. 13–16. Thus, for the remaining sensitivity analyses, when the rebound percentage increases (decreases), the energy takeback rate in the numerator of Eq. (4) increases (decreases) proportionally, and the actual energy savings rate decreases (increases) accordingly.

4.3.2 Effect of capital cost ($\tilde{C}_{cap}$)

The sensitivity of energy rebound to capital cost ($\tilde{C}_{cap}$) is shown in Fig. 13. All other things being equal, as capital cost of the EEU rises, less net savings result from the emplacement effect, leading to smaller income, macro, and total rebound. The same effects would be observed with increasing maintenance and disposal cost rate ($\tilde{C}_{md}$).

A limitation of the capital cost sensitivity analysis is that capital cost ($\tilde{C}_{cap}$) is unlikely to be
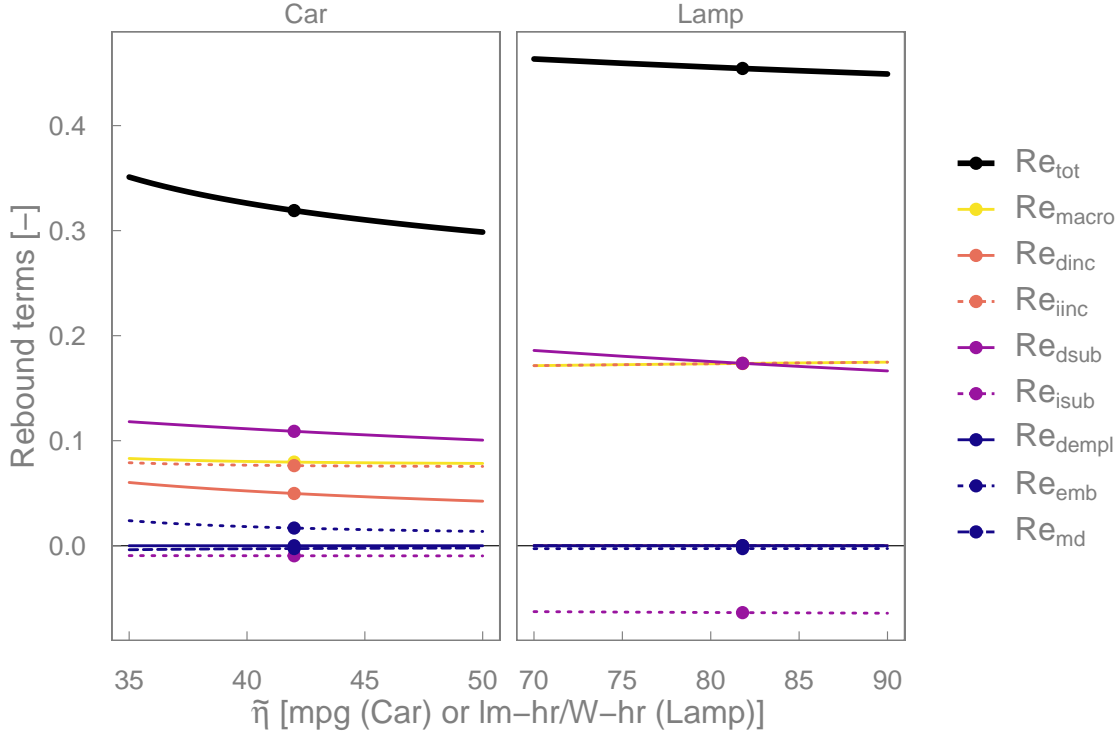
36

Fig. 12: Sensitivity of rebound components to post-EEU efficiency ($\tilde{\eta}$).

independent of $\tilde{\eta}$. Within a given energy efficiency technology (e.g., hybrid cars or LED lamps), greater capital cost ($\tilde{C}_{cap}$) may be associated with greater service efficiency ($\tilde{\eta}$). $\tilde{C}_{cap}$ and $\tilde{\eta}$ should probably be varied jointly, not independently.

4.3.3 Effect of energy price (p_E)

The effect of energy price on rebound is shown in Fig. 14. Increasing energy prices lead to larger total rebound (Re_{tot}), because higher energy prices lead to more net savings ($\hat{N}$) to be spent by the device owner. All other things being equal, more net savings leads to more spending on other goods and services that demand energy.

Fig. 14 also shows the effect of energy price (p_E) on all rebound components. Most rebound components increase with energy price, with the car and lamp examples exhibiting different sensitivities. Substitution effects (Re_{dsub} and Re_{isub}) are the only rebound components that decrease with energy price (p_E). Substitution effects decrease with energy price, because at high energy price, less behavior adjustment is needed to re-equilibrate after emplacement of the efficient device.

In Fig. 14, German energy prices[16] are shown as vertical lines, providing an indication of possible energy price variations. All other things being equal, if U.S. residents paid Germany's energy prices, energy rebound would be 48.8% for the car example and 91.8% for the lamp example, assuming $k = 1$.

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[16]For the car example, the gasoline price in Germany is taken as 1.42 €/liter for the average "super gasoline" (95 octane) price in 2018 (finanzen.net, 2021). For the lamp example, the electricity price in Germany is taken as 0.3 €/kW-hr for the 2018 price of a household using 3.5 MWh/year, an average value for German households (Bundesministerium für Wirtschaft und Energie, 2018). Converting currency (at 1 € = \$1.21) and physical units gives 6.5 \$/US gallon and 0.363 \$/kW-hr.
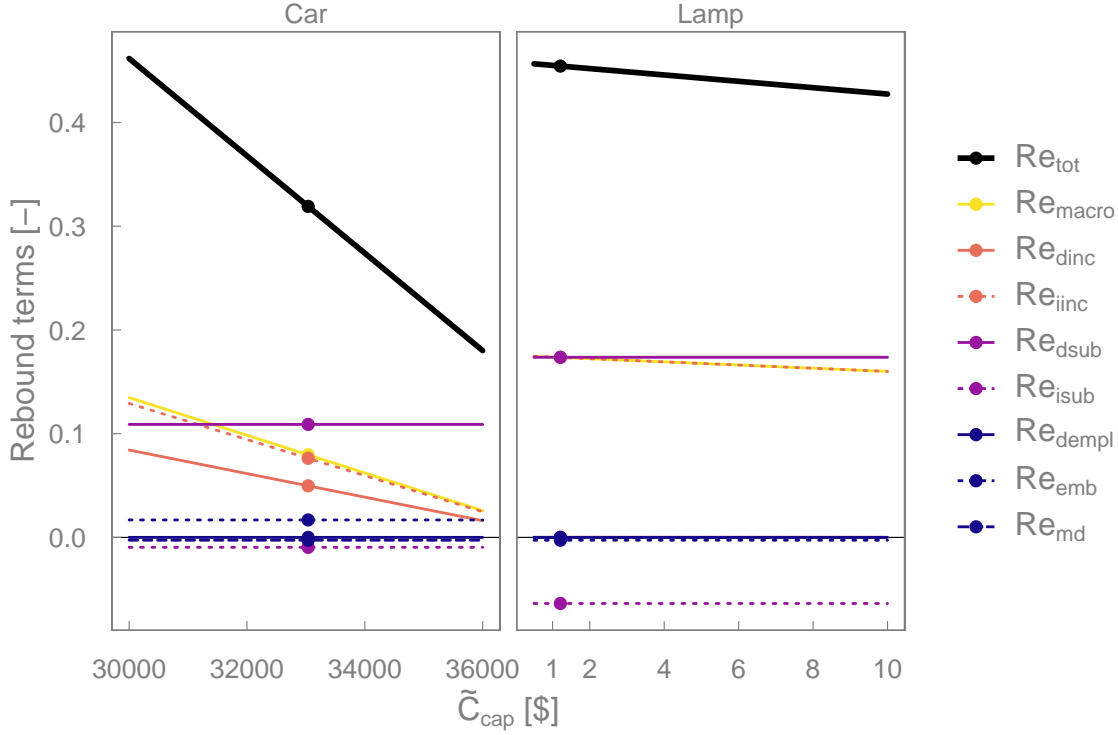
Fig. 13: Sensitivity of rebound components to capital cost ($\tilde{C}_{cap}$).

A limitation of the energy price sensitivity analysis arises from the fact that energy price is not independent of other parameters. Indeed, other rebound parameters would change along with energy price (especially if moving from one country to another), including capital cost ($\tilde{C}_{cap}$), maintenance and disposal expenditure rate ($\tilde{C}_{md}$), energy intensity of the economy (I_E), and energy service consumption rate ($\dot{q}_s^{\circ}$).

4.3.4 Effect of elasticity ($\epsilon_{\dot{q}_s, p_s}$)

Fig. 15 shows the variation of total rebound (Re_{tot}) with the uncompensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s}$). The effect is exponential, and total rebound increases with larger negative values of $\epsilon_{\dot{q}_s, p_s}$, as expected. The lamp example also shows stronger exponential variation than the car example. The main reason that total rebound values are different between the two examples is the larger absolute value of elasticity ($\epsilon_{\dot{q}_s, p_s}$) for the lamp ($-0.4$) compared to the car ($-0.2$). Were the car to have the same elasticity as the lamp (-0.4), total rebound would be approximately the same for both examples ($\sim 50\%$). Fig. 15 shows that direct substitution rebound (Re_{dsub}) is the most sensitive rebound component to changes in $\epsilon_{\dot{q}_s, p_s}$. For the lamp example, indirect income rebound (Re_{iinc}) also increases substantially with $\epsilon_{\dot{q}_s, p_s}$, because net savings increases substantially with $\epsilon_{\dot{q}_s, p_s}$.

A limitation of the elasticity sensitivity study derives from limitations of the CES utility model itself, which constrains price elasticity variation given an elasticity of substitution. Uneconomic conditions should be avoided. For example, negative direct substitution rebound ($Re_{dsub} < 0$) is obtained when $|\epsilon_{\dot{q}_s, p_s}| < f_{\dot{C}_s}^{\circ}$, because the elasticity of substitution goes negative ($\sigma < 0$). (See Eq. (162) in Appendix D.) In reality, smaller negative (closer to 0) price elasticity ($\epsilon_{\dot{q}_s, p_s}$) would correlate with a smaller fraction of expenses spent on the energy service ($f_{\dot{C}_s}^{\circ}$), thereby avoiding
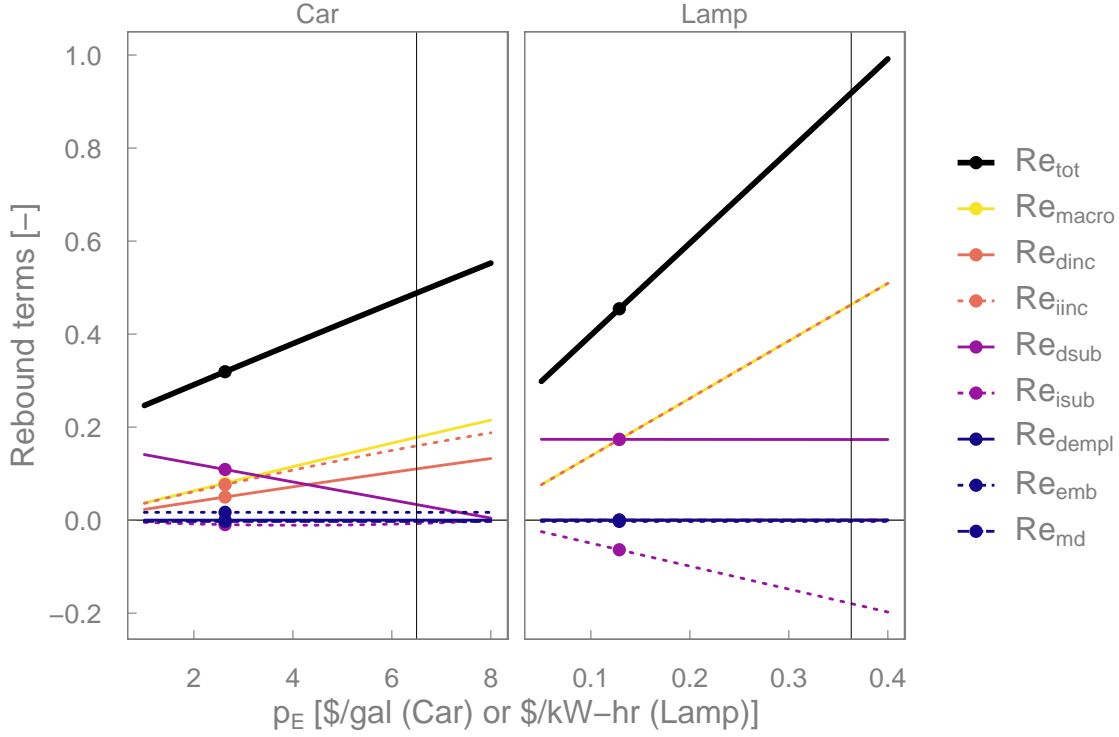
Fig. 14: Sensitivity of rebound components to energy price (p_E). German energy prices denoted by vertical lines.

the uneconomic condition. However, a univariate sensitivity study cannot capture this effect and is best used for smaller variations in $\epsilon_{\dot{q}_s, p_s}$ about a nominal value.

4.3.5 Effect of macro factor (k)

The sensitivity of energy rebound to the macro factor (k) is shown in Fig. 16. The macro factor has a linear effect on total rebound (Re_{tot}) through the macro rebound component (Re_{macro}). All other rebound components are constant when k is varied independently.

A limitation of the macro factor sensitivity analysis is that the macro factor (k) is unlikely to be independent of I_E, because different values of k imply a different macroeconomy.

5 Discussion

In this section, we revisit and discuss the contributions indicated in Section 1. Then we show how the framework enables insights on energy policy, especially policies to encourage energy efficiency. Finally, we discuss and calibrate the macro factor (k).

5.1 Contributions

In Section 1, we claimed four contributions. Here, we reiterate and discuss each.

First, we believe we have developed the first comprehensive rebound analysis framework to include all of (i) capital cost, embodied energy, and maintenance and disposal effects, (ii) non-marginal
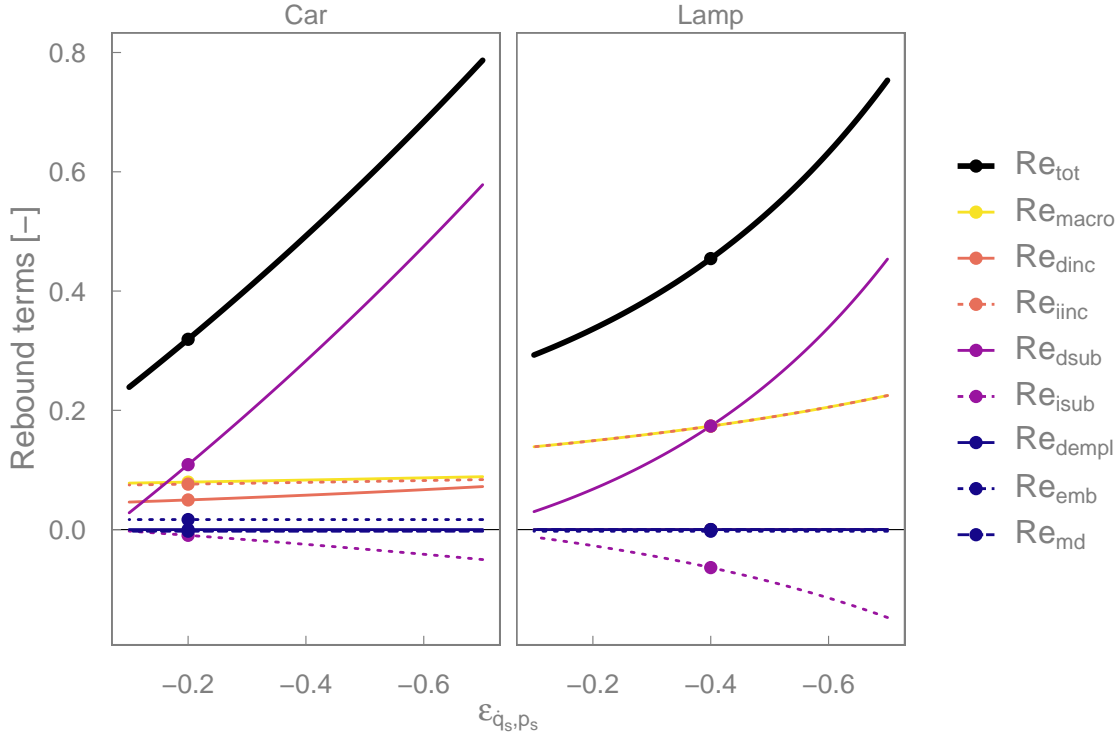
Fig. 15: Sensitivity of rebound components to uncompensated energy service price elasticity of energy demand ($\epsilon_{\dot{q}_s, p_s}$). (Note reversed x-axis scale.)
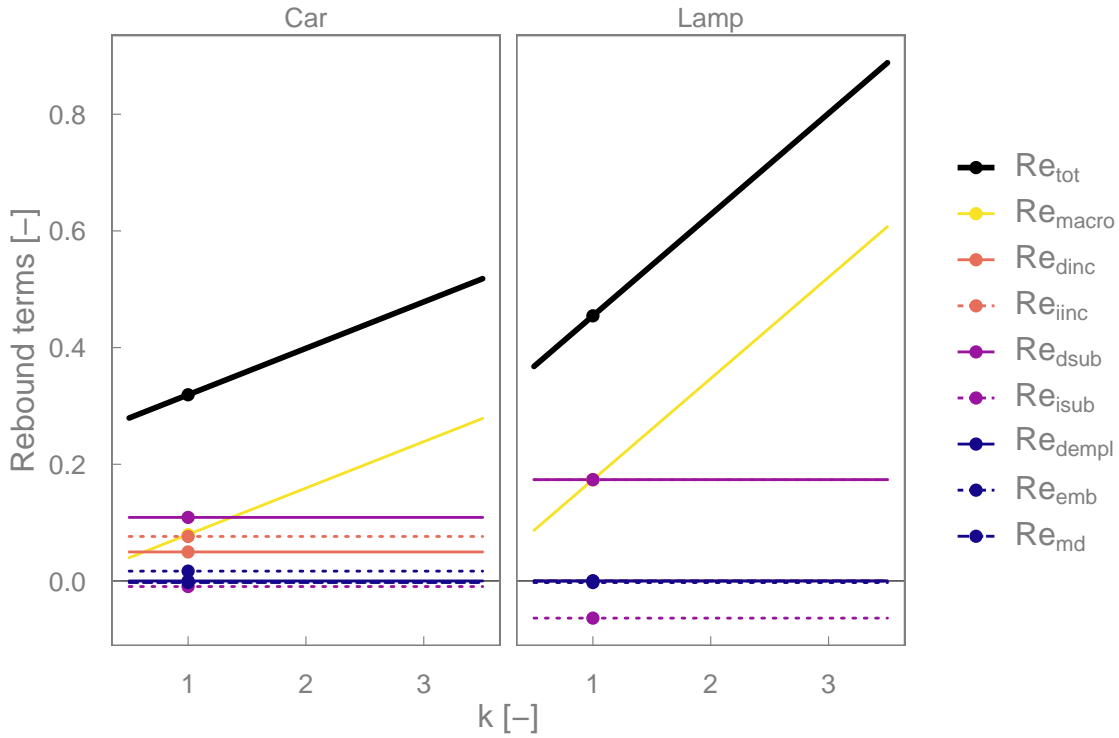

Fig. 16: Sensitivity of rebound components to the macro factor (k).

energy service price decreases, and (iii) non-marginal energy efficiency increases. The embodied energy and maintenance and disposal effects are the energy manifestation of the important difference between costless technology upgrades and (costly) mandated efficiency increases (Gillingham et al., 2013; Fullerton & Ta, 2020). Costly EEUs are not typically included in partial equilibrium rebound frameworks (Azevedo, 2014; Gillingham, 2020); most rebound analysis frameworks assume costless upgrade in both financial and energy terms. One surprising result from including costly upgrades is that accounting for differential capital cost ($\Delta \dot{C}_{cap}^{*}$) can *increase* rebound when the capital cost rate decreases ($\dot{C}_{cap}^{\circ} > \dot{C}_{cap}^{\prime*}$). The lamp example of Section 4.2 illustrates this result. This finding demonstrates that the costless EEU does not provide an upper bound to rebound, as previously suggested by Thomas & Azevedo (2013b).

The accommodation of non-marginal energy efficiency increases and non-marginal energy service price decreases is made possible by direct calculation of consumption positions via the CES utility model of Eqs. 19–22, rather than relying on the approximate utility model and price elasticities.

The possibility of precisely decomposing microeconomic rebound into substitution and income effects in direct and indirect locations was outlined by Khazzoom (1980), spelled out clearly by Greening et al. (2000), and numerically approximated by Borenstein (2015). Yet, authors of previous operational frameworks of microeconomic rebound typically show only stylized graphs of indifference curves in the consumption space, even if they are numerically precise in other respects (Thomas & Azevedo, 2013a; Moshiri & Aliyev, 2017; Li et al., 2018).

The stylized indifference curves of previous frameworks lead to the second contribution of this paper, namely the first visualizations of rebound effects in energy, expenditure, and consumption spaces. We call these visualizations "rebound path graphs," and they can be seen in Figs. 5–10. Our precisely drawn and quantified rebound path graphs through energy, expenditure, and consumption spaces aid comprehension of and intuition about the various rebound effects for both energy analysts and economists..

Segment $-$—$\sim$ in energy path graphs is the macro effect, the third contribution. We have developed the first operational link between rebound effects on macroeconomic and microeconomic scales. We both operationalize the link and show how it should be applied: scaling net savings after the substitution effect. We further draw upon the MPC literature to show how macro effects operate via economic expansion over the long term. This micro-to-macro link can be seen in Eq. (32) and in Figs. 2, 5, and 8. We make the first calibration of the macro factor (k) in Section 5.3 below.

The fourth contribution is tools for calculating rebound for other EEUs with this framework. The tools include an Excel workbook and an R package. See the Leeds University data repository and the ReboundTools R package (Heun, 2021).

5.2 Policy considerations

We take for granted that it is desirable to use energy efficiency to reduce energy consumption and CO_2 emissions. For energy efficiency to be as effective as possible at reducing CO_2 emissions, rebound should be taken into account. The sensitivity analyses of Section 4.3 enable exploration of policy options that are often suggested to promote energy efficiency and reduce CO_2 emissions: energy efficiency standards, rebates or subsidies, and carbon pricing or energy taxes. In the subsections below, we discuss the rebound implications of each policy option with reference to the sensitivity analyses of Section 4.3.

5.2.1 Energy efficiency standards

Energy efficiency standards mandate minimum efficiency levels for selling energy conversion devices into a market. One example of energy efficiency standards is the Corporate Average Fuel Economy (CAFE) standards for U.S. cars.

Fig. 12 shows that rebound falls as post-upgrade energy efficiency ($\tilde{\eta}$) rises. This result is driven mainly by decreasing direct substitution effect rebound (Re_{dsub}) as efficiency goes up. Declining Re_{dsub} is driven by declining (i.e., closer to zero) compensated own price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s, c}$), which leads to an equally declining uncompensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s}$), an observable quantity. (See also Fig. 15.) As a result, raising the minimum efficiency required to participate in a market is beneficial for reducing energy consumption. Indeed, Fig. 11 shows the gap between the expected energy savings rate and the takeback rate grows with post-emplacement efficiency, so absolute energy savings are expected to increase as efficiency grows, at least for the examples in Section 4. (We note in passing that this change in price elasticity is tractable only because the framework accommodates non-marginal efficiency changes by calculating the new consumption bundle directly from the CES utility model.)

However, larger efficiency increases can require higher overnight capital investment within a given technology, which hinders adoption of energy-efficient devices. One measure often employed to boost adoption is financial rebates or subsidies, the subject of the next section.

5.2.2 Rebates or subsidies

Larger efficiency increases can require higher capital investment within a given technology for an EEU. To encourage market penetration of higher-efficiency devices, rebates or subsidies can be offered to device purchasers to incentivize adoption of energy-efficient technologies and grow their markets. (One example of this approach is current subsidies for battery electric vehicles.) Providing rebates or subsidies effectively reduces the capital cost ($\tilde{C}_{cap}$) for the purchaser of energy-efficient devices. However, Fig. 13 shows that reducing capital cost increases energy rebound, mostly due to the income effect in which more net savings (thanks to the rebate) are available to spend on the energy service and other goods. Thus, policymakers should be aware that rebound will reduce the effectiveness of rebates and subsidies to generate actual energy savings.

That said, a rebate or subsidy could be used as a tool to stimulate initial take-up of a new EEU technology. From a rebound perspective, the subsidy-related stimulus should be removed when cost reductions due to scale effects and learning by doing (Wilson, 2012) take hold, and the EEU uptake goes mainstream.

A potential downside to rebates and subsidies is inequities: only those wealthy enough to afford expensive EEUs benefit from the subsidies. To reduce inequities, income-dependent subsidies should be implemented.[17] Moreover, to avoid rebound generated by rebates or subsidies, policymakers may choose to implement energy taxes simultaneously, effectively raising the price of energy, the topic of the next section.

5.2.3 Carbon pricing or energy taxes

An energy tax or other penalty on energy consumption, preferably limited to fossil energy, likely by a price on carbon and enacted alongside the EEU, does two things at the microeconomic scale. First, a fossil-energy tax raises the effective price of fossil energy (p_E) and the price of any fossil-energy-powered energy service (p_s). By raising the relative price of the energy service, substitution

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[17]For a discussion of financing difficulties and the requisite size of rebates, see Fowlie et al. (2018).

effect rebound (Re_{sub}) shrinks, as shown in Fig. 14. Second, a fossil-energy tax raises net financial savings earned by the device owner after emplacing the EEU. The gross savings of the emplacement effect grows with the effective energy price, driving higher income effect rebound (Re_{inc}). Higher energy prices lead to higher macro rebound (Re_{macro}), too, again due to increased gross savings from the emplacement effect, assuming a constant value for the product of the macro factor and the energy intensity of the economy (kI_E). Larger income effect and macro effect rebounds overwhelm the decrease in substitution effect rebound, leading to higher overall rebound from the effectively higher energy price, as shown in Fig. 14. This additional rebound effect moves van den Bergh (2011) to propose a simultaneous cap on total energy consumption (or CO_2 emissions). But so long as backfire is avoided (i.e., total rebound remains less than 100%), absolute energy savings will still occur, though at a lower rate than expected.

That said, another outcome of carbon pricing or fossil-energy taxes could be reduction of the energy intensity of the economy (I_E), a dynamic effect not captured by the univariate sensitivity study of Fig. 14. The energy intensity of the economy could drop because at the macroeconomic scale higher energy prices are likely to induce both energy-saving technical change and structural change toward an overall lower-energy-intensity economy. To avoid the price penalty, consumers might buy other, more-efficient devices or consume substitutes both for the energy service and in other areas of life, thereby contributing to reducing I_E. (See Hart (2018) for a related discussion.) A less-energy-intensive economy could be further encouraged by policymakers with complementary policies to encourage use of efficient substitutes, such as public transport.

If carbon pricing or a fossil-energy tax reduces the energy intensity of the economy (I_E), all rebound terms whose equations include I_E will be affected. Thus, although the introduction of carbon prices or fossil-energy taxes can induce higher income-effect rebound and higher macro-effect rebound, those increases might be less than predicted by the sensitivity study of Fig. 14, due to reduction of the energy intensity of the economy (I_E). And any reduction of I_E due to fossil-energy taxes will tend to reduce total rebound (Re_{tot}), increasing the likelihood that backfire will be avoided and absolute energy savings will occur. Thus, total rebound estimated by this framework (which assumes static I_E) could provide an upper bound on rebound for carbon pricing or fossil-energy taxes.

Carbon pricing or fossil-energy taxes are likely to be an effective policy option to reduce energy consumption, rebound effects, and CO_2 emissions. But the rebound implications of these policies deserve further study, perhaps building on Chan & Gillingham (2015) to untangle the web of interconnected dynamic effects when multiple fuels are involved. One note of caution: across-the-board carbon pricing or fossil-energy taxes could raise further equity issues by limiting the ability of people in low-income groups to purchase energy (Owen & Barrett, 2020). To ameliorate inequities, progressive feebates could be implemented.[18]

5.3   Calibrating k

The framework developed in Section 3 links macroeconomic rebound to microeconomic rebound via a term that scales magnitudes in the microeconomic portion of the framework. Few rebound studies have explored the macroeconomic scale between microeconomic and total rebound. Inspired by Borenstein (2015) and others, we bridge macroeconomic and microeconomic scales with the macro factor (k), as discussed in Section 3.5.4. For the results presented in Section 4, we assumed a placeholder value of $k = 1$, meaning that every \$1 of spending by the device owner in the income effect generates only \$1 of additional economic activity in the broader economy. In combination,

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[18]Feebates route energy tax revenues to lower-income EEU purchasers. See Boyce (2018).

the framework presented in Section 3, the results obtained in Section 4, and recent estimates of total rebound allow, for the first time, a discussion about calibrating k.

To calibrate the macro factor (k), we treat macro rebound (Re_{macro}) as a residual. The macro factor (k) becomes an unknown parameter whose value is to be chosen such that Re_{macro} is sufficient to achieve an expected value for total rebound (Re_{tot}). We take the expected value for Re_{tot} from **?**. Four of 33 studies reviewed by **?** examined total rebound from only consumer EEUs in a computable general equilibrium framework. The average total rebound (Re_{tot}) for the four consumer studies is 54%.[19] The calibrated values of k that give identical $Re_{tot} = 54\%$ for both examples are $k = 3.8$ for the car example and $k = 1.5$ for the lamp example.

Qualitative differences in benefits from EEUs as well as the considerable variance in Re_{tot} in 33 surveyed studies (**?**) indicates that total rebound from one EEU is likely to be different from total rebound from another EEU. For the purposes of illustration we set $k = 3$, being between the values of k estimated from the car and lamp examples. Note that $k = 3$ implies that every \$1 of net savings spent by the device owner generates \$3 of additional economic activity in the broader economy after the income effect. We multiply $k\hat{N}$ by the energy intensity of the economy (I_E) to find the energy implications of macro-effect respending throughout the economy, as shown in Eq. (32).

There are three ways to interpret $k = 3$. First, $k = 3$ can be considered the average long-run economic growth generated by the device owner's spending of freed cash. Efficiency increases in equipment drive a significant part of long-run productivity growth (Greenwood et al., 1997), therefore a large, long-run multiplier is plausible, even if the initial productivity change occurred in household production which is not accounted in GDP. Second, it could be that growth is less than \$3 but that the macroeconomic "energy price effect" (a decline in energy prices due to the fallen demand) induces consumption at a higher energy intensity than that of the pre-EEU economy. Third, from the demand-side perspective entertained by Borenstein (2015), $k = 3$ could be interpreted as $MPC = 0.75$. (See Appendix G.) $MPC = 0.75$ is a reasonable value, being in the upper half of recent estimates by Carroll et al. (2017). Although the cause of the growth in economic activity and energy consumption from an EEU is a supply-side productivity shock, the subsequent demand-side effects may well be interpreted as a multiplier effect, caused by higher real income instead of by higher monetary income.

After calibrating $k = 3$, we see that choosing a placeholder value of $k = 1$ in Section 4 underestimated Re_{macro} and, therefore, Re_{tot}. In Figs. 5 and 8, the macro effect segments ($-\!\!-\!\!\sim$) should be three times longer than they appear. In Tables 7 and 12, the values of macro rebound (Re_{macro}) should triple to 23.9% and 52.1%, and the values of total rebound (Re_{tot}) should increase to 47.8% and 80.1% for the car and lamp examples, respectively.

6 Conclusions

In this paper, we developed a comprehensive, partial equilibrium rebound analysis framework that combines all rebound effects, locations, and scales across energy, expenditure, and consumption spaces with a detailed model of consumer preferences and non-marginal energy service price changes in an operational manner for the simplest case of a single fuel and a single energy service. This paper also contributes the first visualizations of rebound path graphs in energy, expenditure, and consumption spaces to bridge gaps between economists and energy analysts. And we operationalize and calibrate the macro factor ($k = 3$) to provide a link between effects at microeconomic and

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[19]The average total rebound among all 33 studies stood at 63%, supporting the claim by Turner (2013) that consumer and producer rebounds vary.

macroeconomic scales, missing from previous partial equilibrium frameworks. With careful explication of rebound effects, clear derivation of rebound expressions, and novel visualizations of rebound paths, we advance the analytical foundations for empirical estimates of rebound and facilitate interdisciplinary understanding of rebound phenomena toward the goal of enabling of more robust rebound estimates for sound energy and climate policy.

From the development and application of the framework, we draw four important conclusions. First, the car and lamp examples (Section 4) show that the framework enables quantification of rebound magnitudes at microeconomic and macroeconomic scales, including direct and indirect locations for emplacement, substitution, income, and macro effects. Second, the examples show that magnitudes of rebound effects vary with the type of EEU performed. Third, the framework enables evaluation of rebound sensitivities to important parameters (Section 4.3). For the examples in this paper, total rebound is more sensitive to the price of energy (p_E), the elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s}$), and the macro factor (k) than either energy efficiency ($\tilde{\eta}$) or the capital cost ($\tilde{C}_{cap}$) of the upgraded device. Fourth, the discussion (Section 5) shows that rebound is a headwind for efficiency-led reduction of energy consumption and CO_2 mitigation policies. Quantification of rebound effect magnitudes is an important precursor to devising effective energy policies that would encourage energy efficiency, limit rebound effects, and reduce CO_2 emissions.

Future work could be pursued in several areas. (i) Other utility models (besides the CES utility model) could be explored for the substitution effect. (ii) Further empirical studies could be performed to estimate the magnitude of different rebound effects in a variety of real-life EEUs. (iii) Deeper study of macro rebound is needed, including improved estimates for the macro factor (k) and its relation to the MPC. (iv) The rebound implications of the distribution of MPC values across socioeconomic groups (Carroll et al., 2017) could be explored. (v) This framework could be extended to producer-sided energy rebound effects. (vi) The rebound effects of fossil-energy taxes could be studied further, especially for the web of interconnected dynamic effects among rebound components that are functions of the energy intensity of the economy (I_E). (vii) This framework could be embedded in energy-economy models to better include rebound effects in discussions of macro energy modeling, energy policy, and CO_2 emissions mitigation.

Competing interests

Declarations of interest: none.

Author contributions

	MKH	GS	PEB
Conceptualization	●	●	
Methodology	●	●	●
Software	●		
Validation	●		●
Formal analysis	●	●	●
Investigation	●	●	●
Resources	●		
Data curation			●
Writing–original draft	●	●	●
Writing–review & editing	●	●	●
Visualization	●	●	
Supervision	●		
Project administration	●		
Funding acquisition			

Data repository

Data and calculations are stored at Research Data Leeds Repository (`https://doi.org/10.5518/1201`). An `R` package for performing all rebound calculations can be found at `https://github.com/MatthewHeun/ReboundTools`. (See Heun (2021).)

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Table A.1: Symbols and abbreviations.

Symbol	Meaning [example units]
a	a point in the emplacement effect on rebound path graphs or the share parameter in the CES utility model [–]
b	a point in the emplacement effect on rebound path graphs
C	cost [\$]
c	a point in the substitution effect on rebound path graphs
d	a point in the income effect on rebound path graphs
E	final energy [MJ]
f	expenditure share [–]
G	freed cash [\$]
I	energy intensity of economic activity [MJ/\$]
k	macro factor [–]
M	income [\$]
N	net savings [\$]
p	price [\$]
q	quantity [–]
Re	rebound [–]
S	energy cost savings [\$]
t	energy conversion device lifetime [years]
u	utility [utils]

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Appendices

A   Nomenclature

Presentation of the comprehensive rebound analysis framework is aided by a nomenclature that describes energy stages and rebound effects, locations, and scales. Table A.1 shows symbols and abbreviations, their meanings, and example units. Table A.2 shows Greek letters, their meanings, and example units. Table A.3 shows abbreviations and acronyms. Table A.4 shows symbol decorations and their meanings. Table A.5 shows subscripts and their meanings.

Differences are indicated by the Greek letter Δ and always signify subtraction of a quantity at an earlier stage of Fig. 1 from the same quantity at a later stage of Fig. 1. E.g., $\Delta \bar{X} \equiv \bar{X} - \hat{X}$, and $\Delta \tilde{X} \equiv \tilde{X} - \bar{X}$. Lack of decoration on a difference term indicates a difference that spans all stages of Fig. 1. E.g., $\Delta X \equiv \tilde{X} - X^\circ$. ΔX is also the sum of differences across each stage in Fig. 1, as shown below.

$$\Delta X = \Delta \tilde{X} + \Delta \bar{X} + \Delta \hat{X} + \Delta X^*$$
$$\Delta X = (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^*) + (X^* - X^\circ)$$
$$\Delta X = (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^*) + (X^* - X^\circ)$$
$$\Delta X = \tilde{X} - X^\circ \tag{35}$$

Table A.2: Greek letters.

Greek letter	Meaning [example units]
Δ	difference (later quantity less earlier quantity, see Fig. 1)
ϵ	elasticity [–]
$\epsilon_{\dot{q}_s,\dot{M}}$	income ($\dot{M}$) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,\dot{M}}$	income ($\dot{M}$) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s}$	uncompensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s}$	uncompensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s,c}$	compensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s,c}$	compensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
η	final-energy-to-service efficiency [vehicle-km/MJ]
σ	elasticity of substitution between the energy service ($\dot{q}_s^\circ$) and other goods ($\dot{q}_o^\circ$) [–]

Table A.3: Abbreviations.

Abbreviation	Meaning
APF	aggregate production function
CES	constant elasticity of substitution
CV	compensating variation
EEU	energy efficiency upgrade
GDP	gross domestic product
MPC	marginal propensity to consume
mpg	miles per U.S. gallon
U.S.	United States

Table A.4: Decorations.

Decoration	Meaning [example units]
X°	X originally (before the emplacement effect)
X^*	X after the emplacement effect (before the substitution effect)
$\hat{X}$	X after the substitution effect (before the income effect)
$\bar{X}$	X after the income effect (before the macro effect)
$\tilde{X}$	X after the macro effect
$\dot{X}$	rate of X [units of X/year]
M'	effective income [\$]

Table A.5: Subscripts.

Subscript	Meaning
0	quantity at an initial time
1	a specific point on a consumption path graph
c	compensated
cap	capital costs
dev	device
dempl	direct emplacement effect
d	disposal
dinc	direct income effect
dir	direct effects (at the energy conversion device)
dsub	direct substitution effect
emb	embodied
i	index for other goods purchased in the economy
j	one of cap, md, or o in Eq. (8)
iempl	indirect emplacement effects
iinc	indirect income effect
indir	indirect effects (beyond the energy conversion device)
isub	indirect substitution effect
life	lifetime
m	maintenance
md	maintenance and disposal costs
o	other expenditures (besides energy) by the device owner
own	ownership duration
macro	macro effect
s	service stage of the energy conversion chain
tot	sum of all rebound effects in the framework

Table B.1: Lines and curves for rebound path graphs.

Rebound path graph	Lines and curves
Energy	Constant total energy consumption lines 0% and 100% rebound lines
Expenditure	Constant expenditure lines
Consumption	Constant expenditure lines Rays from origin to ∧ point Indifference curves

B Mathematical details of rebound path graphs

Rebound path graphs show the impact of direct and indirect rebound effects in energy space, expenditure space, and consumption space. Notional rebound path graphs can be found in Figs. 2–4. Rebound path graphs for the car example can be found in Figs. 5–7. Graphs for the lamp example can be found in Figs. 8–10.

This appendix shows the mathematical details of rebound path graphs, specifically derivations of equations for lines and curves shown in Table B.1. The lines and curves enable construction of numerically accurate rebound path graphs as shown in Figs. 5–10.

B.1 Energy path graphs

Energy path graphs show direct (on the x-axis) and indirect (on the y-axis) energy consumption associated with the energy conversion device and the device owner. Lines of constant total energy

consumption comprise a scale for total rebound. For example, the 0% and 100% rebound lines are constant total energy consumption lines which pass through the original point ($\circ$) and the post-direct-emplacement-effect point (a) on an energy path graph.

The equation of a constant total energy consumption line is derived from

$$\dot{E}_{tot} = \dot{E}_{dir} + \dot{E}_{indir} \tag{36}$$

at any rebound stage. (See Fig. 1.) Direct energy consumption is energy consumed by the energy conversion device ($\dot{E}_s$), and indirect energy consumption is the sum of embodied energy, energy associated with maintenanace and disposal, and energy associated with expenditures on other goods ($\dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$).

For the energy path graph, direct energy consumption is placed on the x-axis and indirect energy consumption is placed on the y-axis. To derive the equation of a constant energy consumption line, we first rearrange to put the y coordinate on the left of the equation:

$$\dot{E}_{indir} = -\dot{E}_{dir} + \dot{E}_{tot} \ . \tag{37}$$

Next, we substitute y for $\dot{E}_{indir}$, x for $\dot{E}_{dir}$, and $\dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$ for $\dot{E}_{tot}$ to obtain

$$y = -x + \dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E \ , \tag{38}$$

where all of $\dot{E}_s$, $\dot{E}_{emb}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage of Fig. 1.

The constant total energy consumption line that passes through the original point ($\circ$) shows 100% rebound:

$$y = -x + \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ)I_E \ . \tag{39}$$

The 0% rebound line is the constant total energy consumption line that accounts for expected energy savings ($\dot{S}_{dev}$) only:

$$y = -x + (\dot{E}_s^\circ - \dot{S}_{dev}) + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ)I_E \ . \tag{40}$$

The above line passes through the a point on an energy path graph.

B.2 Expenditure path graphs

Expenditure path graphs show direct (on the x-axis) and indirect (on the y-axis) expenses associated with the energy conversion device and the device owner. Lines of constant expenditure are important, because they provide budget constraints for the device owner.

The equation of a constant total expenditure line is derived from the budget constraint

$$\dot{C}_{tot} = \dot{C}_{dir} + \dot{C}_{indir} \tag{41}$$

at any rebound stage. For the expenditure path graph, indirect expenditures are placed on the y-axis and direct expenditures on energy for the energy conversion device are place on the x-axis. Direct expenditure is the cost of energy consumed by the energy conversion device ($\dot{C}_s = p_E \dot{E}_s$), and indirect expenses are the sum of capital costs, maintenanace and disposal costs, and expenditures on other goods ($\dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$). Rearranging to put the y-axis variable on the left side of the equation gives

$$\dot{C}_{indir} = -\dot{C}_{dir} + \dot{C}_{tot} \ . \tag{42}$$

Substituting y for $\dot{C}_{indir}$, x for $\dot{C}_{dir}$, and $\dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$ for $\dot{C}_{tot}$ gives

$$y = -x + \dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o \ , \tag{43}$$

where all of $\dot{C}_s$, $\dot{C}_{cap}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage of Fig. 1.

The constant total expenditure line that passes through the original point ($\circ$) shows the budget constraint for the device owner:

$$y = -x + \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \ , \tag{44}$$

into which Eq. (64) can be substituted with $\dot{C}_s^\circ = p_E \dot{E}_s^\circ$ and $\dot{N}^\circ = 0$ to obtain

$$y = -x + \dot{M}^\circ \ . \tag{45}$$

The constant total expenditure line that accounts for expected energy savings ($\dot{S}_{dev}$) and freed cash ($\dot{G} = p_E \dot{S}_{dev}$) only is given by:

$$y = -x + (\dot{C}_s^\circ - \dot{G}) + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \ , \tag{46}$$

or

$$y = -x + \dot{M}^\circ - \dot{G} \ . \tag{47}$$

The line given by the above equation passes through the a point on an expenditure path graph.

B.3 Consumption path graphs

Consumption path graphs show expenditures in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space to accord with the utility model. (See Appendix D.) Consumption path graphs include (i) constant expenditure lines given prices, (ii) a ray from the origin through the $\wedge$ point, and (iii) indifference curves. Derivations for each are shown in the following subsections.

B.3.1 Constant expenditure lines

There are four constant expenditure lines on the consumption path graphs of Figs. 4, 7, and 10. The constant expenditure lines pass through the original point (line $\circ$—$\circ$), the post-emplacement point (line $*$—$*$), the post-substitution point (line $\wedge$—$\wedge$), and the post-income point (line $-$—$-$). Like the expenditure path graph, lines of constant expenditure on a consumption path graph are derived from the budget constraint of the device owner at each of the four points.

Prior to the EEU, the budget constraint is given by Eq. (64). Substituting $p_s^\circ \dot{q}_s^\circ$ for $p_E \dot{E}_s^\circ$ and recognizing that there is no net savings before the EEU ($\dot{N}^\circ = 0$) gives

$$\dot{M}^\circ = p_s^\circ \dot{q}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \ . \tag{48}$$

To create the line of constant expenditure on the consumption path graph, we allow $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to vary in a compensatory manner: when one increases, the other must decrease. To show that variation along the constant expenditure line, we remove the notation that ties $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to the original point ($\circ$) to obtain

$$\dot{M}^\circ = p_s^\circ \dot{q}_s + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o \,, \tag{49}$$

where all of $\dot{M}^\circ$, p_s°, $\dot{C}_{cap}^\circ$, and $\dot{C}_{md}^\circ$ apply at the same rebound stage of Fig. 1, namely the original point ($\circ$).

To derive the equation of the line representing the original budget constraint in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space (the $\circ$—$\circ$ line through the $\circ$ point in consumption path graphs), we solve for $\dot{C}_o$ to obtain

$$\dot{C}_o = -p_s^\circ \dot{q}_s + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \,. \tag{50}$$

Multiplying judiciously by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ}\dot{C}_o^\circ = -p_s^\circ \frac{\dot{q}_s}{\dot{q}_s^\circ}\dot{q}_s^\circ + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \,. \tag{51}$$

Dividing both sides by $\dot{C}_o^\circ$ yields

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}\frac{\dot{q}_s}{\dot{q}_s^\circ} + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \,. \tag{52}$$

Noting that $\dot{q}_s/\dot{q}_s^\circ$ and $\dot{C}_o/\dot{C}_o^\circ$ are the x-axis and y-axis, respectively, on a consumption path graph gives

$$y = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \,. \tag{53}$$

A similar procedure can be employed to derive the equation of the $*$—$*$ line through the $*$ point after the emplacement effect. The starting point is the budget constraint at the $*$ point (Eq. (66)) with $\dot{M}^\circ$ replacing $\dot{M}^*$, $\tilde{p}_s \dot{q}_s$ replacing $p_E \dot{E}_s^*$, and $\dot{C}_o$ replacing $\dot{C}_o^*$.

$$\dot{M}^\circ = \tilde{p}_s \dot{q}_s + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o + \dot{N}^* \,. \tag{54}$$

Substituting Eq. (75) for $\dot{N}^*$, substituting Eq. (76) for $\dot{G}$, multiplying judiciously by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$, rearranging, and noting that $\dot{q}_s/\dot{q}_s^\circ$ is the x-axis and $\dot{C}_o/\dot{C}_o^\circ$ is the y-axis gives

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G}) \,. \tag{55}$$

Note that the slope of Eq. (55) is less negative than the slope of Eq. (53), because $\tilde{p}_s < p_s^\circ$. The y-intercept of Eq. (55) is less than the y-intercept of Eq. (53), reflecting freed cash. Both effects are seen in consumption path graphs (Figs. 4, 7, and 10). The $\circ$—$\circ$ and $*$—$*$ lines intersect at the coincident $\circ$ and $*$ points.

A similar derivation process can be used to find the equation of line representing the budget constraint after the substitution effect (the $\wedge$—$\wedge$ line through the $\wedge$ point). The starting point is Eq. (79), and the equation for the constant expenditure line is

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G} + \tilde{p}_s \Delta\hat{\dot{q}}_s + \Delta\hat{\dot{C}}_o) \,. \tag{56}$$

Note that the $\wedge$—$\wedge$ line (Eq. (56)) has the same slope as the $*$—$*$ line (Eq. (55)) but a lower y-intercept.

Finally, the corresponding derivation for the equation of the constant expenditure line through the $-$ point (line $-$ — $-$) starts with Eq. (88) and ends with

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^*) \, . \tag{57}$$

B.3.2   Ray from the origin to the ∧ point

On consumption path graphs, the ray from the origin to the ∧ point (line r—r) defines the path along which the income effect (lines ∧—d and d—$-$) operates. The ray from the origin to the ∧ point has slope $(\hat{\dot{C}}_o/\dot{C}_o^\circ)/(\hat{\dot{q}}_s/\dot{q}_s^\circ)$ and a y-intercept of 0. Therefore, the equation of line r—r is

$$y = \frac{\hat{\dot{C}}_o/\dot{C}_o^\circ}{\hat{\dot{q}}_s/\dot{q}_s^\circ} \, x \, . \tag{58}$$

B.3.3   Indifference curves

On a consumption path graph, indifference curves represent lines of constant utility for the energy conversion device owner. In $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, any indifference curve is given by Eq. (144) with $f_{\dot{C}_s}^\circ$ replacing the share parameter a, as shown in Appendix D. Recognizing that $\dot{C}_o/\dot{C}_o^\circ$ is on the y-axis and $\dot{q}_s/\dot{q}_s^\circ$ is on the x-axis leads to substitution of y for $\dot{C}_o/\dot{C}_o^\circ$ and x for $\dot{q}_s/\dot{q}_s^\circ$ to obtain

$$y = \left[\frac{1}{1 - f_{\dot{C}_s}^\circ}\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}(x)^\rho\right]^{(1/\rho)} . \tag{59}$$

At any point in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, namely $(\dot{q}_{s,1}/\dot{q}_s^\circ, \dot{C}_{o,1}/\dot{C}_o^\circ)$, indexed utility $(\dot{u}_1/\dot{u}^\circ)$ is given by Eq. (14) as

$$\frac{\dot{u}_1}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ}\right)^\rho + (1 - f_{\dot{C}_s}^\circ)\left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ}\right)^\rho\right]^{(1/\rho)} . \tag{60}$$

Substituting Eq. (60) into Eq. (59) for $\dot{u}/\dot{u}^\circ$ and simplifying exponents gives

$$y = \left\{\frac{1}{1 - f_{\dot{C}_s}^\circ}\left[f_{\dot{C}_s}^\circ\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ}\right)^\rho + (1 - f_{\dot{C}_s}^\circ)\left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ}\right)^\rho\right] - \frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}(x)^\rho\right\}^{(1/\rho)} . \tag{61}$$

Simplifying further yields the equation of an indifference curve passing through point $(\dot{q}_{s,1}/\dot{q}_s^\circ, \dot{C}_{o,1}/\dot{C}_o^\circ)$:

$$y = \left\{\left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right)\left[\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ}\right)^\rho - (x)^\rho\right] + \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ}\right)^\rho\right\}^{(1/\rho)} . \tag{62}$$

Note that if x is $\dot{q}_{s,1}/\dot{q}_s^\circ$, y becomes $\dot{C}_{o,1}/\dot{C}_o^\circ$, as expected.

C Derivation of comprehensive, consumer-based rebound analysis framework

This appendix provides a detailed derivation of the comprehensive rebound analysis framework, beginning with relationships for each rebound effect.

C.1 Relationships for rebound effects

For each energy rebound effect in Fig. 1, energy and financial analysis must be performed. The purposes of the analyses are to determine for each effect (i) an expression for energy rebound (Re) for the effect and (ii) an equation for net savings ($\dot{N}$) remaining after the effect.

Analysis of each rebound effect involves a set of assumptions and constraints as shown in Table C.1. In Table C.1, relationships for emplacement effect embodied energy rates ($\dot{E}_{emb}^{\circ}$ and $\dot{E}_{emb}^{*}$), capital expenditure rates ($\dot{C}_{cap}^{\circ}$ and $\dot{C}_{cap}^{*}$), and maintenance and disposal expenditure rates ($\dot{C}_{md}^{\circ}$ and $\dot{C}_{md}^{*}$) are typical, and inequalities could switch direction for a specific EEU. Macro effect relationships are given for a single device only. If the EEU is deployed at scale across the economy, the energy service consumption rate ($\tilde{\dot{q}}_s$), device energy consumption rate ($\tilde{\dot{E}}_s$), embodied energy rate ($\tilde{\dot{E}}_{emb}$), capital expenditure rate ($\tilde{\dot{C}}_{cap}$), and maintenance and disposal expenditure rate ($\tilde{\dot{C}}_{md}$) will all increase in proportion to the number of devices emplaced.

Table C.1: Assumptions and constraints for analysis of rebound effects.

Parameter	Emplacement Effect	Substitution Effect	Income Effect	Macro Effect
Energy price	$p_E^\circ = p_E^*$	$p_E^* = \hat{p}_E$	$\hat{p}_E = \bar{p}_E$	$\bar{p}_E = \tilde{p}_E$
Energy service efficiency	$\eta^\circ < \eta^*$	$\eta^* = \hat{\eta}$	$\hat{\eta} = \bar{\eta}$	$\bar{\eta} = \tilde{\eta}$
Energy service price	$p_s^\circ > p_s^*$	$p_s^* = \hat{p}_s$	$\hat{p}_s = \bar{p}_s$	$\bar{p}_s = \tilde{p}_s$
Other goods price	$p_o^\circ = p_o^*$	$p_o^* = \hat{p}_o$	$\hat{p}_o = \bar{p}_o$	$\bar{p}_o = \tilde{p}_o$
Energy service consumption rate	$\dot{q}_s^\circ = \dot{q}_s^*$	$\dot{q}_s^* < \hat{\dot{q}}_s$	$\hat{\dot{q}}_s < \bar{\dot{q}}_s$	$\bar{\dot{q}}_s = \tilde{\dot{q}}_s$
Other goods consumption rate	$\dot{q}_o^\circ = \dot{q}_o^*$	$\dot{q}_o^* > \hat{\dot{q}}_o$	$\hat{\dot{q}}_o < \bar{\dot{q}}_o$	$\bar{\dot{q}}_o = \tilde{\dot{q}}_o$
Device energy consumption rate	$\dot{E}_s^\circ > \dot{E}_s^*$	$\dot{E}_s^* < \hat{\dot{E}}_s$	$\hat{\dot{E}}_s < \bar{\dot{E}}_s$	$\bar{\dot{E}}_s = \tilde{\dot{E}}_s$
Embodied energy rate	$\dot{E}_{emb}^\circ < \dot{E}_{emb}^*$	$\dot{E}_{emb}^* = \hat{\dot{E}}_{emb}$	$\hat{\dot{E}}_{emb} = \bar{\dot{E}}_{emb}$	$\bar{\dot{E}}_{emb} = \tilde{\dot{E}}_{emb}$
Capital expenditure rate	$\dot{C}_{cap}^\circ < \dot{C}_{cap}^*$	$\dot{C}_{cap}^* = \hat{\dot{C}}_{cap}$	$\hat{\dot{C}}_{cap} = \bar{\dot{C}}_{cap}$	$\bar{\dot{C}}_{cap} = \tilde{\dot{C}}_{cap}$
Maint. and disp. expenditure rate	$\dot{C}_{md}^\circ < \dot{C}_{md}^*$	$\dot{C}_{md}^* = \hat{\dot{C}}_{md}$	$\hat{\dot{C}}_{md} = \bar{\dot{C}}_{md}$	$\bar{\dot{C}}_{md} = \tilde{\dot{C}}_{md}$
Energy service expenditure rate	$\dot{C}_s^\circ > \dot{C}_s^*$	$\dot{C}_s^* < \hat{\dot{C}}_s$	$\hat{\dot{C}}_s < \bar{\dot{C}}_s$	$\bar{\dot{C}}_s = \tilde{\dot{C}}_s$
Other goods expenditure rate	$\dot{C}_o^\circ = \dot{C}_o^*$	$\dot{C}_o^* > \hat{\dot{C}}_o$	$\hat{\dot{C}}_o < \bar{\dot{C}}_o$	$\bar{\dot{C}}_o = \tilde{\dot{C}}_o$
Income	$\dot{M}^\circ = \dot{M}^*$	$\dot{M}^* = \hat{\dot{M}}$	$\hat{\dot{M}} = \bar{\dot{M}}$	$\bar{\dot{M}} = \tilde{\dot{M}}$
Net savings	$0 = \dot{N}^\circ < \dot{N}^*$	$\dot{N}^* < \hat{\dot{N}}$	$\hat{\dot{N}} > \bar{\dot{N}} = 0$	$\bar{\dot{N}} = \tilde{\dot{N}} = 0$

Table C.2: Sources for zeroed terms in Tables C.3–C.6.

Zeroed term	Justification (from Table C.1).
$\Delta\dot{C}_o^{*}{}^{0}$	$\dot{C}_o^{\circ} = \dot{C}_o^{*}$ ($\dot{C}_o$ unchanged across emplacement effect.)
$\dot{N}^{\circ}{}^{0}$	$0 = \dot{N}^{\circ}$ (Net savings are zero prior to the EEU.)
$\Delta\dot{\hat{E}}_{emb}{}^{0}$	$\dot{E}_{emb}^{*} = \dot{\hat{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across substitution effect.)
$\Delta\dot{\hat{C}}_{md}{}^{0}$	$\dot{C}_{md}^{*} = \dot{\hat{C}}_{md}$ ($\dot{C}_{md}$ unchanged across substitution effect.)
$\Delta\dot{\bar{E}}_{emb}{}^{0}$	$\dot{\hat{E}}_{emb} = \dot{\bar{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across income effect.)
$\Delta\dot{\bar{C}}_{md}{}^{0}$	$\dot{\hat{C}}_{md} = \dot{\bar{C}}_{md}$ ($\dot{C}_{md}$ unchanged across income effect.)
$\dot{\bar{N}}{}^{0}$	$\dot{\bar{N}} = 0$ (All net savings are spent in the income effect.)

C.2 Derivations

Derivations for rebound definitions and net savings equations are presented in Tables C.3–C.6, one for each rebound effect in Fig. 1. Energy and financial analyses are shown side by side, because each informs the other.

Several terms in Tables C.3–C.6 are zeroed, e.g. $\Delta\dot{C}_o^{*}{}^{0}$. These zeroes can be traced back to Table C.1. Table C.2 highlights the equations in Table C.1 that justify zeroing each term.

Table C.3. **Emplacement Effect**

Energy analysis		*Financial analysis*	
before (○)	$\dot{E}^{\circ} = \dot{E}_s^{\circ} + \dot{E}_{emb}^{\circ} + (\dot{C}_{md}^{\circ} + \dot{C}_o^{\circ})I_E$ (63)	$\dot{M}^{\circ} = p_E\dot{E}_s^{\circ} + \dot{C}_{cap}^{\circ} + \dot{C}_{md}^{\circ} + \dot{C}_o^{\circ} + \dot{N}^{\circ}$ (64)	
after (∗)	$\dot{E}^{*} = \dot{E}_s^{*} + \dot{E}_{emb}^{*} + (\dot{C}_{md}^{*} + \dot{C}_o^{*})I_E$ (65)	$\dot{M}^{*} = p_E\dot{E}_s^{*} + \dot{C}_{cap}^{*} + \dot{C}_{md}^{*} + \dot{C}_o^{*} + \dot{N}^{*}$ (66)	

Take differences to obtain the change in energy consumption, $\Delta\dot{E}^{*} \equiv \dot{E}^{*} - \dot{E}^{\circ}$.

$$\Delta\dot{E}^{*} = \Delta\dot{E}_s^{*} + \Delta\dot{E}_{emb}^{*} + (\Delta\dot{C}_{md}^{*} + \overset{0}{\cancel{\Delta\dot{C}_o^{*}}})I_E \tag{67}$$

Thus,

$$\Delta\dot{E}^{*} = \Delta\dot{E}_s^{*} + \Delta\dot{E}_{emb}^{*} + \Delta\dot{C}_{md}^{*}I_E . \tag{68}$$

Define

$$\dot{S}_{dev} \equiv -\Delta\dot{E}_s^{*} \tag{69}$$

(Also see Eqs. (9) and (10)). Use Eq. (2) to obtain

$$Re_{empl} = 1 - \frac{-\Delta\dot{E}^{*}}{\dot{S}_{dev}} = 1 - \frac{-\Delta\dot{E}_s^{*}}{\dot{S}_{dev}} - \frac{-\Delta\dot{E}_{emb}^{*}}{\dot{S}_{dev}} - \frac{-\Delta\dot{C}_{md}^{*}I_E}{\dot{S}_{dev}} . \tag{70}$$

Define $Re_{dempl} \equiv 1 - \frac{-\Delta\dot{E}_s^{*}}{\dot{S}_{dev}}(=0)$, $Re_{iempl} \equiv Re_{emb} + Re_{md}$, $Re_{emb} \equiv \frac{\Delta\dot{E}_{emb}^{*}}{\dot{S}_{dev}}$, and $Re_{md} \equiv \frac{\Delta\dot{C}_{md}^{*}I_E}{\dot{S}_{dev}}$, such that

$$Re_{empl} = Re_{dempl} + Re_{iempl} . \tag{71}$$

Use the monetary constraint ($\dot{M}^{\circ} = \dot{M}^{*}$) and constant spending on other items ($\dot{C}_o^{\circ} = \dot{C}_o^{*}$) to cancel terms to obtain

$$p_E\dot{E}_s^{\circ} + \dot{C}_{cap}^{\circ} + \dot{C}_{md}^{\circ} + \overset{0}{\cancel{\dot{C}_o^{\circ}}} + \overset{0}{\cancel{\dot{N}^{\circ}}}$$
$$= p_E\dot{E}_s^{*} + \dot{C}_{cap}^{*} + \dot{C}_{md}^{*} + \cancel{\dot{C}_o^{*}} + \dot{N}^{*} . \tag{72}$$

Solving for $\Delta\dot{N}^{*} \equiv \dot{N}^{*} - \overset{0}{\cancel{\dot{N}^{\circ}}}$ gives

$$\Delta\dot{N}^{*} = p_E(\dot{E}_s^{\circ} - \dot{E}_s^{*}) + \dot{C}_{cap}^{\circ} - \dot{C}_{cap}^{*} + \dot{C}_{md}^{\circ} - \dot{C}_{md}^{*} . \tag{73}$$

Rewriting with Δ terms gives

$$\Delta\dot{N}^{*} = -p_E\Delta\dot{E}_s^{*} - \Delta\dot{C}_{cap}^{*} - \Delta\dot{C}_{md}^{*} . \tag{74}$$

Substituting Eq. (69) gives

$$\Delta\dot{N}^{*} = \dot{N}^{*} = p_E\dot{S}_{dev} - \Delta\dot{C}_{cap}^{*} - \Delta\dot{C}_{md}^{*} . \tag{75}$$

Freed cash ($\dot{G}$) resulting from the EEU, before any energy takeback, is given by

$$\dot{G} = p_E\dot{S}_{dev} . \tag{76}$$

Note that Eq. (64) and $\dot{N}^{\circ} = 0$ can be used to calculate $\dot{C}_o^{\circ}$ as

$$\dot{C}_o^{\circ} = \dot{M}^{\circ} - p_E\dot{E}_s^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ} . \tag{77}$$

Table C.4. **Substitution Effect**

Energy analysis		*Financial analysis*	
before ($*$)	$\dot{E}^* = \dot{E}_s^* + \dot{E}_{emb}^* + (\dot{C}_{md}^* + \dot{C}_o^*)I_E \quad (65)$	$\dot{M}^* = p_E\dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o^* + \dot{N}^* \quad (66)$	
after ($\wedge$)	$\hat{\dot{E}} = \hat{\dot{E}}_s + \hat{\dot{E}}_{emb} + (\hat{\dot{C}}_{md} + \hat{\dot{C}}_o)I_E \quad (78)$	$\hat{\dot{M}} = p_E\hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}} \quad (79)$	

Take differences to obtain the change in energy consumption, $\Delta\hat{\dot{E}} \equiv \hat{\dot{E}} - \dot{E}^*$.

$$\Delta\hat{\dot{E}} = \Delta\hat{\dot{E}}_s + \cancel{\Delta\hat{\dot{E}}_{emb}}^{\;0} + (\cancel{\Delta\hat{\dot{C}}_{md}}^{\;0} + \Delta\hat{\dot{C}}_o)I_E \qquad (80)$$

Thus,

$$\Delta\hat{\dot{E}} = \Delta\hat{\dot{E}}_s + \Delta\hat{\dot{C}}_o I_E . \qquad (81)$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\hat{\dot{E}}}{\dot{S}_{dev}} = \frac{\Delta\hat{\dot{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \qquad (82)$$

Define $Re_{sub} \equiv \frac{\Delta\hat{\dot{E}}}{\dot{S}_{dev}}$, $Re_{dsub} \equiv \frac{\Delta\hat{\dot{E}}_s}{\dot{S}_{dev}}$, and $Re_{isub} \equiv \frac{\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{sub} = Re_{dsub} + Re_{isub} . \qquad (83)$$

Use the monetary constraint ($\dot{M}^* = \hat{\dot{M}}$) to obtain

$$p_E\dot{E}_s^* + \cancel{\dot{C}_{cap}^*} + \cancel{\dot{C}_{md}^*} + \dot{C}_o^* + \dot{N}^*$$
$$= p_E\hat{\dot{E}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o + \hat{\dot{N}} . \qquad (84)$$

For the substitution effect, there is no change in capital or maintenance and disposal costs ($\hat{\dot{C}}_{cap} = \dot{C}_{cap}^*$ and $\hat{\dot{C}}_{md} = \dot{C}_{md}^*$). Solving for $\Delta\hat{\dot{N}} \equiv \hat{\dot{N}} - \dot{N}^*$ gives

$$\Delta\hat{\dot{N}} = -p_E\Delta\hat{\dot{E}}_s - \Delta\hat{\dot{C}}_o . \qquad (85)$$

The substitution effect adjusts net savings relative to $\dot{N}^*$ by $\Delta\hat{\dot{N}}$. Thus, $\hat{\dot{N}} = \dot{N}^* + \Delta\hat{\dot{N}}$. Substituting Eqs. (75), (76), and (85) yields

$$\hat{\dot{N}} = \dot{G} - \Delta\dot{C}_{cap}^* - \Delta\dot{C}_{md}^* - p_E\Delta\hat{\dot{E}}_s - \Delta\hat{\dot{C}}_o . \qquad (86)$$

<div align="center">

Table C.5. **Income Effect**

</div>

	Energy analysis		*Financial analysis*	
before ($\wedge$)	$\hat{E} = \hat{\bar{E}}_s + \hat{\bar{E}}_{emb} + (\hat{C}_{md} + \hat{C}_o)I_E$	(78)	$\hat{M} = p_E\hat{\bar{E}}_s + \hat{C}_{cap} + \hat{C}_{md} + \hat{C}_o + \hat{N}$	(79)
after ($-$)	$\bar{E} = \bar{\bar{E}}_s + \bar{\bar{E}}_{emb} + (\bar{C}_{md} + \bar{C}_o)I_E$	(87)	$\bar{M} = p_E\bar{\bar{E}}_s + \bar{C}_{cap} + \bar{C}_{md} + \bar{C}_o + \bar{N}$	(88)

Take differences to obtain the change in energy consumption, $\Delta\bar{\bar{E}} \equiv \bar{\bar{E}} - \hat{\bar{E}}$.

$$\Delta\bar{\bar{E}} = \Delta\bar{\bar{E}}_s + \cancelto{0}{\Delta\bar{\bar{E}}_{emb}} + (\cancelto{0}{\Delta\bar{C}_{md}} + \Delta\bar{C}_o)I_E \tag{89}$$

Thus,

$$\Delta\bar{\bar{E}} = \Delta\bar{\bar{E}}_s + \Delta\bar{C}_o I_E \tag{90}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\bar{\bar{E}}}{\dot{S}_{dev}} = \frac{\Delta\bar{\bar{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\bar{C}_o I_E}{\dot{S}_{dev}} \tag{91}$$

Define $Re_{inc} \equiv \frac{\Delta\bar{\bar{E}}}{\dot{S}_{dev}}$, $Re_{dinc} \equiv \frac{\Delta\bar{\bar{E}}_s}{\dot{S}_{dev}}$, and $Re_{iinc} \equiv \frac{\Delta\bar{C}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{inc} = Re_{dinc} + Re_{iinc} . \tag{92}$$

Use the monetary constraint ($\hat{M} = \bar{M}$) to obtain

$$p_E\hat{\bar{E}}_s + \cancel{\hat{C}_{cap}} + \cancel{\hat{C}_{md}} + \hat{C}_o + \hat{N}$$
$$= p_E\bar{\bar{E}}_s + \cancel{\bar{C}_{cap}} + \cancel{\bar{C}_{md}} + \bar{C}_o + \cancelto{0}{\bar{N}} . \tag{93}$$

For the income effect, there is no change in capital or maintainance and disposal costs ($\hat{C}_{cap} = \dot{C}^*_{cap}$ and $\hat{C}_{md} = \dot{C}^*_{md}$). Notably, $\bar{N} = 0$, because it is assumed that all net monetary savings ($\hat{N}$) are spent on more energy service ($\bar{\bar{E}}_s > \hat{\bar{E}}_s$) and additional purchases in the economy ($\bar{C}_o > \hat{C}_o$). Solving for $\hat{N}$ gives

$$\hat{N} = p_E\Delta\bar{\bar{E}}_s + \Delta\bar{C}_o , \tag{94}$$

the budget constraint for the income effect. By construction, Eq. (94) ensures spending of net savings ($\hat{N}$) on (i) additional energy services ($\Delta\bar{\bar{E}}_s$) and (ii) additional purchases of other goods in the economy ($\Delta\bar{C}_o$) only.

Table C.6. **Macro Effect**

Energy analysis		*Financial analysis*

before $(-)$	$\bar{\tilde{E}}$	(95)	

after $(\sim)$	$\tilde{\tilde{E}}$	(96)	

Take differences to obtain the change in energy consumption,

$$\Delta \tilde{E} \equiv \tilde{\tilde{E}} - \bar{\tilde{E}} \ . \qquad (97)$$

The energy change due to the macro effect $(\Delta \tilde{E})$ is a scalar multiple (k) of net savings $(\hat{N})$, assumed to be spent at the energy intensity of the economy (I_E).

$$\Delta \tilde{E} = k\hat{N}I_E \qquad (98)$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta \tilde{E}}{\dot{S}_{dev}} = \frac{k\hat{N}I_E}{\dot{S}_{dev}} \qquad (99)$$

Define $Re_{macro} \equiv \frac{\Delta \tilde{E}}{\dot{S}_{dev}}$, such that

$$Re_{macro} = \frac{k\hat{N}I_E}{\dot{S}_{dev}} \ . \qquad (32)$$

N/A

C.3 Rebound expressions

All that remains is to determine expressions for each rebound effect. We begin with the device-level expected energy savings rate $(\dot{S}_{dev})$, which appears in the denominator of all rebound expressions.

C.3.1 Expected energy savings $(\dot{S}_{dev})$

$\dot{S}_{dev}$ is the reduction of energy consumption rate by the device due to the EEU. No other effects are considered.

$$\dot{S}_{dev} \equiv \dot{E}_s^{\circ} - \dot{E}_s^{*} \tag{9}$$

The final energy consumption rates $(\dot{E}_s^{\circ}$ and $\dot{E}_s^{*})$ can be written as Eq. (5) in the forms $\dot{E}_s^{\circ} = \dot{q}_s^{\circ}/\eta^{\circ}$ and $\dot{E}_s^{*} = \dot{q}_s^{*}/\eta^{*}$.

$$\dot{S}_{dev} = \frac{\dot{q}_s^{\circ}}{\eta^{\circ}} - \frac{\dot{q}_s^{*}}{\eta^{*}} \tag{100}$$

With reference to Table C.1, we use $\dot{q}_s^{*} = \dot{q}_s^{\circ}$ and $\eta^{*} = \tilde{\eta}$ to obtain

$$\dot{S}_{dev} = \frac{\dot{q}_s^{\circ}}{\eta^{\circ}} - \frac{\dot{q}_s^{\circ}}{\tilde{\eta}} \; . \tag{101}$$

When the EEU increases efficiency such that $\tilde{\eta} > \eta^{\circ}$, expected energy savings grows $(\dot{S}_{dev} > 0)$ as the rate of final energy consumption declines, as expected. As $\tilde{\eta} \to \infty$, all final energy consumption is eliminated $(\dot{E}_s^{*} \to 0)$, and $\dot{S}_{dev} = \dot{q}_s^{\circ}/\eta^{\circ} = \dot{E}_s^{\circ}$. (Of course, $\tilde{\eta} \to \infty$ is impossible. See Paoli & Cullen (2020) for a recent discussion of upper limits to device efficiencies.)

After rearrangement and using $\dot{E}_s^{\circ} = \dot{q}_s^{\circ}/\eta^{\circ}$, we obtain a convenient form

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^{\circ}} - 1 \right) \frac{\eta^{\circ}}{\tilde{\eta}} \dot{E}_s^{\circ} \; . \tag{10}$$

C.3.2 Emplacement effect

The emplacement effect accounts for performance of the EEU only. No behavior changes occur. The direct emplacement effect of the EEU is device energy savings and energy cost savings. The indirect emplacement effects of the EEU produce changes in the embodied energy rate and the maintenance and disposal expenditure rates. By definition, the direct emplacement effect has no rebound. However, indirect emplacement effects may cause energy rebound. Both direct and indirect emplacement effects are discussed below.

Re_{dempl} As shown in Table C.3, the direct rebound from the emplacement effect is $Re_{dempl} = 0$. This result is expected, because, in the absence of behavior changes, there is no takeback of energy savings at the upgraded device.

Re_{iempl} Indirect emplacement rebound effects can occur at any point in the life cycle of an energy conversion device, from manufacturing and distribution to the use phase (maintenance), and finally to disposal. For simplicity, we group maintenance with disposal to form two distinct indirect emplacement rebound effects: (i) an embodied energy effect (Re_{emb}) and (ii) a maintenance and disposal effect (Re_{md}).

Re_{emb} The first component of indirect emplacement effect rebound involves embodied energy. We define embodied energy consistent with the energy analysis literature to be the sum of all final energy consumed in the production of the energy conversion device. The EEU causes the embodied final energy of the device to change from $\dot{E}_{emb}^{\circ}$ to $\dot{E}_{emb}^{*}$.

Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. No energy is embodied in the device while in service. However, for simplicity, we spread all embodied energy over the lifetime of the device, an equal amount assigned to each period. We later take the same approach to capital costs and maintenance and disposal costs. A justification for spreading embodied energy purchase costs comes from considering staggered device replacements by many consumers across several years. In the aggregate, staggered replacements work out to about the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices (t_{life}° and t_{life}^{*}, respectively) to obtain embodied energy rates, such that $\dot{E}_{emb}^{\circ} = E_{emb}^{\circ}/t_{life}^{\circ}$ and $\dot{E}_{emb}^{*} = E_{emb}^{*}/t_{life}^{*}$. The change in embodied final energy due to the EEU (expressed as a rate) is given by $\dot{E}_{emb}^{*} - \dot{E}_{emb}^{\circ}$. After substitution and algebraic rearrangement, the change in embodied energy rate due to the EEU can be expressed as $[(E_{emb}^{*}/E_{emb}^{\circ})(t_{life}^{\circ}/t_{life}^{*}) - 1]\dot{E}_{emb}^{\circ}$, a term that represents energy savings taken back due to embodied energy effects. Thus, Eq. (4) can be employed to write embodied energy rebound as

$$Re_{emb} = \frac{\left(\frac{E_{emb}^{*}}{E_{emb}^{\circ}} \frac{t_{life}^{\circ}}{t_{life}^{*}} - 1 \right) \dot{E}_{emb}^{\circ}}{\dot{S}_{dev}} \ . \tag{12}$$

Embodied energy rebound can be either positive or negative, depending on the sign of the term $(E_{emb}^{*}/E_{emb}^{\circ})(t_{life}^{\circ}/t_{life}^{*}) - 1$. Rising energy efficiency can be associated with increased device complexity and more embodied energy, such that $E_{emb}^{*} > E_{emb}^{\circ}$ and $Re_{emb} > 0$. However, if the upgraded device has longer life than the original device ($t_{life}^{*} > t_{life}^{\circ}$), $\dot{E}_{emb}^{*} - \dot{E}_{emb}^{\circ}$ can be negative, meaning that the upgraded device has a lower embodied energy rate than the original device.

Re_{md} In addition to embodied energy effects, indirect emplacement rebound can be associated with energy demanded by maintenance and disposal (md) expenditures. Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}_m^{\circ}$). Disposal costs (e.g., C_d°) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal expenditures across the lifetime of the original and upgraded devices (t_{life}° and t_{life}^{*}, respectively) to form expenditure rates such that $\dot{C}_{md}^{\circ} = \dot{C}_m^{\circ} + C_d^{\circ}/t_{life}^{\circ}$ and $\dot{C}_{md}^{*} = \dot{C}_m^{*} + C_d^{*}/t_{life}^{*}$.

We assume, for simplicity, that md expenditures indicate energy consumption elsewhere in the economy at its energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in md expenditures is given by $\Delta\dot{C}_{md}^{*}I_E$. This term is an energy takeback rate, so maintenance and disposal rebound is given by

$$Re_{md} = \frac{\Delta\dot{C}_{md}^{*}I_E}{\dot{S}_{dev}} \ , \tag{102}$$

as shown in Table C.3. Slight rearrangement gives

$$Re_{md} = \frac{\left(\frac{\dot{C}_{md}^{*}}{\dot{C}_{md}^{\circ}} - 1 \right) \dot{C}_{md}^{\circ}I_E}{\dot{S}_{dev}} \ . \tag{13}$$

Rebound from maintenance and disposal can be positive or negative, depending on the sign of the term $\dot{C}_{md}^{*}/\dot{C}_{md}^{\circ} - 1$.

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C.3.3 Substitution effect

This section derives expressions for substitution effect rebound. Two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}). Assuming that conditions after the emplacement effect ($*$) are known, both the rate of energy service consumption ($\hat{\dot{q}}_s$) and the rate of other goods consumption ($\hat{\dot{C}}_o$) must be determined as a result of the substitution effect (the $\wedge$ point).

The EEU's energy efficiency increase ($\tilde{\eta} > \eta°$) causes the price of the energy service provided by the device to fall ($\tilde{p}_s < p°_s$). The substitution effect quantifies the amount by which the device owner, in response, increases the consumption rate of the energy service ($\hat{\dot{q}}_s > \dot{q}^*_s$) and decreases the consumption rate of other goods ($\hat{\dot{q}}_o < \dot{q}^*_o$).

The increase in consumption of the energy service substitutes for consumption of other goods in the economy, subject to a utility constraint. The reduction in spending on other goods in the economy is captured by indirect substitution rebound (Re_{isub}).

We begin by deriving an expression for direct and indirect substitution effect rebound (Re_{dsub} and Re_{isub}, respectively). Thereafter, we develop an approximate model and a CES utility model for determining the post-substitution point ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$).

Direct substitution effect rebound expression Direct substitution effect rebound (Re_{dsub}) is given by

$$Re_{dsub} = \frac{\Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} = \frac{\hat{\dot{E}}_s - \dot{E}^*_s}{\dot{S}_{dev}} \ . \tag{15}$$

Substituting the typical relationship of Eq. (5) in the form $\dot{E}_s = \dot{q}_s/\eta$ gives

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\tilde{\eta}} - \frac{\dot{q}^*_s}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{103}$$

Rearranging produces

$$Re_{dsub} = \frac{\left(\frac{\hat{\dot{q}}_s}{\dot{q}°_s} - \frac{\dot{q}^*_s}{\dot{q}°_s}\right) \frac{\dot{q}°_s}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{104}$$

Recognizing that the rate of energy service consumption ($\dot{q}_s$) is unchanged across the emplacement effect leads to $\dot{q}^*_s/\dot{q}°_s = 1$. Furthermore, $\dot{q}°_s/\tilde{\eta} = (\dot{q}°_s/\eta°)(\eta°/\tilde{\eta}) = \dot{E}°_s(\eta°/\tilde{\eta})$, such that

$$Re_{dsub} = \left(\frac{\hat{\dot{q}}_s}{\dot{q}°_s} - 1\right) \frac{\dot{E}°_s \frac{\eta°}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{105}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}°_s} - 1}{\frac{\tilde{\eta}}{\eta°} - 1} \left(\frac{\cancel{\dot{E}°_s} \cancel{\frac{\eta°}{\tilde{\eta}}}}{\cancel{\frac{\eta°}{\tilde{\eta}}} \cancel{\dot{E}°_s}}\right) \ . \tag{106}$$

Canceling terms yields

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} . \tag{16}$$

Eq. (16) is the basis for developing both approximate and CES models of determining direct substitution rebound.

Indirect substitution effect rebound expression Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta\hat{\dot{C}}_o I_E}{\dot{S}_{dev}} = \frac{(\hat{\dot{C}}_o - \dot{C}_o^*)I_E}{\dot{S}_{dev}} . \tag{17}$$

Rearranging gives

$$Re_{isub} = \frac{\left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - \frac{\dot{C}_o^*}{\dot{C}_o^\circ}\right)\dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{107}$$

Recognizing that expenditures on other goods are constant across the emplacement effect gives $\dot{C}_o^*/\dot{C}_o^\circ = 1$ and

$$Re_{isub} = \left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1\right)\frac{\dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{108}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} . \tag{18}$$

Eq. (18) is the basis for developing both approximate and CES models of determining indirect substitution rebound.

Determining the post-substitution effect conditions requires reference to a consumer utility model. We first show an approximate model, often used in the literature. Later, we use a constant elasticity of substitution (CES) utility model. The CES model is used for all calculations and graphs in this paper.

Approximate utility model In the literature, an approximate utility model is often used (Borenstein, 2015, p. 17, footnote 43). In the two examples of this paper (car and electric lamp upgrades), rebound calculated using the approximate utility model (here) differs from rebound calculated using the exact utility model (below) by as much as 5%. Thus, we do not recommend use of the approximate utility model. We discuss the approximate utility model here for completeness only.

In the approximate model, the relationship between energy service price and energy service consumption rate is given by the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\epsilon_{\dot{q}_s,p_s,c}} . \tag{109}$$

Note that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) is assumed constant along an indifference curve in the approximate model. A negative value for the compensated price elasticity of energy service demand is expected ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that when the energy service price decreases ($\tilde{p}_s < p_s^\circ$), the rate of energy service consumption increases ($\hat{\dot{q}}_s > \dot{q}_s^*$).

Substituting Eq. (6) in the form $p_s^\circ = p_E^\circ/\eta^\circ$ and $\tilde{p}_s = p_E^\circ/\tilde{\eta}$ and noting that $\dot{q}_s^\circ = \dot{q}_s^*$ gives

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} . \tag{110}$$

Again, note that the compensated price elasticity of energy service demand is negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), so that as energy service efficiency increases ($\tilde{\eta} > \eta^\circ$), the energy service consumption rate increases ($\hat{\dot{q}}_s > \dot{q}_s^* = \dot{q}_s^\circ$).

Substituting Eq. (110) into Eq. (16) yields the approximate expression for direct substitution rebound.

$$Re_{dsub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{111}$$

The compensated price elasticity of energy service demand is expected to be negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that, e.g. $\epsilon_{\dot{q}_s,p_s,c} = -0.2$ and $\tilde{\eta}/\eta^\circ = 2$ yields $Re_{dsub} = 0.15$.

With $\epsilon_{\dot{q}_s,p_s,c} \in (-1,0)$ expected, the approximate model indicates that direct substitution rebound will never be larger than 1. I.e., the direct substitution effect alone can never cause backfire.

To quantify the substitution effect on other purchases in the approximate model, we introduce another elasticity, the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\epsilon_{\dot{q}_o,p_s,c}} . \tag{112}$$

Because the compensated cross-price elasticity of other goods demand is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$), an energy service price decrease ($\tilde{p}_s < p_s^\circ$) implies a reduction in the rate of consumption of other goods ($\hat{\dot{q}}_o < \dot{q}_o^*$).

The energy service price is inversely proportional to efficiency, yielding

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{113}$$

Assuming that the average price is unchanged across the substitution effect such that $\hat{p}_o = \dot{p}_o^* = p_o^\circ$ (Appendix F), and noting that $\dot{q}_s^* = \dot{q}_s^\circ$ and $\dot{C}_o^* = \dot{C}_o^\circ$, we can write

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{114}$$

Note that Eq. (114) can be used to determine the rate of expenditures on other goods in the economy ($\hat{\dot{C}}_o$) by

$$\hat{\dot{C}}_o = \dot{C}_o^\circ \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{115}$$

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Substituting Eq. (115) into Eq. (18) gives the expression for indirect substitution rebound for the approximate utility model.

$$Re_{isub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{116}$$

Because the compensated cross-elasticity of other goods consumption is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$) and the energy service efficiency ratio is greater than 1 ($\tilde{\eta} > \eta^\circ$), indirect substitution rebound will be negative always ($Re_{isub} < 0$), as expected. Negative rebound indicates that indirect substitution effects reduce the energy takeback rate by direct substitution effects.

CES utility model The approximate utility model assumes that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$) are constant along an indifference curve. These assumptions are approximations that hold only for infinitesimally small energy service price changes ($\Delta p_s^* \equiv p_s^* - p_s^\circ \approx 0$). However, in the case of an energy efficiency upgrade (EEU), the energy service price change is not infinitesimal. Rather, Δp_s^* is finite and may be large.

To determine the new consumption bundle after the substitution effect ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$) and, ultimately, to quantify the direct and indirect substitution rebound effects (Re_{dsub} and Re_{isub}) exactly, we remove the restriction that energy service price elasticities ($\epsilon_{\dot{q}_s,p_s,c}$ and $\epsilon_{\dot{q}_o,p_s,c}$) must be constant along an indifference curve. Instead, we require constancy of only the elasticity of substitution (σ) between the consumption rate of the energy service ($\dot{q}_s$) and the expenditure rate for other goods ($\dot{C}_o$) across the substitution effect. Thus, we employ a CES utility model.

Fig. 4 (especially segments $*$—c and c—$\wedge$) illustrates features of the CES utility model for determining the new consumption bundle. Two equations are helpful for this analysis. First, the slope at any point on indifference curve i$^\circ$—i$^\circ$ is given by Eq. (145) with $\dot{u}/\dot{u}^\circ = 1$ and the share parameter (a) replaced by $f_{\dot{C}_s}^\circ$, as discussed in Appendix D.

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} \left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)}$$

$$\times \left[\left(\frac{1}{1 - f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho}. \tag{117}$$

Second, the equation of the $*$—$*$ expenditure line is

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right) + \frac{1}{\dot{C}_o^\circ}(\dot{M} - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G}). \tag{118}$$

To find the rate of energy service consumption after the substitution effect ($\hat{\dot{q}}_s$), we set the slope of the $*$—$*$ expenditure line (Eq. 118) equal to the slope of the i$^\circ$—i$^\circ$ indifference curve at the original utility rate of $\dot{u}/\dot{u}^\circ = 1$ (Eq. (117)).

$$-\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)}\left[\left(\frac{1}{1 - f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho} \tag{119}$$

Solving for $\dot{q}_s/\dot{q}_s^\circ$ gives $\hat{\dot{q}}_s/\dot{q}_s^\circ$ as

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho}. \tag{19}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an estimate for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

The rate of other goods consumption after the substitution effect ($\hat{\dot{C}}_o$) can be found by substituting Eq. (19) and $\dot{u}/\dot{u}^\circ = 1$ into the functional form of the utility model (Eq. (144)) to obtain

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(\left(\frac{1}{1 - f_{\dot{C}_s}^\circ} \right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} \right) \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1} \right)^{1/\rho}. \tag{120}$$

Simplifying gives

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left[\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho}. \tag{20}$$

Eq. (20) can be substituted into Eq. (18) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left[\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{22}$$

C.3.4 Income effect

Rebound from the income effect rebound quantifies the rate of additional energy demand that arises because the owner of the energy conversion device spends net savings from the EEU. The income rate of the device owner is $\dot{M}^\circ$, which remains unchanged across the rebound effects, such that $\dot{M}^\circ = \dot{M}^* = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Freed cash from the EEU is given by Eq. (76) as $\dot{G} = p_E \dot{S}_{dev}$. In combination, the emplacement effect and the substitution effect leave the device owner with *net savings* ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (86). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables C.3 and C.4.

In this framework, all net savings ($\hat{\dot{N}}$) are spent on either (i) additional energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) or (ii) additional other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$). The income elasticity of energy service demand and the income elasticity of other goods demand ($\epsilon_{\dot{q}_s, \dot{M}}$ and $\epsilon_{\dot{q}_o, \dot{M}}$, respectively) quantify the income preferences of the device owner according to the following expressions:

$$\frac{\bar{\bar{q}}_s}{\hat{\hat{q}}_s} = \left(1 + \frac{\hat{\hat{N}}}{\hat{\hat{M}}'}\right)^{\epsilon_{\dot{q}_s, \dot{M}}} \tag{23}$$

and

$$\frac{\bar{\bar{q}}_o}{\hat{\hat{q}}_o} = \left(1 + \frac{\hat{\hat{N}}}{\hat{\hat{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} , \tag{27}$$

where effective income $(\hat{\hat{M}}')$ is

$$\hat{\hat{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\hat{N}} . \tag{24}$$

Homotheticity means that $\epsilon_{\dot{q}_s, \dot{M}} = 1$ and $\epsilon_{\dot{q}_o, \dot{M}} = 1$.

The budget constraint across the income effect (Eq. (94)) ensures that all net savings available after the substitution effect $(\hat{\hat{N}})$ is re-spent across the income effect, such that $\bar{\bar{N}} = 0$. Appendix E proves that the income preference equations (Eqs. (23) and (27)) satsify the budget constraint (Eq. 94).

The purpose of this section is derivation of expressions for (i) direct income rebound (Re_{dinc}) arising from increased consumption of the energy service $(\bar{\bar{q}}_s > \hat{\hat{q}}_s)$ and (ii) indirect income rebound (Re_{iinc}) arising from increased consumption of other goods $(\bar{\bar{q}}_o > \hat{\hat{q}}_o)$.

But first, we derive a helpful expression to be used later.

Expression for $\hat{\hat{E}}_s$ An expression for $\hat{\hat{E}}_s$ that will be helpful later begins with

$$\hat{\hat{E}}_s = \left(\frac{\hat{\hat{E}}_s}{\dot{E}^*_s}\right) \left(\frac{\dot{E}^*_s}{\dot{E}^\circ_s}\right) \dot{E}^\circ_s . \tag{121}$$

Substituting Eq. (5) and noting efficiency (η) equalities from Table C.1 gives

$$\hat{\hat{E}}_s = \left(\frac{\hat{\hat{q}}_s/\bar{\bar{\eta}}}{\dot{q}^*_s/\tilde{\eta}}\right) \left(\frac{\dot{q}^*_s/\tilde{\eta}}{\dot{q}^\circ_s/\eta^\circ}\right) \dot{E}^\circ_s . \tag{122}$$

Canceling terms yields

$$\hat{\hat{E}}_s = \left(\frac{\hat{\hat{q}}_s}{\dot{q}^*_s}\right) \left(\frac{\dot{q}^*_s}{\dot{q}^\circ_s}\right) \left(\frac{\eta^\circ}{\tilde{\eta}}\right) \dot{E}^\circ_s . \tag{123}$$

Noting energy service consumption rate equalities from Table C.1 $(\dot{q}^*_s = \dot{q}^\circ_s)$ gives

$$\hat{\hat{E}}_s = \frac{\hat{\hat{q}}_s}{\dot{q}^*_s} \frac{\eta^\circ}{\tilde{\eta}} \dot{E}^\circ_s . \tag{124}$$

The next step is to develop an expression for Re_{dinc} using the income preference for energy service consumption.

Expression for Re_{dinc} As shown in Table C.5, direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \bar{\dot{E}}_s}{\dot{S}_{dev}} . \tag{25}$$

Expanding the difference and rearranging gives

$$Re_{dinc} = \frac{\bar{\dot{E}}_s - \hat{\dot{E}}_s}{\dot{S}_{dev}} , \tag{125}$$

and

$$Re_{dinc} = \frac{\left(\frac{\bar{\dot{E}}_s}{\hat{\dot{E}}_s} - 1\right) \hat{\dot{E}}_s}{\dot{S}_{dev}} . \tag{126}$$

Substituting the Eq. (5) as $\bar{\dot{E}}_s = \frac{\bar{\dot{q}}_s}{\tilde{\eta}}$ and $\hat{\dot{E}}_s = \frac{\hat{\dot{q}}_s}{\tilde{\eta}}$ gives

$$Re_{dinc} = \frac{\left(\frac{\bar{\dot{q}}_s/\tilde{\eta}}{\hat{\dot{q}}_s/\tilde{\eta}} - 1\right) \hat{\dot{E}}_s}{\dot{S}_{dev}} . \tag{127}$$

Eliminating terms and substituting Eq. (10) for $\dot{S}_{dev}$ and Eq. (23) for $\bar{\dot{q}}_s/\hat{\dot{q}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1\right] \hat{\dot{E}}_s}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ} . \tag{128}$$

Substituting Eq. (124) for $\hat{\dot{E}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1\right] \frac{\hat{\dot{q}}_s}{\dot{q}_s^*} \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ} . \tag{129}$$

Eliminating terms, recognizing that $\dot{q}_s^\circ = \dot{q}_s^*$, and substituting Eq. (19), which assumes the CES utility model, gives

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} . \tag{26}$$

If there is no net savings ($\hat{\dot{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected.

The next step is to develop an expression for Re_{iinc} using the income preference for other goods consumption.

Expression for Re_{iinc} As shown in Table C.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta \bar{\dot{C}}_o I_E}{\dot{S}_{dev}} . \tag{29}$$

Expanding the difference and rearranging gives

$$Re_{iinc} = \frac{(\bar{\hat{C}}_o - \hat{C}_o)I_E}{\dot{S}_{dev}} \,, \tag{130}$$

and

$$Re_{iinc} = \frac{\left(\frac{\bar{\hat{C}}_o}{\hat{C}_o} - 1\right)\hat{C}_o I_E}{\dot{S}_{dev}} \,. \tag{131}$$

Substituting $\bar{\hat{C}}_o = p_o\bar{\hat{q}}_o$ and $\hat{C}_o = p_o\hat{q}_o$ and cancelling terms gives

$$Re_{iinc} = \frac{\left(\frac{\bar{\hat{q}}_o}{\hat{q}_o} - 1\right)\hat{C}_o I_E}{\dot{S}_{dev}} \,. \tag{132}$$

Substituting the income preference equation for other goods consumption (Eq. (27)) for $\bar{\hat{q}}_o/\hat{q}_o$ and Eq. (10) for $\dot{S}_{dev}$ yields

$$Re_{iinc} = \frac{\left[\left(1 + \frac{\hat{N}}{\hat{M}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} - 1\right]\hat{C}_o I_E}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right)\frac{\eta^\circ}{\tilde{\eta}}\dot{E}_s^\circ} \,. \tag{133}$$

Sutstituting $(\hat{C}_o/\dot{C}_o^\circ)\dot{C}_o^\circ$ for $\hat{C}_o$, recognizing that $\dot{C}_o^* = \dot{C}_o^\circ$, and simplifying gives

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{N}}{\hat{M}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\left(\frac{\tilde{\eta}}{\eta^\circ}\right)\frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ}\left(\frac{\hat{C}_o}{\dot{C}_o^\circ}\right) \,. \tag{134}$$

Substituting Eq. (20) for $\hat{C}_o/\dot{C}_o^\circ$, thereby assuming the CES utility model, gives the final form of the indirect income rebound expression:

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{N}}{\hat{M}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\left(\frac{\tilde{\eta}}{\eta^\circ}\right)\frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ}\left[\left(1 + f_{\dot{C}_s}^\circ\left\{\left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} \,. \tag{30}$$

If there is no net savings ($\hat{N} = 0$), indirect income effect rebound is zero ($Re_{iinc} = 0$), as expected.

C.3.5 <mark>Macro effect</mark>

Macro rebound (Re_{macro}) is given by Eq. (32). Substituting Eq. (86) for net savings ($\hat{N}$) gives

$$Re_{macro} = \frac{k\dot{G}I_E}{\dot{S}_{dev}} - \frac{k\Delta\dot{C}_{cap}^*I_E}{\dot{S}_{dev}} - \frac{k\Delta\dot{C}_{md}^*I_E}{\dot{S}_{dev}} - \frac{kp_E I_E \Delta\hat{\dot{E}}_s}{\dot{S}_{dev}} - \frac{k\Delta\hat{C}_o I_E}{\dot{S}_{dev}} \,. \tag{135}$$

Substituting Eq. (76) for $\dot{G}$ and Eqs. (102), (15), and (17) for rebound terms gives

$$Re_{macro} = \frac{kp_E\cancel{\dot{S}_{dev}}I_E}{\cancel{\dot{S}_{dev}}} - \frac{k\Delta\dot{C}_{cap}^*I_E}{\dot{S}_{dev}} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} \,. \tag{136}$$

Canceling terms and defining Re_{cap} as

$$Re_{cap} \equiv \frac{\Delta \dot{C}^*_{cap} I_E}{\dot{S}_{dev}} \tag{137}$$

gives

$$Re_{macro} = kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} . \tag{33}$$

C.3.6 Rebound sum

The sum of four rebound effects is

$$Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro} . \tag{138}$$

Substituting Eqs. (71), (83), and (92) gives

$$
\begin{aligned}
Re_{tot} = {} & Re_{emb} + Re_{md} && \boxed{\text{emplacement effect}} \\
& + Re_{dsub} + Re_{isub} && \boxed{\text{substitution effect}} \\
& + Re_{dinc} + Re_{iinc} && \boxed{\text{income effect}} \\
& + Re_{macro} && \boxed{\text{macro effect}}
\end{aligned} \tag{139}
$$

Macro effect rebound (Re_{macro}, Eq. (33)) can be expressed in terms of other rebound effects. Substituting Eq. (33) gives

$$
\begin{aligned}
Re_{tot} = {} & Re_{emb} + Re_{md} && \boxed{\text{emplacement effect}} \\
& + Re_{dsub} + Re_{isub} && \boxed{\text{substitution effect}} \\
& + Re_{dinc} + Re_{iinc} && \boxed{\text{income effect}} \\
& + kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} . && \boxed{\text{macro effect}}
\end{aligned} \tag{140}
$$

Rearranging distributes macro effect terms to emplacement and substitution effect terms. This last rearrangement gives the final expression for total rebound.

$$
\begin{aligned}
Re_{tot} = {} & Re_{emb} - kRe_{cap} + (1-k)Re_{md} \\
& + (1 - kp_E I_E)Re_{dsub} + (1-k)Re_{isub} \\
& + Re_{dinc} + Re_{iinc} + kp_E I_E
\end{aligned} \tag{34}
$$

Eq. (34) shows that determining seven rebound values,

- Re_{emb} (Eq. (12)),

- Re_{cap} (Eq. (137)),

- Re_{md} (Eq. (13)),

- Re_{dsub} (Eq. (21)),

- Re_{isub} (Eqs. (22)),

- Re_{dinc} (Eqs. (26)), and

- Re_{iinc} (Eqs. (30)),

is sufficient to calculate total rebound, provided that the macro factor (k), the price of energy (p_E), and the energy intensity of the economy (I_E) are known.

D Utility models and elasticities

As discussed in Section 3.5.2 and Appendix C.3.3, the substitution effect requires a model for device owner utility that compares the perceived benefits of consuming the energy service $(\dot{q}_s)$ to consuming other goods and services $(\dot{q}_o)$. In this appendix, we describe two utility models. The first utility model is an approximate model that applies only for small and marginal changes in energy efficiency and energy service price, such that $\Delta\eta^* \approx 0$ and $\Delta p_s^* \approx 0$. The approximate utility model is discussed for continuity with the literature only. (See, for example, Borenstein (2015).)

We note that larger and non-marginal efficiency gains cause greater rebound (measured in joules) than small and marginal efficiency gains. Thus, any rebound analysis framework needs to accommodate large, non-marginal efficiency changes. The second utility model discussed in this appendix is the Constant Elasticity of Substitution (CES) utility model which does, in fact, accommodate large, non-marginal energy efficiency and energy service price changes. The CES utility model underlies the substitution effect in this framework. (See Section 3.5.2.) Furthermore, the CES utility model is needed for the example EEUs in this paper, which have large, non-marginal percentage increases in energy efficiency. For the car example in Section 4.1, energy efficiency increases from $\eta^\circ = 25$ mpg to $\tilde{\eta} = 42$ mpg, a 68% improvement. For the lamp example in Section 4.2, energy efficiency increases by 826%.

Both the substitution effect and the income effect use elasticities to model consumer preferences, and those elasticities are discussed below. Finally, elasticities for the income effect are discussed.

Before proceeding with the utility models and elasticities, we note briefly that the rate of other goods consumption $(\dot{q}_o)$ is not known independently from the prices of other goods (p_o). With the assumption that the prices of other goods do not change across rebound effects (i.e., p_o is exogenous), the ratio of other goods consumption is equal to the ratio of other goods spending, such that

$$\frac{\dot{q}_o}{\dot{q}_o^\circ} = \frac{\dot{C}_o/\cancel{p_o}}{\dot{C}_o^\circ/\cancel{p_o}} = \frac{\dot{C}_o}{\dot{C}_o^\circ} \tag{141}$$

at all rebound stages. (See Appendix F for details.)

D.1 Utility models for the substitution effect

A utility model gives the ratio of energy service consumption rate and other goods consumption rates across the substitution effect $(\hat{\dot{q}}_s/\dot{q}_s^*$ and $\hat{\dot{q}}_o/\dot{q}_o^*$, respectively). In so doing, utility models quantify the decrease in other goods consumption $(\hat{\dot{q}}_o\dot{q}_o^* < 1)$ caused by the increase of energy service consumption $(\hat{\dot{q}}_s\dot{q}_s^* > 1)$ resulting from the decrease of the energy service price $(p_s^* < p_s^\circ)$. Across the substitution effect, the utility increase of the larger energy service consumption rate must be exactly offset by the utility decrease of the smaller other goods consumption rate.

D.1.1 Approximate utility model

The approximate utility model is given by Eqs. (110) and (114). The equations for the approximate utility model are repeated here for convenience.

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} \tag{110}$$

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} \tag{114}$$

D.1.2 CES utility model

The CES utility model is given by Eq. (14). Here, its derivation is shown.

The CES model for utility ($\dot{u}$) is normalized by (indexed to) conditions prior to emplacement:

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^\rho + (1-a)\left(\frac{\dot{q}_o}{\dot{q}_o^\circ}\right)^\rho\right]^{(1/\rho)}, \tag{142}$$

where $\rho = (\sigma-1)/\sigma$, a is a share parameter (determined below), and σ is the elasticity of substitution between the normalized consumption rate of the energy service and the normalized consumption rate of other goods.

With the assumption of exogenous other goods prices in Eq. (141), we find

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^\rho + (1-a)\left(\frac{\dot{C}_o}{\dot{C}_o^\circ}\right)^\rho\right]^{(1/\rho)}. \tag{143}$$

Eq. (143) is the functional form of the CES utility model, whose share parameter (a) is yet to be determined. The correct expression for the share parameter (a) is found from the equilibrium requirement, namely that the expenditure curve is tangent to the indifference curve in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space prior to the EEU. For example, line o—o is tangent to preference curve i°—i° at point o in Figs. 4, 7, and 10.

To find the slope of the indifference curve, Eq. (143) can be rearranged to find the normalized consumption rate of other goods ($\dot{C}_o/\dot{C}_o^\circ$) as a function of the normalized consumption rate of the energy service ($\dot{q}_s/\dot{q}_s^\circ$) and the normalized utility rate ($\dot{u}/\dot{u}^\circ$):

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = \left[\frac{1}{1-a}\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \frac{a}{1-a}\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1/\rho)}, \tag{144}$$

a form convenient for drawing constant utility rate ($\dot{u}/\dot{u}^\circ$) indifference curves in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space. (See Figs. 4, 7, and 10.) In $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, the slope of an indifference curve is found by taking the first partial derivative of $\dot{C}_o/\dot{C}_o^\circ$ with respect to $\dot{q}_s/\dot{q}_s^\circ$, starting from Eq. (144) and using the chain rule repeatedly. The result is

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{a}{1-a}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)}$$
$$\times \left[\left(\frac{1}{1-a}\right)\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \left(\frac{a}{1-a}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho}. \tag{145}$$

The budget constraint is the starting point for finding the slope of the expenditure line in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space:

$$\dot{M} = p_s\dot{q}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o + \dot{N} , \qquad (146)$$

a generic version of Eqs. (64), (66), (79), and (88) with $p_s\dot{q}_s$ substituted for $p_E\dot{E}_s$. In a manner similar to derivations in Appendix B.3.1, we solve for $\dot{C}_o$ and judiciously multiply by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ to obtain

$$\frac{\dot{C}_o}{\dot{C}_o^\circ}\dot{C}_o^\circ = -p_s\frac{\dot{q}_s}{\dot{q}_s^\circ}\dot{q}_s^\circ + \dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N} . \qquad (147)$$

Solving for $\dot{C}_o/\dot{C}_o^\circ$ and rearranging gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right) + \frac{1}{\dot{C}_o^\circ}(\dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N}) , \qquad (148)$$

from which the slope in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space is taken by inspection to be

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ} . \qquad (149)$$

At any equilibrium point, the expenditure line must be tangent to its indifference curve. Applying the tangency requirement before emplacement enables solving for the correct expression for a. Setting the slope of the expenditure line (Eq. (149)) equal to the slope of the indifference curve (Eq. (145)) gives

$$-\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{a}{1-a}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)}$$
$$\times \left[\left(\frac{1}{1-a}\right)\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^\rho - \left(\frac{a}{1-a}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho} . \qquad (150)$$

For the equilibrium point prior to emplacement, $\dot{q}_s/\dot{q}_s^\circ = 1$, $\dot{u}/\dot{u}^\circ = 1$, and $p_s = p_s^\circ$, which reduces Eq. (150) to

$$-\frac{p_s^\circ\dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{a}{1-a}(1)^{(\rho-1)}\left[\left(\frac{1}{1-a}\right)(1)^\rho - \left(\frac{a}{1-a}\right)(1)^\rho\right]^{(1-\rho)/\rho} . \qquad (151)$$

Simplifying gives

$$\frac{p_s^\circ\dot{q}_s^\circ}{\dot{C}_o^\circ} = \frac{a}{1-a} . \qquad (152)$$

Recognizing that $p_s^\circ\dot{q}_s^\circ = \dot{C}_s^\circ$ and solving for a gives

$$a = \frac{\dot{C}_s^\circ}{\dot{C}_s^\circ + \dot{C}_o^\circ} , \qquad (153)$$

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which is called $f_{\dot{C}_s}^\circ$, the share of energy service expenditure ($\dot{C}_s^\circ$) relative to the sum of energy service and other goods expenditures ($\dot{C}_s^\circ + \dot{C}_o^\circ$) before emplacement. Thus, the CES utility equation (Eq. (143)) becomes

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_o}{\dot{C}_o^\circ} \right)^\rho \right]^{(1/\rho)} , \tag{14}$$

with

$$f_{\dot{C}_s}^\circ \equiv \frac{\dot{C}_s^\circ}{\dot{C}_s^\circ + \dot{C}_o^\circ} . \tag{154}$$

D.2 Elasticities for the substitution effect ($\epsilon_{\dot{q}_s, p_s, c}$, $\epsilon_{\dot{q}_o, p_s, c}$, and σ)

The substitution effect requires a utility model. In this paper, we describe two utility models: an approximate model and a CES model. All utility models require elasticities to describe consumer preferences. In fact, there are three elasticies across the two utility models: $\epsilon_{\dot{q}_s, p_s, c}$, $\epsilon_{\dot{q}_o, p_s, c}$, and σ. The approximate utility model requires the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s, c}$) and the compensated cross-price elasticity of energy service demand ($\epsilon_{\dot{q}_o, p_s, c}$). The CES utility model requires the elasticity of substitution (σ) between energy service consumption and other goods consumption. Because the uncompensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s}$) tends to be more widely estimated, e.g. in studies about driver behavior (Gillingham, 2020), the usual approach is to use $\epsilon_{\dot{q}_s, p_s}$ and the Slutsky equation to derive the compensated elasticity ($\epsilon_{\dot{q}_s, p_s, c}$). The CES utility model then imposes enough restrictions to determine the elasticity of substitution.

D.2.1 Elasticities for the approximate utility model ($\epsilon_{\dot{q}_s, p_s, c}$ and $\epsilon_{\dot{q}_o, p_s, c}$)

Analytical expressions for the compensated elasticities in the approximate utility model can be derived using the Slutsky equation, whereby the price elasticity of the energy service ($\epsilon_{\dot{q}_s, p_s}$) is decomposed into the compensated price elasticity ($\epsilon_{\dot{q}_s, p_s, c}$) and the income elasticity ($\epsilon_{\dot{q}_s, \dot{M}}$) as follows:

$$\epsilon_{\dot{q}_s, p_s} = \epsilon_{\dot{q}_s, p_s, c} - f_{\dot{C}_s}^\circ \epsilon_{\dot{q}_s, \dot{M}} , \tag{155}$$

where $f_{\dot{C}_s}^\circ$ is given by Eq. (154), and $\epsilon_{\dot{q}_s, \dot{M}}$ is given in Section D.3. Solving for the compensated price elasticity ($\epsilon_{\dot{q}_s, p_s, c}$) gives

$$\epsilon_{\dot{q}_s, p_s, c} = \epsilon_{\dot{q}_s, p_s} + f_{\dot{C}_s}^\circ \epsilon_{\dot{q}_s, \dot{M}} . \tag{156}$$

A similar argument allows straightforward derivation of the compensated cross-price elasticity ($\epsilon_{\dot{q}_o, p_s, c}$). With Hicks & Allen (1934), we note that the uncompensated cross-price elasticity ($\epsilon_{\dot{q}_o, p_s}$) can generally be expressed as

$$\epsilon_{\dot{q}_o, p_s} = f_{\dot{C}_s}^\circ (\sigma - \epsilon_{\dot{q}_o, \dot{M}}) , \tag{157}$$

where σ is the elasticity of substitution between the consumption rate of the energy service ($\dot{q}_s$) and the consumption rate of other goods ($\dot{q}_o$). We set Eq. (157) equal to the cross-price version of the Slutsky equation

$$\epsilon_{\dot{q}_o, p_s} = \epsilon_{\dot{q}_o, p_s, c} - f_{\dot{C}_s}^{\circ} \epsilon_{\dot{q}_o, \dot{M}} \ , \tag{158}$$

to obtain

$$f_{\dot{C}_s}^{\circ} (\sigma - \epsilon_{\dot{q}_o, \dot{M}}) = \epsilon_{\dot{q}_o, p_s, c} - f_{\dot{C}_s}^{\circ} \epsilon_{\dot{q}_o, \dot{M}} \ . \tag{159}$$

Solving for $\epsilon_{\dot{q}_o, p_s, c}$ gives

$$\epsilon_{\dot{q}_o, p_s, c} = f_{\dot{C}_s}^{\circ} \sigma \ . \tag{160}$$

Substituting σ from Section D.2.2 (Eq. (162)) gives

$$\epsilon_{\dot{q}_o, p_s, c} = \frac{f_{\dot{C}_s}^{\circ} (f_{\dot{C}_s}^{\circ} + \epsilon_{\dot{q}_s, p_s})}{f_{\dot{C}_s}^{\circ} - 1} \ . \tag{161}$$

D.2.2 Elasticity for the CES utility model (σ)

Gørtz (1977) shows that the elasticity of substitution (σ) in the CES utility model is given by

$$\sigma = \frac{f_{\dot{C}_s}^{\circ} + \epsilon_{\dot{q}_s, p_s}}{f_{\dot{C}_s}^{\circ} - 1} \ . \tag{162}$$

Thus, the elasticity of substitution (σ) can be determined from two pieces of readily available information: (i) the uncompensated own price elasticity ($\epsilon_{\dot{q}_s, p_s}$) and (ii) the share of income spent on the energy service ($f_{\dot{C}_s}^{\circ}$ from Eq. (154)).

D.3 Elasticities for the income effect ($\epsilon_{\dot{q}_s, \dot{M}}$ and $\epsilon_{\dot{q}_o, \dot{M}}$)

The income effect requires two elasticities to estimate the spending of net savings: the income elasticity of energy service consumption ($\epsilon_{\dot{q}_s, \dot{M}}$) and the income elasticity of other goods consumption ($\epsilon_{\dot{q}_o, \dot{M}}$). Due to the homotheticity assumption, both income elasticities are unitary. Thus,

$$\epsilon_{\dot{q}_s, \dot{M}} = 1 \ , \tag{163}$$

and

$$\epsilon_{\dot{q}_o, \dot{M}} = 1 \ . \tag{164}$$

E Proof: Income preference equations satisfy the budget constraint

After the substitution effect, a rate of net savings is available ($\hat{N}$), all of which is spent on additional energy service ($\Delta \bar{\dot{q}}_s, \Delta \bar{\dot{C}}_s = p_E \Delta \bar{\dot{E}}_s$) or additional other goods ($\Delta \bar{\dot{q}}_o, \Delta \bar{\dot{C}}_o$). The income effect must satisfy the budget constraint such that net savings is zero afterward ($\bar{\dot{N}} = 0$). The budget constraint across the income effect is represented by Eq. (94):

$$\hat{N} = p_E \Delta \bar{\dot{E}}_s + \Delta \bar{\dot{C}}_o \ . \tag{94}$$

The additional spending due to the income effect is given by income preference equations

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}s,\dot{M}}} \tag{23}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}o,\dot{M}}} , \tag{27}$$

where

$$\hat{\dot{M}}' \equiv \dot{M}^{\circ} - \dot{C}_{cap}^{*} - \dot{C}_{md}^{*} - \hat{\dot{N}} . \tag{24}$$

This appendix proves that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (94)).

The first step in the proof is to convert the income preference equations to $\dot{C}_s^{\circ}$ and $\dot{C}_o^{\circ}$ ratios. For the energy service income preference equation (Eq. (23)), multiply numerator and denominator of the left-hand side by $\tilde{p}_s = p_E/\tilde{\eta}$ (Eq. (6)) to obtain $\bar{\dot{C}}_s/\hat{\dot{C}}_s$. For the other goods income preference equation (Eq. (27)), multiply numerator and denominator of the left-hand side by p_o to obtain $\bar{\dot{C}}_o/\hat{\dot{C}}_o$. Then, invoke homotheticity to set $\epsilon_{\dot{q}s,\dot{M}} = 1$ and $\epsilon_{\dot{q}o,\dot{M}} = 1$ to obtain

$$\frac{\bar{\dot{C}}_s}{\hat{\dot{C}}_s} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \tag{165}$$

and

$$\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} . \tag{166}$$

The second step in the proof is to obtain expressions for $\Delta\bar{\dot{C}}_s$ and $\Delta\bar{\dot{C}}_o$. Multiply the income preference equations above by $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively. Then, subtract $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively, to obtain

$$\Delta\bar{\dot{C}}_s = \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} \tag{167}$$

and

$$\Delta\bar{\dot{C}}_o = \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} . \tag{168}$$

The above versions of the income preference equations can be substituted into the budget constraint (Eq. (94)) to obtain

$$\hat{\dot{N}} \stackrel{?}{=} \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} + \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} . \tag{169}$$

If equality is demonstrated, the income preference equations satisfy the budget constraint. The remainder of the proof shows the equality of Eq. (169).

Dividing by $\hat{N}$ and multiplying by $\hat{M}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \hat{\dot{M}}' . \tag{170}$$

Substituting Eq. (24) for $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} . \tag{171}$$

Substituting Eq. (79) for $\dot{M}^\circ$, because $\dot{M}^\circ = \hat{\dot{M}}$, gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} p_E \hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}} - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} . \tag{172}$$

Cancelling terms and recognizing that $\dot{C}^*_{cap} = \hat{\dot{C}}_{cap}$, $\dot{C}^*_{md} = \hat{\dot{C}}_{md}$, and $\hat{\dot{C}}_s = p_E \hat{\dot{E}}_s$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \hat{\dot{C}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o - \cancel{\hat{\dot{C}}_{cap}} - \cancel{\hat{\dot{C}}_{md}} . \tag{173}$$

Cancelling terms gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{\checkmark}{=} \hat{\dot{C}}_s + \hat{\dot{C}}_o , \tag{174}$$

thereby completing the proof that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (94)).

F   Other goods expenditures and constant p_o

This framework utilizes a partial equilibrium analysis in which we account for the change of the energy service price due to the EEU ($p_s^\circ \neq p_s^*$), but we do not track the effect of the EEU on prices of other goods. These assumptions have important implications for the relationship between the rate of consumption of other goods ($\dot{q}_o$) and the rate of expenditure on other goods ($\dot{C}_o$).

We assume a basket of other goods (besides the energy service) purchased in the economy, each (i) with its own price ($p_{o,i}$) and rate of consumption ($\dot{q}_{o,i}$), such that the average price of all other goods purchased in the economy prior to the EEU (p_o°) is given by

$$p_o^\circ = \frac{\sum_i p_{o,i}^\circ q_{o,i}^\circ}{\sum_i q_{o,i}^\circ} . \tag{175}$$

Then, the expenditure rate of other purchases in the economy can be given as

$$\dot{C}_o^\circ = p_o^\circ \dot{q}_o^\circ \tag{176}$$

before the EEU and

$$\hat{\dot{C}}_o = \hat{p}_o \hat{\dot{q}}_o \tag{177}$$

after the substitution effect, for example.

We assume that any effects (emplacement, substitution, or income) for a single device are not so large that they cause a measurable change in prices of other goods. Thus,

$$p_o^\circ = p_o^* = \hat{p}_o = \bar{p}_o = \tilde{p}_o \ . \tag{178}$$

In the partial equilibrium analysis, two other goods prices can be equated across any rebound effect to obtain (for the example of the original conditions ($\circ$) and the post-substitution state ($\wedge$))

$$\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} = \frac{\dot{\hat{q}}_o}{\dot{q}_o^\circ} \ . \tag{179}$$

Thus, a ratio of other goods expenditure rates is always equal to a ratio of other goods consumption rates.

G Income and macro effects and relation to the marginal propensity to consume (MPC)

Borenstein (2015) has postulated a demand-side argument that macro effects can be represented by a multiplier, which we call the macro factor (k). Borenstein's formulation and our implementation are reminiscent of the marginal propensity to consume (MPC). In this appendix, we show the relationship between the macro factor (k) and MPC.

The relationship between the macro factor (k) and MPC spans the income and macro effects. In this framework, the device owner's net savings after the substitution effect ($\dot{\hat{N}}$) is respent completely. One may assume that firms and other consumers who receive the net savings have a marginal propensity to re-spend of MPC. The total spending throughout the economy of each year's net savings ($\dot{\hat{N}}$) is given by the infinite series

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\dot{\hat{N}} \ , \tag{180}$$

where the first term ($1 \times \dot{\hat{N}}$) represents spending of net savings by the device owner in the direct and indirect income effects, and the remaining terms $[(MPC + MPC^2 + MPC^3 + \ldots)\dot{\hat{N}}]$ represent macro-effect spending in the broader economy.

The macro effect portion of the spending can be represented by the macro factor (k).

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\dot{\hat{N}} = (1 + k)\dot{\hat{N}} \tag{181}$$

Canceling $\dot{\hat{N}}$ and simplifying the infinite series to its converged fraction (assuming $MPC < 1$) gives

$$\frac{1}{1 - MPC} = 1 + k \ . \tag{182}$$

Solving for k yields

$$k = \frac{1}{\frac{1}{MPC} - 1} \ . \tag{31}$$

With $MPC = 0.5$, as in Section 3.5.4, $k = 1$ is obtained. If $k = 3$, as in Section 5.3, $MPC = 0.75$ is implied. The relationship between k and MPC is given in Fig. G.1.
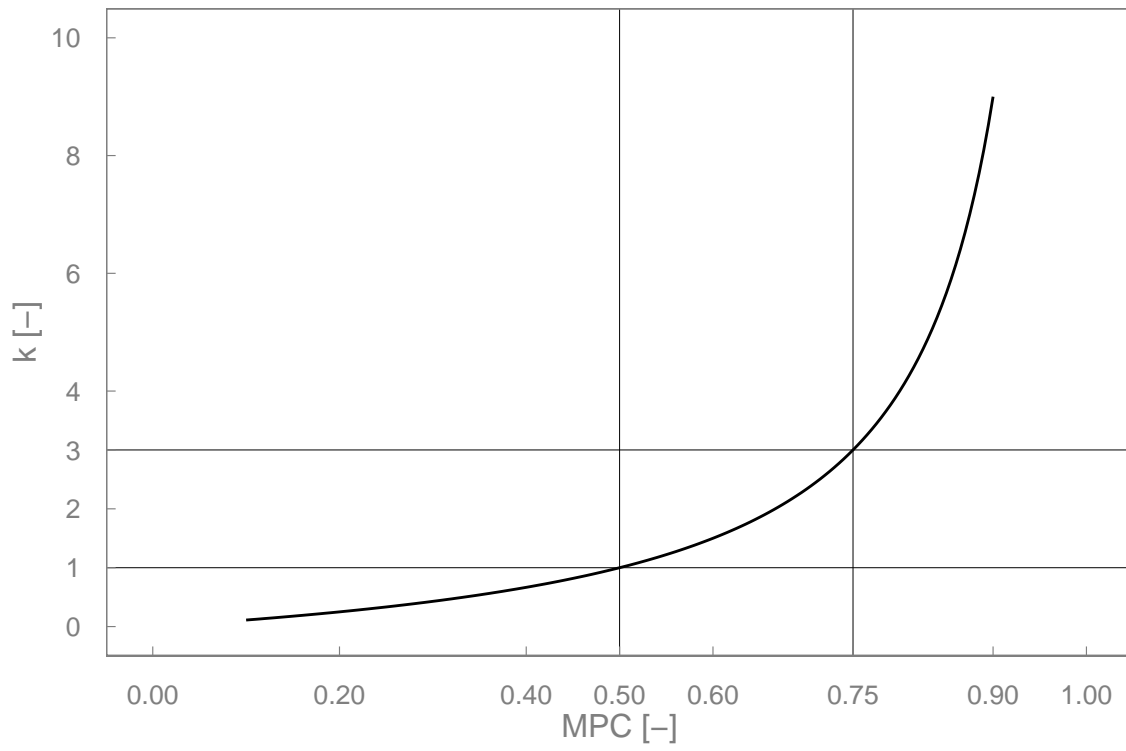
Fig. G.1: The relationship between *MPC* and *k* in Eq. (31).