Advancing the necessary foundations for empirical energy rebound estimates, Part I: A partial equilibrium framework

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Abstract

Widespread implementation of energy efficiency is a key greenhouse gas emissions mitigation measure, but rebound can "take back" energy savings. However, conceptual foundations lag behind empirical estimates of the size of rebound. A new clarity is needed, one that is built upon solid analytical frameworks involving both economics and energy analysis. In this paper (Part I), we help advance a rigorous analytical framework that starts at the microeconomics of rebound and is approachable for both energy analysts and economists alike. We include emplacement effects (including embodied energy and maintenance and disposal effects), substitution effects (direct and indirect), and income effects (direct and indirect) and link them to macro rebound. In Part II, we develop new rebound visualization techniques and exercise the framework by first calibrating the macro factor then by estimating rebound for two examples.

Keywords: Energy efficiency, Energy rebound, Energy services, Microeconomic rebound, Substitution and income effects, Macroeconomic rebound

1 Introduction

Energy rebound is an interdisciplinary challenge that threatens a low-carbon future (van den Bergh, 2017; Brockway et al., 2017). Rebound makes energy efficiency less effective at decreasing energy consumption and reducing carbon emissions by taking back (or reversing, in the case of "backfire") energy savings expected from an energy efficiency improvement (Sorrell, 2009). Recent evidence shows that rebound is both (i) larger than commonly assumed (Stern, 2020) and (ii) mostly missing from large energy and climate models (Brockway et al., 2021), Rebound is one explanation for why energy consumption and carbon emissions have never been absolutely decoupled from economic growth (Haberl et al., 2020; Brockway et al., 2021).

1.1 A short history of rebound

Famously, the roots of energy rebound trace back to Jevons who said "[i]*t is wholly a confusion of ideas to suppose that the economical use of fuel is equivalent to a diminished consumption. The* *very contrary is the truth*" (Jevons, 1865, p. 103, emphasis in original). Less famously, the origins of rebound extend further backward from Jevons to Williams (1840) and Parkes who wrote "[t]he economy of fuel is the secret of the economy of the steam-engine; it is the fountain of its power, and the adopted measure of its effects. Whatever, therefore, conduces to increase the efficiency of coal, and to diminish the cost of its use, directly tends to augment the value of the steam-engine, and to enlarge the field of its operations" (Parkes, 1838, p. 161). For nearly 200 years, then, it has been understood that efficiency gains may be taken back or, paradoxically even, cause growth in energy consumption, as Jevons assumed.

The oil crises of the 1970s shone a light back onto energy efficiency, and research into rebound appeared late in the decade (Madlener & Turner, 2016; Saunders et al., 2021). A modern debate over the magnitude of energy rebound commenced. On one side, scholars including Brookes (1979, 1990) and Khazzoom (1980) suggested rebound could be large. Others, including Lovins (1988) and Grubb (1990, 1992), claimed rebound was likely to be small. Debate over the size of energy rebound continues today. Advocates of small rebound (less than, say, 50%), suggest "the rebound effect is overplayed" (Gillingham et al., 2013, p. 475), while others claim the evidence for large rebound (greater than 50%) is growing (Saunders, 2015), and rebound will negatively impact the effectiveness of energy efficiency to reduce carbon emissions (van den Bergh, 2017).

1.2 Absence of solid analytical foundations

Turner contends that the lack of consensus on the magnitude of rebound in the modern empirical literature is caused by "a rush to empirical estimation in the absence of solid analytical foundations" (Turner, 2013, p. 25). Borenstein (2015) made some progress toward solidifying the analytical foundations ~~for microeconomic rebound~~, but more is needed to support empirical estimation efforts. In the absence of solid analytical foundations, the wide variety of rebound estimation approaches yields a wide range of rebound estimates, giving the appearance of uncertainty and leading energy and climate modelers to either (i) use questionable rebound estimates or (ii) ignore rebound altogether. We suggest that improving the conceptual space around rebound and solidifying the analytical frameworks will (i) help generate more robust estimates of rebound, (ii) lead to rebound being better included in more energy and climate models, and (iii) provide improved policymaking around energy efficiency.

But why is there an "absence of solid analytical foundations?" We propose that development of solid analytical frameworks for rebound is hampered by the fact that rebound is a decidedly interdisciplinary topic, involving both economics and energy analysis. Birol & Keppler (2000, p. 458) note that "different implicit and explicit assumptions of different research communities ('economists', 'engineers') … have in the past led to vastly differing points of view."[1] Turner states that "[d]ifferent definitions of energy efficiency will be appropriate in different circumstances. However, … it is often not clear what different authors mean by energy efficiency" (Turner, 2013, p. 237–38). If authors from the two disciplines cannot even agree on the terms, it is unsurprising that only modest progress has been made on analytical foundations. To fully understand rebound, economists need to have an energy analyst's understanding of energy, and energy analysts need to have an economist's understanding of finance and human behavior. That's a tall order, indeed.

[1] We prefer the term "energy analysts" over "engineers," because "energy analysts" better describes the group of people engaged in "energy analysis." For this paper, we define "energy analysis" as the study of energy transformations from stocks to flows to wastes and energy services along society's energy conversion chain for the purpose of generating energy services, economic activity, and human well-being.

1.3 New clarity is needed

We contend that new clarity is needed. A description of rebound that is both (i) technically rigorous and (ii) approachable from both sides (economics and energy analysis) will be a good starting point toward that clarity. In other words, the finance and human behavior aspects of rebound need to be presented in ways energy analysts can understand. And the energy aspects of rebound need to be presented in ways economists can understand.

Summarizing, we surmise that reducing global carbon emissions has been hampered, in part, by the fact that rebound is not sufficiently included in energy and climate models. We suspect that one reason rebound is not sufficiently included in energy and climate models is the lack of consensus on rebound estimation methods and, hence, rebound magnitude. We agree with Turner that lack of consensus on rebound magnitude is a symptom of the absence of solid analytical foundations for rebound. We posit that developing solid analytical frameworks for rebound is difficult because it is an inherently interdisciplinary topic. We believe that providing a detailed explication of an analytical framework for energy rebound will go some way toward providing additional clarity on the topic.

1.4 Objective, contributions, and structure

The *objective* of this paper is to improve clarity in the rebound space by supporting the development of a rigorous analytical framework, one that (i) starts at the microeconomics of rebound (building upon Borenstein (2015)) and (ii) is approachable for both energy analysts and economists alike.

The key *contributions* of this paper are (i) development of the first (to our knowledge) rebound analysis framework that combines embodied energy effects, maintenance and disposal effects, non–marginal energy efficiency increases, and non-marginal energy service price decreases and (ii) the first operationalized link between rebound effects on microeconomic and macroeconomic scales.

The remainder of this paper is *structured* as follows. Section 2 describes the comprehensive rebound analysis framework. Section 3 discusses the framework in relation to previous frameworks, and Section 4 concludes. Results can be found in Part II.

2 Methods: development of the framework

In this section, we develop a partial equilibrium energy rebound analysis framework (concisely, "this framework" or "our framework").

2.1 Rebound typology

Table 1 shows our topology of rebound effects. We distinguish between microeconomic and macroeconomic rebound effects. Microeconomic rebound occurs at the single device or for a single device owner. ~~It is a partial equilibrium response.~~ Macroeconomic rebound is a broader, economy-wide response to a single device upgrade. It comprises market and price effects, composition effects, and productivity effects (growth, scale, labour supply, and disinvestment effects). (See Greening et al. (2000), Barker et al. (2009), and Vivanco et al. (2018).)

We follow others, including Jenkins et al. (2011) and Walnum et al. (2014), in splitting microeconomic rebound into direct and indirect locations. Like other authors, we recognize many macroeconomic effects, even if we don't later distinguish among them.[2]

[2]For example, Sorrell (2009) sets out five macroeconomic rebound effects: embodied energy effects, respending

Table 1: Rebound typology. Comparison to Sorrell (2009), Jenkins et al. (2011), Thomas & Azevedo (2013a,b), and Walnum et al. (2014) in *italics*.

	Location	
	Direct	Indirect
Microeconomic rebound These mechanisms occur at the single device level within a static economy based on responses to the reduction in implicit price of an energy service.	**Emplacement effect** (Re_{dempl}) Accounts for performance of the Energy Efficeincy Upgrade (EEU) only. No behavior changes occur. The direct energy effect of emplacement of the EEU is expected device-level energy savings. By definition, there is no rebound from direct emplacement effects ($Re_{dempl} \equiv 0$).	**Emplacement effect** (Re_{iempl}) Differential lifecycle energy effects (versus counterfactual) of the EEU, i.e., embodied energy (Re_{emb}), and implied energy demand from maintenance and disposal (Re_{md}). *Other authors include embodied effects but not effects associated with maintenance and disposal.*
	Substitution effect (Re_{dsub}) Spending of freed cash on more of the energy service. *Same as other authors.*	**Substitution effect** (Re_{isub}) Decreased spending on other goods and services. *Other authors typically include indirect substitution effects within re-spending and reinvestment effects.*
	Income effect (Re_{dinc}) Spending of freed cash on more of the energy service. *Same as other authors.*	**Income effect** (Re_{iinc}) Increased spending on other goods and services. *Other authors typically include indirect income effects within re-spending and reinvestment* effects.
Macroeconomic rebound These mechanisms originate from the dynamic response of the economy to reach a stable equilibrium (between supply and demand for goods and energy services). These mechanisms combine various short and long run effects.		**Macroeconomic effect** (Re_{macro}) Comprised of numerous components including: energy market effect, composition effect, growth effect, scale effect, labor supply effect, and disinvestment effect. *We have close alignment with other authors in that (i) we fold scale and investment/ disinvestment effects into an economic growth effect and (ii) we include labor market effects within the indirect factors of production response.*

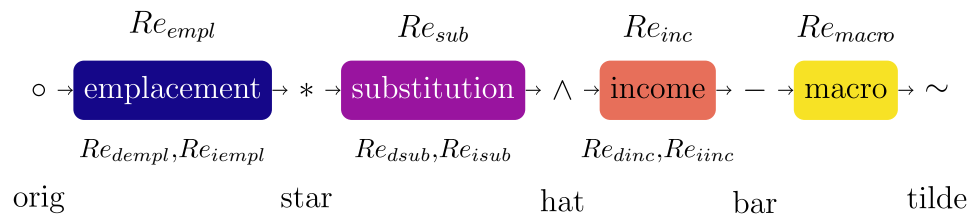

Fig. 1: Flowchart of rebound effects and decorations.

Fig. 1 shows rebound effects arranged in left-to-right order of their discussion in this paper. Rebound symbols are shown above each effect (Re_{empl}, etc.). Nomenclature for direct and indirect locations are shown beneath each effect (Re_{dempl}, etc.). Decorations for each stage are shown between rebound effects ($\circ$, $*$, etc.). Names for the decorations are given at the bottom of the figure ("orig," "star," etc.).[3] The left-to-right order does not necessarily represent the progression of rebound effects through time.

We assume an energy conversion device (say, a car) that consumes final energy[4] (say, gasoline) at a rate $\dot{E}^\circ$ (in MJ/year).[5] The final energy is available at price p_E (in \$/MJ). The original energy conversion device provides a rate of energy service $\dot{q}_s^\circ$ (in vehicle-km/year) with final-to-service efficiency η° (in vehicle-km/MJ). An energy efficiency upgrade (EEU) increases final-to-service efficiency[6] such that $\tilde{\eta} > \eta^\circ$, possibly at an increased cost to emplace (capital cost), maintain, and dispose of the device such that $C_{cap}^\circ < \tilde{C}_{cap}$ and $\dot{C}_{md}^\circ < \dot{\tilde{C}}_{md}$. (This is not a costless EEU.) As final-to-service efficiency increases ($\eta^\circ < \tilde{\eta}$), the price of the energy service declines ($p_s^\circ > \tilde{p}_s$). The final energy price (p_E) is assumed exogenous ($p_E^\circ = p_E^* = \hat{p}_E = \bar{p}_E = \tilde{p}_E$), so the final energy purchaser (the device owner) is a price taker.[7] Initially, the device owner spends income ($\dot{M}^\circ$) on final energy for the device ($\dot{C}_s^\circ = p_E \dot{E}_s^\circ$), annualized capital costs for the device ($\dot{C}_{cap}^\circ$), annualized

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effects, output effects, energy market effects, and composition effects. (We place the embodied energy effect at the microeconomic scale.) Santarius (2016) and Lange et al. (2021) introduce meso (i.e., sectoral) level rebound between the micro and macro levels. van den Bergh (2011) distinguishes 14 types of rebound, providing, perhaps, the greatest complexity.

[3]Note that the vocabulary and mathematical notation for rebound effects is important; Fig. 1 and Appendix A provide guides to notational elements used throughout this paper, including symbols, Greek letters, abbreviations, decorations, and subscripts. The notational elements can be mixed to provide a rich and expressive "language" of energy rebound. In several places, we use colored backgrounds on rebound effects for visual convenience.

[4]Conventionally, stages of the energy conversion chain are primary energy (e.g., coal, oil, natural gas, wind, and solar), final energy (e.g., electricity and refined petroleum), useful energy (e.g., heat, light, and mechanical drive), and energy services (e.g., transport, illumination, and space heating).

[5]We use "rate" to indicate any quantity measured per unit time. None of the rates in this paper indicate %/yr changes. Symbolically, rates are identified by a single dot above the symbol, a convention adopted from the engineering literature where, e.g., $\dot{m}$ often indicates a mass flow rate in kg/s, $\dot{E}$ often indicates an energy flow rate in MJ/yr, and $\dot{x}$ often indicates a velocity in m/s. The overdot is an important way to maintain clarity of nomenclature in this paper. For example, E is a quantity of energy in, say, MJ, while $\dot{E}$ is a rate of energy in, say, MJ/yr. We later annualize capital costs (in \$) and embodied energy (in MJ) to create cost rates ($\dot{C}$ in \$/yr) and embodied energy rates ($\dot{E}_{emb}$ in MJ/yr).

[6]Note that energy service efficiency (η) improves between the original ($\circ$) and post-emplacement ($*$) stages of Fig. 1, remaining constant thereafter. Thus, $\eta^\circ < \eta^* = \hat{\eta} = \bar{\eta} = \tilde{\eta}$, as shown in Table B.1. We refer to all post-emplacement efficiencies (η^*, $\hat{\eta}$, $\bar{\eta}$, and $\tilde{\eta}$) as $\tilde{\eta}$ to match the nomenclature of Borenstein (2015). When convenient, the same approach to nomenclature is taken with other quantities such as the capital cost rate ($\dot{C}_{cap}$) and maintenance and disposal cost rate ($\dot{C}_{md}$).

[7]Relaxing the exogenous final energy price assumption would require a general equilibrium model that is beyond the scope of this paper. Rebound due to energy price change is included in the macro effect.

costs for maintenance and disposal of the device ($\dot{C}_{md}^\circ$), and other goods and services ($\dot{C}_o^\circ$). The budget constraint for the device owner is

$$\dot{M}^\circ = \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ + \cancelto{0}{\dot{N}^\circ} \ , \tag{1}$$

where net savings prior to the EEU ($\dot{N}^\circ$) is zero, by definition.

Later (Sections 2.4.1–2.4.4), we walk through four rebound effects, deriving rebound expressions for each. But first we define rebound relationships (Section 2.2), and show typical energy and cost relationships (Section 2.3). In developing the framework, we endeavor to bring clarity by providing sufficient detail to assist energy analysts to understand the economics and economists to understand the energy analysis.

2.2 Rebound relationships

Energy rebound is defined as

$$Re \equiv 1 - \frac{\text{actual final energy savings rate}}{\text{expected final energy savings rate}} \ , \tag{2}$$

where both actual and expected final energy savings rates are in MJ/year and expected positive. Final energy "takeback" rate is defined as expected final energy savings rate less actual final energy savings rate. Note that the takeback rate can be negative, indicating that the actual final energy savings rate is greater than the expected final energy savings rate, a condition called hyperconservation. Thus,

$$Re = 1 - \frac{\text{expected final energy savings rate} - \text{takeback rate}}{\text{expected final energy savings rate}} \ . \tag{3}$$

Simplifying gives

$$Re = \frac{\text{takeback rate}}{\text{expected final energy savings rate}} \ . \tag{4}$$

Energy rebound can be given by either Eq. (2) or Eq. (4).

We define rebound at the final energy stage of the energy conversion chain, because that is the point of energy purchase by the device owner. To simplify derivations, we choose not to apply final-to-primary energy multipliers to final energy rates in the numerators and denominators of rebound expressions derived from Eqs. (2) and (4); they divide out anyway. Henceforth, we drop the adjective "final" from the noun "energy," unless there is reason to indicate a specific stage of the energy conversion chain.

2.3 Typical energy and cost relationships

With the rebound notation of Appendix A, four typical relationships emerge. First, the consumption rate of the energy service ($\dot{q}_s$) is the product of final-to-service efficiency (η) and the rate of energy consumption by the energy conversion device ($\dot{E}_s$). Typical units for automotive transport and illumination (the examples in Part II) are shown beneath each equation.[8]

[8]Note that "pass" is short for "passenger," and "lm" is the SI notation for the lumen, a unit of lighting energy rate.

$$\dot{q}_s = \eta \dot{E}_s \tag{5}$$
$$[\text{pass-km/yr}] = [\text{pass-km/MJ}][\text{MJ/yr}]$$
$$[\text{lm-hr/yr}] = [\text{lm-hr/MJ}][\text{MJ/yr}]$$

Second, the energy service price (p_s) is the ratio of energy price (p_E) to the final-to-service efficiency (η).

$$p_s = \frac{p_E}{\eta} \tag{6}$$
$$[\$/\text{pass-km}] = \frac{[\$/\text{MJ}]}{[\text{pass-km/MJ}]}$$
$$[\$/\text{lm-hr}] = \frac{[\$/\text{MJ}]}{[\text{lm-hr/MJ}]}$$

Third, energy service expenditure rates ($\dot{C}_s$) are the product of energy price (p_E) and device energy consumption rates ($\dot{E}_s$).

$$\dot{C}_s = p_E \dot{E}_s \tag{7}$$
$$[\$/\text{yr}] = [\$/\text{MJ}][\text{MJ/yr}]$$

Fourth, indirect energy rates ($\dot{E}_j$) for maintenance and disposal ($j = md$) and other goods expenditures ($j = o$) are the product of expenditures rates ($\dot{C}_j$) and energy intensity of the economy (I_E).

$$\dot{E}_j = \dot{C}_j I_E \tag{8}$$
$$[\text{MJ/yr}] = [\$/\text{yr}][\text{MJ}/\$]$$

2.4 Rebound effects

The four rebound effects (emplacement, substitution, income, and macro) shown in the rebound path graphs are discussed in subsections below. In each subsection, we define each effect and show mathematical expressions for rebound (Re) caused by each effect. Detailed derivations of all rebound expressions can be found in Appendix B. See, in particular, Tables B.3–B.6, which provide a parallel structure for energy and financial accounting across all rebound effects. We begin with the emplacement effect.

2.4.1 Emplacement effect

The emplacement effect accounts for performance changes of the device due to the fact that a higher-efficiency device has been put in service (and will need to be decommissioned at a later date); behavior changes are addressed later, in the substitution and income effects.

Direct emplacement effects (Re_{dempl}) The direct emplacement effects of the EEU include device energy savings ($\dot{S}_{dev}$) and device energy cost savings ($\Delta \dot{C}_s^*$). The indirect effects of EEU emplacement are (i) changes in the embodied energy rate ($\Delta \dot{E}_{emb}^*$), (ii) changes in the capital expenditure rate ($\Delta \dot{C}_{cap}^*$), and (iii) changes in the maintenance and disposal energy and expenditure rates ($\Delta \dot{E}_{md}^*$ and $\Delta \dot{C}_{md}^*$).

The rate of energy savings due to the direct emplacement effect ($\dot{S}_{dev}$) is given by

$$\dot{S}_{dev} \equiv \dot{E}_s^{\circ} - \dot{E}_s^{*} = -\Delta \dot{E}_s^{*} \; . \tag{9}$$

$\dot{S}_{dev}$ can be rewritten conveniently as

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^{\circ}} - 1 \right) \frac{\eta^{\circ}}{\tilde{\eta}} \dot{E}_s^{\circ} \; . \tag{10}$$

(See Appendix B.3.1 for the derivation.)

Because behavior changes are not considered in the direct emplacement effect, actual and expected energy savings rates are identical, and there is no takeback. By definition, then, the direct emplacement effect causes no rebound. Thus,

$$Re_{dempl} = 0 \; . \tag{11}$$

Although the direct emplacement effect does not cause rebound, indirect emplacement effects may indeed cause rebound. Indirect emplacement effects account for the life cycle of the energy conversion device, including energy embodied by manufacturing processes (subscript *emb*) and maintenance and disposal activities (subscript *md*).

Embodied energy effects (Re_{emb}) One of the unique features of this framework is that independent analyses of embodied energy and capital costs of the EEU are expected. We note that the different terms (embodied energy rate, $\dot{E}_{emb}$, and capital cost rate, $\dot{C}_{cap}$) might seem to imply different processes, but they actually refer to the same emplacement effect. Purchasing an upgraded device (which likely leads to $\dot{C}_{cap}^{\circ} \neq \dot{C}_{cap}^{*}$) will likely mean a changed embodied energy rate ($\dot{E}_{emb}^{\circ} \neq \dot{E}_{emb}^{*}$) to provide the same energy service. Our names for these aspects of rebound (embodied energy and capital cost) reflect common usage in the energy and economics fields, respectively.

Consistent with the energy analysis literature, we define embodied energy to be the sum of all energy consumed in the production of the energy conversion device, all the way back to resource extraction.[9] Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. No energy is embodied in the device while in service. The EEU causes the embodied energy of the energy conversion device to change from E_{emb}° to E_{emb}^{*}.

For simplicity, we spread all embodied energy over the lifetime of the device to provide a constant embodied energy rate ($\dot{E}_{emb}$). (We later take the same approach to capital costs ($\dot{C}_{cap}$) and maintenance and disposal costs ($\dot{C}_{md}$).) A justification for spreading embodied energy purchase costs comes from considering device replacements by many consumers across several years. In the aggregate, evenly spaced (in time) replacements work out to the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices without discounting (t_{life}° and t_{life}^{*}, respectively) to obtain embodied energy rates, such that $\dot{E}_{emb}^{\circ} = E_{emb}^{\circ}/t_{life}^{\circ}$ and $\dot{E}_{emb}^{*} = E_{emb}^{*}/t_{life}^{*}$. The change in embodied final energy due to the EEU (expressed as a rate) is given by $\Delta\dot{E}_{emb}^{*} = \dot{E}_{emb}^{*} - \dot{E}_{emb}^{\circ}$. The expression for embodied energy rebound is

$$Re_{emb} = \frac{\left(\frac{E_{emb}^{*}}{E_{emb}^{\circ}} \frac{t_{life}^{\circ}}{t_{life}^{*}} - 1 \right) \dot{E}_{emb}^{\circ}}{\dot{S}_{dev}} \; . \tag{12}$$

[9]We take an energy approach here, consistent with the literature on energy rebound. One could use an alternative quantification of energy, such as exergy, the work potential of energy (Sciubba & Wall, 2007) or emergy, the solar content of energy (Brown & Herendeen, 1996). Doing so is left as an exercise for the reader.

(See Appendix B.3.2 for details of the derivation.)

Embodied energy rebound (Re_{emb}) can be either positive or negative, depending on the sign of the term $(E^*_{emb}/E^\circ_{emb})(t^\circ_{life}/t^*_{life})-1$. Rising energy efficiency can be associated with increased device complexity, additional energy consumption in manufacturing, and more embodied energy, such that $E^\circ_{emb} < E^*_{emb}$ and $Re_{emb} > 0$, all other things being equal. However, if the upgraded device has longer life than the original device ($t^*_{life} > t^\circ_{life}$), $\dot{E}^*_{emb} - \dot{E}^\circ_{emb}$ could be negative, meaning that the upgraded device has a lower embodied energy rate than the original device.

Maintenance and disposal effects (Re_{md}) In addition to embodied energy, indirect emplacement effect rebound accounts for energy demanded by maintenance and disposal (md) activities. Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}^\circ_m$). Disposal costs (e.g., C°_d) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal costs across the lifetime of the original and upgraded devices (t°_{life} and t^*_{life}, respectively) to form expenditure rates such that $\dot{C}^\circ_{md} = \dot{C}^\circ_m + C^\circ_d/t^\circ_{life}$ and $\dot{C}^*_{md} = \dot{C}^*_m + C^*_d/t^*_{life}$.

For simplicity, we assume that maintenance and disposal expenditures imply energy consumption elsewhere in the economy at its overall energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in maintenance and disposal expenditures is given by $\Delta\dot{C}^*_{md} I_E = (\dot{C}^*_{md} - \dot{C}^\circ_{md}) I_E$. Rebound from maintenance and disposal activities is given by

$$Re_{md} = \frac{\left(\frac{\dot{C}^*_{md}}{\dot{C}^\circ_{md}} - 1\right)\dot{C}^\circ_{md} I_E}{\dot{S}_{dev}} \ . \tag{13}$$

(See Appendix B.3.2 for details of the derivation.)

2.4.2 Substitution effect

Neoclassical consumer theory decomposes price-induced behavior change into (i) substituting energy service consumption for other goods consumption due to the lower post-EEU price of the energy service (the substitution effect) and (ii) spending the higher real income (the income effect).[10] This section develops mathematical expressions for substitution effect rebound (Re_{sub}), thereby accepting the standard neoclassical microeconomic assumptions about consumer behavior. (The next section addresses income effect rebound, Re_{inc}.) The substitution effect alters compensated demand, which is the demand for the expenditure-minimizing consumption bundle that maintains utility at the pre-EEU level, given the new prices. Compensated demand is a technical term for a thought experiment from welfare economics: the device owner's budget is altered so that the owner is "compensated" for the change in price while maintaining the same level of utility as before. The change in the budget is called "compensating variation" (CV). The substitution effect involves (i) an increase in consumption of the energy service, the direct substitution effect (subscript $dsub$) and (ii) a decrease in consumption of other goods, the indirect substitution effect (subscript $isub$). Thus, two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}).

After emplacement of the more efficient device (but before the substitution effect), the price of the energy service decreases ($p^\circ_s > p^*_s$). ~~This compensating variation translates into a lower budget constraint~~ After compensating variation, consumption at the new prices yields utility at the same

[10] For the original development of the decomposition see Slutsky (1915) and Allen (1936). For a modern introduction see Nicholson & Snyder (2017).

level as prior to the EEU ~~and minimizing expenditure~~, by consuming more of the now-lower-cost energy service and less of the now-relatively-more-expensive other goods.

~~Rebound from the substitution effect is typically decomposed into indirect (the decrease in other goods consumption) and direct (the increase in energy service consumption) components.~~ An approximate utility model is often used in the literature (e.g., see Borenstein (2015, p. 17, footnote 43)) for determining post-substitution effect consumption and therefore Re_{dsub} and Re_{isub}. The approximate utility model assumes that the compensated energy service price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$) are constant along an indifference curve. Whether that is true depends on the utility function used. Constant price elasticities (as in the approximate utility model) are approximations that are in general only applicable to marginal price changes. Appendix B.3.3 contains details of the approximate utility model.

Here, we present an exact utility model that allows the compensated energy service price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$) to vary along an indifference curve, allowing analysis of non-marginal energy service price changes ($p_s^\circ \gg p_s^*$). We employ a fully specified CES utility model, which allows the direct calculation of the utility-maximizing consumption bundle for any constraint. The utility model describes the device owner's behavior as

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right)^{\rho} + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_o}{\dot{C}_o^\circ} \right)^{\rho} \right]^{(1/\rho)} . \tag{14}$$

The device owner's utility rate (relative to the original condition, $\dot{u}/\dot{u}^\circ$) is determined by the consumption rate of the energy service ($\dot{q}_s$) and the consumption rate of other goods and services ($\dot{C}_o$). The share parameter ($f_{\dot{C}_s}^\circ$) between $\dot{q}_s$ and $\dot{C}_o$ is taken from the original (pre-EEU) consumption basket. The exponent ρ is calculated from the (constant) elasticity of substitution (σ) as $\rho \equiv (\sigma - 1)/\sigma$. All quantities are normalized to pre-EEU values so that the cost share of other goods can be used straightforwardly in empirical applications rather than having to construct quantity and price indices. The normalized specification is commonly used in empirical CES *production* function applications (Klump et al., 2012; Temple, 2012; Gechert et al., 2021). See Appendix C for further details of the CES utility model.

Direct substitution effect rebound (Re_{dsub}) is

$$Re_{dsub} = \frac{\Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} , \tag{15}$$

which can be rearranged to

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} . \tag{16}$$

Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} , \tag{17}$$

which can be rearranged to

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} . \tag{18}$$

To find the post-substitution effect point ($\wedge$), we solve for the location on the indifference curve where its slope is equal to the slope of the original expenditure line, assuming the CES utility model.[11] The results are

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} \tag{19}$$

and

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left[\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} . \tag{20}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an expression for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

Eq. (20) can be substituted directly into Eq. (18) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left[\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{22}$$

(See Appendix B.3.3 for details of the derivations of Eqs. (16), (18), and (19)–(22).)

2.4.3 Income effect

The monetary income rate of the device owner ($\dot{M}^\circ$) remains unchanged across the rebound effects, such that $\dot{M}^\circ = \dot{M}^* = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Thanks to the energy service price decline, real income rises, and freed cash from the EEU is given by Eq. (49) as $\dot{G} = p_E \dot{S}_{dev}$. Emplacement effect adjustments and compensating variation modify freed cash to leave the device owner with *net* savings ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (59). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables B.3 and B.4.

[11] Other utility models could be used. However, the Cobb-Douglas utility model is inappropriate for this framework, because it assumes that the sum of substitution and income rebound is 100% *always*. Regardless of the utility model, expressions for $\hat{\dot{q}}_s/\dot{q}_s^\circ$ and $\hat{\dot{C}}_o/\dot{C}_o^\circ$ must be determined and substituted into Eqs. 16 and 18, respectively.

Rebound from the income effect quantifies the rate of additional energy demand that arises when the energy conversion device owner spends net savings from the EEU.

Additional energy demand from the income effect is determined by several constraints. The income effect under utility maximization satisfies the budget constraint, so that net savings are zero after the income effect ($\bar{\dot{N}} = 0$). See Appendix D for a mathematical proof that the income preference equations below (Eqs. (23) and (27)) satisfy the budget constraint.

A second constraint is that net savings are spent completely on (i) additional spending on the energy service ($\hat{\dot{q}}_s < \bar{\dot{q}}_s$) and (ii) additional spending on other goods ($\hat{\dot{q}}_o < \bar{\dot{q}}_o$). This constraint is satisfied by construction below.

The proportions in which income-effect spending is allocated depends on preferences, which prescribe an income expansion path. The CES utility function allocates spending on the energy service and other goods in the same proportion as post-substitution-effect expenditures on the energy service and other goods, due to its homotheticity. This constraint is satisfied by construction below, particularly via an effective income term ($\hat{\dot{M}}'$). However, this framework could accommodate non-homothetic preferences for spending across the income effect (turning the income expansion path into a more general curve).

We next show expressions for direct and indirect income effect rebound.

Direct income effect (Re_{dinc}) The income elasticity of energy service demand ($\epsilon_{\dot{q}_s,\dot{M}}$) quantifies the amount of net savings spent on more of the energy service ($\hat{\dot{q}}_s < \bar{\dot{q}}_s$). (See Appendix C for additional information about elasticities.) Spending of net savings on additional energy service leads to direct income effect rebound (Re_{dinc}).

The ratio of rates of energy service consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} . \tag{23}$$

Homotheticity means that $\epsilon_{\dot{q}_s,\dot{M}} = 1$.

Effective income ($\hat{\dot{M}}'$) is given by

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} . \tag{24}$$

For the purposes of the income effect, the effective income equation (Eq. (24)) adjusts income ($\dot{M}^\circ$) to account for sunk costs ($\dot{C}^*_{cap}$ and $\dot{C}^*_{md}$) and net savings ($\hat{\dot{N}}$).

Direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \bar{\dot{E}}_s}{\dot{S}_{dev}} . \tag{25}$$

(See Table B.5.) After substitution, rearranging, and canceling of terms (Appendix B.3.4), the expression for direct income rebound is

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left\{ f^\circ_{\dot{C}_s} + (1 - f^\circ_{\dot{C}_s}) \left[\left(\frac{1 - f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \right) \frac{\tilde{p}_s \dot{q}^\circ_s}{\dot{C}^\circ_o} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} . \tag{26}$$

If there are no net savings after the substitution effect ($\hat{\dot{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected.[12]

Indirect income effect (Re_{iinc}) ~~But~~ not all net savings ($\hat{\dot{N}}$) are spent on more energy for the energy conversion device. The income elasticity of other goods demand ($\epsilon_{\dot{q}_o,\dot{M}}$) quantifies the amount of net savings spent on additional other goods ($\hat{\dot{q}}_o < \bar{\dot{q}}_o$). Spending of net savings on additional other goods and services leads to indirect income effect rebound (Re_{iinc}).

The ratio of rates of other goods consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} . \tag{27}$$

Under the assumption that prices of other goods are exogenous (see Appendix E), the ratio of rates of other goods consumption ($\bar{\dot{q}}_o/\hat{\dot{q}}_o$) is equal to the ratio of rates of other goods expenditures ($\bar{\dot{C}}_o/\hat{\dot{C}}_o$) such that

$$\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} . \tag{28}$$

Homotheticity means that $\epsilon_{\dot{q}_o,\dot{M}} = 1$. As shown in Table B.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}} . \tag{29}$$

After substitution, rearranging, and canceling of terms, the expression for indirect income rebound is

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left[\left(1 + f_{\dot{C}_s}^\circ \left\{\left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} . \tag{30}$$

(See Appendix B.3.4 for details of the derivation of direct and indirect income effect rebound.)

2.4.4 Macro effect

The previous rebound effects (emplacement effect, substitution effect, and income effect) occur at the microeconomic scale. However, changes at the microeconomic scale can have important implications at the macroeconomic scale, including via aggregate economic growth from the energy-augmenting productivity increase.

Turner (2013) cautions that when households see the productivity of their non-market activities increase, GDP remains unchanged. That may be true in the short run, but the question over longer periods is whether the higher productivity of household production of energy services does not affect also their capacity to contribute to economic growth. One channel could be time-saving. If the EEU saves time, then this time could be spent on more work or on increasing human capital (Sorrell &

[12] Zero net savings ($\hat{\dot{N}} = 0$) could occur if increases in the capital cost rate ($\Delta\dot{C}^*_{cap}$) and/or the maintenance and disposal cost rate ($\Delta\dot{C}^*_{md}$) consume all freed cash ($\dot{G}$) plus savings from the compensating variation.

Dimitropoulos, 2008). If the EEU saves money (but no time), then the freed cash could be spent on human capital-increasing activities or even be used to start a venture. In all cases, it would be rash to conclude that just because some EEUs lead to productivity increases not captured by GDP, it does not eventually lead to additional economic growth.[13]

Borenstein also addressed these macro effects from consumer behavior noting that "income effect rebound will be larger economy-wide than would be inferred from evaluating only the direct income gain from the end user's transaction" (Borenstein, 2015, p. 11) and likening it to the Keynesian macroeconomic multiplier. However, the dynamic macro rebound effect is not an autonomous expansion of expenditure in an otherwise unchanged economy (like the Keynesian multiplier). Rather, macroeconomic rebound is caused by an energy productivity improvement, a supply-side shock. After the EEU, it takes less energy (and therefore less energy cost) to generate the same economic activity, because energy efficiency has improved. That said, Borenstein is right to highlight that supply-side and demand-side effects both play a role as the consequences of the technology shock play themselves out. Furthermore, his approach has the advantage that it can be directly linked to our income effect and its consequences for macroeconomic rebound. Borenstein also notes that scaling from net savings ($\hat{\dot{N}}$) at the device level to productivity-driven growth at the macro level is unexplored territory.

Another novel contribution of this paper (in addition to the framework itself and rebound path graphs) is the first operationalization of the macro rebound multiplier idea. To operationalize the macro rebound effect, we scale the net savings gained by the device owner at the microeconomic scale ($\hat{\dot{N}}$) by a macro factor (k) that represents the economic activity generated by the infinite series of respending of net savings ($\hat{\dot{N}}$) throughout the economy.[14] The macro factor (k) can be likened to the total respending described by a marginal propensity to consume (MPC). k represents respending in the broader economy after the income effect has occurred and is not tied to any particular EEU or economic sector. $k \geq 0$ is expected. $k = 0$ means there is no dynamic effect resulting from the energy efficiency upgrade. $k > 0$ means that productivity-driven macroeconomic growth has occurred with consequent implications for additional energy consumption in the wider economy. The relationship between k and MPC is given by

$$k = \frac{1}{\frac{1}{MPC} - 1} \,. \tag{31}$$

(See Appendix F for the derivation of Eq. (31).)

A further advantage of using the MPC approach is that there are many estimates of its magnitude, though we stress again that MPC is a *representation* of the effect, while the cause is higher energy efficiency on the supply side. A recent review by Carroll et al. (2017) reports that most empirical estimates show MPC between 0.2 and 0.6, with the full range estimates spanning 0.0 to 0.9.

We assume as a first approximation (following Antal & van den Bergh (2014) and Borenstein (2015)) that macro effect respending implies energy consumption according to the average energy intensity of the economy (I_E). Macro rebound is given by

[13] To appreciate the difference, consider one case where an increased mileage leads to the household saving on energy per car trip. The household takes more trips (direct rebound), without effect on GDP. In the other case, the household buys the energy service (transport) directly from a taxi company. Here, the taxi company lowers the price but gains more customers, leading immediately to growth in inflation-adjusted, i.e. real, GDP. Yet, the physical change of more car trips is the same in both cases.

[14] The macro factor (k) appears unitless, but its units are actually \$ of economic growth created per \$ of net savings spent by the device owner in the income effect ($\hat{\dot{N}}$).

$$Re_{macro} = \frac{k\hat{\dot{N}}I_E}{\dot{S}_{dev}} \; . \tag{32}$$

(See Table B.6.) After some algebra (Appendix B.3.5), we arrive at an expression for macro effect rebound:

$$Re_{macro} = kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} \; . \tag{33}$$

2.5 Rebound sum

The sum of all rebound emerges from the four rebound effects (emplacement effect, substitution effect, income effect, and macro effect). Macro effect rebound (Re_{macro}, Eq. (33)) is expressed in terms of other rebound effects. (Derivation details can be found in Appendix B.3.6.) After algebra and canceling of terms, we find

$$\begin{aligned} Re_{tot} = Re_{emb} &- kRe_{cap} + (1-k)Re_{md} \\ &+ (1 - kp_E I_E)Re_{dsub} + (1-k)Re_{isub} \\ &+ Re_{dinc} + Re_{iinc} + kp_E I_E \; . \end{aligned} \tag{34}$$

3 Discussion

We developed above a ~~partial equilibrium~~ rebound framework for consumers. We note that many of its components are similar to those for a producer-sided framework due to the symmetry between neoclassical microeconomic producer and consumer theory. Partial equilibrium frameworks provide detailed assessment of individual EEUs with tractable, easy-to-understand mathematics. Partial equilibrium frameworks are easier to understand, in part, because they constrain price variation to the energy service only; all other prices remain constant. In our partial equilibrium framework, general equilibrium effects are captured by a simplified, one-dimensional, macroeconomic-scale rebound effect discussed in Section 2.4.4.

~~In contrast, general equilibrium frameworks allow prices of all goods and services in an economy to adjust to the EEU. Recent examples include Hart (2018), Lemoine (2020), Fullerton & Ta (2020), and Blackburn & Moreno-Cruz (2020). General equilibrium frameworks provide detail and precision on economy-wide price adjustments, but they give up specificity about individual device upgrades, make assumptions during calibration, and lose simplicity of exposition.~~

We are not the first to develop a partial equilibrium rebound analysis framework, so it is worthwhile to compare our framework with others for key features: analysis of all effects, locations, and scales; analysis in energy, expenditure, and consumption spaces; level of detail in the consumer preference model; allowance for non-marginal energy efficiency changes; and operationality. Only when all of the above characteristics are present can a comprehensive picture of rebound emerge. Table 2 shows our assessment of selected previous partial equilibrium frameworks (in columns) relative to the characteristics discussed above (in rows).

Table 2: Comparison among relevant partial equilibrium rebound analysis frameworks. Empty (white) circles indicate no treatment of a subject by a framework. Partially and fully filled circles indicate partial and comprehensive treatment of a subject by a framework.

	Nässén & Holmberg (2009)	Thomas & Azevedo (2013a,b)	Borenstein (2015)	Chan & Gillingham (2015)	Wang et al. (2021)	This paper (2023)
Effects, locations, and scales						
Direct emplacement effect	●	●	●	●	●	●
Capital cost and embodied energy effect	◒	◒	◒	◔	○	●
Maintenance and disposal effect	○	○	◒	○	○	●
Direct and indirect substitution effects	◒	◒	●	●	●	●
Direct and indirect income effects	◒	◒	●	●	●	●
Macro effect	○	○	◔	○	○	●
Other characteristics						
Presentation of energy, expenditure, and consumption spaces	◕	◕	◒	◕	◒	●
Detailed model of device owner behavior and preferences	◔	◒	◒	●	●	●
Non-marginal energy service price changes	○	○	○	○	○	●
Operationality	●	●	◒	○	○	●

Because all frameworks evaluate the expected decrease in direct energy consumption from the EEU, the "Direct emplacement effect" row contains ● in all columns. Three early papers (Nässén & Holmberg, 2009; Thomas & Azevedo, 2013a,b) estimate rebound quantitatively, earning high marks (●) in the "Operationality" row. Both motivate their frameworks at least partially with microeconomic theory (consumer preferences and substitution and income effects) but use simple linear demand functions in their empirical analyses. Thus, the connection between economic theory and empirics is tenuous, leading to intermediate ratings (◒ or less) in the "substitution effects," "income effects," and "Detailed model of consumer preferences" rows. More recently, Chan & Gillingham (2015) and Wang et al. (2021) anchor the rebound effect firmly in consumer theory, earning high ratings (●) in the "substitution effects," "income effects," and "Detailed model of consumer preferences" rows. They extend their frameworks to advanced topics such as multiple fuels, energy services, and nested utility functions with intermediate inputs. However, neither framework exhibits operationality, earning ○ in the last row of Table 2. In the middle of the table (and between the other studies in time), the framework by Borenstein (2015) touches on nearly all important characteristics. However, the Borenstein framework cannot separate substitution and income effects cleanly in empirical analysis, reverting to partial analyses of both, leading to a ◒ rating in the "Detailed model of consumer preferences" row.

No previous framework engages fully with either the differential financial effects or the differential energetic effects of the upfront purchase of the upgraded device, leading to low ratings across all previous frameworks in the "Capital cost and embodied energy effect" row. In fact, except for Nässén & Holmberg (2009), no framework engages with capital costs, although all note its importance. (Nässén & Holmberg note that capital costs and embodied energy can have very strong effects on rebound.) Thomas & Azevedo (2013a,b) provide the only framework that traces embodied energy effects of every consumer good using input-output methods, but they do not analyze embodied energy of the upgraded device. Borenstein (2015) notes the embodied energy of the upgraded device and the embodied energy of other goods but does not integrate embodied energy or financing costs into the framework for empirical analysis. Borenstein is the only author to treat the financial side of embodied energy or maintenance and disposal effects. Borenstein (2015) postulates the macro effect, but does not develop the idea into an operational theory, earning ◒ in the "Macro effect" row. No other frameworks even discuss the link between macro and micro rebound effects, leading to ○ in the "Macro effect" row for all other previous frameworks. Finally, all previous frameworks assume constant price elasticities and implicitly "marginal" or small improvements in efficiency, excluding the analysis of important non-incremental upgrades where price elasticities are likely to vary. Therefore, all previous frameworks earn ○ in the "Non-marginal energy service price changes" row.

Table 2 shows that previous frameworks contain many key pieces, providing a substantial base from which to develop a comprehensive and operational rebound analysis framework. A left-to-right reading of the table demonstrates that previous frameworks start from microeconomic consumer theory and move towards more rigorous theoretical treatment over time, with recent frameworks making important advanced theoretical contributions at the expense of operationality. In the end, no previous rebound analysis framework combines all rebound effects, locations, and scales across energy, expenditure, and consumption spaces with a detailed model of consumer preferences and non-marginal energy service price changes in an operational manner for the simplest case (understandable across disciplines) of a single fuel and a single energy service. In particular, non-marginal price changes and the macro effect require conceptual development and operationalization. This paper aims addresses the gaps in Table 2, completing the "This paper" column with filled circles (●).

4 Conclusions

In this paper (Part I), we developed a partial equilibrium rebound analysis framework that combines all rebound effects, locations, and scales across energy, expenditure, and consumption spaces with a detailed model of consumer preferences and non-marginal energy service price changes in an operational manner for the simplest case of a single fuel and a single energy service. With careful explication of rebound effects and clear derivation of rebound expressions, we advance the analytical foundations for empirical estimates of rebound and facilitate interdisciplinary understanding of rebound phenomena toward the goal of enabling of more robust rebound estimates for sound energy and climate policy.

Future work could be pursued in several areas. (i) Other utility models (besides the CES utility model, but not a Cobb-Douglas utility model) could be explored for the substitution effect. (ii) This framework could be extended to producer-sided energy rebound effects. (iii) This framework could be embedded in energy-economy models to better include rebound effects in discussions of macro energy modeling, energy policy, and CO_2 emissions mitigation.

In Part II of this paper, we attempt to bring further clarity to rebound analysis in three ways. First, we develop a way to visualize rebound effects at all locations (direct and indirect) in energy, expenditure, and consumption spaces. Second, we apply the partial equilibrium framework to two EEUs: an upgraded electric lamp and an upgraded automobile. Finally, we provide rebound estimates for the two examples.

Competing interests

Declarations of interest: none.

Author contributions

	MKH	GS	PEB
Conceptualization	●	●	
Methodology	●	●	●
Software	●		
Validation	●		●
Formal analysis	●	●	●
Investigation	●	●	●
Resources	●		
Data curation			●
Writing–original draft	●	●	●
Writing–review & editing	●	●	●
Visualization	●	●	
Supervision	●		
Project administration	●		
Funding acquisition			

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References

Allen, R. G. D. (1936). Professor Slutsky's Theory Choice of Consumers' Choice. Review of Economic Studies, 3(2), 120–129.

Antal, M., & van den Bergh, J. C. (2014). Re-spending rebound: A macro-level assessment for OECD countries and emerging economies. Energy Policy, 68, 585–590.

Barker, T., Dagoumas, A., & Rubin, J. (2009). The macroeconomic rebound effect and the world economy. Energy Efficiency, 2, 411–427.

Birol, F., & Keppler, J. H. (2000). Prices, technology development, and the rebound e!ect. Energy Policy, 28, 457–469.

Blackburn, C. J., & Moreno-Cruz, J. (2020). Energy efficiency in general equilibrium with input-output linkages. BEA Working Paper Series WP2020-1, Bureau of Economic Analysis.
URL `https://www.bea.gov/index.php/system/files/papers/WP2020-1.pdf`

Borenstein, S. (2015). A microeconomic framework for evaluating energy efficiency rebound and some implications. The Energy Journal, 36(1), 1–21.

Brockway, P. E., Saunders, H., Heun, M. K., Foxon, T. J., Steinberger, J. K., Barrett, J. R., & Sorrell, S. (2017). Energy rebound as a potential threat to a low-carbon future: Findings from a new exergy-based national-level rebound approach. Energies, 10(51), 1–24.

Brockway, P. E., Sorrell, S., Semieniuk, G., Heun, M. K., & Court, V. (2021). Energy efficiency and economy-wide rebound effects: A review of the evidence and its implications. Renewable and Sustainable Energy Reviews, 141(110781), 1–20.

Brookes, L. (1979). A low energy strategy for the UK. Atom, 269(73-78).

Brookes, L. (1990). The greenhouse effect: the fallacies in the energy efficiency solution. Energy Policy, 18(2), 199–201.

Brown, M., & Herendeen, R. (1996). Embodied Energy Analysis and EMERGY Analysis: a Comparative View. Ecological Economics, 19, 219–235.

Carroll, C., Slacalek, J., Tokuoka, K., & White, M. N. (2017). The distribution of wealth and the marginal propensity to consume. Quantitative Economics, 8(3), 977–1020.

Chan, N. W., & Gillingham, K. (2015). The microeconomic theory of the rebound effect and its welfare implications. Journal of the Association of Environmental and Resource Economists, 2(1), 133–159.

Fullerton, D., & Ta, C. L. (2020). Costs of energy efficiency mandates can reverse the sign of rebound. Journal of Public Economics, 188, 104225.

Gechert, S., Havranek, T., Irsova, Z., & Kolcunova, D. (2021). Measuring capital-labor substitution: The importance of method choices and publication bias. Review of Economic Dynamics.
URL `https://www.sciencedirect.com/science/article/pii/S1094202521000387`

Gillingham, K., Kotchen, M. J., Rapson, D. S., & Wagner, G. (2013). The rebound effect is overplayed. Nature, 493.

Gillingham, K. T. (2020). The rebound effect and the proposed rollback of U.S. fuel economy standards. Review of Environmental Economics and Policy, 14(1), 136–142.

Gørtz, E. (1977). An identity between price elasticities and the elasticity of substitution of the utility function. The Scandinavian Journal of Economics, 79(4), 497–499.

Greening, L. A., Greene, D. L., & Difiglio, C. (2000). Energy efficiency and consumption—the rebound effect—a survey. Energy policy, 28(6-7), 389–401.

Grubb, M. (1990). Energy efficiency and economic fallacies. Energy Policy, 18(8), 783–785.

Grubb, M. (1992). Reply to Brookes. Energy Policy, (May), 392–393.

Haberl, H., Wiedenhofer, D., Virág, D., Kalt, G., Plank, B., Brockway, P., Fishman, T., Hausknost, D., Krausmann, F., Leon-Gruchalski, B., Mayer, A., Pichler, M., Schaffartzik, A., Sousa, T., Streeck, J., & Creutzig, F. (2020). A systematic review of the evidence on decoupling of GDP, resource use and GHG emissions, part II: synthesizing the insights. Environmental Research Letters, 15(065003), 1–42.

Hart, R. (2018). Rebound, directed technological change, and aggregate demand for energy. Journal of Environmental Economics and Management, 89, 218–234.

Hicks, J. R., & Allen, R. G. D. (1934). A reconsideration of the theory of value. Part II. A mathematical theory of individual demand functions. Economica, 1(2), 196–219.

Jenkins, J., Nordhaus, T., & Shellenberger, M. (2011). Energy emergence: Rebound and backfire as emergent phenomena. Tech. rep., Breakthrough Institute, Oakland, California, USA.
URL `https://s3.us-east-2.amazonaws.com/uploads.thebreakthrough.org/legacy/blog/Energy{_}Emergence.pdf`

Jevons, W. S. (1865). The Coal Question: An Inquiry Concerning the Progress of the Nation and the Probable Exhaustion of our Coal Mines. London: Macmillan.

Khazzoom, J. D. (1980). Economic implications of mandated efficiency in standards for household appliances. The Energy Journal, 1(4).

Klump, R., Mcadam, P., & Willman, A. (2012). The normalized CES production function: Theory and empirics. Journal of Economic Surveys, 26(5), 769–799.
Lange, S., Kern, F., Peuckert, J., & Santarius, T. (2021). The Jevons paradox unravelled: A multi-level typology of rebound effects and mechanisms. Energy Research and Social Science, 74, 101982.
Lemoine, D. (2020). General equilibrium rebound from energy efficiency innovation. European Economic Review, 125, 1–20.
Lovins, A. B. (1988). Energy saving resulting from the adoption of more efficient appliances: Another view. The Energy Journal, (pp. 155–162).
Madlener, R., & Turner, K. (2016). After 35 Years of Rebound Research in Economics: Where Do We Stand?, chap. 1, (pp. 17–36). Rethinking Climate and Energy Policies New Perspectives on the Rebound Phenomenon. Cham, Switzerland: Springer.
Nässén, J., & Holmberg, J. (2009). Quantifying the rebound effects of energy efficiency improvements and energy conserving behaviour in Sweden. Energy Efficiency, 2(3), 221–231.
Nicholson, W., & Snyder, C. (2017). Microeconomic Theory: Basic Principles & Extensions. Boston: Cengage Learning.
Paoli, L., & Cullen, J. (2020). Technical limits for energy conversion efficiency. Energy, 192, 1–12.
Parkes, J. (1838). On the evaporation of water from steam boilers. Transactions of the Institution of Civil Engineers, 2(1), 161–179.
Santarius, T. (2016). Investigating meso-economic rebound effects: production-side effects and feedback loops between the micro and macro level. Journal of Cleaner Production, 134, 406–413.
Saunders, H. D. (2015). Recent evidence for large rebound: Elucidating the drivers and their implications for climate change models. The Energy Journal, 36(1), 23–48.
Saunders, H. D., Roy, J., Azevedo, I. M., Chakravart, D., Dasgupta, S., de la Rue du Can, S., Druckman, A., Fouquet, R., Grubb, M., Lin, B., Lowe, R., Madlener, R., McCoy, D. M., Mundaca, L., Oreszczyn, T., Sorrell, S., Stern, D., Tanaka, K., & Wei, T. (2021). Energy efficiency: What has research delivered in the last 40 years? Annual Review of Environment and Resources, 46, 135–165.
Sciubba, E., & Wall, G. (2007). A brief commented history of exergy from the beginnings to 2004. International Journal of Thermodynamics, 10(1), 1–26.
Slutsky, E. (1915). Sulla teoria del bilancio del consumatore. Giornale degli Economisti e Rivista di Statistica, 53(1), 1–26.
Sorrell, S. (2009). Jevons' paradox revisited: The evidence for backfire from improved energy efficiency. Energy Policy, 37(4), 1456–1469.
Sorrell, S., & Dimitropoulos, J. (2008). The rebound effect: Microeconomic definitions, limitations and extensions. Ecological Economics, 65(3), 636–649.
Stern, D. I. (2020). How Large is the Economy-Wide Rebound Effect? Energy Policy, 147, 111870.
Temple, J. (2012). The calibration of CES production functions. Journal of Macroeconomics, 34, 294–303.
Thomas, B. A., & Azevedo, I. L. (2013a). Estimating direct and indirect rebound effects for U.S. households with input–output analysis. Part 1: Theoretical framework. Ecological Economics, 86, 199–210.
Thomas, B. A., & Azevedo, I. L. (2013b). Estimating direct and indirect rebound effects for U.S. households with input–output analysis. Part 2: Simulation. Ecological Economics, 86, 188–198.
Turner, K. (2013). "rebound" effects from increased energy efficiency: A time to pause and reflect. The Energy Journal, 34(4), 25–42. URL https://www.jstor.org/stable/41969250
van den Bergh, J. C. (2017). Rebound policy in the Paris agreement: instrument comparison and climate-club revenue offsets. Climate Policy, 17(6), 801–813.
van den Bergh, J. C. J. M. (2011). Energy conservation more effective with rebound policy. Environmental and Resource Economics, 48(1), 43–58.
Vivanco, D. F., Sala, S., & McDowall, W. (2018). Roadmap to rebound: How to address rebound effects from resource efficiency policy. Sustainability (Switzerland), 10(6), 1–17.
Walnum, H. J., Aall, C., & Løkke, S. (2014). Can rebound effects explain why sustainable mobility has not been achieved? Sustainability, 6(12), 9510–9537.
Wang, J., Yu, S., & Liu, T. (2021). A theoretical analysis of the direct rebound effect caused by energy efficiency improvement of private consumers. Economic Analysis and Policy, 69(145), 171–181.
Williams, C. W. (1840). The combustion of coal and the prevention of smoke: chemically and practically considered. London: J. Weale, 1st ed.

Appendices

A Nomenclature

Presentation of the comprehensive rebound analysis framework is aided by a nomenclature that describes energy stages and rebound effects, locations, and scales. Table A.1 shows symbols and abbreviations, their meanings, and example units. Table A.2 shows Greek letters, their meanings, and example units. Table A.3 shows abbreviations and acronyms. Table A.4 shows symbol decorations and their meanings. Table A.5 shows subscripts and their meanings.

Differences are indicated by the Greek letter Δ and always signify subtraction of a quantity at an earlier stage of Fig. 1 from the same quantity at a later stage of Fig. 1. E.g., $\Delta\bar{X} \equiv \bar{X} - \hat{X}$, and $\Delta\tilde{X} \equiv \tilde{X} - \bar{X}$. Lack of decoration on a difference term indicates a difference that spans all stages of Fig. 1. E.g., $\Delta X \equiv \tilde{X} - X^{\circ}$. ΔX is also the sum of differences across each stage in Fig. 1, as shown below.

Table A.1: Symbols and abbreviations.

Symbol	Meaning [example units]
a	a point in the emplacement effect on rebound path graphs or the share parameter in the CES utility model [–]
b	a point in the emplacement effect on rebound path graphs
C	cost [$]
c	a point in the substitution effect on rebound path graphs
d	a point in the income effect on rebound path graphs
E	final energy [MJ]
f	expenditure share [–]
G	freed cash [$]
I	energy intensity of economic activity [MJ/$]
k	macro factor [–]
M	income [$]
N	net savings [$]
p	price [$]
q	quantity [–]
Re	rebound [–]
S	energy cost savings [$]
t	energy conversion device lifetime [years]
u	utility [utils]

Table A.2: Greek letters.

Greek letter	Meaning [example units]
Δ	difference (later quantity less earlier quantity, see Fig. 1)
ϵ	elasticity [–]
$\epsilon_{\dot{q}_s,\dot{M}}$	income ($\dot{M}$) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,\dot{M}}$	income ($\dot{M}$) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s}$	uncompensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s}$	uncompensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s,c}$	compensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s,c}$	compensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
η	final-energy-to-service efficiency [vehicle-km/MJ]
σ	elasticity of substitution between the energy service ($\dot{q}_s^\circ$) and other goods ($\dot{q}_o^\circ$) [–]

Table A.3: Abbreviations.

Abbreviation	Meaning
APF	aggregate production function
CES	constant elasticity of substitution
CV	compensating variation
EEU	energy efficiency upgrade
GDP	gross domestic product
MPC	marginal propensity to consume
mpg	miles per U.S. gallon
U.S.	United States

Table A.4: Decorations.

Decoration	Meaning [example units]
X°	X originally (before the emplacement effect)
X^{*}	X after the emplacement effect (before the substitution effect)
$\hat{X}$	X after the substitution effect (before the income effect)
$\bar{X}$	X after the income effect (before the macro effect)
$\tilde{X}$	X after the macro effect
$\dot{X}$	rate of X [units of X/year]
M'	effective income [$]

Table A.5: Subscripts.

Subscript	Meaning
0	quantity at an initial time
1	a specific point on a consumption path graph
c	compensated
cap	capital costs
dev	device
$dempl$	direct emplacement effect
d	disposal
$dinc$	direct income effect
dir	direct effects (at the energy conversion device)
$dsub$	direct substitution effect
emb	embodied
i	index for other goods purchased in the economy
j	one of cap, md, or o in Eq. (8)
$iempl$	indirect emplacement effects
$iinc$	indirect income effect
$indir$	indirect effects (beyond the energy conversion device)
$isub$	indirect substitution effect
$life$	lifetime
m	maintenance
md	maintenance and disposal costs
o	other expenditures (besides energy) by the device owner
own	ownership duration
$macro$	macro effect
s	service stage of the energy conversion chain
tot	sum of all rebound effects in the framework

$$
\begin{aligned}
\Delta X &= \Delta\tilde{X} + \Delta\bar{X} + \Delta\hat{X} + \Delta X^* \\
\Delta X &= (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^*) + (X^* - X^\circ) \\
\Delta X &= (\tilde{X} - \cancel{\bar{X}}) + (\cancel{\bar{X}} - \cancel{\hat{X}}) + (\cancel{\hat{X}} - \cancel{X^*}) + (\cancel{X^*} - X^\circ) \\
\Delta X &= \tilde{X} - X^\circ
\end{aligned} \tag{35}
$$

B Derivation of the rebound analysis framework

This appendix provides a detailed derivation of the comprehensive rebound analysis framework, beginning with relationships for each rebound effect.

B.1 Relationships for rebound effects

For each energy rebound effect in Fig. 1, energy and financial analysis must be performed. The purposes of the analyses are to determine for each effect (i) an expression for energy rebound (Re) for the effect and (ii) an equation for net savings ($\dot{N}$) remaining after the effect.

Analysis of each rebound effect involves a set of assumptions and constraints as shown in Table B.1. In Table B.1, relationships for emplacement effect embodied energy rates ($\dot{E}^\circ_{emb}$ and $\dot{E}^*_{emb}$), capital expenditure rates ($\dot{C}^\circ_{cap}$ and $\dot{C}^*_{cap}$), and maintenance and disposal expenditure rates ($\dot{C}^\circ_{md}$ and $\dot{C}^*_{md}$) are typical, and inequalities could switch direction for a specific EEU. Macro effect relationships are given for a single device only. If the EEU is deployed at scale across the economy, the energy service consumption rate ($\tilde{\dot{q}}_s$), device energy consumption rate ($\tilde{\dot{E}}_s$), embodied energy rate ($\tilde{\dot{E}}_{emb}$), capital expenditure rate ($\tilde{\dot{C}}_{cap}$), and maintenance and disposal expenditure rate ($\tilde{\dot{C}}_{md}$) will all increase in proportion to the number of devices emplaced.

Table B.1: Assumptions and constraints for analysis of rebound effects.

Parameter	Emplacement Effect	Substitution Effect	Income Effect	Macro Effect
Energy price	$p_E^\circ = p_E^*$	$p_E^* = \hat{p}_E$	$\hat{p}_E = \bar{p}_E$	$\bar{p}_E = \tilde{p}_E$
Energy service efficiency	$\eta^\circ < \eta^*$	$\eta^* = \hat{\eta}$	$\hat{\eta} = \bar{\eta}$	$\bar{\eta} = \tilde{\eta}$
Energy service price	$p_s^\circ > p_s^*$	$p_s^* = \hat{p}_s$	$\hat{p}_s = \bar{p}_s$	$\bar{p}_s = \tilde{p}_s$
Other goods price	$p_o^\circ = p_o^*$	$p_o^* = \hat{p}_o$	$\hat{p}_o = \bar{p}_o$	$\bar{p}_o = \tilde{p}_o$
Energy service consumption rate	$\dot{q}_s^\circ = \dot{q}_s^*$	$\dot{q}_s^* < \dot{\hat{q}}_s$	$\dot{\hat{q}}_s < \dot{\bar{q}}_s$	$\dot{\bar{q}}_s = \dot{\tilde{q}}_s$
Other goods consumption rate	$\dot{q}_o^\circ = \dot{q}_o^*$	$\dot{q}_o^* > \dot{\hat{q}}_o$	$\dot{\hat{q}}_o < \dot{\bar{q}}_o$	$\dot{\bar{q}}_o = \dot{\tilde{q}}_o$
Device energy consumption rate	$\dot{E}_s^\circ > \dot{E}_s^*$	$\dot{E}_s^* < \dot{\hat{E}}_s$	$\dot{\hat{E}}_s < \dot{\bar{E}}_s$	$\dot{\bar{E}}_s = \dot{\tilde{E}}_s$
Embodied energy rate	$\dot{E}_{emb}^\circ < \dot{E}_{emb}^*$	$\dot{E}_{emb}^* = \dot{\hat{E}}_{emb}$	$\dot{\hat{E}}_{emb} = \dot{\bar{E}}_{emb}$	$\dot{\bar{E}}_{emb} = \dot{\tilde{E}}_{emb}$
Capital expenditure rate	$\dot{C}_{cap}^\circ < \dot{C}_{cap}^*$	$\dot{C}_{cap}^* = \dot{\hat{C}}_{cap}$	$\dot{\hat{C}}_{cap} = \dot{\bar{C}}_{cap}$	$\dot{\bar{C}}_{cap} = \dot{\tilde{C}}_{cap}$
Maint. and disp. expenditure rate	$\dot{C}_{md}^\circ < \dot{C}_{md}^*$	$\dot{C}_{md}^* = \dot{\hat{C}}_{md}$	$\dot{\hat{C}}_{md} = \dot{\bar{C}}_{md}$	$\dot{\bar{C}}_{md} = \dot{\tilde{C}}_{md}$
Energy service expenditure rate	$\dot{C}_s^\circ > \dot{C}_s^*$	$\dot{C}_s^* < \dot{\hat{C}}_s$	$\dot{\hat{C}}_s < \dot{\bar{C}}_s$	$\dot{\bar{C}}_s = \dot{\tilde{C}}_s$
Other goods expenditure rate	$\dot{C}_o^\circ = \dot{C}_o^*$	$\dot{C}_o^* > \dot{\hat{C}}_o$	$\dot{\hat{C}}_o < \dot{\bar{C}}_o$	$\dot{\bar{C}}_o = \dot{\tilde{C}}_o$
Income	$\dot{M}^\circ = \dot{M}^*$	$\dot{M}^* = \dot{\hat{M}}$	$\dot{\hat{M}} = \dot{\bar{M}}$	$\dot{\bar{M}} = \dot{\tilde{M}}$
Net savings	$0 = \dot{N}^\circ < \dot{N}^*$	$\dot{N}^* < \dot{\hat{N}}$	$\dot{\hat{N}} > \dot{\bar{N}} = 0$	$\dot{\bar{N}} = \dot{\tilde{N}} = 0$

Table B.2: Justification for zeroed terms in Tables B.3–B.6.

Zeroed term	Justification (from Table B.1).
$\cancelto{0}{\Delta \dot{C}_o^*}$	$\dot{C}_o^\circ = \dot{C}_o^*$ ($\dot{C}_o$ unchanged across emplacement effect.)
$\cancelto{0}{\dot{N}^\circ}$	$0 = \dot{N}^\circ$ (Net savings are zero prior to the EEU.)
$\cancelto{0}{\Delta \hat{\dot{E}}_{emb}}$	$\dot{E}_{emb}^* = \hat{\dot{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \hat{\dot{C}}_{md}}$	$\dot{C}_{md}^* = \hat{\dot{C}}_{md}$ ($\dot{C}_{md}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \bar{\dot{E}}_{emb}}$	$\hat{\dot{E}}_{emb} = \bar{\dot{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across income effect.)
$\cancelto{0}{\Delta \bar{\dot{C}}_{md}}$	$\hat{\dot{C}}_{md} = \bar{\dot{C}}_{md}$ ($\dot{C}_{md}$ unchanged across income effect.)
$\cancelto{0}{\bar{\dot{N}}}$	$\bar{\dot{N}} = 0$ (All net savings are spent in the income effect.)

B.2 Derivations

Derivations for rebound definitions and net savings equations are presented in Tables B.3–B.6, one for each rebound effect in Fig. 1. Energy and financial analyses are shown side by side, because each informs the other.

Several terms in Tables B.3–B.6 are zeroed, e.g. $\cancelto{0}{\Delta \dot{C}_o^*}$. These zeroes can be traced back to Table B.1. Table B.2 highlights the equations in Table B.1 that justify zeroing each term.

Table B.3. **Emplacement Effect**

	Energy analysis		*Financial analysis*	
before ($\circ$)	$\dot{E}^\circ = \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ) I_E$	(36)	$\dot{M}^\circ = p_E \dot{E}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ + \dot{N}^\circ$	(37)
after ($*$)	$\dot{E}^* = \dot{E}_s^* + \dot{E}_{emb}^* + (\dot{C}_{md}^* + \dot{C}_o^*) I_E$	(38)	$\dot{M}^* = p_E \dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o^* + \dot{N}^*$	(39)

Take differences to obtain the change in energy consumption, $\Delta \dot{E}^* \equiv \dot{E}^* - \dot{E}^\circ$.

$$\Delta \dot{E}^* = \Delta \dot{E}_s^* + \Delta \dot{E}_{emb}^* + (\Delta \dot{C}_{md}^* + \cancelto{0}{\Delta \dot{C}_o^*}) I_E \tag{40}$$

Thus,

$$\Delta \dot{E}^* = \Delta \dot{E}_s^* + \Delta \dot{E}_{emb}^* + \Delta \dot{C}_{md}^* I_E \ . \tag{41}$$

Define

$$\dot{S}_{dev} \equiv -\Delta \dot{E}_s^* \tag{42}$$

(Also see Eqs. (9) and (10)). Use Eq. (2) to obtain

$$Re_{empl} = 1 - \frac{-\Delta \dot{E}^*}{\dot{S}_{dev}} = 1 - \frac{-\Delta \dot{E}_s^*}{\dot{S}_{dev}} - \frac{-\Delta \dot{E}_{emb}^*}{\dot{S}_{dev}} - \frac{-\Delta \dot{C}_{md}^* I_E}{\dot{S}_{dev}} \ . \tag{43}$$

Define $Re_{dempl} \equiv 1 - \frac{-\Delta \dot{E}_s^*}{\dot{S}_{dev}} (= 0)$, $Re_{iempl} \equiv Re_{emb} + Re_{md}$, $Re_{emb} \equiv \frac{\Delta \dot{E}_{emb}^*}{\dot{S}_{dev}}$, and $Re_{md} \equiv \frac{\Delta \dot{C}_{md}^* I_E}{\dot{S}_{dev}}$, such that

$$Re_{empl} = Re_{dempl} + Re_{iempl} \ . \tag{44}$$

Use the monetary constraint ($\dot{M}^\circ = \dot{M}^*$) and constant spending on other items ($\dot{C}_o^\circ = \dot{C}_o^*$) to cancel terms to obtain

$$\begin{aligned} p_E \dot{E}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \cancel{\dot{C}_o^\circ} + \cancelto{0}{\dot{N}^\circ} \\ = p_E \dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \cancel{\dot{C}_o^*} + \dot{N}^* \ . \end{aligned} \tag{45}$$

Solving for $\Delta \dot{N}^* \equiv \dot{N}^* - \cancelto{0}{\dot{N}^\circ}$ gives

$$\Delta \dot{N}^* = p_E (\dot{E}_s^\circ - \dot{E}_s^*) + \dot{C}_{cap}^\circ - \dot{C}_{cap}^* + \dot{C}_{md}^\circ - \dot{C}_{md}^* \ . \tag{46}$$

Rewriting with Δ terms gives

$$\Delta \dot{N}^* = -p_E \Delta \dot{E}_s^* - \Delta \dot{C}_{cap}^* - \Delta \dot{C}_{md}^* \ . \tag{47}$$

Substituting Eq. (42) gives

$$\Delta \dot{N}^* = \dot{N}^* = p_E \dot{S}_{dev} - \Delta \dot{C}_{cap}^* - \Delta \dot{C}_{md}^* \ . \tag{48}$$

Freed cash ($\dot{G}$) resulting from the EEU, before any energy takeback, is given by

$$\dot{G} = p_E \dot{S}_{dev} \ . \tag{49}$$

Note that Eq. (37) and $\dot{N}^\circ = 0$ can be used to calculate $\dot{C}_o^\circ$ as

$$\dot{C}_o^\circ = \dot{M}^\circ - p_E \dot{E}_s^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \ . \tag{50}$$

Table B.4. **Substitution Effect**

	Energy analysis		*Financial analysis*	
before $(*)$	$\dot{E}^* = \dot{E}^*_s + \dot{E}^*_{emb} + (\dot{C}^*_{md} + \dot{C}^*_o)I_E$	(38)	$\dot{M}^* = p_E\dot{E}^*_s + \dot{C}^*_{cap} + \dot{C}^*_{md} + \dot{C}^*_o + \dot{N}^*$	(39)
after $(\wedge)$	$\dot{\hat{E}} = \dot{\hat{E}}_s + \dot{\hat{E}}_{emb} + (\dot{\hat{C}}_{md} + \dot{\hat{C}}_o)I_E$	(51)	$\dot{\hat{M}} = p_E\dot{\hat{E}}_s + \dot{\hat{C}}_{cap} + \dot{\hat{C}}_{md} + \dot{\hat{C}}_o + \dot{\hat{N}}$	(52)

Take differences to obtain the change in energy consumption, $\Delta\dot{\hat{E}} \equiv \dot{\hat{E}} - \dot{E}^*$.

$$\Delta\dot{\hat{E}} = \Delta\dot{\hat{E}}_s + \cancelto{0}{\Delta\dot{\hat{E}}_{emb}} + (\cancelto{0}{\Delta\dot{\hat{C}}_{md}} + \Delta\dot{\hat{C}}_o)I_E \tag{53}$$

Thus,

$$\Delta\dot{\hat{E}} = \Delta\dot{\hat{E}}_s + \Delta\dot{\hat{C}}_o I_E\ . \tag{54}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\dot{\hat{E}}}{\dot{S}_{dev}} = \frac{\Delta\dot{\hat{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\dot{\hat{C}}_o I_E}{\dot{S}_{dev}} \tag{55}$$

Define $Re_{sub} \equiv \frac{\Delta\dot{\hat{E}}}{\dot{S}_{dev}}$, $Re_{dsub} \equiv \frac{\Delta\dot{\hat{E}}_s}{\dot{S}_{dev}}$, and $Re_{isub} \equiv \frac{\Delta\dot{\hat{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{sub} = Re_{dsub} + Re_{isub}\ . \tag{56}$$

Use the monetary constraint ($\dot{M}^* = \dot{\hat{M}}$) to obtain

$$\begin{aligned} p_E\dot{E}^*_s &+ \cancel{\dot{C}^*_{cap}} + \cancel{\dot{C}^*_{md}} + \dot{C}^*_o + \dot{N}^* \\ &= p_E\dot{\hat{E}}_s + \cancel{\dot{\hat{C}}_{cap}} + \cancel{\dot{\hat{C}}_{md}} + \dot{\hat{C}}_o + \dot{\hat{N}}\ . \end{aligned} \tag{57}$$

For the substitution effect, there is no change in capital or maintenance and disposal costs ($\dot{\hat{C}}_{cap} = \dot{C}^*_{cap}$ and $\dot{\hat{C}}_{md} = \dot{C}^*_{md}$). Solving for $\Delta\dot{\hat{N}} \equiv \dot{\hat{N}} - \dot{N}^*$ gives

$$\Delta\dot{\hat{N}} = -p_E\Delta\dot{\hat{E}}_s - \Delta\dot{\hat{C}}_o\ . \tag{58}$$

The substitution effect adjusts net savings relative to $\dot{N}^*$ by $\Delta\dot{\hat{N}}$. Thus, $\dot{\hat{N}} = \dot{N}^* + \Delta\dot{\hat{N}}$. Substituting Eqs. (48), (49), and (58) yields

$$\dot{\hat{N}} = \dot{G} - \Delta\dot{C}^*_{cap} - \Delta\dot{C}^*_{md} - p_E\Delta\dot{\hat{E}}_s - \Delta\dot{\hat{C}}_o\ . \tag{59}$$

Table B.5. **Income Effect**

	Energy analysis		*Financial analysis*	
before ($\wedge$)	$\hat{\dot{E}} = \hat{\dot{E}}_s + \hat{\dot{E}}_{emb} + (\hat{\dot{C}}_{md} + \hat{\dot{C}}_o) I_E$	(51)	$\hat{\dot{M}} = p_E \hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}}$	(52)
after ($-$)	$\bar{\dot{E}} = \bar{\dot{E}}_s + \bar{\dot{E}}_{emb} + (\bar{\dot{C}}_{md} + \bar{\dot{C}}_o) I_E$	(60)	$\bar{\dot{M}} = p_E \bar{\dot{E}}_s + \bar{\dot{C}}_{cap} + \bar{\dot{C}}_{md} + \bar{\dot{C}}_o + \bar{\dot{N}}$	(61)

Take differences to obtain the change in energy consumption, $\Delta\bar{\dot{E}} \equiv \bar{\dot{E}} - \hat{\dot{E}}$.

$$\Delta\bar{\dot{E}} = \Delta\bar{\dot{E}}_s + \cancelto{0}{\Delta\bar{\dot{E}}_{emb}} + (\cancelto{0}{\Delta\bar{\dot{C}}_{md}} + \Delta\bar{\dot{C}}_o) I_E \qquad (62)$$

Thus,

$$\Delta\bar{\dot{E}} = \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o I_E \qquad (63)$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\bar{\dot{E}}}{\dot{S}_{dev}} = \frac{\Delta\bar{\dot{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}} \qquad (64)$$

Define $Re_{inc} \equiv \frac{\Delta\bar{\dot{E}}}{\dot{S}_{dev}}$, $Re_{dinc} \equiv \frac{\Delta\bar{\dot{E}}_s}{\dot{S}_{dev}}$, and $Re_{iinc} \equiv \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{inc} = Re_{dinc} + Re_{iinc} \; . \qquad (65)$$

Use the monetary constraint ($\hat{\dot{M}} = \bar{\dot{M}}$) to obtain

$$p_E \hat{\dot{E}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o + \hat{\dot{N}}$$
$$= p_E \bar{\dot{E}}_s + \cancel{\bar{\dot{C}}_{cap}} + \cancel{\bar{\dot{C}}_{md}} + \bar{\dot{C}}_o + \cancelto{0}{\bar{\dot{N}}} \; . \qquad (66)$$

For the income effect, there is no change in capital or maintainance and disposal costs ($\hat{\dot{C}}_{cap} = \dot{C}^*_{cap}$ and $\hat{\dot{C}}_{md} = \dot{C}^*_{md}$). Notably, $\bar{\dot{N}} = 0$, because it is assumed that all net monetary savings ($\hat{\dot{N}}$) are spent on more energy service ($\bar{\dot{E}}_s > \hat{\dot{E}}_s$) and additional purchases in the economy ($\bar{\dot{C}}_o > \hat{\dot{C}}_o$). Solving for $\hat{\dot{N}}$ gives

$$\hat{\dot{N}} = p_E \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o \; , \qquad (67)$$

the budget constraint for the income effect. By construction, Eq. (67) ensures spending of net savings ($\hat{\dot{N}}$) on (i) additional energy services ($\Delta\bar{\dot{E}}_s$) and (ii) additional purchases of other goods in the economy ($\Delta\bar{\dot{C}}_o$) only.

Table B.6. **Macro Effect**

	Energy analysis		*Financial analysis*
before (−)	$\bar{\dot{E}}$	(68)	
after (∼)	$\tilde{\dot{E}}$	(69)	
	Take differences to obtain the change in energy consumption,		N/A

$$\Delta\tilde{\dot{E}} \equiv \tilde{\dot{E}} - \bar{\dot{E}} \; . \tag{70}$$

The energy change due to the macro effect ($\Delta\tilde{\dot{E}}$) is a scalar multiple (k) of net savings ($\hat{\dot{N}}$), assumed to be spent at the energy intensity of the economy (I_E).

$$\Delta\tilde{\dot{E}} = k\hat{\dot{N}} I_E \tag{71}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}} = \frac{k\hat{\dot{N}} I_E}{\dot{S}_{dev}} \tag{72}$$

Define $Re_{macro} \equiv \frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}}$, such that

$$Re_{macro} = \frac{k\hat{\dot{N}} I_E}{\dot{S}_{dev}} \; . \tag{32}$$

B.3 Rebound expressions

All that remains is to determine expressions for each rebound effect. We begin with the device-level expected energy savings rate ($\dot{S}_{dev}$), which appears in the denominator of all rebound expressions.

B.3.1 Expected energy savings ($\dot{S}_{dev}$)

$\dot{S}_{dev}$ is the reduction of energy consumption rate by the device due to the EEU. No other effects are considered.

$$\dot{S}_{dev} \equiv \dot{E}_s^\circ - \dot{E}_s^* \tag{9}$$

The final energy consumption rates ($\dot{E}_s^\circ$ and $\dot{E}_s^*$) can be written as Eq. (5) in the forms $\dot{E}_s^\circ = \dot{q}_s^\circ/\eta^\circ$ and $\dot{E}_s^* = \dot{q}_s^*/\eta^*$.

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^*}{\eta^*} \tag{73}$$

With reference to Table B.1, we use $\dot{q}_s^* = \dot{q}_s^\circ$ and $\eta^* = \tilde{\eta}$ to obtain

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^\circ}{\tilde{\eta}} \,. \tag{74}$$

When the EEU increases efficiency such that $\tilde{\eta} > \eta^\circ$, expected energy savings grows ($\dot{S}_{dev} > 0$) as the rate of final energy consumption declines, as expected. As $\tilde{\eta} \to \infty$, all final energy consumption is eliminated ($\dot{E}_s^* \to 0$), and $\dot{S}_{dev} = \dot{q}_s^\circ/\eta^\circ = \dot{E}_s^\circ$. (Of course, $\tilde{\eta} \to \infty$ is impossible. See Paoli & Cullen (2020) for a recent discussion of upper limits to device efficiencies.)

After rearrangement and using $\dot{E}_s^\circ = \dot{q}_s^\circ/\eta^\circ$, we obtain a convenient form

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right)\frac{\eta^\circ}{\tilde{\eta}}\dot{E}_s^\circ \,. \tag{10}$$

B.3.2 Emplacement effect

The emplacement effect accounts for performance of the EEU only. No behavior changes occur. The direct emplacement effect of the EEU is device energy savings and energy cost savings. The indirect emplacement effects of the EEU produce changes in the embodied energy rate and the maintenance and disposal expenditure rates. By definition, the direct emplacement effect has no rebound. However, indirect emplacement effects may cause energy rebound. Both direct and indirect emplacement effects are discussed below.

Re_{dempl} As shown in Table B.3, the direct rebound from the emplacement effect is $Re_{dempl} = 0$. This result is expected, because, in the absence of behavior changes, there is no takeback of energy savings at the upgraded device.

Re_{iempl} Indirect emplacement rebound effects can occur at any point in the life cycle of an energy conversion device, from manufacturing and distribution to the use phase (maintenance), and finally to disposal. For simplicity, we group maintenance with disposal to form two distinct indirect emplacement rebound effects: (i) an embodied energy effect (Re_{emb}) and (ii) a maintenance and disposal effect (Re_{md}).

Re_{emb} The first component of indirect emplacement effect rebound involves embodied energy. We define embodied energy consistent with the energy analysis literature to be the sum of all final energy consumed in the production of the energy conversion device. The EEU causes the embodied final energy of the device to change from $\dot{E}^{\circ}_{emb}$ to $\dot{E}^{*}_{emb}$.

Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. No energy is embodied in the device while in service. However, for simplicity, we spread all embodied energy over the lifetime of the device, an equal amount assigned to each period. We later take the same approach to capital costs and maintenance and disposal costs. A justification for spreading embodied energy purchase costs comes from considering staggered device replacements by many consumers across several years. In the aggregate, staggered replacements work out to about the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices without discounting (t°_{life} and t^{*}_{life}, respectively) to obtain embodied energy rates, such that $\dot{E}^{\circ}_{emb} = E^{\circ}_{emb}/t^{\circ}_{life}$ and $\dot{E}^{*}_{emb} = E^{*}_{emb}/t^{*}_{life}$. The change in embodied final energy due to the EEU (expressed as a rate) is given by $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$. After substitution and algebraic rearrangement, the change in embodied energy rate due to the EEU can be expressed as $[(E^{*}_{emb}/E^{\circ}_{emb})(t^{\circ}_{life}/t^{*}_{life}) - 1]\dot{E}^{\circ}_{emb}$, a term that represents energy savings taken back due to embodied energy effects. Thus, Eq. (4) can be employed to write embodied energy rebound as

$$Re_{emb} = \frac{\left(\frac{E^{*}_{emb}}{E^{\circ}_{emb}}\frac{t^{\circ}_{life}}{t^{*}_{life}} - 1\right)\dot{E}^{\circ}_{emb}}{\dot{S}_{dev}} . \tag{12}$$

Embodied energy rebound can be either positive or negative, depending on the sign of the term $(E^{*}_{emb}/E^{\circ}_{emb})(t^{\circ}_{life}/t^{*}_{life}) - 1$. Rising energy efficiency can be associated with increased device complexity and more embodied energy, such that $E^{*}_{emb} > E^{\circ}_{emb}$ and $Re_{emb} > 0$. However, if the upgraded device has longer life than the original device ($t^{*}_{life} > t^{\circ}_{life}$), $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$ can be negative, meaning that the upgraded device has a lower embodied energy rate than the original device.

Re_{md} In addition to embodied energy effects, indirect emplacement rebound can be associated with energy demanded by maintenance and disposal (md) expenditures. Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}^{\circ}_{m}$). Disposal costs (e.g., C°_{d}) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal expenditures across the lifetime of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) to form expenditure rates such that $\dot{C}^{\circ}_{md} = \dot{C}^{\circ}_{m} + C^{\circ}_{d}/t^{\circ}_{life}$ and $\dot{C}^{*}_{md} = \dot{C}^{*}_{m} + C^{*}_{d}/t^{*}_{life}$.

We assume, for simplicity, that md expenditures indicate energy consumption elsewhere in the economy at its energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in md expenditures is given by $\Delta\dot{C}^{*}_{md}I_E$. This term is an energy takeback rate, so maintenance and disposal rebound is given by

$$Re_{md} = \frac{\Delta\dot{C}^{*}_{md}I_E}{\dot{S}_{dev}} , \tag{75}$$

as shown in Table B.3. Slight rearrangement gives

$$Re_{md} = \frac{\left(\frac{\dot{C}^{*}_{md}}{\dot{C}^{\circ}_{md}} - 1\right)\dot{C}^{\circ}_{md}I_E}{\dot{S}_{dev}} . \tag{13}$$

Rebound from maintenance and disposal can be positive or negative, depending on the sign of the term $\dot{C}^*_{md}/\dot{C}^\circ_{md} - 1$.

B.3.3 Substitution effect

This section derives expressions for substitution effect rebound. Two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}). Assuming that conditions after the emplacement effect ($*$) are known, both the rate of energy service consumption ($\hat{\dot{q}}_s$) and the rate of other goods consumption ($\hat{\dot{C}}_o$) must be determined as a result of the substitution effect (the $\wedge$ point).

The EEU's energy efficiency increase ($\tilde{\eta} > \eta^\circ$) causes the price of the energy service provided by the device to fall ($\tilde{p}_s < p_s^\circ$). The substitution effect quantifies the amount by which the device owner, in response, increases the consumption rate of the energy service ($\hat{\dot{q}}_s > \dot{q}_s^*$) and decreases the consumption rate of other goods ($\hat{\dot{q}}_o < \dot{q}_o^*$).

The increase in consumption of the energy service substitutes for consumption of other goods in the economy, subject to a utility constraint. The reduction in spending on other goods in the economy is captured by indirect substitution rebound (Re_{isub}).

We begin by deriving an expression for direct and indirect substitution effect rebound (Re_{dsub} and Re_{isub}, respectively). Thereafter, we develop an approximate model and a CES utility model for determining the post-substitution point ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$).

Direct substitution effect rebound expression Direct substitution effect rebound (Re_{dsub}) is given by

$$Re_{dsub} = \frac{\Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} = \frac{\hat{\dot{E}}_s - \dot{E}_s^*}{\dot{S}_{dev}} \ . \tag{15}$$

Substituting the typical relationship of Eq. (5) in the form $\dot{E}_s = \dot{q}_s/\eta$ gives

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\tilde{\eta}} - \frac{\dot{q}_s^*}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{76}$$

Rearranging produces

$$Re_{dsub} = \frac{\left(\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - \frac{\dot{q}_s^*}{\dot{q}_s^\circ}\right)\frac{\dot{q}_s^\circ}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{77}$$

Recognizing that the rate of energy service consumption ($\dot{q}_s$) is unchanged across the emplacement effect leads to $\dot{q}_s^*/\dot{q}_s^\circ = 1$. Furthermore, $\dot{q}_s^\circ/\tilde{\eta} = (\dot{q}_s^\circ/\eta^\circ)(\eta^\circ/\tilde{\eta}) = \dot{E}_s^\circ(\eta^\circ/\tilde{\eta})$, such that

$$Re_{dsub} = \left(\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1\right)\frac{\dot{E}_s^\circ \frac{\eta^\circ}{\tilde{\eta}}}{\dot{S}_{dev}} \ . \tag{78}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\left(\frac{\cancel{\dot{E}_s^\circ}\,\cancel{\frac{\eta^\circ}{\tilde{\eta}}}}{\cancel{\frac{\eta^\circ}{\tilde{\eta}}}\,\cancel{\dot{E}_s^\circ}}\right) \ . \tag{79}$$

Canceling terms yields

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} . \tag{16}$$

Eq. (16) is the basis for developing both approximate and CES models of determining direct substitution rebound.

Indirect substitution effect rebound expression Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} = \frac{(\hat{\dot{C}}_o - \dot{C}_o^*) I_E}{\dot{S}_{dev}} . \tag{17}$$

Rearranging gives

$$Re_{isub} = \frac{\left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - \frac{\dot{C}_o^*}{\dot{C}_o^\circ} \right) \dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{80}$$

Recognizing that expenditures on other goods are constant across the emplacement effect gives $\dot{C}_o^* / \dot{C}_o^\circ = 1$ and

$$Re_{isub} = \left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1 \right) \frac{\dot{C}_o^\circ I_E}{\dot{S}_{dev}} . \tag{81}$$

Substituting Eq. (10) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, \frac{\tilde{\eta}}{\eta^\circ} \, \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} . \tag{18}$$

Eq. (18) is the basis for developing both approximate and CES models of determining indirect substitution rebound.

Determining the post-substitution effect conditions requires reference to a consumer utility model. We first show an approximate model, often used in the literature. Later, we use a constant elasticity of substitution (CES) utility model. The CES model is used for all calculations and graphs in this paper.

Approximate utility model In the literature, an approximate utility model is often used (Borenstein, 2015, p. 17, footnote 43). In the two examples of this paper (car and electric lamp upgrades), rebound calculated using the approximate utility model (here) differs from rebound calculated using the exact utility model (below) by as much as 5%. Thus, we do not recommend use of the approximate utility model. We discuss the approximate utility model here for completeness only.

In the approximate model, the relationship between energy service price and energy service consumption rate is given by the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s, p_s, c}$), such that

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^*} = \left(\frac{\tilde{p}_s}{p_s^\circ} \right)^{\epsilon_{\dot{q}_s, p_s, c}} . \tag{82}$$

Note that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) is assumed constant along an indifference curve in the approximate model. A negative value for the compensated price elasticity of energy service demand is expected ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that when the energy service price decreases ($\tilde{p}_s < p_s^\circ$), the rate of energy service consumption increases ($\hat{\dot{q}}_s > \dot{q}_s^*$).

Substituting Eq. (6) in the form $p_s^\circ = p_E^\circ/\eta^\circ$ and $\tilde{p}_s = p_E^\circ/\tilde{\eta}$ and noting that $\dot{q}_s^\circ = \dot{q}_s^*$ gives

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} . \tag{83}$$

Again, note that the compensated price elasticity of energy service demand is negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), so that as energy service efficiency increases ($\tilde{\eta} > \eta^\circ$), the energy service consumption rate increases ($\hat{\dot{q}}_s > \dot{q}_s^* = \dot{q}_s^\circ$).

Substituting Eq. (83) into Eq. (16) yields the approximate expression for direct substitution rebound.

$$Re_{dsub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{84}$$

The compensated price elasticity of energy service demand is expected to be negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that, e.g. $\epsilon_{\dot{q}_s,p_s,c} = -0.2$ and $\tilde{\eta}/\eta^\circ = 2$ yields $Re_{dsub} = 0.15$.

With $\epsilon_{\dot{q}_s,p_s,c} \in (-1, 0)$ expected, the approximate model indicates that direct substitution rebound will never be larger than 1. I.e., the direct substitution effect alone can never cause backfire.

To quantify the substitution effect on other purchases in the approximate model, we introduce another elasticity, the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\epsilon_{\dot{q}_o,p_s,c}} . \tag{85}$$

Because the compensated cross-price elasticity of other goods demand is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$), an energy service price decrease ($\tilde{p}_s < p_s^\circ$) implies a reduction in the rate of consumption of other goods ($\hat{\dot{q}}_o < \dot{q}_o^*$).

The energy service price is inversely proportional to efficiency, yielding

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{86}$$

Assuming that the average price is unchanged across the substitution effect such that $\hat{p}_o = \dot{p}_o^* = p_o^\circ$ (Appendix E), and noting that $\dot{q}_s^* = \dot{q}_s^\circ$ and $\dot{C}_o^* = \dot{C}_o^\circ$, we can write

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{87}$$

Note that Eq. (87) can be used to determine the rate of expenditures on other goods in the economy ($\hat{\dot{C}}_o$) by

$$\hat{\dot{C}}_o = \dot{C}_o^\circ \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{88}$$

Substituting Eq. (88) into Eq. (18) gives the expression for indirect substitution rebound for the approximate utility model.

$$Re_{isub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, \frac{\tilde{\eta}}{\eta^\circ} \, \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{89}$$

Because the compensated cross-elasticity of other goods consumption is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$) and the energy service efficiency ratio is greater than 1 ($\tilde{\eta} > \eta^\circ$), indirect substitution rebound will be negative always ($Re_{isub} < 0$), as expected. Negative rebound indicates that indirect substitution effects reduce the energy takeback rate by direct substitution effects.

CES utility model The approximate utility model assumes that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$) are constant along an indifference curve. These assumptions are approximations that hold only for infinitesimally small energy service price changes ($\Delta p_s^* \equiv p_s^* - p_s^\circ \approx 0$). However, in the case of an energy efficiency upgrade (EEU), the energy service price change is not infinitesimal. Rather, Δp_s^* is finite and may be large.

To determine the new consumption bundle after the substitution effect ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$) and, ultimately, to quantify the direct and indirect substitution rebound effects (Re_{dsub} and Re_{isub}) exactly, we remove the restriction that energy service price elasticities ($\epsilon_{\dot{q}_s,p_s,c}$ and $\epsilon_{\dot{q}_o,p_s,c}$) must be constant along an indifference curve. Instead, we require constancy of only the elasticity of substitution (σ) between the consumption rate of the energy service ($\dot{q}_s$) and the expenditure rate for other goods ($\dot{C}_o$) across the substitution effect. Thus, we employ a CES utility model.

Two equations are helpful for this analysis. First, the slope at any point on indifference curve i°—i° is given by Eq. (118) with $\dot{u}/\dot{u}^\circ = 1$ and the share parameter (a) replaced by $f_{\dot{C}_s}^\circ$, as discussed in Appendix C.

$$\begin{aligned}\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = &-\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \\ &\times \left[\left(\frac{1}{1 - f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho} .\end{aligned} \tag{90}$$

Second, the equation of the $*$—$*$ expenditure line is

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right) + \frac{1}{\dot{C}_o^\circ}(\dot{M} - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ - \dot{G}) . \tag{91}$$

To find the rate of energy service consumption after the substitution effect ($\hat{\dot{q}}_s$), we set the slope of the $*$—$*$ expenditure line (Eq. 91) equal to the slope of the i°—i° indifference curve at the original utility rate of $\dot{u}/\dot{u}^\circ = 1$ (Eq. (90)).

$$-\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)}\left[\left(\frac{1}{1 - f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^\rho\right]^{(1-\rho)/\rho} \tag{92}$$

Solving for $\dot{q}_s/\dot{q}_s^\circ$ gives $\hat{\dot{q}}_s/\dot{q}_s^\circ$ as

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1-f_{\dot{C}_s}^\circ)\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\frac{\rho}{1-\rho}}\right\}^{-1/\rho} . \tag{19}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an estimate for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1-f_{\dot{C}_s}^\circ)\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\frac{\rho}{1-\rho}}\right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

The rate of other goods consumption after the substitution effect ($\hat{\dot{C}}_o$) can be found by substituting Eq. (19) and $\dot{u}/\dot{u}^\circ = 1$ into the functional form of the utility model (Eq. (117)) to obtain

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(\left(\frac{1}{1-f_{\dot{C}_s}^\circ}\right) - \left(\frac{f_{\dot{C}_s}^\circ}{1-f_{\dot{C}_s}^\circ}\right)\left\{ f_{\dot{C}_s}^\circ + (1-f_{\dot{C}_s}^\circ)\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{\frac{\rho}{1-\rho}}\right\}^{-1}\right)^{1/\rho} . \tag{93}$$

Simplifying gives

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left[\left(1 + f_{\dot{C}_s}^\circ\left\{\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} . \tag{20}$$

Eq. (20) can be substituted into Eq. (18) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left[\left(1 + f_{\dot{C}_s}^\circ\left\{\left[\left(\frac{1-f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right)\frac{\tilde{p}_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\frac{\tilde{\eta}}{\eta^\circ}\frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{22}$$

B.3.4 Income effect

Rebound from the income effect rebound quantifies the rate of additional energy demand that arises because the owner of the energy conversion device spends net savings from the EEU. The income rate of the device owner is $\dot{M}^\circ$, which remains unchanged across the rebound effects, such that $\dot{M}^\circ = \dot{M}^* = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Freed cash from the EEU is given by Eq. (49) as $\dot{G} = p_E\dot{S}_{dev}$. In combination, the emplacement effect and the substitution effect leave the device owner with *net* savings ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (59). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables B.3 and B.4.

In this framework, all net savings ($\hat{\dot{N}}$) are spent on either (i) additional energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) or (ii) additional other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$). The income elasticity of energy service demand and the income elasticity of other goods demand ($\epsilon_{\dot{q}_s,\dot{M}}$ and $\epsilon_{\dot{q}_o,\dot{M}}$, respectively) quantify the income preferences of the device owner according to the following expressions:

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} \tag{23}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} , \tag{27}$$

where effective income ($\hat{\dot{M}}'$) is

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} . \tag{24}$$

Homotheticity means that $\epsilon_{\dot{q}_s,\dot{M}} = 1$ and $\epsilon_{\dot{q}_o,\dot{M}} = 1$.

The budget constraint across the income effect (Eq. (67)) ensures that all net savings available after the substitution effect ($\hat{\dot{N}}$) is re-spent across the income effect, such that $\bar{\dot{N}} = 0$. Appendix D proves that the income preference equations (Eqs. (23) and (27)) satsify the budget constraint (Eq. 67).

The purpose of this section is derivation of expressions for (i) direct income rebound (Re_{dinc}) arising from increased consumption of the energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) and (ii) indirect income rebound (Re_{iinc}) arising from increased consumption of other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$).

But first, we derive a helpful expression to be used later.

Expression for $\hat{\dot{E}}_s$ An expression for $\hat{\dot{E}}_s$ that will be helpful later begins with

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{E}}_s}{\dot{E}^*_s}\right)\left(\frac{\dot{E}^*_s}{\dot{E}^\circ_s}\right)\dot{E}^\circ_s . \tag{94}$$

Substituting Eq. (5) and noting efficiency (η) equalities from Table B.1 gives

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{q}}_s/\cancel{\tilde{\eta}}}{\dot{q}^*_s/\cancel{\tilde{\eta}}}\right)\left(\frac{\dot{q}^*_s/\tilde{\eta}}{\dot{q}^\circ_s/\eta^\circ}\right)\dot{E}^\circ_s . \tag{95}$$

Canceling terms yields

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{q}}_s}{\dot{q}^*_s}\right)\left(\frac{\cancel{\dot{q}^*_s}}{\cancel{\dot{q}^\circ_s}}\right)\left(\frac{\eta^\circ}{\tilde{\eta}}\right)\dot{E}^\circ_s . \tag{96}$$

Noting energy service consumption rate equalities from Table B.1 ($\dot{q}^*_s = \dot{q}^\circ_s$) gives

$$\hat{\dot{E}}_s = \frac{\hat{\dot{q}}_s}{\dot{q}^*_s}\frac{\eta^\circ}{\tilde{\eta}}\dot{E}^\circ_s . \tag{97}$$

The next step is to develop an expression for Re_{dinc} using the income preference for energy service consumption.

Expression for Re_{dinc} As shown in Table B.5, direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \dot{\bar{E}}_s}{\dot{S}_{dev}} \; . \tag{25}$$

Expanding the difference and rearranging gives

$$Re_{dinc} = \frac{\dot{\bar{E}}_s - \dot{\hat{E}}_s}{\dot{S}_{dev}} \; , \tag{98}$$

and

$$Re_{dinc} = \frac{\left(\frac{\dot{\bar{E}}_s}{\dot{\hat{E}}_s} - 1\right) \dot{\hat{E}}_s}{\dot{S}_{dev}} \; . \tag{99}$$

Substituting the Eq. (5) as $\dot{\bar{E}}_s = \frac{\dot{\bar{q}}_s}{\tilde{\eta}}$ and $\dot{\hat{E}}_s = \frac{\dot{\hat{q}}_s}{\tilde{\eta}}$ gives

$$Re_{dinc} = \frac{\left(\frac{\dot{\bar{q}}_s/\cancel{\tilde{\eta}}}{\dot{\hat{q}}_s/\cancel{\tilde{\eta}}} - 1\right) \dot{\hat{E}}_s}{\dot{S}_{dev}} \; . \tag{100}$$

Eliminating terms and substituting Eq. (10) for $\dot{S}_{dev}$ and Eq. (23) for $\dot{\bar{q}}_s/\dot{\hat{q}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1\right] \dot{\hat{E}}_s}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ} \; . \tag{101}$$

Substituting Eq. (97) for $\dot{\hat{E}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1\right] \frac{\dot{\hat{q}}_s}{\dot{q}_s^*} \frac{\cancel{\eta^\circ}}{\cancel{\tilde{\eta}}} \cancel{\dot{E}_s^\circ}}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\cancel{\eta^\circ}}{\cancel{\tilde{\eta}}} \cancel{\dot{E}_s^\circ}} \; . \tag{102}$$

Eliminating terms, recognizing that $\dot{q}_s^\circ = \dot{q}_s^*$, and substituting Eq. (19), which assumes the CES utility model, gives

$$Re_{dinc} = \frac{\left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} \; . \tag{26}$$

If there is no net savings ($\dot{\hat{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected.

The next step is to develop an expression for Re_{iinc} using the income preference for other goods consumption.

Expression for Re_{iinc} As shown in Table B.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta \dot{\bar{C}}_o I_E}{\dot{S}_{dev}} \; . \tag{29}$$

Expanding the difference and rearranging gives

$$Re_{iinc} = \frac{(\bar{\dot{C}}_o - \hat{\dot{C}}_o) I_E}{\dot{S}_{dev}} \ , \tag{103}$$

and

$$Re_{iinc} = \frac{\left(\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} - 1\right) \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \ . \tag{104}$$

Substituting $\bar{\dot{C}}_o = p_o \bar{\dot{q}}_o$ and $\hat{\dot{C}}_o = p_o \hat{\dot{q}}_o$ and cancelling terms gives

$$Re_{iinc} = \frac{\left(\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} - 1\right) \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \ . \tag{105}$$

Substituting the income preference equation for other goods consumption (Eq. (27) for $\bar{\dot{q}}_o / \hat{\dot{q}}_o$ and Eq. (10) for $\dot{S}_{dev}$ yields

$$Re_{iinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1\right] \hat{\dot{C}}_o I_E}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ} \ . \tag{106}$$

Sutstituting $(\hat{\dot{C}}_o / \dot{C}_o^\circ) \dot{C}_o^\circ$ for $\hat{\dot{C}}_o$, recognizing that $\dot{C}_o^* = \dot{C}_o^\circ$, and simplifying gives

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ}\right) \ . \tag{107}$$

Substituting Eq. (20) for $\hat{\dot{C}}_o / \dot{C}_o^\circ$, thereby assuming the CES utility model, gives the final form of the indirect income rebound expression:

$$Re_{iinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left[\left(1 + f_{\dot{C}_s}^\circ \left\{\left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ}\right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} \ . \tag{30}$$

If there is no net savings ($\hat{\dot{N}} = 0$), indirect income effect rebound is zero ($Re_{iinc} = 0$), as expected.

B.3.5 Macro effect

Macro rebound (Re_{macro}) is given by Eq. (32). Substituting Eq. (59) for net savings ($\hat{\dot{N}}$) gives

$$Re_{macro} = \frac{k \dot{G} I_E}{\dot{S}_{dev}} - \frac{k \Delta \dot{C}_{cap}^* I_E}{\dot{S}_{dev}} - \frac{k \Delta \dot{C}_{md}^* I_E}{\dot{S}_{dev}} - \frac{k p_E I_E \Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} - \frac{k \Delta \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} \ . \tag{108}$$

Substituting Eq. (49) for $\dot{G}$ and Eqs. (75), (15), and (17) for rebound terms gives

$$Re_{macro} = \frac{k p_E \cancel{\dot{S}_{dev}} I_E}{\cancel{\dot{S}_{dev}}} - \frac{k \Delta \dot{C}_{cap}^* I_E}{\dot{S}_{dev}} - k Re_{md} - k p_E I_E Re_{dsub} - k Re_{isub} \ . \tag{109}$$

Canceling terms and defining Re_{cap} as

$$Re_{cap} \equiv \frac{\Delta \dot{C}^{*}_{cap} I_E}{\dot{S}_{dev}} \tag{110}$$

gives

$$Re_{macro} = kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub}\ . \tag{33}$$

B.3.6 Rebound sum

The sum of four rebound effects is

$$Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro}\ . \tag{111}$$

Substituting Eqs. (44), (56), and (65) gives

$$\begin{aligned} Re_{tot} = Re_{emb} &+ Re_{md} && \text{emplacement effect} \\ &+ Re_{dsub} + Re_{isub} && \text{substitution effect} \\ &+ Re_{dinc} + Re_{iinc} && \text{income effect} \\ &+ Re_{macro} && \text{macro effect} \end{aligned} \tag{112}$$

Macro effect rebound (Re_{macro}, Eq. (33)) can be expressed in terms of other rebound effects. Substituting Eq. (33) gives

$$\begin{aligned} Re_{tot} = Re_{emb} &+ Re_{md} && \text{emplacement effect} \\ &+ Re_{dsub} + Re_{isub} && \text{substitution effect} \\ &+ Re_{dinc} + Re_{iinc} && \text{income effect} \\ &+ kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub}\ . && \text{macro effect} \end{aligned} \tag{113}$$

Rearranging distributes macro effect terms to emplacement and substitution effect terms. This last rearrangement gives the final expression for total rebound.

$$\begin{aligned} Re_{tot} = Re_{emb} &- kRe_{cap} + (1-k)Re_{md} \\ &+ (1 - kp_E I_E)Re_{dsub} + (1-k)Re_{isub} \\ &+ Re_{dinc} + Re_{iinc} + kp_E I_E \end{aligned} \tag{34}$$

Eq. (34) shows that determining seven rebound values,

- Re_{emb} (Eq. (12)),
- Re_{cap} (Eq. (110)),
- Re_{md} (Eq. (13)),
- Re_{dsub} (Eq. (21)),
- Re_{isub} (Eqs. (22)),

- Re_{dinc} (Eqs. (26)), and
- Re_{iinc} (Eqs. (30)),

is sufficient to calculate total rebound, provided that the macro factor (k), the price of energy (p_E), and the energy intensity of the economy (I_E) are known.

C Utility models and elasticities

As discussed in Section 2.4.2 and Appendix B.3.3, the substitution effect requires a model for device owner utility that compares the perceived benefits of consuming the energy service ($\dot{q}_s$) to consuming other goods and services ($\dot{q}_o$). In this appendix, we describe two utility models. The first utility model is an approximate model that applies only for small and marginal changes in energy efficiency and energy service price, such that $\Delta\eta^* \approx 0$ and $\Delta p_s^* \approx 0$. The approximate utility model is discussed for continuity with the literature only. (See, for example, Borenstein (2015).)

We note that larger and non-marginal efficiency gains cause greater rebound (measured in joules) than small and marginal efficiency gains. Thus, any rebound analysis framework needs to accommodate large, non-marginal efficiency changes. The second utility model discussed in this appendix is the Constant Elasticity of Substitution (CES) utility model which does, in fact, accommodate large, non-marginal energy efficiency and energy service price changes. The CES utility model underlies the substitution effect in this framework. (See Section 2.4.2.) Furthermore, the CES utility model is needed for the example EEUs in this paper, which have large, non-marginal percentage increases in energy efficiency.

Both the substitution effect and the income effect use elasticities to model consumer preferences, and those elasticities are discussed below. Finally, elasticities for the income effect are discussed.

Before proceeding with the utility models and elasticities, we note briefly that the rate of other goods consumption ($\dot{q}_o$) is not known independently from the prices of other goods (p_o). With the assumption that the prices of other goods do not change across rebound effects (i.e., p_o is exogenous), the ratio of other goods consumption is equal to the ratio of other goods spending, such that

$$\frac{\dot{q}_o}{\dot{q}_o^\circ} = \frac{\dot{C}_o/\cancel{p_o}}{\dot{C}_o^\circ/\cancel{p_o}} = \frac{\dot{C}_o}{\dot{C}_o^\circ} \tag{114}$$

at all rebound stages. (See Appendix E for details.)

C.1 Utility models for the substitution effect

A utility model gives the ratio of energy service consumption rate and other goods consumption rates across the substitution effect ($\hat{\dot{q}}_s/\dot{q}_s^*$ and $\hat{\dot{q}}_o/\dot{q}_o^*$, respectively). In so doing, utility models quantify the decrease in other goods consumption ($\hat{\dot{q}}_o\dot{q}_o^* < 1$) caused by the increase of energy service consumption ($\hat{\dot{q}}_s\dot{q}_s^* > 1$) resulting from the decrease of the energy service price ($p_s^* < p_s^\circ$). Across the substitution effect, the utility increase of the larger energy service consumption rate must be exactly offset by the utility decrease of the smaller other goods consumption rate.

C.1.1 Approximate utility model

The approximate utility model is given by Eqs. (83) and (87). The equations for the approximate utility model are repeated here for convenience.

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s, p_s, c}} \tag{83}$$

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o, p_s, c}} \tag{87}$$

C.1.2 CES utility model

The CES utility model is given by Eq. (14). Here, its derivation is shown.

The CES model for utility ($\dot{u}$) is normalized by (indexed to) conditions prior to emplacement:

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{\rho} + (1-a)\left(\frac{\dot{q}_o}{\dot{q}_o^\circ}\right)^{\rho}\right]^{(1/\rho)} , \tag{115}$$

where $\rho = (\sigma - 1)/\sigma$, a is a share parameter (determined below), and σ is the elasticity of substitution between the normalized consumption rate of the energy service and the normalized consumption rate of other goods.

With the assumption of exogenous other goods prices in Eq. (114), we find

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[a\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{\rho} + (1-a)\left(\frac{\dot{C}_o}{\dot{C}_o^\circ}\right)^{\rho}\right]^{(1/\rho)} . \tag{116}$$

Eq. (116) is the functional form of the CES utility model, whose share parameter (a) is yet to be determined. The correct expression for the share parameter (a) is found from the equilibrium requirement, namely that the expenditure curve is tangent to the indifference curve in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space prior to the EEU.

To find the slope of the indifference curve, Eq. (116) can be rearranged to find the normalized consumption rate of other goods ($\dot{C}_o/\dot{C}_o^\circ$) as a function of the normalized consumption rate of the energy service ($\dot{q}_s/\dot{q}_s^\circ$) and the normalized utility rate ($\dot{u}/\dot{u}^\circ$):

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = \left[\frac{1}{1-a}\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^{\rho} - \frac{a}{1-a}\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^{\rho}\right]^{(1/\rho)} , \tag{117}$$

a form convenient for drawing constant utility rate ($\dot{u}/\dot{u}^\circ$) indifference curves in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space. In $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, the slope of an indifference curve is found by taking the first partial derivative of $\dot{C}_o/\dot{C}_o^\circ$ with respect to $\dot{q}_s/\dot{q}_s^\circ$, starting from Eq. (117) and using the chain rule repeatedly. The result is

$$\begin{aligned}\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = &-\frac{a}{1-a}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \\ &\times\left[\left(\frac{1}{1-a}\right)\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^{\rho} - \left(\frac{a}{1-a}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^{\rho}\right]^{(1-\rho)/\rho} .\end{aligned} \tag{118}$$

The budget constraint is the starting point for finding the slope of the expenditure line in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space:

$$\dot{M} = p_s\dot{q}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o + \dot{N} , \tag{119}$$

a generic version of Eqs. (37), (39), (52), and (61) with $p_s\dot{q}_s$ substituted for $p_E\dot{E}_s$. We solve for $\dot{C}_o$ and judiciously multiply by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ to obtain

$$\frac{\dot{C}_o}{\dot{C}_o^\circ}\dot{C}_o^\circ = -p_s\frac{\dot{q}_s}{\dot{q}_s^\circ}\dot{q}_s^\circ + \dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N} \; . \tag{120}$$

Solving for $\dot{C}_o/\dot{C}_o^\circ$ and rearranging gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right) + \frac{1}{\dot{C}_o^\circ}(\dot{M} - \dot{C}_{cap} - \dot{C}_{md} - \dot{N}) \; , \tag{121}$$

from which the slope in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space is taken by inspection to be

$$\frac{\partial(\dot{C}_o/\dot{C}_o^\circ)}{\partial(\dot{q}_s/\dot{q}_s^\circ)} = -\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ} \; . \tag{122}$$

At any equilibrium point, the expenditure line must be tangent to its indifference curve. Applying the tangency requirement before emplacement enables solving for the correct expression for a. Setting the slope of the expenditure line (Eq. (122)) equal to the slope of the indifference curve (Eq. (118)) gives

$$\begin{aligned} -\frac{p_s\dot{q}_s^\circ}{\dot{C}_o^\circ} = &- \frac{a}{1-a}\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{(\rho-1)} \\ &\times \left[\left(\frac{1}{1-a}\right)\left(\frac{\dot{u}}{\dot{u}^\circ}\right)^{\rho} - \left(\frac{a}{1-a}\right)\left(\frac{\dot{q}}{\dot{q}_s^\circ}\right)^{\rho}\right]^{(1-\rho)/\rho} \; . \end{aligned} \tag{123}$$

For the equilibrium point prior to emplacement, $\dot{q}_s/\dot{q}_s^\circ = 1$, $\dot{u}/\dot{u}^\circ = 1$, and $p_s = p_s^\circ$, which reduces Eq. (123) to

$$-\frac{p_s^\circ\dot{q}_s^\circ}{\dot{C}_o^\circ} = -\frac{a}{1-a}(1)^{(\rho-1)}\left[\left(\frac{1}{1-a}\right)(1)^{\rho} - \left(\frac{a}{1-a}\right)(1)^{\rho}\right]^{(1-\rho)/\rho} \; . \tag{124}$$

Simplifying gives

$$\frac{p_s^\circ\dot{q}_s^\circ}{\dot{C}_o^\circ} = \frac{a}{1-a} \; . \tag{125}$$

Recognizing that $p_s^\circ\dot{q}_s^\circ = \dot{C}_s^\circ$ and solving for a gives

$$a = \frac{\dot{C}_s^\circ}{\dot{C}_s^\circ + \dot{C}_o^\circ} \; , \tag{126}$$

which is called $f_{\dot{C}_s}^\circ$, the share of energy service expenditure ($\dot{C}_s^\circ$) relative to the sum of energy service and other goods expenditures ($\dot{C}_s^\circ+\dot{C}_o^\circ$) before emplacement. Thus, the CES utility equation (Eq. (116)) becomes

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ\left(\frac{\dot{q}_s}{\dot{q}_s^\circ}\right)^{\rho} + (1 - f_{\dot{C}_s}^\circ)\left(\frac{\dot{C}_o}{\dot{C}_o^\circ}\right)^{\rho}\right]^{(1/\rho)} \; , \tag{14}$$

with

$$f^{\circ}_{\dot{C}_s} \equiv \frac{\dot{C}^{\circ}_s}{\dot{C}^{\circ}_s + \dot{C}^{\circ}_o} \; . \tag{127}$$

C.2 Elasticities for the substitution effect ($\epsilon_{\dot{q}_s,p_s,c}$, $\epsilon_{\dot{q}_o,p_s,c}$, and σ)

The substitution effect requires a utility model. In this paper, we describe two utility models: an approximate model and a CES model. All utility models require elasticities to describe consumer preferences. In fact, there are three elasticies across the two utility models: $\epsilon_{\dot{q}_s,p_s,c}$, $\epsilon_{\dot{q}_o,p_s,c}$, and σ. The approximate utility model requires the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated cross-price elasticity of energy service demand ($\epsilon_{\dot{q}_o,p_s,c}$). The CES utility model requires the elasticity of substitution (σ) between energy service consumption and other goods consumption. Because the uncompensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s}$) tends to be more widely estimated, e.g. in studies about driver behavior (Gillingham, 2020), the usual approach is to use $\epsilon_{\dot{q}_s,p_s}$ and the Slutsky equation to derive the compensated elasticity ($\epsilon_{\dot{q}_s,p_s,c}$). The CES utility model then imposes enough restrictions to determine the elasticity of substitution.

C.2.1 Elasticities for the approximate utility model ($\epsilon_{\dot{q}_s,p_s,c}$ and $\epsilon_{\dot{q}_o,p_s,c}$)

Analytical expressions for the compensated elasticities in the approximate utility model can be derived using the Slutsky equation, whereby the price elasticity of the energy service ($\epsilon_{\dot{q}_s,p_s}$) is decomposed into the compensated price elasticity ($\epsilon_{\dot{q}_s,p_s,c}$) and the income elasticity ($\epsilon_{\dot{q}_s,\dot{M}}$) as follows:

$$\epsilon_{\dot{q}_s,p_s} = \epsilon_{\dot{q}_s,p_s,c} - f^{\circ}_{\dot{C}_s} \epsilon_{\dot{q}_s,\dot{M}} \; , \tag{128}$$

where $f^{\circ}_{\dot{C}_s}$ is given by Eq. (127), and $\epsilon_{\dot{q}_s,\dot{M}}$ is given in Section C.3. Solving for the compensated price elasticity ($\epsilon_{\dot{q}_s,p_s,c}$) gives

$$\epsilon_{\dot{q}_s,p_s,c} = \epsilon_{\dot{q}_s,p_s} + f^{\circ}_{\dot{C}_s} \epsilon_{\dot{q}_s,\dot{M}} \; . \tag{129}$$

A similar argument allows straightforward derivation of the compensated cross-price elasticity ($\epsilon_{\dot{q}_o,p_s,c}$). With Hicks & Allen (1934), we note that the uncompensated cross-price elasticity ($\epsilon_{\dot{q}_o,p_s}$) can generally be expressed as

$$\epsilon_{\dot{q}_o,p_s} = f^{\circ}_{\dot{C}_s} (\sigma - \epsilon_{\dot{q}_o,\dot{M}}) \; , \tag{130}$$

where σ is the elasticity of substitution between the consumption rate of the energy service ($\dot{q}_s$) and the consumption rate of other goods ($\dot{q}_o$). We set Eq. (130) equal to the cross-price version of the Slutsky equation

$$\epsilon_{\dot{q}_o,p_s} = \epsilon_{\dot{q}_o,p_s,c} - f^{\circ}_{\dot{C}_s} \epsilon_{\dot{q}_o,\dot{M}} \; , \tag{131}$$

to obtain

$$f^{\circ}_{\dot{C}_s} (\sigma - \epsilon_{\dot{q}_o,\dot{M}}) = \epsilon_{\dot{q}_o,p_s,c} - f^{\circ}_{\dot{C}_s} \epsilon_{\dot{q}_o,\dot{M}} \; . \tag{132}$$

Solving for $\epsilon_{\dot{q}_o,p_s,c}$ gives

$$\epsilon_{\dot{q}_o,p_s,c} = f^{\circ}_{\dot{C}_s} \sigma \; . \tag{133}$$

Substituting σ from Section C.2.2 (Eq. (135)) gives

$$\epsilon_{\dot{q}_o,p_s,c} = \frac{f^{\circ}_{\dot{C}_s}\left(f^{\circ}_{\dot{C}_s} + \epsilon_{\dot{q}_s,p_s}\right)}{f^{\circ}_{\dot{C}_s} - 1} \ . \tag{134}$$

C.2.2 Elasticity for the CES utility model (σ)

Gørtz (1977) shows that the elasticity of substitution (σ) in the CES utility model is given by

$$\sigma = \frac{f^{\circ}_{\dot{C}_s} + \epsilon_{\dot{q}_s,p_s}}{f^{\circ}_{\dot{C}_s} - 1} \ . \tag{135}$$

Thus, the elasticity of substitution (σ) can be determined from two pieces of readily available information: (i) the uncompensated own price elasticity ($\epsilon_{\dot{q}_s,p_s}$) and (ii) the share of income spent on the energy service ($f^{\circ}_{\dot{C}_s}$ from Eq. (127)).

C.3 Elasticities for the income effect ($\epsilon_{\dot{q}_s,\dot{M}}$ and $\epsilon_{\dot{q}_o,\dot{M}}$)

The income effect requires two elasticities to estimate the spending of net savings: the income elasticity of energy service consumption ($\epsilon_{\dot{q}_s,\dot{M}}$) and the income elasticity of other goods consumption ($\epsilon_{\dot{q}_o,\dot{M}}$). Due to the homotheticity assumption, both income elasticities are unitary. Thus,

$$\epsilon_{\dot{q}_s,\dot{M}} = 1 \ , \tag{136}$$

and

$$\epsilon_{\dot{q}_o,\dot{M}} = 1 \ . \tag{137}$$

D Proof: Income preference equations satisfy the budget constraint

After the substitution effect, a rate of net savings is available ($\hat{\dot{N}}$), all of which is spent on additional energy service ($\Delta\bar{\dot{q}}_s$, $\Delta\bar{\dot{C}}_s = p_E \Delta\bar{\dot{E}}_s$) or additional other goods ($\Delta\bar{\dot{q}}_o$, $\Delta\bar{\dot{C}}_o$). The income effect must satisfy the budget constraint such that net savings is zero afterward ($\bar{\dot{N}} = 0$). The budget constraint across the income effect is represented by Eq. (67):

$$\hat{\dot{N}} = p_E \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o \ . \tag{67}$$

The additional spending due to the income effect is given by income preference equations

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} \tag{23}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} \ , \tag{27}$$

where

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \ . \tag{24}$$

This appendix proves that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (67)).

The first step in the proof is to convert the income preference equations to $\dot{C}^\circ_s$ and $\dot{C}^\circ_o$ ratios. For the energy service income preference equation (Eq. (23)), multiply numerator and denominator of the left-hand side by $\tilde{p}_s = p_E/\tilde{\eta}$ (Eq. (6)) to obtain $\bar{\dot{C}}_s/\hat{\dot{C}}_s$. For the other goods income preference equation (Eq. (27)), multiply numerator and denominator of the left-hand side by p_o to obtain $\bar{\dot{C}}_o/\hat{\dot{C}}_o$. Then, invoke homotheticity to set $\epsilon_{\dot{q}_s,\dot{M}} = 1$ and $\epsilon_{\dot{q}_o,\dot{M}} = 1$ to obtain

$$\frac{\bar{\dot{C}}_s}{\hat{\dot{C}}_s} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \tag{138}$$

and

$$\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \ . \tag{139}$$

The second step in the proof is to obtain expressions for $\Delta\bar{\dot{C}}_s$ and $\Delta\bar{\dot{C}}_o$. Multiply the income preference equations above by $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively. Then, subtract $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively, to obtain

$$\Delta\bar{\dot{C}}_s = \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} \tag{140}$$

and

$$\Delta\bar{\dot{C}}_o = \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \ . \tag{141}$$

The above versions of the income preference equations can be substituted into the budget constraint (Eq. (67)) to obtain

$$\hat{\dot{N}} \stackrel{?}{=} \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} + \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \ . \tag{142}$$

If equality is demonstrated, the income preference equations satisfy the budget constraint. The remainder of the proof shows the equality of Eq. (142).

Dividing by $\hat{\dot{N}}$ and multiplying by $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \stackrel{?}{=} \hat{\dot{M}}' \ . \tag{143}$$

Substituting Eq. (24) for $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \stackrel{?}{=} \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \ . \tag{144}$$

Substituting Eq. (52) for $\dot{M}^\circ$, because $\dot{M}^\circ = \dot{\hat{M}}$, gives

$$\dot{\hat{C}}_s + \dot{\hat{C}}_o \stackrel{?}{=} p_E \dot{\hat{E}}_s + \dot{\hat{C}}_{cap} + \dot{\hat{C}}_{md} + \dot{\hat{C}}_o + \cancel{\dot{\hat{N}}} - \dot{C}^*_{cap} - \dot{C}^*_{md} - \cancel{\dot{\hat{N}}} \; . \tag{145}$$

Cancelling terms and recognizing that $\dot{C}^*_{cap} = \dot{\hat{C}}_{cap}$, $\dot{C}^*_{md} = \dot{\hat{C}}_{md}$, and $\dot{\hat{C}}_s = p_E \dot{\hat{E}}_s$ gives

$$\dot{\hat{C}}_s + \dot{\hat{C}}_o \stackrel{?}{=} \dot{\hat{C}}_s + \cancel{\dot{\hat{C}}_{cap}} + \cancel{\dot{\hat{C}}_{md}} + \dot{\hat{C}}_o - \cancel{\dot{\hat{C}}_{cap}} - \cancel{\dot{\hat{C}}_{md}} \; . \tag{146}$$

Cancelling terms gives

$$\dot{\hat{C}}_s + \dot{\hat{C}}_o \stackrel{\checkmark}{=} \dot{\hat{C}}_s + \dot{\hat{C}}_o \; , \tag{147}$$

thereby completing the proof that the income preference equations (Eqs. (23) and (27)) satisfy the budget constraint (Eq. (67)).

E Other goods expenditures and constant p_o

This framework utilizes a partial equilibrium analysis in which we account for the change of the energy service price due to the EEU ($p_s^\circ \neq p_s^*$), but we do not track the effect of the EEU on prices of other goods. These assumptions have important implications for the relationship between the rate of consumption of other goods ($\dot{q}_o$) and the rate of expenditure on other goods ($\dot{C}_o$).

We assume a basket of other goods (besides the energy service) purchased in the economy, each (i) with its own price ($p_{o,i}$) and rate of consumption ($\dot{q}_{o,i}$), such that the average price of all other goods purchased in the economy prior to the EEU (p_o°) is given by

$$p_o^\circ = \frac{\sum\limits_i p_{o,i}^\circ q_{o,i}^\circ}{\sum\limits_i q_{o,i}^\circ} \; . \tag{148}$$

Then, the expenditure rate of other purchases in the economy can be given as

$$\dot{C}_o^\circ = p_o^\circ \dot{q}_o^\circ \tag{149}$$

before the EEU and

$$\dot{\hat{C}}_o = \hat{p}_o \dot{\hat{q}}_o \tag{150}$$

after the substitution effect, for example.

We assume that any effects (emplacement, substitution, or income) for a single device are not so large that they cause a measurable change in prices of other goods. Thus,

$$p_o^\circ = p_o^* = \hat{p}_o = \bar{p}_o = \tilde{p}_o \; . \tag{151}$$

In the partial equilibrium analysis, two other goods prices can be equated across any rebound effect to obtain (for the example of the original conditions ($\circ$) and the post-substitution state ($\wedge$))

$$\frac{\dot{\hat{C}}_o}{\dot{C}_o^\circ} = \frac{\dot{\hat{q}}_o}{\dot{q}_o^\circ} \; . \tag{152}$$

Thus, a ratio of other goods expenditure rates is always equal to a ratio of other goods consumption rates.

F Income and macro effects and relation to the marginal propensity to consume (MPC)

Borenstein (2015) has postulated a demand-side argument that macro effects can be represented by a multiplier, which we call the macro factor (k). Borenstein's formulation and our implementation are reminiscent of the marginal propensity to consume (MPC). In this appendix, we show the relationship between the macro factor (k) and MPC.

The relationship between the macro factor (k) and MPC spans the income and macro effects. In this framework, the device owner's net savings after the substitution effect ($\hat{\dot{N}}$) is respent completely. One may assume that firms and other consumers who receive the net savings have a marginal propensity to re-spend of MPC. The total spending throughout the economy of each year's net savings ($\hat{\dot{N}}$) is given by the infinite series

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\hat{\dot{N}} \ , \tag{153}$$

where the first term ($1 \times \hat{\dot{N}}$) represents spending of net savings by the device owner in the direct and indirect income effects, and the remaining terms $[(MPC + MPC^2 + MPC^3 + \ldots)\hat{\dot{N}}]$ represent macro-effect spending in the broader economy.

The macro effect portion of the spending can be represented by the macro factor (k).

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\hat{\dot{N}} = (1 + k)\hat{\dot{N}} \tag{154}$$

Canceling $\hat{\dot{N}}$ and simplifying the infinite series to its converged fraction (assuming $MPC < 1$) gives

$$\frac{1}{1 - MPC} = 1 + k \ . \tag{155}$$

Solving for k yields

$$k = \frac{1}{\frac{1}{MPC} - 1} \ . \tag{31}$$

With $MPC = 0.5$, as in Section 2.4.4, $k = 1$ is obtained. If $k = 3$, as in Part II, $MPC = 0.75$ is implied. The relationship between k and MPC is given in Fig. F.1.

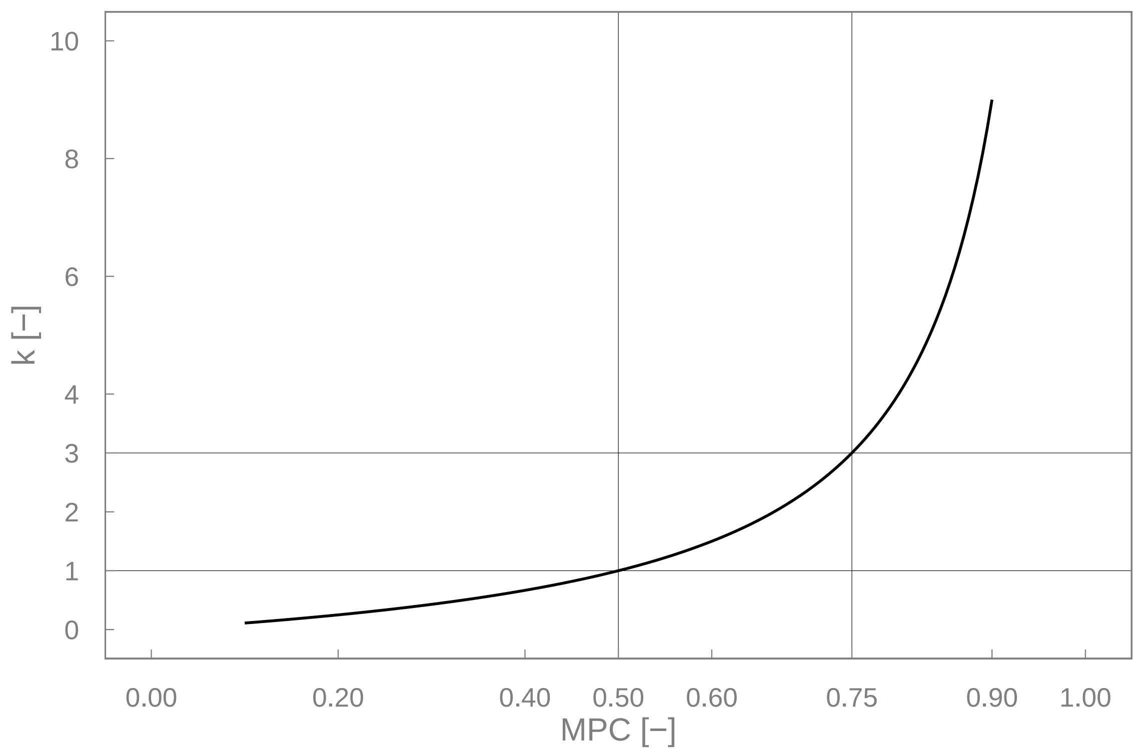

Fig. F.1: The relationship between MPC and k in Eq. (31).