PB comment 2 Dec 2022 → (*) denotes more than a 1 line/word change, if implemented. I have numbered these. Instructions/queries are (in brackets)

Advancing the necessary foundations for empirical energy rebound estimates, Part II: Visualization, examples, and results

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(Same comment as Part I)

Abstract

Widespread implementation of energy efficiency is a key greenhouse gas emissions mitigation measure, but rebound can "take back" energy savings. However, ~~conceptual foundations lag behind~~ empirical estimates of the size of rebound. A new clarity is needed, one that is built upon solid analytical frameworks involving both economics and energy analysis. In Part I, we developed a rigorous analytical framework built upon the microeconomics of rebound that is approachable for both energy analysts and economists alike. In this paper (Part II), we bring further clarity to energy rebound by developing rebound path graphs, a novel way to visualize and illustrate rebound phenomena through energy, expenditure, and consumption spaces. Further, we perform the first calibration of the macro factor for macroeconomic rebound, finding $k \approx 3$. Finally, we apply the framework developed in Part I to provide estimates of total rebound for two case studies: upgrades of a car (48%) and an electric lamp (80%). Comparison of the estimates to previously estimated values is provided.

(See my query later asking for old 4.3 in WP to be reinstated as 4.4. Showing the effects of key variables could even make it into the abstract)

Keywords: Energy efficiency, Energy rebound, Energy services, Microeconomic rebound, Substitution and income effects, Macroeconomic rebound

1 Introduction

In Part I of this two-part paper, we argued that improved clarity is needed about the effects and scales of energy rebound. We said that

> a description of rebound is needed that is both (i) technically rigorous and (ii) approachable from both sides (economics and energy analysis). In other words, the finance and human behavior aspects of rebound need to be presented in ways energy analysts can understand. And the energy aspects of rebound need to be presented in ways economists can understand.

To move improve clarity in the rebound space, in Part I of this two-part paper we developed a partial equilibrium framework for analyzing energy rebound, one that is tractable for energy

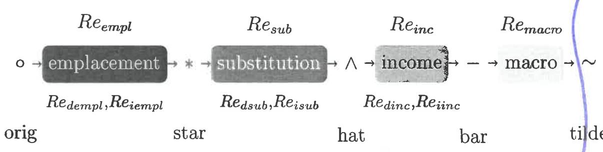

Fig. 1: Flowchart of rebound effects and decorations.

(not sure I like the 'locations'. (they only apply at subst + income stage?) delete for simplicity.)

analysts and economists alike. The framework contains ~~two locations (direct, d, and indirect, i) and~~ four rebound effects (emplacement, substitution, income, and macro) between five stages (o, *, ∧, –, and ~). Rebound terms and symbol decorations are shown in Fig. 1. (See Part I for details.)

In this paper (Part II), we make further progress toward the goal of clarity in three ways. First, we develop a new way to visualize components and mechanisms of rebound (rebound path graphs). Second, we apply the framework to two energy efficiency upgrades (EEUs) (an electric lamp and an automobile) with detailed explication of the examples. Finally, we provide estimates of rebound magnitudes for both examples.

(Also point to Table 1?)

(+ develop standard terms 'plane')

The key contributions of Part II are (i) development of the first (to our knowledge) visualizations of rebound effects in energy, expenditure, and consumption spaces, (ii) documentation of new estimates of rebound for automobile and lighting upgrades, and (iii) creation of open source software tools to calculate and visualize rebound for any EEU.

The remainder of this paper is structured as follows. Section 2 describes data for examples, our method of visualizing rebound, and software tools for calculating and visualizing rebound. Section 3 provides two examples: energy efficiency upgrades to a car and an electric lamp. Section 4 discusses the results, and Section 5 concludes.

2 Data and methods

(Sect 2.1)

The section contains data for the examples, an explication of our method for visualizing rebound effects and magnitudes, and a description of software tools for rebound calculations and visualization. (Sect 2.3) (Sect 2.2)

2.1 Data

To demonstrate application of the rebound analysis framework developed ~~in Section 2 of~~ in Part I, we analyze two case studies: energy efficiency upgrades to a car and an electric lamp. The case studies are presented with much detail here to support our goal of bringing clarity to the process of estimating the magnitude of rebound effects. Here, we collect parameter values for the equations for eight rebound components: Re_{dempl}, Re_{emb}, Re_{md}, Re_{dsub}, Re_{isub}, Re_{dinc}, Re_{iinc}, and Re_{macro}. The ~~total rebound~~ (Re_{tot}) is given by the sum of the above components.

2.1.1 Data for car example

For the first example, we consider the purchase of a more fuel efficient car, namely a gasoline-electric Ford Fusion Hybrid car, to replace a conventional gasoline Ford Fusion car. The cars are matched as closely as possible, except for the inclusion of an electric battery in the hybrid car. The car

((*1) Confusing maybe from this. As a reader (1) eqn (34) Part I is different. I was expecting the same? (2) lines 867–874 (Part I) says we only need 7 values, not 8.)

economic

case study features a larger initial capital investment ($C^{\circ}_{cap} < \tilde{C}_{cap}$) versus the long-term benefit of decreased energy service costs ($\dot{C}^{\circ}_{s} > \dot{\tilde{C}}_{s}$).

We require three sets of data. First, basic car parameters are summarized in Table 1. Second, we require several general parameters, mainly relating to the U.S. economy and personal finances of the average U.S. citizen, shown in Table 2. Third, we require elasticity parameters, as given in Table 3.

Table 1

Table 1: Car example: Vehicle parameters.

Description Parameters [units]	Ford Fusion (gasoline)	Ford Fusion (hybrid EV)	Data sources and notes
Fuel economy η°, $\tilde{\eta}$ [mpg]	25	42	Combined cycle mpg value taken from Thecarconnection.com (2020), for Titanium FWD 2020 model with Intercooled I-4, 2.0 L engine. Combined cycle mpg value taken from Thecarconnection.com (2020), for Titanium FWD 2020 model with Gas/Electric I-4, 2.0 L engine.
Capital expenditure rate $\dot{C}^{\circ}_{cap}$, $\dot{\tilde{C}}_{cap}$ [\$/year]	4,778	4,720	Seven year annual, averaged capital costs = purchase + finance costs − resale value (purchase − depreciation). Ford Fusion gasoline costs from Edmunds.com (2020a). Ford Fusion Hybrid car costs from Edmunds.com (2020b).
Ownership duration t°_{own}, $\tilde{t}_{own}$ [years]	7	7	U.S. car ownership (from new) length from Businesswire.com (2015), and has risen from 52 months (2005) to 79 months (2015), so taken as 84 months in 2020 (7 years) for our example.
Lifespan t°_{life}, $\tilde{t}_{life}$ [years]	14	14	Lifetime taken as 14 years, based on 13–17 years for U.S. cars from Berla.com (2016) and 14 years for UK cars from Society of Motor Manufacturers and Traders (2020).
Embodied energy E°_{emb}, $\tilde{E}_{emb}$ [MJ]	34,000	40,000	34,000 MJ for conventional Ford Fusion gasoline car taken from Argonne National Laboratory, Energy Systems Division (2010). We assume an additional 6,000 MJ added for Ford Fusion Hybrid Electric Vehicle (HEV) battery, as HEV typically adds 10–25% to total LCA energy of vehicle manufacture (Onat et al., 2015). Battery lifetime assumed same as car lifetime, based on Nordelöf et al. (2014) and Onat et al. (2015).
Maintenance and disposal expenditure rate $\dot{C}^{\circ}_{md}$, $\dot{\tilde{C}}_{md}$ [\$/year]	2,731	2,710	Seven year annual, averaged maintenance costs = sum of insurance, maintenance, repairs, taxes, and fees (excluding financing, depreciation, fuel). Ford Fusion maintenance costs from Edmunds.com (2020a). Ford Fusion Hybrid maintenance costs from Edmunds.com (2020b).

Table 2: Car example: Economic parameters (2020).

Description Parameter [units]	Value	Data sources and notes
Distance driven prior to upgrade $\dot{q}_s^\circ$ [miles/year]	12,416	Average U.S. vehicle miles/year, calculated from Carinsurance.com (2019). This is slightly lower than the average driver miles/year (13,476) (US Department of Transportation, 2018), as there are more registered U.S. vehicles than drivers.
Real median personal income U.S., in 2018 [$/year]	34,317	Taken from Federal Reserve Bank of St Louis (2019).
U.S. 2018 disposable income / real income (minus current taxes) [–]	0.88319	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition (US Bureau of Economic Analysis, 2020).
Share of savings from 2018 disposable income [–]	0.07848	Taken from U.S. Bureau of Economic Analysis (BEA) National and Products Accounts (NIPA) Table 2.1. Personal Income and Its Disposition (US Bureau of Economic Analysis, 2020).
Personal consumption in 2018 $\dot{M}$ [$/year]	27,929.83	Calculation: ($34,317/year)(0.88319)(1 − 0.07848)
Price of gasoline p_E [$/gallon]	2.63	Source: US Energy Information Administration (2020b)
Fractional spend on original energy service $f_{\dot{C}_s}$ [–]	0.064	Calculation: $1,306 (spend on energy service) / [$19,115 (other goods) + $1,306 (energy service)] = 0.064, where spend on energy service = 12,416 miles / 25 mpg × $2.63/gallon = $1,306.
Macro factor k [–]	1.0	Assumed value.

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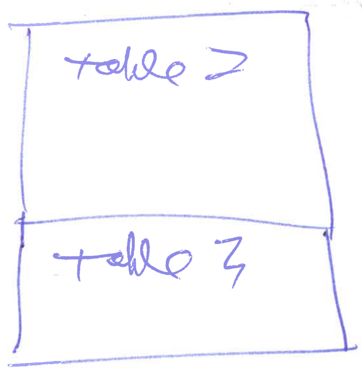
(should we say 'initial value'? as later we calibrate k.)

Table 3: Car example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Price elasticity of car use demand $\epsilon_{\dot{q}_s, p_s}$ [–]	−0.2	We adopt −0.2 as our baseline value, based on U.S. studies including Gillingham (2020) who estimated a value of −0.1, Goetzke & Vance (2018) who estimated values between −0.05 and −0.23, and Parry & Small (2005) who estimated values between −0.1 and −0.3. For comparison, Borenstein (2015) uses values of −0.1 to −0.4 based on Parry & Small (2005).
Compensated price elasticity of car use demand $\epsilon_{\dot{q}_s, p_s, c}$ [–]	−0.136	Calculated via the Slutsky Equation (Eq. (128) in Part I).
Compensated cross-price elasticity of demand for other goods $\epsilon_{\dot{q}_o, p_s, c}$ [–]	0.009	Calculated via Eq. (134) in Part I.
Income elasticity of demand for car use $\epsilon_{\dot{q}_s, \dot{M}}$ [–]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\epsilon_{\dot{q}_o, \dot{M}}$ [–]	1.0	Follows from CES utility function.



2.1.2 Data for lamp example

For the second example, we consider purchasing a Light Emitting Diode (LED) electric lamp to replace a baseline incandescent electric lamp. Both lamps are matched as closely as possible in terms of energy service delivery (measured in lumen output per lamp), the key difference being the energy required to provide that energy service. The LED lamp has a low initial capital investment rate (less than the incumbent incandescent lamp, actually) and a long-term benefit of decreased direct energy expenditures at the same energy service delivery rate (lm-hr/year).

Again, three sets of data are required. First, basic lamp parameters are summarized in Table 4. Second, several general parameters, mainly relating to the U.S. economy and personal finances of the average U.S. citizen are given in Table 5. Third, we require the elasticity parameters, as shown in Table 6.

economic

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Table 4: Lamp example: Electric lamp parameters.

Description Parameters [units]	Incandescent lamp	LED lamp	Data sources and notes
Lamp efficacy η°, $\tilde{\eta}$ [lm-hr/W-hr]	8.83	81.8	Incandescent: 530 lm output / 60 W energy input. LED: 450 lm output / 5.5 W energy input.
Capital expenditure rate $\dot{C}^{\circ}_{cap}$, $\dot{C}^{*}_{cap}$ [$/year]	1.044	0.121	Purchase costs: $1.88 for incandescent lamp from HomeDepot.com (2020b), and $1.21 for LED lamp from HomeDepot.com (2020a).
Ownership duration t°_{own}, t^{*}_{own} [years]	1.8	10	Assumed same as lamp lifespan
Lifespan t_{life}, t^{*}_{life} [years]	1.8	10	Based on assumed 3 hours/day from HomeDepot.com (2020b) and HomeDepot.com (2020a).
Life cycle analysis (LCA) embodied energy E°_{emb}, E^{*}_{emb} [MJ]	2.20	6.50	Base document: Table 4.5 Manufacturing Phase Primary Energy (MJ/20 million lumen-hours), contained in U.S. DoE Life-cycle assessment of energy and environmental impacts of LED lighting products (US Department of Energy, 2012). Incandescent lamp: LCA energy = 42.2 MJ/20 million lumen-hours. Lifetime output = 530 lumens × 3 hours/day × 365 days/year × 1.8 years = 1,044,630 lumen-hrs. Thus LCA energy / lamp = 42.2 × 1.0446/20 = 2.21 MJ. LED lamp: LCA energy = 132 MJ/20 Million lumen-hours for pack of 5 LED lamps. Lifetime output = 450 lumens × 3 hours/day × 365 days/year × 10 years = 4,926,405 lumen-hrs. Thus LCA energy / lamp = 132 MJ/5 × 4.9264/20 = 6.5 MJ.
Maintenance and disposal expenditure rate $\dot{C}^{\circ}_{md}$, $\dot{C}^{*}_{md}$ [$/year]	0.00	0.00	Assumed negligible.

Table 5: Lamp example: Economic parameters (2020).

Description Parameter [units]	Value	Data sources and notes
Lighting consumption prior to upgrade $\dot{q}_s^{\circ}$ [lm-hr/year]	580,350	Calculation: (530 lm) (3 hrs/day) (365 days/year).
Real median personal income U.S. in 2018 [$/year]	34,317	Refer to Table 2.
U.S. 2018 disposable income / real income (minus current taxes) [–]	0.88319	Refer to Table 2.
Share of savings from 2018 disposable income [–]	0.07848	Refer to Table 2.
Personal consumption in 2018 $\dot{M}$ [$/year]	27,929.83	Calculation: ($34,317/year)(0.88319)(1 − 0.07848).
Price of electricity p_E [$/kW-hr]	0.1287	U.S. 2018 average U.S. household electricity price (US Energy Information Administration, 2020a).
Fractional spend on original energy service $f_{C_s}^{\circ}$ [–]	0.0003028	Calculation: $8.5/year (spend on energy service) / [$27,920/year (other goods) + $8.5/year (energy service)] = 0.00030, where spend on energy service = 580,350 lm-hrs/year / 8.83 lm/W / 1000 W/kW × $0.1287/kW-hr = $8.5/year.
Macro factor k [–]	1.0	Assumed value.

(move to 1 page of 2 Tables?)

Table 6

(Should we say 'initial' value? later we calibrate k?)

Table 6: Lamp example: Elasticity parameters.

Description Parameter [units]	Value	Data sources and notes
Price elasticity of lighting demand $\epsilon_{\dot{q}_s,p_s}$ [–]	−0.4	We adopt −0.4 as our baseline value, as the average of last 50 years from Fouquet (2014, Fig. 4). For comparison, Borenstein (2015) uses a range of −0.4 to −0.8, based on Fouquet & Pearson (2011).
Compensated price elasticity of lighting demand $\epsilon_{\dot{q}_s,p_s,c}$ [–]	−0.3997	Calculated via the Slutsky Equation (Eq. (128) in Part I).
Compensated cross-price elasticity of demand for other goods $\epsilon_{\dot{q}_o,p_s,c}$ [–]	0.00012	Calculated via Eq. (134) in Part I.
Income elasticity of lighting demand $\epsilon_{\dot{q}_s,\dot{M}}$ [–]	1.0	Follows from CES utility function.
Income elasticity of demand for other goods $\epsilon_{\dot{q}_o,\dot{M}}$ [–]	1.0	Follows from CES utility function.

(#7) (my idea here is to space out so the a, b, *, c, ^, d align vertically) (to address JUDF query)

Table 7: Rebound path graph segments.

Segments
o—a
a--b
b—*
*-c
c-^
^-d

Segment	Rebound effect	Symbol
$\circ$ — a	Direct emplacement	Re_{dempl}
$a \cdots b$	Embodied energy	Re_{emb}
b — $*$	Maintenance and disposal	Re_{md}
$*$ — c	Indirect substitution	Re_{isub}
c — $\wedge$	Direct substitution	Re_{dsub}
$\wedge$ — d	Direct income	Re_{dinc}
d — –	Indirect income	Re_{iinc}
– — $\sim$	Macro	Re_{macro}

2.2 Visualization

Any rebound analysis should track energy, expenditure, and consumption at the device (direct location) and elsewhere in the economy (indirect location) across all adjustments for all rebound effects. Doing so involves many terms and much complexity. Until now, visualizing the energy, expenditure, and consumption aspects of rebound phenomena has not been possible in single graphs.

We introduce rebound path graphs to bring clarity to the rebound locations ~~(direct and indirect)~~ ~~and adjustments~~ rebound effects (via emplacement, substitution, income, and macro effects) across all ~~spaces~~ (energy, expenditure, and consumption). In rebound path graphs, each space is represented by a plane that contains a path that shows adjustments to energy, expenditure, or consumption in response to the EEU.

Axes for the rebound path graphs are formed from important rebound quantities. Effects ~~at the direct location~~ direct are placed on the x-axis, and ~~effects at the indirect location~~ indirect effect are placed on the y-axis. Thus, (i) direct and indirect energy consumption rates ($\dot{E}_{dir}$, $\dot{E}_{indir}$) are placed on the x- and y-axes of the energy path graph, respectively; (ii) direct and indirect expenditure rates ($\dot{C}_{dir}$ and $\dot{C}_{indir}$) are placed on the x- and y-axes of the expenditure path graph, respectively; and (iii) the consumption rate of the energy service ($\dot{q}_s$) and the expenditure rate on other consumption goods ($\dot{C}_o$) are placed on the x- and y-axes of the consumption path graph, respectively. Paths through energy, expenditure, and consumption ~~spaces~~ consist of segments that represent changes due to the various rebound effects. Table 7 provides the key for rebound path graph segments. (See Appendix B for detailed mathematical descriptions of rebound path graphs.)

placed on each + placed

Figs. 2–4 show notional rebound path graphs in energy, expenditure, and consumption spaces, respectively. The notional path graphs are not quantified, i.e. there are no scales on the axes. Later (Section 3), rebound path graphs (with scales) illustrate the numerical examples.

2.2.1 Energy path graphs

Fig. 2 shows a notional energy path graph, with the direct energy consumption rate ($\dot{E}_{dir}$) on the x-axis and the indirect energy consumption rate ($\dot{E}_{indir}$) on the y-axis. Points $\circ$, $*$, $\wedge$, $-$, and $\sim$ represent the rebound stages between the rebound effects. Points a, b, c, and d represent intermediate stages. Lines with negative slope through points $\circ$, a, $*$, $\wedge$, $-$, and $\sim$ indicate energy consumption isoquants (sum of direct and indirect components) at key points. Table 7 shows segments and rebound effects for all rebound path graphs. Note that segment $-$ — $\sim$ appears only on energy path graphs, because the framework tracks energy consumption but not expenditures or consumption in the macro effect.

In the notional energy path graph of Fig. 2, point a lies on the $Re_{tot} = 0\%$ line indicating that

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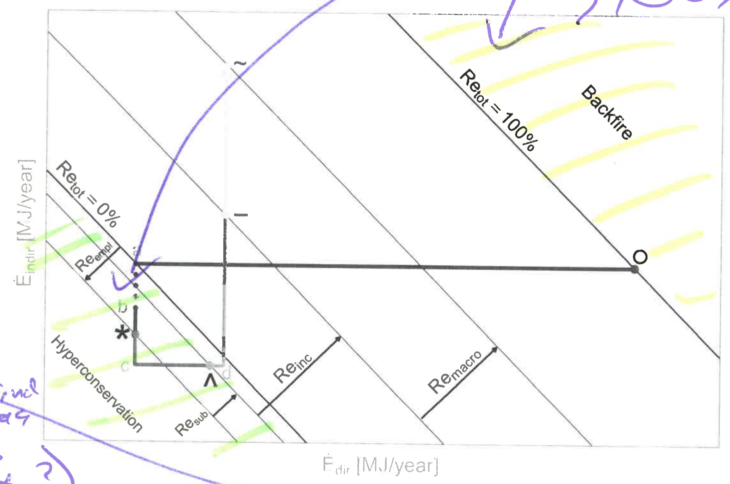

Fig. 2: Notional energy path graph.

point a (and the $Re_{tot} = 0\%$ line) is the point from which all rebound effects (Re_{empl}, Re_{sub}, Re_{inc}, and Re_{macro}) are measured. If rebound effects cause total energy demand to return to the original energy consumption level (negative sloping line through the $\circ$ point), all expected energy savings are taken back by rebound effects. Thus, the line of constant energy consumption through the $\circ$ point is labeled $Re_{tot} = 100\%$. The contribution of each rebound effect to total rebound is represented by the distance that each component's segment moves across the rebound isoquants. Total rebound (Re_{tot}) is measured linearly between and beyond the $Re_{tot} = 0\%$ and $Re_{tot} = 100\%$ lines, with direct rebound in the x direction and indirect rebound in the y direction. The region below and to the left of the $Re_{tot} = 0\%$ line in Fig. 2 exhibits negative rebound, indicating hyperconservation. The region above and to the right of the $Re_{tot} = 100\%$ line shows backfire, i.e. greater total energy consumption after the EEU than before it.

In the notional energy path graph (Fig. 2), emplacement rebound is negative ($Re_{empl} < 0$), because the upgraded device has a lesser embodied energy rate ($\dot{E}^{\circ}_{emb} > \dot{E}^{*}_{emb}$) and a lesser maintenance and disposal expenditure rate ($\dot{C}^{\circ}_{md} > \dot{C}^{*}_{md}$) than the original device. Fig. 3 shows segments $a \cdots b$ and b — $*$ moving in the negative y direction, consistent with a lower capital expenditure rate ($\dot{C}^{\circ}_{cap} > \dot{C}^{*}_{cap}$) and a reduced maintenance and disposal expenditure rate ($\dot{C}^{\circ}_{md} > \dot{C}^{*}_{md}$).

In the notional energy path graph of Fig. 2, the upgraded device has a lower embodied energy rate than the original device, as shown by point b being below point a.

2.2.2 Expenditure path graphs

A notional expenditure path graph is shown in Fig. 3, with the direct expenditure rate on the energy service ($\dot{C}_{dir}$) on the x-axis and the indirect expenditure rate ($\dot{C}_{indir}$) on the y-axis. Lines with negative slope through points $\circ$, a, $*$, and $\wedge$ indicate expenditure isoquants (sum of direct and

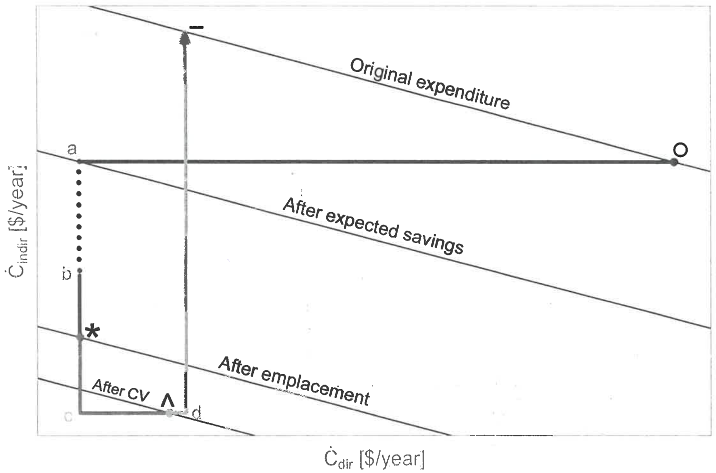

Fig. 3: Notional expenditure path graph. CV is compensating variation. (i.e. . . .)

indirect components).

In the notional graphs of Figs. 2 and 3, embodied energy rates and capital cost rates (represented by segments $a \cdots b$) move in the same direction (both in the negative y direction). However, both segments $a \cdots b$ could move in the positive y direction, or they could move in opposite directions, depending on the results of the independent analyses for embodied energy and capital cost rates.

2.2.3 Consumption path graphs

A notional consumption path graph is shown in Fig. 4. The indexed rate of energy service demand ($\dot{q}_s/\dot{q}_s^\circ$) is shown on the x-axis, and the indexed rate of other goods demand ($\dot{C}_o/\dot{C}_o^\circ$) is shown on the y-axis. Iso-cost loci of energy service and other goods demand are shown as lines with negative slope.

The substitution effect is shown by segments $*$—c (the indirect component) and c—$\wedge$ (the direct component) in Figs. 2–4. Indifference curves are denoted by i°—i° and $\bar{\text{i}}$—$\bar{\text{i}}$. A ray from the origin through the $\wedge$ point is denoted r—r. Note that points $\circ$, a, b, and $*$ collapse together on consumption path graphs, because both the rate of energy service consumption and the rate of other goods consumption are unchanged across the emplacement effect ($\dot{q}_s^\circ = \dot{q}_s^*$ and $\dot{C}_o^\circ = \dot{C}_o^*$, respectively).

Prior to the EEU, the consumption basket (of the energy service and other goods) is represented by the $\circ$ point. The budget constraint, i.e. the amount of money available for final goods purchases, is shown as isoquant $\circ$—$\circ$. The $\circ$—$\circ$ line is tangent to the lower indifference curve (i°—i°) at point $\circ$, the optimal consumption bundle prior to the EEU. The budget line $*$—$*$ indicates the cost of purchasing the original consumption bundle at the new prices. The substitution effect indicates the cheaper, optimal consumption bundle at the $\wedge$ point.

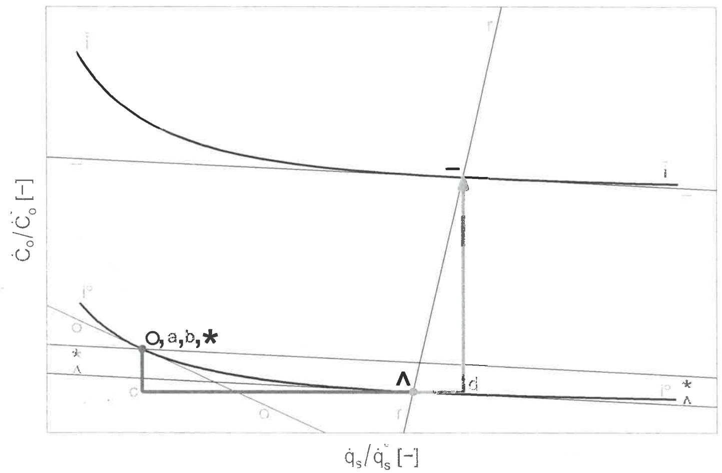

Fig. 4: Notional consumption path graph.

Rebound from the substitution effect is typically decomposed into indirect (the decrease in other goods consumption, segment $*$—c) and direct (the increase in energy service consumption, segment c—$\wedge$) components. The impact of the substitution effect on energy consumption rates and expenditure rates can be seen in Figs. 2 and 3, respectively. The income expansion path is a ray (r—r) on consumption path graphs. In the consumption path graph, the pre- and post-income-effect points ($\wedge$ and $-$, respectively) lie along ray r—r from the origin through point $\wedge$ in Fig. 4.

The increased consumption rate of the energy service is represented by segments $\wedge$—d in Figs. 2–4.

2.3 Software tools

We developed an open source `R` package `ReboundTools` to standardize and distribute our methods for calculating rebound magnitudes. `ReboundTools` can be found at `https://github.com/MatthewHeun/ReboundTools`. (See Heun (2021).) `ReboundTools` provides functions for (i) reading input data from a spreadsheet, (ii) performing rebound calculations, and (iii) generating rebound path graphs. `ReboundTools` was used for all calculations and all rebound path graphs in this paper.

In addition, an Excel workbook that performs rebound calculations using the framework of this paper can be found at the data repository for this paper at `https://doi.org/10.5518/1201`. (See Brockway et al. (2022).)

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3 Results

3.1 Case 1: Purchase of a new car

Armed with the parameter values from Tables 1–3, and the equations in Section 2 of Part I, we calculate important values at each rebound stage, as shown in Table 8. Note that Table 8 applies to the car owner. Across the macro effect (segment – ~ in Fig. 5), changes occur only in the macroeconomy. For the car owner, no changes are observed across the macro effect. Thus, the – (bar) and ~ (tilde) columns of Table 8 are identical.

Table 8: Results for car example with macro factor (k) assumed to be 1.

	∘ (orig)	∗ (star)	∧ (hat)	– (bar)	~ (tilde)
η [mile/gal]	25.0	42.0	42.0	42.0	42.0
η [mile/MJ]	0.197	0.332	0.332	0.332	0.332
p_s [$/mile]	0.105	0.063	0.063	0.063	0.063
$\dot{q}_s$ [mile/year]	12,416	12,416	13,336	13,756	13,756
$\dot{E}_s$ [MJ/year]	62,885	37,432	40,204	41,470	41,470
$\dot{E}_{emb}$ [MJ/year]	2,429	2,857	2,857	2,857	2,857
$\dot{C}_s$ [$/year]	1,306	777	835	861	861
$\dot{C}_{cap}$ [$/year]	4,778	4,720	4,720	4,720	4,720
$\dot{C}_{md}$ [$/year]	2,731	2,710	2,710	2,710	2,710
$\dot{C}_o$ [$/year]	19,115	19,115	19,040	19,639	19,639
$\dot{N}$ [$/year]	0	608	626	0	0
$\dot{M}$ [$/year]	27,930	27,930	27,930	27,930	27,930

Results are represented graphically in quantified energy, expenditure, and consumption path graphs in Figs. 5–7. The energy path graph (Fig. 5) shows the size of each rebound effect for the car example.

Rebound components for the car upgrade are shown in Table 9.

Table 9: Car example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	1.7
Re_{md}	−0.3
Re_{dsub}	10.9
Re_{isub}	−1.0
Re_{dinc}	5.0
Re_{iinc}	7.6
Re_{macro}	8.0
Re_{tot}	31.9

The emplacement effect has three components: the direct emplacement effect, the embodied energy effect, and the maintenance and disposal effect. Rebound from the direct emplacement effect (Re_{dempl}) is 0.0% always, because energy takeback (and, therefore, rebound) occurs after the EEU is emplaced. Indirect rebound due to the embodied energy effect (Re_{emb}) is 1.7%, due to the higher embodied energy rate ($\Delta\dot{E}^*_{emb}$ = 429 MJ/year) stemming from the electric battery in the hybrid EV

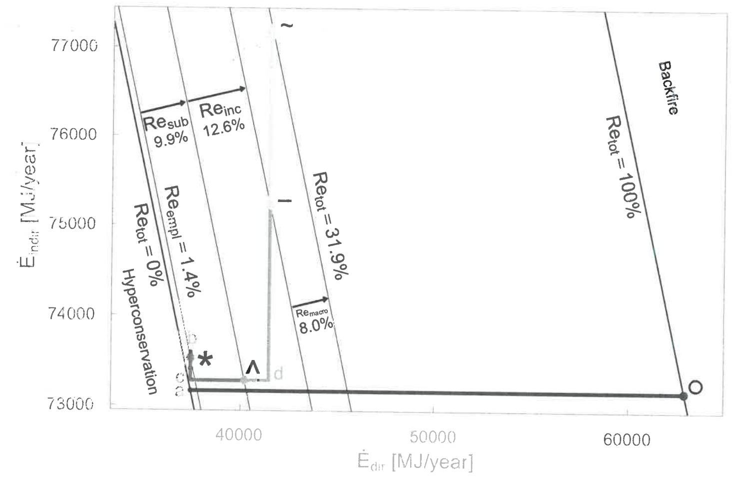

Fig. 5: Energy path graph for the car example. Macro factor (k) is assumed to be 1.

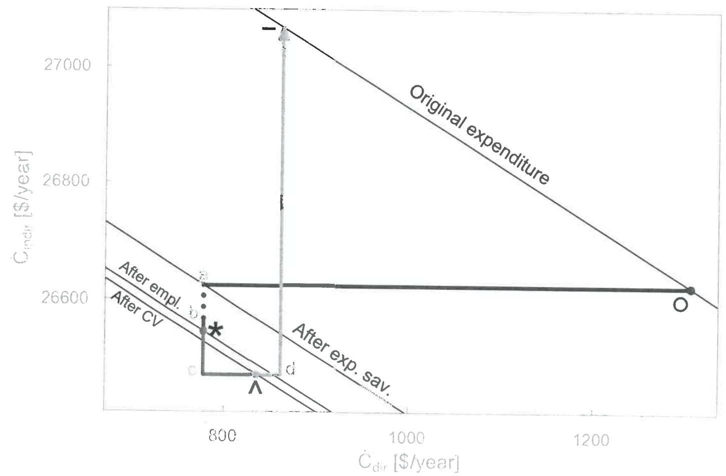

Fig. 6: Expenditure path graph for the car example. CV is compensating variation (ie, - - -)

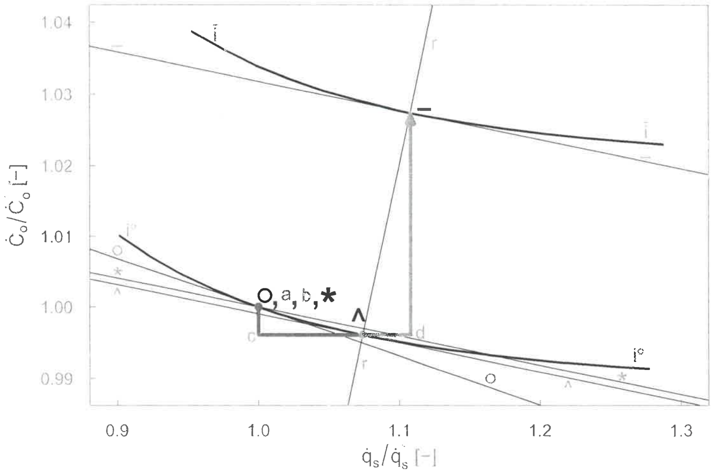

Fig. 7: Consumption path graph for the car example.

car. Rebound due to the maintenance and disposal effect (Re_{md}) is small and negative (-0.3%), because of the slightly lower maintenance and disposal costs for the hybrid EV car.

The substitution effect has two components: direct and indirect substitution effect rebound. Rebound from direct substitution (Re_{dsub}) is positive, as expected (10.9%). The car owner will, on average, prefer more driving, because of fuel economy enhancements (42 mpg > 25 mpg). In other words, with no other changes, the more fuel-efficient car is driven 10.9% further per year. Conversely, the indirect substitution effect (Re_{isub}) is slightly negative (-1.0%) to achieve the same level of utility after increased driving. Indeed, less money is spent on other goods ($\Delta\hat{\dot{C}}_o = -75.18$ \$/year).

The income effect also has two components: direct and indirect income effect rebound. The direct income effect (Re_{dinc}) is positive (5.0%), because the car owner allocates some net savings to additional driving. Rebound from the indirect income effect (Re_{iinc}) is positive (7.6%) due to higher spending on other goods. Thus, the net savings after the substitution effect ($\hat{\dot{N}} = 625.79$ \$/year) translates into positive direct and indirect income rebound at the microeconomic scale. Total microeconomic rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 24.0\%$.

Finally, we note that the link between macroeconomic and microeconomic rebound is largely unexplored, and we assume a value of $k = 1$ for both case studies, initially. We return to the matter of calibrating k in the Discussion (Section 4.1).

With k assumed to be 1 the macro effect leads to macroeconomic rebound (Re_{macro}) of 8.0%, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (transportation).

3.2 Case 2: Purchase of a new electric lamp

With the parameter values from Tables 4–6 and the equations in Section 2 of Part I in hand, we calculate important values at each rebound stage, as shown in Table 10. Similar to Table 8, Table 10 applies to the lamp owner, so no changes are observed across the macro effect, and the − (bar) and ∼ (tilde) columns of Table 10 are identical.

Table 10: Results for lamp example with macro factor (k) assumed to be 1.

	∘ (orig)	∗ (star)	∧ (hat)	− (bar)	∼ (tilde)
η [lm-hr/kW-hr]	8,833	81,800	81,800	81,800	81,800
η [lm-hr/MJ]	2,454	22,722	22,722	22,722	22,722
p_s [\$/lm-hr]	0.00001457	0.00000157	0.00000157	0.00000157	0.00000157
$\dot{q}_s$ [lm-hr/year]	580,350	580,350	1,412,867	1,413,439	1,413,439
$\dot{E}_s$ [MJ/year]	236.5	25.5	62.2	62.2	62.2
$\dot{E}_{emb}$ [MJ/year]	1.222	0.650	0.650	0.650	0.650
$\dot{C}_s$ [\$/year]	8.46	0.91	2.22	2.22	2.22
$\dot{C}_{cap}$ [\$/year]	1.04	0.12	0.12	0.12	0.12
$\dot{C}_{md}$ [\$/year]	0.00	0.00	0.00	0.00	0.00
$\dot{C}_o$ [\$/year]	27,920	27,920	27,916	27,927	27,927
$\dot{N}$ [\$/year]	0.00	8.47	11.31	0.00	0.00
$\dot{M}$ [\$/year]	27,930	27,930	27,930	27,930	27,930

Results are represented graphically in energy, expenditure, and consumption path graphs in Figs. 8–10. The energy path graph (Fig. 8) shows the size of each rebound effect for the lamp example. Rebound components for the lamp upgrade are shown in Table 11.

The emplacement effect rebound components start with the direct emplacement effect (Re_{dempl}), which is always 0.0%. Indirect rebound due to the embodied energy effect (Re_{emb}) is -0.3%. Although the LED lamp has higher embodied energy ($E^*_{emb} = 6.50$ MJ) than the incandescent lamp ($E^{\circ}_{emb} = 2.20$ MJ), the LED lamp has a much longer lifetime, meaning that the LED embodied energy *rate* ($\dot{E}^*_{emb} = 0.65$ MJ/year) is less than the incandescent embodied energy rate ($\dot{E}^{\circ}_{emb} = 1.22$ MJ/year). Thus, the change in embodied energy rate ($\Delta\dot{E}^*_{emb}$) is -0.57 MJ/year, and embodied energy rebound is negative ($Re_{emb} = -0.3\%$). Rebound due to the maintenance and disposal effect (Re_{md}) is 0.0%, because we assume no difference in maintenance and disposal costs between the incandescent lamp and the LED lamp.[1]

Direct substitution effect rebound (Re_{dsub}) is 17.4% due to the much higher LED lamp efficacy ($\tilde{\eta} = 81.8$ lm/W) compared to the incandescent lamp ($\eta^{\circ} = 8.83$ lm/W), leading to increased demand for lighting (from $\dot{q}^*_s = 580,350$ lm-hr/year to $\hat{\dot{q}}_s = 1,412,867$ lm-hr/year) as shown by segment c—∧ in Fig. 10. To maintain constant utility, consumption of other goods is reduced ($\Delta\hat{\dot{C}}_o = -4.15$ \$/year), as shown by segment ∗—c in Fig. 10, yielding indirect substitution effect rebound (Re_{isub}) of -6.4%.

Income effect rebound arises from spending net energy cost savings associated with converting from the incandescent lamp to the LED lamp ($\hat{\dot{N}} = 11.31$ \$/year). Direct income effect rebound (Re_{dinc}) is 0.01%, positive but small, as the lamp owner allocates some of the net savings to increased

[1]Maintenance cost rates for both incandescent and LED lamps are likely to be equal and negligible; lamps are usually installed and forgotten. Real-world disposal cost differences between the incandescent and LED technologies are also likely to be negligible. However, if "disposal" includes recycling processes, cost rates may be different between the two technologies due to the wide variety of materials in LED lamps compared to incandescent lamps.

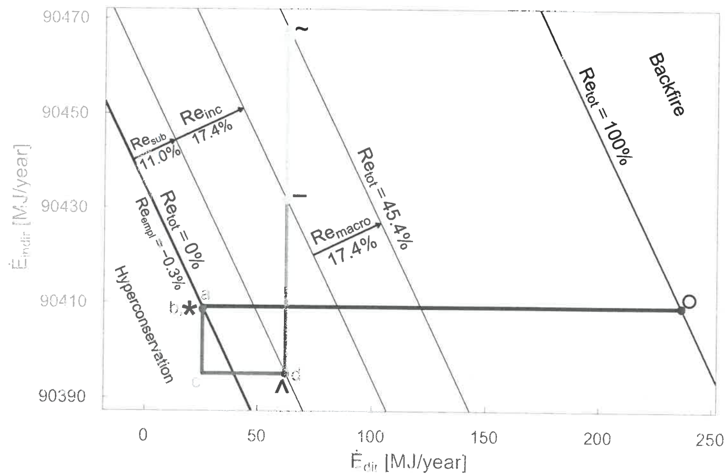

Fig. 8: Energy path graph for the lamp example. Macro factor (k) is assumed to be 1.

demand for lighting. The indirect income effect rebound is large (Re_{iinc} = 17.4%), due to the energy implications of increased spending on other goods. Total microeconomic scale rebound (emplacement, substitution, and income effects) sums to $Re_{micro} = 28.1\%$.

Finally, macro effect rebound (Re_{macro}) is coincidentally also 17.4% with k assumed to be 1, due to economic expansion caused by productivity enhancements arising from the more-efficient provision of the energy service (lighting).

Table 11: Lamp example: rebound results with macro factor (k) assumed to be 1.

Rebound term	Value [%]
Re_{dempl}	0.0
Re_{emb}	−0.3
Re_{md}	0.0
Re_{dsub}	17.4
Re_{isub}	−6.4
Re_{dinc}	0.0
Re_{iinc}	17.4
Re_{macro}	17.4
Re_{tot}	45.4

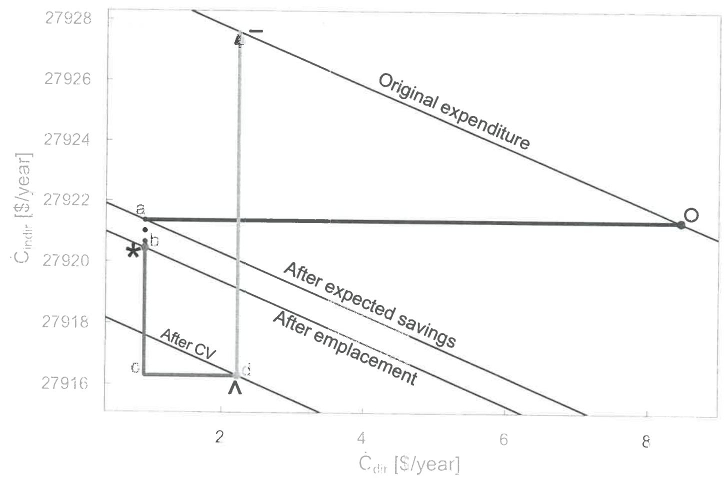

Fig. 9: Expenditure pathgraph for the lamp example. CV is compensating variation.

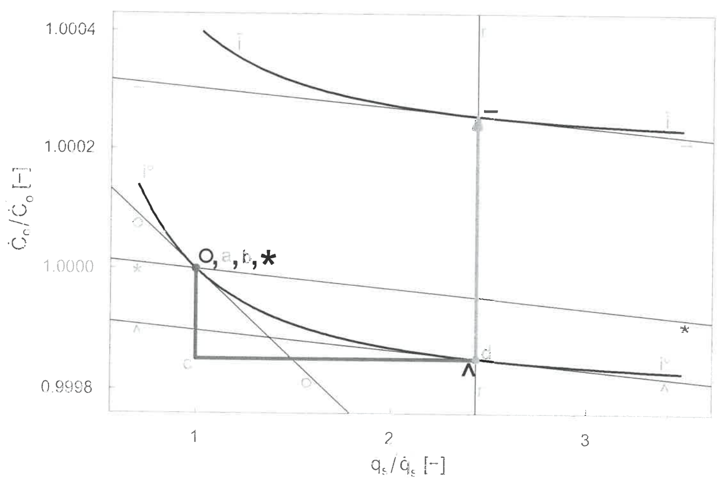

Fig. 10: Consumption path for the lamp example.

A first attempt at

4 Discussion

4.1 Calibrating k

The framework developed in Section 2 of Part I links macroeconomic rebound to microeconomic rebound via a term that scales magnitudes in the microeconomic portion of the framework. Few rebound studies have explored the macroeconomic scale between microeconomic and total rebound. Inspired by Borenstein (2015) and others, we bridge macroeconomic and microeconomic scales with the macro factor (k), as discussed in Section 2.4.4 of Part I. For the results presented in Section 3, we assumed a placeholder value of $k = 1$, meaning that every \$1 of spending by the device owner in the income effect generates only \$1 of additional economic activity in the broader economy. In combination, the framework presented in Section 2 of Part I, the results obtained in Section 3, and recent estimates of total rebound allow, for the first time, a discussion about calibrating k. After calibrating k, macro rebound and total rebound can be estimated.

To calibrate the macro factor (k), we treat macro rebound (Re_{macro}) as a residual. The macro factor (k) becomes an unknown parameter whose value is to be chosen such that Re_{macro} is sufficient to achieve an expected value for total rebound (Re_{tot}). We take the expected value for Re_{tot} from Brockway et al. (2021). Four of 33 studies reviewed by Brockway et al. (2021) examined total rebound from only consumer EEUs in a computable general equilibrium framework. The average total rebound (Re_{tot}) for the four consumer studies is 54%.[2] The calibrated values of k that give identical $Re_{tot} = 54\%$ for both examples are $k = 3.8$ for the car example and $k = 1.5$ for the lamp example.

Qualitative differences in benefits from EEUs as well as the considerable variance in Re_{tot} in 33 surveyed studies (Brockway et al., 2021) indicates that total rebound from one EEU is likely to be different from total rebound from another EEU. For a first approximation of a calibration for k, we take $k \approx 3$, being between the values of k estimated from the car and lamp examples. Note that $k \approx 3$ implies that every \$1 of net savings spent by the device owner generates \$3 of economic activity in the broader economy due to the income effect. We multiply $k\hat{N}$ by the energy intensity of the economy (I_E) to find the energy implications of macro-effect respending throughout the economy.

There are three ways to interpret $k \approx 3$. First, $k \approx 3$ can be considered the average long-run economic growth generated by the device owner's spending of freed cash. Efficiency increases in equipment drive a significant part of long-run productivity growth (Greenwood et al., 1997), therefore a large long-run multiplier is plausible, even if the initial productivity change occurred in household production which is not accounted in GDP. Second, it could be that growth is less than \$3 but that the macroeconomic "energy price effect" (a decline in energy prices due to the fallen demand) induces consumption at a higher energy intensity than that of the pre-EEU economy. Third, from the demand-side perspective entertained by Borenstein (2015), $k \approx 3$ could be interpreted as $MPC \approx 0.75$. (See Fig. F.1 in Appendix F of Part I.) $MPC \approx 0.75$ is a reasonable value, being in the upper half of recent estimates by Carroll et al. (2017). Although the cause of the growth in economic activity and energy consumption from an EEU is a supply-side productivity shock, the subsequent demand-side effects may well be interpreted as a multiplier effect, caused by higher real income instead of by higher monetary income.

After calibrating $k \approx 3$, we can estimate all rebound components in our framework. Emplacement (Re_{empl}), substitution (Re_{sub}), and income (Re_{inc}) rebound magnitudes are unchanged after

[2]The average total rebound among all 33 studies stood at 63%, supporting the claim by Turner (2013) that consumer and producer rebounds vary.

calibrating $k \approx 3$. However, we see that choosing a placeholder value of $k = 1$ in Section 3 underestimated Re_{macro} and, therefore, Re_{tot} in Section 3. In Figs. 5 and 8, the macro effect segments (— ~) should be three times longer than they appear. In Tables 9 and 11, the values of macro rebound (Re_{macro}) should triple to 23.9% and 52.1%, and the values of total rebound (Re_{tot}) should increase to 47.8% and 80.1% for the car and lamp examples, respectively. For the rest of this paper, we assume $k \approx 3$.

4.2 Comparison between the car and lamp case studies

Fuller

Tables 9 and 11 and our calibration of $k \approx 3$ in Section 4.1 enable comparisons between the car and lamp examples. Several points can be made.

First, the magnitude of every rebound effect is different between the two examples, the exception being direct emplacement rebound (Re_{dempl}) which is always 0.0 by definition. The implication is that every EEU needs to be analyzed separately. Estimates of the magnitudes of rebound effects for one EEU should never be assumed to apply to a different EEU.

Second, one cannot know *a-priori* which rebound effects will be large and which will be small for a given EEU. Furthermore, some rebound effects are dependent on economic parameters, such as energy intensity (I_E). Thus, it is important to estimate the magnitude of all rebound effects for each EEU in each economy.

Third, the two examples illustrate the fact that embodied energy rebound (Re_{emb}) can be positive or negative, as discussed in Section 2.4.1 of Part I. The car's embodied energy rebound is positive (1.7%) because of the high embodied energy of the EV's battery relative to the internal combustion engine vehicle. Although the LED lamp's embodied energy is larger than the incandescent lamp's embodied energy, the LED lamp's embodied energy rebound is negative (−0.3%), due to the longer life of the LED lamp compared to the incandescent lamp. Thus, each EEU should be analyzed independently for its embodied energy rebound.

Fourth, macro effect rebound is quite different between the two examples, owing to differences in freed cash. (For the car, Re_{macro} is 23.9%. For the lamp, Re_{macro} is 52.1%.) The efficiency gain for the lamp is far greater than the efficiency gain for the car, leading to much different rates of freed cash ($\dot{G}$) and different macro rebound estimates.

Fifth, in both examples the macro effect rebound (Re_{macro}) is much larger than any single micro rebound component. However, the sum of micro rebound components ($Re_{micro} = Re_{empl} + Re_{sub} + Re_{inc}$) is close in magnitude to the macro rebound (Re_{macro}). These observations are likely to be the result of Re_{macro} being the sum of several macroeconomic sub-effects, among which we don't discriminate in this framework. (See Table 1 in Part I for a list of macro rebound effects.)

4.3 Comparison to previous rebound estimates

With $k \approx 3$

Tables 12 and 13 compare car and lamp results from Section 4.1 to results from previous studies. The comparison studies in Tables 12 and 13 are neither comprehensive nor definitive of car and lamp EEUs; rather, they are examples that show the sort of estimations carried out in the general literature. That said, many of the comparison studies are highly cited in their field and carry reasonable academic weight suitable for our purposes. Tables 12 and 13 and their associated references enable two types of observations: (a) on the coverage of rebound components, and (b) on their magnitudes and associated estimation methods.

Firstly, we can say that none of the comparison studies estimate all rebound effect components, as we have done in Sections 3 and 4.1. Further, none of the studies decompose rebound into both

(I will look again at this text + revise PB action)

Table 12: Rebound magnitude comparisons for the car example. All numbers in %. Note that $Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro}$, and $Re_{tot} = Re_{dir} + Re_{indir}$.

		Re_{empl}	Re_{sub}	Re_{inc}	Re_{macro}	Re_{dir}	Re_{indir}	Re_{tot}
0	This paper (2022)	1.4	9.9	12.6	23.9	15.9	32.0	47.8
1	Greene et al. (1998)					20		
2	Small & Van Dender (2005)					22		
3	Koesler (2013)							49
4	Thomas & Azevedo (2013)					10	6	
5	Borenstein (2015)		13	11				
6	Chitnis & Sorrell (2015)					55	23	
7	Dimitropoulos et al. (2018)					26–29		
8	Duarte et al. (2018)							51

(PB action also to update tables)

Table 13: Rebound magnitude comparisons for the lamp example. All numbers in %. Note that $Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro}$, and $Re_{tot} = Re_{dir} + Re_{indir}$.

		Re_{empl}	Re_{sub}	Re_{inc}	Re_{macro}	Re_{dir}	Re_{indir}	Re_{tot}
0	This paper (2022)	−0.3	11.0	17.4	52.1	17.4	62.8	80.1
1	Guertin et al. (2003)					32–49		
2	Freire-González (2011)					49	16	
3	Thomas & Azevedo (2013)					10	10	
4	Borenstein (2015)		14	6				
5	Chitnis & Sorrell (2015)					41	8	
6	Duarte et al. (2018)							51
7	Barkhordar (2019)					28		43

#3 4.4 Sensitivity study

(I think we could bring in old section 4.3 from the WP? I liked that section. I felt it added depth to the paper)

#4 not really. maybe this becomes a further work/next steps section?

locations and both effects of microeconomic rebound that we think are important: direct and indirect locations and substitution and income effects. Thus, our framework is bringing conceptual clarity to this space. In addition, none of the studies estimate emplacement rebound (Re_{empl}) or macro rebound (Re_{macro}). In fact, only ****X**** of the 10 studies estimate total rebound (Re_{tot}). We also observe that studies that estimate total rebound are based on a top-down estimation of overall, economy-wide rebound, rather than a bottom-up "sum-of-components" approach that we employ. That finding is instructive—it supports the view that a comprehensive framework that sets out individual rebound components has been missing—something that informed the objective for Part I of this paper. Top-down estimations of total, economy-wide rebound may also inhibit comparison, as the effects included/excluded are not clear, giving an appearance of a "black box" estimation approach, where the magnitude of overall rebound can depend on model/analysis assumptions. **** Do we want to be quite so critical? This paper relies on external estimates of total rebound to calibrate k. ****

Secondly, helpful insights can be gained from comparison of rebound magnitudes and estimation methods. Highest alignment between our estimates and earlier estimates of rebound magnitudes appears within the direct (microeconomic) rebound (Re_{dir}). Our car (15.9%) and lamp (17.4%) estimates are in the lower half of the comparison studies for both the case of the car (10% to 49%) and lamp (10% to 55%). This alignment may be due to the easier estimation of direct rebound, from either empirical data (e.g., Small & Van Dender (2005)) or via own-price elasticities (e.g., Chitnis & Sorrell (2015)). The empirical data approach appears to give lower rebound estimates, closer to our values.

For indirect (microeconomic) rebound (Re_{indir}), there is little agreement on the magnitude of rebound effects. Our estimates for the car (32.0%) and lamp (62.8%) are higher than those found in the comparison studies for either the car (6% to 23%) or lamp (8% to 16%) cases. The most likely cause of our larger indirect rebound estimates is that we include both micro and macro rebound scales, whereas the comparison studies focus mainly on microeconomic rebound only (commonly via cross-price elasticities). In other words, comparisons of our indirect rebound estimates with the comparison studies may be unfair, as we include macro-scale effects.

For total rebound (Re_{tot}), our estimates of 47.8% (car) and 80.1% (lamp) are very close to the comparison studies for the car example (49% to 51%), but nearly double estimates from the comparison studies for the lamp example (43% to 51%). Beyond that, comparison (as noted earlier) is inhibited by existing top-down methods versus our bottom up approach to estimate total rebound. **** Except that our bottom-up method relies upon a top-down calibration for k. How do we resolve this tension? —MKH ****

4.4 Implications for CO_2 emissions

**** This section is a response to Referee 1. Do we like it/want to keep it? Is it in the right location? —MKH ****

To the extent that energy rebound takes back energy savings from EEUs, it is a threat to a low carbon future, as noted by van den Bergh (2017) and Brockway et al. (2017). However, the results above highlight two aspects that make it difficult to determine the precise impact of rebound on CO_2 emissions.

First, we see that rebound is measured at the final energy stage, while CO_2 emissions are the result of primary energy consumption (coal, oil, natural gas, wind, solar, hydro, etc.). Determining the CO_2 implications of rebound requires untangling the amount of each primary energy source associated with each type of final energy consumed. Such untangling will be different for each

we could bring in the 'need to integrate into models' from Part I + put in sect 5. (Better models = better evidence for CO_2 policy)

(I think this is overstretching the paper. We can put one line in future work in section 5)

economy. Furthermore, this untangling would need to be accomplished for each rebound effect, something that is beyond the scope of this framework.

Second, we note that the various rebound effects take place at different times throughout the life of a device. E.g., the emplacement effect (Re_{empl}) includes embodying of energy during manufacture and distribution, but its rebound effect (Re_{emb}) also takes account of device longevity. The emplacement effect also includes maintenance and disposal activities (Re_{md}); maintenance occurs throughout the life of the device, but disposal occurs at end of life. The macro effect also occurs occurs throughout the life of the device, but it depends on the magnitude of adjustments in the substitution and income effects. It will be difficult to know, *a-priori*, the source primary energies for each adjustment for each rebound effect during societal energy transitions.

Thus, further work will be required to utilize this rebound framework (and other frameworks that aspire to describe both direct and indirect rebound effects) for precise determination of the CO_2 emissions impacts of rebound.

5 Conclusions

In this paper (Part II), we attempt to bring clarity to energy rebound by (i) developing of the first (to our knowledge) visualizations of rebound effects in energy, expenditure, and consumption spaces (ii) documenting new estimates of rebound for automobile and lighting upgrades, and (iii) creating open source software tools to calculate and visualize rebound for any energy efficiency upgrade. To the extent that Part I and Part II provide clarity, we trust that the gap between economists and energy analysts will be reduced, leading to better interdisciplinary understanding of rebound phenomena. We encourage energy analysts and economists to use visualizations like the rebound path graphs to document their rebound estimates going forward. We hope that greater clarity about and understanding of energy rebound will bring about sound energy and climate policy.

From the development and application of the framework in Part II, we draw two important conclusions. First, the car and lamp examples (Section 3) show that the framework enables quantification of rebound magnitudes at microeconomic and macroeconomic scales, including direct ~~and~~ indirect ~~locations~~ for emplacement, substitution, income, and macro effects. Second, the examples show that magnitudes of rebound effects vary with the type of EEU performed.

Future work could be pursued in several areas. (i) Further empirical studies could be performed to estimate the magnitude of different rebound effects in a variety of real-life EEUs. (ii) Deeper study of macro rebound is needed, including improved estimates for the macro factor (k) and its relation to the MPC. (iii) The rebound implications of the distribution of *MPC* values across socioeconomic groups (Carroll et al., 2017) could be explored. (iv) The rebound effects of fossil-energy taxes could be studied, especially for the web of interconnected dynamic effects among rebound components that are functions of the energy intensity of the economy (I_E). (v) Sensitivities of rebound components to model parameters could be investigated, although this will be challenging work because many rebound parameters are covariant. For example, post-EEU efficiency ($\tilde{\eta}$) is unlikely to be independent of post-EEU capital cost ($\tilde{C}_{cap}$).

Competing interests

Declarations of interest: none.

Author contributions

	MKH	GS	PEB
Conceptualization	●	●	
Methodology	●	●	●
Software	●		
Validation	●		●
Formal analysis	●	●	●
Investigation	●	●	●
Resources	●		
Data curation			●
Writing–original draft	●	●	●
Writing–review & editing	●	●	●
Visualization	●	●	
Supervision	●		
Project administration	●		
Funding acquisition			●

(Gonna guess it's Paul)

Data repository

Data and example calculations in spreadsheet format are stored at the Research Data Leeds Repository (`https://doi.org/10.5518/1201`).

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Table A.1: Symbols and abbreviations.

Symbol	Meaning [example units]
a	a point in the emplacement effect on rebound path graphs or the share parameter in the CES utility model [–]
b	a point in the emplacement effect on rebound path graphs
C	cost [$]
c	a point in the substitution effect on rebound path graphs
d	a point in the income effect on rebound path graphs
E	final energy [MJ]
f	expenditure share [–]
G	freed cash [$]
I	energy intensity of economic activity [MJ/$]
k	macro factor [–]
M	income [$]
N	net savings [$]
p	price [$]
q	quantity [–]
Re	rebound [–]
S	energy cost savings [$]
t	energy conversion device lifetime [years]
u	utility [utils]

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Appendices

A Nomenclature

Presentation of the comprehensive rebound analysis framework is aided by a nomenclature that describes energy stages and rebound effects, locations, and scales. Table A.1 shows symbols and abbreviations, their meanings, and example units. Table A.2 shows Greek letters, their meanings, and example units. Table A.3 shows abbreviations and acronyms. Table A.4 shows symbol decorations and their meanings. Table A.5 shows subscripts and their meanings.

Differences are indicated by the Greek letter Δ and always signify subtraction of a quantity at an earlier stage of Fig. 1 from the same quantity at a later stage of Fig. 1. E.g., $\Delta\bar{X} \equiv \bar{X} - \hat{X}$, and $\Delta\tilde{X} \equiv \tilde{X} - \bar{X}$. Lack of decoration on a difference term indicates a difference that spans all stages of Fig. 1. E.g., $\Delta X \equiv \tilde{X} - X^{\circ}$. ΔX is also the sum of differences across each stage in Fig. 1, as shown below.

Table A.2: Greek letters.

Greek letter	Meaning [example units]
Δ	difference (later quantity less earlier quantity, see Fig. 1)
ϵ	elasticity [–]
$\epsilon_{\dot{q}_s,\dot{M}}$	income ($\dot{M}$) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,\dot{M}}$	income ($\dot{M}$) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s}$	uncompensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s}$	uncompensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s,c}$	compensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s,c}$	compensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
η	final-energy-to-service efficiency [vehicle-km/MJ]
σ	elasticity of substitution between the energy service ($\dot{q}_s^\circ$) and other goods ($\dot{q}_o^\circ$) [–]

Table A.3: Abbreviations.

Abbreviation	Meaning
APF	aggregate production function
CES	constant elasticity of substitution
CV	compensating variation
EEU	energy efficiency upgrade
GDP	gross domestic product
MPC	marginal propensity to consume
mpg	miles per U.S. gallon
U.S.	United States

Table A.4: Decorations.

Decoration	Meaning [example units]
X°	X originally (before the emplacement effect)
X^*	X after the emplacement effect (before the substitution effect)
$\hat{X}$	X after the substitution effect (before the income effect)
$\bar{X}$	X after the income effect (before the macro effect)
$\tilde{X}$	X after the macro effect
$\dot{X}$	rate of X [units of X/year]
M'	effective income [$]

Table A.5: Subscripts.

Subscript	Meaning
0	quantity at an initial time
1	a specific point on a consumption path graph
c	compensated
cap	capital costs
dev	device
dempl	direct emplacement effect
d	disposal
dinc	direct income effect
dir	direct effects (at the energy conversion device)
dsub	direct substitution effect
emb	embodied
i	index for other goods purchased in the economy
j	one of *cap*, *md*, or *o* in Eq. (??)
iempl	indirect emplacement effects
iinc	indirect income effect
indir	indirect effects (beyond the energy conversion device)
isub	indirect substitution effect
life	lifetime
m	maintenance
md	maintenance and disposal costs
o	other expenditures (besides energy) by the device owner
own	ownership duration
macro	macro effect
s	service stage of the energy conversion chain
tot	sum of all rebound effects in the framework

$$
\begin{aligned}
\Delta X &= \Delta\tilde{X} + \Delta\bar{X} + \Delta\hat{X} + \Delta X^* \\
\Delta X &= (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^*) + (X^* - X^\circ) \\
\Delta X &= (\tilde{X} - \cancel{\bar{X}}) + (\cancel{\bar{X}} - \cancel{\hat{X}}) + (\cancel{\hat{X}} - \cancel{X^*}) + (\cancel{X^*} - X^\circ) \\
\Delta X &= \tilde{X} - X^\circ
\end{aligned} \tag{1}
$$

B Mathematical details of rebound path graphs

Rebound path graphs show the impact of direct and indirect rebound effects in energy space, expenditure space, and consumption space. Notional rebound path graphs can be found in Figs. 2–4. Rebound path graphs for the car example can be found in Figs. 5–7. Graphs for the lamp example can be found in Figs. 8–10.

This appendix shows the mathematical details of rebound path graphs, specifically derivations of equations for lines and curves shown in Table B.1. The lines and curves enable construction of numerically accurate rebound path graphs as shown in Figs. 5–10.

B.1 Energy path graphs

Energy path graphs show direct (on the x-axis) and indirect (on the y-axis) energy consumption associated with the energy conversion device and the device owner. Lines of constant total energy consumption comprise a scale for total rebound. For example, the 0% and 100% rebound lines are constant total energy consumption lines which pass through the original point (o) and the post-direct-emplacement-effect point (a) on an energy path graph.

plane (update throughout)

Table B.1: Lines and curves for rebound path graphs.

Rebound path graph	Lines and curves
Energy	Constant total energy consumption lines 0% and 100% rebound lines
Expenditure	Constant expenditure lines
Consumption	Constant expenditure lines Rays from origin to $\wedge$ point Indifference curves

The equation of a constant total energy consumption line is derived from

$$\dot{E}_{tot} = \dot{E}_{dir} + \dot{E}_{indir} \tag{2}$$

at any rebound stage. Direct energy consumption is energy consumed by the energy conversion device ($\dot{E}_s$), and indirect energy consumption is the sum of embodied energy, energy associated with maintenanace and disposal, and energy associated with expenditures on other goods ($\dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$).

For the energy path graph, direct energy consumption is placed on the x-axis and indirect energy consumption is placed on the y-axis. To derive the equation of a constant energy consumption line, we first rearrange to put the y coordinate on the left of the equation:

$$\dot{E}_{indir} = -\dot{E}_{dir} + \dot{E}_{tot} \ . \tag{3}$$

Next, we substitute y for $\dot{E}_{indir}$, x for $\dot{E}_{dir}$, and $\dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$ for $\dot{E}_{tot}$ to obtain

$$y = -x + \dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E \ , \tag{4}$$

where all of $\dot{E}_s$, $\dot{E}_{emb}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage.

The constant total energy consumption line that passes through the original point ($\circ$) shows 100% rebound:

$$y = -x + \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ)I_E \ . \tag{5}$$

The 0% rebound line is the constant total energy consumption line that accounts for expected energy savings ($\dot{S}_{dev}$) only:

$$y = -x + (\dot{E}_s^\circ - \dot{S}_{dev}) + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ)I_E \ . \tag{6}$$

The above line passes through the a point on an energy path graph.

B.2 Expenditure path graphs

Expenditure path graphs show direct (on the x-axis) and indirect (on the y-axis) expenses associated with the energy conversion device and the device owner. Lines of constant expenditure are important, because they provide budget constraints for the device owner.

The equation of a constant total expenditure line is derived from the budget constraint

$$\dot{C}_{tot} = \dot{C}_{dir} + \dot{C}_{indir} \tag{7}$$

at any rebound stage. For the expenditure path graph, indirect expenditures are placed on the y-axis and direct expenditures on energy for the energy conversion device are place on the x-axis. Direct expenditure is the cost of energy consumed by the energy conversion device ($\dot{C}_s = p_E\dot{E}_s$), and indirect expenses are the sum of capital costs, maintenanace and disposal costs, and expenditures on other goods ($\dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$). Rearranging to put the y-axis variable on the left side of the equation gives

$$\dot{C}_{indir} = -\dot{C}_{dir} + \dot{C}_{tot} \ . \tag{8}$$

Substituting y for $\dot{C}_{indir}$, x for $\dot{C}_{dir}$, and $\dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$ for $\dot{C}_{tot}$ gives

$$y = -x + \dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o \ , \tag{9}$$

where all of $\dot{C}_s$, $\dot{C}_{cap}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage.

The constant total expenditure line that passes through the original point ($\circ$) shows the budget constraint for the device owner:

$$y = -x + \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \ , \tag{10}$$

into which Eq. (37) of Part I can be substituted with $\dot{C}_s^\circ = p_E\dot{E}_s^\circ$ and $\dot{N}^\circ = 0$ to obtain

$$y = -x + \dot{M}^\circ \ . \tag{11}$$

The constant total expenditure line that accounts for expected energy savings ($\dot{S}_{dev}$) and freed cash ($\dot{G} = p_E\dot{S}_{dev}$) only is given by:

$$y = -x + (\dot{C}_s^\circ - \dot{G}) + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \ , \tag{12}$$

or

$$y = -x + \dot{M}^\circ - \dot{G} \ . \tag{13}$$

The line given by the above equation passes through the a point on an expenditure path graph.

B.3 Consumption path graphs

Consumption path graphs show expenditures in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space to accord with the utility model. (See Appendix C of Part I.) Consumption path graphs include (i) constant expenditure lines given prices, (ii) a ray from the origin through the $\wedge$ point, and (iii) indifference curves. Derivations for each are shown in the following subsections.

B.3.1 Constant expenditure lines

There are four constant expenditure lines on the consumption path graphs of Figs. 4, 7, and 10. The constant expenditure lines pass through the original point (line $\circ$—$\circ$), the post-emplacement point (line $*$—$*$), the post-substitution point (line $\wedge$—$\wedge$), and the post-income point (line $-$ — $-$). Like the expenditure path graph, lines of constant expenditure on a consumption path graph are derived from the budget constraint of the device owner at each of the four points.

Prior to the EEU, the budget constraint is given by Eq. (37) of Part I. Substituting $p_s^\circ\dot{q}_s^\circ$ for $p_E\dot{E}_s^\circ$ and recognizing that there is no net savings before the EEU ($\dot{N}^\circ = 0$) gives

$$\dot{M}^{\circ} = p_s^{\circ}\dot{q}_s^{\circ} + \dot{C}_{cap}^{\circ} + \dot{C}_{md}^{\circ} + \dot{C}_o^{\circ} \,. \tag{14}$$

To create the line of constant expenditure on the consumption path graph, we allow $\dot{q}_s^{\circ}$ and $\dot{C}_o^{\circ}$ to vary in a compensatory manner: when one increases, the other must decrease. To show that variation along the constant expenditure line, we remove the notation that ties $\dot{q}_s^{\circ}$ and $\dot{C}_o^{\circ}$ to the original point ($\circ$) to obtain

$$\dot{M}^{\circ} = p_s^{\circ}\dot{q}_s + \dot{C}_{cap}^{\circ} + \dot{C}_{md}^{\circ} + \dot{C}_o \,, \tag{15}$$

where all of $\dot{M}^{\circ}$, p_s°, $\dot{C}_{cap}^{\circ}$, and $\dot{C}_{md}^{\circ}$ apply at the same rebound stage, namely the original point ($\circ$) in this instance.

To derive the equation of the line representing the original budget constraint in $\dot{C}_o/\dot{C}_o^{\circ}$ vs. $\dot{q}_s/\dot{q}_s^{\circ}$ space (the $\circ$—$\circ$ line through the $\circ$ point in consumption path graphs), we solve for $\dot{C}_o$ to obtain

$$\dot{C}_o = -p_s^{\circ}\dot{q}_s + \dot{M}^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ} \,. \tag{16}$$

Multiplying judiciously by $\dot{C}_o^{\circ}/\dot{C}_o^{\circ}$ and $\dot{q}_s^{\circ}/\dot{q}_s^{\circ}$ gives

$$\frac{\dot{C}_o}{\dot{C}_o^{\circ}}\dot{C}_o^{\circ} = -p_s^{\circ}\frac{\dot{q}_s}{\dot{q}_s^{\circ}}\dot{q}_s^{\circ} + \dot{M}^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ} \,. \tag{17}$$

Dividing both sides by $\dot{C}_o^{\circ}$ yields

$$\frac{\dot{C}_o}{\dot{C}_o^{\circ}} = -\frac{p_s^{\circ}\dot{q}_s^{\circ}}{\dot{C}_o^{\circ}}\frac{\dot{q}_s}{\dot{q}_s^{\circ}} + \frac{1}{\dot{C}_o^{\circ}}(\dot{M}^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ}) \,. \tag{18}$$

Noting that $\dot{q}_s/\dot{q}_s^{\circ}$ and $\dot{C}_o/\dot{C}_o^{\circ}$ are the x-axis and y-axis, respectively, on a consumption path graph gives

$$y = -\frac{p_s^{\circ}\dot{q}_s^{\circ}}{\dot{C}_o^{\circ}}x + \frac{1}{\dot{C}_o^{\circ}}(\dot{M}^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ}) \,. \tag{19}$$

A similar procedure can be employed to derive the equation of the $*$—$*$ line through the $*$ point after the emplacement effect. The starting point is the budget constraint at the $*$ point (Eq. (39) of Part I) with $\dot{M}^{\circ}$ replacing $\dot{M}^{*}$, $\tilde{p}_s\dot{q}_s$ replacing $p_E\dot{E}_s^{*}$, and $\dot{C}_o$ replacing $\dot{C}_o^{*}$.

$$\dot{M}^{\circ} = \tilde{p}_s\dot{q}_s + \dot{C}_{cap}^{*} + \dot{C}_{md}^{*} + \dot{C}_o + \dot{N}^{*} \,. \tag{20}$$

Substituting Eq. (48) of Part I for $\dot{N}^{*}$, substituting Eq. (49) of Part I to obtain $\dot{G}$, multiplying judiciously by $\dot{C}_o^{\circ}/\dot{C}_o^{\circ}$ and $\dot{q}_s^{\circ}/\dot{q}_s^{\circ}$, rearranging, and noting that $\dot{q}_s/\dot{q}_s^{\circ}$ is the x-axis and $\dot{C}_o/\dot{C}_o^{\circ}$ is the y-axis gives

$$y = -\frac{\tilde{p}_s\dot{q}_s^{\circ}}{\dot{C}_o^{\circ}}x + \frac{1}{\dot{C}_o^{\circ}}(\dot{M}^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ} - \dot{G}) \,. \tag{21}$$

Note that the slope of Eq. (21) is less negative than the slope of Eq. (19), because $\tilde{p}_s < p_s^{\circ}$. The y-intercept of Eq. (21) is less than the y-intercept of Eq. (19), reflecting freed cash. Both effects are seen in consumption path graphs (Figs. 4, 7, and 10). The $\circ$—$\circ$ and $*$—$*$ lines intersect at the coincident $\circ$ and $*$ points.

A similar derivation process can be used to find the equation of the line representing the budget constraint after the substitution effect (the $\wedge$—$\wedge$ line through the $\wedge$ point). The starting point is Eq. (52) of Part I, and the equation for the constant expenditure line is

$$y = -\frac{\tilde{p}_s\dot{q}_s^{\circ}}{\dot{C}_o^{\circ}}x + \frac{1}{\dot{C}_o^{\circ}}(\dot{M}^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ} - \dot{G} + \tilde{p}_s\Delta\hat{\dot{q}}_s + \Delta\hat{\dot{C}}_o) \,. \tag{22}$$

Note that the $\wedge$—$\wedge$ line (Eq. (22)) has the same slope as the $*$—$*$ line (Eq. (21)) but a lower y-intercept.

Finally, the corresponding derivation for the equation of the constant expenditure line through the $-$ point (line $-$—$-$) starts with Eq. (61) of Part I and ends with

$$y = -\frac{\tilde{p}_s\dot{q}_s^{\circ}}{\dot{C}_o^{\circ}}x + \frac{1}{\dot{C}_o^{\circ}}(\dot{M}^{\circ} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ} - \Delta\dot{C}_{cap}^{*} - \Delta\dot{C}_{md}^{*}) \,. \tag{23}$$

B.3.2 Ray from the origin to the $\wedge$ point

On consumption path graphs, the ray from the origin to the $\wedge$ point (line r—r) defines the path along which the income effect (lines $\wedge$—d and d—$-$) operates. The ray from the origin to the $\wedge$ point has slope $(\hat{\dot{C}}_o/\dot{C}_o^{\circ})/(\hat{\dot{q}}_s/\dot{q}_s^{\circ})$ and a y-intercept of 0. Therefore, the equation of line r—r is

$$y = \frac{\hat{\dot{C}}_o/\dot{C}_o^{\circ}}{\hat{\dot{q}}_s/\dot{q}_s^{\circ}}x \,. \tag{24}$$

B.3.3 Indifference curves

On a consumption path graph, indifference curves represent lines of constant utility for the energy conversion device owner. In $\dot{C}_o/\dot{C}_o^{\circ}$ vs. $\dot{q}_s/\dot{q}_s^{\circ}$ space, any indifference curve is given by Eq. (117) of Part I with $f_{\dot{C}_s}^{\circ}$ replacing the share parameter a, as shown in Appendix C of Part I. Recognizing that $\dot{C}_o/\dot{C}_o^{\circ}$ is on the y-axis and $\dot{q}_s/\dot{q}_s^{\circ}$ is on the x-axis leads to substitution of y for $\dot{C}_o/\dot{C}_o^{\circ}$ and x for $\dot{q}_s/\dot{q}_s^{\circ}$ to obtain

$$y = \left[\frac{1}{1-f_{\dot{C}_s}^{\circ}}\left(\frac{\dot{u}}{\dot{u}^{\circ}}\right)^{\rho} - \frac{f_{\dot{C}_s}^{\circ}}{1-f_{\dot{C}_s}^{\circ}}(x)^{\rho}\right]^{(1/\rho)} \,. \tag{25}$$

At any point in $\dot{C}_o/\dot{C}_o^{\circ}$ vs. $\dot{q}_s/\dot{q}_s^{\circ}$ space, namely $(\dot{q}_{s,1}/\dot{q}_s^{\circ}, \dot{C}_{o,1}/\dot{C}_o^{\circ})$, indexed utility $(\dot{u}_1/\dot{u}^{\circ})$ is given by Eq. (14) of Part I as

$$\frac{\dot{u}_1}{\dot{u}^{\circ}} = \left[f_{\dot{C}_s}^{\circ}\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^{\circ}}\right)^{\rho} + (1-f_{\dot{C}_s}^{\circ})\left(\frac{\dot{C}_{o,1}}{\dot{C}_o^{\circ}}\right)^{\rho}\right]^{(1/\rho)} \,. \tag{26}$$

Substituting Eq. (26) into Eq. (25) for $\dot{u}/\dot{u}^{\circ}$ and simplifying exponents gives

$$y = \left\{\frac{1}{1-f_{\dot{C}_s}^{\circ}}\left[f_{\dot{C}_s}^{\circ}\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^{\circ}}\right)^{\rho} + (1-f_{\dot{C}_s}^{\circ})\left(\frac{\dot{C}_{o,1}}{\dot{C}_o^{\circ}}\right)^{\rho}\right] - \frac{f_{\dot{C}_s}^{\circ}}{1-f_{\dot{C}_s}^{\circ}}(x)^{\rho}\right\}^{(1/\rho)} \,. \tag{27}$$

Simplifying further yields the equation of an indifference curve passing through point $(\dot{q}_{s,1}/\dot{q}_s^{\circ}, \dot{C}_{o,1}/\dot{C}_o^{\circ})$:

$$y = \left\{ \left(\frac{f^{\circ}_{\dot{C}_s}}{1 - f^{\circ}_{\dot{C}_s}} \right) \left[\left(\frac{\dot{q}_{s,1}}{\dot{q}^{\circ}_s} \right)^{\rho} - (x)^{\rho} \right] + \left(\frac{\dot{C}_{o,1}}{\dot{C}^{\circ}_o} \right)^{\rho} \right\}^{(1/\rho)} . \tag{28}$$

Note that if x is $\dot{q}_{s,1}/\dot{q}^{\circ}_s$, y becomes $\dot{C}_{o,1}/\dot{C}^{\circ}_o$, as expected.