Energy, expenditure, and consumption aspects of rebound, Part I: A rigorous analytical framework

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Abstract

Widespread implementation of energy efficiency is a key greenhouse gas emissions mitigation measure, but rebound can "take back" energy savings. However, the absence of solid analytical foundations hinders empirical determination of the size of rebound. A new clarity is needed, one that involves both economics and energy analysis. In this paper (Part I of a two-part paper), we advance a rigorous analytical framework that starts at the microeconomic level and is approachable for both energy analysts and economists. We develop the first (to our knowledge) rebound analysis framework that (i) clarifies the energy, expenditure, and consumption aspects of rebound, (ii) combines embodied energy effects with maintenance and disposal effects (under a new "emplacement effect" term), and (iii) allows exact analytical determination of the effects of non-marginal energy efficiency increases and non-marginal energy service price decreases. Furthermore, we provide the first operationalized link between rebound effects on microeconomic and macroeconomic levels.

Keywords: Energy efficiency, Energy rebound, Energy services, Microeconomic rebound, Substitution and income effects, Macroeconomic rebound

JEL codes: O13, Q40, Q43

1 Introduction

Energy efficiency is often considered to be the most important means of reducing energy consumption and CO_2 emissions (International Energy Agency, 2017, Fig. 3.15, p. 139). But energy rebound makes energy efficiency less effective at decreasing energy consumption by taking back (or reversing, in the case of "backfire") energy savings expected from energy efficiency improvements (Sorrell, 2009). As such, energy rebound is a threat to a low-carbon future (van den Bergh, 2017; Brockway et al., 2017).

Recent evidence shows that rebound is both larger than commonly assumed (Stern, 2020) and mostly missing from large energy and climate models (Brockway et al., 2021). Thus, rebound could be an important reason why energy consumption and carbon emissions have never been absolutely decoupled from economic growth (Haberl et al., 2020; Brockway et al., 2021).

1.1 A short history of rebound

Famously, the roots of energy rebound trace back to Jevons who said "[*i*]*t is wholly a confusion of ideas to suppose that the economical use of fuel is equivalent to a diminished consumption. The very contrary is the truth*" (Jevons, 1865, p. 103, emphasis in original). Less famously, the origins of rebound extend further backward from Jevons to Williams (1840) and Parkes who wrote "[t]he economy of fuel is the secret of the economy of the steam-engine; it is the fountain of its power, and the adopted measure of its effects. Whatever, therefore, conduces to increase the efficiency of coal, and to diminish the cost of its use, directly tends to augment the value of the steam-engine, and to enlarge the field of its operations" (Parkes, 1838, p. 161). For nearly 200 years, then, it has been understood that efficiency gains may be taken back or, paradoxically even, cause growth in energy consumption, as Jevons suggested.

The oil crises of the 1970s shone a light back onto energy efficiency, and research into rebound appeared late in the decade (Madlener & Turner, 2016; Saunders et al., 2021). A modern debate over the magnitude of energy rebound commenced. On one side, scholars including Brookes (1979, 1990) and Khazzoom (1980) suggested rebound could be large. Others, including Lovins (1988) and Grubb (1990, 1992), claimed rebound was likely to be small. Debate over the size of energy rebound continues today. Advocates of small rebound (less than, say, 50%), suggest “the rebound effect is overplayed” (Gillingham et al., 2013, p. 475), while others claim (i) that the evidence for large rebound (greater than 50%) is growing (Saunders, 2015; Berner et al., 2022) and (ii) that rebound will reduce the effectiveness of energy efficiency to decrease carbon emissions (van den Bergh, 2017).

1.2 Absence of solid analytical foundations

Turner contends that the lack of consensus on the magnitude of energy rebound in the modern empirical literature is caused by “a rush to empirical estimation in the absence of solid analytical foundations” (Turner, 2013, p. 25). Progress has been made recently on how price changes affect economy-wide rebound in general equilibrium frameworks (Lemoine, 2020; Fullerton & Ta, 2020; Blackburn & Moreno-Cruz, 2020). Arguments from microeconomics (i.e., at sectoral and individual level) have been used from the outset of the modern debate (e.g., Khazzoom (1980) and Greening et al. (2000)), and Borenstein (2015) and Chan & Gillingham (2015) recently made progress toward solidifying the microeconomic analytical foundations.

Yet more is needed to support empirical efforts. For instance, while the microeconomic categories of substitution and income effects provide analytical clarity about how behavior changes affect energy service consumption, it has been unclear how they could be used for precise numerical rebound calculations. Where previous numerical calculations were made, they tended approximate the substitution effect from other goods to the cheaper energy service, without maintaining constant utility for the device user. They also used constant price elasticities for non-marginal efficiency improvements, even though constant price elasticities typically provide only approximations of substitution and income effects for small efficiency changes. Further, previous analytical studies have stressed the importance of the cost of buying an upgraded device as well as the energy embodied in the device. Yet, there is no clearly formulated approach for how to incorporate these cost and energy components into rebound calculations. And rebound involves simultaneous changes in energy, expenditure, and consumption aspects, and keeping an overview of all aspects is hard, with no approach to our knowledge documenting all changes in a straightforward and consistent manner. Finally, while recent general equilibrium rebound modeling has led to important insights about the effects of changing prices, dynamic aspects of a macroeconomic rebound have been neglected by these approaches.

In the absence of solid analytical foundations, the wide variety of rebound calculation approaches contributes to a wide range of rebound values, giving the appearance of uncertainty and leading some energy and climate modelers to either (i) use questionable rebound values or (ii) ignore rebound altogether. Insufficient inclusion of rebound in energy and climate models could lead to overly optimistic projections of the capability of energy efficiency to reduce carbon emissions (Brockway et al., 2021). We suggest that improving the conceptual foundations of rebound and solidifying the analytical frameworks will (i) help generate more robust estimates of rebound, (ii) lead to better rebound calculations in energy and climate models, and (iii) provide improved evidence for policymaking around energy efficiency.

But why is there an "absence of solid analytical foundations?" We propose that development of solid analytical frameworks for rebound is hampered by the fact that rebound is a decidedly interdisciplinary topic, involving both economics and energy analysis. Birol & Keppler (2000, p. 458) note that "different implicit and explicit assumptions of different research communities ('economists', 'engineers') ... have in the past led to vastly differing points of view."[1] Turner states that "[d]ifferent definitions of energy efficiency will be appropriate in different circumstances. However, ... it is often not clear what different authors mean by energy efficiency" (Turner, 2013, p. 237–38). If authors from the two disciplines cannot even agree on the key terms, it is unsurprising that only modest progress has been made on analytical foundations. To fully understand rebound, economists need to have an energy analyst's understanding of energy, and energy analysts need to have an economist's understanding of finance and human behavior.[2] Developing the knowledge and skills required to assess and calculate, let alone mitigate, rebound effects is a tall order, indeed.

[1] We prefer the term "energy analysts" over "engineers," because "energy analysts" better describes the group of people engaged in "energy analysis." For this paper, we define "energy analysis" to be the study of energy transformations from stocks to flows and wastes along society's energy conversion chain for the purpose of generating energy services, economic activity, and human well-being.

[2] Indeed, this is why the authors for these papers come from the energy analysis (MKH, PEB) and economics (GS) disciplines.

1.3 New clarity is needed

We contend that new clarity is needed. A description of rebound that is (i) consistent across energy, expenditure, and consumption aspects, (ii) technically rigorous, and (iii) approachable from both sides (economics and energy analysis) will be a good starting point toward that clarity. In other words, the finance and human behavior aspects of rebound need to be presented in ways energy analysts can understand. And the energy aspects of rebound need to be presented in ways economists can understand.

Summarizing, we surmise that reducing global carbon emissions has been hampered, in part, by the fact that rebound is not sufficiently included in energy and climate models. We suspect that one reason rebound is not sufficiently included is the lack of consensus on rebound calculation methods and, hence, rebound magnitude. We agree with Turner that lack of consensus on rebound magnitude is a symptom of the absence of solid analytical foundations for rebound. We posit that developing solid analytical frameworks is difficult because energy rebound is an inherently interdisciplinary topic. We believe that providing a detailed explication of a rigorous analytical framework for energy rebound, which is approachable by both energy analysts and economists alike, will go some way toward providing additional clarity in the field.

1.4 Objective, contributions, and structure

The *objective* of this paper is to improve clarity in the field of energy rebound by supporting the development of a rigorous analytical framework, one that (i) starts at the microeconomics of rebound (building especially upon Borenstein (2015)) and (ii) is approachable for both energy analysts and economists. We strive to keep the framework as simple as possible and in this spirit limit our attention to a model of consumer demand for energy services, while noting that the approach is transferable to a producer model with few modifications.

The key *contributions* of this paper are (i) a novel and clear explication of interrelated energy, expenditure, and consumption aspects of energy rebound, (ii) development of the first (to our knowledge) rebound analysis framework that combines embodied energy effects, maintenance and disposal effects, non-marginal energy efficiency increases, and non-marginal energy service price decreases, and (iii) the first operationalized link between rebound effects on microeconomic and macroeconomic levels.

The remainder of this paper is *structured* as follows. Section 2 describes the rebound analysis framework. Section 3 discusses this framework relative to previous frameworks, and Section 4 concludes. Results from the application of our framework to energy efficiency upgrades to a car and an electric lamp can be found in Part II.

2 Methods: development of the framework

In this section, we develop an energy rebound framework for an individual consumer who upgrades the energy efficiency of a single device (concisely, "the framework," "this framework," or "our framework"). We endeavor to bring clarity to the field of energy rebound by providing sufficient detail to assist energy analysts to understand the economics and economists to understand the energy analysis.

2.1 Rebound typology

Table 1 shows our typology of rebound effects. We follow others, including Jenkins et al. (2011) and Walnum et al. (2014), in identifying and including both direct and indirect rebound effects, which occur at (direct) and beyond (indirect) the level of the device and its user. Again following others, such as Gillingham et al. (2016), we distinguish between rebound effects at the microeconomic and macroeconomic levels.

Microeconomic rebound occurs at the level of the single device and its user and in our framework comprises three effects: an emplacement effect, a substitution effect, and an income effect, each of which partitions direct and indirect rebound effects. All combinations are possible. The direct rebound effect can be partitioned into a direct emplacement effect, a direct substitution effect, and a direct income effect. At the level of the device, all of the direct rebound effects change the consumption of energy by the device whose efficiency has been upgraded, according to a microeconomic behavioral model of the consumer who responds to the cheaper energy service. Similarly, the indirect rebound effect can be partitioned into an indirect emplacement effect, an

Table 1: Rebound typology for our framework.

	Direct rebound (Re_{dir})	**Indirect rebound** (Re_{indir})
Microeconomic rebound (Re_{micro}) These mechanisms occur at the single device/user level within a static economy based on responses to the reduction in implicit price of an energy service.	**Emplacement effect** (Re_{dempl}) Accounts for performance of the Energy Efficiency Upgrade (EEU) only. No behavior changes occur. The direct energy effect of emplacement of the EEU is expected device-level energy savings. By definition, there is no rebound from direct emplacement effects ($Re_{dempl} \equiv 0$).	**Emplacement effect** (Re_{iempl}) Differential energy adjustments beyond the usage of the upgraded device, via (i) the embodied energy associated with the manufacturing phase (Re_{emb}) and (ii) the implied energy demand from maintenance and disposal (Re_{md}). Re_{iempl} can be > 0 or < 0, depending on the characteristics of the EEU.
	Substitution effect (Re_{dsub}) Change in preference toward the energy service relative to other goods as a result of the EEU. Excludes by definition the effects of freed cash (income effects). $Re_{dsub} > 0$ is typical due to greater consumption of the energy service.	**Substitution effect** (Re_{isub}) Change in preference away from other goods relative to the energy service as a result of the EEU. Excludes by definition the effects of freed cash (income effects). $Re_{isub} < 0$ is typical due to reduced consumption of other goods and services.
	Income effect (Re_{dinc}) Spending of some of the freed cash to obtain more of the energy service. $Re_{dinc} > 0$ is typical due to increased consumption of the energy service.	**Income effect** (Re_{iinc}) Spending of some of the freed cash on other goods and services. $Re_{iinc} > 0$ is typical due to increased consumption of other goods and services.
Macroeconomic rebound (Re_{macro}) These mechanisms originate from the dynamic response of the economy to reach a stable equilibrium (between supply and demand for energy services and other goods). These mechanisms combine various short and long run effects.		**Macroeconomic effect** (Re_{macro}) Increased energy consumption in the broader macroeconomic system, i.e., beyond responses at the micro-economic (device/user) level. $Re_{macro} > 0$ is typical, due to spending of freed cash (at the micro-economic level) causing greater consumption in the wider economy.

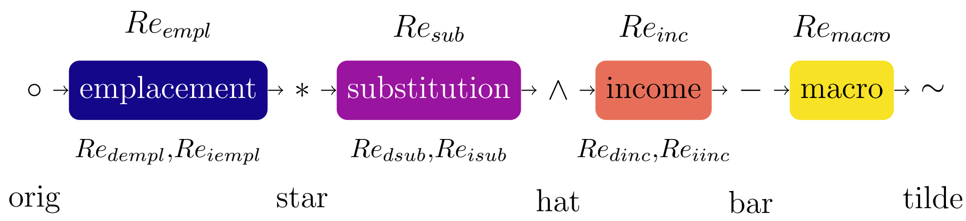

Fig. 1: Flowchart of rebound effects and decorations.

indirect substitution effect, and an indirect income effect. All of the indirect effects change the induced energy consumption beyond the upgraded device, again according to a microeconomic behavioral model. We assume a *partial equilibrium* response to the energy efficiency upgrade (EEU) at the microeconomic level; other prices in the economy (p_o) remain unchanged in response to the EEU.

In contrast, macroeconomic rebound is a broader, economy-wide response to the single device upgrade. Like other authors, we recognize many macroeconomic rebound effects, even if we don't later distinguish among them.[3] At the macroeconomic level, *general equilibrium* effects can occur as prices for all goods and services (even energy) may change in response to the EEU. Further treatment of macroeconomic rebound can be found in Section 2.5.4 of this paper (Part I) and in Section 4.1 of Part II.

Fig. 1 shows rebound effects arranged in the left-to-right order of their discussion in this paper. The left-to-right order does not necessarily represent the progression of rebound effects through time. Rebound symbols are shown above each effect (Re_{empl}, etc.). Nomenclature for partitions of direct and indirect rebound is shown beneath each effect (Re_{dempl}, etc.). Decorations for each stage are shown between rebound effects ($\circ$, $*$, etc.). Names for the decorations are given at the bottom of the figure ("orig," "star," etc.).[4]

[3] For example, Sorrell (2009) sets out five macroeconomic rebound effects: embodied energy effects, respending effects, output effects, energy market effects, and composition effects. (We place the embodied energy effect at the microeconomic level.) Santarius (2016) and Lange et al. (2021) introduce meso (i.e., sectoral) level rebound between the micro and macro levels. van den Bergh (2011) distinguishes 14 types of rebound, providing, perhaps, the greatest complexity.

[4] Note that the vocabulary and mathematical notation for rebound effects is important; Fig. 1 and Appendix A provide guides to notational elements used throughout this paper, including symbols, Greek letters, abbreviations, decorations, and subscripts. The notational elements can be mixed to provide a rich and expressive symbolic "language" for energy rebound. In several places, including Fig. 1, we use colored backgrounds on rebound effects for visual convenience.

2.2 Rebound relationships

Energy rebound is defined as

$$Re \equiv 1 - \frac{\text{actual final energy savings rate}}{\text{expected final energy savings rate}} , \tag{1}$$

where both actual and expected final energy savings rates are in MJ/yr (megajoules per year) and expected positive. The final energy "takeback" rate is defined as the expected final energy savings rate less the actual final energy savings rate.[5] Rewriting Eq. (1) with the definition of takeback gives

$$Re = 1 - \frac{\text{expected final energy savings rate} - \text{takeback rate}}{\text{expected final energy savings rate}} . \tag{2}$$

Simplifying gives

$$Re = \frac{\text{takeback rate}}{\text{expected final energy savings rate}} . \tag{3}$$

We define rebound at the final energy[6] stage of the energy conversion chain, because the final energy stage is the point of energy purchase by the device user. To simplify derivations, we choose not to apply final-to-primary energy multipliers to final energy rates in the numerators and denominators of rebound expressions derived from Eqs. (1) and (3); they divide out anyway.[7] Henceforth, we drop the adjective "final" from the noun "energy," unless there is reason to indicate a specific stage of the energy conversion chain.

2.3 The energy conversion device and energy efficiency upgrade (EEU)

We assume an energy conversion device (say, a car) that consumes energy (say, gasoline) at a rate $\dot{E}^{\circ}$ (in MJ/yr). We use "rate" to indicate any quantity measured per unit time, such as a flow of energy per year or a flow of income per year. None of the rates in this paper indicate exponential (%/yr) changes. Symbolically, rates are identified by a single dot above the symbol, a convention

[5]Note that the takeback rate can be negative, indicating that the actual final energy savings rate is greater than the expected final energy savings rate, a condition called hyperconservation.

[6]Conventionally, stages of the energy conversion chain are primary energy (e.g., coal, oil, natural gas, wind, and solar), final energy (e.g., electricity and refined petroleum), useful energy (e.g., heat, light, and mechanical drive), and energy services (e.g., transport, illumination, and space heating).

[7]Primary energy may be important when the upgraded device consumes a different final energy carrier compared to the original device, i.e., when fuel-switching occurs (Chan & Gillingham, 2015).

adopted from the engineering literature where, e.g., $\dot{x}$ often indicates a velocity in m/s (meters per second), $\dot{m}$ often indicates a mass flow rate in kg/s (kilograms per second), and $\dot{E}$ often indicates an energy flow rate in kW (kilowatts). The overdot is an important notational element in this paper, as it provides clarity between stocks (without overdots) and flows (with overdots). For example, E is a quantity of energy in, say, MJ, while $\dot{E}$ is a rate of energy in, say, MJ/yr. We later annualize capital costs (C_{cap} in \$) and energy embodied in the device during its production (E_{emb} in MJ) to create cost rates ($\dot{C}_{cap}$ in \$/yr) and embodied energy rates ($\dot{E}_{emb}$ in MJ/yr).

Energy is available at price p_E (in \$/MJ). The original energy conversion device provides a rate of energy service $\dot{q}_s^\circ$ (in, say, vehicle-km/yr) with final-to-service efficiency η° (in, say, vehicle-km/MJ). An energy efficiency upgrade (EEU) increases final-to-service efficiency[8] such that $\eta^\circ < \tilde{\eta}$. The EEU is not costless, so the upgraded device may be more expensive to purchase than a like-for-like replacement of the original device. We call this increased "capital cost" ($C_{cap}^\circ < \tilde{C}_{cap}$). It may also be more costly to maintain and dispose of the upgraded device ($\dot{C}_{md}^\circ < \tilde{\dot{C}}_{md}$). However, the opposite may hold, too. As final-to-service efficiency increases ($\eta^\circ < \tilde{\eta}$), the price of the energy service declines ($p_s^\circ > \tilde{p}_s$). The energy price (p_E) is assumed exogenous at the microeconomic level ($p_E^\circ = p_E^* = \hat{p}_E = \bar{p}_E = \tilde{p}_E$), so the energy purchaser (the device user) is a price taker.[9] Initially, the device user spends income ($\dot{M}^\circ$) on energy for the device ($\dot{C}_s^\circ = p_E \dot{E}_s^\circ$), annualized capital costs for the device ($\dot{C}_{cap}^\circ$), annualized costs for maintenance and disposal of the device ($\dot{C}_{md}^\circ$), and other goods and services ($\dot{C}_o^\circ$). The budget constraint for the device user is

$$\dot{M}^\circ = \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ + \cancelto{0}{\dot{N}^\circ} , \tag{4}$$

where net savings prior to the EEU ($\dot{N}^\circ$) is zero, by definition.

Later (Sections 2.5.1–2.5.4), we walk through the four rebound effects (emplacement, substitution, income, and macro), deriving rebound expressions for each, but first we show typical energy and cost relationships (Section 2.4).

[8] Note that energy service efficiency (η) improves between the original ($\circ$) and post-emplacement ($*$) stages of Fig. 1, remaining constant thereafter. Thus, $\eta^\circ < \eta^* = \hat{\eta} = \bar{\eta} = \tilde{\eta}$, as shown in Table B.1. We refer to all post-emplacement efficiencies (η^*, $\hat{\eta}$, $\bar{\eta}$, and $\tilde{\eta}$) as $\tilde{\eta}$ to match the nomenclature of Borenstein (2015). When convenient, the same approach to nomenclature is taken with other quantities such as the capital cost rate ($\dot{C}_{cap}$) and maintenance and disposal cost rate ($\dot{C}_{md}$).

[9] Relaxing the exogenous energy price assumption would require a general equilibrium model that is beyond the scope of this paper.

2.4 Typical energy and cost relationships

With the rebound notation of Appendix A, four typical relationships emerge. First, the consumption rate of the energy service ($\dot{q}_s$) is the product of final-to-service efficiency (η) and the rate of energy consumption by the energy conversion device ($\dot{E}_s$). Typical units for automotive transport and illumination (the examples in Part II) are shown beneath each equation.[10]

$$\dot{q}_s = \eta \dot{E}_s \tag{5}$$

$$[\text{pass·km/yr}] = [\text{pass·km/MJ}][\text{MJ/yr}]$$

$$[\text{lm·hr/yr}] = [\text{lm·hr/MJ}][\text{MJ/yr}]$$

Second, the energy service price (p_s) is the ratio of energy price (p_E) to the final-to-service efficiency (η).

$$p_s = \frac{p_E}{\eta} \tag{6}$$

$$[\$/\text{pass·km}] = \frac{[\$/\text{MJ}]}{[\text{pass·km/MJ}]}$$

$$[\$/\text{lm·hr}] = \frac{[\$/\text{MJ}]}{[\text{lm·hr/MJ}]}$$

Third, energy service expenditure rates ($\dot{C}_s$) are the product of energy price (p_E) and device energy consumption rates ($\dot{E}_s$).

$$\dot{C}_s = p_E \dot{E}_s \tag{7}$$

$$[\$/\text{yr}] = [\$/\text{MJ}][\text{MJ/yr}]$$

Fourth, indirect energy rates for maintenance and disposal ($\dot{E}_{md}$) and other goods expenditures ($\dot{E}_o$) are the product of expenditures rates ($\dot{C}_{md}$ and $\dot{C}_o$) and the energy intensity of the economy (I_E).

$$\dot{E}_{md} = \dot{C}_{md} I_E \tag{8}$$

$$\dot{E}_o = \dot{C}_o I_E \tag{9}$$

$$[\text{MJ/yr}] = [\$/\text{yr}][\text{MJ}/\$]$$

[10] Note that "pass" is short for "passenger," and "lm" is the SI notation for the lumen, a unit of lighting energy rate.

2.5 Rebound effects

The four rebound effects (emplacement, substitution, income, and macro) are discussed in subsections below. In each subsection, we define the effect and show mathematical expressions for rebound (Re) caused by the effect. Detailed derivations of all rebound expressions can be found in Appendix B. See, in particular, Tables B.3–B.6, which provide a parallel structure for energy and financial accounting across all rebound effects. We begin with the emplacement effect.

2.5.1 Emplacement effect

The emplacement effect accounts for performance changes of the device due to the fact that a higher-efficiency device has been put in service (and will need to be decommissioned at a later date); behavior changes are addressed later, in the substitution and income effects.

Direct emplacement effect (Re_{dempl}) The direct emplacement effects of the EEU include device energy savings ($\dot{S}_{dev}$) and device energy cost savings ($\Delta\dot{C}_s^*$). The indirect effects of EEU emplacement are (i) changes in the embodied energy rate ($\Delta\dot{E}_{emb}^*$), (ii) changes in the capital expenditure rate ($\Delta\dot{C}_{cap}^*$), and (iii) changes in the maintenance and disposal energy and expenditure rates ($\Delta\dot{E}_{md}^*$ and $\Delta\dot{C}_{md}^*$). $\dot{S}_{dev}$ can be written conveniently as

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right)\frac{\eta^\circ}{\tilde{\eta}}\dot{E}_s^\circ \ . \tag{10}$$

(See Appendix B.3.1 for the derivation.)

Because the original and upgraded device are assumed to have equal performance[11] and because behavior changes are not considered in the direct emplacement effect, actual and expected energy savings rates are identical, and there is no takeback. By definition, then, the direct emplacement effect causes no rebound. Thus,

$$Re_{dempl} = 0 \ . \tag{11}$$

[11]Of course, it is often the case that the original and upgraded devices have small performance differences. E.g., a high-efficiency LED lamp may have slightly greater or slightly lesser lumen output than the incandescent lamp it replaces. For the purpose of explicating this framework, we assume that the performance of the upgraded device can be matched closely enough to the performance of the original device such that the differences are immaterial to the user.

Indirect emplacement effects (Re_{iempl}) Although the direct emplacement effect does not cause rebound, indirect emplacement effects may indeed cause rebound. Indirect emplacement effects account for the life cycle of the energy conversion device, including energy embodied by manufacturing processes (subscript *emb*) and maintenance and disposal activities (subscript *md*).

Embodied energy effect (Re_{emb}) One of the unique features of this framework is that independent analyses of embodied energy and capital costs of the EEU are required. We note that the different terms (embodied energy rate, $\dot{E}_{emb}$, and capital cost rate, $\dot{C}_{cap}$) might seem to imply different processes, but they actually refer to the same emplacement effect. Purchasing an upgraded device (which likely leads to $\dot{C}^{\circ}_{cap} \neq \dot{C}^{*}_{cap}$) will likely mean a changed embodied energy rate ($\dot{E}^{\circ}_{emb} \neq \dot{E}^{*}_{emb}$) to provide the same energy service. Our names for these aspects of rebound (embodied energy and capital cost) reflect common usage in the energy and economics fields, respectively.

Consistent with the energy analysis literature, we define embodied energy to be the sum of all energy consumed in the production of the energy conversion device, all the way back to resource extraction.[12] Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. We assume no energy is embodied in the device while in service. The EEU causes the embodied energy of the energy conversion device to change from E°_{emb} to E^{*}_{emb}.

For simplicity, we spread all embodied energy over the lifetime of the device to provide a constant embodied energy rate ($\dot{E}_{emb}$). (We later take the same approach to capital costs ($\dot{C}_{cap}$) and maintenance and disposal costs ($\dot{C}_{md}$).) A justification for spreading embodied energy and purchase costs comes from considering device replacements by many consumers across several years. In the aggregate, evenly spaced (in time) replacements work out to the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) without discounting to obtain embodied energy rates, such that $\dot{E}^{\circ}_{emb} = E^{\circ}_{emb}/t^{\circ}_{life}$ and $\dot{E}^{*}_{emb} = E^{*}_{emb}/t^{*}_{life}$. The change in embodied final energy due to the EEU (expressed as a rate) is

[12] We take an energy approach here, consistent with the literature on energy rebound. One could use an alternative quantification of energy, such as exergy, the work potential of energy (Sciubba & Wall, 2007) or emergy, the solar content of energy (Brown & Herendeen, 1996).

given by $\Delta \dot{E}^*_{emb} = \dot{E}^*_{emb} - \dot{E}^\circ_{emb}$. The expression for embodied energy rebound is

$$Re_{emb} = \frac{\left(\frac{E^*_{emb}}{E^\circ_{emb}} \frac{t^\circ_{life}}{t^*_{life}} - 1\right) \dot{E}^\circ_{emb}}{\dot{S}_{dev}} . \tag{12}$$

(See Appendix B.3.2 for details of the derivation.)

Embodied energy rebound (Re_{emb}) can be either positive or negative, depending on the sign of the term $(E^*_{emb}/E^\circ_{emb})(t^\circ_{life}/t^*_{life}) - 1$. Rising energy efficiency can be associated with increased device complexity, additional energy consumption in manufacturing, and more embodied energy, such that $E^\circ_{emb} < E^*_{emb}$ and $Re_{emb} > 0$, all other things being equal. However, if the upgraded device has longer life than the original device ($t^*_{life} > t^\circ_{life}$), $\dot{E}^*_{emb} - \dot{E}^\circ_{emb}$ could be negative, meaning that the upgraded device has a lower embodied energy rate than the original device.

Maintenance and disposal effect (Re_{md}) In addition to embodied energy, indirect emplacement effect rebound accounts for energy demanded by maintenance and disposal (md) activities. Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}^\circ_m$). Disposal costs (e.g., C°_d) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal costs across the lifetime of the original and upgraded devices (t°_{life} and t^*_{life}, respectively) to form expenditure rates such that $\dot{C}^\circ_{md} = \dot{C}^\circ_m + C^\circ_d/t^\circ_{life}$ and $\dot{C}^*_{md} = \dot{C}^*_m + C^*_d/t^*_{life}$.

For simplicity, we assume that maintenance and disposal expenditures imply energy consumption elsewhere in the economy at its overall energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in maintenance and disposal expenditures is given by $\Delta \dot{C}^*_{md} I_E = (\dot{C}^*_{md} - \dot{C}^\circ_{md}) I_E$. Rebound from maintenance and disposal activities is given by

$$Re_{md} = \frac{\left(\frac{\dot{C}^*_{md}}{\dot{C}^\circ_{md}} - 1\right) \dot{C}^\circ_{md} I_E}{\dot{S}_{dev}} . \tag{13}$$

(See Appendix B.3.2 for details of the derivation.)

2.5.2 Substitution effect

Neoclassical consumer theory decomposes price-induced behavior change into (i) substituting energy service consumption for other goods consumption due to the lower post-EEU price of the energy service (the substitution effect) and (ii) spending the higher real income (the income effect).[13] This section develops mathematical expressions for substitution effect rebound (Re_{sub}), thereby accepting the standard neoclassical microeconomic assumptions about consumer behavior.[14] (The next section addresses income effect rebound, Re_{inc}.) The substitution effect determines compensated demand, which is the demand for the expenditure-minimizing consumption bundle that maintains utility at the pre-EEU level, given the new prices. Compensated demand is a technical term for a thought experiment from welfare economics: the device user's budget is altered so that the user is "compensated" for the change in price so as to maintain the same level of utility as before. In the case of an EEU, this implies the budget is reduced because the energy service price has fallen, so that it becomes cheaper to maintain a given level of utility. The change in the budget is called "compensating variation" (CV). The substitution effect involves (i) an increase in consumption of the energy service, the direct substitution effect (subscript $dsub$) and (ii) a decrease in consumption of other goods, the indirect substitution effect (subscript $isub$). Thus, two terms comprise substitution effect rebound: direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}).

After emplacement of the more efficient device (but before the substitution effect), the price of the energy service decreases ($p_s^\circ > p_s^*$). After compensating variation tightens the budget constraint, consumption at the new prices yields utility at the same level as prior to the EEU by consuming more of the now-lower-cost energy service and less of the now-relatively-more-expensive other goods.

A constant price elasticity (CPE) utility model is often used in the literature (e.g., see Borenstein (2015, p. 17, footnote 43)) for determining post-substitution effect consumption and therefore Re_{dsub} and Re_{isub}. By definition, the CPE utility model assumes that compensated and uncompensated, own and cross price elasticities remain constant along an indifference curve. (See Appendix C.) Typically, constant price elasticities (as in the CPE utility model) are approximations that are

[13] For the original development of the decomposition see Slutsky (1915) and Allen (1936). For a modern introduction see Nicholson & Snyder (2017).

[14] Alternative assumptions on behavior would arise from, e.g., adopting a behavioral economic framework (Dütschke et al., 2018; Dorner, 2019) or an informational entropy-constrained economic framework (Foley, 2020).

applicable only to marginal price changes. Appendix B.3.3 contains details of the CPE utility model.

Here, we present a constant elasticity of substitution (CES) utility model that allows all of the uncompensated own price elasticity ($\varepsilon_{\dot{q}_s,p_s}$), the uncompensated cross price elasticity ($\varepsilon_{\dot{q}_o,p_s}$), the compensated own price elasticity ($\varepsilon_{\dot{q}_s,p_s,c}$), and the compensated cross price elasticity ($\varepsilon_{\dot{q}_o,p_s,c}$) to vary along an indifference curve, thereby enabling numerically precise analysis of non-marginal energy service price changes ($p_s^\circ \gg p_s^*$). The CES utility model allows the direct calculation of the utility-maximizing consumption bundle for any constraint, describing the device user's behavior as

$$\frac{\dot{u}}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_s}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_o}{\dot{C}_o^\circ} \right)^\rho \right]^{(1/\rho)} . \tag{14}$$

The device user's utility rate (relative to the original condition, $\dot{u}/\dot{u}^\circ$) is determined by the consumption rate of the energy service ($\dot{q}_s$) and the consumption rate of other goods and services ($\dot{C}_o$). The share parameter ($f_{\dot{C}_s}^\circ$) between $\dot{q}_s$ and $\dot{C}_o$ is taken from the original (pre-EEU) consumption basket. The exponent ρ is calculated from the (constant) elasticity of substitution (σ) as $\rho \equiv (\sigma - 1)/\sigma$. All quantities are normalized to pre-EEU values so that the cost share of other goods can be used straightforwardly in empirical applications rather than having to construct quantity and price indices. The normalized specification is commonly used in empirical CES *production* function applications (Klump et al., 2012; Temple, 2012; Gechert et al., 2021). See Appendix C for further details of the CES utility model.

Direct substitution effect rebound (Re_{dsub}) is

$$Re_{dsub} = \frac{\Delta \hat{\dot{E}}_s}{\dot{S}_{dev}} , \tag{15}$$

which can be rearranged to

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} . \tag{16}$$

Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} , \tag{17}$$

which can be rearranged to

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, \frac{\tilde{\eta}}{\eta^\circ} \, \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \ . \tag{18}$$

To find the post-substitution effect point ($\wedge$), we solve for the location on the indifference curve where its slope is equal to the slope of the expenditure line after the EEU, assuming the CES utility model.[15] The results are

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\rho/(1-\rho)} \right\}^{-1/\rho} \tag{19}$$

and

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\rho/(\rho-1)} - 1 \right\} \right)^{-1/\rho} \ . \tag{20}$$

Eq. (19) can be substituted directly into Eq. (16) to obtain an expression for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\rho/(1-\rho)} \right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{21}$$

Eq. (20) can be substituted directly into Eq. (18) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\rho/(\rho-1)} - 1 \right\} \right)^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \, \frac{\tilde{\eta}}{\eta^\circ} \, \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \tag{22}$$

(See Appendix B.3.3 for details of the derivations of Eqs. (16), (18), and (19)–(22).)

2.5.3 Income effect

The monetary income rate of the device user ($\dot{M}^\circ$) remains unchanged across the rebound effects, such that $\dot{M}^\circ = \dot{M}^* = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Thanks to the energy service price decline, real income

[15] Other utility models could be used; however, the Cobb-Douglas utility model is inappropriate for this framework, because it assumes that the sum of substitution and income rebound is 100% *always*. Regardless of the utility model, expressions for $\hat{\dot{q}}_s/\dot{q}_s^\circ$ and $\hat{\dot{C}}_o/\dot{C}_o^\circ$ must be determined and substituted into Eqs. (16) and (18), respectively.

rises, and freed cash from the EEU is given by as $\dot{G} = p_E \dot{S}_{dev}$. (See Eq. (49) in Appendix B.2.) Emplacement effect adjustments and compensating variation modify freed cash to leave the device user with *net* savings ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (59) in Appendix B.2. (Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables B.3 and B.4.) Rebound from the income effect quantifies the rate of additional energy demand that arises when the energy conversion device user spends net savings from the EEU.

Additional energy demand from the income effect is determined by several constraints. The income effect under utility maximization satisfies the budget constraint, so that net savings are zero after the income effect ($\bar{\dot{N}} = 0$). (See Appendix D for a mathematical proof that the income preference equations below (Eqs. (23) and (27)) satisfy the budget constraint.)

A second constraint is that net savings are spent completely on (i) additional consumption of the energy service ($\hat{\dot{q}}_s < \bar{\dot{q}}_s$) and (ii) additional consumption of other goods ($\hat{\dot{q}}_o < \bar{\dot{q}}_o$). The proportions in which income-effect spending is allocated depends on the utility model, which prescribes the income expansion path for consumption. Given post-EEU prices, maximized CES utility means spending in the same proportion on the energy service and other goods across the income effect, a property known as homotheticity. This constraint is satisfied by construction below, particularly via an effective income term ($\hat{\dot{M}}'$). However, this framework could accommodate non-homothetic preferences for spending across the income effect (turning the income expansion path into a more general curve instead of a line).

We next show expressions for direct and indirect income effect rebound.

Direct income effect (Re_{dinc}) The income elasticity of energy service demand ($\varepsilon_{\dot{q}_s,\dot{M}}$) quantifies the amount of net savings spent on more of the energy service ($\hat{\dot{q}}_s < \bar{\dot{q}}_s$). (See Appendix C for additional information about elasticities.) Spending of net savings on additional energy service consumption leads to direct income effect rebound (Re_{dinc}).

The ratio of rates of energy service consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_s,\dot{M}}} . \tag{23}$$

Homotheticity means that $\varepsilon_{\dot{q}_s,\dot{M}} = 1$.

Effective income ($\hat{\dot{M}}'$) is given by

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \; . \tag{24}$$

For the purposes of the income effect, effective income (Eq. (24)) adjusts original income ($\dot{M}^\circ$) to account for sunk costs ($\dot{C}^*_{cap}$ and $\dot{C}^*_{md}$) and net savings ($\hat{\dot{N}}$).

Direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \bar{\dot{E}}_s}{\dot{S}_{dev}} \; . \tag{25}$$

(See Table B.5.) After substitution, rearranging, and canceling of terms (Appendix B.3.4), the expression for direct income rebound is

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left\{ f^\circ_{\dot{C}_s} + (1 - f^\circ_{\dot{C}_s}) \left[\left(\frac{1 - f^\circ_{\dot{C}_s}}{f^\circ_{\dot{C}_s}} \right) \frac{\tilde{p}_s \dot{q}^\circ_s}{\dot{C}^\circ_o} \right]^{\rho/(1-\rho)} \right\}^{-1/\rho} \; . \tag{26}$$

If there are no net savings after the substitution effect ($\hat{\dot{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected.[16]

Indirect income effect (Re_{iinc}) Not all net savings ($\hat{\dot{N}}$) are spent on more energy for the energy conversion device. The income elasticity of other goods demand ($\varepsilon_{\dot{q}_o,\dot{M}}$) quantifies the amount of net savings spent on additional other goods ($\hat{\dot{q}}_o < \bar{\dot{q}}_o$). Spending of net savings on additional other goods and services leads to indirect income effect rebound (Re_{iinc}).

The ratio of rates of other goods consumed across the income effect is given by

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} \; . \tag{27}$$

Under the assumption that prices of other goods are exogenous (see Appendix E), the ratio of rates of other goods consumption ($\bar{\dot{q}}_o/\hat{\dot{q}}_o$) is equal to the ratio of rates of other goods expenditures ($\bar{\dot{C}}_o/\hat{\dot{C}}_o$) such that

[16]Zero net savings ($\hat{\dot{N}} = 0$) could occur if increases in the capital cost rate ($\Delta\dot{C}^*_{cap}$) and/or the maintenance and disposal cost rate ($\Delta\dot{C}^*_{md}$) consume all freed cash ($\dot{G}$) plus savings from the compensating variation.

$$\frac{\dot{\bar{\hat{C}}}_o}{\dot{\hat{C}}_o} = \left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} . \tag{28}$$

Homotheticity means that $\varepsilon_{\dot{q}_o,\dot{M}} = 1$. As shown in Table B.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta \dot{\bar{C}}_o I_E}{\dot{S}_{dev}} . \tag{29}$$

After substitution, rearranging, and canceling of terms, the expression for indirect income rebound is

$$Re_{iinc} = \frac{\left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\varepsilon_{\dot{q}_o,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left(\frac{\tilde{\eta}}{\eta^\circ}\right) \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\rho/(\rho-1)} - 1 \right\} \right)^{-1/\rho} . \tag{30}$$

(See Appendix B.3.4 for details of the derivation of direct and indirect income effect rebound.)

2.5.4 Macro effect

The previous rebound effects (emplacement effect, substitution effect, and income effect) occur at the microeconomic level. However, changes at the microeconomic level can have important impacts at the macroeconomic or economy-wide level. In the short run, macroeconomic changes include price changes in goods other than the energy service. For instance, other goods to which the energy service is an input could become cheaper, and changes in demand from cross price elasticities could alter other prices as quantities supplied adjust to the new demand schedule. The most notable price change is the price of energy itself which could fall due to lower demand. The energy price or market effect is accordingly typically noted as an important macroeconomic rebound effect (Gillingham et al., 2016). In the long-run, i.e., when capital stock can be replaced in response to changes in relative costs and demand, rebound could change further. These kinds of rebounds can be captured by a general equilibrium model (Stern, 2020).

In addition, there are dynamic effects that arise from economic growth and structural change. It is one of the basic tenets of economics that productivity gains have been the main long-run driver of economic growth in the last couple of centuries (Smith, 1776; Marx, 1867; Solow, 1957) and that such growth is accompanied by structural changes, i.e., a changing composition of economic activity (Schumpeter, 1939; Kuznets, 1971). Structural changes pose complicated problems for rebound, as network effects can lead to path-dependencies in using low- or high-energy intensity technologies

(Arthur, 1989; Fouquet, 2016). Structural changes also interact with economic growth. We propose a simple rule for incorporating these dynamic effects into our framework below.

Before establishing a formalism for Re_{macro}, we clarify the link between consumer theory and economic growth. Turner (2013) cautions that when households see the productivity of their non-market activities increase, GDP remains unchanged.[17] That may be true in the short run. But the question over longer periods is whether the more productive household energy services do not also feed through into economic growth accounted for by GDP. People in affluent countries spend about as much time on unpaid (i.e., non-market) work as on paid work (Folbre, 2021). Therefore productivity improvements in unpaid work can spill over into paid work, which enters GDP. One channel could be time-saving. If the EEU saves time, then saved time could be spent on more paid work or on increasing human capital (Sorrell & Dimitropoulos, 2008; Gautham & Folbre, 2022). If the EEU saves money (but no time), then the freed cash could be spent on more effective (and more costly) human capital-increasing activities or even be used to start a venture. In all cases, it would be rash to conclude that just because some EEUs lead to productivity increases not captured directly by GDP, they do not eventually lead to additional economic growth.

Borenstein also addressed these macro effects from consumer behavior noting that "income effect rebound will be larger economy-wide than would be inferred from evaluating only the direct income gain from the end user's transaction" (Borenstein, 2015, p. 11) and likening it to the Keynesian macroeconomic multiplier. However, the dynamic macro rebound effect is not an autonomous expansion of expenditure, a demand-side shock, in an otherwise unchanged economy, like the Keynesian multiplier (Kahn, 1931; Keynes, 1936). Rather, macroeconomic rebound is caused by an energy productivity improvement, a supply-side shock. After the EEU, it takes less energy (and therefore less energy cost) to generate the same economic activity, because energy efficiency has improved. That said, Borenstein is right to highlight that supply-side and demand-side effects both play a role as the consequences of the technology shock play themselves out. Furthermore, his

[17] To appreciate the difference between production for the market and production for the household, consider the case where increased mileage leads to the household saving on energy per car trip. The household takes more trips (direct rebound), without effect on GDP. In the other case, the household buys the energy service (transport) directly from a taxi company. Here, the taxi company lowers the price but gains more customers, leading immediately to growth in inflation-adjusted (i.e., real) GDP, as more driving services are produced. Yet, the physical change of more car trips is the same in both cases.

approach has the advantage that it can be directly linked to the income effect (minus compensating variation) and its consequence for macroeconomic rebound. Borenstein also notes that scaling from net savings ($\dot{N}^*$) at the device level to productivity-driven growth at the macro level is unexplored territory.

Another novel contribution of this paper (in addition to the framework itself) is the first operationalization of the macro rebound multiplier idea. We stress that such a multiplier stands for the cumulative productivity growth triggered by the initial productivity increase in the EEU. But to operationalize the macro rebound multiplier, we note that the net savings gained by the device user at the microeconomic level ($\dot{N}^*$) are spent on new goods that create new incomes and, according to the marginal propensity to consume (MPC), expenditures throughout the economy. Over time, and allowing for temporary contractions (Basu et al., 2006), this leads to the infinite series of respending of net savings ($\dot{N}^*$), a multiplier which we represent by a macro factor (k).[18]

The macro factor (k) represents respending in the broader economy after the emplacement effect has occurred and is not tied to any particular EEU or economic sector. $k \geq 0$ is expected. $k = 0$ means there is no dynamic effect resulting from the energy efficiency upgrade. $k > 0$ means that productivity-driven macroeconomic growth has occurred with consequent implications for additional energy consumption in the wider economy. The relationship between k and MPC is given by the multiplier relationship

$$k = \frac{1}{\frac{1}{MPC} - 1} \; . \tag{31}$$

(See Appendix F for the derivation of Eq. (31).)

A further advantage of using the macro factor approach is that there are many estimates of the magnitude of MPC, though we stress again that using consumption multipliers is a *representation* of the effect, while the cause is not a demand-side fiscal expansion, but rather energy efficiency on the supply side.[19] A recent review by Carroll et al. (2017) reports that most empirical estimates show MPC between 0.2 and 0.6, with the full range of estimates spanning 0.0 to 0.9.

[18]The macro factor (k) appears unitless, but its units are actually \$ of economic expansion created per \$ of net savings gained by the device user in the emplacement effect ($\dot{N}^*$) throughout the economy.

[19]In particular, this approach avoids the problem of crowding out, since productive capacity expands, not just expenditure (Gillingham et al., 2016).

We assume as a first approximation (following Antal & van den Bergh (2014) and Borenstein (2015)) that macro effect respending implies energy consumption according to the average energy intensity of the economy (I_E). Macro rebound is therefore given by

$$Re_{macro} = \frac{k\dot{N}^* I_E}{\dot{S}_{dev}} \; . \tag{32}$$

(See Table B.6.) After some algebra (Appendix B.3.5), we arrive at an expression for macro effect rebound:

$$Re_{macro} = k(p_E I_E - Re_{cap} - Re_{md}) \; . \tag{33}$$

2.6 Rebound sum

The sum of all rebound emerges from the four rebound effects (emplacement effect, substitution effect, income effect, and macro effect). Macro effect rebound (Re_{macro} in Eq. (33)) is expressed in terms of other rebound effects. (Derivation details can be found in Appendix B.3.6.) After algebra and canceling of terms, we find

$$Re_{tot} = \; Re_{emb} + k(p_E I_E - Re_{cap}) + (1-k)Re_{md} + Re_{dsub} + Re_{isub} + Re_{dinc} + Re_{iinc} \; . \tag{34}$$

3 Discussion

We developed above a rebound framework for consumers. We note that many of its components are similar to those for a producer-sided framework due to the symmetry between neoclassical microeconomic producer and consumer theory. Ours is a partial equilibrium framework at the microeconomic level that provides a detailed assessment of individual EEUs with tractable, easy-to-understand mathematics. Partial equilibrium frameworks are easier to understand, in part, because they constrain price variation to the energy service only; all other prices remain constant (at least at the microeconomic level).[20] In our framework, general equilibrium effects and other dynamic effects at the macroeconomic level are captured by a simplified, one-dimensional rebound effect discussed in Section 2.5.4.

[20] General equilibrium frameworks provide detail and precision on economy-wide price adjustments, but they give up specificity about individual device upgrades, make assumptions during calibration, and lose simplicity of exposition.

Table 2: Comparison among relevant rebound analysis frameworks. Empty (white) circles indicate no treatment of a subject by a framework. Partly and fully filled circles indicate partial and comprehensive treatment of a subject by a framework.

	Nässén & Holmberg (2009)	Thomas & Azevedo (2013a,b)	Borenstein (2015)	Chan & Gillingham (2015)	Wang et al. (2021)	This paper (2023)
Rebound effects						
Direct emplacement effect	●	●	●	●	●	●
Capital cost and embodied energy effect	◒	◒	◒	◒	○	●
Maintenance and disposal effect	○	○	◒	○	○	●
Direct and indirect substitution effects	◒	◒	●	●	●	●
Direct and indirect income effects	◒	◒	●	●	●	●
Macro effect	○	○	◒	○	○	◒
Other characteristics						
Analysis on energy, expenditure, and consumption planes	◒	◒	◒	◒	◒	●
Detailed model of device user behavior and preferences	◒	◒	◒	●	●	●
Non-marginal energy service price changes	○	○	○	○	○	●
Empirical application	●	●	◒	○	○	●

We are not the first to develop a rebound analysis framework, so it is worthwhile to compare our framework to others for key features: analysis of all rebound effects; analysis of energy, expenditure, and consumption aspects of rebound; level of detail in the consumer preference model; allowance for non-marginal energy efficiency changes; and empirical application. When all of the above characteristics are present, a fuller picture of rebound can emerge.[21] Table 2 shows our assessment of selected previous partial equilibrium frameworks (in columns) relative to the characteristics discussed above (in rows).

Because all frameworks evaluate the expected decrease in direct energy consumption from the EEU, the "Direct emplacement effect" row contains ● in all columns. Three early papers (Nässén & Holmberg, 2009; Thomas & Azevedo, 2013a,b) estimate rebound quantitatively, earning high marks (●) in the "Empirical application" row. Both Nässén & Holmberg and Thomas & Azevedo motivate their frameworks at least partially with microeconomic theory (consumer preferences and substitution and income effects) but use simple linear demand functions in their empirical analyses.

[21]See Section 2.2 of Part II for literal pictures of rebound in energy, expenditure, and consumption planes.

Thus, the connection between economic theory and empirics is tenuous, leading to intermediate ratings (◒ or less) in the "substitution effects," "income effects," and "Detailed model of consumer preferences" rows. More recently, Chan & Gillingham (2015) and Wang et al. (2021) anchor the rebound effect firmly in consumer theory, earning high ratings (●) in the "substitution effects," "income effects," and "Detailed model of consumer preferences" rows. They extend their frameworks to advanced topics that our framework does not presently incorporate, such as multiple fuels, energy services, and nested utility functions with intermediate inputs. However, neither Chan & Gillingham nor Wang et al. provide empirical applications, earning ○ in the last row of Table 2. In the middle of the table (and between the other studies in time), the framework by Borenstein (2015) touches on nearly all important characteristics. However, the Borenstein framework cannot separate substitution and income effects cleanly in empirical analysis, reverting to partial analyses of both, leading to a ◒ rating in the "Detailed model of consumer preferences" and "Empirical application" rows.

No previous framework engages fully with either the differential financial effects or the differential energetic effects of the upfront purchase of the upgraded device, leading to low ratings across all previous frameworks in the "Capital cost and embodied energy effect" row. In fact, except for Nässén & Holmberg (2009), no framework engages with capital costs, although all note its importance. (Nässén & Holmberg note that capital costs and embodied energy can have very strong effects on rebound.) Thomas & Azevedo (2013a,b) provide the only framework that traces embodied energy effects of every consumer good using input-output methods, but they do not analyze embodied energy of the upgraded device. Borenstein (2015) notes the embodied energy of the upgraded device and the embodied energy of other goods but does not integrate embodied energy or financing costs into the framework for empirical analysis. Borenstein is, however, the only author to treat the financial side of embodied energy or maintenance and disposal effects. Borenstein (2015) postulates the macro effect, but does not operationalize the link between micro and macro levels, earning ◒ in the "Macro effect" row. No other framework even discusses the link between macro and micro rebound effects, leading to ○ in the "Macro effect" row for all previous frameworks (apart from Borenstein (2015)). Our framework operationalizes the link between micro and macro levels, via the macro factor (k), but more work can be done in this area. Thus, "This paper (2023)" earns ◒ in the "Macro effect" row. Finally, all previous frameworks assume constant price elasticities and implicitly marginal or small improvements in efficiency, excluding the numerically precise analysis of important non-incremental upgrades where price elasticities are likely to vary. Therefore, all previous frameworks earn ○ in the "Non-marginal energy service price changes" row.

Table 2 shows that previous frameworks contain many key pieces, providing starting points from which to develop our rebound analysis framework. A left-to-right reading of the table demonstrates that previous frameworks start from microeconomic consumer theory and move towards more rigorous theoretical treatment over time, with recent frameworks making important advanced theoretical contributions at the expense of empirical applicability. In the end, no previous rebound analysis framework combines all rebound effects across energy, expenditure, and consumption aspects with a detailed model of consumer preferences, non-marginal energy service price changes, and empirical applicability for the simplest case (understandable across disciplines) of a single fuel and a single energy service. In particular, assessing the rebound implications of differential capital costs, non-marginal price changes, and the macro effect required conceptual development as in Section 2.5.4 and Appendix B.3.5. (Development of empirical applications is left for Part II.) This paper addresses most of the gaps in Table 2; hence we fill the "This paper (2023)" column with filled circles (●) in nearly all rows. By so doing, we enhance clarity in the field of energy rebound.

4 Conclusions

In this paper (Part I), we developed a rigorous analytical framework that includes all rebound effects across energy, expenditure, and consumption aspects with a detailed model of consumer preferences and non-marginal energy service price changes in an operational manner for the simplest case of a single fuel and a single energy service. With careful explication of rebound effects and clear derivation of rebound expressions, we advance the analytical foundations for empirical analyses and facilitate interdisciplinary understanding of rebound phenomena toward the goal of enhancing clarity in the field of energy rebound and enabling more robust rebound calculations for sound energy and climate policy.

Future work could be pursued in several areas. (i) Other utility models (besides the CES utility model, but not a Cobb-Douglas utility model) could be explored for the substitution effect. (ii) This framework could be extended to producer-sided energy rebound effects. (iii) This framework could be extended to include some of the advanced topics in Chan & Gillingham (2015) and Wang et al. (2021), such as multiple fuels or energy services, more than one other consumption good, and nested utility functions with intermediate inputs. (iv) This framework could be extended to include fuel-switching EEUs, wherein the upgraded device uses a different fuel from the original device. (v) The greenhouse gas emissions implications of energy rebound could be evaluated using this framework, provided that the primary energy associated with final energy purchases were available. Borenstein (2015) went some way to analyzing emissions and could provide a starting point for such work. The capability to analyze fuel-switching EEUs will be important for analyzing the greenhouse gas emissions implications of many EEUs that involve electrification, such as the transition to all-electric vehicles and the conversion of natural gas and oil furnaces to heat pumps for home heating.

In Part II of this paper, we attempt to bring further clarity to rebound analysis in three ways. First, we develop a way to visualize the energy, expenditure, and consumption aspects of rebound effects. Second, we apply the framework to two EEUs: an upgraded car and an upgraded electric lamp. Finally, we provide results of rebound calculations for the two examples.

Competing interests

Declarations of interest: none.

Author contributions

Author contributions for this paper (Part I of the two-part paper) are shown in Table 3.

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Table 3: Author contributions.

	MKH	GS	PEB
Conceptualization	●	●	
Methodology	●	●	●
Software			
Validation	●		●
Formal analysis			
Investigation	●	●	
Resources	●	●	●
Data curation			
Writing–original draft	●	●	
Writing–review & editing	●	●	●
Visualization			
Supervision	●		
Project administration	●		
Funding acquisition			●

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