Advancing the necessary foundations for empirical energy rebound estimates, Part II: Examples and results

Matthew Kuperus Heun[1,*], Gregor Semieniuk[2], and Paul E. Brockway[3]

[1]*Engineering Department, Calvin University, 3201 Burton St. SE, Grand Rapids, MI, 49546*
[2]*Political Economy Research Institute and Department of Economics, UMass Amherst*
[3]*Sustainability Research Institute, School of Earth and Environment, University of Leeds*
[*]Corresponding author: `mkh2@calvin.edu`

Abstract

**** Abstract ****

Keywords: Energy efficiency, Energy rebound, Energy services, Microeconomic rebound, Substitution and income effects, Macroeconomic rebound, **** Update for split paper ****

1 Introduction

**** Introduction ****

2 Methods: development of the comprehensive rebound framework

**** Methods ****

3 Results: Two applications of the rebound framework

**** Results ****

4 Discussion

**** Discussion ****

5 Conclusions

**** Conclusions ****

Competing interests

Declarations of interest: none.

Author contributions

	MKH	GS	PEB
Conceptualization	●	●	
Methodology	●	●	●
Software	●		
Validation	●		●
Formal analysis	●	●	●
Investigation	●	●	●
Resources	●		
Data curation			●
Writing–original draft	●	●	●
Writing–review & editing	●	●	●
Visualization	●	●	
Supervision	●		
Project administration	●		
Funding acquisition			

Data repository

Data and calculations are stored at Research Data Leeds Repository (`https://doi.org/10.5518/1201`). An `R` package for performing all rebound calculations can be found at `https://github.com/MatthewHeun/ReboundTools`. (See Heun (2021).)

Acknowledgements

Paul Brockway's time was funded by the UK Research and Innovation (UKRI) Council, supported under EPSRC Fellowship award EP/R024254/1. The authors are grateful for comments from internal reviewers Becky Haney and Jeremy Van Antwerp (Calvin University); Nathan Chan (University of Massachusetts at Amherst); and Zeke Marshall (University of Leeds). The authors benefitted from discussions with Daniele Girardi (University of Massachusetts at Amherst) and Christopher Blackburn (Bureau of Economic Analysis). Finally, the authors thank the students of MKH's Fall 2019 Thermal Systems Design course (ENGR333) at Calvin University who studied energy rebound for many energy conversion devices using an early version of this framework.

References

Borenstein, S. (2015). A microeconomic framework for evaluating energy efficiency rebound and some implications. The Energy Journal, 36(1), 1–21.

Heun, M. K. (2021). ReboundTools: an `R` package that provides functions to analyze energy rebound. V0.1.32. URL `https://doi.org/10.5281/zenodo.4999846`

Paoli, L., & Cullen, J. (2020). Technical limits for energy conversion efficiency. Energy, 192, 1–12.

Table A.1: Symbols and abbreviations.

Symbol	Meaning [example units]
a	a point in the emplacement effect on rebound path graphs or the share parameter in the CES utility model [–]
b	a point in the emplacement effect on rebound path graphs
C	cost [$]
c	a point in the substitution effect on rebound path graphs
d	a point in the income effect on rebound path graphs
E	final energy [MJ]
f	expenditure share [–]
G	freed cash [$]
I	energy intensity of economic activity [MJ/$]
k	macro factor [–]
M	income [$]
N	net savings [$]
p	price [$]
q	quantity [–]
Re	rebound [–]
S	energy cost savings [$]
t	energy conversion device lifetime [years]
u	utility [utils]

Appendices

A Nomenclature

Presentation of the comprehensive rebound analysis framework is aided by a nomenclature that describes energy stages and rebound effects, locations, and scales. Table A.1 shows symbols and abbreviations, their meanings, and example units. Table A.2 shows Greek letters, their meanings, and example units. Table A.3 shows abbreviations and acronyms. Table A.4 shows symbol decorations and their meanings. Table A.5 shows subscripts and their meanings.

Differences are indicated by the Greek letter Δ and always signify subtraction of a quantity at an earlier stage of Fig. **??** from the same quantity at a later stage of Fig. **??**. E.g., $\Delta\bar{X} \equiv \bar{X} - \hat{X}$, and $\Delta\tilde{X} \equiv \tilde{X} - \bar{X}$. Lack of decoration on a difference term indicates a difference that spans all stages of Fig. **??**. E.g., $\Delta X \equiv \tilde{X} - X^{\circ}$. ΔX is also the sum of differences across each stage in Fig. **??**, as shown below.

$$
\begin{aligned}
\Delta X &= \Delta\tilde{X} + \Delta\bar{X} + \Delta\hat{X} + \Delta X^{*} \\
\Delta X &= (\tilde{X} - \bar{X}) + (\bar{X} - \hat{X}) + (\hat{X} - X^{*}) + (X^{*} - X^{\circ}) \\
\Delta X &= (\tilde{X} - \cancel{\bar{X}}) + (\cancel{\bar{X}} - \cancel{\hat{X}}) + (\cancel{\hat{X}} - \cancel{X^{*}}) + (\cancel{X^{*}} - X^{\circ}) \\
\Delta X &= \tilde{X} - X^{\circ}
\end{aligned} \tag{1}
$$

Table A.2: Greek letters.

Greek letter	Meaning [example units]
Δ	difference (later quantity less earlier quantity, see Fig. **??**)
ϵ	elasticity [–]
$\epsilon_{\dot{q}_s,\dot{M}}$	income ($\dot{M}$) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,\dot{M}}$	income ($\dot{M}$) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s}$	uncompensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s}$	uncompensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
$\epsilon_{\dot{q}_s,p_s,c}$	compensated energy service price (p_s) elasticity of energy service demand ($\dot{q}_s$) [–]
$\epsilon_{\dot{q}_o,p_s,c}$	compensated energy service price (p_s) elasticity of other goods demand ($\dot{q}_o$) [–]
η	final-energy-to-service efficiency [vehicle-km/MJ]
σ	elasticity of substitution between the energy service ($\dot{q}_s^\circ$) and other goods ($\dot{q}_o^\circ$) [–]

Table A.3: Abbreviations.

Abbreviation	Meaning
APF	aggregate production function
CES	constant elasticity of substitution
CV	compensating variation
EEU	energy efficiency upgrade
GDP	gross domestic product
MPC	marginal propensity to consume
mpg	miles per U.S. gallon
U.S.	United States

Table A.4: Decorations.

Decoration	Meaning [example units]
X°	X originally (before the emplacement effect)
X^*	X after the emplacement effect (before the substitution effect)
$\hat{X}$	X after the substitution effect (before the income effect)
$\bar{X}$	X after the income effect (before the macro effect)
$\tilde{X}$	X after the macro effect
$\dot{X}$	rate of X [units of X/year]
M'	effective income [$]

Table A.5: Subscripts.

Subscript	Meaning
0	quantity at an initial time
1	a specific point on a consumption path graph
c	compensated
cap	capital costs
dev	device
$dempl$	direct emplacement effect
d	disposal
$dinc$	direct income effect
dir	direct effects (at the energy conversion device)
$dsub$	direct substitution effect
emb	embodied
i	index for other goods purchased in the economy
j	one of cap, md, or o in Eq. (**??**)
$iempl$	indirect emplacement effects
$iinc$	indirect income effect
$indir$	indirect effects (beyond the energy conversion device)
$isub$	indirect substitution effect
$life$	lifetime
m	maintenance
md	maintenance and disposal costs
o	other expenditures (besides energy) by the device owner
own	ownership duration
$macro$	macro effect
s	service stage of the energy conversion chain
tot	sum of all rebound effects in the framework

Table B.1: Lines and curves for rebound path graphs.

Rebound path graph	Lines and curves
Energy	Constant total energy consumption lines 0% and 100% rebound lines
Expenditure	Constant expenditure lines
Consumption	Constant expenditure lines Rays from origin to $\wedge$ point Indifference curves

B Mathematical details of rebound path graphs

Rebound path graphs show the impact of direct and indirect rebound effects in energy space, expenditure space, and consumption space. Notional rebound path graphs can be found in Figs. **??**–**??**. Rebound path graphs for the car example can be found in Figs. **??**–**??**. Graphs for the lamp example can be found in Figs. **??**–**??**.

This appendix shows the mathematical details of rebound path graphs, specifically derivations of equations for lines and curves shown in Table B.1. The lines and curves enable construction of numerically accurate rebound path graphs as shown in Figs. **??**–**??**.

B.1 Energy path graphs

Energy path graphs show direct (on the x-axis) and indirect (on the y-axis) energy consumption associated with the energy conversion device and the device owner. Lines of constant total energy consumption comprise a scale for total rebound. For example, the 0% and 100% rebound lines are constant total energy consumption lines which pass through the original point ($\circ$) and the post-direct-emplacement-effect point (a) on an energy path graph.

The equation of a constant total energy consumption line is derived from

$$\dot{E}_{tot} = \dot{E}_{dir} + \dot{E}_{indir} \tag{2}$$

at any rebound stage. (See Fig. **??**.) Direct energy consumption is energy consumed by the energy conversion device ($\dot{E}_s$), and indirect energy consumption is the sum of embodied energy, energy associated with maintenanace and disposal, and energy associated with expenditures on other goods ($\dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$).

For the energy path graph, direct energy consumption is placed on the x-axis and indirect energy consumption is placed on the y-axis. To derive the equation of a constant energy consumption line, we first rearrange to put the y coordinate on the left of the equation:

$$\dot{E}_{indir} = -\dot{E}_{dir} + \dot{E}_{tot} \; . \tag{3}$$

Next, we substitute y for $\dot{E}_{indir}$, x for $\dot{E}_{dir}$, and $\dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E$ for $\dot{E}_{tot}$ to obtain

$$y = -x + \dot{E}_s + \dot{E}_{emb} + (\dot{C}_{md} + \dot{C}_o)I_E \; , \tag{4}$$

where all of $\dot{E}_s$, $\dot{E}_{emb}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage of Fig. **??**.

The constant total energy consumption line that passes through the original point (∘) shows 100% rebound:

$$y = -x + \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ) I_E \; . \tag{5}$$

The 0% rebound line is the constant total energy consumption line that accounts for expected energy savings ($\dot{S}_{dev}$) only:

$$y = -x + (\dot{E}_s^\circ - \dot{S}_{dev}) + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ) I_E \; . \tag{6}$$

The above line passes through the a point on an energy path graph.

B.2 Expenditure path graphs

Expenditure path graphs show direct (on the x-axis) and indirect (on the y-axis) expenses associated with the energy conversion device and the device owner. Lines of constant expenditure are important, because they provide budget constraints for the device owner.

The equation of a constant total expenditure line is derived from the budget constraint

$$\dot{C}_{tot} = \dot{C}_{dir} + \dot{C}_{indir} \tag{7}$$

at any rebound stage. For the expenditure path graph, indirect expenditures are placed on the y-axis and direct expenditures on energy for the energy conversion device are place on the x-axis. Direct expenditure is the cost of energy consumed by the energy conversion device ($\dot{C}_s = p_E \dot{E}_s$), and indirect expenses are the sum of capital costs, maintenanace and disposal costs, and expenditures on other goods ($\dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$). Rearranging to put the y-axis variable on the left side of the equation gives

$$\dot{C}_{indir} = -\dot{C}_{dir} + \dot{C}_{tot} \; . \tag{8}$$

Substituting y for $\dot{C}_{indir}$, x for $\dot{C}_{dir}$, and $\dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o$ for $\dot{C}_{tot}$ gives

$$y = -x + \dot{C}_s + \dot{C}_{cap} + \dot{C}_{md} + \dot{C}_o \; , \tag{9}$$

where all of $\dot{C}_s$, $\dot{C}_{cap}$, $\dot{C}_{md}$, and $\dot{C}_o$ apply at the same rebound stage of Fig. **??**.

The constant total expenditure line that passes through the original point (∘) shows the budget constraint for the device owner:

$$y = -x + \dot{C}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \; , \tag{10}$$

into which Eq. (30) can be substituted with $\dot{C}_s^\circ = p_E \dot{E}_s^\circ$ and $\dot{N}^\circ = 0$ to obtain

$$y = -x + \dot{M}^\circ \; . \tag{11}$$

The constant total expenditure line that accounts for expected energy savings ($\dot{S}_{dev}$) and freed cash ($\dot{G} = p_E \dot{S}_{dev}$) only is given by:

$$y = -x + (\dot{C}_s^\circ - \dot{G}) + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \; , \tag{12}$$

or

$$y = -x + \dot{M}^\circ - \dot{G} \; . \tag{13}$$

The line given by the above equation passes through the a point on an expenditure path graph.

B.3 Consumption path graphs

Consumption path graphs show expenditures in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space to accord with the utility model. (See Appendix D.) Consumption path graphs include (i) constant expenditure lines given prices, (ii) a ray from the origin through the $\wedge$ point, and (iii) indifference curves. Derivations for each are shown in the following subsections.

B.3.1 Constant expenditure lines

There are four constant expenditure lines on the consumption path graphs of Figs. **??**, **??**, and **??**. The constant expenditure lines pass through the original point (line $\circ$—$\circ$), the post-emplacement point (line $*$—$*$), the post-substitution point (line $\wedge$—$\wedge$), and the post-income point (line $-$—$-$). Like the expenditure path graph, lines of constant expenditure on a consumption path graph are derived from the budget constraint of the device owner at each of the four points.

Prior to the EEU, the budget constraint is given by Eq. (30). Substituting $p_s^\circ \dot{q}_s^\circ$ for $p_E \dot{E}_s^\circ$ and recognizing that there is no net savings before the EEU ($\dot{N}^\circ = 0$) gives

$$\dot{M}^\circ = p_s^\circ \dot{q}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ \; . \tag{14}$$

To create the line of constant expenditure on the consumption path graph, we allow $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to vary in a compensatory manner: when one increases, the other must decrease. To show that variation along the constant expenditure line, we remove the notation that ties $\dot{q}_s^\circ$ and $\dot{C}_o^\circ$ to the original point ($\circ$) to obtain

$$\dot{M}^\circ = p_s^\circ \dot{q}_s + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o \; , \tag{15}$$

where all of $\dot{M}^\circ$, p_s°, $\dot{C}_{cap}^\circ$, and $\dot{C}_{md}^\circ$ apply at the same rebound stage of Fig. **??**, namely the original point ($\circ$).

To derive the equation of the line representing the original budget constraint in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space (the $\circ$—$\circ$ line through the $\circ$ point in consumption path graphs), we solve for $\dot{C}_o$ to obtain

$$\dot{C}_o = -p_s^\circ \dot{q}_s + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{16}$$

Multiplying judiciously by $\dot{C}_o^\circ/\dot{C}_o^\circ$ and $\dot{q}_s^\circ/\dot{q}_s^\circ$ gives

$$\frac{\dot{C}_o}{\dot{C}_o^\circ}\dot{C}_o^\circ = -p_s^\circ \frac{\dot{q}_s}{\dot{q}_s^\circ}\dot{q}_s^\circ + \dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{17}$$

Dividing both sides by $\dot{C}_o^\circ$ yields

$$\frac{\dot{C}_o}{\dot{C}_o^\circ} = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}\frac{\dot{q}_s}{\dot{q}_s^\circ} + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \; . \tag{18}$$

Noting that $\dot{q}_s/\dot{q}_s^\circ$ and $\dot{C}_o/\dot{C}_o^\circ$ are the x-axis and y-axis, respectively, on a consumption path graph gives

$$y = -\frac{p_s^\circ \dot{q}_s^\circ}{\dot{C}_o^\circ}x + \frac{1}{\dot{C}_o^\circ}(\dot{M}^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ) \; . \tag{19}$$

A similar procedure can be employed to derive the equation of the $*$—$*$ line through the $*$ point after the emplacement effect. The starting point is the budget constraint at the $*$ point (Eq. (32)) with $\dot{M}^\circ$ replacing $\dot{M}^*$, $\tilde{p}_s \dot{q}_s$ replacing $p_E \dot{E}_s^*$, and $\dot{C}_o$ replacing $\dot{C}_o^*$.

$$\dot{M}^\circ = \tilde{p}_s \dot{q}_s + \dot{C}^*_{cap} + \dot{C}^*_{md} + \dot{C}_o + \dot{N}^* \; . \tag{20}$$

Substituting Eq. (41) for $\dot{N}^*$, substituting Eq. (42) for $\dot{G}$, multiplying judiciously by $\dot{C}_o^\circ / \dot{C}_o^\circ$ and $\dot{q}_s^\circ / \dot{q}_s^\circ$, rearranging, and noting that $\dot{q}_s / \dot{q}_s^\circ$ is the x-axis and $\dot{C}_o / \dot{C}_o^\circ$ is the y-axis gives

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} x + \frac{1}{\dot{C}_o^\circ} (\dot{M}^\circ - \dot{C}^\circ_{cap} - \dot{C}^\circ_{md} - \dot{G}) \; . \tag{21}$$

Note that the slope of Eq. (21) is less negative than the slope of Eq. (19), because $\tilde{p}_s < p_s^\circ$. The y-intercept of Eq. (21) is less than the y-intercept of Eq. (19), reflecting freed cash. Both effects are seen in consumption path graphs (Figs. **??**, **??**, and **??**). The $\circ$—$\circ$ and $*$—$*$ lines intersect at the coincident $\circ$ and $*$ points.

A similar derivation process can be used to find the equation of line representing the budget constraint after the substitution effect (the $\wedge$—$\wedge$ line through the $\wedge$ point). The starting point is Eq. (45), and the equation for the constant expenditure line is

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} x + \frac{1}{\dot{C}_o^\circ} (\dot{M}^\circ - \dot{C}^\circ_{cap} - \dot{C}^\circ_{md} - \dot{G} + \tilde{p}_s \Delta \hat{\dot{q}}_s + \Delta \hat{\dot{C}}_o) \; . \tag{22}$$

Note that the $\wedge$—$\wedge$ line (Eq. (22)) has the same slope as the $*$—$*$ line (Eq. (21)) but a lower y-intercept.

Finally, the corresponding derivation for the equation of the constant expenditure line through the $-$ point (line $-$—$-$) starts with Eq. (54) and ends with

$$y = -\frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} x + \frac{1}{\dot{C}_o^\circ} (\dot{M}^\circ - \dot{C}^\circ_{cap} - \dot{C}^\circ_{md} - \Delta \dot{C}^*_{cap} - \Delta \dot{C}^*_{md}) \; . \tag{23}$$

B.3.2 Ray from the origin to the $\wedge$ point

On consumption path graphs, the ray from the origin to the $\wedge$ point (line r—r) defines the path along which the income effect (lines $\wedge$—d and d—$-$) operates. The ray from the origin to the $\wedge$ point has slope $(\hat{\dot{C}}_o / \dot{C}_o^\circ)/(\hat{\dot{q}}_s / \dot{q}_s^\circ)$ and a y-intercept of 0. Therefore, the equation of line r—r is

$$y = \frac{\hat{\dot{C}}_o / \dot{C}_o^\circ}{\hat{\dot{q}}_s / \dot{q}_s^\circ} x \; . \tag{24}$$

B.3.3 Indifference curves

On a consumption path graph, indifference curves represent lines of constant utility for the energy conversion device owner. In $\dot{C}_o / \dot{C}_o^\circ$ vs. $\dot{q}_s / \dot{q}_s^\circ$ space, any indifference curve is given by Eq. (**??**) with $f^\circ_{\dot{C}_s}$ replacing the share parameter a, as shown in Appendix D. Recognizing that $\dot{C}_o / \dot{C}_o^\circ$ is on the y-axis and $\dot{q}_s / \dot{q}_s^\circ$ is on the x-axis leads to substitution of y for $\dot{C}_o / \dot{C}_o^\circ$ and x for $\dot{q}_s / \dot{q}_s^\circ$ to obtain

$$y = \left[\frac{1}{1 - f^\circ_{\dot{C}_s}} \left(\frac{\dot{u}}{\dot{u}^\circ} \right)^\rho - \frac{f^\circ_{\dot{C}_s}}{1 - f^\circ_{\dot{C}_s}} (x)^\rho \right]^{(1/\rho)} \; . \tag{25}$$

At any point in $\dot{C}_o/\dot{C}_o^\circ$ vs. $\dot{q}_s/\dot{q}_s^\circ$ space, namely $(\dot{q}_{s,1}/\dot{q}_s^\circ,\ \dot{C}_{o,1}/\dot{C}_o^\circ)$, indexed utility $(\dot{u}_1/\dot{u}^\circ)$ is given by Eq. (**??**) as

$$\frac{\dot{u}_1}{\dot{u}^\circ} = \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ} \right)^\rho \right]^{(1/\rho)} . \tag{26}$$

Substituting Eq. (26) into Eq. (25) for $\dot{u}/\dot{u}^\circ$ and simplifying exponents gives

$$y = \left\{ \frac{1}{1 - f_{\dot{C}_s}^\circ} \left[f_{\dot{C}_s}^\circ \left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ} \right)^\rho + (1 - f_{\dot{C}_s}^\circ) \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ} \right)^\rho \right] - \frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} (x)^\rho \right\}^{(1/\rho)} . \tag{27}$$

Simplifying further yields the equation of an indifference curve passing through point $(\dot{q}_{s,1}/\dot{q}_s^\circ,\ \dot{C}_{o,1}/\dot{C}_o^\circ)$:

$$y = \left\{ \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} \right) \left[\left(\frac{\dot{q}_{s,1}}{\dot{q}_s^\circ} \right)^\rho - (x)^\rho \right] + \left(\frac{\dot{C}_{o,1}}{\dot{C}_o^\circ} \right)^\rho \right\}^{(1/\rho)} . \tag{28}$$

Note that if x is $\dot{q}_{s,1}/\dot{q}_s^\circ$, y becomes $\dot{C}_{o,1}/\dot{C}_o^\circ$, as expected.

C Derivation of comprehensive, consumer-based rebound analysis framework

This appendix provides a detailed derivation of the comprehensive rebound analysis framework, beginning with relationships for each rebound effect.

C.1 Relationships for rebound effects

For each energy rebound effect in Fig. **??**, energy and financial analysis must be performed. The purposes of the analyses are to determine for each effect (i) an expression for energy rebound (Re) for the effect and (ii) an equation for net savings ($\dot{N}$) remaining after the effect.

Analysis of each rebound effect involves a set of assumptions and constraints as shown in Table C.1. In Table C.1, relationships for emplacement effect embodied energy rates ($\dot{E}_{emb}^\circ$ and $\dot{E}_{emb}^*$), capital expenditure rates ($\dot{C}_{cap}^\circ$ and $\dot{C}_{cap}^*$), and maintenance and disposal expenditure rates ($\dot{C}_{md}^\circ$ and $\dot{C}_{md}^*$) are typical, and inequalities could switch direction for a specific EEU. Macro effect relationships are given for a single device only. If the EEU is deployed at scale across the economy, the energy service consumption rate ($\tilde{\dot{q}}_s$), device energy consumption rate ($\tilde{\dot{E}}_s$), embodied energy rate ($\tilde{\dot{E}}_{emb}$), capital expenditure rate ($\tilde{\dot{C}}_{cap}$), and maintenance and disposal expenditure rate ($\tilde{\dot{C}}_{md}$) will all increase in proportion to the number of devices emplaced.

Table C.1: Assumptions and constraints for analysis of rebound effects.

Parameter	Emplacement Effect	Substitution Effect	Income Effect	Macro Effect
Energy price	$p_E^\circ = p_E^*$	$p_E^* = \hat{p}_E$	$\hat{p}_E = \bar{p}_E$	$\bar{p}_E = \tilde{p}_E$
Energy service efficiency	$\eta^\circ < \eta^*$	$\eta^* = \hat{\eta}$	$\hat{\eta} = \bar{\eta}$	$\bar{\eta} = \tilde{\eta}$
Energy service price	$p_s^\circ > p_s^*$	$p_s^* = \hat{p}_s$	$\hat{p}_s = \bar{p}_s$	$\bar{p}_s = \tilde{p}_s$
Other goods price	$p_o^\circ = p_o^*$	$p_o^* = \hat{p}_o$	$\hat{p}_o = \bar{p}_o$	$\bar{p}_o = \tilde{p}_o$
Energy service consumption rate	$\dot{q}_s^\circ = \dot{q}_s^*$	$\dot{q}_s^* < \hat{\dot{q}}_s$	$\hat{\dot{q}}_s < \bar{\dot{q}}_s$	$\bar{\dot{q}}_s = \tilde{\dot{q}}_s$
Other goods consumption rate	$\dot{q}_o^\circ = \dot{q}_o^*$	$\dot{q}_o^* > \hat{\dot{q}}_o$	$\hat{\dot{q}}_o < \bar{\dot{q}}_o$	$\bar{\dot{q}}_o = \tilde{\dot{q}}_o$
Device energy consumption rate	$\dot{E}_s^\circ > \dot{E}_s^*$	$\dot{E}_s^* < \hat{\dot{E}}_s$	$\hat{\dot{E}}_s < \bar{\dot{E}}_s$	$\bar{\dot{E}}_s = \tilde{\dot{E}}_s$
Embodied energy rate	$\dot{E}_{emb}^\circ < \dot{E}_{emb}^*$	$\dot{E}_{emb}^* = \hat{\dot{E}}_{emb}$	$\hat{\dot{E}}_{emb} = \bar{\dot{E}}_{emb}$	$\bar{\dot{E}}_{emb} = \tilde{\dot{E}}_{emb}$
Capital expenditure rate	$\dot{C}_{cap}^\circ < \dot{C}_{cap}^*$	$\dot{C}_{cap}^* = \hat{\dot{C}}_{cap}$	$\hat{\dot{C}}_{cap} = \bar{\dot{C}}_{cap}$	$\bar{\dot{C}}_{cap} = \tilde{\dot{C}}_{cap}$
Maint. and disp. expenditure rate	$\dot{C}_{md}^\circ < \dot{C}_{md}^*$	$\dot{C}_{md}^* = \hat{\dot{C}}_{md}$	$\hat{\dot{C}}_{md} = \bar{\dot{C}}_{md}$	$\bar{\dot{C}}_{md} = \tilde{\dot{C}}_{md}$
Energy service expenditure rate	$\dot{C}_s^\circ > \dot{C}_s^*$	$\dot{C}_s^* < \hat{\dot{C}}_s$	$\hat{\dot{C}}_s < \bar{\dot{C}}_s$	$\bar{\dot{C}}_s = \tilde{\dot{C}}_s$
Other goods expenditure rate	$\dot{C}_o^\circ = \dot{C}_o^*$	$\dot{C}_o^* > \hat{\dot{C}}_o$	$\hat{\dot{C}}_o < \bar{\dot{C}}_o$	$\bar{\dot{C}}_o = \tilde{\dot{C}}_o$
Income	$\dot{M}^\circ = \dot{M}^*$	$\dot{M}^* = \hat{\dot{M}}$	$\hat{\dot{M}} = \bar{\dot{M}}$	$\bar{\dot{M}} = \tilde{\dot{M}}$
Net savings	$0 = \dot{N}^\circ < \dot{N}^*$	$\dot{N}^* < \hat{\dot{N}}$	$\hat{\dot{N}} > \bar{\dot{N}} = 0$	$\bar{\dot{N}} = \tilde{\dot{N}} = 0$

Table C.2: Sources for zeroed terms in Tables C.3–C.6.

Zeroed term	Justification (from Table C.1).
$\cancelto{0}{\Delta \dot{C}_o^*}$	$\dot{C}_o^\circ = \dot{C}_o^*$ ($\dot{C}_o$ unchanged across emplacement effect.)
$\cancelto{0}{\dot{N}^\circ}$	$0 = \dot{N}^\circ$ (Net savings are zero prior to the EEU.)
$\cancelto{0}{\Delta \hat{\dot{E}}_{emb}}$	$\dot{E}_{emb}^* = \hat{\dot{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \hat{\dot{C}}_{md}}$	$\dot{C}_{md}^* = \hat{\dot{C}}_{md}$ ($\dot{C}_{md}$ unchanged across substitution effect.)
$\cancelto{0}{\Delta \bar{\dot{E}}_{emb}}$	$\hat{\dot{E}}_{emb} = \bar{\dot{E}}_{emb}$ ($\dot{E}_{emb}$ unchanged across income effect.)
$\cancelto{0}{\Delta \bar{\dot{C}}_{md}}$	$\hat{\dot{C}}_{md} = \bar{\dot{C}}_{md}$ ($\dot{C}_{md}$ unchanged across income effect.)
$\cancelto{0}{\bar{\dot{N}}}$	$\bar{\dot{N}} = 0$ (All net savings are spent in the income effect.)

C.2 Derivations

Derivations for rebound definitions and net savings equations are presented in Tables C.3–C.6, one for each rebound effect in Fig. **??**. Energy and financial analyses are shown side by side, because each informs the other.

Several terms in Tables C.3–C.6 are zeroed, e.g. $\cancelto{0}{\Delta \dot{C}_o^*}$. These zeroes can be traced back to Table C.1. Table C.2 highlights the equations in Table C.1 that justify zeroing each term.

Table C.3. **Emplacement Effect**

	Energy analysis		*Financial analysis*	
before ($\circ$)	$\dot{E}^\circ = \dot{E}_s^\circ + \dot{E}_{emb}^\circ + (\dot{C}_{md}^\circ + \dot{C}_o^\circ) I_E$	(29)	$\dot{M}^\circ = p_E \dot{E}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \dot{C}_o^\circ + \dot{N}^\circ$	(30)
after ($*$)	$\dot{E}^* = \dot{E}_s^* + \dot{E}_{emb}^* + (\dot{C}_{md}^* + \dot{C}_o^*) I_E$	(31)	$\dot{M}^* = p_E \dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \dot{C}_o^* + \dot{N}^*$	(32)

Take differences to obtain the change in energy consumption, $\Delta \dot{E}^* \equiv \dot{E}^* - \dot{E}^\circ$.

$$\Delta \dot{E}^* = \Delta \dot{E}_s^* + \Delta \dot{E}_{emb}^* + (\Delta \dot{C}_{md}^* + \cancelto{0}{\Delta \dot{C}_o^*}) I_E \tag{33}$$

Thus,

$$\Delta \dot{E}^* = \Delta \dot{E}_s^* + \Delta \dot{E}_{emb}^* + \Delta \dot{C}_{md}^* I_E \; . \tag{34}$$

Define

$$\dot{S}_{dev} \equiv -\Delta \dot{E}_s^* \tag{35}$$

(Also see Eqs. (**??**) and (**??**)). Use Eq. (**??**) to obtain

$$Re_{empl} = 1 - \frac{-\Delta \dot{E}^*}{\dot{S}_{dev}} = 1 - \frac{-\Delta \dot{E}_s^*}{\dot{S}_{dev}} - \frac{-\Delta \dot{E}_{emb}^*}{\dot{S}_{dev}} - \frac{-\Delta \dot{C}_{md}^* I_E}{\dot{S}_{dev}} \; . \tag{36}$$

Define $Re_{dempl} \equiv 1 - \frac{-\Delta \dot{E}_s^*}{\dot{S}_{dev}} (= 0)$, $Re_{iempl} \equiv Re_{emb} + Re_{md}$, $Re_{emb} \equiv \frac{\Delta \dot{E}_{emb}^*}{\dot{S}_{dev}}$, and $Re_{md} \equiv \frac{\Delta \dot{C}_{md}^* I_E}{\dot{S}_{dev}}$, such that

$$Re_{empl} = Re_{dempl} + Re_{iempl} \; . \tag{37}$$

Use the monetary constraint ($\dot{M}^\circ = \dot{M}^*$) and constant spending on other items ($\dot{C}_o^\circ = \dot{C}_o^*$) to cancel terms to obtain

$$\begin{aligned} p_E \dot{E}_s^\circ + \dot{C}_{cap}^\circ + \dot{C}_{md}^\circ + \cancel{\dot{C}_o^\circ} + \cancelto{0}{\dot{N}^\circ} \\ = p_E \dot{E}_s^* + \dot{C}_{cap}^* + \dot{C}_{md}^* + \cancel{\dot{C}_o^*} + \dot{N}^* \; . \end{aligned} \tag{38}$$

Solving for $\Delta \dot{N}^* \equiv \dot{N}^* - \cancelto{0}{\dot{N}^\circ}$ gives

$$\Delta \dot{N}^* = p_E (\dot{E}_s^\circ - \dot{E}_s^*) + \dot{C}_{cap}^\circ - \dot{C}_{cap}^* + \dot{C}_{md}^\circ - \dot{C}_{md}^* \; . \tag{39}$$

Rewriting with Δ terms gives

$$\Delta \dot{N}^* = -p_E \Delta \dot{E}_s^* - \Delta \dot{C}_{cap}^* - \Delta \dot{C}_{md}^* \; . \tag{40}$$

Substituting Eq. (35) gives

$$\Delta \dot{N}^* = \dot{N}^* = p_E \dot{S}_{dev} - \Delta \dot{C}_{cap}^* - \Delta \dot{C}_{md}^* \; . \tag{41}$$

Freed cash ($\dot{G}$) resulting from the EEU, before any energy takeback, is given by

$$\dot{G} = p_E \dot{S}_{dev} \; . \tag{42}$$

Note that Eq. (30) and $\dot{N}^\circ = 0$ can be used to calculate $\dot{C}_o^\circ$ as

$$\dot{C}_o^\circ = \dot{M}^\circ - p_E \dot{E}_s^\circ - \dot{C}_{cap}^\circ - \dot{C}_{md}^\circ \; . \tag{43}$$

Table C.4. **Substitution Effect**

	Energy analysis		*Financial analysis*	
before $(*)$	$\dot{E}^* = \dot{E}^*_s + \dot{E}^*_{emb} + (\dot{C}^*_{md} + \dot{C}^*_o) I_E$	(31)	$\dot{M}^* = p_E \dot{E}^*_s + \dot{C}^*_{cap} + \dot{C}^*_{md} + \dot{C}^*_o + \dot{N}^*$	(32)
after $(\wedge)$	$\dot{\hat{E}} = \dot{\hat{E}}_s + \dot{\hat{E}}_{emb} + (\dot{\hat{C}}_{md} + \dot{\hat{C}}_o) I_E$	(44)	$\dot{\hat{M}} = p_E \dot{\hat{E}}_s + \dot{\hat{C}}_{cap} + \dot{\hat{C}}_{md} + \dot{\hat{C}}_o + \dot{\hat{N}}$	(45)

Take differences to obtain the change in energy consumption, $\Delta\dot{\hat{E}} \equiv \dot{\hat{E}} - \dot{E}^*$.

$$\Delta\dot{\hat{E}} = \Delta\dot{\hat{E}}_s + \cancelto{0}{\Delta\dot{\hat{E}}_{emb}} + (\cancelto{0}{\Delta\dot{\hat{C}}_{md}} + \Delta\dot{\hat{C}}_o) I_E \tag{46}$$

Thus,

$$\Delta\dot{\hat{E}} = \Delta\dot{\hat{E}}_s + \Delta\dot{\hat{C}}_o I_E \ . \tag{47}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\dot{\hat{E}}}{\dot{S}_{dev}} = \frac{\Delta\dot{\hat{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\dot{\hat{C}}_o I_E}{\dot{S}_{dev}} \tag{48}$$

Define $Re_{sub} \equiv \frac{\Delta\dot{\hat{E}}}{\dot{S}_{dev}}$, $Re_{dsub} \equiv \frac{\Delta\dot{\hat{E}}_s}{\dot{S}_{dev}}$, and $Re_{isub} \equiv \frac{\Delta\dot{\hat{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{sub} = Re_{dsub} + Re_{isub} \ . \tag{49}$$

Use the monetary constraint ($\dot{M}^* = \dot{\hat{M}}$) to obtain

$$\begin{aligned} &p_E \dot{E}^*_s + \cancel{\dot{C}^*_{cap}} + \cancel{\dot{C}^*_{md}} + \dot{C}^*_o + \dot{N}^* \\ &\quad = p_E \dot{\hat{E}}_s + \cancel{\dot{\hat{C}}_{cap}} + \cancel{\dot{\hat{C}}_{md}} + \dot{\hat{C}}_o + \dot{\hat{N}} \ . \end{aligned} \tag{50}$$

For the substitution effect, there is no change in capital or maintenance and disposal costs ($\dot{\hat{C}}_{cap} = \dot{C}^*_{cap}$ and $\dot{\hat{C}}_{md} = \dot{C}^*_{md}$). Solving for $\Delta\dot{\hat{N}} \equiv \dot{\hat{N}} - \dot{N}^*$ gives

$$\Delta\dot{\hat{N}} = -p_E \Delta\dot{\hat{E}}_s - \Delta\dot{\hat{C}}_o \ . \tag{51}$$

The substitution effect adjusts net savings relative to $\dot{N}^*$ by $\Delta\dot{\hat{N}}$. Thus, $\dot{\hat{N}} = \dot{N}^* + \Delta\dot{\hat{N}}$. Substituting Eqs. (41), (42), and (51) yields

$$\dot{\hat{N}} = \dot{G} - \Delta\dot{C}^*_{cap} - \Delta\dot{C}^*_{md} - p_E \Delta\dot{\hat{E}}_s - \Delta\dot{\hat{C}}_o \ . \tag{52}$$

Table C.5. **Income Effect**

	Energy analysis		*Financial analysis*	
before ($\wedge$)	$\hat{\dot{E}} = \hat{\dot{E}}_s + \hat{\dot{E}}_{emb} + (\hat{\dot{C}}_{md} + \hat{\dot{C}}_o)I_E$	(44)	$\hat{\dot{M}} = p_E\hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \hat{\dot{N}}$	(45)
after ($-$)	$\bar{\dot{E}} = \bar{\dot{E}}_s + \bar{\dot{E}}_{emb} + (\bar{\dot{C}}_{md} + \bar{\dot{C}}_o)I_E$	(53)	$\bar{\dot{M}} = p_E\bar{\dot{E}}_s + \bar{\dot{C}}_{cap} + \bar{\dot{C}}_{md} + \bar{\dot{C}}_o + \bar{\dot{N}}$	(54)

Take differences to obtain the change in energy consumption, $\Delta\bar{\dot{E}} \equiv \bar{\dot{E}} - \hat{\dot{E}}$.

$$\Delta\bar{\dot{E}} = \Delta\bar{\dot{E}}_s + \cancelto{0}{\Delta\bar{\dot{E}}_{emb}} + (\cancelto{0}{\Delta\bar{\dot{C}}_{md}} + \Delta\bar{\dot{C}}_o)I_E \tag{55}$$

Thus,

$$\Delta\bar{\dot{E}} = \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o I_E \tag{56}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\bar{\dot{E}}}{\dot{S}_{dev}} = \frac{\Delta\bar{\dot{E}}_s}{\dot{S}_{dev}} + \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}} \tag{57}$$

Define $Re_{inc} \equiv \frac{\Delta\bar{\dot{E}}}{\dot{S}_{dev}}$, $Re_{dinc} \equiv \frac{\Delta\bar{\dot{E}}_s}{\dot{S}_{dev}}$, and $Re_{iinc} \equiv \frac{\Delta\bar{\dot{C}}_o I_E}{\dot{S}_{dev}}$, such that

$$Re_{inc} = Re_{dinc} + Re_{iinc}\ . \tag{58}$$

Use the monetary constraint ($\hat{\dot{M}} = \bar{\dot{M}}$) to obtain

$$\begin{aligned} & p_E\hat{\dot{E}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o + \hat{\dot{N}} \\ & \quad = p_E\bar{\dot{E}}_s + \cancel{\bar{\dot{C}}_{cap}} + \cancel{\bar{\dot{C}}_{md}} + \bar{\dot{C}}_o + \cancelto{0}{\bar{\dot{N}}}\ . \end{aligned} \tag{59}$$

For the income effect, there is no change in capital or maintainance and disposal costs ($\hat{\dot{C}}_{cap} = \dot{C}^*_{cap}$ and $\hat{\dot{C}}_{md} = \dot{C}^*_{md}$). Notably, $\bar{\dot{N}} = 0$, because it is assumed that all net monetary savings ($\hat{\dot{N}}$) are spent on more energy service ($\bar{\dot{E}}_s > \hat{\dot{E}}_s$) and additional purchases in the economy ($\bar{\dot{C}}_o > \hat{\dot{C}}_o$). Solving for $\hat{\dot{N}}$ gives

$$\hat{\dot{N}} = p_E\Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o\ , \tag{60}$$

the budget constraint for the income effect. By construction, Eq. (60) ensures spending of net savings ($\hat{\dot{N}}$) on (i) additional energy services ($\Delta\bar{\dot{E}}_s$) and (ii) additional purchases of other goods in the economy ($\Delta\bar{\dot{C}}_o$) only.

Table C.6. **Macro Effect**

	Energy analysis	*Financial analysis*
before (−)	$\bar{\dot{E}}$ (61)	
after (∼)	$\tilde{\dot{E}}$ (62)	

Energy analysis:

Take differences to obtain the change in energy consumption,

$$\Delta\tilde{\dot{E}} \equiv \tilde{\dot{E}} - \bar{\dot{E}} \ . \tag{63}$$

The energy change due to the macro effect ($\Delta\tilde{\dot{E}}$) is a scalar multiple (k) of net savings ($\hat{\dot{N}}$), assumed to be spent at the energy intensity of the economy (I_E).

$$\Delta\tilde{\dot{E}} = k\hat{\dot{N}}I_E \tag{64}$$

All terms are energy takeback rates. Divide by $\dot{S}_{dev}$ to create rebound terms.

$$\frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}} = \frac{k\hat{\dot{N}}I_E}{\dot{S}_{dev}} \tag{65}$$

Define $Re_{macro} \equiv \frac{\Delta\tilde{\dot{E}}}{\dot{S}_{dev}}$, such that

$$Re_{macro} = \frac{k\hat{\dot{N}}I_E}{\dot{S}_{dev}} \ . \qquad \textbf{(??)}$$

Financial analysis: N/A

C.3 Rebound expressions

All that remains is to determine expressions for each rebound effect. We begin with the device-level expected energy savings rate ($\dot{S}_{dev}$), which appears in the denominator of all rebound expressions.

C.3.1 Expected energy savings ($\dot{S}_{dev}$)

$\dot{S}_{dev}$ is the reduction of energy consumption rate by the device due to the EEU. No other effects are considered.

$$\dot{S}_{dev} \equiv \dot{E}_s^\circ - \dot{E}_s^* \tag{??}$$

The final energy consumption rates ($\dot{E}_s^\circ$ and $\dot{E}_s^*$) can be written as Eq. (**??**) in the forms $\dot{E}_s^\circ = \dot{q}_s^\circ/\eta^\circ$ and $\dot{E}_s^* = \dot{q}_s^*/\eta^*$.

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^*}{\eta^*} \tag{66}$$

With reference to Table C.1, we use $\dot{q}_s^* = \dot{q}_s^\circ$ and $\eta^* = \tilde{\eta}$ to obtain

$$\dot{S}_{dev} = \frac{\dot{q}_s^\circ}{\eta^\circ} - \frac{\dot{q}_s^\circ}{\tilde{\eta}} \; . \tag{67}$$

When the EEU increases efficiency such that $\tilde{\eta} > \eta^\circ$, expected energy savings grows ($\dot{S}_{dev} > 0$) as the rate of final energy consumption declines, as expected. As $\tilde{\eta} \to \infty$, all final energy consumption is eliminated ($\dot{E}_s^* \to 0$), and $\dot{S}_{dev} = \dot{q}_s^\circ/\eta^\circ = \dot{E}_s^\circ$. (Of course, $\tilde{\eta} \to \infty$ is impossible. See Paoli & Cullen (2020) for a recent discussion of upper limits to device efficiencies.)

After rearrangement and using $\dot{E}_s^\circ = \dot{q}_s^\circ/\eta^\circ$, we obtain a convenient form

$$\dot{S}_{dev} = \left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right)\frac{\eta^\circ}{\tilde{\eta}}\dot{E}_s^\circ \; . \tag{??}$$

C.3.2 Emplacement effect

The emplacement effect accounts for performance of the EEU only. No behavior changes occur. The direct emplacement effect of the EEU is device energy savings and energy cost savings. The indirect emplacement effects of the EEU produce changes in the embodied energy rate and the maintenance and disposal expenditure rates. By definition, the direct emplacement effect has no rebound. However, indirect emplacement effects may cause energy rebound. Both direct and indirect emplacement effects are discussed below.

Re_{dempl} As shown in Table C.3, the direct rebound from the emplacement effect is $Re_{dempl} = 0$. This result is expected, because, in the absence of behavior changes, there is no takeback of energy savings at the upgraded device.

Re_{iempl} Indirect emplacement rebound effects can occur at any point in the life cycle of an energy conversion device, from manufacturing and distribution to the use phase (maintenance), and finally to disposal. For simplicity, we group maintenance with disposal to form two distinct indirect emplacement rebound effects: (i) an embodied energy effect (Re_{emb}) and (ii) a maintenance and disposal effect (Re_{md}).

Re_{emb} The first component of indirect emplacement effect rebound involves embodied energy. We define embodied energy consistent with the energy analysis literature to be the sum of all final energy consumed in the production of the energy conversion device. The EEU causes the embodied final energy of the device to change from $\dot{E}^{\circ}_{emb}$ to $\dot{E}^{*}_{emb}$.

Energy is embodied in the device within manufacturing and distribution supply chains prior to consumer acquisition of the device. No energy is embodied in the device while in service. However, for simplicity, we spread all embodied energy over the lifetime of the device, an equal amount assigned to each period. We later take the same approach to capital costs and maintenance and disposal costs. A justification for spreading embodied energy purchase costs comes from considering staggered device replacements by many consumers across several years. In the aggregate, staggered replacements work out to about the same embodied energy in every period.

Thus, we allocate embodied energy over the life of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) to obtain embodied energy rates, such that $\dot{E}^{\circ}_{emb} = E^{\circ}_{emb}/t^{\circ}_{life}$ and $\dot{E}^{*}_{emb} = E^{*}_{emb}/t^{*}_{life}$. The change in embodied final energy due to the EEU (expressed as a rate) is given by $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$. After substitution and algebraic rearrangement, the change in embodied energy rate due to the EEU can be expressed as $[(E^{*}_{emb}/E^{\circ}_{emb})(t^{\circ}_{life}/t^{*}_{life}) - 1]\dot{E}^{\circ}_{emb}$, a term that represents energy savings taken back due to embodied energy effects. Thus, Eq. (**??**) can be employed to write embodied energy rebound as

$$Re_{emb} = \frac{\left(\frac{E^{*}_{emb}}{E^{\circ}_{emb}}\frac{t^{\circ}_{life}}{t^{*}_{life}} - 1\right)\dot{E}^{\circ}_{emb}}{\dot{S}_{dev}} \,. \qquad (\textbf{??})$$

Embodied energy rebound can be either positive or negative, depending on the sign of the term $(E^{*}_{emb}/E^{\circ}_{emb})(t^{\circ}_{life}/t^{*}_{life}) - 1$. Rising energy efficiency can be associated with increased device complexity and more embodied energy, such that $E^{*}_{emb} > E^{\circ}_{emb}$ and $Re_{emb} > 0$. However, if the upgraded device has longer life than the original device ($t^{*}_{life} > t^{\circ}_{life}$), $\dot{E}^{*}_{emb} - \dot{E}^{\circ}_{emb}$ can be negative, meaning that the upgraded device has a lower embodied energy rate than the original device.

Re_{md} In addition to embodied energy effects, indirect emplacement rebound can be associated with energy demanded by maintenance and disposal (md) expenditures. Maintenance expenditures are typically modeled as a per-year expense, a rate (e.g., $\dot{C}^{\circ}_{m}$). Disposal costs (e.g., C°_{d}) are one-time expenses incurred at the end of the useful life of the energy conversion device. Like embodied energy, we spread disposal expenditures across the lifetime of the original and upgraded devices (t°_{life} and t^{*}_{life}, respectively) to form expenditure rates such that $\dot{C}^{\circ}_{md} = \dot{C}^{\circ}_{m} + C^{\circ}_{d}/t^{\circ}_{life}$ and $\dot{C}^{*}_{md} = \dot{C}^{*}_{m} + C^{*}_{d}/t^{*}_{life}$.

We assume, for simplicity, that md expenditures indicate energy consumption elsewhere in the economy at its energy intensity (I_E). Therefore, the change in energy consumption rate caused by a change in md expenditures is given by $\Delta\dot{C}^{*}_{md}I_E$. This term is an energy takeback rate, so maintenance and disposal rebound is given by

$$Re_{md} = \frac{\Delta\dot{C}^{*}_{md}I_E}{\dot{S}_{dev}} \,, \qquad (68)$$

as shown in Table C.3. Slight rearrangement gives

$$Re_{md} = \frac{\left(\frac{\dot{C}^{*}_{md}}{\dot{C}^{\circ}_{md}} - 1\right)\dot{C}^{\circ}_{md}I_E}{\dot{S}_{dev}} \,. \qquad (\textbf{??})$$

Rebound from maintenance and disposal can be positive or negative, depending on the sign of the term $\dot{C}^{*}_{md}/\dot{C}^{\circ}_{md} - 1$.

C.3.3 Substitution effect

This section derives expressions for substitution effect rebound. Two terms comprise substitution effect rebound, direct substitution rebound (Re_{dsub}) and indirect substitution rebound (Re_{isub}). Assuming that conditions after the emplacement effect ($*$) are known, both the rate of energy service consumption ($\dot{\hat{q}}_s$) and the rate of other goods consumption ($\dot{\hat{C}}_o$) must be determined as a result of the substitution effect (the $\wedge$ point).

The EEU's energy efficiency increase ($\tilde{\eta} > \eta^\circ$) causes the price of the energy service provided by the device to fall ($\tilde{p}_s < p_s^\circ$). The substitution effect quantifies the amount by which the device owner, in response, increases the consumption rate of the energy service ($\dot{\hat{q}}_s > \dot{q}_s^*$) and decreases the consumption rate of other goods ($\dot{\hat{q}}_o < \dot{q}_o^*$).

The increase in consumption of the energy service substitutes for consumption of other goods in the economy, subject to a utility constraint. The reduction in spending on other goods in the economy is captured by indirect substitution rebound (Re_{isub}).

We begin by deriving an expression for direct and indirect substitution effect rebound (Re_{dsub} and Re_{isub}, respectively). Thereafter, we develop an approximate model and a CES utility model for determining the post-substitution point ($\dot{\hat{q}}_s$ and $\dot{\hat{C}}_o$).

Direct substitution effect rebound expression Direct substitution effect rebound (Re_{dsub}) is given by

$$Re_{dsub} = \frac{\Delta \dot{\hat{E}}_s}{\dot{S}_{dev}} = \frac{\dot{\hat{E}}_s - \dot{E}_s^*}{\dot{S}_{dev}} \; . \tag{??}$$

Substituting the typical relationship of Eq. (**??**) in the form $\dot{E}_s = \dot{q}_s/\eta$ gives

$$Re_{dsub} = \frac{\frac{\dot{\hat{q}}_s}{\tilde{\eta}} - \frac{\dot{q}_s^*}{\tilde{\eta}}}{\dot{S}_{dev}} \; . \tag{69}$$

Rearranging produces

$$Re_{dsub} = \frac{\left(\frac{\dot{\hat{q}}_s}{\dot{q}_s^\circ} - \frac{\dot{q}_s^*}{\dot{q}_s^\circ}\right)\frac{\dot{q}_s^\circ}{\tilde{\eta}}}{\dot{S}_{dev}} \; . \tag{70}$$

Recognizing that the rate of energy service consumption ($\dot{q}_s$) is unchanged across the emplacement effect leads to $\dot{q}_s^*/\dot{q}_s^\circ = 1$. Furthermore, $\dot{q}_s^\circ/\tilde{\eta} = (\dot{q}_s^\circ/\eta^\circ)(\eta^\circ/\tilde{\eta}) = \dot{E}_s^\circ(\eta^\circ/\tilde{\eta})$, such that

$$Re_{dsub} = \left(\frac{\dot{\hat{q}}_s}{\dot{q}_s^\circ} - 1\right)\frac{\dot{E}_s^\circ \frac{\eta^\circ}{\tilde{\eta}}}{\dot{S}_{dev}} \; . \tag{71}$$

Substituting Eq. (**??**) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{dsub} = \frac{\frac{\dot{\hat{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1}\left(\frac{\cancel{\dot{E}_s^\circ}\cancel{\frac{\eta^\circ}{\tilde{\eta}}}}{\cancel{\frac{\eta^\circ}{\tilde{\eta}}}\cancel{\dot{E}_s^\circ}}\right) \; . \tag{72}$$

Canceling terms yields

$$Re_{dsub} = \frac{\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \; . \tag{??}$$

Eq. (??) is the basis for developing both approximate and CES models of determining direct substitution rebound.

Indirect substitution effect rebound expression Indirect substitution effect rebound (Re_{isub}) is given by

$$Re_{isub} = \frac{\Delta \hat{\dot{C}}_o I_E}{\dot{S}_{dev}} = \frac{(\hat{\dot{C}}_o - \dot{C}_o^*) I_E}{\dot{S}_{dev}} \; . \tag{??}$$

Rearranging gives

$$Re_{isub} = \frac{\left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - \frac{\dot{C}_o^*}{\dot{C}_o^\circ} \right) \dot{C}_o^\circ I_E}{\dot{S}_{dev}} \; . \tag{73}$$

Recognizing that expenditures on other goods are constant across the emplacement effect gives $\dot{C}_o^*/\dot{C}_o^\circ = 1$ and

$$Re_{isub} = \left(\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1 \right) \frac{\dot{C}_o^\circ I_E}{\dot{S}_{dev}} \; . \tag{74}$$

Substituting Eq. (??) for $\dot{S}_{dev}$ and rearranging gives

$$Re_{isub} = \frac{\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \; . \tag{??}$$

Eq. (??) is the basis for developing both approximate and CES models of determining indirect substitution rebound.

Determining the post-substitution effect conditions requires reference to a consumer utility model. We first show an approximate model, often used in the literature. Later, we use a constant elasticity of substitution (CES) utility model. The CES model is used for all calculations and graphs in this paper.

Approximate utility model In the literature, an approximate utility model is often used (Borenstein, 2015, p. 17, footnote 43). In the two examples of this paper (car and electric lamp upgrades), rebound calculated using the approximate utility model (here) differs from rebound calculated using the exact utility model (below) by as much as 5%. Thus, we do not recommend use of the approximate utility model. We discuss the approximate utility model here for completeness only.

In the approximate model, the relationship between energy service price and energy service consumption rate is given by the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^*} = \left(\frac{\tilde{p}_s}{p_s^\circ} \right)^{\epsilon_{\dot{q}_s,p_s,c}} \; . \tag{75}$$

Note that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) is assumed constant along an indifference curve in the approximate model. A negative value for the compensated price elasticity of energy service demand is expected ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that when the energy service price decreases ($\tilde{p}_s < p_s^\circ$), the rate of energy service consumption increases ($\hat{\dot{q}}_s > \dot{q}_s^*$).

Substituting Eq. (**??**) in the form $p_s^\circ = p_E^\circ/\eta^\circ$ and $\tilde{p}_s = p_E^\circ/\tilde{\eta}$ and noting that $\dot{q}_s^\circ = \dot{q}_s^*$ gives

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} . \tag{76}$$

Again, note that the compensated price elasticity of energy service demand is negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), so that as energy service efficiency increases ($\tilde{\eta} > \eta^\circ$), the energy service consumption rate increases ($\hat{\dot{q}}_s > \dot{q}_s^* = \dot{q}_s^\circ$).

Substituting Eq. (76) into Eq. (**??**) yields the approximate expression for direct substitution rebound.

$$Re_{dsub} = \frac{\left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_s,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \tag{77}$$

The compensated price elasticity of energy service demand is expected to be negative ($\epsilon_{\dot{q}_s,p_s,c} < 0$), such that, e.g. $\epsilon_{\dot{q}_s,p_s,c} = -0.2$ and $\tilde{\eta}/\eta^\circ = 2$ yields $Re_{dsub} = 0.15$.

With $\epsilon_{\dot{q}_s,p_s,c} \in (-1, 0)$ expected, the approximate model indicates that direct substitution rebound will never be larger than 1. I.e., the direct substitution effect alone can never cause backfire.

To quantify the substitution effect on other purchases in the approximate model, we introduce another elasticity, the compensated energy service cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$), such that

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{p}_s}{p_s^\circ}\right)^{\epsilon_{\dot{q}_o,p_s,c}} . \tag{78}$$

Because the compensated cross-price elasticity of other goods demand is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$), an energy service price decrease ($\tilde{p}_s < p_s^\circ$) implies a reduction in the rate of consumption of other goods ($\hat{\dot{q}}_o < \dot{q}_o^*$).

The energy service price is inversely proportional to efficiency, yielding

$$\frac{\hat{\dot{q}}_o}{\dot{q}_o^*} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{79}$$

Assuming that the average price is unchanged across the substitution effect such that $\hat{p}_o = \dot{p}_o^* = p_o^\circ$ (Appendix F), and noting that $\dot{q}_s^* = \dot{q}_s^\circ$ and $\dot{C}_o^* = \dot{C}_o^\circ$, we can write

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} = \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{80}$$

Note that Eq. (80) can be used to determine the rate of expenditures on other goods in the economy ($\hat{\dot{C}}_o$) by

$$\hat{\dot{C}}_o = \dot{C}_o^\circ \left(\frac{\tilde{\eta}}{\eta^\circ}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} . \tag{81}$$

Substituting Eq. (81) into Eq. (**??**) gives the expression for indirect substitution rebound for the approximate utility model.

$$Re_{isub} = \frac{\left(\frac{\tilde{\eta}}{\eta^{\circ}}\right)^{-\epsilon_{\dot{q}_o,p_s,c}} - 1}{\frac{\tilde{\eta}}{\eta^{\circ}} - 1} \frac{\tilde{\eta}}{\eta^{\circ}} \frac{\dot{C}_o^{\circ} I_E}{\dot{E}_s^{\circ}} \tag{82}$$

Because the compensated cross-elasticity of other goods consumption is positive ($\epsilon_{\dot{q}_o,p_s,c} > 0$) and the energy service efficiency ratio is greater than 1 ($\tilde{\eta} > \eta^{\circ}$), indirect substitution rebound will be negative always ($Re_{isub} < 0$), as expected. Negative rebound indicates that indirect substitution effects reduce the energy takeback rate by direct substitution effects.

CES utility model The approximate utility model assumes that the compensated price elasticity of energy service demand ($\epsilon_{\dot{q}_s,p_s,c}$) and the compensated cross-price elasticity of other goods demand ($\epsilon_{\dot{q}_o,p_s,c}$) are constant along an indifference curve. These assumptions are approximations that hold only for infinitesimally small energy service price changes ($\Delta p_s^* \equiv p_s^* - p_s^{\circ} \approx 0$). However, in the case of an energy efficiency upgrade (EEU), the energy service price change is not infinitesimal. Rather, Δp_s^* is finite and may be large.

To determine the new consumption bundle after the substitution effect ($\hat{\dot{q}}_s$ and $\hat{\dot{C}}_o$) and, ultimately, to quantify the direct and indirect substitution rebound effects (Re_{dsub} and Re_{isub}) exactly, we remove the restriction that energy service price elasticities ($\epsilon_{\dot{q}_s,p_s,c}$ and $\epsilon_{\dot{q}_o,p_s,c}$) must be constant along an indifference curve. Instead, we require constancy of only the elasticity of substitution (σ) between the consumption rate of the energy service ($\dot{q}_s$) and the expenditure rate for other goods ($\dot{C}_o$) across the substitution effect. Thus, we employ a CES utility model.

Fig. **??** (especially segments $*$—c and c—$\wedge$) illustrates features of the CES utility model for determining the new consumption bundle. Two equations are helpful for this analysis. First, the slope at any point on indifference curve i°—i° is given by Eq. (**??**) with $\dot{u}/\dot{u}^{\circ} = 1$ and the share parameter (a) replaced by $f_{\dot{C}_s}^{\circ}$, as discussed in Appendix D.

$$\begin{aligned}\frac{\partial(\dot{C}_o/\dot{C}_o^{\circ})}{\partial(\dot{q}_s/\dot{q}_s^{\circ})} = &-\frac{f_{\dot{C}_s}^{\circ}}{1 - f_{\dot{C}_s}^{\circ}} \left(\frac{\dot{q}_s}{\dot{q}_s^{\circ}}\right)^{(\rho-1)} \\ &\times \left[\left(\frac{1}{1 - f_{\dot{C}_s}^{\circ}}\right) - \left(\frac{f_{\dot{C}_s}^{\circ}}{1 - f_{\dot{C}_s}^{\circ}}\right)\left(\frac{\dot{q}}{\dot{q}_s^{\circ}}\right)^{\rho}\right]^{(1-\rho)/\rho} .\end{aligned} \tag{83}$$

Second, the equation of the $*$—$*$ expenditure line is

$$\frac{\dot{C}_o}{\dot{C}_o^{\circ}} = -\frac{\tilde{p}_s \dot{q}_s^{\circ}}{\dot{C}_o^{\circ}} \left(\frac{\dot{q}_s}{\dot{q}_s^{\circ}}\right) + \frac{1}{\dot{C}_o^{\circ}}(\dot{M} - \dot{C}_{cap}^{\circ} - \dot{C}_{md}^{\circ} - \dot{G}) . \tag{84}$$

To find the rate of energy service consumption after the substitution effect ($\hat{\dot{q}}_s$), we set the slope of the $*$—$*$ expenditure line (Eq. 84) equal to the slope of the i°—i° indifference curve at the original utility rate of $\dot{u}/\dot{u}^{\circ} = 1$ (Eq. (83)).

$$-\frac{\tilde{p}_s \dot{q}_s^{\circ}}{\dot{C}_o^{\circ}} = -\frac{f_{\dot{C}_s}^{\circ}}{1 - f_{\dot{C}_s}^{\circ}} \left(\frac{\dot{q}_s}{\dot{q}_s^{\circ}}\right)^{(\rho-1)} \left[\left(\frac{1}{1 - f_{\dot{C}_s}^{\circ}}\right) - \left(\frac{f_{\dot{C}_s}^{\circ}}{1 - f_{\dot{C}_s}^{\circ}}\right)\left(\frac{\dot{q}}{\dot{q}_s^{\circ}}\right)^{\rho}\right]^{(1-\rho)/\rho} \tag{85}$$

Solving for $\dot{q}_s/\dot{q}_s^{\circ}$ gives $\hat{\dot{q}}_s/\dot{q}_s^{\circ}$ as

$$\frac{\hat{\dot{q}}_s}{\dot{q}_s^\circ} = \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} . \qquad (??)$$

Eq. (**??**) can be substituted directly into Eq. (**??**) to obtain an estimate for direct substitution rebound (Re_{dsub}) via the CES utility model.

$$Re_{dsub} = \frac{\left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \qquad (??)$$

The rate of other goods consumption after the substitution effect ($\hat{\dot{C}}_o$) can be found by substituting Eq. (**??**) and $\dot{u}/\dot{u}^\circ = 1$ into the functional form of the utility model (Eq. (**??**)) to obtain

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left(\left(\frac{1}{1 - f_{\dot{C}_s}^\circ} \right) - \left(\frac{f_{\dot{C}_s}^\circ}{1 - f_{\dot{C}_s}^\circ} \right) \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1} \right)^{1/\rho} . \qquad (86)$$

Simplifying gives

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \left[\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} . \qquad (??)$$

Eq. (**??**) can be substituted into Eq. (**??**) to obtain an expression for indirect substitution rebound (Re_{isub}) via the CES utility model.

$$Re_{isub} = \frac{\left[\left(1 + f_{\dot{C}_s}^\circ \left\{ \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{1-\sigma} - 1 \right\} \right)^{-1} \right]^{1/\rho} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \frac{\tilde{\eta}}{\eta^\circ} \frac{\dot{C}_o^\circ I_E}{\dot{E}_s^\circ} \qquad (??)$$

C.3.4 Income effect

Rebound from the income effect rebound quantifies the rate of additional energy demand that arises because the owner of the energy conversion device spends net savings from the EEU. The income rate of the device owner is $\dot{M}^\circ$, which remains unchanged across the rebound effects, such that $\dot{M}^\circ = \dot{M}^* = \hat{\dot{M}} = \bar{\dot{M}} = \tilde{\dot{M}}$. Freed cash from the EEU is given by Eq. (42) as $\dot{G} = p_E \dot{S}_{dev}$. In combination, the emplacement effect and the substitution effect leave the device owner with *net* savings ($\hat{\dot{N}}$) from the EEU, as shown in Eq. (52). Derivations of expressions for freed cash from the emplacement effect ($\dot{G}$) and net savings after the substitution effect ($\hat{\dot{N}}$) are presented in Tables C.3 and C.4.

In this framework, all net savings ($\hat{\dot{N}}$) are spent on either (i) additional energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) or (ii) additional other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$). The income elasticity of energy service demand and the income elasticity of other goods demand ($\epsilon_{\dot{q}_s,\dot{M}}$ and $\epsilon_{\dot{q}_o,\dot{M}}$, respectively) quantify the income preferences of the device owner according to the following expressions:

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} \tag{??}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} , \tag{??}$$

where effective income ($\hat{\dot{M}}'$) is

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} . \tag{??}$$

Homotheticity means that $\epsilon_{\dot{q}_s,\dot{M}} = 1$ and $\epsilon_{\dot{q}_o,\dot{M}} = 1$.

The budget constraint across the income effect (Eq. (60)) ensures that all net savings available after the substitution effect ($\hat{\dot{N}}$) is re-spent across the income effect, such that $\bar{\dot{N}} = 0$. Appendix E proves that the income preference equations (Eqs. (**??**) and (**??**)) satsify the budget constraint (Eq. 60).

The purpose of this section is derivation of expressions for (i) direct income rebound (Re_{dinc}) arising from increased consumption of the energy service ($\bar{\dot{q}}_s > \hat{\dot{q}}_s$) and (ii) indirect income rebound (Re_{iinc}) arising from increased consumption of other goods ($\bar{\dot{q}}_o > \hat{\dot{q}}_o$).

But first, we derive a helpful expression to be used later.

Expression for $\hat{\dot{E}}_s$ An expression for $\hat{\dot{E}}_s$ that will be helpful later begins with

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{E}}_s}{\dot{E}^*_s}\right)\left(\frac{\dot{E}^*_s}{\dot{E}^\circ_s}\right)\dot{E}^\circ_s . \tag{87}$$

Substituting Eq. (**??**) and noting efficiency (η) equalities from Table C.1 gives

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{q}}_s/\cancel{\tilde{\eta}}}{\dot{q}^*_s/\cancel{\tilde{\eta}}}\right)\left(\frac{\dot{q}^*_s/\tilde{\eta}}{\dot{q}^\circ_s/\eta^\circ}\right)\dot{E}^\circ_s . \tag{88}$$

Canceling terms yields

$$\hat{\dot{E}}_s = \left(\frac{\hat{\dot{q}}_s}{\dot{q}^*_s}\right)\left(\frac{\cancel{\dot{q}^*_s}}{\cancel{\dot{q}^\circ_s}}\right)\left(\frac{\eta^\circ}{\tilde{\eta}}\right)\dot{E}^\circ_s . \tag{89}$$

Noting energy service consumption rate equalities from Table C.1 ($\dot{q}^*_s = \dot{q}^\circ_s$) gives

$$\hat{\dot{E}}_s = \frac{\hat{\dot{q}}_s}{\dot{q}^*_s}\frac{\eta^\circ}{\tilde{\eta}}\dot{E}^\circ_s . \tag{90}$$

The next step is to develop an expression for Re_{dinc} using the income preference for energy service consumption.

Expression for Re_{dinc} As shown in Table C.5, direct income rebound is defined as

$$Re_{dinc} \equiv \frac{\Delta \bar{\dot{E}}_s}{\dot{S}_{dev}} \; . \tag{??}$$

Expanding the difference and rearranging gives

$$Re_{dinc} = \frac{\bar{\dot{E}}_s - \hat{\dot{E}}_s}{\dot{S}_{dev}} \; , \tag{91}$$

and

$$Re_{dinc} = \frac{\left(\frac{\bar{\dot{E}}_s}{\hat{\dot{E}}_s} - 1\right) \hat{\dot{E}}_s}{\dot{S}_{dev}} \; . \tag{92}$$

Substituting the Eq. (**??**) as $\bar{\dot{E}}_s = \frac{\bar{\dot{q}}_s}{\tilde{\eta}}$ and $\hat{\dot{E}}_s = \frac{\hat{\dot{q}}_s}{\tilde{\eta}}$ gives

$$Re_{dinc} = \frac{\left(\frac{\bar{\dot{q}}_s/\cancel{\tilde{\eta}}}{\hat{\dot{q}}_s/\cancel{\tilde{\eta}}} - 1\right) \hat{\dot{E}}_s}{\dot{S}_{dev}} \; . \tag{93}$$

Eliminating terms and substituting Eq. (**??**) for $\dot{S}_{dev}$ and Eq. (**??**) for $\bar{\dot{q}}_s/\hat{\dot{q}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1\right] \hat{\dot{E}}_s}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\eta^\circ}{\tilde{\eta}} \dot{E}_s^\circ} \; . \tag{94}$$

Substituting Eq. (90) for $\hat{\dot{E}}_s$ gives

$$Re_{dinc} = \frac{\left[\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1\right] \frac{\hat{\dot{q}}_s}{\dot{q}_s^*} \frac{\cancel{\eta^\circ}}{\cancel{\tilde{\eta}}} \cancel{\dot{E}_s^\circ}}{\left(\frac{\tilde{\eta}}{\eta^\circ} - 1\right) \frac{\cancel{\eta^\circ}}{\cancel{\tilde{\eta}}} \cancel{\dot{E}_s^\circ}} \; . \tag{95}$$

Eliminating terms, recognizing that $\dot{q}_s^\circ = \dot{q}_s^*$, and substituting Eq. (**??**), which assumes the CES utility model, gives

$$Re_{dinc} = \frac{\left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^\circ} - 1} \left\{ f_{\dot{C}_s}^\circ + (1 - f_{\dot{C}_s}^\circ) \left[\left(\frac{1 - f_{\dot{C}_s}^\circ}{f_{\dot{C}_s}^\circ} \right) \frac{\tilde{p}_s \dot{q}_s^\circ}{\dot{C}_o^\circ} \right]^{\frac{\rho}{1-\rho}} \right\}^{-1/\rho} \; . \tag{??}$$

If there is no net savings ($\hat{\dot{N}} = 0$), direct income effect rebound is zero ($Re_{dinc} = 0$), as expected.

The next step is to develop an expression for Re_{iinc} using the income preference for other goods consumption.

Expression for Re_{iinc} As shown in Table C.5, indirect income rebound is defined as

$$Re_{iinc} \equiv \frac{\Delta \bar{\dot{C}}_o I_E}{\dot{S}_{dev}} \; . \tag{??}$$

Expanding the difference and rearranging gives

$$Re_{iinc} = \frac{(\dot{\bar{C}}_o - \dot{\hat{C}}_o) I_E}{\dot{S}_{dev}} \; , \tag{96}$$

and

$$Re_{iinc} = \frac{\left(\frac{\dot{\bar{C}}_o}{\dot{\hat{C}}_o} - 1\right) \dot{\hat{C}}_o I_E}{\dot{S}_{dev}} \; . \tag{97}$$

Substituting $\dot{\bar{C}}_o = p_o \dot{\bar{q}}_o$ and $\dot{\hat{C}}_o = p_o \dot{\hat{q}}_o$ and cancelling terms gives

$$Re_{iinc} = \frac{\left(\frac{\dot{\bar{q}}_o}{\dot{\hat{q}}_o} - 1\right) \dot{\hat{C}}_o I_E}{\dot{S}_{dev}} \; . \tag{98}$$

Substituting the income preference equation for other goods consumption (Eq. (**??**) for $\dot{\bar{q}}_o / \dot{\hat{q}}_o$ and Eq. (**??**) for $\dot{S}_{dev}$ yields

$$Re_{iinc} = \frac{\left[\left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1\right] \dot{\hat{C}}_o I_E}{\left(\frac{\tilde{\eta}}{\eta^{\circ}} - 1\right) \frac{\eta^{\circ}}{\tilde{\eta}} \dot{E}_s^{\circ}} \; . \tag{99}$$

Sutstituting $(\dot{\hat{C}}_o / \dot{C}_o^{\circ}) \dot{C}_o^{\circ}$ for $\dot{\hat{C}}_o$, recognizing that $\dot{C}_o^{*} = \dot{C}_o^{\circ}$, and simplifying gives

$$Re_{iinc} = \frac{\left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^{\circ}} - 1} \left(\frac{\tilde{\eta}}{\eta^{\circ}}\right) \frac{\dot{C}_o^{\circ} I_E}{\dot{E}_s^{\circ}} \left(\frac{\dot{\hat{C}}_o}{\dot{C}_o^{\circ}}\right) \; . \tag{100}$$

Substituting Eq. (**??**) for $\dot{\hat{C}}_o / \dot{C}_o^{\circ}$, thereby assuming the CES utility model, gives the final form of the indirect income rebound expression:

$$Re_{iinc} = \frac{\left(1 + \frac{\dot{\hat{N}}}{\dot{\hat{M}}'}\right)^{\epsilon_{\dot{q}_o, \dot{M}}} - 1}{\frac{\tilde{\eta}}{\eta^{\circ}} - 1} \left(\frac{\tilde{\eta}}{\eta^{\circ}}\right) \frac{\dot{C}_o^{\circ} I_E}{\dot{E}_s^{\circ}} \left[\left(1 + f_{\dot{C}_s}^{\circ} \left\{\left[\left(\frac{1 - f_{\dot{C}_s}^{\circ}}{f_{\dot{C}_s}^{\circ}}\right) \frac{\tilde{p}_s \dot{q}_s^{\circ}}{\dot{C}_o^{\circ}}\right]^{1-\sigma} - 1\right\}\right)^{-1}\right]^{1/\rho} \; . \tag{??}$$

If there is no net savings ($\dot{\hat{N}} = 0$), indirect income effect rebound is zero ($Re_{iinc} = 0$), as expected.

C.3.5 Macro effect

Macro rebound (Re_{macro}) is given by Eq. (**??**). Substituting Eq. (52) for net savings ($\dot{\hat{N}}$) gives

$$Re_{macro} = \frac{k \dot{G} I_E}{\dot{S}_{dev}} - \frac{k \Delta \dot{C}_{cap}^{*} I_E}{\dot{S}_{dev}} - \frac{k \Delta \dot{C}_{md}^{*} I_E}{\dot{S}_{dev}} - \frac{k p_E I_E \Delta \dot{\hat{E}}_s}{\dot{S}_{dev}} - \frac{k \Delta \dot{\hat{C}}_o I_E}{\dot{S}_{dev}} \; . \tag{101}$$

Substituting Eq. (42) for $\dot{G}$ and Eqs. (68), (**??**), and (**??**) for rebound terms gives

$$Re_{macro} = \frac{k p_E \cancel{\dot{S}_{dev}} I_E}{\cancel{\dot{S}_{dev}}} - \frac{k \Delta \dot{C}_{cap}^{*} I_E}{\dot{S}_{dev}} - k Re_{md} - k p_E I_E Re_{dsub} - k Re_{isub} \; . \tag{102}$$

Canceling terms and defining Re_{cap} as

$$Re_{cap} \equiv \frac{\Delta \dot{C}^*_{cap} I_E}{\dot{S}_{dev}} \tag{103}$$

gives

$$Re_{macro} = kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} \ . \tag{??}$$

C.3.6 Rebound sum

The sum of four rebound effects is

$$Re_{tot} = Re_{empl} + Re_{sub} + Re_{inc} + Re_{macro} \ . \tag{104}$$

Substituting Eqs. (37), (49), and (58) gives

$$\begin{aligned} Re_{tot} = Re_{emb} + Re_{md} & \quad \text{emplacement effect} \\ + Re_{dsub} + Re_{isub} & \quad \text{substitution effect} \\ + Re_{dinc} + Re_{iinc} & \quad \text{income effect} \\ + Re_{macro} & \quad \text{macro effect} \end{aligned} \tag{105}$$

Macro effect rebound (Re_{macro}, Eq. (**??**)) can be expressed in terms of other rebound effects. Substituting Eq. (**??**) gives

$$\begin{aligned} Re_{tot} = Re_{emb} + Re_{md} & \quad \text{emplacement effect} \\ + Re_{dsub} + Re_{isub} & \quad \text{substitution effect} \\ + Re_{dinc} + Re_{iinc} & \quad \text{income effect} \\ + kp_E I_E - kRe_{cap} - kRe_{md} - kp_E I_E Re_{dsub} - kRe_{isub} \ . & \quad \text{macro effect} \end{aligned} \tag{106}$$

Rearranging distributes macro effect terms to emplacement and substitution effect terms. This last rearrangement gives the final expression for total rebound.

$$\begin{aligned} Re_{tot} = Re_{emb} & - kRe_{cap} + (1-k)Re_{md} \\ & + (1 - kp_E I_E)Re_{dsub} + (1-k)Re_{isub} \\ & + Re_{dinc} + Re_{iinc} + kp_E I_E \end{aligned} \tag{??}$$

Eq. (**??**) shows that determining seven rebound values,

- Re_{emb} (Eq. (**??**)),
- Re_{cap} (Eq. (103)),
- Re_{md} (Eq. (**??**)),
- Re_{dsub} (Eq. (**??**)),
- Re_{isub} (Eqs. (**??**)),

- Re_{dinc} (Eqs. (**??**)), and
- Re_{iinc} (Eqs. (**??**)),

is sufficient to calculate total rebound, provided that the macro factor (k), the price of energy (p_E), and the energy intensity of the economy (I_E) are known.

D Utility models and elasticities

E Proof: Income preference equations satisfy the budget constraint

After the substitution effect, a rate of net savings is available ($\hat{\dot{N}}$), all of which is spent on additional energy service ($\Delta\bar{\dot{q}}_s$, $\Delta\bar{\dot{C}}_s = p_E \Delta\bar{\dot{E}}_s$) or additional other goods ($\Delta\bar{\dot{q}}_o$, $\Delta\bar{\dot{C}}_o$). The income effect must satisfy the budget constraint such that net savings is zero afterward ($\bar{\dot{N}} = 0$). The budget constraint across the income effect is represented by Eq. (60):

$$\hat{\dot{N}} = p_E \Delta\bar{\dot{E}}_s + \Delta\bar{\dot{C}}_o \; . \tag{60}$$

The additional spending due to the income effect is given by income preference equations

$$\frac{\bar{\dot{q}}_s}{\hat{\dot{q}}_s} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_s,\dot{M}}} \tag{??}$$

and

$$\frac{\bar{\dot{q}}_o}{\hat{\dot{q}}_o} = \left(1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'}\right)^{\epsilon_{\dot{q}_o,\dot{M}}} \; , \tag{??}$$

where

$$\hat{\dot{M}}' \equiv \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \; . \tag{??}$$

This appendix proves that the income preference equations (Eqs. (**??**) and (**??**)) satisfy the budget constraint (Eq. (60)).

The first step in the proof is to convert the income preference equations to $\dot{C}_s^\circ$ and $\dot{C}_o^\circ$ ratios. For the energy service income preference equation (Eq. (**??**)), multiply numerator and denominator of the left-hand side by $\tilde{p}_s = p_E/\tilde{\eta}$ (Eq. (**??**)) to obtain $\bar{\dot{C}}_s/\hat{\dot{C}}_s$. For the other goods income preference equation (Eq. (**??**)), multiply numerator and denominator of the left-hand side by p_o to obtain $\bar{\dot{C}}_o/\hat{\dot{C}}_o$. Then, invoke homotheticity to set $\epsilon_{\dot{q}_s,\dot{M}} = 1$ and $\epsilon_{\dot{q}_o,\dot{M}} = 1$ to obtain

$$\frac{\bar{\dot{C}}_s}{\hat{\dot{C}}_s} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \tag{107}$$

and

$$\frac{\bar{\dot{C}}_o}{\hat{\dot{C}}_o} = 1 + \frac{\hat{\dot{N}}}{\hat{\dot{M}}'} \ . \tag{108}$$

The second step in the proof is to obtain expressions for $\Delta\bar{\dot{C}}_s$ and $\Delta\bar{\dot{C}}_o$. Multiply the income preference equations above by $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively. Then, subtract $\Delta\hat{\dot{C}}_s$ and $\Delta\hat{\dot{C}}_o$, respectively, to obtain

$$\Delta\bar{\dot{C}}_s = \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} \tag{109}$$

and

$$\Delta\bar{\dot{C}}_o = \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \ . \tag{110}$$

The above versions of the income preference equations can be substituted into the budget constraint (Eq. (60)) to obtain

$$\hat{\dot{N}} \overset{?}{=} \frac{\hat{\dot{C}}_s}{\hat{\dot{M}}'}\hat{\dot{N}} + \frac{\hat{\dot{C}}_o}{\hat{\dot{M}}'}\hat{\dot{N}} \ . \tag{111}$$

If equality is demonstrated, the income preference equations satisfy the budget constraint. The remainder of the proof shows the equality of Eq. (111).

Dividing by $\hat{\dot{N}}$ and multiplying by $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \hat{\dot{M}}' \ . \tag{112}$$

Substituting Eq. (**??**) for $\hat{\dot{M}}'$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \dot{M}^\circ - \dot{C}^*_{cap} - \dot{C}^*_{md} - \hat{\dot{N}} \ . \tag{113}$$

Substituting Eq. (45) for $\dot{M}^\circ$, because $\dot{M}^\circ = \hat{\dot{M}}$, gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} p_E\hat{\dot{E}}_s + \hat{\dot{C}}_{cap} + \hat{\dot{C}}_{md} + \hat{\dot{C}}_o + \cancel{\hat{\dot{N}}} - \dot{C}^*_{cap} - \dot{C}^*_{md} - \cancel{\hat{\dot{N}}} \ . \tag{114}$$

Cancelling terms and recognizing that $\dot{C}^*_{cap} = \hat{\dot{C}}_{cap}$, $\dot{C}^*_{md} = \hat{\dot{C}}_{md}$, and $\hat{\dot{C}}_s = p_E\hat{\dot{E}}_s$ gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{?}{=} \hat{\dot{C}}_s + \cancel{\hat{\dot{C}}_{cap}} + \cancel{\hat{\dot{C}}_{md}} + \hat{\dot{C}}_o - \cancel{\hat{\dot{C}}_{cap}} - \cancel{\hat{\dot{C}}_{md}} \ . \tag{115}$$

Cancelling terms gives

$$\hat{\dot{C}}_s + \hat{\dot{C}}_o \overset{\checkmark}{=} \hat{\dot{C}}_s + \hat{\dot{C}}_o \ , \tag{116}$$

thereby completing the proof that the income preference equations (Eqs. (**??**) and (**??**)) satisfy the budget constraint (Eq. (60)).

F Other goods expenditures and constant p_o

This framework utilizes a partial equilibrium analysis in which we account for the change of the energy service price due to the EEU ($p_s^\circ \neq p_s^*$), but we do not track the effect of the EEU on prices of other goods. These assumptions have important implications for the relationship between the rate of consumption of other goods ($\dot{q}_o$) and the rate of expenditure on other goods ($\dot{C}_o$).

We assume a basket of other goods (besides the energy service) purchased in the economy, each (i) with its own price ($p_{o,i}$) and rate of consumption ($\dot{q}_{o,i}$), such that the average price of all other goods purchased in the economy prior to the EEU (p_o°) is given by

$$p_o^\circ = \frac{\sum_i p_{o,i}^\circ q_{o,i}^\circ}{\sum_i q_{o,i}^\circ} \; . \tag{117}$$

Then, the expenditure rate of other purchases in the economy can be given as

$$\dot{C}_o^\circ = p_o^\circ \dot{q}_o^\circ \tag{118}$$

before the EEU and

$$\hat{\dot{C}}_o = \hat{p}_o \hat{\dot{q}}_o \tag{119}$$

after the substitution effect, for example.

We assume that any effects (emplacement, substitution, or income) for a single device are not so large that they cause a measurable change in prices of other goods. Thus,

$$p_o^\circ = p_o^* = \hat{p}_o = \bar{p}_o = \tilde{p}_o \; . \tag{120}$$

In the partial equilibrium analysis, two other goods prices can be equated across any rebound effect to obtain (for the example of the original conditions ($\circ$) and the post-substitution state ($\wedge$))

$$\frac{\hat{\dot{C}}_o}{\dot{C}_o^\circ} = \frac{\hat{\dot{q}}_o}{\dot{q}_o^\circ} \; . \tag{121}$$

Thus, a ratio of other goods expenditure rates is always equal to a ratio of other goods consumption rates.

G Income and macro effects and relation to the marginal propensity to consume (MPC)

Borenstein (2015) has postulated a demand-side argument that macro effects can be represented by a multiplier, which we call the macro factor (k). Borenstein's formulation and our implementation are reminiscent of the marginal propensity to consume (MPC). In this appendix, we show the relationship between the macro factor (k) and MPC.

The relationship between the macro factor (k) and MPC spans the income and macro effects. In this framework, the device owner's net savings after the substitution effect ($\hat{\dot{N}}$) is respent completely. One may assume that firms and other consumers who receive the net savings have a marginal propensity to re-spend of MPC. The total spending throughout the economy of each year's net savings ($\hat{\dot{N}}$) is given by the infinite series

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\hat{\dot{N}} \ , \tag{122}$$

where the first term ($1 \times \hat{\dot{N}}$) represents spending of net savings by the device owner in the direct and indirect income effects, and the remaining terms [$(MPC + MPC^2 + MPC^3 + \ldots)\hat{\dot{N}}$] represent macro-effect spending in the broader economy.

The macro effect portion of the spending can be represented by the macro factor (k).

$$(1 + MPC + MPC^2 + MPC^3 + \ldots)\hat{\dot{N}} = (1 + k)\hat{\dot{N}} \tag{123}$$

Canceling $\hat{\dot{N}}$ and simplifying the infinite series to its converged fraction (assuming $MPC < 1$) gives

$$\frac{1}{1 - MPC} = 1 + k \ . \tag{124}$$

Solving for k yields

$$k = \frac{1}{\frac{1}{MPC} - 1} \ . \qquad \textbf{(??)}$$

With $MPC = 0.5$, as in Section **??**, $k = 1$ is obtained. If $k = 3$, as in Section **??**, $MPC = 0.75$ is implied. The relationship between k and MPC is given in Fig. G.1.

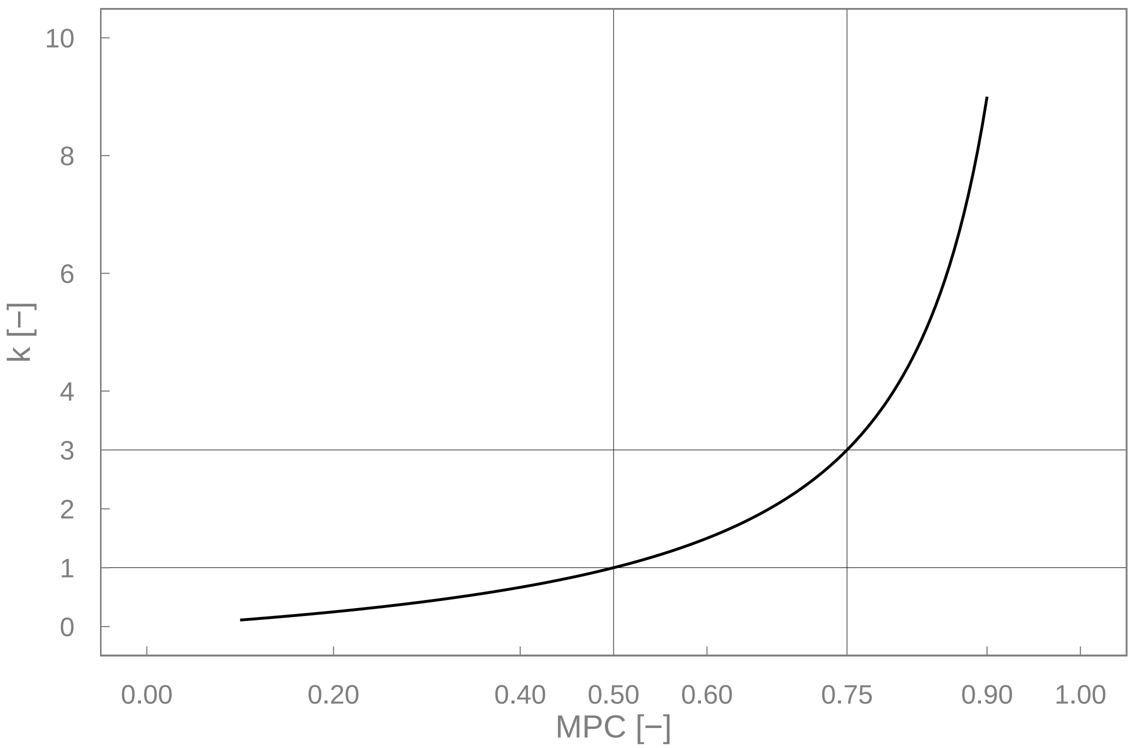

Fig. G.1: The relationship between MPC and k in Eq. (**??**).