The Quantum Zeno Effect

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Abstract

The quantum Zeno effect is derived in a general quantum systems, making only minimal assumptions about the measurement process. In the limit of arbitrarily frequent measurement, the effect freezes the evolution of the system, a result which seems paradoxical. However, through the use of a system-probe model, the spectrum of the interaction Hamiltonian is shown to be at least proportional to the measurement frequency, demonstrating the non-physicality of this limit. The quantum Zeno effect is then argued to be a dynamical effect, originating from the interaction between a system and probe needed to perform measurement in quantum mechanics.

1 Introduction

The quantum Zeno effect (QZE) refers to the suppression of the evolution of a quantum system through frequent measurement. Althought this effect has been noticed by many, including von Neumann [1] and Turing [2], the seminal paper on the topic was written by Misra and Sudershan in 1977 [3]. In this paper they proved rigourously that unstable particles must decay non-exponentially for small time periods, and that this could be exploited through rapid measurement to slow the particle's decay. They were so distrubed by the implications that a continuously measured particle could never decay that they labelled this the quantum Zeno paradox, akin to the paradoxes of the Greek philosopher Zeno who believed that motion was illusionary.

The quantum Zeno effect has received constant theoretical interest since, and in 1988, an experimental test involving an oscillating system was suggested by Cook [4]. This experiment was then successfully performed in 1990 by Itano *et al.*, providing experimental confirmation of the effect [5]. Since then, the effect has been demonstrated for unstable systems [6], and has found many applications such as in quantum computing [7] and quantum 'bang-bang' control [8]. A more complete discussion of extensions and applications of the QZE can be found in [1] and [9].

Unsurprisingly for an issue connected to quantum measurement and interpretations of quantum mechanics, the QZE has long been mired in controversy and confusion. Indeed one of the central issues is that, while many authors consider the effect to be paradoxical, which aspect they consider paradoxical varies. In this essay we will discuss two different but related, aspects of the Zeno effect.

As the frequency of the measurements is taken to infinity, the evolution of a quantum system is completely frozen. Misra and Sudarshan considered this 'continuous' measurement of the system state to simply be an idealization of the monitoring of a particle in a gas chamber. Hence they believed it to be paradoxical that quantum mechanics predicted in this case that the particle would never decay. This issue was partial resolved by calculating the frequency of measurements required to observe the Zeno effect. For the case of a charged pion decay, it was found by Chiu *et al.* that measurements would need to occur at intervals 10^{14} times smaller than the lifetime of the particle [10], corresponding to measurements every 10^{-22} seconds. However, while this explains why the Zeno effect has not experimentally been noticed in particle physics, the issue of continuous measurement still presents a conceptual challenge. For this reason, many authors such as Nakazato *et al.* [11] reserve the term quantum Zeno *paradox* for this situation. We will call this problem the first Zeno paradox.

A second seemingly paradoxical aspect of the Zeno effect is that the presence of a measurement device dramatically changes the behaviour of a system. We label this the second Zeno paradox, to distinguish it from the previous concern. Authors such as Home and Whitaker [12] argue that the term quantum Zeno effect should be restricted to experimental setups where an externally sepearted macroscopic measuring device affects the dynamics of a system. For these authors, the essence of the quantum Zeno effect is the fact that a nonlocal negative-result (interaction-free) measurement can affect the evolution of a system so dramatically. Since the experiments mentioned previously are not of this type, the authors do not consider these true examples of the QZE. Experimentally verifying the QZE for this type of measurement has proved quite challenging. The first experimental realization of an interaction-free Zeno effect was only published in 2015, by Peise *et al.* [13].

We aim to discuss both aspects of the quantum Zeno effect using quantum measurement theory. Most introductions to the QZE focus on the case of decaying particles, and use the projection postulate to describe the measurement process. This has had the unfortunate effect of obscuring the generality of the QZE. It has also led to confusion as to the exact relationship between measurement and the QZE [12]. For this reason we shall begin with a proof of the QZE for general quantum systems, whilst making minimal assumptions about the measurement process.

We then turn to explaining the two paradox associated with the QZE. By examining the interaction Hamiltonian required to produce arbitrarily quick measurements, we will find that the energy spectrum of the Hamiltonian must be made sufficiently large. As the frequency of the measurements becomes infinite, the Hamilontian spectra is also required to become infinite, and so we conclude that continuously measure the state of a system is impossible in quantum mechanics. This resolves the first paradox.

To resolve the second paradox, we note that as the interaction Hamiltonian is required to become large in order to produce measurements, then the QZE should be considered a result of this. Rather than being specific to the measurement process, the QZE is a dynamical result of the interaction process. The only role of measurement is that it necessitates the strong interaction. We then discuss interaction-free measurements, and shows that the term is a misnomer. Since interaction-free measurements require interactions like every other measurement, our arguments still apply.

2 General Proof of the QZE

Whilst many discussions of the quantum Zeno effect focus on decaying particles, we will demonstrate that the effect in a completely general setup. Another problem with early discussions of the quantum Zeno effect was a reliance on the projection postulate. As noted by [12], this led to confusion as to the relevance of the projection postulate to the QZE. We shall show that the projection postulate is not needed at all for the QZE. Instead, given two general assumptions about the measurement process, the QZE inevitably follows.

Say we have a system with density matrix ρ and Hamiltonian H. If no measurements occur, then the in the Schródinger picture, the system satisfies the equation

$$\frac{d}{dt}\rho = -i[H, \rho].$$

For an observable M, we will require a measurement of M to be some process, lasting a finite period and ending at a time τ, which changes ρ while satisfying

1. The value of $\langle M(t)\rangle$ for $t \leq \tau$ is not affected by the measurement process.

2. At end of the measurement, M and $\rho(\tau)$ commute.

These are necessary but not sufficient; clearly there are other properties that measurements desirably should posses. The first assumption simply states that our measurement shouldn't change the value of the observable being measured. The second measurement requires our measurement to result in a density matrix which is a mixture of eigenstates of M, so that M has a well-defined value. Both requirements are trivially satisfied by the von Neumann process, and also by any system-probe model which approximates the von Neumann postulate. We shall now prove that under these assumptions, along with the assumption that M has bounded second derivative, the QZE occurs. We begin with a lemma.

Lemma 1: For a quantum system, if the density matrix ρ commutes with an observable M at time $t = 0$, then

$$\frac{d}{dt}\langle M\rangle\bigg|_{t=0} = 0.$$

Proof: Let the Hamiltonian of the system be H. We can then calculate

$$\frac{d}{dt}\langle M\rangle = \frac{d}{dt}\mathrm{Tr}(\rho M) = \mathrm{Tr}([H, \rho]M) = \mathrm{Tr}(H\rho M - \rho HM)$$

$$= \mathrm{Tr}(H\rho M - HM\rho) = \mathrm{Tr}(H[\rho, M]).$$

Since at $t = 0$, $[\rho(0), M] = 0$, we conclude that

$$\frac{d}{dt}\langle M\rangle\bigg|_{t=0} = \mathrm{Tr}(H[\rho(0), M]) = 0.$$

Theorem 1: Take any quantum system with a Hamiltonian H and an observable M so that $[H,[H,M]]$ is bounded by some $A \in \mathbb{R}^+$, and say that we initially measure M at $t = 0$, and then measure M again at time intervals τ in a way that satisfies our measurement assumptions. If we take any $a > \tau$, then

$$|\langle M(a)\rangle - \langle M(0)\rangle| \leq aA\tau.$$

Proof: First note that

$$\left|\frac{d^2}{dt^2}\langle M(t)\rangle\right| = |[H,[H,M]]| \leq A$$

and so the condition that $[H,[H,M]]$ is bounded is equivalent to stating that $\langle M(t)\rangle$ has a bounded second derivative.

We now will calculate the change in M between two measurements, say between $t = n\tau$ and $t = (n+1)\tau$. Since $\rho(n\tau)$ commutes with M by our assumptions about the measurement process, Lemma 1 allows us to conclude that

$$\frac{d}{dt}\langle M\rangle\bigg|_{t=n\tau} = 0.$$

Since it is always true that

$$\left|\frac{d^2}{dt^2}\langle M(t)\rangle\right| \leq A,$$

we get the inequality

$$M(n\tau) - \frac{1}{2}t^2A \leq M(n\tau + t) \leq M(n\tau) + \frac{1}{2}t^2A.$$

Hence

$$|M((n+1)\tau) - M(\tau)| \leq \frac{1}{2}\tau^2 A.$$

Let us call the total number of intervals between measurements $N \in \mathbb{N}$. This number must satisfy

$$N\tau \leq a < (N+1)\tau$$

and so as $\tau < a$, is at least 1. Now

$$|\langle M(a)\rangle - \langle M(0)\rangle| = \left|\langle M(a)\rangle - \langle M((N+1)\tau)\rangle + \sum_{k=0}^{N}\langle M((k+1)\tau)\rangle - \langle M(k\tau))\rangle\right|$$

$$\leq \frac{N+1}{2}\tau^2 A \leq N\tau^2 A \leq aA\tau$$

and this completes the proof.

Theorem 1 allows us to linearly bound the value of an observable M by time intervals τ between measurements. By taking τ to be sufficiently small, we can hence force the change in $\langle M \rangle$ to be arbitrarily small. This theorem also demonstrates the QZE is unavoidable if we wish to make frequent measurements of a system.

Other then the conditions we placed on measurements, the only condition we required was that $[H, [H, M]]$ was totally bounded. As noted in the proof, this just means that the second derivative of $\langle M \rangle$ cannot change arbitrarily quickly, and so in most physical situations this is reasonable. For instance, for finite Hilbert spaces all observables must be totally bounded and so this condition is always true.

It is also worth noting that while the condition that $[H, [H, M]]$ is totally bounded is sufficient for the QZE, it is by no means necessary, and so other conditions could instead be used. One such (rather abstract) condition is that the density matrix ρ is a mixture of pure states restricted to a compact set of the Hilbert space. As $[H, [H, M]]$ is continuous, it would be totally bounded on this set and so the QZE would occur. A concrete application of this condition would be to states of the harmonic oscillator with a totally bounded energy.

One of the most important aspects of the QZE is the suppression of transitions from an initial state. If a system were initially in state $|\psi\rangle$, and if we were to repeatedly measure the projection operator $|\psi\rangle \langle\psi|$, then applying Theorem 1 it is clear that the system will be prevented from evolving away from this state. This is a specific manifestation of a more general corollary of Theorem 1.

Corollary 1: Given a quantum system with Hilbert space $\mathcal{H}$, say that the system is initial in an N-dimensional subspace $\mathcal{H}_s$ with the property that there is a K so that for every $|\phi\rangle \in \mathcal{H}$ and $|\psi\rangle \in \mathcal{H}_s$,

$$|\langle\phi| H^2 |\psi\rangle| \leq K.$$

If we measure if the system is in $\mathcal{H}_s$ at time intervals of τ, then the probability p that the system is still in $\mathcal{H}_s$ after a time has passed satisfies the inequality

$$p \geq 1 - 4NKa\tau.$$

Proof: We measure whether the system is in the subspace $\mathcal{H}_s$ by measuring the projection P_s onto this subspace, so that

$$p = \langle P_s \rangle.$$

Let $|\psi\rangle_k$ be an orthonormal basis of $\mathcal{H}_s$. We now note that

$$[H, [H, P_s]] = H^2 P_s + P_s H^2 - 2HP_sH$$

and so

$$\sup_{|\phi\rangle} |\langle [H, [H, P_s]] \rangle| = |2\mathrm{Re}(\langle\phi| H^2 P_s |\phi\rangle) - 2\langle\phi| HP_sH |\phi\rangle|$$

$$\leq 2\sum_k |\langle\phi|H^2|\psi_k\rangle\langle\psi_k|\phi\rangle| + |\langle\phi|H|\psi_k\rangle\langle\psi_k|H|\phi\rangle|$$

$$\leq 2NK + 2\sum_k |\langle\psi_k|H|\phi\rangle\langle\phi|H|\psi_k\rangle| \leq 4NK$$

Hence, as $[H,[H,P_\psi]]$ is bounded by $4NK$, we can apply Theorem 1 to deduce that

$$|\langle P_\psi(a)\rangle - \langle P_\psi(0)\rangle| = |p-1| \leq 4NKa\tau.$$

We conclude that

$$p \geq 1 - 4NKa\tau.$$

Corollary 1 shows that the QZE can induce superselection rules in the evolution of a system. This phenomenon is known as quantum Zeno dynamics [14], and was experimentally realised in 2014 by Schäffer *et al.* using a Bose-Einstein condensate [15].

3 Can measurement occur arbitrarily frequently?

In the last section we proved that given a few general assumptions, the QZE occurs. The most striking aspect of this effect is that as the time between measurements, τ is taken to zero, then the measured variable cannot change:

$$|\langle M(a)\rangle - \langle M(0)\rangle| \leq aA\tau \underset{\tau\to 0}{\longrightarrow} 0.$$

This inevitably occurs for any system satisfying the assumptions of Theorem 1. For the case of a system transitioning from a state, in the limit of $\tau \to 0$ will completely prevent the state evolving. This bizarre but universal behaviour is the subject of the first Zeno paradox, to which we now turn.

In order to take τ to zero, we need to be able to perform measurements arbitrarily quickly. However, in order to correctly treat measurement using quantum mechanics, we must treat the measuring probe as a quantum system, and the measurement process as an interaction between the system and the device. Say we are measuring some variable M which has eigenspaces given by the orthogonal set of projections $\{P_m\}$. Before the interaction, our probe is in some initial state $|i\rangle$, and so the system-probe state is given by

$$\rho \otimes |i\rangle\langle i|$$

where ρ is the density matrix of the system.

The measurement process is some unitary U_M which when applied to the system-probe, entangles eigenspaces of M with orthonormal states of the probe:

$$U_M^* \rho \otimes |i\rangle\langle i| U_M = \sum_m P_m \rho P_m \otimes |m\rangle\langle m|$$

Measuring the state of the probe then allows us to deduce the state of the system.

The unitary measurement process does not occur instantaneously, but is instead the result of some interaction $H_M(t)$ between the system and probe over a time interval t_M:

$$U_M = e^{\int_0^{t_M} iH_M(t)dt}.$$

In the previous section our measurements were required to happen at time intervals of τ. This would require that $t_M \leq \tau$, so that one measurement could be completed before the next began. Hence the limit of $\tau \to 0$ requires $t_M \to 0$. We now prove simply need to show that the latter limit is non-physical.

Theorem 2: At some time during a measurement process distingiushing between n possible eigenspaces, the largest and smallest eigenvalues of the interaction Hamiltonian, E_1 and E_0, satisfy the inequality

$$E_1 - E_0 \geq \frac{1}{t_M}\left(1 - \frac{1}{n^2}\right).$$

Proof: Say that our measurement starts at $t = 0$. We shall define the observable

$$C = I \otimes |i\rangle \langle i|$$

which measures whether the probe is in its initial state. Clearly,

$$\langle C(0) \rangle = 1.$$

After the measurement, the value of C is

$$\langle C(t_M) \rangle = \mathrm{Tr}(C \sum_m P_m \rho P_m \otimes |m\rangle \langle m|) = \sum_m \mathrm{Tr}(P_m \rho) |\langle m|i\rangle|^2$$

a value which is minimised by choosing ρ to be a pure state $|n\rangle \langle n|$ which minimises $|\langle n|i\rangle|^2$. But in general,

$$|i\rangle = \beta |\phi\rangle + \sum_m \alpha_m |m\rangle$$

where $|\phi\rangle$ is a state which is orthonormal to every $|m\rangle$. Since this state must be normalised,

$$\langle i|i\rangle = 1 = |\beta|^2 + \sum_m |\alpha_m|^2$$

and so the maximum minimum of $|\langle n|i\rangle|^2$ is $\frac{1}{n^2}$. Therefore, we conclude that there is a system state $|n\rangle$ so that after the measurement process,

$$C(t_M) \leq \frac{1}{n^2}.$$

Using the generalised uncertainty principle, we have that for any two operators

$$\sigma_A \sigma_B \geq \frac{1}{2} |\langle [A, B] \rangle|.$$

If we apply this to the observables C and $H_M(t)$ at time t during the measurement process, then we know that

$$\sigma_M(t)\sigma_{H_M}(t) \geq \frac{1}{2}|\langle[C(t), H_M(t)]\rangle| = \frac{1}{2}|\langle\dot{C}(t)\rangle|.$$

The mean-value theorem says that there is a time $\xi \in [0, t_M]$ so that

$$\langle\dot{C}(\xi)\rangle = \frac{C(t_M) - C(0)}{t_M} \leq \frac{1}{t_M}\left(\frac{1}{n^2} - 1\right).$$

At this time,

$$\sigma_{C(\xi)} = \sqrt{\langle C(\xi)^2\rangle - \langle C(\xi)\rangle^2} = \sqrt{\langle C(\xi)\rangle - \langle C(\xi)\rangle^2} \leq \sqrt{\langle C(\xi)\rangle}\sqrt{1 - \langle C(\xi)\rangle} \leq \frac{1}{2}$$

as $\langle C(\xi)\rangle \in [0, 1]$. We hence have

$$\sigma_{H_M(\xi)} \geq \frac{|\langle\dot{C}(\xi)\rangle|}{2\sigma_{C(\xi)}} \geq \frac{1}{t_M}\left(1 - \frac{1}{n^2}\right).$$

Now noting that

$$\sigma_{H_M(\xi)} = \sqrt{\langle H_M(\xi)^2\rangle - \langle H_M(\xi)\rangle^2}$$
$$= \sqrt{\langle(H_M(\xi) - E_0 I)^2\rangle - \langle(H_M(\xi) - E_0 I)\rangle^2}$$

and that

$$\langle(H_M(\xi) - E_0 I)^2\rangle \leq (E_1 - E_0)^2, \qquad \langle(H_M(\xi) - E_0 I)\rangle^2 \geq 0$$

we have

$$\sigma_{H_M(\xi)} \leq \sqrt{(E_1 - E_0)^2} = E_1 - E_0.$$

We have finally arrived at the inequality we sought,

$$E_1 - E_0 \geq \frac{1}{t_M}\left(1 - \frac{1}{n^2}\right).$$

The above theorem demonstrates the non-physical nature of the limit where $\tau \to 0$, as it shows the spectrum of the interaction Hamiltonian must become unbounded at some point. This resolves the first paradox; quantum mechanics does not allow arbitrarily rapid measurements and so the concept of continuously measuring a quantum system in this sense is not possible.

4 Is measurement needed for the QZE?

The second paradox raised with by QZE is to explain how the presence of a measuring device can impact the evolution of a system so dramatically. From our discussion of system-probe models however, the answer is quite clear. A measuring device needs to interact with the system if it is to extract information from the system. For finite but small τ, Theorem 2 shows that the spectrum of the interaction Hamiltonian must become large, and when this occurs the interaction Hamiltonian dominates the system's Hamiltonian. From this perspective, the QZE is a purely dynamical effect; the fact that the interaction implements a measurement is irrelevant.

We can strengthen this argument by noting that, whilst entangling a system with a probe allows us to measure system observables, the QZE occurs regardless of whether or not we then perform measurements on the probe. In Theorem 1 we required that at the end of a measure, the density matrix ρ of the system commutes with the observable M measured. Yet simply the process of entangling ρ with the probe

$$U_M^* \rho \otimes |i\rangle \langle i| U_M = \sum_m P_m \rho P_m \otimes |m\rangle \langle m|$$

satisfies this, a fact we can easily verify by calculating

$$\left[\sum_m P_m \rho P_m \otimes |m\rangle \langle m|, M \otimes I \right] = \sum_m [P_m \rho P_m, M] \otimes |m\rangle \langle m| = 0.$$

So as long as the other hypotheses of Theorem 1 are met, entanglement is sufficient to trigger the QZE.

A specific instance of the second paradox which has caused debate is that of negative-result measurements and so it is to this class of measurements we now turn. The idea behind negative-result measurements is an attempt to perform a measurement without interacting with the measured system. For instance, say that we have two spacially separated potentials with states $|a\rangle$ and $|b\rangle$ refering to states where a particle is in the first or second potential repectively. One way to check which well the particle is in is to fire a photon at the first well. If the photon interacts with something, then we know the particle was in state $|a\rangle$. Otherwise if the photon passes through unimpeded then we deduce that no particle was in the first well and hence the particle is actually in the second well. Since the photon did not interact with the particle, we have managed to measure something without an interaction, and hence this kind of measurement is often called an 'interaction-free' measurement, a term first used in a paper by Dicke [16]. It then seems very strange that effects such as the QZE could be induced by negative-result measurements.

This issue is resolved by a more careful analysis of the notion of what it means to be interaction free in quantum mechanics. Thinking classically, if we found a particle in state $|b\rangle$ we would conclude that the particle was in state $|b\rangle$ prior to the measurement. Hence, no particle was ever present in $|a\rangle$ and so the photon interacted with nothing. However, in quantum mechanics this argument is a non-sequitar. If we were to say examine the value of an observable

$$Q = |a\rangle \langle b| + |b\rangle \langle a|$$

then prior to our measurement, $\langle Q \rangle$ may have had values between -1 and 1. However, after the measurement, the particle is now in state $|a\rangle$ or $|b\rangle$ and in both situations, $\langle Q \rangle = 0$. Clearly our 'interaction-free' measurement very much changed the value of an observable. For this reason, it seems that the name ‘interaction-free’ is a clear misnomer, a result of a misapplication of classical ideas to the quantum domain. A more detailed discussion of this issue can be found in [17]. In general, interaction-free measurements need non-trivial interaction Hamiltonians [18][19] and so our arguments apply to them as much as to any other measurement process. For these reasons, we disagree with the arguments of Home and Whitaker in [12] that the quantum Zeno effect should be reserved for interaction-free measurements, as there seems to be no meaningful distinction between these and other measurements.

5 Conclusion

We have explored the quantum Zeno effects and its associated paradoxes using quantum measurement theory. We showed that the QZE is a very general effect, and that by placing only two sensible assumptions on our measurement process along with a dynamical assumption, the QZE could be demonstrated rigourously. In the specific situation where we were testing whether a system is in a subspace, we showed that transitions from that subspace could be arbitrarily inhibited by sufficiently quick measurements.

The first paradox associated with the Zeno effect is the fact that when the frequency of measurement occurs arbitrarily quickly, the evolution of a system can be stopped entirely. By examining a system-probe model, we proved that to measure something within a time interval τ, the interaction Hamiltonian must be such that at some point, its maximum eigenvalue E_1 and minimum eigenvalue E_0 must satisfy

$$E_1 - E_0 \geq \frac{1}{\tau}\left(1 - \frac{1}{n^2}\right).$$

Here n^2 is the number of possibilities that the measurement must distinguish between, a value which is always at least 2. Our inequality shows that interaction must become unphysical as $\tau \to 0$ and so we conclude that instantaneous measurements are impossible. So while we like to think that we are constantly monitoring our world, quantum measurement theory tells us that instead all we have is a series of still frames.

The second paradox is that the presence of a measuring device seems to have a signficant impact on the behaviour of a system. To resolve this paradox we need to remember that measurement requires interaction, and so rather than being a result of measurement *per se*, the quantum Zeno effect is a result of the interaction process needed for a measurement to be possible.

So the two ‘paradoxes’ we have explored in relation to the QZE have not been problems with quantum mechanics, but have instead been caused by the inappropriate use of classical concepts. ‘Classical measurement theory’ doesn’t exist as a topic because in the classical world, we usually think of observables as having a definite value which, with sufficient skill, we can obtain to arbitrary accuracy. In quantum mechanics however, to measure anything we need to interact with our system, sometimes quite brutally, and in general there is no way to avoid this. If this perspective is taken, then the QZE no longer seems so mysterious.

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