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Testing for a Unit Root in a Time Series With a Changing Mean

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This study considers testing for a unit root in a time series characterized by a structural change in its mean level. My approach follows the "intervention analysis" of Box and Tiao (1975) in the sense that I consider the change as being exogenous and as occurring at a known date. Standard unit-root tests are shown to be biased toward nonrejection of the hypothesis of a unit root when the full sample is used. Since tests using split sample regressions usually have low power, I design test statistics that allow the presence of a change in the mean of the series under both the null and alternative hypotheses. The limiting distribution of the statistics is derived and tabulated under the null hypothesis of a unit root. My analysis is illustrated by considering the behavior of various univariate time series for which the unit-root hypothesis has been advanced in the literature. This study complements that of Perron (1989), which considered time series with trends.

KEY WORDS: Functional weak convergence; Hypothesis testing; Intervention analysis; Nonstationarity; Structural change.

1. INTRODUCTION

Testing for the presence of a unit root has become a problem of great concern to economists. Theoretical advances by, among others, Dickey and Fuller (1979, 1981), Fuller, Hasza, and Goebel (1981), Said and Dickey (1984), Phillips (1987), and Phillips and Perron (1988) have permitted the development and applications of formal tests of this hypothesis. Useful reviews and applications of these procedures can be found in the work of Dickey, Bell, and Miller (1986) and Perron (1988). The unit-root hypothesis in a time series of data has indeed far-reaching implications with respect to economic theory and the interpretation of empirical evidence. Since the seminal study of Nelson and Plosser (1982), the view that most macroeconomic time series are best construed as exhibiting some kind of stochastic nonstationarity has become prevalent. It is argued as well that the total variability of a series over time is explained in greater part by variations in permanent shocks than by variations in transitory components.

Sometimes, however, a quick glance at the graph of a time series reveals the presence of a sudden change in the mean level of the series at a given time period. This change may appear so big and sudden compared to the variability exhibited over the rest of the sample period that one may wish to isolate its effect and consider this particular period as an "outlier event" or as exogenous. Following the "intervention analysis" of Box and Tiao (1975), one may wish to remove a particular sudden change from the noise function and introduce it in the deterministic part of the series. The noise function is then analyzed without this particular "extraordinary event."

As an example, consider the behavior of the U.S. ex post real-interest rate over the period 1961:1–1986:3 constructed using the rate on three-month treasury bills deflated by the consumer price index inflation rate. Figure 1 is a graph of this series. It exhibits a marked discontinuity in its mean around the year 1980. Before 1980:3, the average real rate was close to 0 ranging from +4% to −6%. Over the period 1980:4–1986:3, however, the average real rate was close to 6%, ranging from 3% to 11%, approximately. This sudden change in mean can, therefore, be viewed as "extraordinary" and sudden, given the general historical pattern of the series. Furthermore, it can be associated (with a slight delay) with the often-mentioned change in monetary policy initiated by the Federal Reserve Bank in October 1979. In the terminology of Box and Tiao (1975), the change in monetary policy is the "intervention" that may have caused the sudden change in the mean of the series.

The case of the ex post real-interest rate has not escaped the concern of economists investigating the prevalence of stochastic nonstationarity. Recently, Walsh (1987) and Rose (1988) analyzed related series and concluded that they were characterized by the presence of a unit root. The same appears to hold with the series analyzed here. Consider a Dickey–Fuller (1979) type regression of the form

$$y_t = \mu + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + e_t. \qquad (1)$$

Table 1 presents split-sample and full-sample Dickey–Fuller statistics for the real-interest-rate series. For the period 1961:1–1986:3 and a truncation lag parameter

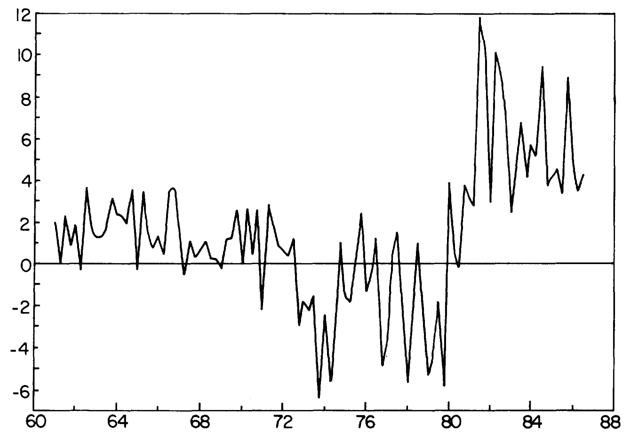

Figure 1. Ex Post Real-Interest Rate (1961:1–1986:3, quarterly).

k equal to 3, the ordinary least squares (OLS) estimate of α is .872 with a t statistic for $\alpha = 1$ of -1.51; one cannot reject the null hypothesis of a unit root. If split samples are considered, however, a different picture emerges. For the period 1961:1–1980:3 and $k = 3$, $\hat{\alpha}$ is .742 with a t statistic for $\alpha = 1$ of -1.92; for the period 1980:4–1986:3 and $k = 5$, $\hat{\alpha}$ is $-.248$ with a t statistic for $\alpha = 1$ of -2.19. The estimates of the sum of the autoregressive coefficients are markedly less using any split sample than the estimate using the full sample. The split samples are not large enough to permit rejection of the null hypothesis of a unit root, however.

Figure 2 presents a graph of the quarterly U.S. unemployment series. This series has also attracted some attention, most notably in a study by Evans (1989). It can be seen that the mean of the series appears to have increased sometime in the mid-seventies. Evans considered splitting the sample into two episodes with the 1974:1 quarter as the break point. I follow the same strategy. As can be observed from the split-sample and full-sample Dickey–Fuller statistics presented in Table 1, a pattern similar to the case of the real-interest-rate series emerges. The split-sample estimates of α are below the full-sample estimate, and none of the cases allow rejection of the unit-root hypothesis. One feature that is different from the real-interest-rate case is that

Table 1. Split-Sample and Full-Sample Dickey–Fuller Statistics

Series/period	k	$\tilde{\mu}$	$t_{\tilde{\mu}}$	$\tilde{\alpha}$	$t_{\tilde{\alpha}}$
Real-interest rate					
1961:1–1986:3	3	.247	.91	.872	−1.51
1961:1–1980:3	3	−.028	−.11	.742	−1.92
1980:4–1986:3	5	7.02	2.04	−.248	−2.19
Unemployment rate					
1948:1–1988:3	9	.212	2.06	.963	−2.10
1948:1–1973:4	9	.473	2.49	.900	−2.54
1974:1–1988:3	6	.682	2.26	.905	−2.33
Terms-of-trade index					
1900–1983	0	.019	1.26	.838	−2.64
1900–1920	4	.126	1.31	.784	−.94
1921–1983	6	.033	2.13	.171	−4.15

NOTE: The regression is $y_t = \tilde{\mu} + \tilde{\alpha} y_{t-1} + \sum_{i=1}^{k} \tilde{c}_i \Delta y_{t-i} + \tilde{e}_t$.

the change in the mean of the series is more gradual. I shall come back to the implications of this difference for my analysis.

Figure 3 presents a graph of the Grilli–Yang real-commodity-price index over the period 1900–1983. This series was analyzed by Cuddington and Urzúa (1989), who investigated issues of persistence. As they pointed out, the series exhibits a marked change in mean in the year 1920. Table 1 shows that the unit-root hypothesis cannot be rejected using the full-sample estimates. It can, however, be rejected using the 1921–1983 sample at a high confidence level, but not with the 1900–1920 sample (due to a lack of power given only 21 observations).

These examples suggest the following line of investigation. First, on the supposition that the series is stationary except for this change in the mean, what are the properties of tests for a unit root as obtained, for example, from Regression (1)? Second, given that the use of split samples often implies tests with low power, can we derive test statistics for the null hypothesis of a unit root that allow the use of the full sample and permit the presence of such a change in the deterministic component? The first issue was also analyzed in a different framework by Chen and Tiao (1990). They considered random level-shift autoregressive moving average (RLARMA) models and showed how standard Box–Jenkins model-identification procedures would suggest an integrated process with the usual diagnostics indicating no misspecification. Their analysis suggests that estimated autoregressive integrated moving average models would produce forecasts with substantially higher mean squared error than an estimated RLARMA if the rate of occurrence of the shifts is small. My analysis deals with the first issue differently and puts more emphasis on the second point; that is, I concentrate on the issue of deciding whether a particular series is characterized by stationary deviations around a shifting mean function or by an integrated process in the important case in which only one shift occurs in the series.

The plan of this article is as follows: Section 2 in-

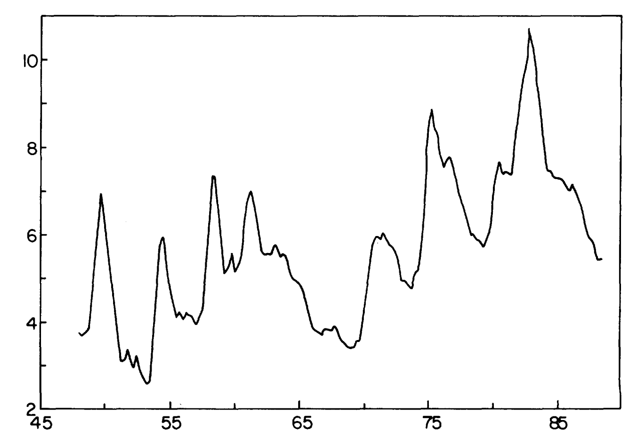

Figure 2. U.S. Unemployment Rate (1948:1–1988:3, quarterly).

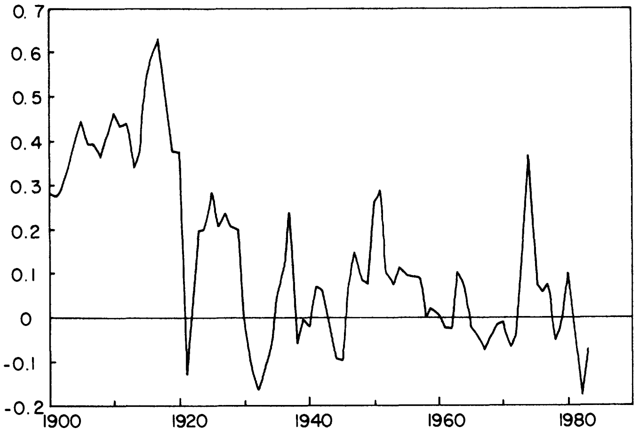

Figure 3. Grilli–Yang Commodity-Price Index (1900–1983, annual).

vestigates the behavior of the OLS estimate of α in Regression (1) when the data is supposed to be stationary except for a sudden change in the mean. I provide Monte Carlo evidence on the finite-sample behavior and derive the limiting distribution, which shows a nonvanishing bias. Section 3 proposes test statistics that allow such a change and derives their limiting distribution under the null hypothesis of a unit root. My method is illustrated by analyzing the series described previously.

2. EFFECTS OF A CHANGING MEAN ON TESTS FOR A UNIT ROOT

Under the null hypothesis, the series $\{y_t\}$ (of which a sample of size $T + 1$ is available) is a realization of a time series process characterized by the presence of a unit root. The usual characterization is generalized, however, to allow a one-time change in the structure of the series occurring at a time $T_B(1 < T_B < T)$. This hypothesis can be parameterized as follows:

$$y_t = \gamma D(TB)_t + y_{t-1} + w_t, \qquad t = 1, \ldots, T, \qquad (2)$$

where $D(TB)_t = 1$ if $t = T_B + 1$ and 0 otherwise and $y_0 = y(0)$ is either a fixed constant or a random variable. The conditions on the sequence of innovations $\{w_t\}$ are specified such that one can use a functional central limit theorem for partial sums $\{S_t = \sum_{j=1}^{t} w_j\}$. Readers can refer to Phillips (1987) and Phillips and Perron (1988) for further details concerning these conditions. They are general enough to permit a series $\{w_t\}$ generated by any finite-order ARMA(p, q) process with Gaussian innovations.

Under the null hypothesis (2), the mean of the series $\{y_t\}$, when conditioning on the initial observation y_0, is given by y_0 up to time T_B and by $y_0 + \gamma$ afterward. Under the alternative hypothesis that the series does not contain a unit root, the model is given by

$$y_t = \mu + \gamma DU_t + e_t, \qquad t = 1, \ldots, T, \qquad (3)$$

where $DU_t = 0$ if $t \leq T_B$ and 1 otherwise. Again, the conditions on $\{e_t\}$ are general enough to permit an ARMA($p + 1$, q) representation consistent with the process (2).

To assess the effects of a shift in the level of the series on tests for the presence of a unit root, I first present a small Monte Carlo experiment. Consider the behavior of the least squares estimator $\hat{\alpha}$ in the following regression:

$$y_t = \hat{\mu} + \hat{\alpha} y_{t-1} + \hat{e}_t. \qquad (4)$$

The preceding regression is one that would be used to test for a unit root ($\alpha = 1$) if the errors were uncorrelated. Suppose, however, that the series is generated by (3) with $\mu = 0$, $T_B = 50$, $T = 100$, and $e_t \sim$ NID(0, 1). We generated 10,000 replications of such a series and for each replication calculated $\hat{\alpha}$ in (4). This exercise was performed for $\gamma = 0, 1, 2, 5$, and 10. Figure 4 presents the cumulative distribution function (cdf) of $\hat{\alpha}$ in each case. The experiment reveals that as the magnitude of the change in the mean—that is, γ—increases, the cdf of $\hat{\alpha}$ becomes more concentrated at a value ever closer to 1. The corresponding means and variances of the samples of $\hat{\alpha}$ generated are shown in Table 2.

What emerges from this simulation experiment is that, if the magnitude of the change is significant, one could hardly reject the unit-root hypothesis even if the series would consist of iid disturbances around a deterministic component (albeit one with a shift in mean). In particular, one would conclude that the shocks have a permanent effect. Here, the shocks clearly have no permanent effect; only the one-time shift in the trend function is permanent. The problem is one of model misspecification.

To analyze the effect on the distribution of $\hat{\alpha}$ of an increase in the sample size with a shift of a given magnitude, I shall derive the asymptotic limit of $\hat{\alpha}$. To carry out the asymptotic analysis requires that the prebreak and postbreak samples increase at the same rate as the total number of observations, T, increases. To this effect, assume, for simplicity, that $T_B = \lambda T$ for all T. I

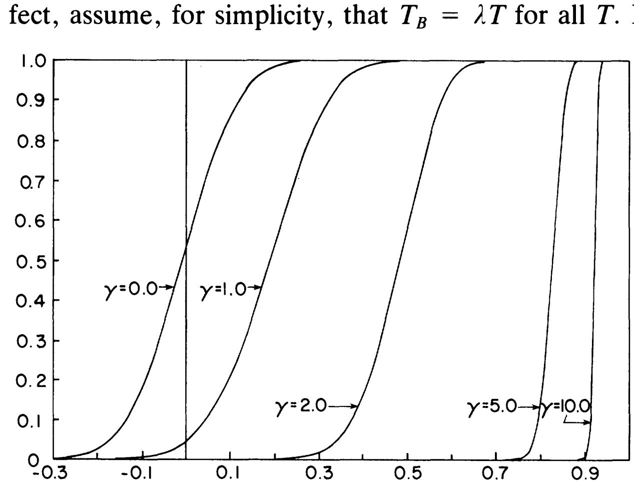

Figure 4. Cumulative Distribution Function of $\hat{\alpha}$. This is the empirical cdf of $\hat{\alpha}$ in the regression $y_t = \hat{\mu} + \hat{\alpha} y_{t-1} + e_t$ when the data are generated by $y_t = \gamma DU_t + e_t$ ($t = 1, \ldots, 100$); $DU_t = 1$ if $t > 50$ and 0 otherwise; and $e_t \sim$ iid N(0, 1). Simulations are based on 10,000 replications.

Table 2. Mean and Variance of $\hat{\alpha}$

γ	Mean	Variance
.0	−.008	.0100
1.0	.186	.0131
2.0	.478	.0065
5.0	.828	.0006
10.0	.923	.0001

NOTE: The empirical mean and variance of $\hat{\alpha}$ are obtained from 10,000 replications, where $\hat{\alpha}$ is the OLS estimate in the regression $y_t = \hat{\mu} + \hat{\alpha} y_{t-1} + \hat{e}_t$. The data were generated by $y_t = \gamma DU_t + e_t$ [$t = 1, \ldots, 100$, $DU_t = 1$ if $t \geq 50$ and 0 otherwise, e_t is iid $N(0, 1)$].

refer to λ as the *break fraction*. The asymptotic limits are taken as T increases to infinity in a sequence that ensures an integer value for T_B for a given λ. This type of increasing sequence is assumed throughout the article.

Given these specifications and the "mixing conditions" imposed on the sequence $\{e_t\}$, it can be shown, following Perron (1989), that, as $T \uparrow \infty$,

$$\hat{\alpha} \rightarrow [\lambda(1-\lambda)\gamma^2 + \rho_1]/[\lambda(1-\lambda)\gamma^2 + \sigma_e^2], \quad (5)$$

where $\rho_1 = \lim_{T\to\infty} T^{-1} \sum_1^T E(e_t e_{t-1})$, $\sigma_e^2 = \lim_{T\to\infty} T^{-1} \sum_1^T E(e_t^2)$, and $\rightarrow$ denotes almost sure convergence.

What the result in (5) shows is that $\hat{\alpha}$ is not a consistent estimate of the true first-order correlation coefficient of the nondeterministic part of the series $\{y_t\}$, ρ_1/σ_e^2, unless $\gamma = 0$. In particular, $\hat{\alpha}$ converges to a value greater than ρ_1/σ_e^2. This limit value approaches 1 as γ increases. Since $\hat{\alpha}$ does not converge to 1 for any fixed γ, however, the usual test statistics for testing that $\alpha = 1$, such as $T(\hat{\alpha} - 1)$ or the t statistic on $\hat{\alpha}$, would eventually reject the null hypothesis of a unit root. Nevertheless, added to the general poor power properties of tests for a unit root against stationary alternatives is now the consideration that the limit of $\hat{\alpha}$ is inflated above the true first-order correlation coefficient of the stochastic part.

The simulation experiment, along with the asymptotic result, points to the need to develop alternative statistical procedures that could distinguish a process with a unit root from a stationary series around a deterministic function with a break.

3. TESTS THAT ALLOW FOR A CHANGE IN MEAN

In this section, I extend the Dickey–Fuller testing strategy to ensure a consistent procedure when a time series is subject to a shift in its mean. I shall present several ways to do so, all of which are asymptotically equivalent, and discuss the main differences between them.

Consider first subtracting a mean from the raw series $\{y_t\}$ by allowing a change at time T_B. Let $\{\tilde{y}_t\}$ be the residuals from a regression of y_t on a constant and DU_t. Furthermore, let $\tilde{\alpha}$ be the least squares estimator of α in the following regression:

$$\tilde{y}_t = \tilde{\alpha}\tilde{y}_{t-1} + \tilde{e}_t, \qquad t = 1, \ldots, T. \quad (6)$$

Up to this point the extensions from the no-break model are straightforward enough. Matters are not so simple, however, concerning the distribution of the statistics of interest—namely, the normalized bias $T(\tilde{\alpha} - 1)$ and the t statistic for testing the null hypothesis that $\alpha = 1$, $t_{\tilde{\alpha}}$. Needless to say, the only manageable analytical distribution theory is asymptotic in nature. But two features are added over the standard Dickey–Fuller approach—(a) the presence of the extra regressor DU_t and (b) the split-sample nature of this extra regressor. To this effect, I derive the asymptotic distribution of $T(\tilde{\alpha} - 1)$ and $t_{\tilde{\alpha}}$ under the null hypothesis of a unit root. As in Section 2, I require that the break point T_B increase at the same rate as the total sample size T. Again, for simplicity, it is assumed that $T_B = \lambda T$ with both T and T_B integer valued.

The method of proof is similar to that of Phillips (1987) and Phillips and Perron (1988). I use weak convergence results that hold for normalized functions of the sum of the innovations when the latter are assumed to satisfy some appropriate "mixing conditions." The limiting distributions obtained under this general setting are then specialized to the iid case. The asymptotic distribution in the iid case are evaluated using simulations of functionals of Wiener processes, and critical values are tabulated. I then show how the results can be extended to innovations $\{e_t\}$ that follow the general ARMA(p, q) process.

I let $w(r)$ be a unit Wiener process defined on $C[0, 1]$, the space of all real-valued functions defined on the interval $[0, 1]$, and $\sigma^2 = \lim_{T\to\infty} E[T^{-1}S_T^2]$, $S_T = \sum_1^T e_t$, and $\sigma_e^2 = \lim_{T\to\infty} E[T^{-1}\sum_1^T e_t^2]$. Denoting weak convergence in distribution by $\Rightarrow$, for $0 < \lambda < 1$,

$$T(\tilde{\alpha} - 1) \Rightarrow H/K \quad (7)$$

and

$$t_{\tilde{\alpha}} \Rightarrow (\sigma/\sigma_e)H/[\lambda(1-\lambda)K]^{1/2}, \quad (8)$$

where $H = [(1-\lambda)\lambda/2][w(1)^2 - \sigma_e^2/\sigma^2] - (1-\lambda)w(1)\int_0^\lambda w(r)\,dr + [w(1) - w(\lambda)][\int_0^\lambda w(r)\,dr - \lambda\int_0^1 w(r)\,dr]$ and $K = (1-\lambda)\lambda\int_0^1 w(r)^2\,dr - [\int_\lambda^1 w(r)\,dr]^2 - (1-\lambda)[\int_0^1 w(r)\,dr]^2 + 2(1-\lambda)\int_0^1 w(r)\,dr \int_\lambda^1 w(r)\,dr$.

The method of proof is similar to that of Perron (1989) and is therefore omitted. Details can be found in the working-paper version of this study available on request. Equations (7) and (8) provide a representation for the limiting distribution of the normalized least squares estimator and its t statistic. These limiting distributions are functions of the parameter λ, the ratio of the prebreak sample size to total sample size. It is easy to verify that, when λ is either 0 or 1, the limiting distributions are given by $T(\tilde{\alpha} - 1) \Rightarrow H'/K'$ and $t_{\tilde{\alpha}} \Rightarrow (\sigma/\sigma_e)H'/(K')^{1/2}$, where $H' = (1/2)[w(1)^2 - \sigma_e^2/\sigma^2] - w(1)\int_0^1 w(r)\,dr$ and $K' = \int_0^1 w(r)^2\,dr - [\int_0^1 w(r)\,dr]^2$.

These latter asymptotic distributions were derived by Phillips and Perron (1988) in the specific case in which no dummy variables are included. When $\lambda = .5$, the asymptotic distributions in (7) and (8) also correspond to the asymptotic distributions of the statistics $T(\hat{\alpha}_{\mu d} - 1)$ and $\hat{\tau}_{\mu d}$ when $d = 2$ that were analyzed by Dickey, Hasza, and Fuller (1984) in the context of testing for a seasonal unit root in univariate time series.

The expressions for the limiting distributions in (7) and (8) depend on additional nuisance parameters apart from λ—namely, σ^2 and σ_e^2. As in Phillips (1987) and Phillips and Perron (1988), σ_e^2 is the variance of the innovations and σ^2 is, in the case of weakly stationary innovations, equal to $2\pi f(0)$, where $f(0)$ is the spectral density of $\{e_t\}$ evaluated at frequency 0. When the innovations $\{e_t\}$ are martingale differences, $\sigma^2 = \sigma_e^2$ and the limiting distributions are invariant with respect to nuisance parameters, except λ.

Therefore, when $\sigma^2 = \sigma_e^2$, percentage points of the limiting distributions can be tabulated for given values of λ. Furthermore, in this case, as pointed out by an associate editor, the limiting distributions are symmetric around $\lambda = .5$. Hence the asymptotic distributions are the same for λ and $(1 - \lambda)$, and only one set of critical values needs to be tabulated. To see this, it is more useful to write the asymptotic distributions as (with $\sigma^2 = \sigma_e^2$) $T(\tilde{\alpha} - 1) \Rightarrow H^*/K^*$ and $t_{\tilde{\alpha}} \Rightarrow H^*/(\lambda(1-\lambda)K^*)^{1/2}$, where $H^* = \lambda N_1 + (1-\lambda)N_2$ and $K^* = \lambda^2 D_1 + (1-\lambda)^2 D_2$; with $N_i = \int_0^1 w_i^d(r)\, dw_i(r)$ and $D_i = \int_0^1 w_i^d(r)^2\, dr$, $w_i^d(r) = w_i(r) - \int_0^1 w_i(r)\, dr$ is the demeaned version of the Wiener process $w_i(r)$ and $w_1(r)$ and $w_2(r)$ are independent Wiener processes. Hence the limiting distributions involve linear combinations of functionals of independent Wiener processes, the weight of the linear combination depending on the ratio λ. The asymptotic distributions are also symmetric in the general case in which $\sigma^2 \neq \sigma_e^2$ if additional conditions are imposed on the limiting variances in each subsample.

Tables 3 and 4 present selected percentage points that permit hypothesis testing. The critical values in the asymptotic case are obtained via simulation methods as in Perron (1989). I use 20,000 replications of the functionals involved in the limiting distribution (7) and (8). See that article for more details. To assess the adequacy of the asymptotic approximation, I have also included in these tables critical values of $T(\tilde{\alpha} - 1)$ and $t_{\tilde{\alpha}}$ in the finite-sample case. These were obtained using 5,000 replications of (6). In general, the asymptotic distribution is an adequate approximation to the finite-sample distribution.

Several features are worth mentioning with respect to these critical values. First, as expected, for a given size of the test, the critical values are larger (in absolute value) than the standard Dickey–Fuller critical values in the left tail of the distribution (see Fuller 1976). One would, therefore, expect a loss in power when the alternative is a stationary process. This feature could imply that when λ is close to 0 or 1 a more powerful testing procedure could be obtained by applying the Dickey–Fuller statistics on the larger subsample. A case in point is the example of the Grilli–Yang commodity-price index discussed in Section 1. As Table 1 showed, the unit-root hypothesis is easily rejected using the 1921–1983 sample (with a p value lower than .01). As will be seen in Section 4, one can still reject the unit-root hypothesis using a test allowing for a changing mean but at a lower significance level (the exact p value depending on the statistic used).

Second, again in the left tail of the distribution, the critical values attain a maximum (in absolute value) around the value $\lambda = .5$—that is, for a break at mid-

Table 3. Percentage Points of the Distribution of $T(\tilde{\alpha} - 1)$

	1.0%	2.5%	5.0%	10.0%	90.0%	95.0%	97.5%	99.0%
$\lambda = .1, .9$								
$T = 50$	−21.76	−18.07	−15.04	−12.09	−1.12	−.33	.30	.99
$T = 100$	−21.85	−18.77	−16.14	−13.00	−1.09	−.28	.45	.98
$T = 200$	−22.32	−18.80	−15.99	−12.72	−1.02	−.29	.35	1.04
$T = \infty$	−23.79	−19.96	−16.64	−13.36	−1.17	−.34	.27	1.00
$\lambda = .2, .8$								
$T = 50$	−22.95	−19.33	−16.51	−13.68	−1.44	−.51	.14	.78
$T = 100$	−24.19	−20.08	−17.20	−14.33	−1.52	−.56	.18	.91
$T = 200$	−24.76	−20.23	−17.05	−14.21	−1.37	−.53	.16	1.10
$T = \infty$	−25.03	−21.12	−18.02	−14.69	−1.55	−.61	.10	.90
$\lambda = .3, .7$								
$T = 50$	−23.79	−20.10	−17.04	−14.12	−1.81	−.79	−.03	.76
$T = 100$	−24.78	−20.76	−17.69	−14.84	−1.92	−.86	−.08	.73
$T = 200$	−25.11	−21.01	−18.34	−15.06	−1.72	−.78	−.01	.95
$T = \infty$	−25.90	−21.66	−18.55	−15.37	−1.94	−.93	−.14	.71
$\lambda = .4, .6$								
$T = 50$	−24.33	−19.83	−17.00	−14.40	−2.07	−1.00	−.32	.80
$T = 100$	−23.98	−21.09	−18.11	−15.06	−2.07	−1.11	−.27	.58
$T = 200$	−24.90	−21.78	−18.50	−15.21	−2.13	−1.10	−.25	.57
$T = \infty$	−26.21	−22.24	−18.97	−15.71	−2.24	−1.19	−.36	.54
$\lambda = .5$								
$T = 50$	−23.45	−20.10	−17.50	−14.57	−2.18	−1.18	−.20	.89
$T = 100$	−25.38	−21.11	−18.41	−15.20	−2.34	−1.29	−.37	.54
$T = 200$	−25.10	−21.39	−18.50	−15.41	−2.27	−1.17	−.32	.71
$T = \infty$	−26.07	−22.06	−18.95	−15.76	−2.39	−1.35	−.52	.38

NOTE: The entries for $\lambda = .5$ and $T = \infty$ are taken from Dickey et al. (1984, table 6). The other entries corresponding to $T = \infty$ were obtained using 20,000 simulations based on the asymptotic distribution (7). The remaining entries, corresponding to the finite-sample cases, were obtained by directly simulating $T(\tilde{\alpha} - 1)$ from (6). In each case, 5,000 replications of a unit-root process with iid $N(0, 1)$ innovations were used.

Table 4. Percentage Points of the Distribution of $t_{\hat{\alpha}}$

	1.0%	2.5%	5.0%	10.0%	90.0%	95.0%	97.5%	99.0%
λ = .1, .9								
T = 50	−3.90	−3.46	−3.12	−2.76	−.51	−.15	.15	.48
T = 100	−3.77	−3.40	−3.09	−2.78	−.51	−.15	.21	.60
T = 200	−3.58	−3.32	−3.06	−2.75	−.48	−.14	.21	.57
$T = \infty$	−3.67	−3.37	−3.10	−2.78	−.55	−.17	.14	.53
λ = .2, .8								
T = 50	−4.04	−3.65	−3.30	−2.92	−.60	−.24	.06	.36
T = 100	−3.86	−3.54	−3.22	−2.91	−.64	−.25	.05	.45
T = 200	−3.85	−3.50	−3.20	−2.89	−.63	−.24	.07	.52
$T = \infty$	−3.80	−3.49	−3.23	−2.92	−.67	−.28	.04	.44
λ = .3, .7								
T = 50	−4.14	−3.76	−3.39	−3.05	−.66	−.32	−.01	.34
T = 100	−4.05	−3.66	−3.33	−3.02	−.73	−.35	−.03	.34
T = 200	−3.91	−3.58	−3.34	−3.00	−.71	−.33	−.00	.39
$T = \infty$	−3.88	−3.56	−3.30	−2.99	−.76	−.40	−.06	.31
λ = .4, .6								
T = 50	−4.11	−3.71	−3.43	−3.08	−.74	−.37	−.11	.26
T = 100	−4.03	−3.68	−3.38	−3.05	−.74	−.42	−.10	.23
T = 200	−3.95	−3.65	−3.34	−3.02	−.76	−.43	−.11	.32
$T = \infty$	−3.92	−3.60	−3.35	−3.05	−.81	−.46	−.15	.22
λ = .5								
T = 50	−4.09	−3.72	−3.45	−3.08	−.77	−.41	−.07	.31
T = 100	−4.04	−3.70	−3.38	−3.08	−.82	−.49	−.15	.21
T = 200	−4.12	−3.61	−3.34	−3.03	−.79	−.45	−.11	.30
$T = \infty$	−3.90	−3.60	−3.34	−3.04	−.84	−.50	−.20	.15

NOTE: The entries for λ = .5 and $T = \infty$ are taken from Dickey et al. (1984, table 7). The other entries corresponding to $T = \infty$ were obtained using 20,000 simulations based on the asymptotic distribution (8). The remaining entries, corresponding to the finite-sample cases, were obtained by directly simulating $t_{\hat{\alpha}}$ from (6). For each case, 5,000 replications of a unit-root process with iid $N(0, 1)$ innovations were used.

sample. Critical values of the statistics are smallest when λ is close to 0 or 1. This is to be expected, since, as previously mentioned, the critical values are identical to those of Dickey and Fuller (1979) when λ = 0, 1.

These sets of results can be used to perform hypothesis testing. One simply picks the critical value corresponding to the sample value of λ at the chosen signficance level. Since I only provide critical values for a selected grid of λ's, the procedure suggested is to choose the critical value corresponding to the value of λ nearest its sample value—that is, T_B/T. Given that the differences in the critical values over adjacent values of λ in the tables are not substantially different, this procedure should not produce misleading inferences.

3.1 Extensions to More General Error Processes

Inferences based on Regression (6), with critical values given in Tables 3 and 4, are valid only in the case in which the innovation sequence $\{e_t\}$ is uncorrelated. When there is additional correlation, as would often be expected, extensions are necessary. The first extension concerns the way in which the dynamics of the process are reflected in the change in the mean of the series. Insights into this problem can be obtained using the literature on specification of time series processes in the presence of outliers (e.g., see Tiao 1985; Tsay 1986). As stated in Equations (2) and (3), the model corresponds to the case of an additive outlier. In the case of a unit-root process, there is a single outlier at time T_B that instantaneously changes the level of the series in a permanent way. In the case of a stationary process, there is also an instantaneous change that is permanent in the level of the series. This implies that the change in the mean of the process is not affected by its dynamics.

On the other hand, the change in the mean of the series need not be instantaneous and may be affected by the dynamic specification of the noise process. In analogy with the literature on outliers in time series, such a case can be modeled by an innovational-outlier model. Under the null hypothesis, this model takes the form

$$y_t = y_{t-1} + A(L)^{-1}B(L)[v_t + \gamma D(TB)_t], \qquad t = 1, \ldots, T, \quad (9)$$

where we specify an ARMA(p, q) process for e_t of the form $A(L)e_t = B(L)v_t$ with $A(L)$ and $B(L)$, pth- and qth-order polynomials in L, respectively, and $v_t \sim$ iid $(0, \sigma_v^2)$. Under the alternative hypothesis that the process does not contain a unit root, the process (3) becomes

$$y_t = \mu + A(L)^{-1}B(L)[v_t + \gamma DU_t], \qquad t = 1, \ldots, T. \quad (10)$$

Equations (9) and (10) specify that the change in the mean of the series is not instantaneous but depends on the dynamic specification of the error process. The immediate impact of the change is given by γ and the long-term change is $A(1)^{-1}B(1)\gamma$.

The rest of this section specifies the appropriate testing procedure to be used in what I shall refer to as the *additive-outlier model* and the *innovational-outlier model*. In each case, however, the asymptotic distributions of the test statistics are the same and require only the critical values in Tables 3 and 4.

3.2 The Additive-Outlier Model

The first approach adopts the procedure suggested by Dickey and Fuller (1979) and Said and Dickey (1984) that adds extra lags of the first differences of the data as regressors in Equation (6). This extended framework is characterized by the following regression (again estimated by OLS):

$$\tilde{y}_t = \alpha\tilde{y}_{t-1} + \sum_{j=1}^{k} c_j \Delta\tilde{y}_{t-j} + v_t, \qquad t = k+1, \ldots, T, \quad (11)$$

where $\Delta\tilde{y}_t = \tilde{y}_t - \tilde{y}_{t-1}$.

In the preceding representation, α is the sum of the autoregressive coefficients and the test is again that $\alpha = 1$. The parameter k specifies the number of extra regressors added. In a simple AR(p) process, $k = p$. In a more general ARMA(p, q) process with p and q unknown, k must increase at a controlled rate with the sample size. We denote by α^* the OLS estimator of α obtained from Regression (11) and by t_{α^*} its associated t statistic for testing the null hypothesis that $\alpha = 1$. Arguments similar to those developed by Said and Dickey (1984) can be used to show that the limiting distribution of the t statistic t_{α^*} is the same when the innovation sequence is an ARMA(p, q) process and Regression (11) is used as it is when the t statistic $t_{\tilde{\alpha}}$ from Regression (6) is used and we have iid errors, provided k increases at a suitable rate with the sample size. See Said and Dickey (1984) for the exact set of conditions under which this equivalence holds.

The second approach considers extensions to the procedure suggested by Phillips (1987) and Phillips and Perron (1988). It is useful first to write the limiting distributions in (7) and (8) in a different, more compact form. To do so, I adopt the framework suggested by Ouliaris, Park, and Phillips (1989). Define $w^*(r)$ as a stochastic process on $C[0, 1]$, the space of all real-valued continuous functions on the interval [0, 1], such that $w^*(r)$ is the projection residual of a Wiener process $w(r)$ on the subspace generated by the functions $\{1, du(r)\}$, where $du(r) = 1$ if $r > \lambda$ and 0 otherwise. Adopting this notation, an alternative representation of the limiting distributions in (7) and (8) is given by

$$T(\tilde{\alpha} - 1) \Rightarrow \left(\int_0^1 w^*(r)\, dw(r) + \delta\right)\left(\int_0^1 w^*(r)^2\, dr\right)^{-1}$$

and

$$t_{\tilde{\alpha}} \Rightarrow (\sigma/\sigma_e)\left(\int_0^1 w^*(r)\, dw(r) + \delta\right)\left(\int_0^1 w^*(r)^2\, dr\right)^{-1/2},$$

where $\delta = (\sigma^2 - \sigma_e^2)/(2\sigma^2)$.

Now define $\tilde{\sigma}^2$ and $\tilde{\sigma}_e^2$ as, respectively, any consistent estimator of σ^2 and σ_e^2 based on the estimated residuals from Regression (6). [See, among others, Perron (1988) for a discussion about the construction of such estimators.] Moreover, define S_*^2 as the residual sum of squares from the regression of y_{t-1} on a constant and DU_t. We then define the transformed statistics as

$$Z(\tilde{\alpha}) = T(\tilde{\alpha} - 1) - T^2(\tilde{\sigma}^2 - \tilde{\sigma}_e^2)/(2S_*^2) \quad (12)$$

and

$$Z(t_{\tilde{\alpha}}) = (\tilde{\sigma}_e/\tilde{\sigma})t_{\tilde{\alpha}} - T(\tilde{\sigma}^2 - \tilde{\sigma}_e^2)/(2\tilde{\sigma}S_*). \quad (13)$$

Following Ouliaris et al. (1989), it is straightforward to show that

$$Z(\tilde{\alpha}) \Rightarrow \left(\int_0^1 w^*(r)\, dw(r)\right)\left(\int_0^1 w^*(r)^2\, dr\right)^{-1} \quad (14)$$

and

$$Z(t_{\tilde{\alpha}}) \Rightarrow \left(\int_0^1 w^*(r)\, dw(r)\right)\left(\int_0^1 w^*(r)^2\, dr\right)^{-1/2} \quad (15)$$

The limiting distributions in (14) and (15) are those whose critical values are presented in Tables 3 and 4, derived using the representations given by (7) and (8).

3.3 The Innovational-Outlier Model

Testing the null hypothesis of a unit root in the innovational-outlier model specified by Equations (9) and (10) can be achieved by considering the following regression, estimated by OLS:

$$y_t = \mu + \gamma DU_t + dD(TB)_t + \alpha y_{t-1} + \sum_{j=1}^{k} c_j \Delta y_{t-j} + v_t. \quad (16)$$

We denote by $\hat{\alpha}$ the OLS estimator of α in (16) and by $t_{\hat{\alpha}}$ its associated t statistic for testing $\alpha = 1$. Equation (16) is similar to the two-steps regression procedure given by (6) and the prior detrending. It involves only a one-step regression by estimating the trend function and the dynamics of the process simultaneously, however. Such a specification implies that the change in the mean of the series does not occur instantaneously, and its effect on the level of y_t depends on the dynamics of the process. From standard arguments used in proving (7) and (8) and the results of Said and Dickey (1984), it is clear that the asymptotic distribution of $t_{\hat{\alpha}}$ in (16) is the same as the asymptotic distribution of t_{α^*} in (11); hence the critical values from Table 4 can again be used for inferences.

4. APPLICATIONS AND DISCUSSIONS

I applied my testing procedure to the three series discussed in Section 1. Results are presented in Tables 5, 6, and 7 for the statistics t_{α^*}, $t_{\hat{\alpha}}$, $Z(\tilde{\alpha})$, and $Z(t_{\tilde{\alpha}})$ applied to the real-interest-rate series, the unemployment series, and the terms-of-trade-index series. For the statistics $Z(\tilde{\alpha})$ and $Z(t_{\tilde{\alpha}})$, I need consistent esti-

Table 5. Full-Sample Unit-Root Tests With Changing Mean: Additive-Outlier Method

Series	T	$\tilde{\mu}$	$t_{\tilde{\mu}}$	$\tilde{\gamma}$	$t_{\tilde{\gamma}}$	k	α^*	t_{α^*}
Interest rate	103	.079	.03	5.56	9.45	3	.632	−2.68
Unemployment rate	163	4.77	40.23	2.47	12.54	6	.875	−3.87*
Terms-of-trade index	84	.412	16.59	−.37	−12.73	1	.498	−5.25*

NOTE: The regressions are $y_t = \tilde{\mu} + \tilde{\gamma}DU_t + \tilde{y}_t$ and $\tilde{y}_t = \alpha^*\tilde{y}_{t-1} + \sum_{i=1}^{k} c_i^*\Delta\tilde{y}_{t-1} + e_t^*$.
* A statistic significant at the 5% level.

mators of σ^2 and σ_e^2. I used $\tilde{\sigma}_e^2 = T^{-1}\sum_1^T \tilde{e}_t^2$, *where* $\tilde{e}_t^2$ are the residuals from Regression (6). To construct a consistent estimator of σ^2, I follow a procedure similar to that of Newey and West (1987), using a triangular Bartlett window. The statistic takes the form

$$\tilde{\sigma}^2 = T^{-1}\sum_1^T \tilde{e}_t^2 + 2T^{-1}\sum_{\tau=1}^{l} \omega(\tau, l) \sum_{t=\tau+1}^{T} \tilde{e}_t\tilde{e}_{t-\tau},$$

where $\omega(\tau, l) = 1 - [\tau/(1 + l)]$ and l is the truncation lag parameter. Since there is no general agreement on the appropriate choice of l, I present the results for various values of this parameter. The parameter k in the construction of the t_{α^*} and $t_{\tilde{\alpha}}$ statistics is chosen by a test of significance on the lagged first differences of the data in the appropriate regression.

Consider first the ex post real-interest-rate series. The sample is 1961:1–1986:3 and the time of break is 1980:3. Therefore, $\lambda = 79/103 \approx .8$, and, from Tables 3 and 4, the 5% critical value for the $Z(\tilde{\alpha})$ statistic is -18.02, and for the $Z(t_{\tilde{\alpha}})$, $t_{\tilde{\alpha}}$, and t_{α^*} statistics, it is -3.23. Using the additive-outlier method, the results are mixed. One can easily reject the null hypothesis of a unit root using either the $Z(\tilde{\alpha})$ or $Z(t_{\tilde{\alpha}})$ statistics but not with the extended Dickey–Fuller t_{α^*}. The latter result is, however, quite sensitive to the value of k chosen. The t_{α^*} statistic with $k = 2$ (not reported) yields essentially the same estimate of α^* but the t statistic jumps to -3.69, which is highly significant. I have reported the results with $k = 3$ because the third lag of the first differences was significant using conventional significance level. Using the innovational-outlier method, the t statistic $t_{\hat{\alpha}}$ is -3.71, which permits rejection at the 1% level. Furthermore, the estimate of the sum of the autoregressive coefficients is very low at .530, suggesting a rather different picture than the one obtained when no break is allowed.

Consider now the unemployment-rate series. The sample is 1948:1–1988:3 and the time of break is 1974:1. Therefore, $\lambda = 103/163 \approx .6$ and the 5% critical values are, respectively, -18.97 and -3.35 for the normalized least squares estimator and its t statistic. Using the additive-outlier method, the results are again mixed. Here the extended Dickey–Fuller statistic t_{α^*} permits rejection at even the 1% level, but, for most values of the truncation lag parameter l, the $Z(\tilde{\alpha})$ and $Z(t_{\tilde{\alpha}})$ statistics do not permit rejecting the null hypothesis of a unit root. Using the innovational-outlier method, the unit-root hypothesis is easily rejected at even the 1% level. These results are to be expected, given that the smooth transition to a higher mean for the unemployment series suggests, if anything, that such a change would better be modeled by an innovational-outlier model. My results agree with those reached by Evans (1989) using a similar methodology.

Finally, consider the terms-of-trade-index series. The sample is annual from 1900–1983 and the time of break is 1920. Hence $\lambda = 21/84 = .25$, and the 5% critical values for $\lambda = .30$ are -18.55 and -3.30. For this series, the null hypothesis of a unit root is easily rejected considering the additive-outlier method with either of the three statistics $Z(\tilde{\alpha})$, $Z(t_{\tilde{\alpha}})$, and t_{α^*}. The estimated sum of the autoregressive coefficients is quite low at .498, suggesting a behavior quite different from a unit-root process. The unit-root hypothesis is not rejected using the innovational-outlier method, since the t statistic is estimated at -2.46. This may be due to the poor power properties of this testing procedure when a large value of the truncation lag parameter is used; here $k = 8$. It is, however, most probably due to the

Table 6. Full-Sample Unit-Root Tests With Changing Mean: Additive-Outlier Method, Using Phillips–Perron Statistics

		l				
Series	Statistic	1	6	12	18	24
Interest rate	$Z(\tilde{\alpha})$	−69.32*	−90.09*	−114.12*	−128.55*	−141.97*
	$Z(t_{\tilde{\alpha}})$	−7.35*	−7.87*	−8.51*	−8.90*	−9.25*
Unemployment rate	$Z(\tilde{\alpha})$	−18.83*	−21.82*	−16.76	−12.50	−11.08
	$Z(t_{\tilde{\alpha}})$	−3.05	−3.28	−2.87	−2.47	−2.32
Terms-of-trade index	$Z(\tilde{\alpha})$	−35.57*	−27.41*	−21.26*	−19.69*	−16.93
	$Z(t_{\tilde{\alpha}})$	−4.64*	−4.23*	−3.92*	−3.86*	−3.78*

* A statistic significant at the 5% level.

Table 7. Full-Sample Unit-Root Tests With Changing Mean: Innovational-Outlier Method

Series	k	$\hat{\mu}$	$t_{\hat{\mu}}$	$\hat{\gamma}$	$t_{\hat{\gamma}}$	$\hat{d}$	$t_{\hat{d}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$
Interest rate	2	.003	.01	2.78	3.10	.130	.05	.530	−3.71*
Unemployment rate	9	.538	3.74	.284	3.19	.079	.25	.886	−3.84*
Terms-of-trade index	8	.173	2.57	−.162	−2.69	−.424	−3.87	.652	−2.46

NOTE: The regression is $y_t = \hat{\mu} + \hat{\gamma} DU_t + \hat{d} D(TB)_t + \hat{\alpha} y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + \hat{e}_t$.
* A statistic significant at the 5% level.

small-sample bias introduced by using an innovational-outlier method when the additive-outlier method appears more appropriate (by inspection of the very sharp drop in Fig. 3). Hence, contrary to Cuddington and Urzúa (1989), we would favor rejecting the null hypothesis of a unit root for this terms-of-trade-index series.

These applications show how the unit-root hypothesis is sensitive to minor changes in the specification of the deterministic component of a time series. As in the previous study of Perron (1989) it appears that many macroeconomic time series are better construed as stationary fluctuations around a deterministic component that occasionally (but rarely) changes in a dramatic way. Only these one-time changes appear to have any permanent effects.

The present study provides critical values for testing for a unit root when a time series is subject to a more or less sudden change in its mean. It is important to understand the nature of the postulated behavior needed to implement the tests. In the spirit of the Box–Tiao intervention analysis, I view these changes as exogenous—that is, as due to factors other than the stochastic structure of the noise process. It is, therefore, part of the maintained hypothesis and must be permitted under both the null and alternative hypotheses. Inference is conditional on such a change.

Issues of concern may arise over the choice of the break point T_B. Usually, visual inspection is sufficient, since the method is better suited for sudden changes. More formal methods can be developed, however, using, for example, cumulative sum analysis or the behavior of the recursive autocorrelation coefficients. Moreover, MacNeill (1978) presented a methodology for dealing with changes in general polynomials. His analysis postulated uncorrelated errors in the polynomial regression and hence cannot be used to address the issue of concern in this article—namely, whether the series are characterized by stationary or nonstationary fluctuations around a possibly changing trend function. Although it appears possible to extend MacNeill's analysis to allow for serial correlation in the residuals, it is less transparent how it could be extended to allow for possible nonstationarity of the unit-root type. Future work along these lines is clearly desirable.

The charge of data mining could be raised in that the choice of T_B is selected after a look at the data and that, in this sense, the inference might be biased in favor of the alternative hypothesis. Such an argument, however, overlooks the fact that the inference is conditional on such a change and that it is imposed under both the null and alternative hypotheses. The question that can be answered using the methodology described here is really in the spirit of the Box–Tiao intervention analysis: If one chooses to overlook a particular event that occurred during a very short period, is the remaining noise in the series consistent with the hypothesis that a unit root is present?

Since the procedure is conditional on a given exogenous change, the method purposefully does not explain such a change nor does it provide a stochastic structure describing their occurrences. For the change to be plausibly taken out of the noise function, however, it is preferable to relate it to "major" events that are known to have occurred and may have caused the structural change in the behavior of the series. In some of the cases considered in this article, these major events are the change in monetary policy by the Federal Reserve in October 1979 for the real-interest rate and the oil-price shock of 1973 and the ensuing slowdown in growth for the unemployment rate (or the change in monetary policy with respect to higher rates of inflation that occurred at this time).

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