An Approach to Camera-based Contact-less Breathing Rate Monitoring

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Abstract—Respiratory rate (RR), measured by breaths per minute, is one of the four human vital signs. It is recommended to check the RR pattern regularly as it provides early signs of various common cardio-vascular diseases across different age groups. To facilitate a ubiquitous contact-less RR monitoring system, we propose to use off-the-shelf video cameras to monitor RR instead of using special pressure-based wearables. We aim to capture the breathing-induced body parts (shoulder, abdomen, chest) movement via a regular video camera and design a robust video processing mechanism to track the movements to infer the breathing rate from the video. In developing such a system, we plan to collect a large RR dataset to train and validate contactless RR methods and develop a lightweight robust approach to extract RR from the input video for ubiquitous contact-less RR applications. Firstly, we aim to collect quality RR datasets from diverse subjects by considering realistic situations like low RR, high RR, exercise, force RR, natural RR, variance in clothes, lighting conditions, backgrounds, different RR-induced body components, and body posture. In this study, we will make our data open source to validate RR methods and attract wider research. Secondly, we create a spatiotemporal model to localize the RR-induced body parts and track their subtle temporal movement due to RR to infer the underlying breathing rate. Currently, we are exploring edge detection and edge movement tracking by calculating their volumetric changes and edge-energy shifts, which are more computationally efficient than their datahungry deep-learning-based video processing counterparts. In the future, we also plan to develop data-efficient deep-learning approaches to learn breathing rates from video automatically.

Index Terms—Spatiotemporal signal, Respiratory rate, Edge detection, Regions of Interest

I. INTRODUCTION

There are several human vital signs crucial to monitoring a person's physical health, such as heart rate, blood pressure, respiratory rate (RR), and body temperature [1]. As the development of technologies in computing, digital photography, and medicine has grown significantly, so too has the desire to move medical technology into the realm of video-based health services. One such service is to estimate vital signs via a video-based medium, without the physical intervention of some hardware device or a person to administer them.

In the past, there have been several issues discovered with the methods traditionally used to find a person's RR. Contactbased methods to measure breathing rates have suffered difficulties with intrusion for those with physical conditions and disabilities, losses in contact during use, and complex hardware setups. For instance, it can be challenging to fit a physical respiratory monitoring device for an infant or even someone with severe burns.

This motivates the significant use of video-based RR measurement in clinical settings. It can provide constant and non-invasive RR monitoring for those needing inpatient care in clinical environments like hospitals. The use of this technology is even more helpful because it may update in real-time, allowing doctors and other hospital staff to react to changes in RR that may indicate potential health issues for patients; contact-based methods require close administration and are not convenient for the patient or the time of physicians and nursing staff.

Employing contactless methods has been vital in various healthcare settings. Specifically, it has been instrumental in preventing respiratory issues in the post-anesthesia care unit [2], identifying abnormal respiratory patterns early on in the neonatal intensive care unit [3], [4], and facilitating rapid and reliable patient assessment in emergency triage rooms, thereby lessening the burden on nurses [5]. Furthermore, these approaches have led to the development of systems and mobile applications that monitor respiratory rates during telerehabilitation sessions at home, especially when using stationary bikes [6]. Moreover, they have also been adapted for outdoor usage in mobile scenarios [7].

With developing such a video-based system, there must also be an extensive, yet holistic dataset to capture the wide breadth of RR. To that point, this study constructs a dataset designed specifically for remote RR estimation, curated in a controlled laboratory environment. Samples from a diverse body of willing participants are collected with a range of varying respiration rates, clothing types, lighting, and postures.

Though recent advances in technology have allowed for

contact-less approaches to measuring RR, several of these methods rely on computationally-costly machine learning models to compute it effectively. This study proposes unique mathematically-based solutions to avoid the added computational complexity of machine learning-based approaches. The specific approach requires video signal analysis, including performing edge detection onto the video frames, to monitor the shift of detected edges in several regions of interest (ROIs) between the shoulder, abdomen, and chest. The merit each ROI provides to the overall measure of RR must be considered carefully. Afterward, signal processing techniques are applied to eliminate unwanted artifacts and gain a better understanding of the RR signal across time. By developing an efficient videobased technique to measure RR, the efficacy of video-based RR monitoring can be proven for practical use in clinical settings.

II. RELATED WORK

There have been several attempts in recent years to monitor RR using a video-based technique. One major issue with the use of such techniques has been in detecting specific ROIs from which to extract meaningful data to calculate the RR. Some studies have relied on cropping the ROIs by manually cropping them from specific video frames [8]. This practice involves manually annotating specific regions in a video visually by a human user. This generally produces highly useful ROIs that provide high levels of accuracy, but without automation, is impractical for real-time usage in clinical settings.

In one particular study using an RGB-depth camera, it was found that using the Viola-Jones face detection algorithm could help identify ROIs in the face and chest regions [9]. Another study utilized the Viola-Jones algorithm to first identify the subject's face and then subsequently their chest ROI [10]. The Viola-Jones algorithm has been used as a tool to identify ROIs by first identifying the facial region and approximating different ROIs, like in the chest, from that initial location. This approach can be useful and computationally efficient, but as it was used in these studies, it assumes the subjects will always sit in an upright position. As a result, it neglects the complexities associated with extracting ROIs from positions that are not solely from the front view of the body in an upright sitting position.

Some studies have addressed the issue of ROI detection by eliminating the need to identify ROIs at all by relying on the use of video processing techniques like Hermite transforms and Eulerian video magnification (EVM), along with either Bayesian artificial hydrocarbon networks or convolutional neural networks (CNNs) [11] [12]. EVM operates by magnifying specific ROIs very closely in order to extract a breathing signal by noticing subtle changes [11]. After magnification has been performed, the next step in calculating the respiratory signal is to train neural networks by feeding magnified video frames that depict the different stages of breathing when the chest has risen and when it has fallen so that models may recognize when breaths are taken [11]. Although these techniques reach high levels of accuracy in predicting the RR [11] [12], their use

of computationally-heavy deep-learning networks hurts their efficacy in real-time clinical usage because they are not the most resource or time-efficient.

Other techniques deal with the issue of finding the RR differently. In the study utilizing the RGB-depth camera, the camera's depth values were used to compute the RR, which calculated a relatively accurate result in constant time [9]. Another study relied solely on a smartphone camera to simultaneously predict heart rate and RR [8]. The RR was extracted by utilizing a Welch periodogram to determine how much power and energy were shifting at different frequencies in the video breathing signal [8]. The uniqueness of this approach lies in its use of a smartphone camera and Welch periodogram. Though unique, the particular tactic faces substantial limitations when subjects wear non-plain clothing or move, and only utilizes manual ROI selection [8].

Many techniques have only been deployed on limited datasets featuring a small variety of subject postures, two or fewer ROIs only considering the chest and face, and few varying perspectives from which to orient the acquisition of the video signal [9] [8] [11] [12] [10]. However, there are interesting benefits explored by studies to include data from patients with conditions like Cerebral palsy for wider subject representation and more thorough testing [9]. The lack of variation in the datasets used could affect the accuracy of their results in more diverse circumstances, such as a wide variety of testing environments and subject positions.

Altogether, there are some drawbacks across the existing methods to compute RR. One prevalent issue includes the heavy cost of computation a few methods employed in their calculation of RR. Another issue is detailed by sensitivities to light or movements from patients that make it difficult to calculate the RR [13]. For methods that rely on ROI tracking via feature extraction, typically using the face, movements can lead to losing the ROI.

III. METHODOLOGY

In this study, data had to be collected in order to build a dataset to extract the RR in a setting that closely resembled a clinical laboratory. The dataset was meant to be holistic, representing a diverse group of test subjects in several poses, with several different rates of respiration, independent of the lighting or type of clothing the subjects wore. After collecting data, the videos were processed to extract the RR, and the signals were cleaned to better understand the RR.

A. Data Collection

The first stage in finding the RR was to build a dataset. This required taking several samples across a wide range of willing participants. The experiments conducted in the study had taken place at the CARDS research center [14] and the Information Technology and Engineering (ITE) building at the University of Maryland, Baltimore County (UMBC) [15], involving 30 students as participants. The rooms were set up similarly to a clinical environment with simplistic furnishings including tables, chairs, and couches. The setting was meant to resemble

a room that might include a hospital bed where a patient would lie or sit down. There were few obstructions or people in the background of the testing sites. The data samples collected were about 3 minutes in length on average, during which the students were asked to assume both sitting and lying positions. More specifically, as shown in Figure 1, subjects were recorded from a sitting position facing toward the front, a sitting position taken from the side profile, and from a position lying. These positions allow the RR to be extracted from regions of interest (ROIs) like the shoulder, chest, and stomach. A smartphone's front-facing camera, positioned approximately 4 feet away from the participants, was used to record each session. Students were told not to speak and to remain still throughout the duration of the recordings, similar to how a physician might direct a patient when admitted to a hospital for testing.



Fig. 1. (From left to right) Subject positions from the front sitting position, side sitting position, and lying down positions.

In order to verify whether any results produced from this study had any efficacy, a device was utilized the record the ground truth (GT) signal, which is an accurate measure of the actual RR from the subject that is collected from a contactbased device. In this study, the Vernier Go Direct Respiration Belt [16] scientific sensor device created by Vernier Software and Technology was used, pictured in Figure 2. It was created with the purpose to monitor human RR and relative breathing pressure wirelessly. The device includes a pressure sensor integrated into a comfortable elastic belt, which is worn around the chest or abdomen. This setup allows it to capture and analyze respiratory data by tracking the expansion and contraction of the chest or abdomen during breathing. Realtime measurements of respiration rate and relative breathing pressure are provided by the Go Direct Respiration Belt. The Vernier device was connected via Bluetooth to a device managed by the lab member administering the data collection. The GT signal and the data of RR were saved as CSV and .GAMBL files to the mobile device once a test concluded. A plot of the GT signal and its own subsequently calculated RR from one sample are shown in Figure 3.

The respiration rate was calculated by the Vernier device using a sample window of 30 seconds and an advance interval of 10 seconds, while the step rate calculation utilized the same sample window and advance interval of 10 seconds. The RR was calculated based on data taken within the last 30 seconds.



Fig. 2. The Vernier Go Direct Respiration Belt used in the study.

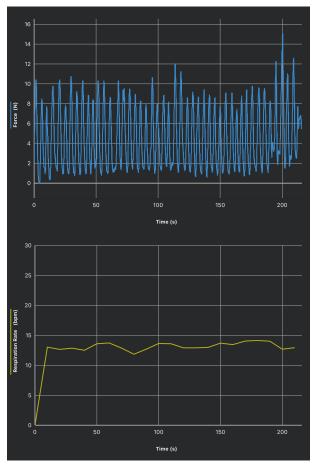


Fig. 3. (Top) Sample graphical data from the Vernier Go device plotting the force of the subject's breathing in Newtons (N) against time in seconds (s). (Bottom) Sample respiratory data from the Vernier Go device measuring the respiration rate in breaths per minute (BPM) against time in seconds (s).

requiring a minimum of 10 seconds of collected data to make a prediction.

However, before starting the process of recording any samples, participants were provided with a detailed explanation of the purpose and nature of the data collection. Informed consent was obtained from all participants to make sure they fully understood and willingly agreed to take part in the study.

After each recording session, the students had the opportunity to review the video footage. Since the sample footage was produced using the student's mobile device, if any participant had objections or concerns about their recorded video, they could choose to delete it from their mobile device before it ever was used as part of the research study. Furthermore, if any participant requested it, the sensor data recorded on the data collector's mobile device was deleted. The data was saved for further analysis only after obtaining the participants' consent and ensuring their satisfaction with the recorded information. This method emphasized ethical principles and maintained the participants' privacy concerns throughout the entire experiment.

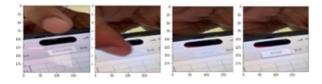


Fig. 4. Images of frames showing the data alignment process.

There was an issue concerning data synchronization with the data from the GT device with the video data, as they were sampled at different rates. To guarantee precise analysis, preprocessing was carried out to align the video frames with the corresponding sensor readings. This process required manual identification of specific moments in the recorded video when data collection commenced. The Vernier Go device's app provided an interface in which the data collection could begin by pressing a "collect" button. In the recorded video, the frame number at which the "collect" button was pressed, as shown in Figure 4, served as a reference point. To ensure accuracy, ten random frames were manually checked and matched with the corresponding force values from the sensor. The video was recorded at 30 frames per second (fps), whereas the sensor data was collected at a rate of 10 fps. As a result, it was important to match the timing of both devices to produce meaningful results.

After synchronizing the video frame rate with the sensor frame rate, a clear relationship emerged between the respiratory cycle's inhalation and exhalation phases and the sensor's force data. The lower values in the sensor force data were observed during inhalation, while exhalation led to an increase in force data. This proportional relationship between the video frame rate and the sensor force data demonstrated a consistent pattern, shown clearly in Figure 3. Using effective data preprocessing, particularly by aligning the video frames with the corresponding sensor force values, a harmonized and reliable

dataset was achieved, poised for more in-depth analysis and RR estimation.

B. Canny Edge Detection

In this study, the use of the Canny edge detection algorithm [17], shown in Figure 5, was instrumental in determining the RR from the given video sample data. In the specific application used for this experiment, it was necessary to apply the algorithm to each frame of a sample video, since it works directly on images. The first step in utilizing Canny edge detection was to take an input image from the video, which required breaking the video into its corresponding frames and then converting each image frame to a gray-scaled image. This helps to reduce computational complexity since the image only has one channel (brightness) compared with a color image's three red, green, and blue (RGB) channels.

The next step was to apply a Gaussian filter to every frame. A Gaussian filter reduces noise in an image by smoothing out its edges for processing. A Gaussian filter will remove the sharpness and fine details present in an image using a bell-shaped curve, known as a Gaussian distribution [18]. The distribution is applied over all the pixels in an image, and depending on the strength of their color values and their neighboring pixels', will blend those pixels more closely, "smoothing" them out. This is imperative for reducing false edges in the final result.

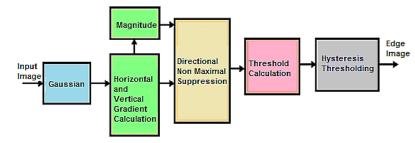


Fig. 5. Graphical representation of the steps utilized to make Canny Edge Detection.

The following step is to calculate gradients in the image. The gradients are calculated in both the vertical and horizontal directions. This requires the use of Sobel filters [19], which compute the derivative of the image in both the vertical and horizontal directions. Once these filters are applied, the gradient magnitude and direction of every pixel can be determined. These metrics yield important information regarding every pixel's strength and regions where pixel intensity is changing the most. With this information, non-maximal suppression can be performed. This technique produces a map of thinly-defined edges because only local maxima from the pixels are chosen as edges, whereas all other surrounding pixels are ignored.

The final steps in the process use thresholding, which defines high and low cutoff values for pixels from which to classify edges as being strong, weak, not an edge, etc. The selection of threshold values is crucial in the study because they determined the strength and number of edges appearing

in the video data. The strong edges are determined to be edges because their pixel gradient magnitudes are higher than the higher part of the specified threshold. Edges that are inbetween threshold values are classified as weak and checked to determine whether or not they belong to a strong edge. Anything below the threshold is ignored as being part of an edge.

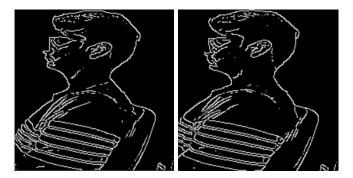


Fig. 6. Two edge detected frames demonstrating a subject's breathing during one full inhalation.

The Canny edge detection algorithm allows for the sample videos to be expressed only in terms of their edges, highlighted as white lines against a black background shown in Figure 6. This isolates data related to RR by revealing only the edges of their body moving, making even the smaller respiratory movements calculable via mathematical computing tools.

C. Signal Processing, Analysis, and Filtering

Once Canny edge detection was deployed to remove unnecessary video noise inconsequential to measuring RR, it was important to isolate the specific regions of interest (ROIs) in the video that held respiratory information. The regions that appear to most visually provide respiratory information were the shoulder, abdomen, and chest, which appear to all move significantly in a pattern when people breathe. Figure 7 shows each of these ROIs closely cropped. In this study, the ROIs were manually selected so that the efficacy of the RR monitoring technique could be proven. This involved examining each of the sample videos and cropping a specific region of the video from which to measure the RR.



Fig. 7. Specific ROIs extracted (left to right) from the shoulder, chest, and stomach.

Once specific regions were cropped and applied to the video feed, it was necessary to collect all of the edge data from each frame and store it for further examination. In this step, collected edge data was transformed in 1 of 3 different ways. The first method involved summing the edge-detected points in each given frame with respect to the x-axis (frame rows).

Then, only the elements with the maximum number of points in a given row along the x-axis were chosen. The second method worked similarly but utilized a sum of all those points with respect to the y-axis (frame columns). The data was represented as the row or column numbers that carried the maximum number of edge points plotted against time, with an example shown in Figure 8. The third method summed every point present in a given frame. This data was plotted as the number of points represented in a frame against time. Among different sample videos, a graphical pattern emerged that demonstrated the flow of movement of the subject's breathing, but each method would yield minutely-different results. A relationship was observed between a particular method working better for specific ROIs than for others. For instance, the point-sum in the x-direction typically yielded stronger signals for the shoulder ROI, but not the others. In several samples, the signal captured was too noisy or rough for one given method but not others. Thus, a frequency analysis was performed using a Fast Fourier Transform (FFT) for each method on every sample to automatically select the highest quality signal for RR extraction.

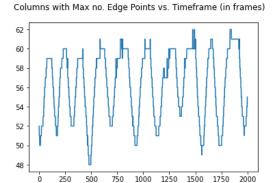


Fig. 8. A sample video's raw noisy signal plotted as the column numbers featuring the maximum number of edge points against the timeframe, specified by the number of frames.

Once a signal was selected using the 3 methods, the initial plot of the raw signal was very noisy and yielded little information from which RR could be drawn. The signal was filtered using a carefully-designed low-pass Butterworth filter. To design the filter, there were several considerations made to produce an accurate and clean signal, such as the video sampling rate, the Nyquist frequency, the polynomial approximation's order, and the cutoff frequency. With this information, a plot of the row or column numbers that carry the maximum number of edge points, or the approximated summed points in a frame, can be plotted more meaningfully against time, as shown in Figure 9.

With a clean signal produced depicting the subject's subtle movements clearly, the RR may be calculated in relation to time.

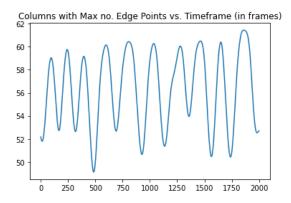


Fig. 9. The same sample video signal is featured in Figure 8, but processed with the Butterworth filter.

IV. EXPERIMENTAL RESULTS

As of now, a qualitative analysis has been conducted to determine how effective this method may be at deriving the RR. Thus far, examining the dynamic edge changes has yielded patterns in movements that are visually similar to the GT device's capture of movement by the measure of force. This capture of movement appears significantly promising to yield the RR in future work.

V. CONCLUSION

In this study, a technique was proposed to extract the RR from a video-based medium. A variety of subjects were ethically sampled to create a dataset of breathing in controlled laboratory environments, measured with a smartphone video camera and the Vernier Go device. The proposed technique utilized the Canny edge detection algorithm along with several functions performed to capture the pattern of edge movements from subjects breathing during controlled trials. The functions used in this work involved taking edge data and summing the corresponding points present in frames of the video data along different axes. Signal processing was done to clean the raw signal, from which the RR could be calculated. The signal and an estimated RR will be compared with the signal and RR acquired from the GT Vernier Go device during each trial.

A. Limitations and Future Work

The use of the energy shift video-based technique is reserved for use in clinical settings only. In the research conducted, it is assumed subjects are to be sitting or lying down still as if instructed to do so by a physician or resting at an inpatient facility. In the trials to build the dataset, subjects are instructed not to speak as these artifacts make the processing of the RR very difficult. In the future, the dataset will be diversified further to be even more inclusive of various breathing and health conditions. There will also be future work done to distinctly formulate the RR and compare it with the GT signal using RMSE and accuracy measures.

The approach in this study currently uses manuallyannotated ROIs for the purpose of bolstering the information that can be gathered from each ROI. However, in the near future, the plan is to utilize a variation of the Viola-Jones or Histogram of Oriented Gradients (HoG) algorithms to efficiently detect ROIs in real-time, but robustly from various positions by optimizing what landmarks the algorithm uses to detect faces. The goal is to be able to extract ROIs accurately, even from a side profile when someone is lying down or sitting.

Another limitation of this study is the current use of recorded video data to measure RR. The use of a recording provides a control for camera distance and subject positioning but lacks dynamic use in real-world scenarios. Such scenarios include using the provided solution to measure the RR of patients at a hospital in real-time while they may be lying or sitting in a bed or chair.

In the future, the use of lightweight machine learning techniques will be explored to be compared with the techniques used in this study. More noise reduction technologies will be investigated in the hopes of producing even more accurate data. The goal is to continuously improve video-based RR techniques for their practical use in the real world.

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